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AUTHOR	Herman, Daniel L.
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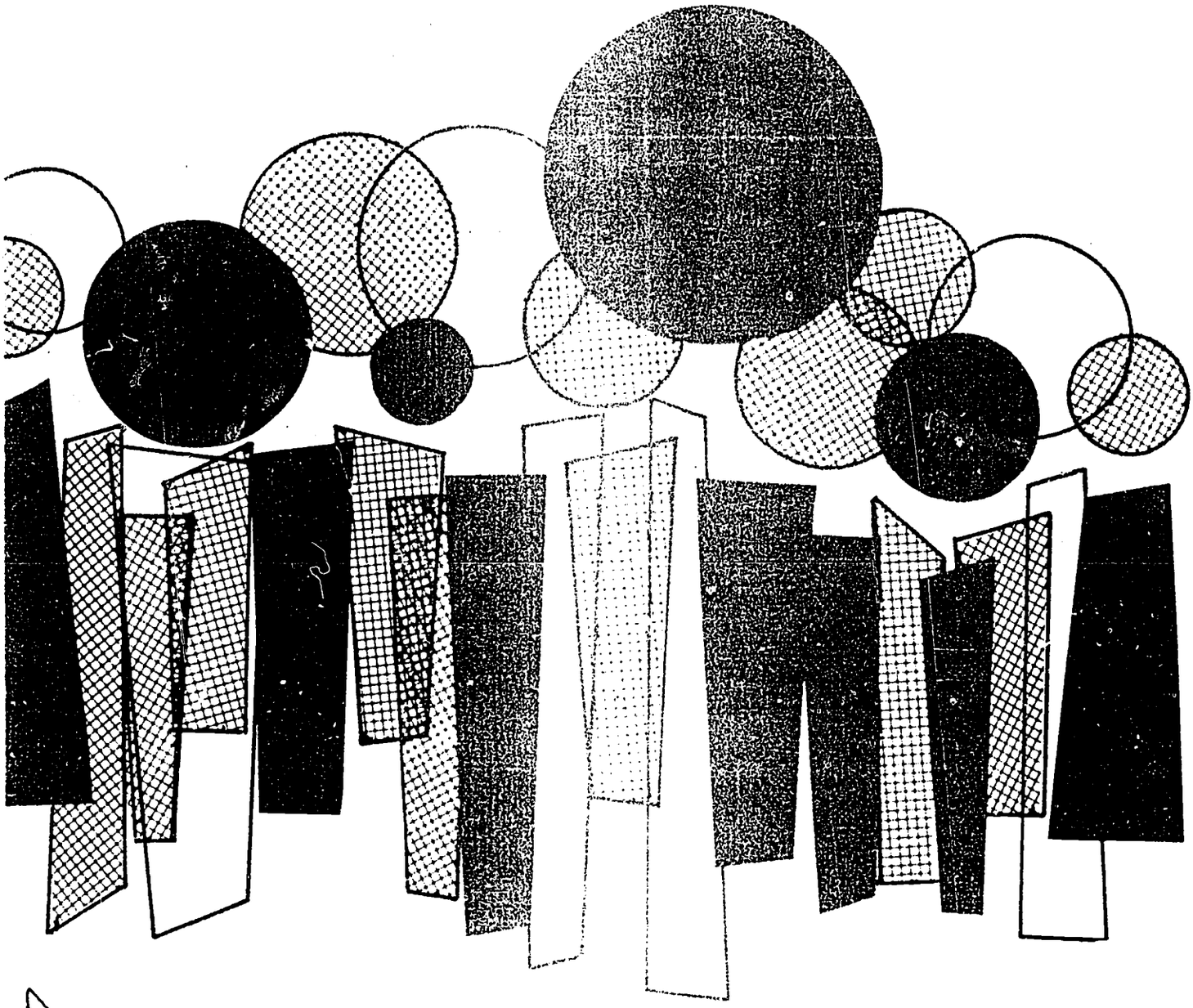
ABSTRACT

This guide to accompany "Similarity and Congruence" contains all of the student information in SE 015 346 plus supplemental teacher materials. A summary of terminal objectives and teaching aids and equipment is given. With each section are listings of objectives, teaching aids, suggested approaches, and discussion questions. Related documents are SE 015 334 - SE 015 346. This work was prepared under an ESEA Title III contract. (LS)

SIMILARITY AND CONGRUENCE



TEACHER'S GUIDE

OAKLAND COUNTY MATHEMATICS PROJECT STAFF

Dr. Albert P. Shulte, Project Director
Mr. Terrence G. Coburn, Assistant Director and Project Writer
Dr. David W. Wells, Project Coordinator
Mr. Stuart A. Choate, Project Writer
Mr. Philip L. Cox, Project Writer
Mr. Daniel L. Herman, Project Writer
Mr. Robert E. Peterson, Project Writer
Miss Diane M. Ptak, Project Writer
Miss Joyce Sweet, Project Writer
Mrs. Kathleen Danielson, Special Project Writer
Mrs. Lawanda Beane, Secretary
Miss Deborah J. Davis, Secretary
Mrs. Shirley Greene, Secretary
Mrs. Judy Orr, Secretary
Miss Linda Tee, Secretary

SIMILARITY AND CONGRUENCE

OAKLAND COUNTY MATHEMATICS PROJECT

All units have benefited from the combined attention of the entire project staff. The major writing responsibility for this unit was handled by

DANIEL L. HERMAN

ILLUSTRATIONS BY KATHLEEN DANIELSON

Oakland Schools
2100 Pontiac Lake Road
Pontiac, Michigan 48054

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PREFACE

BASIC ASSUMPTIONS

This unit has been written for the student with no previous exposure to the concepts of similarity and congruence. Besides skills in whole number computation and elementary measurement, the student should have some background in geometric terminology and ratio and proportion. The extent of the background is outlined below.

1. The following list of items should be familiar to the student to the extent that he can identify each.

   a. angle
   b. square
   c. triangle
   d. rectangle
   e. five-sided geometric figure
   f. vertex
   g. line segment

2. Given a set of line segments and their measures, the student should be able to set up ratios between measures of the segments and determine whether or not these ratios are equivalent.

OBJECTIVES

A summary of the terminal objectives appears after the TABLE OF CONTENTS. The Teacher's Guide for each lesson contains the objectives for that particular lesson. The objectives that are listed should not limit the scope of the teaching process, but should be used as a basis for evaluating student progress. Discuss the objectives for each lesson with your students.

OVERVIEW

This booklet is a study of the properties of similar and congruent figures. Each lesson contains a variety of activities and/or exercises which focus on one of these properties. Size and shape are used to determine whether figures are similar, congruent or neither. The ASA, SSS, etc. theorems are not mentioned (they will be discussed in a future booklet). Some applications of similarity are presented in the last chapter but a major emphasis on application will be reserved for a future booklet.

BOOKLET ORGANIZATION

Most activities in the student booklet are labeled EXERCISES, CLASS ACTIVITY, DISCUSSION QUESTIONS or ✓POINT.

Exercises are intended primarily for individual supervised study. It is not anticipated that much homework will be assigned, although brief sections of exercises may well serve that function.

Class activity sections are problem situations to be handled on a cooperative or group basis. They are to be in-class activities and are <u>not</u> appropriate homework exercises. The role of the teacher is that of a resource person and a director of learning. Students should be allowed to gather information and make discoveries on their own.

Discussion questions are to be used for in-class discussion rather than homework. They serve as an aid in clarifying concepts.

The student has an opportunity to check his own progress in those sections labeled ✓POINT. The intent is for the student to use them to evaluate his own progress rather than a quiz section to be given by the teacher.

Material in the TEACHER'S GUIDE is arranged, for each lesson, under the headings: Objectives, Equipment and Teaching Aids, Content and Approach, Things to Discuss, and Answers.

The information under Content and Approach is designed to give the teacher assistance in interpreting the emphasis and direction of the lesson. The questions and comments listed under Things to Discuss are meant to be suggestive of the type of class discussion which would be helpful in reviewing and extending the lesson. Most answers are over-printed on the student pages and the others can be found on the teacher pages in proximity to the exercises.

ALL MATERIAL FOR THE TEACHER IS PRINTED IN RED.

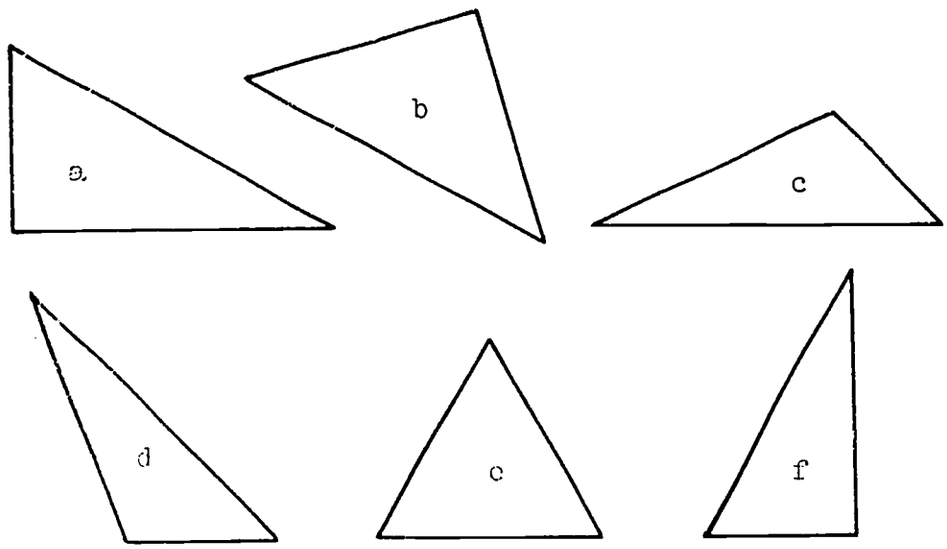
SUMMARY OF TERMINAL OBJECTIVES

1. The students will be able to define:

   a. similar figures b. congruent figures, and c. coincide. The following definitions would be acceptable.

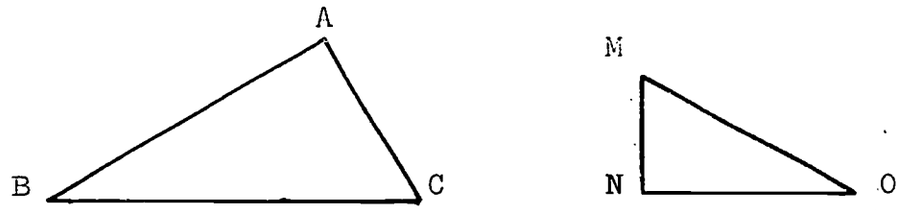
      a. Figures which have the same shape are called similar figures.
      b. Figures which have the same size and shape are called congruent figures.
      c. Figures which fit on each other exactly are said to coincide.

2.



Given a set of triangles such as the above, the student will be able to select the pair that is similar and the pair that is congruent. The student may use tracing paper in making the decision.

3.



Given similar triangles ABC and MNO. The student will be able to give a correct response to the following.

a. Name a similarity correspondence between the triangles.

b. Name the corresponding angles.

c. Name the corresponding sides.

d. Which of the following is true?

1. $\frac{m(\overline{AB})}{m(\overline{MN})} = \frac{m(\overline{BC})}{m(\overline{NO})} = \frac{m(\overline{CA})}{m(\overline{OM})}$ 2. $\frac{m(\overline{AB})}{m(\overline{NO})} = \frac{m(\overline{BC})}{m(\overline{OM})} = \frac{m(\overline{CA})}{m(\overline{MN})}$

4. Given a similarity correspondence for a pair of similar triangles ($\triangle RST \sim \triangle HIJ$), the student will be able to name the corresponding angles and sides of the similar triangles.

5-9 The student will be able to indicate whether each of the following is a property of similar figures, congruent figures or both.

5. Corresponding angles are always congruent.

6. Corresponding sides are always the same length.

7. The figures are always the same shape.

8. The figures are always the same shape and size.

9. Corresponding sides always have equal ratios. ("Both" would be the best answer for this property, but "similar figures" would be acceptable.)

EQUIPMENT AND TEACHING AIDS

A.	STUDENT	LESSON
1.	Scissors	1, 6, 7
*2.	Tracing Paper	1, 2, 3, 4, 5, 6
*3.	Riffle Cards	1
4.	Ruler	4
5.	Centimeter Ruler	7
*6.	Grid Paper	7

B.	TEACHER	LESSON
1.	Overhead Projector and Screen	1, 2, 4, 5, 6
2.	Clear Acetate	5
*3.	Prepared Overhead Transparencies	1, 5, 6

* materials provided with the booklets

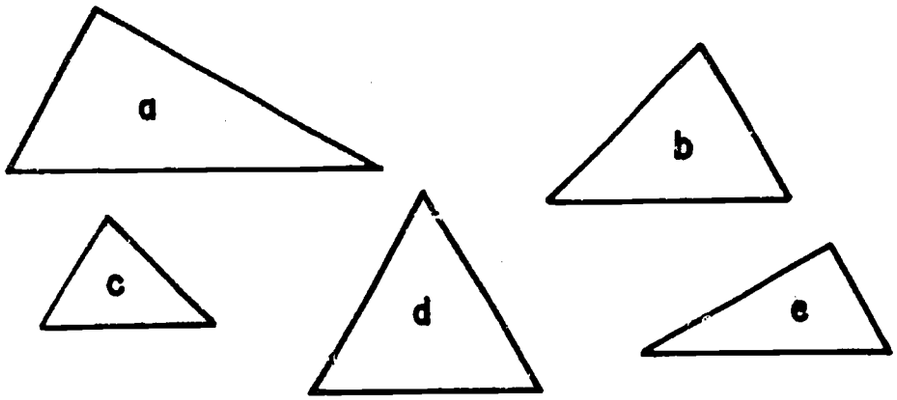
NOTE: The TEACHER'S GUIDE for each lesson lists the equipment and teaching aids needed for that lesson.

TABLE OF CONTENTS

OBJECTIVES

An introduction to the concept of similarity.
After completing this lesson the student shall be able to:

1. define similar figures--"figures that have the same shape" would be acceptable.
2. identify similar figures given a set such as--



use of tracing paper would be acceptable.

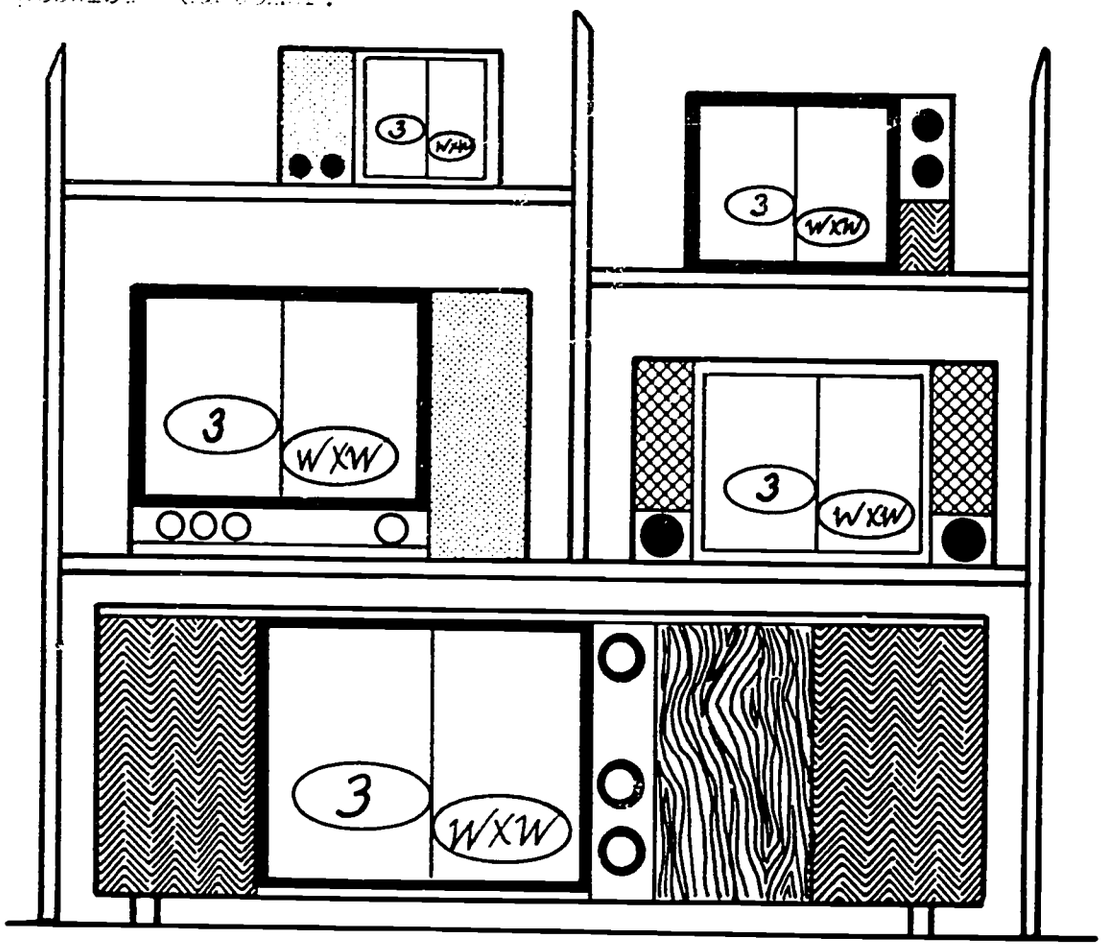
EQUIPMENT AND TEACHING AIDS

1. Overhead Projector
2. Projection Screen
3. Prepared Transparencies S1-S10 (A and B) and S11.
   S11 is a duplicate of page 4a with cutouts of the figures on page 4b.
   S1-S10 (A and B) are familiar signs and outlines of their shapes
4. Scissors - one for each student
5. Tracing Paper
6. "Riffle cards" - ten packs per class

LESSON 1

SIZE AND SHAPE

The manager of Polly's Appliance Store frequently tunes all the television sets in his store to the same station. This never fails to attract the attention of those who walk by and usually provides his salesmen with an opportunity to talk to a possible customer.



The manager and the salesman disagree on whether the pictures are actually the same. The salesman claims the pictures are exactly the same and the manager says they are different. Who do you agree with? Can you name some reasons why each man feels he is right?

CONTENT AND APPROACH

The questions at the bottom of the page should prompt some discussion. If not and the entire class feels the pictures are different, ask them "How can they be different if all the pictures are coming from the same T.V. camera?" If the entire class feels the pictures are the same, ask "How can you say they are the same when they are all different sizes?"

THE PICTURES ON THE TELEVISIONS ARE EXACTLY THE SAME SHAPE.

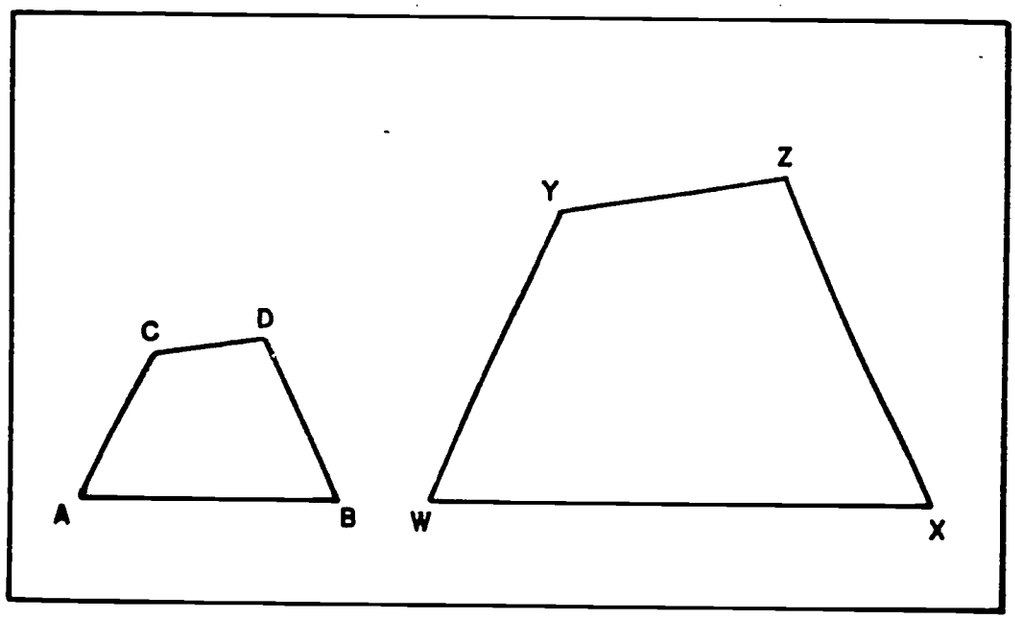
CLASS DISCUSSION

1. Do you agree with the statement above? What does it mean to say two pictures or objects have the same shape?

2. Photographs can be enlarged, but they remain the same shape. Can you name some other objects which vary in size but maintain the same shape?

3. Do all maps of Michigan have the same shape? How can you be sure the maps give an accurate picture of the actual shape of the state? Do you have any idea how maps are made?

GEOMETRIC FIGURES WHICH ARE EXACTLY THE SAME SHAPE ARE CALLED SIMILAR FIGURES.



CONTENT AND APPROACH

The statement at the top of the page should resolve the discussion from page one.

Some answers to Discussion Question #2 may suggest objects which are the same shape in a general sense of the term (people, books, etc.). Separate these from answers which suggest objects which are precisely the same shape (U.S. flags, maps, squares, etc.).

In the definition of similar figures, the use of the word "exactly" may be a point of discussion among mathematicians but its use gives more meaning to the definition from the student's point of view.

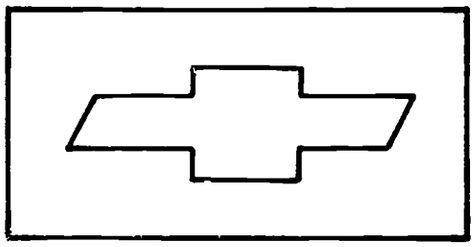
THINGS TO DISCUSS

What is the difference between saying two figures are exactly the same shape and saying two figures are exactly alike?

How does the cover of this book illustrate similar figures?

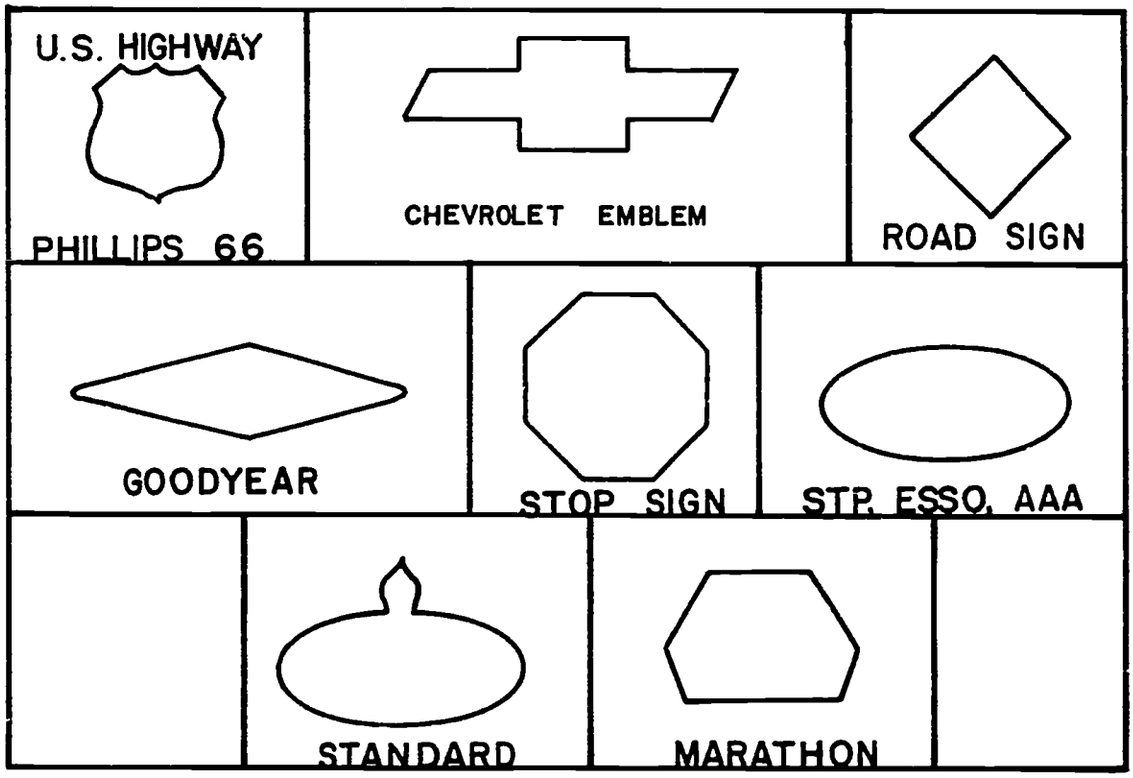
Two objects have the same shape. Does this mean they must have different sizes?

HAVE YOU SEEN THIS FIGURE BEFORE?



General Motors uses this figure as an emblem to identify and promote Chevrolet automobiles and trucks. You've seen the emblem displayed in several places including newspapers, television and billboards. Is the emblem on a billboard the same size as one in a newspaper? Are they both the same shape? The Chevrolet emblem may be very large or very small, but it is always the same shape. Geometrically speaking, the emblems are similar figures.

Can you identify a familiar sign whose shape is similar to the figures below?



CONTENT AND APPROACH

Use transparencies S1-S10 (A and B) in conjunction with this page. Transparency 1a is the outline of the Chevrolet emblem and 1b is a picture of the emblem. Leave 1a on the overhead and use 1b as an overlay to illustrate the "same shape". Follow this procedure for the remaining transparencies.

Allow the students to "guess" the sign by its shape.

Some outlines suggest more than one sign (oval - STP, AAA, EXXON, ENCO, etc.).

Students may wish to suggest other shapes with which some product is identified.

Suggested home work: Have students bring ads containing these shapes from newspapers and magazines.

Tear out pages 4a and 4b and cut along the dotted lines. Are figures 1a and 1b similar? _____ Try placing one over the other and holding them to the light. Move them around, flip one over, compare the angles; continue until you can see the similarity or are satisfied that no matter how you line them up, the figures are not similar. Use this method to compare 2a and 2b, 3a and 3b etc. Use the chart to indicate your decision.

Figures	Similar	Not Similar
1a and 1b		X
2a and 2b		X
3a and 3b	X	
4a and 4b	X	
5a and 5b		X
6a and 6b	X	
7a and 7b	X	
8a and 8b	X	
9a and 9b		X
10a and 10b	X	

CONTENT AND APPROACH

Pages 4a and 4b should be torn from the book one page at a time and rather quickly. This should prevent the binding from coming loose.

Ability to identify similar figures is one objective of this lesson. The activity outlined on page 4 suggests a method which could help the student make his decision.

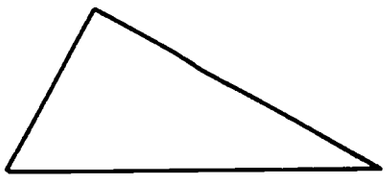
Some pairs (1a and 1b for example) are not similar but are close enough to cause some discussion. Refer the students to the word "exactly" in the definition of similar figures on page 2.

Display transparency S-11 and allow students to use the cutouts to demonstrate the similarity or lack of it.

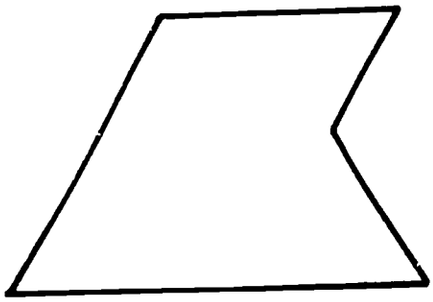
If a student uses "angle" in making his decision, suggest he make a conjecture concerning the angles of similar figures; but do not make a big issue of the matter at this time--it will be discussed in Lesson 2.

THINGS TO DISCUSS

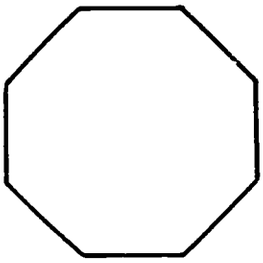
Suppose two figures were drawn on pieces of cardboard and you could not use this procedure to determine if they were similar or not. Can you suggest other methods which could be used?



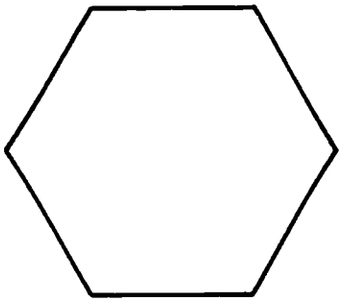
1a



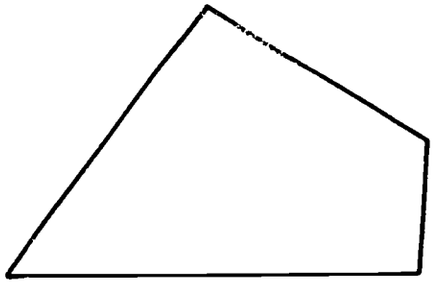
6a



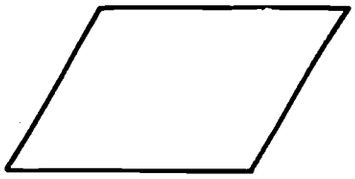
2a



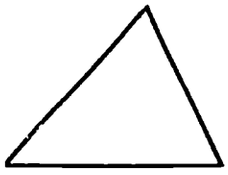
7a



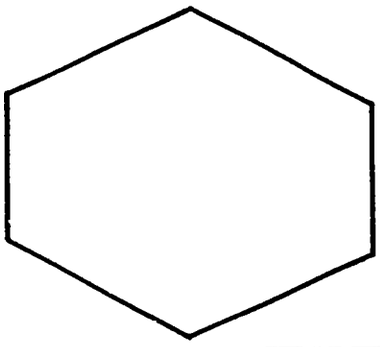
3a



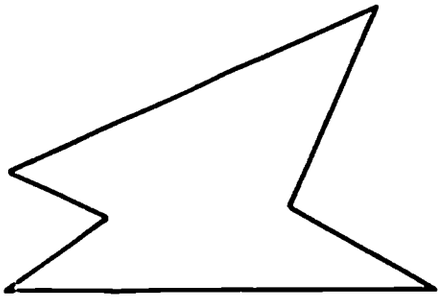
8a



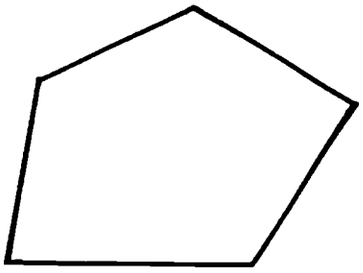
4a



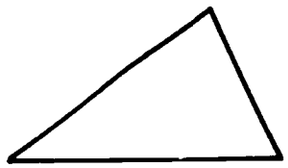
9a



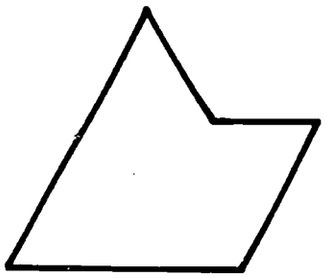
5a



10a



1b



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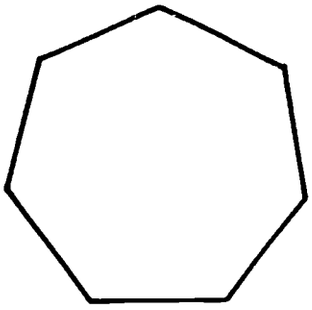
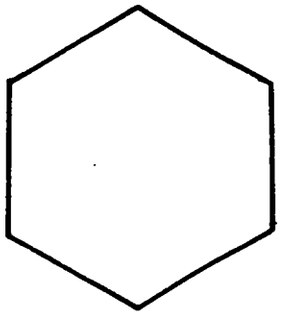
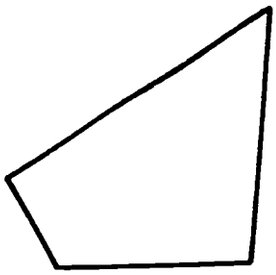


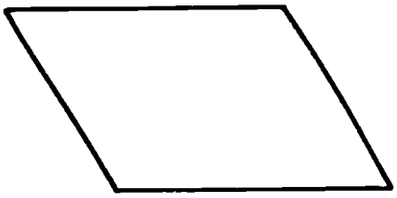
2b



7b



3b



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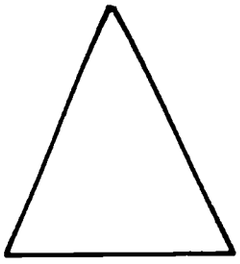
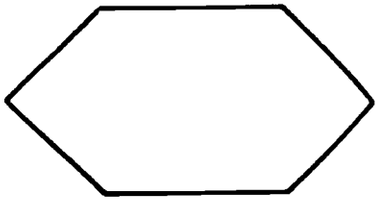


4b



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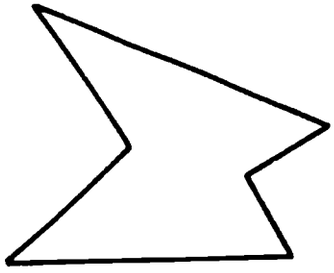
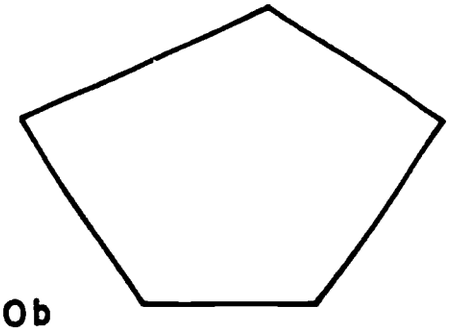
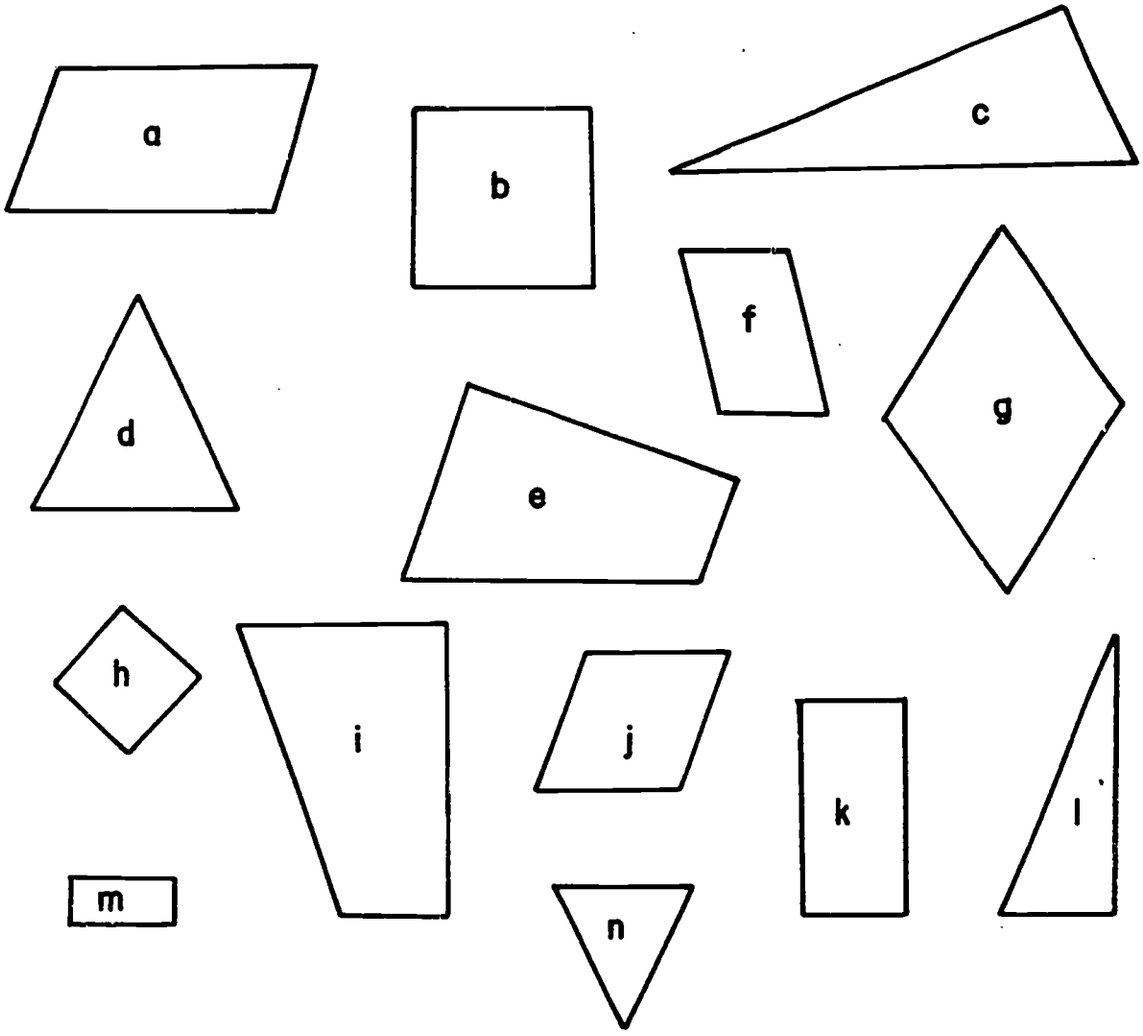


5b



10b

FIND THE SIMILAR FIGURES



__a__ is similar to __f__

__b__ is similar to __h__

__c__ is similar to __l__

__d__ is similar to __n__

__e__ is similar to __i__

__g__ is similar to __j__

__k__ is similar to __m__

CONTENT AND APPROACH

This exercise requires the student to match similar figures simply by looking at them. Before assigning the page, distribute the riffle cards. They will convey the idea of mentally turning the figure about until they recognize a similarity.

THINGS TO DISCUSS

Figures k and b have four square corners each. Why aren't they similar?

Figures g and h are both diamond shaped. Why aren't they similar?

Cut out insert A and carefully cut out each figure. Which figures are similar to each other? B,C,D,E,G Show how figures B and D can be combined to form a figure the same size and shape as figure A, E or F. Name some combinations which will form a figure the same size and shape as figure C.

Are the statements true or false?

1. Figures A, B and D will form a figure similar to figure F.
2. Figures B, D and E will form a figure similar to figure A.
3. Figures A and B will form a figure similar to figure E.
4. Figures C and G will form a figure similar to figure F.
5. Figures B, D and F will form a figure similar to figure A.
6. Figures D, E and F will form a figure similar to figure A.
7. Figures G and C will form a figure similar to figure E.
8. Figures A, B, D, E and F will form a figure similar to figure C.
9. Figures B, C, D and E will form a figure similar to figure F.
10. Figures D, B, E and G will form a figure similar to figure F.
11. All the figures will form a figure similar to figure A.
12. All the figures will form a figure similar to figure C.

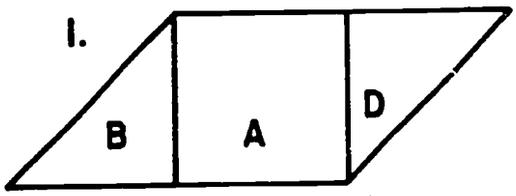
CONTENT AND APPROACH

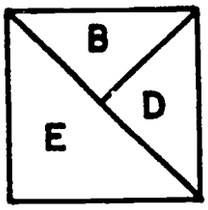
Do not attempt to tear out insert A. The booklet binding will give before this heavy paper will tear.

Remind students to cut very carefully.

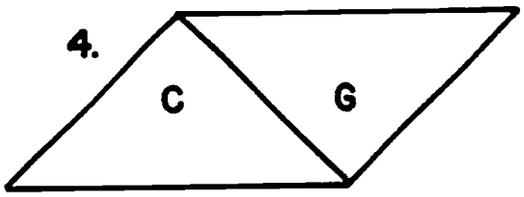
Have the students make a sketch of their solutions showing the placement of each piece. The overhead projector can be used to illustrate the solutions. Lay the pieces on the stage and leave a small gap between them. The image on the screen will afford each student an opportunity to see a solution. Let the students demonstrate their solutions on the overhead.

A solution is given for each statement that is true. Several statements have more than one solution. Use the solutions only as a last resort. Students enjoy seeing the teacher "stumped" on occasion.

1.


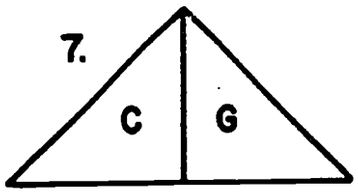
2.


3. FALSE

4.


5. FALSE

6. FALSE

7.


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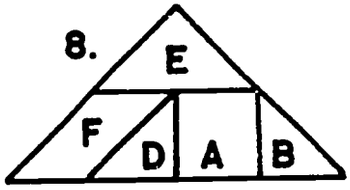

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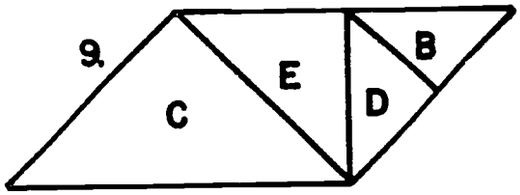

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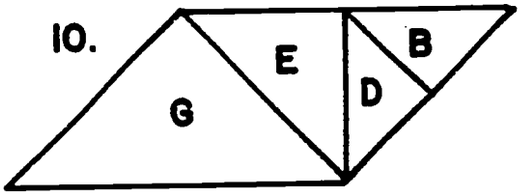
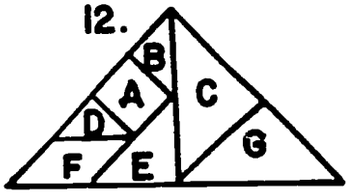


11.


12.


OBJECTIVES

1. Given a pair of similar triangles such as:

the student will be able to name the corresponding angles.

2. Given the sentence: The corresponding angles of similar figures

   1. have the same vertex
   2. are always the same size
   3. are never the same size

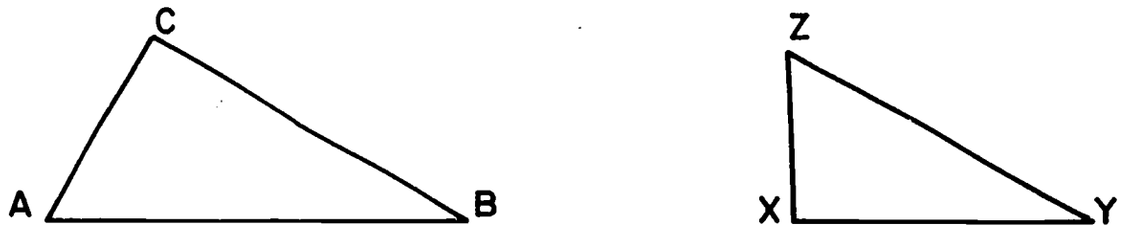
the student will select number two as the best response.

EQUIPMENT AND TEACHING AIDS

1. Overhead projector
2. Projection screen
3. Tracing paper

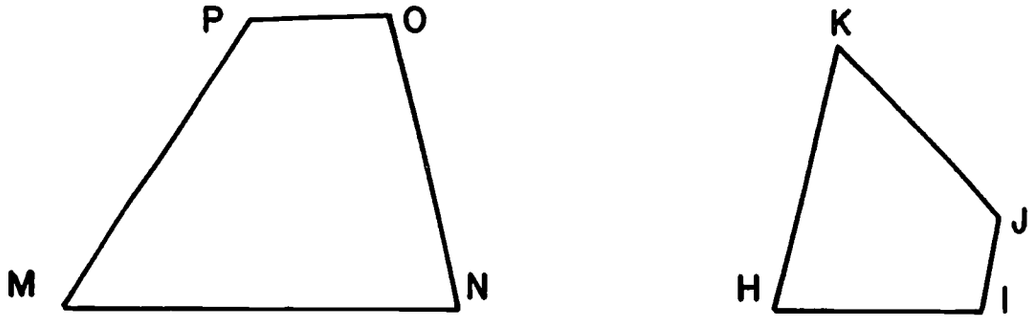
COMPARE THE ANGLES

Triangle ABC is similar to triangle ZYX ($\triangle ABC \sim \triangle ZYX$).



Angle C is the largest angle of triangle ABC and angle X is the largest angle of triangle ZYX. We say that angle C corresponds to angle X. Angle B is the smallest angle of triangle ABC. Which angle of triangle ZYX corresponds to angle B? $\angle Y$ Which angle of triangle ZYX corresponds to angle A? $\angle Z$

Figure MNOP is similar to Figure KHIJ (fig. MNOP$\sim$fig. KHIJ).



Angle M corresponds to angle K. Use a compass, protractor or tracing paper to compare their sizes. Is angle M less than, greater than, or the same size as angle K? SAME SIZE Angle N corresponds to angle H. How do their sizes compare? SAME SIZE Angle O corresponds to angle I. How do their sizes compare? Is angle P the same size as angle J? YES Compare the sizes of the corresponding angles of the triangles at the top of this page.

CONTENT AND APPROACH

The students should be informed that the symbol $\sim$ replaces the words "is similar to".

Students that are having trouble recognizing the similarity of figures such as those on page 7 will not be able to succeed in naming corresponding angles. To these students, suggest tracing one of the figures and laying the tracing on the other figure so they "line up". The angles which "line up" are the corresponding angles. This can be demonstrated using the overhead projector.

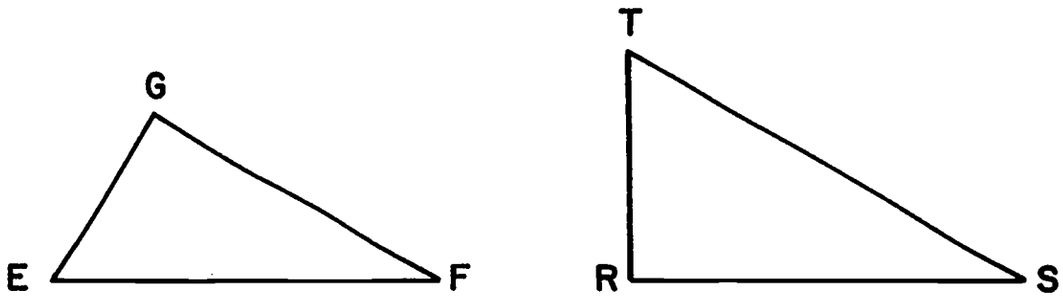
The directions and questions at the bottom should lead to a discovery that corresponding angles of similar figures are the same size.

THINGS TO DISCUSS

The corresponding angles of the similar figures on this page are the same size. Are the corresponding angles of similar figures always the same size? Can you use some examples from lesson one to support your answer?

THE CORRESPONDING ANGLES OF SIMILAR FIGURES ARE THE SAME SIZE.

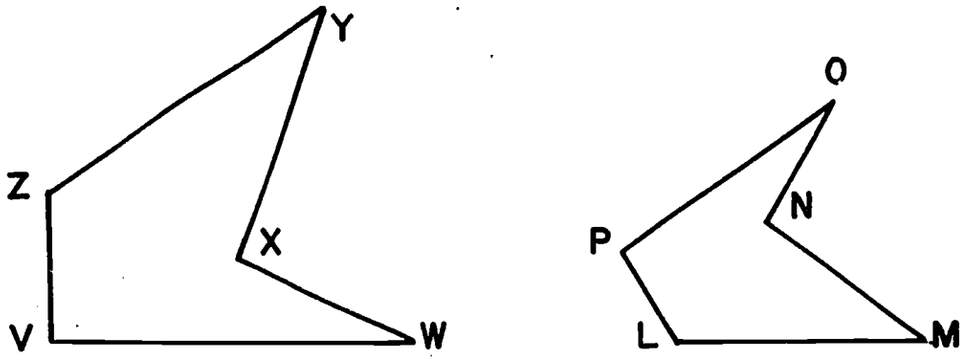
Triangle EFG is similar to triangle TSR ($\triangle$EFG $\sim$ $\triangle$TSR)



Which angle corresponds to angle E? (Find the angle which is the same size as angle E.) ANGLE T

Which angle corresponds to angle F? ANGLE S

Angle G corresponds to angle ANGLE R



Find the corresponding angles of the similar figures above.

Angle V corresponds to angle P
Angle W corresponds to angle O
Angle X corresponds to angle N
Angle Y corresponds to angle M
Angle Z corresponds to angle L

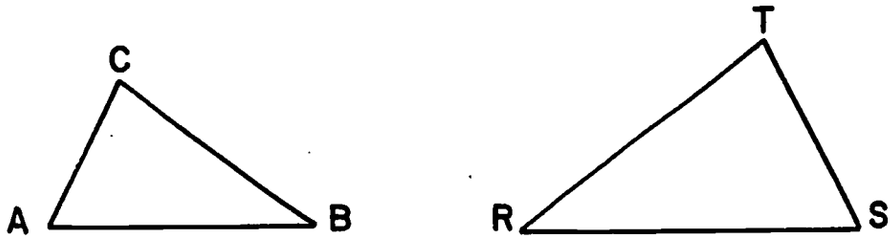
CONTENT AND APPROACH

Remind students that the statement at the top of the page is a property of all similar figures.

Most students are probably ready to work the exercises on the next four pages. Use page 8 to work individually or in a small group with those still having trouble.

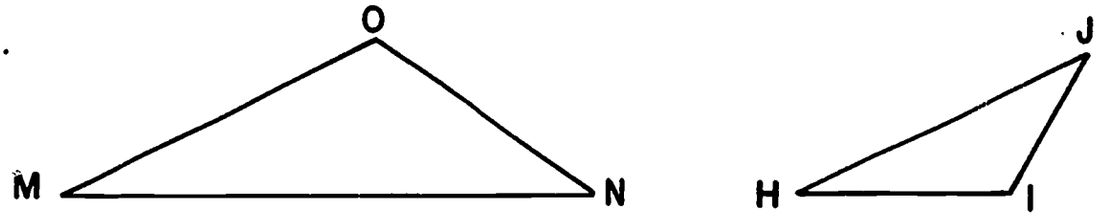
Find the corresponding angles of the similar figures.

1.



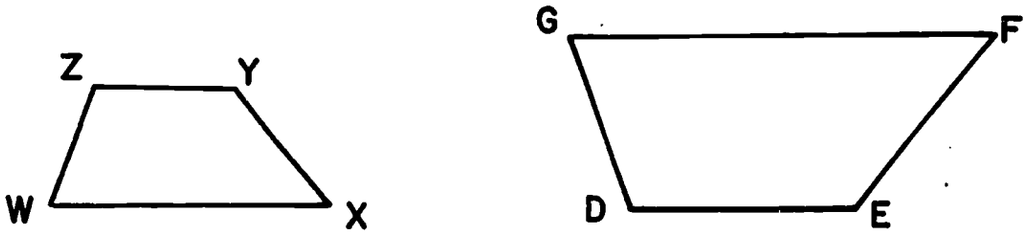
Angle A ⟷ Angle S
Angle B ⟷ Angle R
Angle C ⟷ Angle T

2.



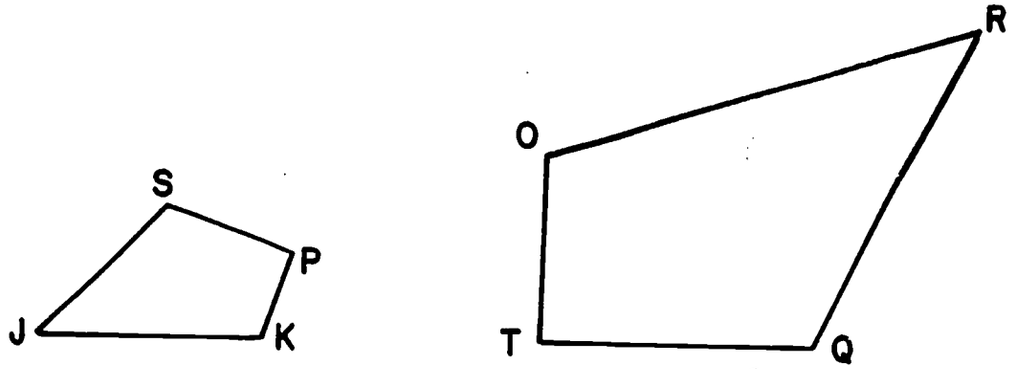
Angle M ⟷ Angle H
Angle O ⟷ Angle I
Angle N ⟷ Angle J

3.



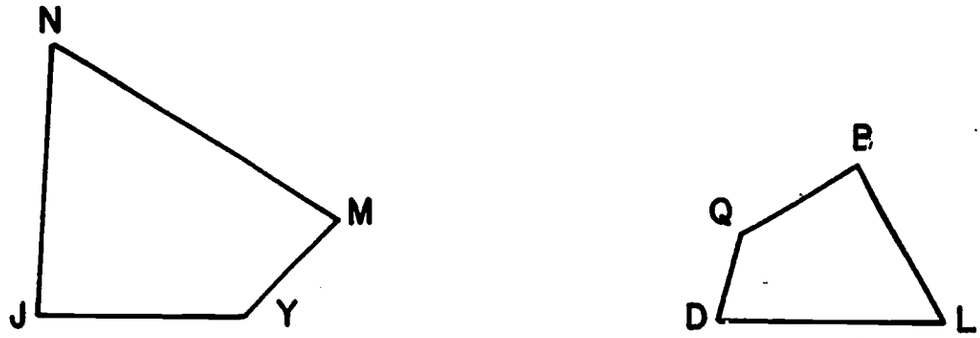
Angle W ⟷ Angle G
Angle X ⟷ Angle F
Angle Y ⟷ Angle E
Angle Z ⟷ Angle D

4.



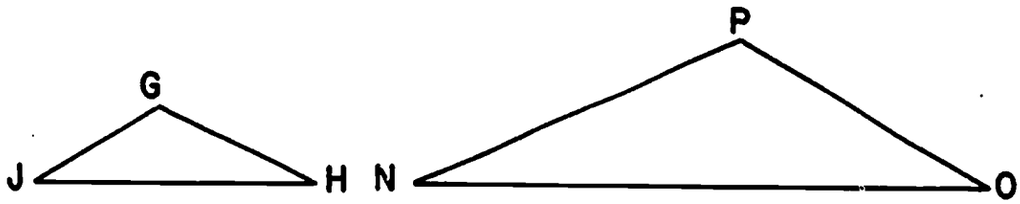
Angle S ⟷ Angle Q
Angle J ⟷ Angle R
Angle K ⟷ Angle O
Angle P ⟷ Angle T

5.



Angle J ⟷ Angle B
Angle Y ⟷ Angle Q
Angle M ⟷ Angle D
Angle N ⟷ Angle L

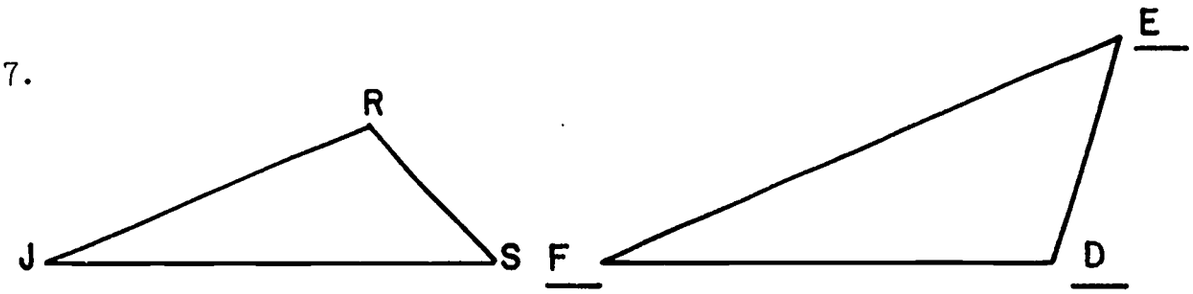
6.



Angle H ⟷ Angle N
Angle G ⟷ Angle P
Angle J ⟷ Angle O

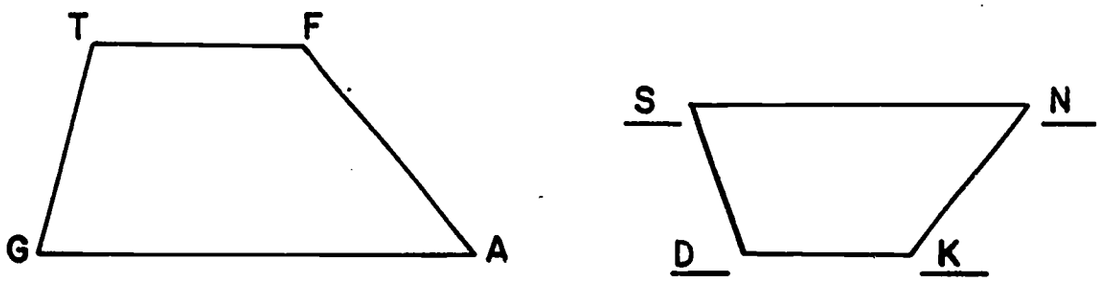
The corresponding angles are given. Use this information to correctly label the vertices of the second figure.

7.



Angle R ⟷ Angle D
Angle J ⟷ Angle F
Angle S ⟷ Angle E

8.

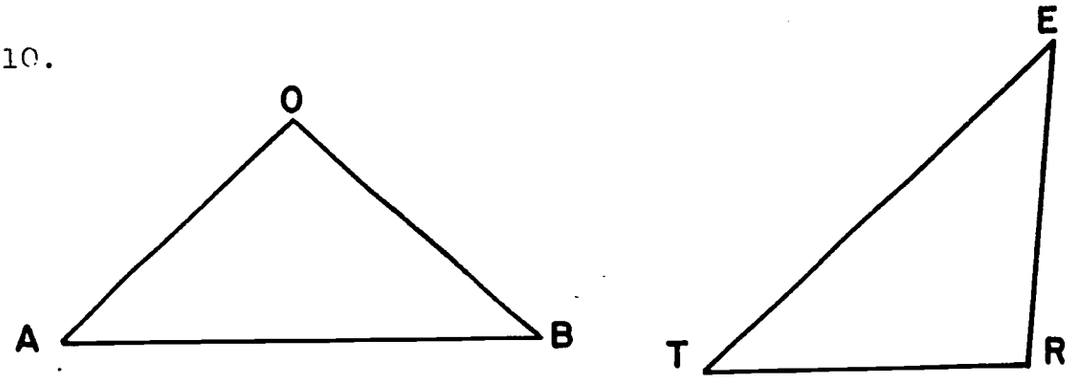


Angle G ⟷ Angle S
Angle A ⟷ Angle N
Angle F ⟷ Angle K
Angle T ⟷ Angle D

9.

Angle A ⟷ Angle Z
Angle B ⟷ Angle Y
Angle C ⟷ Angle X

10.



The similar triangles above have two correspondences which pair up angles of the same size. One is given. Can you find the other?

Angle A ⟷ Angle T
Angle B ⟷ Angle E
Angle O ⟷ Angle R

Angle A ⟷ Angle E
Angle B ⟷ Angle T
Angle O ⟷ Angle R

11. Describe a pair of triangles having three correspondences which pair up angles of the same size.

EQUILATERAL TRIANGLES

✓ **POINT**

1. Can you give a brief definition of similar figures?

2. What relationship do corresponding angles of similar figures have?

3. Explain your procedure for determining the corresponding angles of two similar triangles.

ANSWERS TO ✓ **POINT**

1. Answers will vary. Expected answer: Figures which have exactly the same shape are called similar figures.
2. They are the same size.
3. Answers will vary.

OBJECTIVES

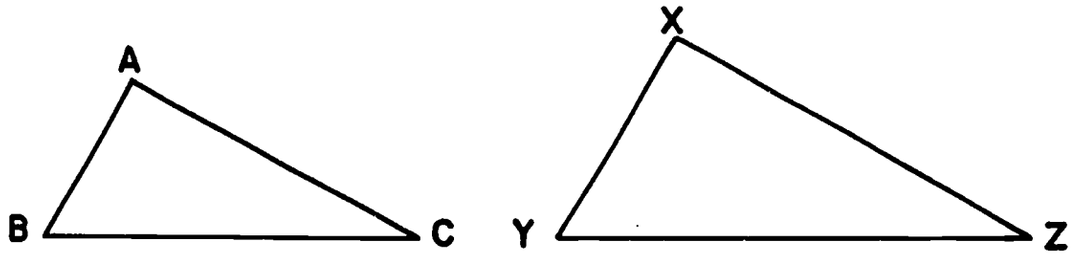
1. Given a pair of similar triangles such as:

the student will be able to name a similarity correspondence for the triangles.

2. Given a similarity correspondence such as: $\triangle DEF \leftrightarrow \triangle SBT$, the student will be able to name the corresponding angles of the similar triangles.

SIMILARITY CORRESPONDENCE

Triangle ABC is similar to triangle XYZ ($\triangle ABC \sim \triangle XYZ$).



Since the triangles are similar, the corresponding angles are the same size. The corresponding angles are:

$$\angle A \longleftrightarrow \angle X$$
$$\angle B \longleftrightarrow \angle Y$$
$$\angle C \longleftrightarrow \angle Z$$

A shorter way to represent the corresponding angles of the triangles is

$$\triangle ABC \longleftrightarrow \triangle XYZ$$

Notice how the corresponding angles are represented in this expression

$$\triangle A\ B\ C \longleftrightarrow \triangle X\ Y\ Z$$

We have made a correspondence between the similar triangles which also indicates the corresponding angles.

A correspondence between two similar figures which names the corresponding angles is called a <u>SIMILARITY CORRESPONDENCE</u>.

CONTENT AND APPROACH

Six different correspondences can be named between the similar triangles on page 13.

They are:

1. $\triangle ABC \longleftrightarrow \triangle XYZ$
2. $\triangle ABC \longleftrightarrow \triangle XZY$
3. $\triangle ABC \longleftrightarrow \triangle YXZ$
4. $\triangle ABC \longleftrightarrow \triangle YZX$
5. $\triangle ABC \longleftrightarrow \triangle ZXY$
6. $\triangle ABC \longleftrightarrow \triangle ZYX$

Note: The correspondence $\triangle CBA \longleftrightarrow \triangle ZYX$ is the same as number one above.

Naming a correspondence between two figures also names a set of corresponding angles. The first letters of each figure name a pair of corresponding angles, the second letters of each figure name a pair of corresponding angles, etc. (See the diagram at the bottom of page 13).

Since correspondence number one above ($\triangle ABC \longleftrightarrow \triangle XYZ$) is the only correspondence which associates the congruent angles of the similar triangles, it is called a <u>similarity correspondence</u>.

When we say triangle ABC is similar to triangle XYZ or write $\triangle ABC \sim \triangle XYZ$, we understand this to mean that:

1. Triangle ABC corresponds to triangle XYZ ($\triangle ABC \leftrightarrow \triangle XYZ$)
2. The corresponding angles are the same size.

EXERCISES

1. Name the corresponding angles of the similar triangles named below.

a) $\triangle RST \sim \triangle DEF$ c) $\triangle ACD \sim \triangle WYX$

b) $\triangle UVW \sim \triangle JHI$ d) $\triangle MNO \sim \triangle ONP$

2. The corresponding angles are given. Name the similar triangles.

a) $\angle A \longleftrightarrow \angle O$
$\angle B \longleftrightarrow \angle M$
$\angle C \longleftrightarrow \angle N$
$\triangle ABC \sim \triangle OMN$

b) $\angle W \longleftrightarrow \angle H$
$\angle X \longleftrightarrow \angle K$
$\angle Y \longleftrightarrow \angle J$
$\triangle WXY \sim \triangle HKJ$

3. The corresponding angles of a pair of similar triangles are:

$\angle B \longleftrightarrow \angle O$
$\angle C \longleftrightarrow \angle N$
$\angle D \longleftrightarrow \angle P$

Which of the following names a similarity correspondence between these triangles? (More than one may be correct.)

(a) $\triangle BCD \longleftrightarrow \triangle ONP$ (d) $\triangle DBC \longleftrightarrow \triangle PON$

(b) $\triangle DCB \longleftrightarrow \triangle PNO$ e) $\triangle BDC \longleftrightarrow \triangle ONP$

c) $\triangle CDB \longleftrightarrow \triangle NOP$

CONTENT AND APPROACH

The statement at the top of the page should be carefully analyzed with the students. The symbol $\sim$ has taken on additional meaning. Besides replacing the words "is similar to", it names a similarity correspondence. This means that

$$\triangle ABC \sim \triangle XYZ$$
$$\text{and} \quad \triangle ABC \sim \triangle ZYX$$

are not equivalent statements (Different corresponding angles are named).

This notation is widely accepted, is clear, and has the advantage that one can tell the corresponding angles from the notation, without referring to figures.

By this time, most students should be able to function without the aid of tracing paper. Encourage those still using tracing paper to try their skill without it, but do not insist that they do so.

THINGS TO DISCUSS

To say $\triangle ABC \sim \triangle XYZ$ is the same as saying $\triangle CBA \sim \triangle ZYX$; but not the same as saying $\triangle ABC \sim \triangle ZYX$. Can you explain why?

ANSWERS TO EXERCISE 1

a. $\angle R \longleftrightarrow \angle D$
$\angle S \longleftrightarrow \angle E$
$\angle T \longleftrightarrow \angle F$

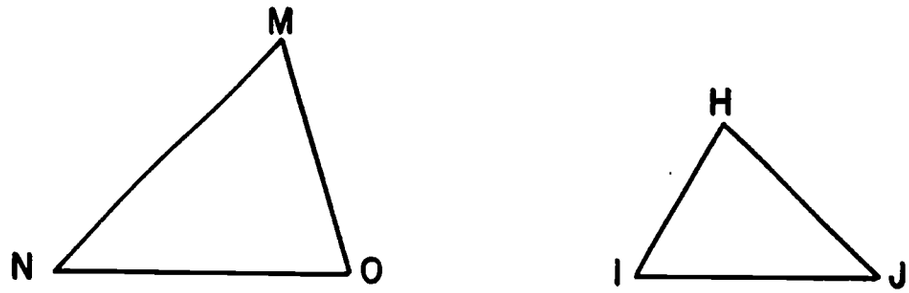
b. $\angle U \longleftrightarrow \angle J$
$\angle V \longleftrightarrow \angle H$
$\angle W \longleftrightarrow \angle I$

c. $\angle A \longleftrightarrow \angle W$
$\angle C \longleftrightarrow \angle Y$
$\angle D \longleftrightarrow \angle X$

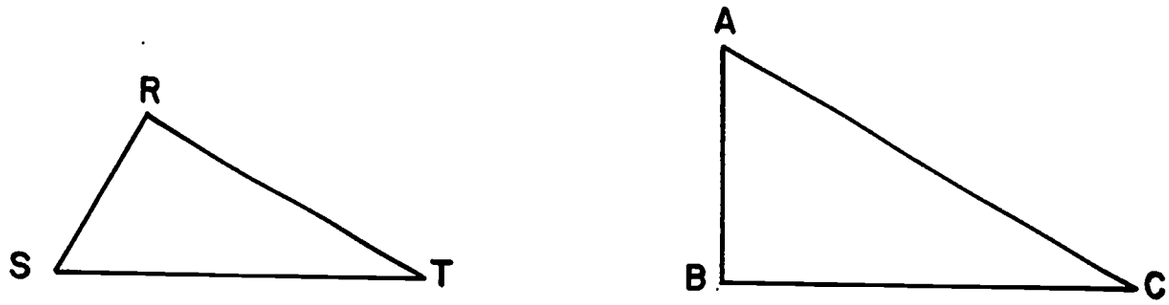
d. $\angle M \longleftrightarrow \angle O$
$\angle N \longleftrightarrow \angle N$
$\angle O \longleftrightarrow \angle P$

4. In each part below, the figures are similar. Name a similarity correspondence for each pair of figures.

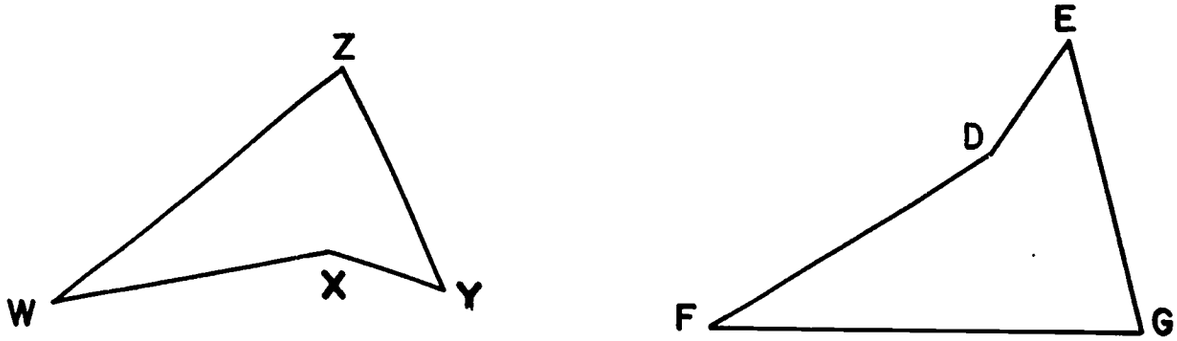
a. $\triangle$ MNO $\sim$ $\triangle$ IJH



b. $\triangle$ RST $\sim$ $\triangle$ BAC

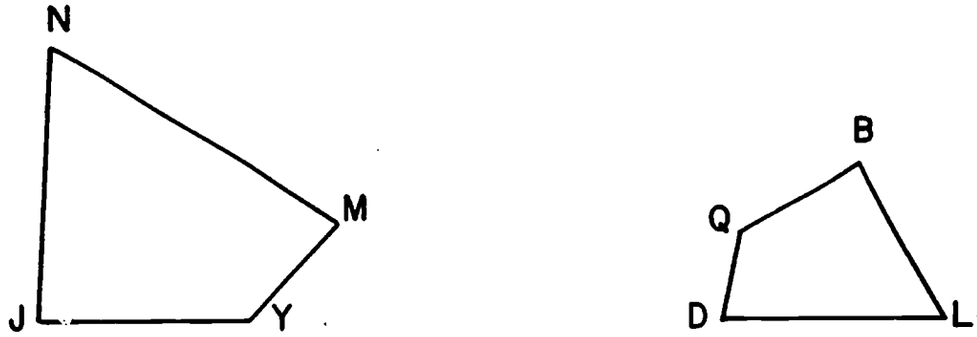


c. fig. WXYZ $\sim$ fig. FDEG

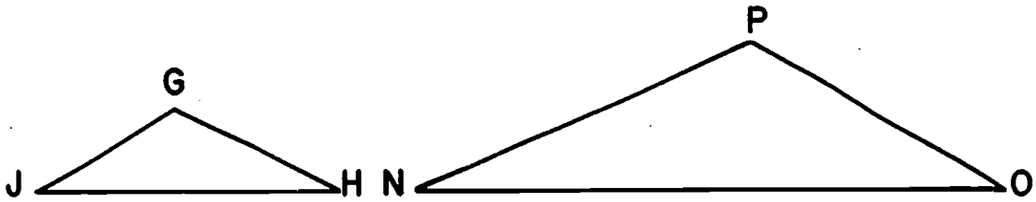


d. fig. ABCD $\sim$ fig. SRQP

e. fig. NJYM $\sim$ fig. LBQD

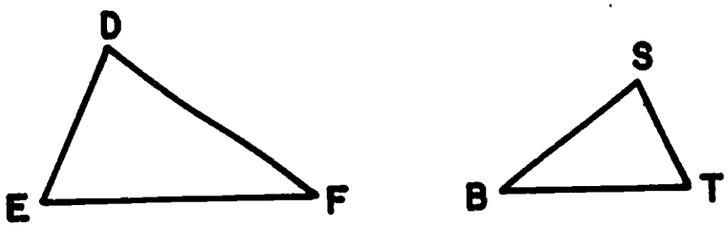


f. $\triangle$NOP $\sim$ $\triangle$HJG



OBJECTIVES

1. Given a pair of similar triangles such as:



the students will be able to name the corresponding sides.

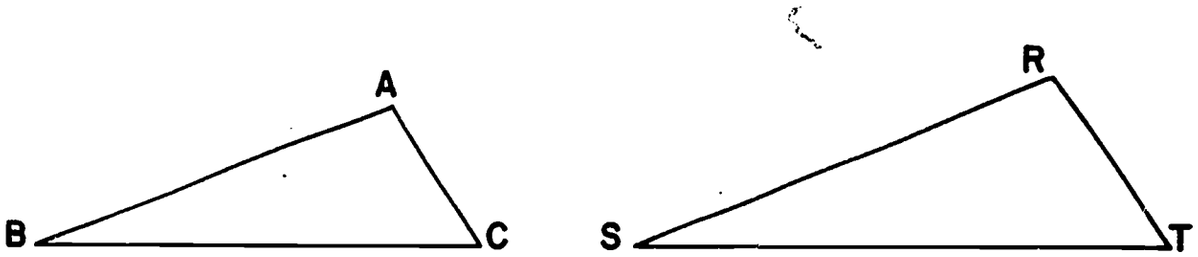
2. Given a similarity correspondence such as: $\triangle DEF \longleftrightarrow \triangle SBT$, the student will be able to name the corresponding sides of the similar triangles.

EQUIPMENT AND TEACHING AIDS

1. Overhead projector
2. Projection screen
3. Tracing paper
4. Ruler

CORRESPONDING SIDES

Besides naming the corresponding angles, a similarity correspondence between two figures also names the <u>corresponding sides</u> of the figures.



Triangle ABC is similar to triangle RST ($\triangle ABC \sim \triangle RST$)

A B C $\sim$ R S T — Side AB corresponds to side RS ($\overline{AB} \longleftrightarrow \overline{RS}$)

A B C $\sim$ R S T — Side BC corresponds to side ST ($\overline{BC} \longleftrightarrow \overline{ST}$)

A B C $\sim$ R S T — Side AC corresponds to side RT ($\overline{AC} \longleftrightarrow \overline{RT}$)

DISCUSSION QUESTIONS

1. Given any similarity correspondence, make a rule for naming corresponding sides of the similar figures.
2. The corresponding angles of similar figures are the same size. What relationship do corresponding sides have?
3. Given two similar figures, describe a procedure for finding a similarity correspondence.
4. If the corresponding angles of a pair of similar figures are known, is it possible to determine the corresponding sides? Explain your answer.

CONTENT AND APPROACH

In a similarity correspondence, the first and second letters of each figure name a pair of corresponding sides, the second and third letters of each figure name a pair of corresponding sides, etc. The last and first letters of each figure also name a pair of corresponding sides (see the diagrams below the figures on page 17).

As with corresponding angles, tracing can also be used to find the corresponding sides of similar figures. Trace one of the figures and lay the tracing on the other so they "line up". The sides that "line up" are the corresponding sides. Use the overhead projector to demonstrate this procedure.

ANSWERS TO DISCUSSION QUESTIONS

1. Answers will vary.
2. Possible answers may include:
   a. they are same length
   b. they are not the same length
   c. they are not related
   d. they are related by ratio.

By demonstration, show that answers a and b are correct for some but not all similar figures, therefore both are poor answers. Answer c is the most logical answer, but suggest they withhold judgment of this kind till later in the booklet. Some students may have studied similarity before and give answer d. Tell these students they are correct and you may need them to help explain the relationship when it is presented later in the booklet.

3. Answers will vary.
4. Yes, if the figure is a triangle. Yes, if the figures have more than three sides and the angles are named in succession. No, if the figures have more than three sides and the angles are named in random order.

EXERCISES

1) Name the corresponding sides of the similar triangles below.

a. $\triangle RST \sim \triangle DEF$ c. $\triangle ACD \sim \triangle WYZ$

b. $\triangle UVW \sim \triangle JHI$ d. $\triangle MNO \sim \triangle ONP$

2) The corresponding sides are given. Name the similar figures.

a) $\overline{AB} \longleftrightarrow \overline{PQ}$

$\overline{BC} \longleftrightarrow \overline{QN}$

$\overline{AC} \longleftrightarrow \overline{PN}$

b) $\overline{WX} \longleftrightarrow \overline{HK}$

$\overline{XY} \longleftrightarrow \overline{KJ}$

$\overline{YZ} \longleftrightarrow \overline{JI}$

$\overline{WZ} \longleftrightarrow \overline{HI}$

3) The corresponding angles of a pair of similar triangles are

$\angle B \longleftrightarrow \angle N$

$\angle A \longleftrightarrow \angle T$

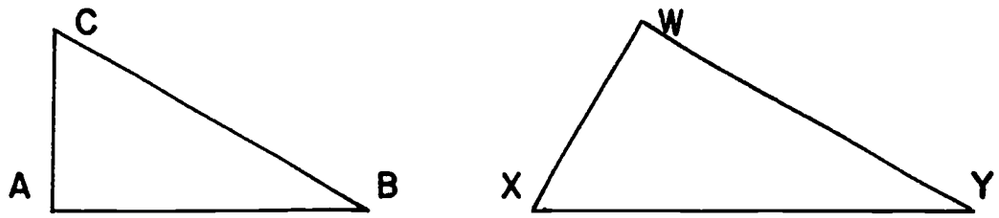
$\angle D \longleftrightarrow \angle X$

a) Name the similar triangles.

b) Name the corresponding sides.

4) Name a similarity correspondence for the pairs of similar figures and complete the tables naming the corresponding sides.

a. $\triangle ABC \sim \triangle WYX$



$\overline{AB} \longleftrightarrow \overline{WY}$

$\overline{BC} \longleftrightarrow \overline{YX}$

$\overline{CA} \longleftrightarrow \overline{XW}$

ANSWERS TO EXERCISES ON PAGE 18

1. a. $\overline{RS} \longleftrightarrow \overline{DE}$

   $\overline{ST} \longleftrightarrow \overline{EF}$

   $\overline{TR} \longleftrightarrow \overline{FD}$

   b. $\overline{UV} \longleftrightarrow \overline{JH}$

   $\overline{VW} \longleftrightarrow \overline{HI}$

   $\overline{WU} \longleftrightarrow \overline{IJ}$

   c. $\overline{AC} \longleftrightarrow \overline{WY}$

   $\overline{CD} \longleftrightarrow \overline{YZ}$

   $\overline{DA} \longleftrightarrow \overline{ZW}$

   d. $\overline{MN} \longleftrightarrow \overline{ON}$

   $\overline{NO} \longleftrightarrow \overline{NP}$

   $\overline{OM} \longleftrightarrow \overline{PO}$

2. a. $\triangle ABC \sim \triangle OMN$

   b. fig. WXYZ $\sim$ fig. HKJI

3. a. $\triangle BAD \sim \triangle NTX$

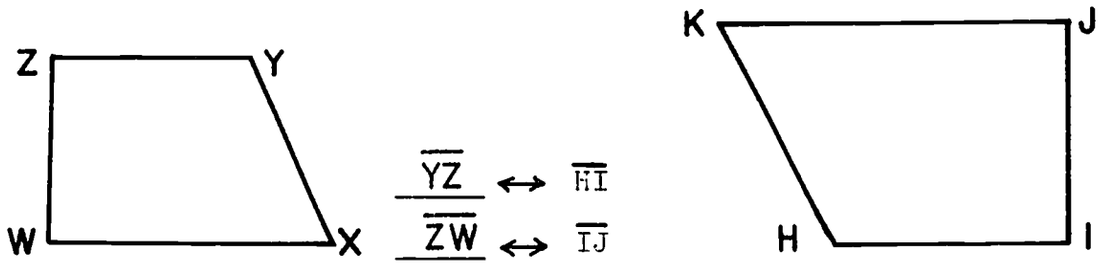
   b. $\overline{BA} \longleftrightarrow \overline{NT}$

   $\overline{AD} \longleftrightarrow \overline{TX}$

   $\overline{DB} \longleftrightarrow \overline{XN}$

4. answers on page 18

4b. fig. WXYZ ~ fig. JKHI



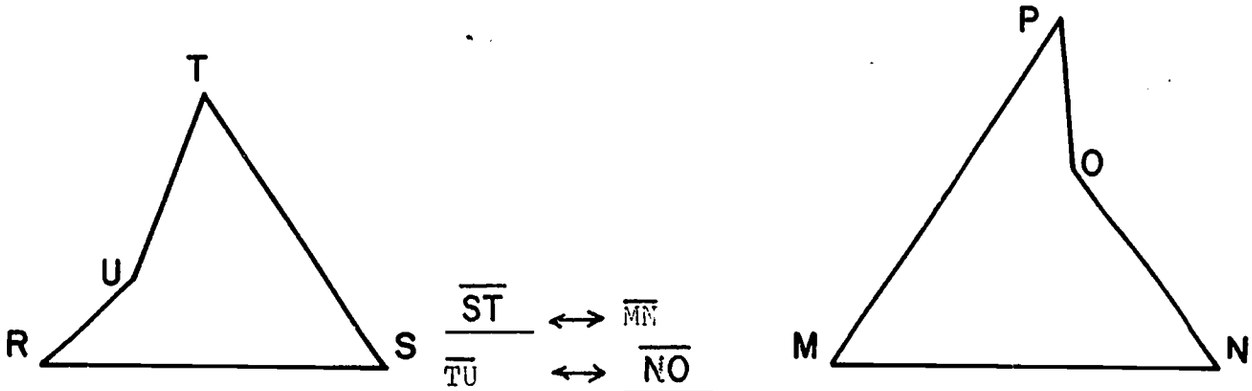
$\overline{YZ} \leftrightarrow \overline{HI}$

$\overline{ZW} \leftrightarrow \overline{IJ}$

$\overline{WX} \leftrightarrow \overline{JK}$

$\overline{XY} \leftrightarrow \overline{KH}$

4c. fig. TURS ~ fig. NOPM



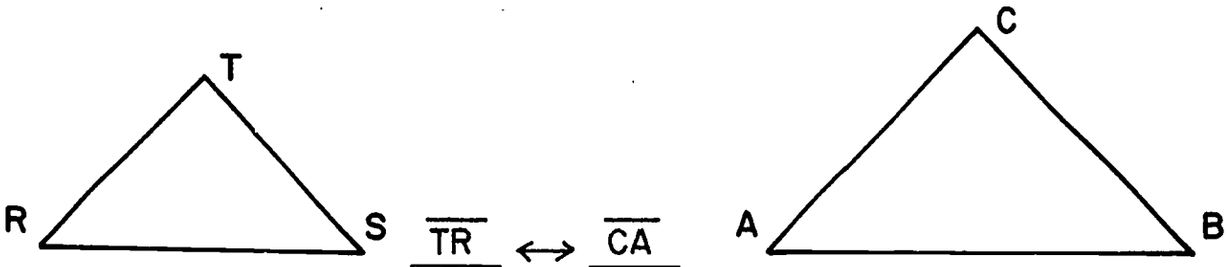
$\overline{ST} \leftrightarrow \overline{MN}$

$\overline{TU} \leftrightarrow \overline{NO}$

$\overline{UR} \leftrightarrow \overline{OP}$

$\overline{RS} \leftrightarrow \overline{PM}$

4d. $\triangle$ TRS ~ $\triangle$ CAB



$\overline{TR} \leftrightarrow \overline{CA}$

$\overline{RS} \leftrightarrow \overline{AB}$

$\overline{ST} \leftrightarrow \overline{BC}$

CLASS ACTIVITY

A) 1. Draw a triangle - as large as your drawing area will allow.

2. Label the vertices R, S, and T.

3. Find the midpoint of $\overline{RS}$ and label it "A".
Find the midpoint of $\overline{ST}$ and label it "B".
Find the midpoint of $\overline{TR}$ and label it "C".

4. Connect A, B, and C.

B) 1. Make a list of all the similar triangles in the drawing.

$\triangle RST \sim \triangle ASB \sim \triangle BCA \sim \triangle CBT \sim \triangle RAC$

2. Name the corresponding angles of triangles ABC and RST.

$\angle R \leftrightarrow \angle B$; $\angle S \leftrightarrow \angle C$; $\angle T \leftrightarrow \angle A$

3. Name the corresponding sides of the same triangles.

$\overline{RS} \longleftrightarrow \overline{BC}$; $\overline{ST} \longleftrightarrow \overline{CA}$; $\overline{TR} \longleftrightarrow \overline{BA}$

4. Name a similarity correspondence for the two triangles.

$\triangle RST \sim \triangle BCA$

C) 1. Find the midpoints of the sides of triangle ABC and connect them. Is this triangle similar to all the others? **YES**

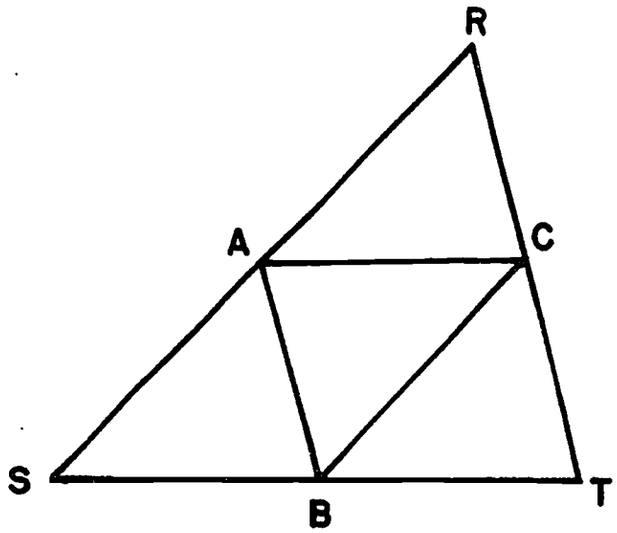
2. Connect the midpoints of the sides of any triangle in the drawing. Do you always get similar triangles? **YES**

3. Finish this sentence: The triangle formed by connecting the midpoints of the sides of any given triangle is

SIMILAR TO THE GIVEN TRIANGLE

CONTENT AND APPROACH

The figure at the right is an example of what the drawing could look like after completing Section A of the activity. Connecting the midpoints of the sides of any given triangle will divide the triangle into four congruent triangles similar to the given triangle.



Section A can be used as a group activity. Divide the class into groups. Each group has an area of the blackboard to draw the figure described in Section A and sends its members (one at a time) to complete one (or a part of one) of the instructions in A. If the figures are drawn accurately it will afford the students an opportunity to see and compare the results using different shaped triangles. Section B can be used by individuals as a checkpoint and C can be used for class discussion.

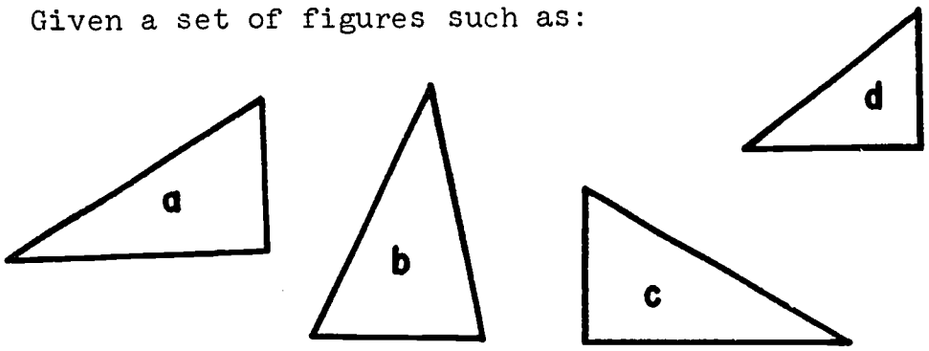
THINGS TO DISCUSS

On what rectangular piece of paper is it possible to draw a triangle which covers more than half the area of the paper?

OBJECTIVES

The student will be able to

1. give a definition of coincide. The following would be an acceptable definition: If two figures will fit exactly on each other, we say they coincide.
2. Given a set of figures such as:

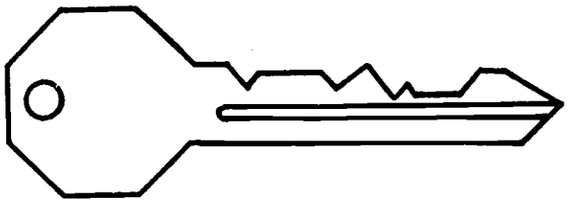


the student will be able to identify those which could be made to coincide.

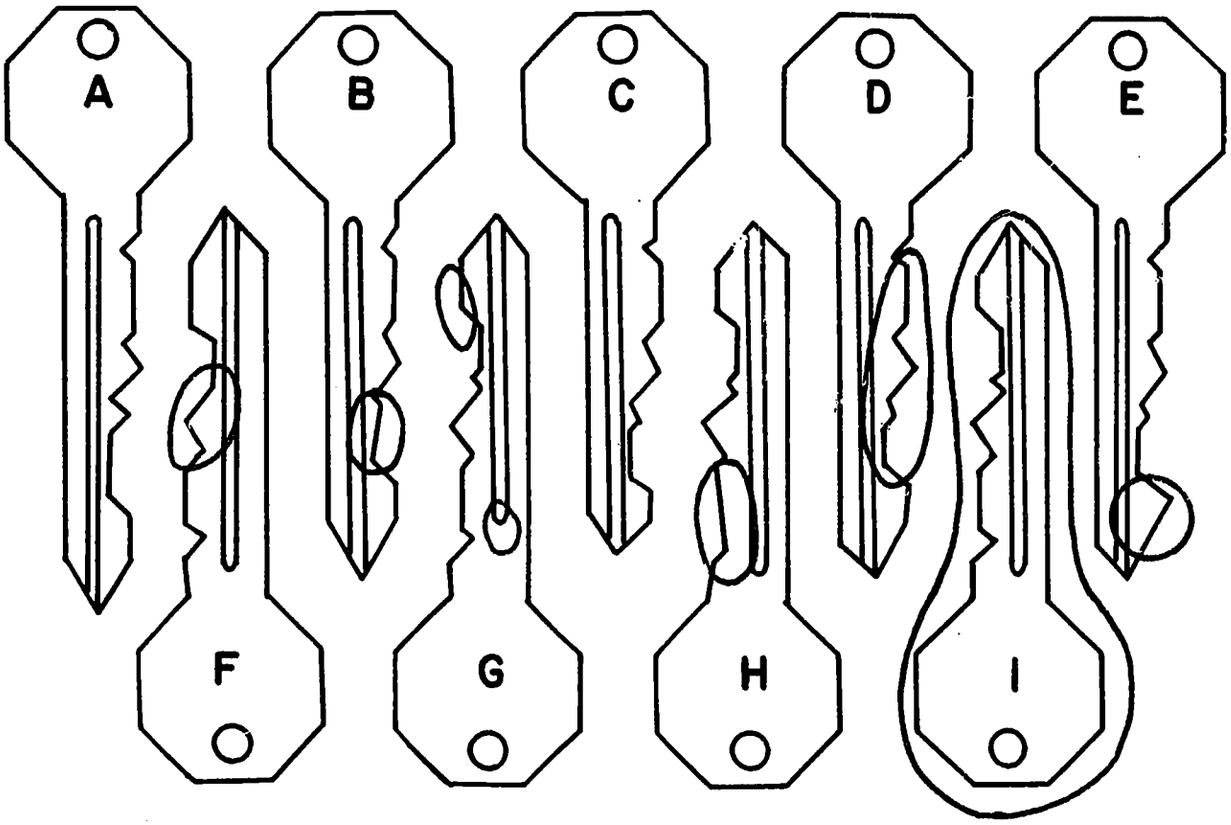
EQUIPMENT AND TEACHING AIDS

1. Overhead Projector
2. Projection Screen
3. Transparencies S12 and S13
4. Clear Sheet of Acetate
5. Felt Tip Pen or Grease Pencil
6. Tracing Paper

A PERFECT MATCH



One of the keys below will unlock the same door as the key above. Can you find it? Place a circle around your suggestion.



DISCUSSION QUESTIONS

1. Why do you think the key you selected will unlock the door?
2. Can you think of a way to prove your selection is correct?
3. Explain your reasons for "ruling out" each of the other keys.

CONTENT AND APPROACH

Before using the transparency

1. Give students time to select the duplicate key.
2. Discuss the questions at the bottom of the page.

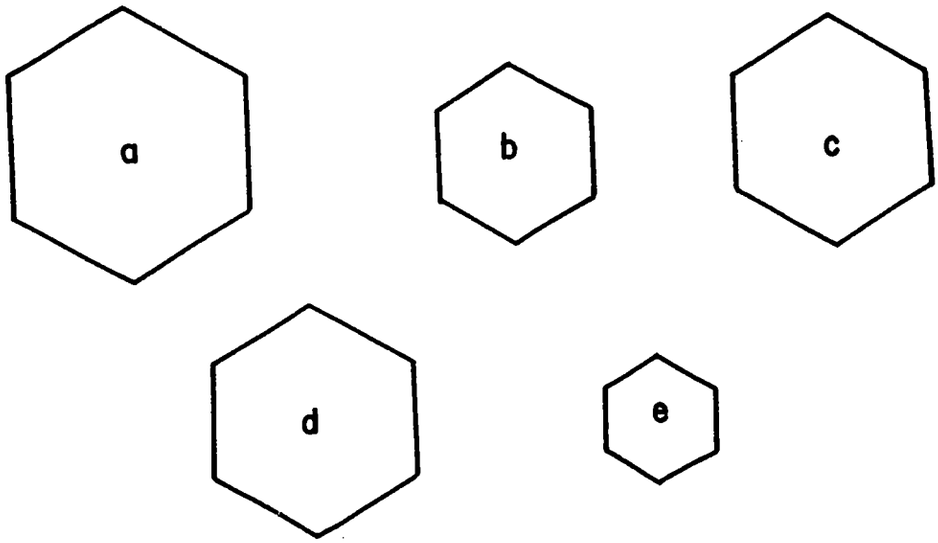
Transparency S12 is a duplicate of the keys on page 21 with a cutout of the key at the top. First lay the cutout on the key at the top of the page to verify that it is a duplicate and then use the cutout to check the key selected by the students as a duplicate. Lay the cutout on each of the remaining keys to show why they are not duplicates. Some students may claim the transparency is different from his book. Hand him the cutout or transparency so he can check for himself.

POSSIBLE ANSWERS TO DISCUSSION QUESTIONS

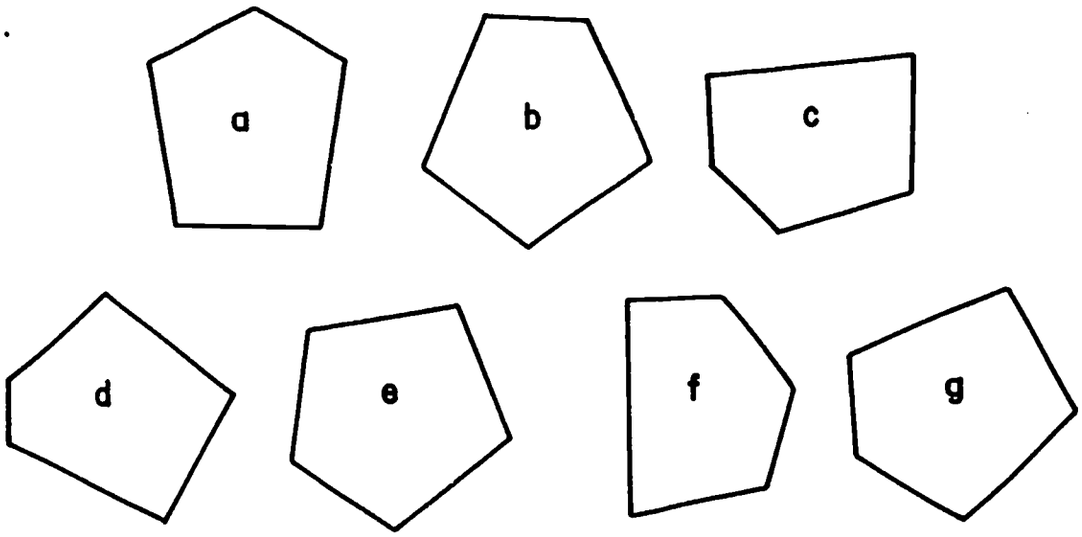
1. The size and shape of the key is exactly the same as the key at the top of the page.
2. a. cut out one and lay it over the other
   b. trace one and lay the tracing over the other
3. A. too long
   B. see student page 21
   C. too short
   D. see student page 21
   E. see student page 21
   F. see student page 21
   G. see student page 21
   H. see student page 21

In each set of figures, find the pair or pairs which are exactly the same shape and size.

1.



2.



CONTENT AND APPROACH

In exercises 1-3, students should make their selections without the aid of tracing paper. Tracing paper should be used to check their decisions.
Transparency S13 is a copy of exercise 2 on page 22 and the exercises on page 23. A clear piece of acetate can be used as tracing paper for the transparency.

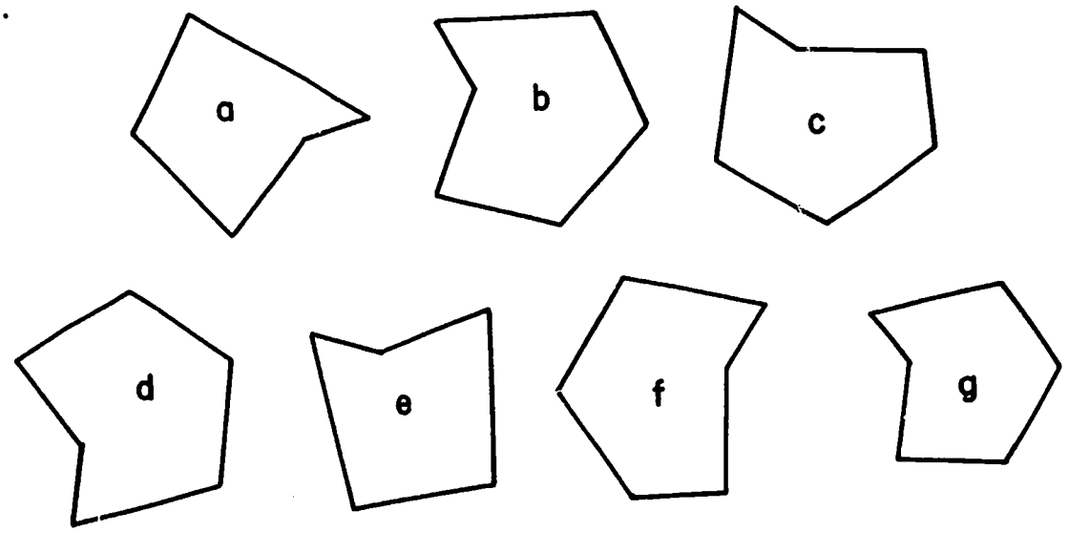
THINGS TO DISCUSS

Some students will adopt a method of selecting the pairs (process of elimination, comparing sides, comparing angles, etc.). It would be interesting and valuable to discuss these methods.

ANSWERS

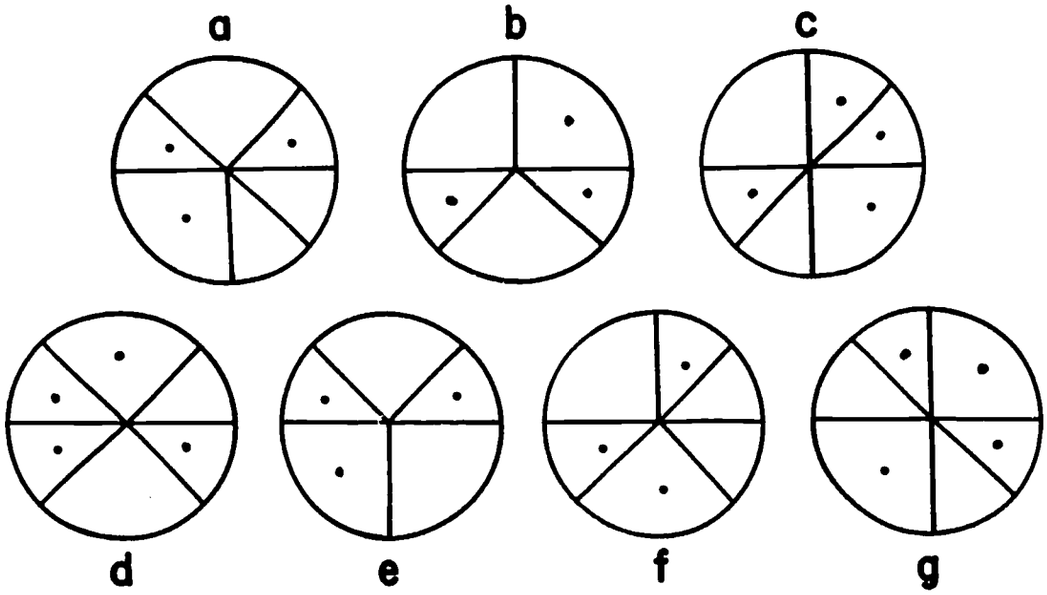
1. c and d
2. a and g

3.



If two figures are placed one over the other, and all their corresponding parts agree exactly, we say the figures coincide.

In the following sets of points, find the pairs which could be made to coincide.



CONTENT AND APPROACH

Make special emphasis of the term coincide if it has not been mentioned before and especially if the term is new to the students.

ANSWERS

3. b and d; c and f

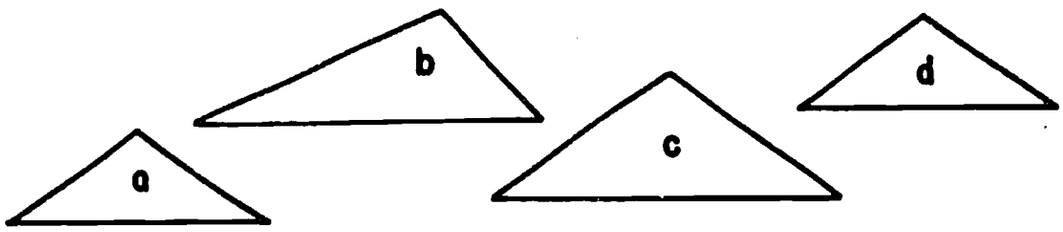
bottom of page. a and f; c and d

SECTION 6

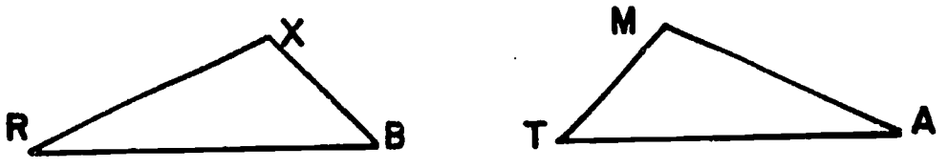
OBJECTIVES

1. The student will be able to give a definition or description of "congruent figures". The following would be considered an adequate definition: Two figures which are exactly the same shape and size are called congruent figures.

2. Given the following set of figures, the student will be able to identify the pair which is congruent.



3. Given a pair of congruent figures, such as:



the student will be able to name

   a. the corresponding angles.
   b. the corresponding sides.
   c. a congruence correspondence.

4. From a list of statements about congruent figures, the student will be able to select the following true statements:

   a. their corresponding angles are congruent.
   b. their corresponding sides have the same length.
   c. a tracing of one could be made to coincide with the other.

5. Given a list of statements about congruent figures, the students will be able to indicate whether each statement is: (a) always true; (b) sometimes true; (c) never true.

EQUIPMENT AND TEACHING AIDS

1. Overhead Projector
2. Projection Screen
3. Transparency S14
4. Scissors
5. Tracing Paper

CONGRUENCE

GEOMETRIC FIGURES WHICH ARE EXACTLY THE SAME SHAPE AND SIZE ARE CALLED CONGRUENT FIGURES.

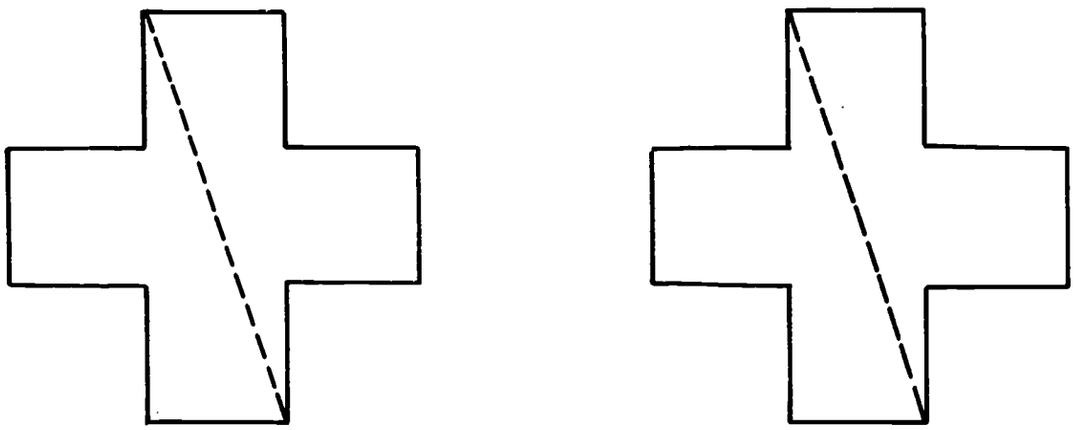
Which of the following figures would always be congruent?

a. squares
b. right angles
c. rectangles
d. similar figures
e. acute angles
f. similar figures of the same size
g. right triangles with the same area
h. circles with the same radius
i. squares with the same area
j. rectangles with the same area

What applications would congruence have in the following list of items?

a. gaskets
b. coins
c. a rubber stamp
d. light bulbs
e. sports
f. an assembly line
g. maps
h. footprints

Make cutouts of the congruent figures below. Cut along the dotted lines. Are the four figures which result congruent? Use these four figures to form a square.



CONTENT AND APPROACH

Call attention to the definition of congruence at the top of page 24. Ask the students to explain how this differs from the definition of similar figures on page 2.

The questions on page 24 are intended for class or small group discussion.

If something is always true then no counter-example exists. When there is disagreement about whether a set of figures are always congruent, the issue can be settled if a counter-example can be produced.

Suggestions about applications of congruence will vary. Some examples are listed below.

a. gaskets - Most gaskets line up with the surface they come between (location of holes is based on congruence).

b. coins - The perimeters of pennies are congruent. Except for the date many coin faces are congruent.

c. a rubber stamp - Makes congruent impressions.

d. light bulbs - Socket ends are congruent.

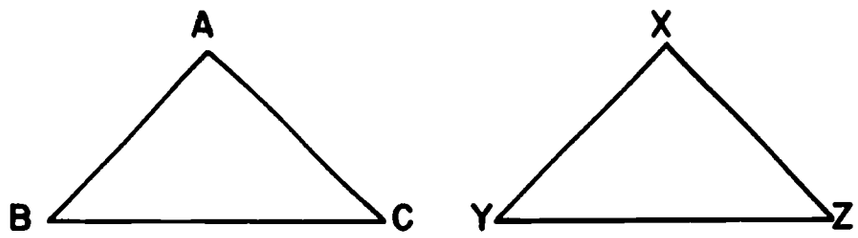
e. sports - Football playing fields are congruent. Basketball courts are seldom congruent but the hoops are congruent.

f. an assembly line - An assembly line operates on the principle of interchangeable parts and interchangeable parts are congruent.

h. footprints - A person's footprints are usually congruent.

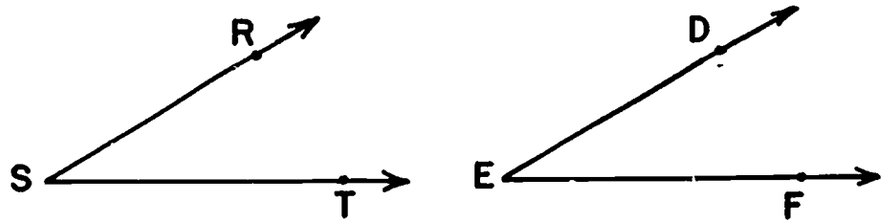
POLYGONS THAT ARE THE SAME SHAPE AND SIZE ARE CONGRUENT.

Triangle ABC is congruent to triangle XYZ ($\triangle ABC \cong \triangle XYZ$).



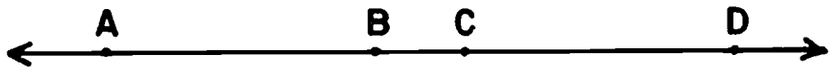
ANGLES WHICH CAN BE MADE TO COINCIDE ARE CONGRUENT.

Angle RST is congruent to angle DEF ($\angle RST \cong \angle DEF$).



LINE SEGMENTS WHOSE END POINTS CAN BE MADE TO COINCIDE ARE CONGRUENT.

Segment AB is congruent to segment CD ($\overline{AB} \cong \overline{CD}$).



THINGS TO DISCUSS

1. What is a polygon?
2. How would you define the word "congruent" to a friend? Would you use the word coincide?
3. Can you reword the definition of congruent polygons using the word coincide?
4. If two angles are congruent, what can you say about their measures? about their "sides"?
5. How is an angle measured?
6. If two segments are congruent, what can you say about their measures?
7. Segment AB is congruent to segment CD. (student page 25) Name another pair of congruent segments in the illustration. Can you tell why they must be congruent?

Triangle RST is congruent to triangle LKM ($\triangle RST \cong \triangle LKM$).

When we say triangle RST is congruent to triangle LKM or write $\triangle RST \cong \triangle LKM$, we understand this to mean that:

1. triangle RST corresponds to triangle LKM ($\triangle RST \leftrightarrow \triangle LKM$).
2. the corresponding angles are congruent.
3. the corresponding sides are congruent.

Since triangle RST is congruent to triangle LKM, the corresponding angles and sides are congruent.

$\angle R \cong \angle L$	$\overline{RS} \cong \overline{LK}$
$\angle S \cong \angle K$	$\overline{ST} \cong \overline{KM}$
$\angle T \cong \angle M$	$\overline{TR} \cong \overline{ML}$

If you have forgotten how corresponding angles and sides are represented in a correspondence between two figures, turn back to pages 13 and 17.

DISCUSSION QUESTIONS

1. Compare the statement above concerning what is meant when we say two figures are congruent with the statement at the top of page 14. What is the same? What is different?
2. Are all congruent figures similar? Are all similar figures congruent?
3. $\triangle RST \leftrightarrow \triangle KLM$ does not show that the triangles above are congruent. Can you explain why?

CONTENT AND APPROACH

Like the similarity symbol, the congruence symbol ($\cong$) also implies a correspondence. Explanation is on page 26.

Possible answers to discussion questions

1. Same - The similarity ($\sim$) and congruent ($\cong$) symbols both imply correspondence. The corresponding angles of both similar and congruent figures are congruent.

   Different - The corresponding sides of congruent figures are always congruent but the corresponding sides of similar figures are not necessarily congruent.

2. yes; no.

3. The corresponding angles are not congruent.

EXERCISES

1. Name the congruent angles and sides of the congruent triangles named below.

   a. $\triangle HKJ \cong \triangle MNP$
   b. $\triangle BDF \cong \triangle XTZ$
   c. $\triangle RTV \cong \triangle LMN$

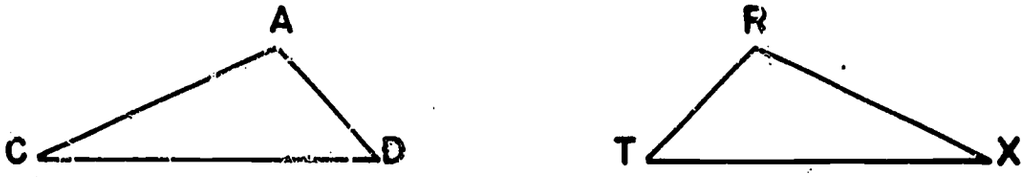
2. The congruent sides are given. Name the congruent figures.

   a. $\overline{WX} \cong \overline{TU}$, $\overline{XY} \cong \overline{UV}$, $\overline{YW} \cong \overline{VT}$

   b. $\overline{BC} \cong \overline{HI}$, $\overline{CD} \cong \overline{IJ}$, $\overline{DA} \cong \overline{JK}$, $\overline{AB} \cong \overline{KH}$

3. Name the pairs of congruent triangles drawn below.

   a. $\triangle ACD \cong \triangle RXT$



   b. $\triangle HKL \cong \triangle EDF$

   c. $\triangle TOB \cong \triangle FEC$

ANSWERS

1. a. $\angle H \longleftrightarrow \angle M$ $\quad \overline{HK} \longleftrightarrow \overline{MN}$
   $\angle K \longleftrightarrow \angle N$ $\quad \overline{KJ} \longleftrightarrow \overline{NP}$
   $\angle J \longleftrightarrow \angle P$ $\quad \overline{JH} \longleftrightarrow \overline{PM}$

   b. $\angle B \longleftrightarrow \angle X$ $\quad \overline{BD} \longleftrightarrow \overline{XT}$
   $\angle D \longleftrightarrow \angle T$ $\quad \overline{DF} \longleftrightarrow \overline{TZ}$
   $\angle F \longleftrightarrow \angle Z$ $\quad \overline{FB} \longleftrightarrow \overline{ZX}$

   c. $\angle R \longleftrightarrow \angle L$ $\quad \overline{RT} \longleftrightarrow \overline{LM}$
   $\angle T \longleftrightarrow \angle M$ $\quad \overline{TV} \longleftrightarrow \overline{MN}$
   $\angle V \longleftrightarrow \angle N$ $\quad \overline{VR} \longleftrightarrow \overline{NL}$

2. a. $\triangle WXY \cong \triangle TUV$
   b. fig. BCDA $\cong$ fig. HIJK

3. ANSWERS ON STUDENT PAGE

SIMILAR OR CONGRUENT?

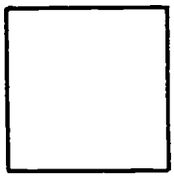
Study the pairs of figures. Indicate whether you think they are similar, similar and congruent, or not similar by underlining the proper word or words.

If you cannot tell just by looking, try tracing one of the figures on a piece of scrap paper and laying it on top of the other.

similar

congruent

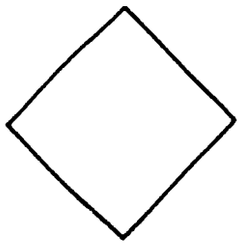
not similar



similar

congruent

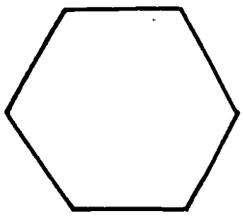
not similar



similar

congruent

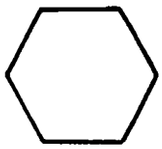
not similar



similar

congruent

not similar

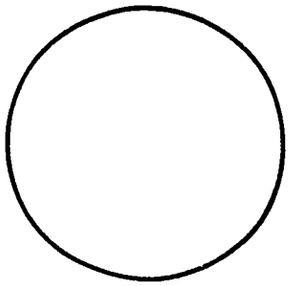


CONTENT AND APPROACH

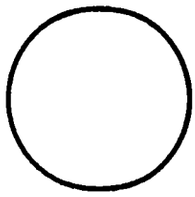
Use the overhead projector if you wish to discuss the answers with the entire class. Make a transparency of pages 28 and 29 and use a clear piece of acetate as tracing paper.

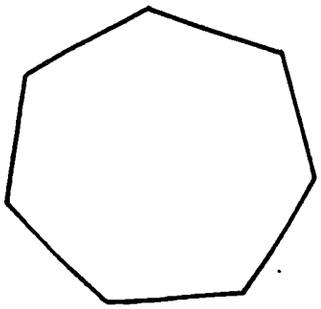
THINGS TO DISCUSS

If the figures are congruent, should the word "similar" be also underlined?

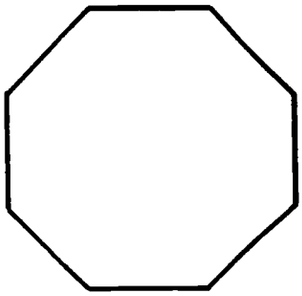


[✓ similar]
congruent
not similar

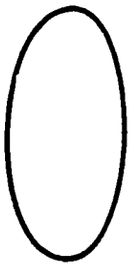


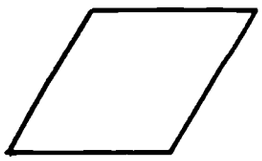


similar
congruent
[✓ not similar]

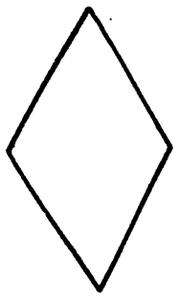


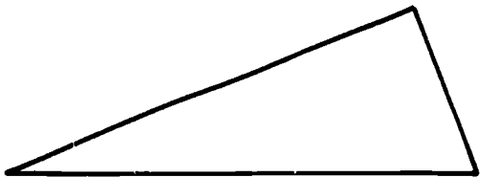
[✓ similar]
congruent
not similar



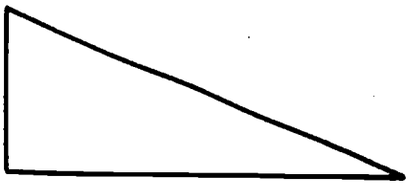


[✓ similar]
[✓ congruent]
not similar

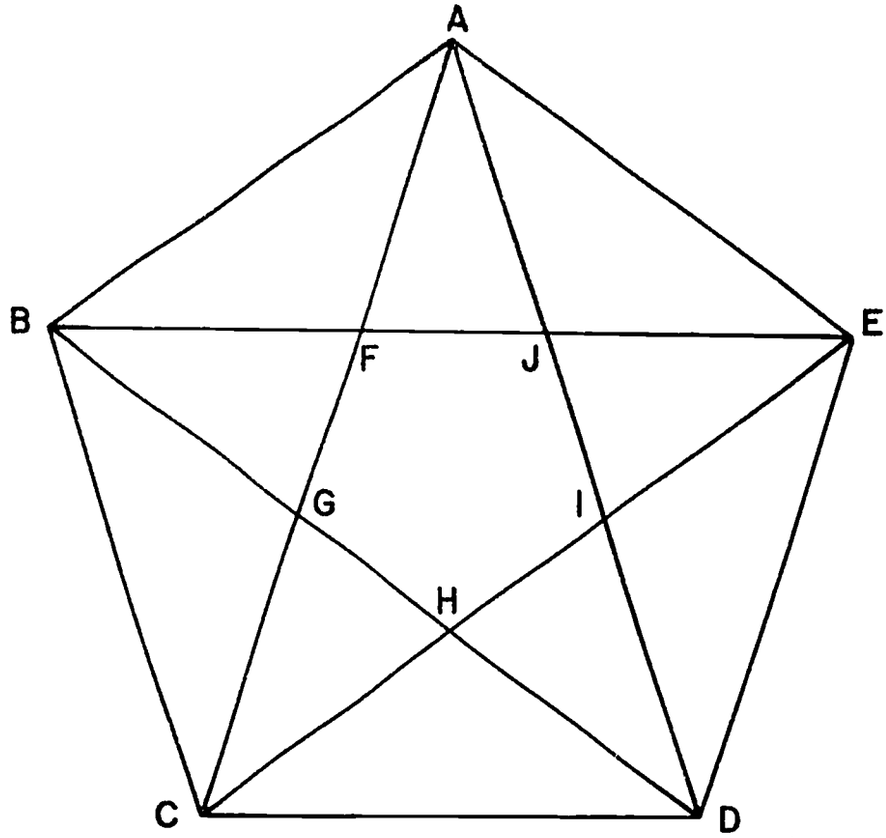




[✓ similar]
congruent
not similar



HOW MANY TRIANGLES CAN YOU FIND?



Make a list of the triangles as you find them. (Remember: △ABC and △CBA are the same triangle.)

△AFJ	△DIE	△AFE	△EJD	△DBJ
△BFG	△EJA	△BFC	△ACD	△ECF
△CGH	△ABE	△BGA	△BDE	△ADG
△DHI	△BCA	△CGD	△CEA	
△EIJ	△CDB	△CHB	△DAB	
△AFB	△DEC	△DHE	△EBC	
△BGC	△EAD	△DIC	△BEH	
△CHD	△AJB	△EIA	△CAI	

CONTENT AND APPROACH

Locating and listing the triangles on a random basis will, almost certainly, lead to duplication, omission, confusion and eventual frustration. Encourage the students to adopt a logical method of locating the triangles.

Use transparency S14 (and the cutouts) to identify and locate the various triangles in the figure on page 30.

THINGS TO DISCUSS

Explain your method of locating the triangles.

How many different shaped triangles are in the figure?

1. Name some triangles that are congruent to $\triangle$ABC._____

2. Name some triangles that are congruent to $\triangle$AFE._____

3. Name some triangles that are congruent to $\triangle$FCE._____

4. Name some triangles that are similar but not congruent to $\triangle$ACD._____

5. Name some triangles that are similar but not congruent to $\triangle$ABE._____

6. Name a figure that is similar to figure ABCDE._____
   What is the name for a five-sided figure?_____

7. Make a small sketch of a figure similar to figure AGDE.

8. Name some figures that are congruent to figure AGDE._____

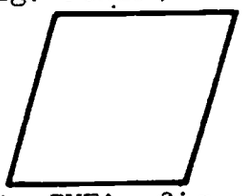
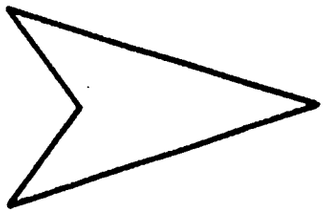
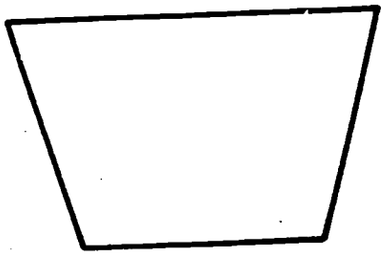
9. Make a small sketch of a figure similar to figure BGCE.

10. Make a small sketch of a figure similar to figure BCDE.

CONTENT AND APPROACH

The questions on page 31 refer to the figure on page 30.

ANSWERS

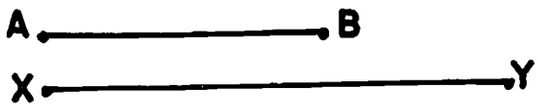
1. $\triangle$BCD, $\triangle$CDE, $\triangle$DEA, $\triangle$EAB, $\triangle$AGD, $\triangle$BHE, $\triangle$CIA, $\triangle$DJB, $\triangle$EFC.
2. $\triangle$AJB, $\triangle$BFC, $\triangle$BGA, $\triangle$CGD, $\triangle$CHB, $\triangle$DHE, $\triangle$DIC, $\triangle$EIA, $\triangle$EJD.
3. Same set as #1
4. $\triangle$AFJ, $\triangle$BFG, $\triangle$CGH, $\triangle$DHI, $\triangle$EIJ.
5. $\triangle$FAB, $\triangle$GBC, $\triangle$HCD, $\triangle$IDE, $\triangle$JEA.
6. fig. FGHIJ, Pentagon
7. 
8. fig BHEA, fig. CIAB, fig. DJBC, fig. EFCD
9. 
10. 

LESSON 7

OBJECTIVES

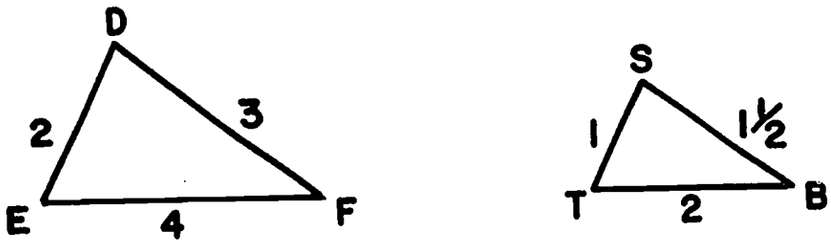
1. The student will read the symbol $m(\overline{AB})$ as "the measure of segment AB".

2. Given two segments such as:



and a foot ruler (graduated in eighths), the student will be able to determine the ratio $\frac{m(\overline{AB})}{m(\overline{XY})}$.

3. Given a pair of similar figures such as:

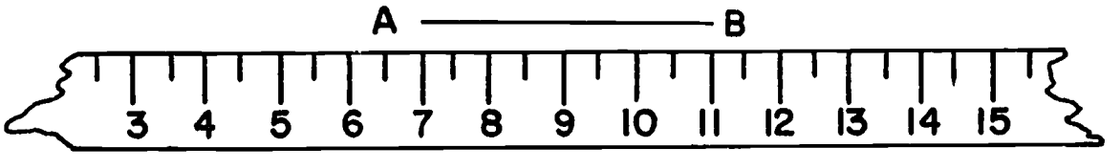


the student will be able to give the ratios of the measures of the sides of triangle DEF to the corresponding sides of triangle STB.

EQUIPMENT AND TEACHING AIDS

1. centimeter ruler
2. scissors

MEASURE AND RATIO



The measure, to the nearest centimeter, of segment AB is four centimeters. We write: $m(\overline{AB}) = 4cm$. Measure each of the following segments to the nearest centimeter and write the ratio of their measure to the measure of $\overline{AB}$.

C ———————— D

$m(\overline{CD})$ = 8, Ratio: $\frac{m(\overline{CD})}{m(\overline{AB})} = \frac{8}{4}$ or $\frac{2}{1}$

X ———— Y

$m(\overline{XY})$ = 2, Ratio: $\frac{m(\overline{XY})}{m(\overline{AB})} = \frac{2}{4}$ or $\frac{1}{2}$

R ———————————— S

$m(\overline{RS})$ = 12, Ratio: $\frac{m(\overline{RS})}{m(\overline{AB})} = \frac{12}{4}$ or $\frac{3}{1}$

K ———————— L

$m(\overline{KL})$ = 6, Ratio: $\frac{m(\overline{KL})}{m(\overline{AB})} = \frac{6}{4}$ or $\frac{3}{2}$

M ———————————— N

$m(\overline{MN})$ = 10, Ratio: $\frac{m(\overline{MN})}{m(\overline{AB})} = \frac{10}{4}$ or $\frac{5}{2}$

CONTENT AND APPROACH

The exercises on page 32 and 33 require the student to read a centimeter ruler. If the student has this ability the exercises will serve as reinforcement; if not, the student should be assisted in finding the measures so that poor measurement skills do not interfere with the major emphasis of the lesson: writing ratios of the measures of two segments.

It is not necessary that the ratios be reduced to lowest terms ($\frac{4}{8} = \frac{1}{2}$, etc.), but those who wish to reduce them should not be discouraged.

Make sure the students understand the symbol $m(\overline{AB})$ to mean: the measure of segment AB.

U ———————— V

$m(\overline{UV})$ = 5, Ratio: $\frac{m(\overline{UV})}{m(\overline{AB})} = \frac{5}{4}$

T ———————————— F

$m(\overline{TF})$ = 9, Ratio: $\frac{m(\overline{TF})}{m(\overline{AB})} = \frac{9}{4}$

G ———— H

$m(\overline{GH})$ = 3, Ratio: $\frac{m(\overline{GH})}{m(\overline{AB})} = \frac{3}{4}$

EXERCISES

1. From the segments above and on page 32, name the pair whose measures are in the ratio $\frac{5}{8}$. $\frac{M(\overline{UV})}{M(\overline{CD})}$

2. The ratio of the measures of $\overline{XY}$ to $\overline{AB}$ is $\frac{2}{4}$. Is the ratio equal to the ratio $\frac{1}{2}$? Name at least three other pairs of segments whose measures are in the ratio $\frac{1}{2}$.
$\frac{M(\overline{KL})}{M(\overline{RS})}$, $\frac{M(\overline{UV})}{M(\overline{MN})}$, $\frac{M(\overline{GH})}{M(\overline{KL})}$,

3. The ratio of the measures of $\overline{AB}$ to $\overline{RS}$ is $\frac{1}{3}$. This is recorded in the chart below. Complete the chart by placing an "X" in the proper space of each column.

	$\frac{m(\overline{AB})}{m(\overline{RS})}$	$\frac{m(\overline{UV})}{m(\overline{KL})}$	$\frac{m(\overline{TF})}{m(\overline{RS})}$	$\frac{m(\overline{GH})}{m(\overline{RS})}$	$\frac{m(\overline{GH})}{m(\overline{TF})}$	$\frac{m(\overline{GH})}{m(\overline{AB})}$	$\frac{m(\overline{XY})}{m(\overline{KL})}$	$\frac{m(\overline{MN})}{m(\overline{RS})}$	$\frac{m(\overline{KL})}{m(\overline{CD})}$	$\frac{m(\overline{XY})}{m(\overline{CD})}$
$\frac{1}{3}$	X				X		X			
$\frac{1}{4}$				X						X
$\frac{5}{6}$		X						X		
$\frac{3}{4}$			X			X			X	

CONTENT AND APPROACH

Review the answers given for the ratios on pages 32 and 33 before assigning the exercises at the bottom of page 33. To avoid further mistakes and confusion, each student should have correct answers (identical except for reducing) for the ratios to facilitate working and checking these exercises.

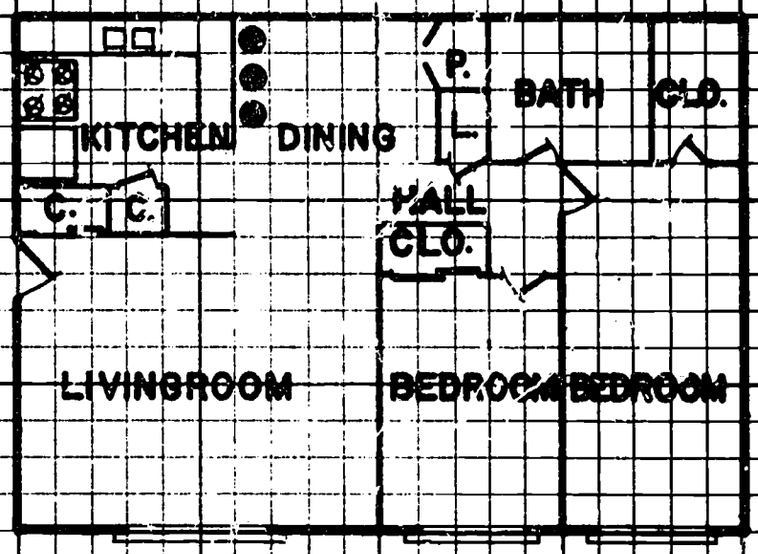


FIGURE 1

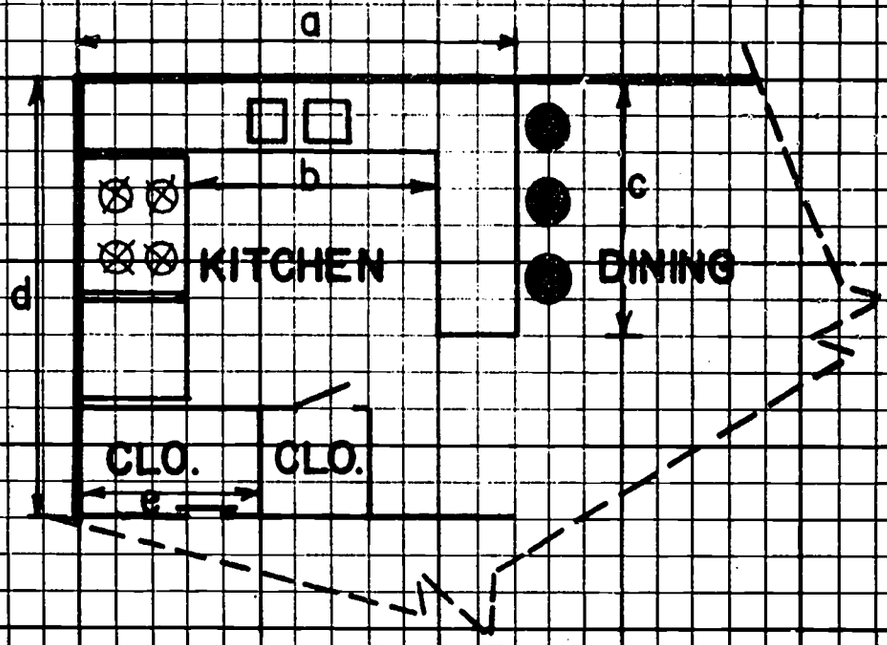


FIGURE 2

The diagram in figure 1 at the top of page 34 represents a floor plan for an apartment and figure 2 is an enlargement of the kitchen area. Using the side of one of the squares on the grid as a unit of length, find the following:

1. the lengths represented by a, b, c, d, and e in the enlargement (figure 2).
2. the lengths of the corresponding parts in the total floor plan (figure 1).
3. the ratios of the lengths in 1 to the lengths in 2.

Record the information in the table below.

	ENLARGEMENT	TOTAL PLAN	RATIO
a	12	6	$\frac{12}{6}$ or $\frac{2}{1}$
b	7	$3\frac{1}{2}$	$\frac{7}{3\frac{1}{2}}$ or $\frac{2}{1}$
c	7	$3\frac{1}{2}$	$\frac{7}{3\frac{1}{2}}$ or $\frac{2}{1}$
d	12	6	$\frac{12}{6}$ or $\frac{2}{1}$
e	5	$2\frac{1}{2}$	$\frac{5}{2\frac{1}{2}}$ or $\frac{2}{1}$

DISCUSSION QUESTIONS

1. An architect would say the ratio of the enlargement to the total floor plan is $\frac{2}{1}$. Can you explain why?
2. Are the drawings of the kitchen areas congruent? Are they similar? Explain your answers.
3. The dimensions of the living room in fig. 1 are 8 by 10. What would the dimensions be if an enlargement were made in the ratio $\frac{2}{1}$?
4. Using the ratio $\frac{2}{1}$ to enlarge the total floor plan, what would be the new dimensions of the bathroom, small bedroom, large bedroom and walk-in closet?
5. If each unit of length in fig. 1 represented two feet, what would be the dimensions of the actual living room? The other rooms?

POSSIBLE ANSWERS TO DISCUSSION QUESTIONS

1. The ratio of any dimension in the enlargement to its corresponding dimension in the total floor plan is $\frac{2}{1}$.

2. The drawings represent the same kitchen area, but the drawings themselves are not congruent because they are not the same size. They are similar because they are exactly the same shape (corresponding angles are also congruent).

3. 16 by 20

4.

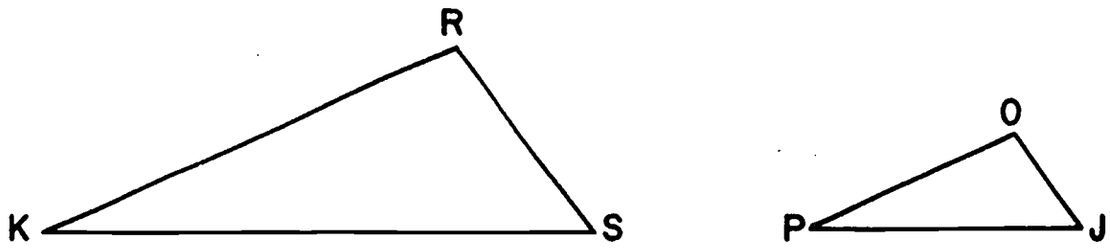
bathroom	8' × 9'
small bedroom	10' × 14'
large bedroom	10' × 20'
walk-in closet	5' × 8'

5.

living room	16' × 20'
kitchen	9' × 12'
broom closet	3' × 3'
dining room	11' × 12'
pantry	3' × 4'
linen closet	3' × 4'
bathroom	8' × 9'
walk-in closet	5' × 8'
large bedroom	10' × 20'
small bedroom	10' × 14'
small bedroom closet	3' × 6'
hall	3' × 7'
entry closet	3' × 5'

In each exercise, the figures are similar. Cut out the ruler at the bottom of this page and measure the sides of the figures to the nearest unit. Name a similarity correspondence and compare the ratios of the corresponding sides.

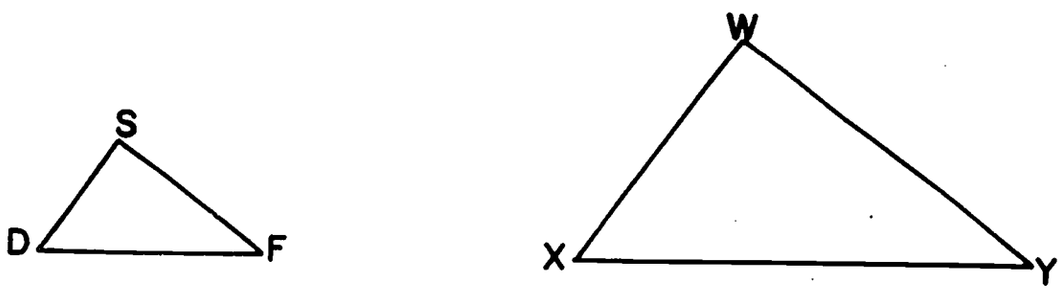
1.



$\triangle RKS \sim \triangle OPJ$

$$\frac{m(\overline{RK})}{m(\overline{OP})} = \frac{20}{10} \text{ or } \frac{2}{1} \quad \frac{m(\overline{KS})}{m(\overline{PJ})} = \frac{24}{12} \text{ or } \frac{2}{1} \quad \frac{m(\overline{SR})}{m(\overline{JO})} = \frac{10}{5} \text{ or } \frac{2}{1}$$

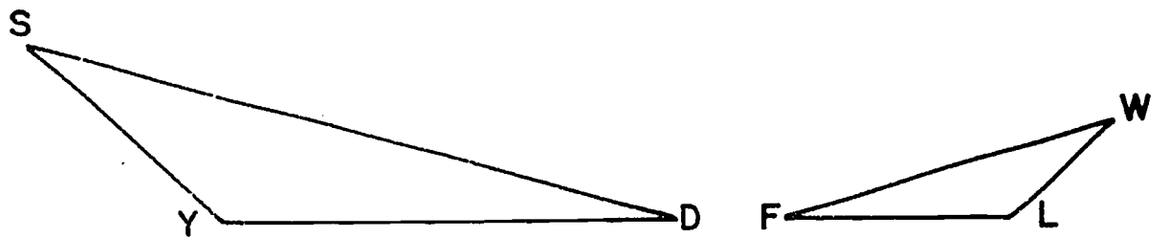
2.



$\triangle DFS \sim \triangle XYW$

$$\frac{m(\overline{DF})}{m(\overline{XY})} = \frac{10}{20} \text{ or } \frac{1}{2} \quad \frac{m(\overline{FS})}{m(\overline{YW})} = \frac{8}{16} \text{ or } \frac{1}{2} \quad \frac{m(\overline{SD})}{m(\overline{WX})} = \frac{6}{12} \text{ or } \frac{1}{2}$$

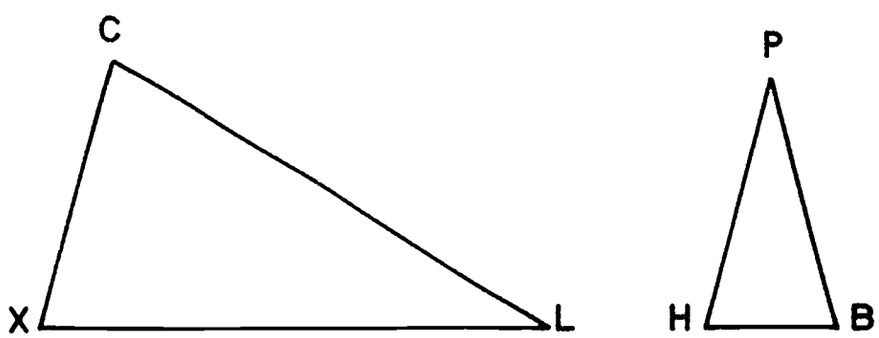
3.



$\triangle SYD \sim \triangle WLF$

$$\frac{m(\overline{SY})}{m(\overline{WL})} = \frac{12}{6} \text{ or } \frac{2}{1} \quad \frac{m(\overline{YD})}{m(\overline{FL})} = \frac{20}{10} \text{ or } \frac{2}{1} \quad \frac{m(\overline{DS})}{m(\overline{FW})} = \frac{30}{15} \text{ or } \frac{2}{1}$$

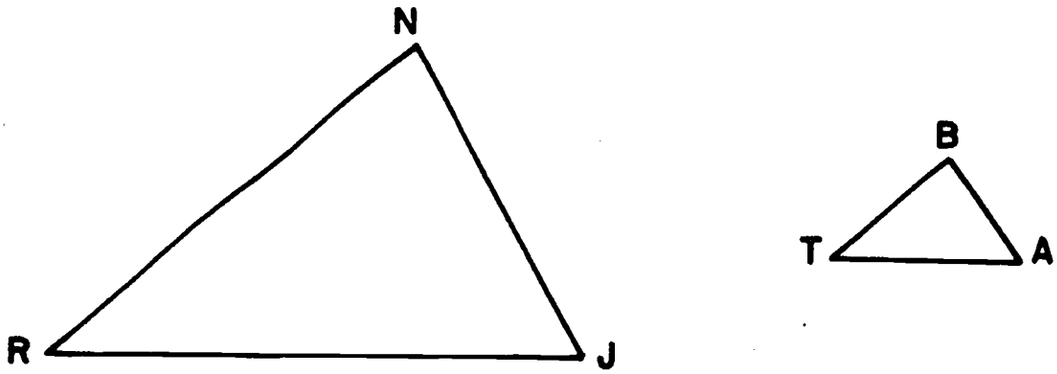
4.



$\triangle XLC \sim \triangle BPH$ or $\triangle HPB$

$$\frac{m(\overline{XL})}{m(\overline{BP})} = \frac{22}{11} \text{ or } \frac{2}{1} \quad \frac{m(\overline{LC})}{m(\overline{PH})} = \frac{22}{11} \text{ or } \frac{2}{1} \quad \frac{m(\overline{CX})}{m(\overline{HB})} = \frac{12}{6} \text{ or } \frac{2}{1}$$

5.



$\triangle NRJ \sim \triangle BTA$

$\frac{m(\overline{NR})}{m(\overline{BT})} = \frac{21}{7}$ or $\frac{3}{1}$ $\quad \frac{m(\overline{RJ})}{m(\overline{TA})} = \frac{24}{8}$ or $\frac{3}{1}$ $\quad \frac{m(\overline{JN})}{m(\overline{AB})} = \frac{15}{5}$ or $\frac{3}{1}$

6.

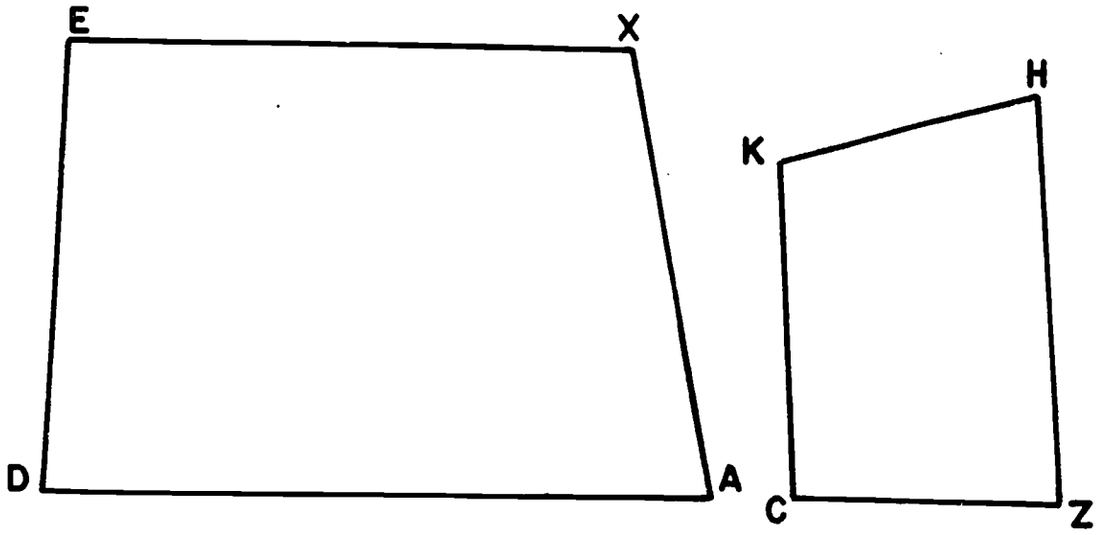


fig. EDAX $\sim$ fig. CZHK

$\frac{m(\overline{ED})}{m(\overline{CZ})} = \frac{20}{12}$ $\quad \frac{m(\overline{DA})}{m(\overline{ZH})} = \frac{30}{18}$ $\quad \frac{m(\overline{AX})}{m(\overline{HK})} = \frac{20}{12}$ $\quad \frac{m(\overline{XE})}{m(\overline{KC})} = \frac{25}{15}$ or $\frac{5}{3}$

for all of these

DISCUSSION QUESTIONS

1. In exercises 1 - 6, you were asked to compare the ratios of the corresponding sides of the similar figures. Discuss the results of your comparisons. Give examples.

2. How can you tell if two ratios are equal?

3. The lengths of segments a, b, c and d are:

$$a = 6 \text{ units}$$
$$b = 24 \text{ units}$$
$$c = 8 \text{ units}$$
$$d = 2 \text{ units}$$

Which of the following ratios are equal?

a) $\frac{a}{b}$ and $\frac{d}{c}$

b) $\frac{c}{b}$ and $\frac{d}{a}$

c) $\frac{a}{d}$ and $\frac{c}{b}$

d) $\frac{a}{b}$ and $\frac{c}{d}$

e) $\frac{b}{c}$ and $\frac{a}{d}$

f) $\frac{d}{b}$ and $\frac{a}{c}$

CLASS ACTIVITY

Use the grid paper at the back of this booklet to make a diagram of the entire floor plan on page 34, in the ratio of $\frac{2}{1}$.

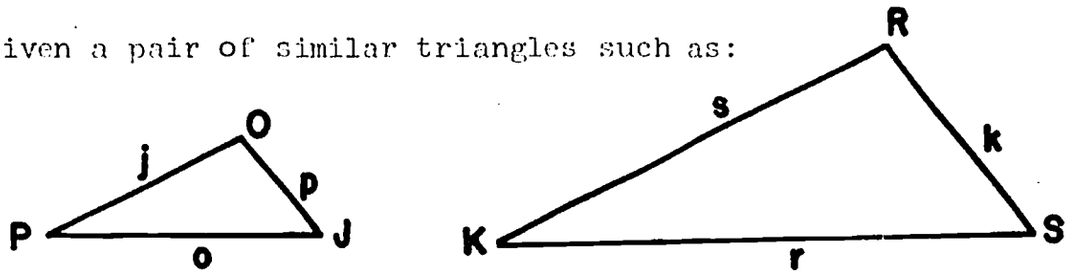
POSSIBLE ANSWERS TO DISCUSSION QUESTIONS

1. The ratios of the corresponding sides of similar figures are equal.

2. If the cross products are equal, the ratios are equal. If the ratios reduce to the same fraction, the ratios are equal.

OBJECTIVES

1. Given a pair of similar triangles such as:



the student will be able to select the correct statements from a list such as:

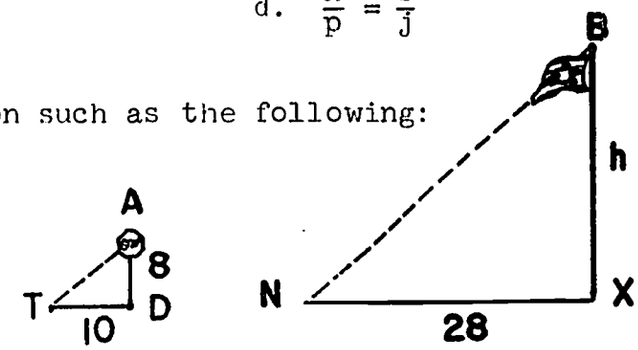
a. $\frac{r}{o} = \frac{s}{j} = \frac{k}{p}$

b. $\frac{o}{s} = \frac{j}{r} = \frac{p}{k}$

c. $\frac{r}{o} = \frac{j}{s}$

d. $\frac{k}{p} = \frac{s}{j}$

2. Given a situation such as the following:



A flagpole, whose height is not known, casts a shadow 28 feet in length; and a stop sign, 8 feet tall, casts a shadow 10 feet in length. In the illustration of the problem, two similar triangles have been drawn. The student will be able to select the correct statements from a list such as:

a. $\overline{TD}$ represents the length of the stop sign shadow.

b. $\overline{BX}$ represents the length of the flagpole's shadow.

c. the ratio $\frac{8}{10}$ is equal to $\frac{h}{28}$.

d. $\frac{m(\overline{TD})}{m(\overline{AD})} = \frac{m(\overline{NX})}{M(\overline{BX})}$

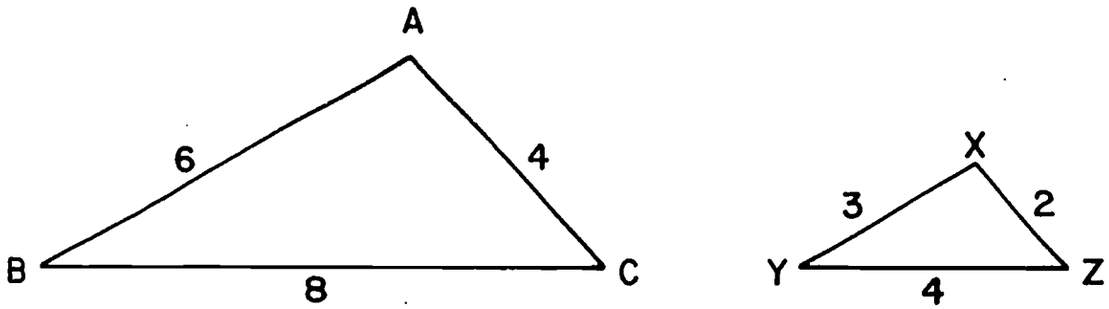
e. the ratio $\frac{10}{8}$ is equal to the ratio $\frac{h}{28}$

EQUAL RATIOS

The exercises in Lesson 7 are an illustration of the relationship between the corresponding sides of similar figures. This relationship can be stated:

THE CORRESPONDING SIDES OF SIMILAR FIGURES HAVE EQUAL RATIOS.

The measures (to the nearest centimeter) are given for each of the sides of the similar triangles below.



Compare the ratios of the corresponding sides.

$\overline{AB} \longleftrightarrow \overline{XY}$ RATIO $\dfrac{m(\overline{AB})}{m(\overline{XY})} = \dfrac{6}{3}$

$\overline{BC} \longleftrightarrow \overline{YZ}$ RATIO $\dfrac{m(\overline{BC})}{m(\overline{YZ})} = \dfrac{8}{4}$

$\overline{CA} \longleftrightarrow \overline{ZX}$ RATIO $\dfrac{m(\overline{CA})}{m(\overline{ZX})} = \dfrac{4}{2}$

CONTENT AND APPROACH

The last property of similar figures to be discussed in this book is stated on page 40. The properties which the students should be familiar with are listed in the chart below.

	SIMILAR FIGURES	CONGRUENT FIGURES
shape and size of figures	exactly the same shape	exactly the same shape and size
corresponding angles	congruent	congruent
corresponding sides	have equal ratios	congruent

THINGS TO DISCUSS

Omit the word "corresponding" in the statement "The corresponding sides of similar figures have equal ratios". Is the statement still valid? Use the figures on page 40 to illustrate your answer.

Is $\frac{6}{3}$ equal to $\frac{8}{4}$? **YES**

Is $\frac{m(\overline{AB})}{m(\overline{XY})} = \frac{m(\overline{BC})}{m(\overline{YZ})}$? **YES**

Is $\frac{8}{4}$ equal to $\frac{4}{2}$? **YES**

Is $\frac{m(\overline{BC})}{m(\overline{YZ})} = \frac{m(\overline{CA})}{m(\overline{ZX})}$? **YES**

Is $\frac{4}{2}$ equal to $\frac{6}{3}$? **YES**

Is $\frac{m(\overline{CA})}{m(\overline{ZX})} = \frac{m(\overline{AB})}{m(\overline{XY})}$? **YES**

Is $\frac{m(\overline{AB})}{m(\overline{XY})} = \frac{m(\overline{BC})}{m(\overline{YZ})} = \frac{m(\overline{CA})}{m(\overline{ZX})}$? **YES**

Notice that $\frac{m(\overline{BC})}{m(\overline{XY})}$ is <u>not</u> equal to $\frac{m(\overline{YZ})}{m(\overline{BC})}$. Can you explain why?

HINT! Why isn't $\frac{m(\overline{AB})}{m(\overline{XY})}$ equal to $\frac{m(\overline{XY})}{m(\overline{AB})}$?

When comparing ratios of corresponding sides of similar figures A and B, if the first ratio compares a side of figure A to a side of figure B, each other ratio must compare a side of figure A to a side of figure B.

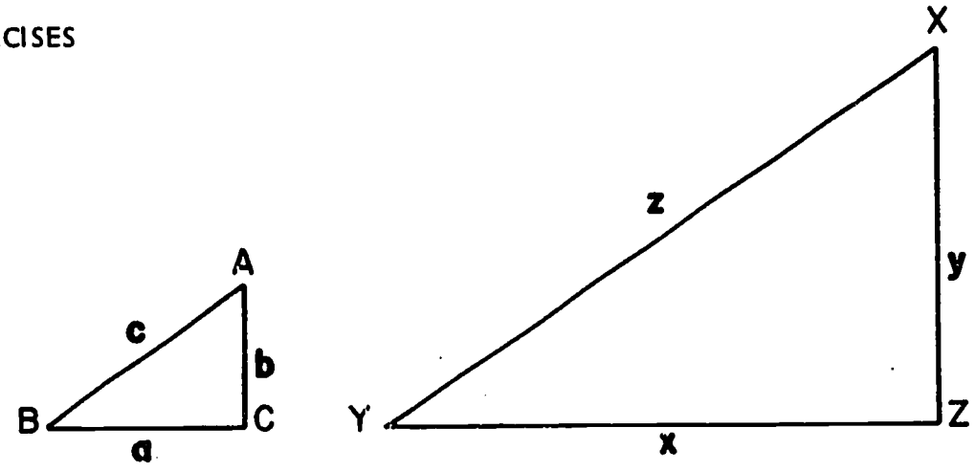
If the first ratio compares a side of figure B to a side of figure A, what must the other ratios compare?

CONTENT AND APPROACH

Draw a pair of similar figures, label them A and B. Use the figures to illustrate and clarify the rule for comparing ratios of corresponding sides of similar figures.

EXERCISES

1.



The sides of triangle ABC have lengths a, b and c; and the sides of triangle XYZ have lengths x, y and z. If the triangles are similar, which of the following statements are true?

a) $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$

b) $\frac{a}{z} = \frac{b}{y} = \frac{c}{x}$

c) $\frac{y}{b} = \frac{x}{a} = \frac{z}{c}$

d) $\frac{x}{b} = \frac{y}{a}$

e) $\frac{y}{b} = \frac{x}{a}$

f) $\frac{c}{z} = \frac{b}{y}$

If b = 5 and y = 12, then $\frac{b}{y} = \frac{5}{12}$. Name the other two ratios which would be equal to $\frac{5}{12}$. $\frac{a}{x}$ and $\frac{c}{z}$

If b = 5 and y = 12, name the ratios which would be equal to $\frac{12}{5}$. $\frac{y}{b}$, $\frac{z}{c}$ and $\frac{x}{a}$

CONTENT AND APPROACH

Assist those students having difficulty with pages 42 and 43 by asking:

1. "Is each ratio comparing corresponding sides?"
2. "Are the ratios constructed in the proper order?"

EXAMPLE - (using 1a. $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$)

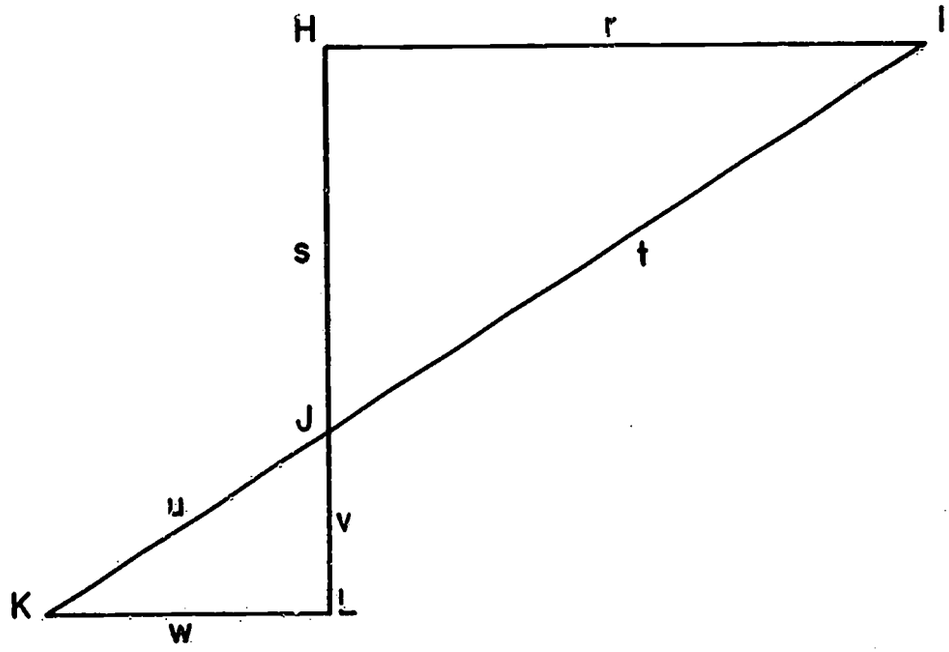
Does the ratio $\frac{a}{x}$ compare corresponding sides?

Does the ratio $\frac{b}{y}$ compare corresponding sides?

Does the ratio $\frac{c}{z}$ compare corresponding sides?

Are the three ratios constructed in agreement with the rule at the bottom of page 41?

2.



Two sides of triangle HIJ have lengths r, s and t; and the sides of triangle JKL have lengths u, v and w. The triangles are similar.

Name a similarity correspondence._____

Name the corresponding sides. _____

Which of the following statements are true?

a) $\frac{r}{u} = \frac{s}{v} = \frac{t}{w}$

b) $\frac{r}{w} = \frac{s}{v} = \frac{t}{u}$

c) $\frac{u}{r} = \frac{v}{t} = \frac{w}{s}$

d) $\frac{r}{w} = \frac{t}{u}$

e) $\frac{r}{u} = \frac{u}{t}$

f) $\frac{s}{v} = \frac{r}{w}$

CONTENT AND APPROACH

See page T-42.

ANSWERS

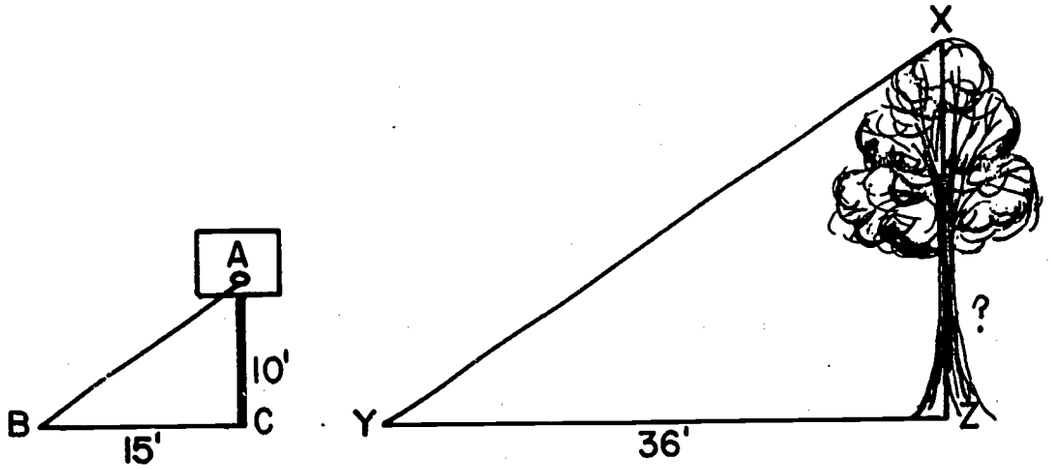
ANY OF THE FOLLOWING SIMILARITY CORRESPONDENCES ARE CORRECT

$\triangle HIJ \sim \triangle LKJ$
$\triangle HJI \sim \triangle LJK$
$\triangle IJH \sim \triangle KJL$
$\triangle IHJ \sim \triangle KLJ$
$\triangle JHI \sim \triangle JLK$
$\triangle JIH \sim \triangle JKL$

THE CORRESPONDING SIDES ARE

$\overline{HI} \longleftrightarrow \overline{LK}$
$\overline{IJ} \longleftrightarrow \overline{KJ}$
$\overline{JH} \longleftrightarrow \overline{JL}$

3.



A tree casts a shadow of 36 feet at a time when a nearby basketball hoop's shadow is 15 feet from the base of the supporting pole. By using the ratios of the measures of corresponding sides of similar triangles, it is possible to find the height of the tree.

In the illustration of the problem, two similar triangles have been drawn. Compare the triangles with those in exercise 1 on page 42.

Segment **XZ** represents the height of the tree.

Segment **AC** represents the height of the basketball hoop.

Segment **YZ** represents the length of the tree's shadow.

Segment **BC** represents the length of the hoop's shadow.

What does $\overline{AB}$ represent? **The distance from the basketball hoop to the end of the hoop's shadow.**

What does $\overline{XY}$ represent? **The distance from the top of the tree to the end of the tree's shadow.**

CONTENT AND APPROACH

Pages 44-47 present two applications of similar figures. A future booklet will be devoted to applications of this sort.

Be sure to compare the illustration on page 44 with the figures on page 42.

Why is the measure of $\overline{AC}$ equal to 10 feet? **THE REGULATION HEIGHT OF A BASKETBALL HOOP IS 10 FEET.**

Place an "h" beside the question mark to represent the height of the tree.

Name the corresponding sides of the triangles.

$\overline{AB} \longleftrightarrow \overline{XY}$

$\overline{BC} \longleftrightarrow \overline{YZ}$

$\overline{CA} \longleftrightarrow \overline{ZX}$

Since the triangles are similar, the following statement is true. Why? **THE CORRESPONDING SIDES OF SIMILAR FIGURES HAVE EQUAL RATIOS.**

$$\frac{m(\overline{BC})}{m(\overline{YZ})} = \frac{m(\overline{AC})}{m(\overline{XZ})}$$

Substitute and solve (can you explain each step?).

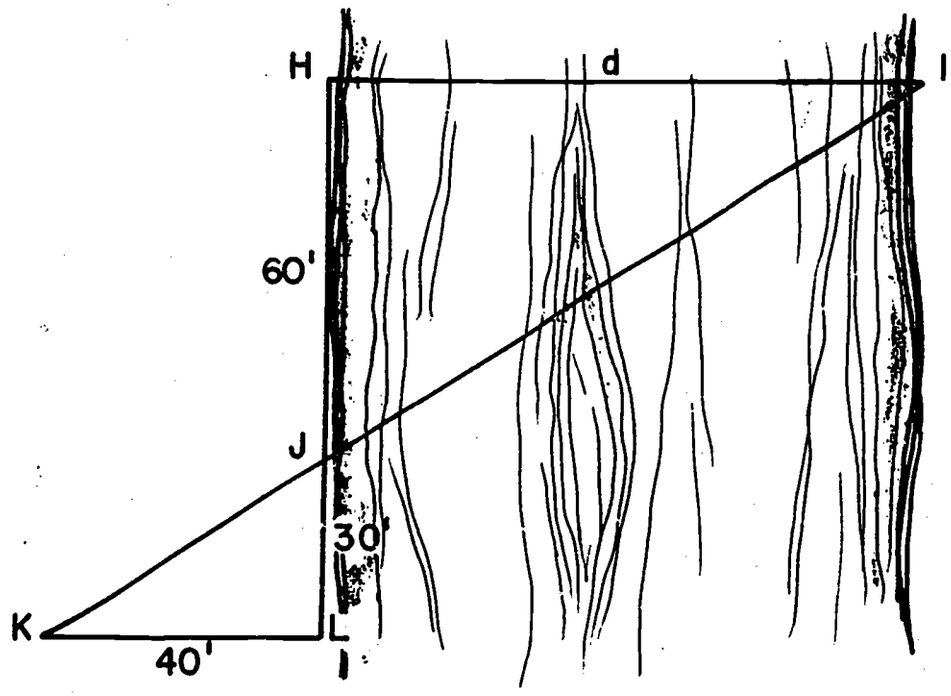
$\frac{15}{36} = \frac{10}{h}$ **SUBSTITUTE VALUES.**

$15h = 360$ **MULTIPLY BOTH SIDES BY 36h.**

$h = 24'$ **DIVIDE BOTH SIDES BY 15.**

DISCUSSION QUESTIONS-

1.



By using surveying instruments and the ratios of the corresponding sides of similar triangles, a team of civil engineers was able to determine the length of a bridge required to cross a river.

In the illustration of the problem above, triangle HIJ is similar to triangle LKJ (compare the illustration with the triangles in problem 2 on page 43).

Since the triangles are similar, the following statement is true. Why? **THE CORRESPONDING SIDES OF SIMILAR FIGURES HAVE EQUAL RATIOS.** $\frac{m(\overline{HI})}{m(\overline{LK})} = \frac{m(\overline{HJ})}{m(\overline{LJ})}$

Substitute and solve (can you explain each step)?

$\frac{d}{40} = \frac{60}{30}$ **SUBSTITUTE VALUES.**

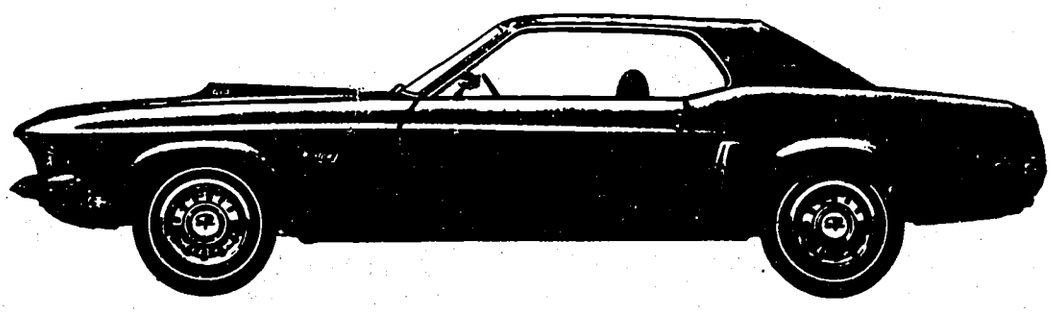
$30d = 2400$ **MULTIPLY BOTH SIDES BY 1200 (40 x 30)**

$d = 80'$ **DIVIDE BOTH SIDES BY 30.**

CONTENT AND APPROACH

See page T-44. Be sure to compare the illustration on page 46 with the figures on page 43.

2.



The length of the Mustang pictured above is $5\frac{3}{4}$ inches. Ford Motor Company reports that the length of an actual Mustang is 187 inches. The ratio of the length of the car in the picture of the length of an actual car is $\frac{5\frac{3}{4}}{187}$.

$\frac{5\frac{3}{4}}{187}$ is approximately equal to $\frac{1}{32}$.

We write: $\frac{5\frac{3}{4}}{187} \approx \frac{1}{32}$.

a. If the ratio of the length of the picture to the actual car is $\frac{1}{32}$, would the ratio of the heights also be $\frac{1}{32}$? **YES**
Would the ratio of the lengths of the doors be $\frac{1}{32}$? **YES**
Name several other items whose ratios would be $\frac{1}{32}$. **WHEELS, FENDERS, ETC.**

b. Would the ratio of the length of the car in the picture to the height of an actual car be $\frac{1}{32}$? **NO** Why? **NOT CORRESPONDING PARTS.**

c. "The corresponding sides of similar figures have equal ratios". Explain how this statement could be used to verify the correct answers to questions a and b.