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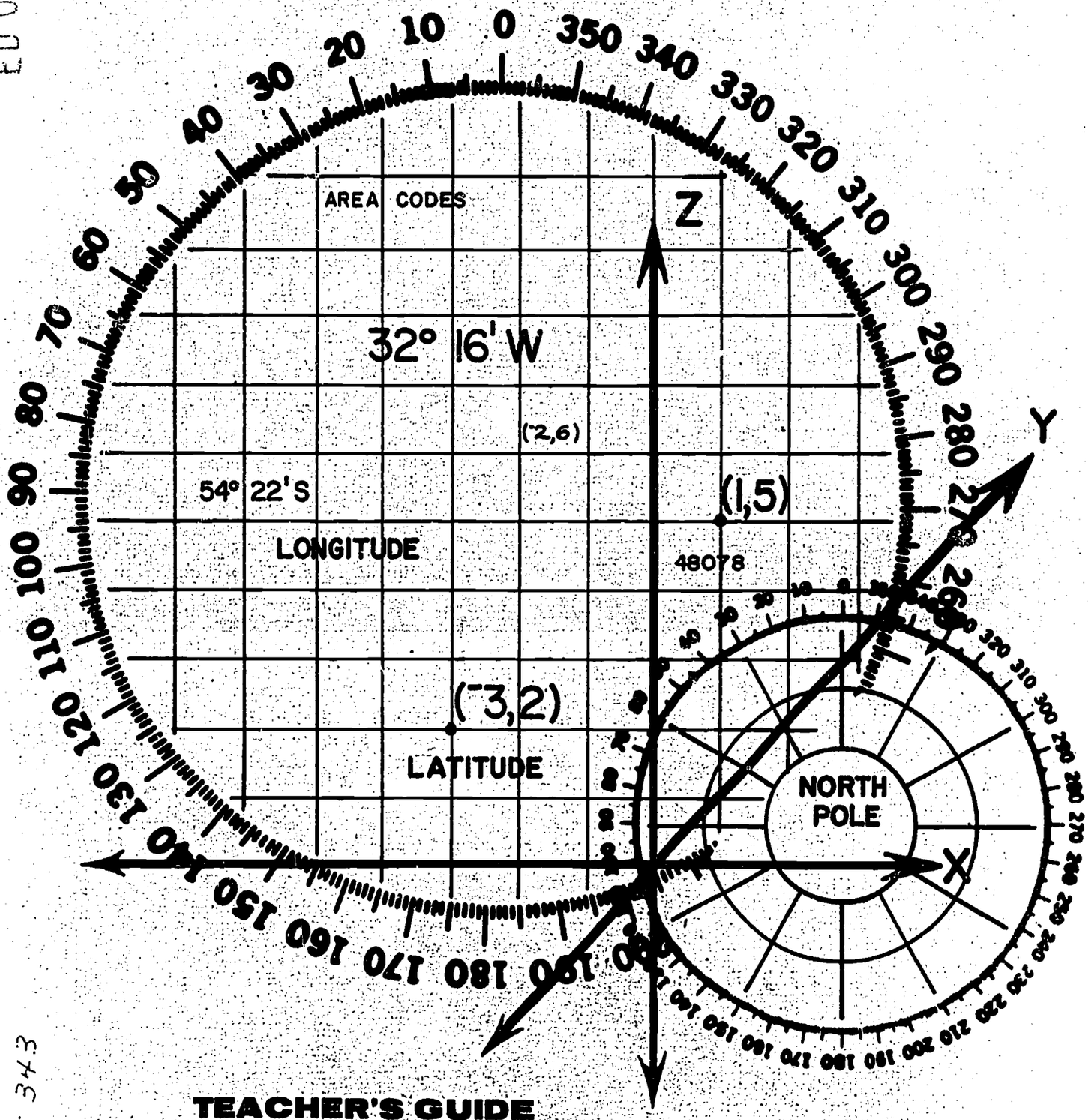
ABSTRACT

This guide accompanies "Where is the Point?"; it contains all of the student materials in SE 015 342 plus supplemental teacher materials. With each lesson there is a list of objectives and equipment and teaching aids, suggested approaches, discussion questions, and answers. Appendices include transparency masters and supplemental activities. Related documents are SE 015 334 - SE 015 342 and SE 015 344 - SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

WHERE IS THE POINT?



TEACHER'S GUIDE

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WHERE IS THE POINT?

OAKLAND COUNTY MATHEMATICS PROJECT

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2100 Pontiac Lake Road
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PREFACE

Essentially, a coordinate system is a scheme used to establish a correspondence between a set of numbers and a set of points.

The need for an orderly plan to describe the position of objects (e.g., points, regions on a map, parts on an assembly line, or satellites in space) is basic to science, technology and many areas of everyday life.

The study of coordinate systems provides the student with necessary background for the later development of some basic algebraic and geometric concepts. Coordinate systems enable one to relate the algebra of numbers to the geometry of sets of points.

The student will study a variety of coordinate systems. The main emphasis, however, is on the rectangular coordinate system in the plane. The information in Table I indicates this emphasis.

TABLE I

Type of Coordinate System	Lesson Number
Rectangular coordinates in the plane	1, 2, 7, 8, 9, 10, 11
Rectangular coordinates in space	5, 11
Line coordinates	11
Polar coordinates (informal)	4
Oblique axes in the plane	9
Global or geographic coordinates	3
Spherical coordinates (informal)	11
Cylindrical coordinates (indirectly)	11

OBJECTIVES

The objectives for each lesson are listed separately lesson by lesson. Do not limit your teaching to these objectives, but rather use them as a guide to the intent of the lessons. Discuss the objectives for each lesson with your students. Let them know what they are expected to be able to do.

The study of coordinate systems involves the student in interesting mathematical situations. We feel that he will develop and practice basic problem solving and critical thinking skills. Specific behavioral objectives relative to problem solving or critical thinking are difficult to formulate, however, and are not spelled out in this booklet.

The main purpose of this booklet is for the student to be able to answer the question: "What is a coordinate system?" Lesson 11 is the key lesson in terms of evaluating the student's success in achieving this purpose.

Subsumed under the above purpose is the ability to construct and use the familiar rectangular coordinate system in the plane. Specific terminal objectives related to this purpose are:

1. The student will be able to draw the axes for a rectangular coordinate system.

2. Given a table of ordered pairs of rational numbers to plot, the student will determine an appropriate unit distance for each axis.

3. Given a pair of axes properly scaled and labeled, the student will be able to plot the point (a,b) where a and b are rational numbers.

4. Given a pair of axes properly scaled and labeled, with a point indicated, the student will be able to describe its coordinates.

- The student will be able to determine the [illegible] in certain rectangular coordinate planes [illegible] given a description of the coordinates of the points in that region. For example, given a description like: All points (x,y) where $x < 1$ and $x > -1$ and $y \geq -2$ and $y \leq 4$: the student will respond by shading in the proper region.

BASIC ASSUMPTIONS

Very little computation is necessary for the completion of this booklet. However, regarding understandings relative to the rational number line, it is assumed that [illegible] will be a review for the student.

Lessons 3,4 and Part 4 of Lesson 11 involve angle measure. Lesson 4 also involves the use of a ruler. It is assumed that the student can: (1) use a full circle protractor-reading it to the nearest degree: (2) convert degree measure to minutes and vice versa: and (3) read a ruler to the nearest $\frac{1}{8}$ inch.

BOOKLET ORGANIZATION

Material in the Teacher's Guide is arranged, for each lesson, under the headings: Objectives, Equipment and Teaching Aids, Content and Approach, Things To Discuss, and Activities.

The information under Content and Approach is to give the teacher assistance in interpreting the emphasis and direction of the lesson. The questions and comments listed under Things To Discuss are meant to be suggestive of the type of class discussion which would be helpful in clarifying and extending the lesson.

In addition to a lesson by lesson description of equipment and teaching aids, a summary list is given on pages vi and vii.

Answers are printed in proximity to the exercises. Exercises denote those problems that are for supervised study in the classroom. Portions of these problems can be used as brief homework assignments.

The exercises labeled ✓POINT, are for use by the student in evaluating his progress as he studies the material.

Power Questions are found at the end of most lessons. These questions generally require more problem solving ability than the typical exercise.

Appendix C. contains supplementary activities which the teacher may wish to utilize.

EQUIPMENT AND TEACHING AIDS

A. STUDENT		LESSON
*1.	Official Highway Map of Oakland County. One for each 4-5 students. (See Appendix A.)	1
2.	Yardstick. One for each 3-4 students.	1
*3.	Highway Map of Michigan. One for every 4-5 students.	4
4.	Full circle protractor. (See Appendix B.)	4,11
5.	Foot ruler (transparent plastic if possible).	4
*6.	Model of the first Octant of the three-dimensional rectangular coordinate system.	5,11

*Indicates those materials provided with the booklets.

7.	Ruler or compass.	6,7,8,9,10,11
8.	Lattice paper. (See Appendix B.)	7,8,9,11
9.	Rectangular coordinate paper (See Appendix B.)	8,9,10,11
10.	Physical models: cubes, cylinders, cones and spheres. Classroom demonstration size. A variety for each 3-4 students.	11
11.	Flexible rulers (plastic or cloth tape). One for every 3-4 students.	11
12.	Construction paper, tape, scissors, etc., depending on the extent of display work done.	11

B. TEACHER

*1.	Prepared transparencies. (In cases where the transparencies are not provided the transparency masters are given in Appendix B.)	1-11
2.	Overhead projector.	1-11

* Indicates those materials provided with the booklet.

NOTE: The Teacher's Guide for each lesson describes the transparencies to be used in that lesson.

The information in Table II is given to help the teacher focus on those lessons which are essential to the development of the main ideas in the booklet.

TABLE II Proposed Teaching Schedule

LESSON NUMBER		Main Idea	Comment	TEACHING DAYS
1.	Part I	Basic notion of location via a symbol-region correspondence.	Essential	1
	Part II	Extension of Part I	Essential	$\frac{1}{2}$
	Part III	Grid resolution.	Optional	$\frac{1}{2}$
2.	Part I	Coordinate systems in everyday life.	Essential	2
	Part II	Ordered pairs of whole numbers and axes.		
3.		An everyday coordinate scheme which can be contrasted with the rectangular coordinate system.	Not Essential	2
4.	Parts I, II	A type of polar coordinate system and an application.	Not Essential	2
	Secret Code	A supplementary activity	Optional	1
5.		3-Dimensional rectangular coordinates.	Essential	3
6.		Order and position on the rational number line.	Essential	1
7.		Plotting ordered pairs of integers.	Essential	$2\frac{1}{2}$
8.		Plotting ordered pairs of rational numbers.	Essential	1
9.		Graph distortion.	Essential	2
10.	Parts I, II	Associating regions in the plane with inequality statements.	Essential	3
11.	Part I	Extending ordered triples into all octants.	Essential	1
		Line coordinates.		
	Part II	Space coordinates with angles.	Optional	1
	Part III	Applying knowledge of coordinatization.	Essential	2
			Total	$25\frac{1}{2}$

*Not essential means cover if time allows but do not attempt to teach for mastery.

TABLE OF CONTENTS

MAP COORDINATES

Part I Where is Graham Lake?

The map that you are using in this lesson is the Official Highway Map of Oakland County.

The map shows lakes, cities, highways, airports, parks and many places of interest. Each place has a location on the map. A location is given by using the letters and numbers on the sides of the map.

The location of Graham Lake is Z-24. Find this on your map.

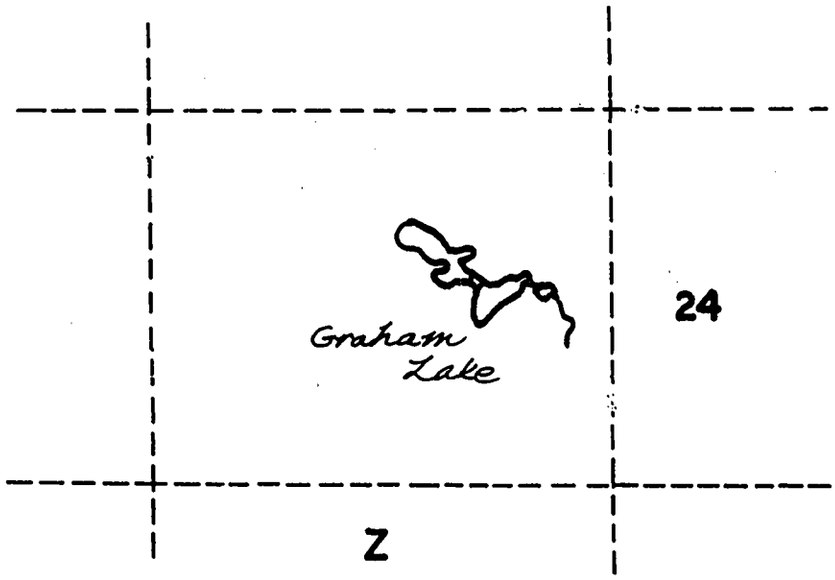


Figure 1

EXERCISES

1-3. A location is given in the right hand column below. Give the name of the lake, city or major intersection you find at that location.

	PLACE	LOCATION
1.	_____	O-9
2.	_____	W-4
3.	_____	BB-29

MAP COORDINATES

OBJECTIVES

1. To introduce the concept of coordination by developing the idea of region to symbol correspondence.

2. The student shall be able to:
   a. locate a region identified by a pair of map coordinates.
   b. determine the coordinates of a given region on the map.

EQUIPMENT AND TEACHING AIDS

The map to be used with this lesson is the Official Highway Map of Oakland County, Michigan. It is issued by the Oakland County Road Commission. A yardstick is helpful in aligning some of the map locations. (See Appendix A.)

ANSWERS TO EXERCISES

PART I

1. Woodpecker Lake
2. Lathrup Village
3. Leonard

4-9 Find the location of the places listed in the left hand column.

PLACE	LOCATION
Example 1. Clarkston	N-21
4. Parker Lake (Oxford Township)	_____
5. Square Lake	_____
6. I-75, M-59 Intersection	_____
7. Pontiac Municipal Airport	_____
8. Your house	_____
9. Your school	_____

Figure 2 shows a grid. A grid has vertical lines which form columns and horizontal lines which forms rows. The rows and columns intersect to form cells. If the rows and columns are labeled with letters or numbers, then each cell can be named.

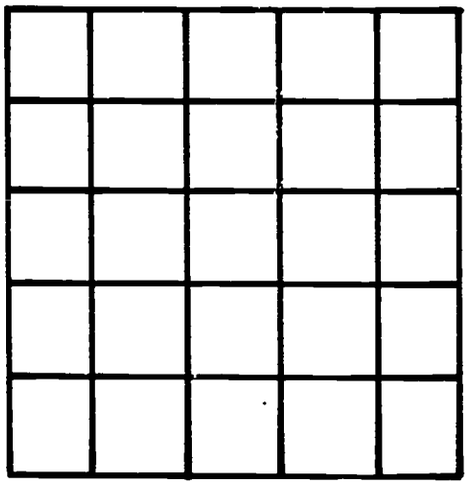


Figure 2

ANSWERS CONTINUED

A. W-[illegible]
[illegible]. S-1 [illegible] U-[illegible]
[illegible]. W-[illegible]
[illegible]. N-16
[illegible]. Answers will vary from student to student.
[illegible]. Answers will vary from student to student.

TRANSPARENCIES

1-1. A tourist type map of Washington, D.C. (An optional transparency. See Appendix A.)

Purpose

A backup procedure in case the county map is not available.

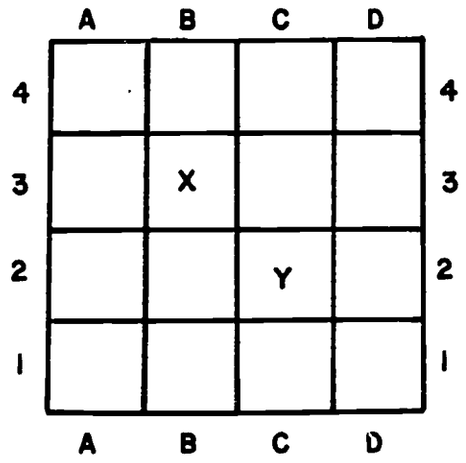
Project the map on the chalkboard so that grid lines and coordinates can be chalked in.

1-[illegible]. A single 6" by 6" grid containing 36 s[illegible] [illegible].

PURPOSES

1. For Lesson 1 illustrating the township section numbering.
2. For the Secret Code Exercises following Lesson 4.
3. For the seating chart illustration in Lesson 2.

Figure 3



EXAMPLE : The X, in Figure 3, is in cell B-3.

10. Give the location of the cell containing the letter Y.

11. Print the letter W in cell A-1.

Vertical and horizontal lines are not drawn on the Oakland County map, but letters and numbers are used to locate cells just as in the grid shown in Figure 3.

12. The numbers are printed in a North-South Line. The letters are printed in a (an) ____?____ line.

The letters and numbers used to give locations are called <u>coordinates</u>. W-4 are the <u>coordinates</u> of a certain section in Southfield Township.

The letter(s) is the <u>East-West coordinate</u> and the number is <u>the North-South coordinate</u>.

By using coordinates we can give another person the location of any object on the map and he can find the position of that object.

ANSWERS CONTINUED

10. C-2
11. (Check student's answer on Figure 3, page 1.)
12. East-West

CONTENT AND APPROACH

It is suggested that the students be allowed to work in small groups of from three to four students each. This cuts down on the required number of maps and the students can help each other locate coordinates.

Map location provides an excellent springboard for the concept of coordinatization. The main purpose of the lesson is to introduce the basic notion of a region to symbol correspondence and how this correspondence is useful.

The term coordinate is introduced in this lesson, but it should not be mastered. The question of arbitrariness or convention in giving coordinates is raised in Lesson 2.

Draw the students' attention to the fact that it takes two coordinates to "fix" or locate a position. Compare the locating of a place on the map to two lines intersecting at a point.

In Part 2, the student should discover that there is more than one coordinate scheme possible on the map. The exercises are arranged so that he will see how township name-section number coordinates could be used.

Part 3 is an optional section. An attempt is made to show the student how grid size is related to resolution. Have the students suggest how they may use finer or coarser grids to obtain greater or lesser degrees of resolution.

The first coordinate tells us how far to go horizontally on the map. The second coordinate tells us how far to go vertically on the map.

✓ **POINT**

Which coordinate, the East-West or the North-South, tells how many units to move horizontally?

PART 2 TOWNSHIP–SECTION NUMBER

A township is a square parcel of land six miles on a side. Each township is divided into thirty-six sections.

EXERCISES

1. Give the name of a city in Avon Township.
2. What township do you live in?
3. Each township is how long on a side?
4. The area of each section is ___?___ .
5. The mile roads run East-West, in Oakland County, starting with ___?___ road at the southern end of the County.

✓ **POINT ANSWER**

The East-West coordinate.

THINGS TO DISCUSS

1. Does each region, on the map, have a symbol which corresponds to it?

2. Township name-section number as an alternate location scheme. Class discussion should bring out advantages and disadvantages of each scheme.

   EXAMPLE A disadvantage of the township-section scheme is that there is no referent for the first coordinate. One has to hunt for the township name.

3. The implied grid. Each cell of the grid corresponds to one square mile. For most map uses this degree of resolution is acceptable. How do you achieve finer resolution?

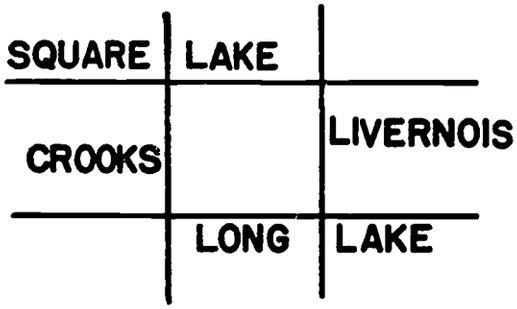
   EXAMPLE Suppose that the scale on the map [illegible] = 1 mile. Each pair of map coordinates [illegible] one square mile. If a transparent grid [illegible] four squares per inch was used as an overlay, then an area $\frac{1}{16}$ of a square mile could be located.

   Have the students make suggestions as to [illegible] newly created coordinates.

PART 2 ANSWERS

1. Answers will vary from student to student.
2. Answers will vary from student to student.
3. Six miles.
4. One square mile.
5. Eight MILE.

--F. Give the location of each section by giving the map coordinates of the section and the name of the township that the section is in.

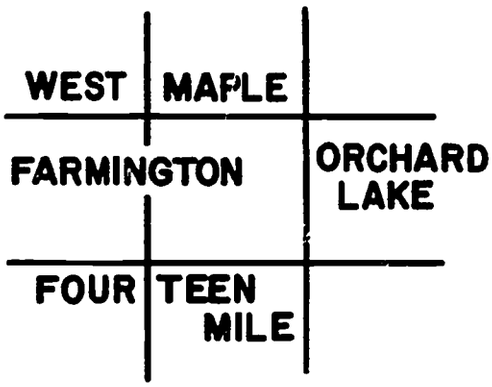


MAP COORDINATES

_____

NAME OF TOWNSHIP

_____

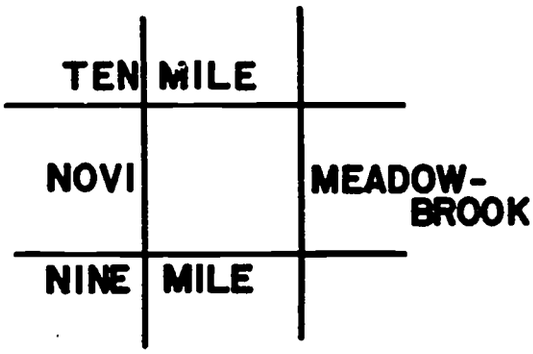


MAP COORDINATES

_____

NAME OF TOWNSHIP

_____



MAP COORDINATES

_____

NAME OF TOWNSHIP

_____

Exercises 6-17 are designed to enable the student to see how township name, Section Number coordinates could be used as an alternate map location scheme.

ANSWERS CONTINUED

6.	AA-11	Troy
7.	P-7	West Bloomfield
8.	K-2	Novi

What are numbers like those shown in Figure 4 used for? Look at the legend on the map.

9	10
16	15

Figure 4

9. Township sections are numbered in a special way. Complete the numbering of each section in Figure 5.

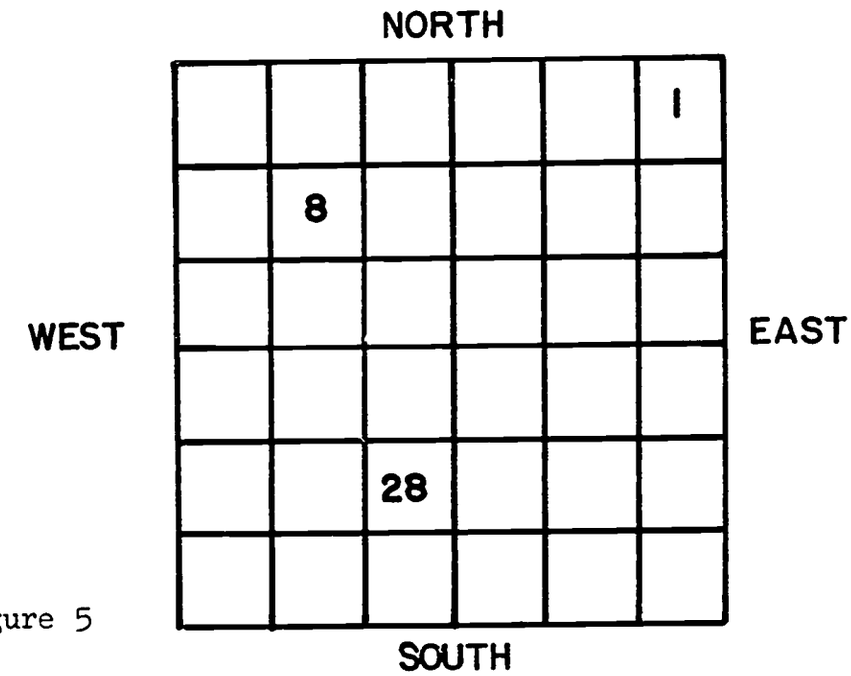
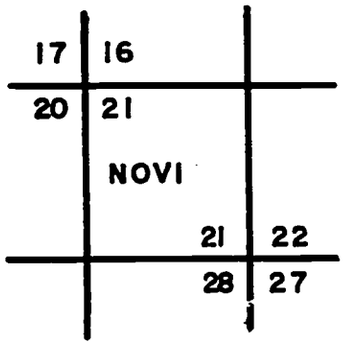


Figure 5

10.



MAP COORDINATES

_____

NAME OF TOWNSHIP

_____

ANSWERS CONTINUED

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

10. 1-4 Devi

11. Find section 9 in Troy Township. Give its map coordinates.

12. How many miles is section Y-9 from Y-2?

13-16. Complete the table in Figure 6 so that each township section corresponds to its proper location.

	Township name and Section number	Map Location
13.	Pontiac-9	
14.		V-10
15.	Waterford-21	
16.		F-1

Figure 6

Township names and section numbers can also be used to give map locations. The township name acts as one coordinate and the section number acts as the other coordinate.

17. In giving the location of Childs Lake you now have two choices. You can give its coordinates as

a. _____ (Letter(s), Number) Or

b. _____ (Township Name, Section Number)

Do you see any advantages in using township names and section numbers for locations? Any disadvantages?

ANSWERS CONTINUED

11. AA-11
12. Seven miles
13. U-17
14. Bloomfield-15
15. O-15
16. Lyon-36
17. F-7 or Milford-36

Part 3 (Optional) Grid Size

Many places on the map (especially cities, large lakes and parks) do not lie entirely within the boundaries of any one section. This is not a real disadvantage if you are just interested in general location.

EXAMPLE: Your friend asks, "Where is South Lyon?" Would you say it's at A-2, B-2, A-3, or B-3? Your friend should be able to find South Lyon by using any one of these four locations.

Each location like BB-17 names a section on the map. Each section measures one mile on a side. Thus, each symbol like BB-17 or F-7, etc., locates one square mile. Think of a way to locate areas on the map which are smaller than one square mile.

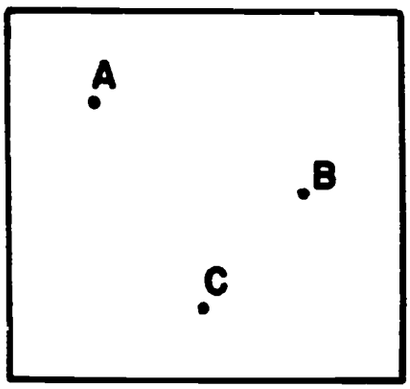


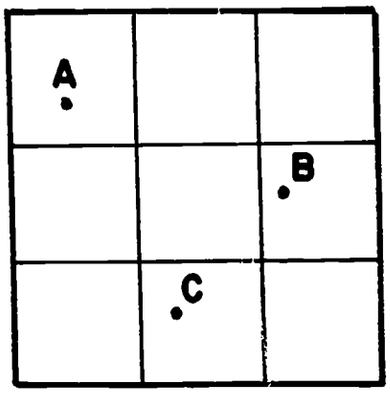
Figure 7

EXERCISES

1. Let points A, B, and C, in Figure 7, represent places in one of the square mile township sections. Draw a grid over Figure 7 so that each point is in a separate cell.

PART 3 ANSWERS

1. Answers will vary from student to student.
   A possible answer is a 3 by 3 grid.



2. Think of a way to label the rows and columns on the grid shown in Figure 8. Use your labels to give the location of the corner of Seventh and Ludlow.

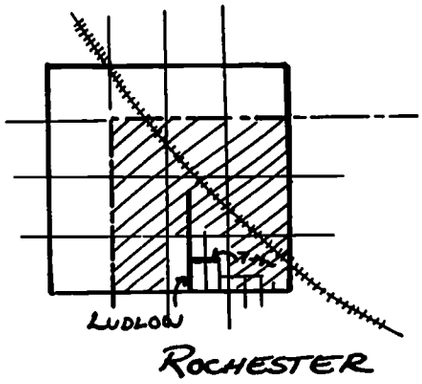


Figure 8

3. What is the area, in square miles, of each cell shown in Figure 8?

ANSWERS CONTINUED

2. answers will vary from student to student.
3. $\frac{1}{16}$ of a square mile.

COORDINATE SYSTEMS

Part I Knowing the label can you locate the object?

In Lesson 1, an object was located on the map by using its map coordinates.

Different types of coordinate systems are used in many places every day.

A coordinate system assigns a label to an object. Knowing the label you can locate the object that is named by the label.

A ticket to a football game gives the coordinates of a seat in the stadium.

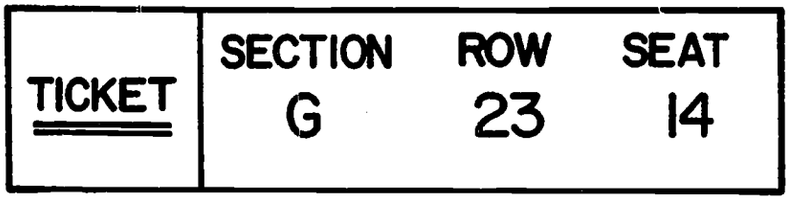


Figure 1

Knowing the coordinates, G-23-14, you can locate the seat.

A shopping center parking lot uses a coordinate system as an aid in locating your car.

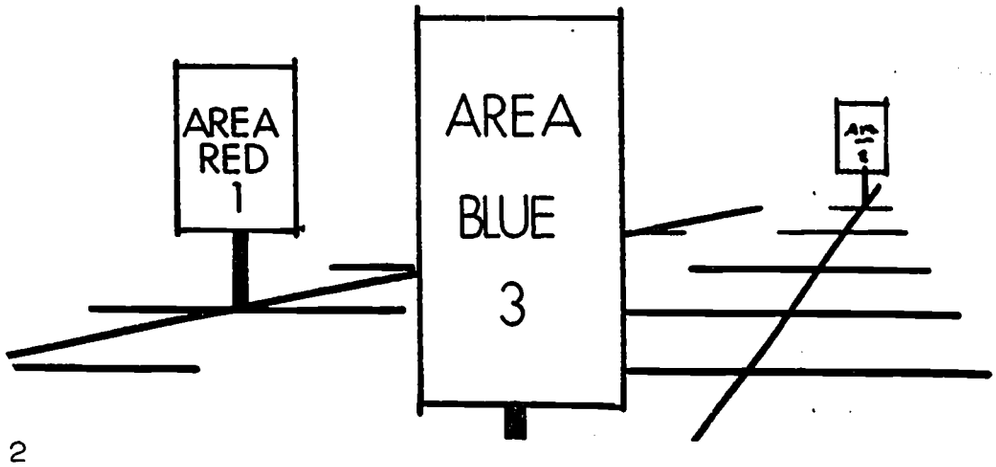


Figure 2

COORDINATE SYSTEMS

OBJECTIVES

1. To review and extend Lesson 1. The student will be able to describe various numbering of numeral-letter combination schemes which are used for location purposes.

2. The student is given the problem of improving a coat checking system. He is to apply the criterion question: knowing the label can I locate the object that is named by the label?

3. The student will discuss the coordinatization of a seating chart using the terms origin, axes, axis and coordinate(s).

   These terms are used in an introductory way and it is not necessary to master them at this time.

4. The student understands the need for an agreement on the order used in giving coordinates. He realizes, for example, that the object located at (6,2) is not the same as the object located at (2,6).

EQUIPMENT AND TEACHING AIDS

TRANSPARENCIES (See Appendix B)

2-1. A single 6" by 6" grid containing 36 sq. inch cells. (Also used in Lesson I)

PURPOSE

1. For the seating chart illustration.
2. For the Secret Code Exercises following Lesson 4.

2-2. Zip code

2-3. Centerville City (Streets, Avenues and house numbers)

2-4. Parking lot

PURPOSE

1. To generate class discussion about various coordinate schemes.

The parking lot shown in Figure 2 is divided into sections and each section is divided into rows.

The coordinates Red-G pinpoint a certain row in a certain section. How would you improve this coordinate system so that the coordinates would locate an individual parking space in a row?

Zip code is a five-digit coordinate system that locates areas within the United States and its possessions for purposes of simplifying the distribution of mail.

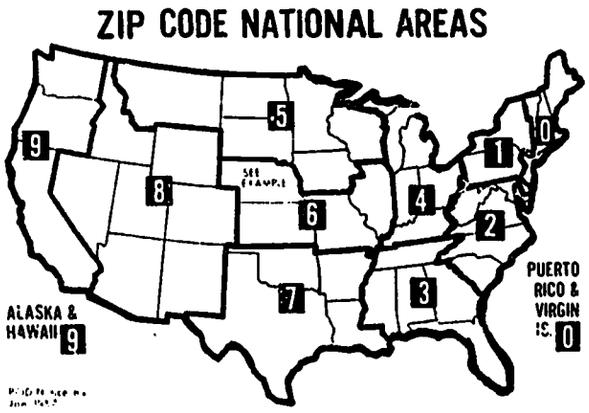


Figure 3

In the Zip code, the United States and its possessions are divided up into 10 large geographic areas. Each area consists of three or more states or possessions and is given a number between 0 and 9, inclusive. What number area is Michigan in?

Each of these large Zip code areas is divided into two or more sub-areas.

Together, the first three digits of any Zip code number stand for either a particular Sectional Center or a metropolitan city. (See Figure 4.)

CONTENT AND APPROACH

The students discuss various common everyday location schemes, e.g. football tickets, parking lot signs and zip codes. They are then asked to design an improved coat checking system.

The principal notion in Part One of this lesson is that of location. An object is labeled with a symbol. This symbol may only identify the object, but it can also serve to locate or fix the object's position.

In many instances it is desirable that the correspondence between the objects and their labels be one-to-one. It should not be inferred, however, that such a correspondence is necessary.

The parking lot coordinate system could be improved by numbering the individual parking stalls. Then the coordinates Red-G-26 would pinpoint the exact location of a particular car. The students will argue that this need not be done because if a person remembers Red-G, they can usually see their car when they get to this area. The point is, however, the relationship between the coordinates and the location. That is, how the coordinates "pin point" or "fix" the exact location of an object.

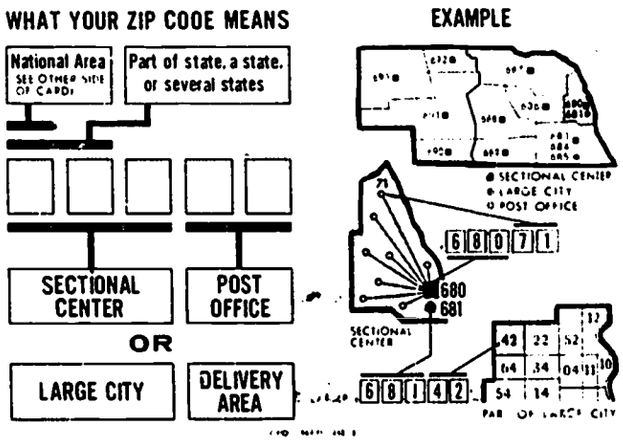


Figure 4

The last two digits stand for a post office or a delivery area. What is your Zip code? Do you think a letter could be delivered that has just a name and the Zip code for an address? Why?

DISCUSSION QUESTIONS

1. Explain how each of the following are used as a coordinate system. What is located? How is it located?
   a. House numbers in a city
   b. Room numbers in a hotel
   c. Dewey decimal system (in the library)
   d. Ticket to a play
   e. Latitude and longitude on a world map
2. Can you suggest other types of coordinate systems?

DISCUSSION QUESTIONS ANSWERS

1.

a. In most American cities the house numbers are arranged in such a way that:

(1) They are even on one side of the street and odd on the other side.

(2) They increase in magnitudes of 100 for each block.

(3) They increase in size as one is leaving the center of the city.

Of course, there are all kinds of exceptions, but overall, house numbers can be thought of as a coordinate scheme.

Using the "Centerville City" transparency we see how house numbers combine with a street - avenue rectangular grid to produce a nice coordinate system.

The address: 252 Second Avenue N.E. tells you that the house is in the third block north of First Street and east of First Avenue and on the right hand side of the avenue going north.

b. Room numbers in a hotel usually correspond to the floor number that the room is on.

Students may be encouraged to suggest schemes which would better coordinatize hotel rooms.

c. Have the students discuss how books are classified and located in their library.

d. Section Number, Row Number, Seat Number is a typical scheme used in large theaters or stadiums.

e. Many students will remember latitude and longitude from geography class. If they do not remember, then postpone this discussion until Lesson 3.

EXERCISES

Scott Wright has been chosen to be chairman of the coatroom committee for the Annual Junior Senior Dance. The chief part of his job will be to organize a way to check coats in and out.

Scott wants to make it easy to locate and return a coat when the check stub is turned in.

The coatroom will be in Mr. Rolfe's classroom.

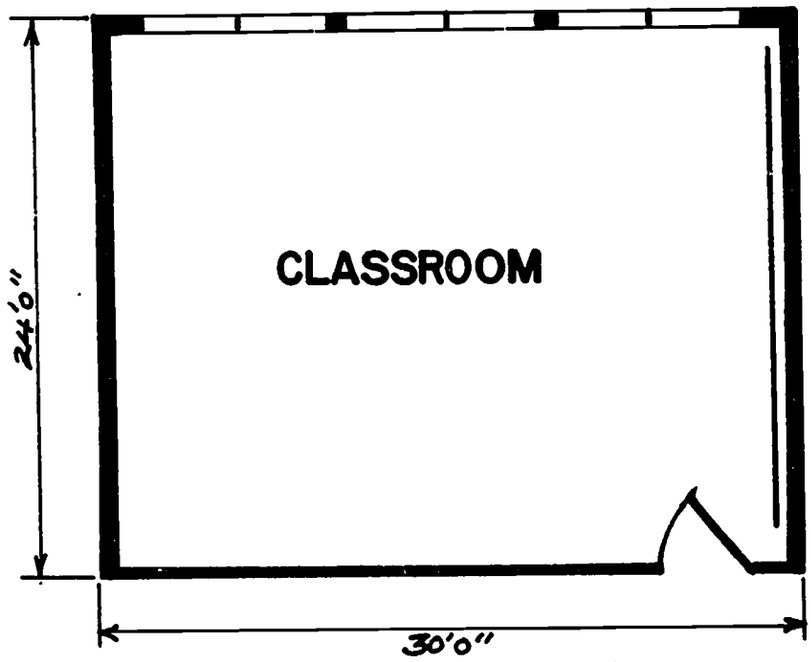


Figure 5

Nine coatracks are to be used.

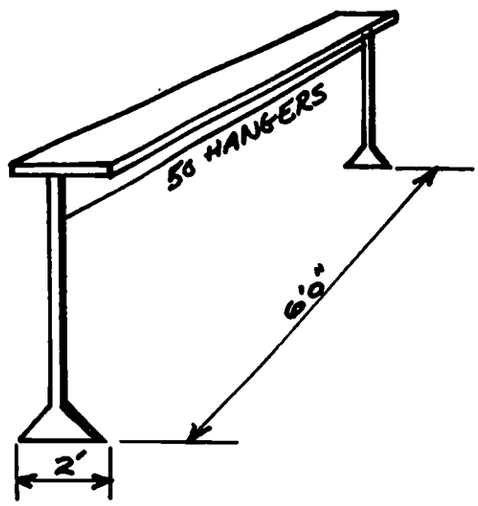


Figure 6

2. Ask the students to suggest labels which are used for location purposes. The criterion is: knowing the label can I locate the object that is named by the label? You will get responses such as:

1. Coat checks
2. Zip code
3. Parking lot signs
4. Automobile license numbers
5. Telephone number
6. Social Security number

Arguments will arise over some of these suggestions. For example, knowing a car's license number may not help you locate the car. It should, however, help you locate the name and address of the owner.

The general idea of a coordinate scheme should be made clear: knowing the scheme one can locate the object's position, using the label assigned to the object, and determine the label given the object's location.

Some students may be familiar with the Tornado Plotting Map shown on page 37. Others may know something about numerically controlled machine tools like the one illustrated on page 78.

One of the purposes of this discussion is to impress the students with the general nature of coordinate systems and their utility.

NOTE: See Items 3 and 4 in Appendix C for supplementary activities.

Help Scott think of a smooth way to check coats in and out of the coatroom.

1. Place the coatracks in Mr. Rolfe's room. Use your own design. Show your answer by drawing the coatracks, on the picture in Figure 5.
2. The numbering used on the coatchecks should be part of your coordinate system for locating the coats. Show how you would label the coatchecks. Answer by labeling a sample ticket below.

NUMBER _____	ANNUAL JUNIOR SENIOR WINTER DANCE coat check	NUMBER _____

3. Mark an "x" on your design in Figure 5 to show where the coat is located that goes with your answer to Exercise 2.
4. Compare your coordinate system with others in your class. Can there be more than one answer?

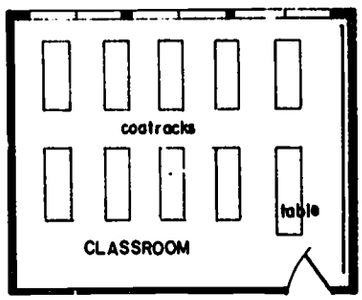
Part 2 Seating Chart Numbers

(Your teacher will handle this part of the lesson using an overhead transparency of the seating chart for your class.)

The coat checking exercise is an open ended problem. The students are expected to design a coordinate system which will involve the placement of the coatracks in Mr. Rolfe's classroom and the corresponding labels on the coatchecks.

ANSWERS TO EXERCISES

1. A possible answer.



Check to see if the students used the dimensions of the classroom and the coatracks, allowing for adequate aisle space and the width and length of the racks.

2. Answers will vary. The numbering on the coatracks should governed by an overall coordinate scheme.

3. Answer depends on the students' answer to exercises one and two above.

4. Have the students display their systems.

EXERCISES

Sandra	David	Al	Judy M.	Fred	Harry
John		June	Phil	Mark	Lynn
Mike B.	Donna	Steve	Diane		Lonnie
Evelyn	Bill		Dan	Judy R.	Roy
Martha	Carol	Debbie	Jean	Terry	Bob
Peter	Mary	Stu	Phyllis	Ginny	Mike W.

The following exercises use the seating chart shown above.

1. Jean's seating chart coordinates are (3,1). Her row coordinate is 3. One is her ? coordinate.
2. Peter is sitting at the origin of the seating chart coordinate system. What are his coordinates?
3. Everyone sitting on the seat axis has what number for their row coordinate?
4. Everyone sitting on the row axis has what number for their seat coordinate?
5. Give Steve's coordinates.
6. Who is sitting at (3,2)?
7. Do the coordinates (3,2) locate the same position as the coordinates (2,3)?

PART 2 SEATING CHART NUMBERS

Having discussed some basic principles of location, the class is now ready to apply them and obtain their seat coordinates.

Use an overhead transparency of your seating chart.

(Sample chart used on page 15)

S ↑

5	Sandra	David	Al	Judy M.	Fred	Harry
4	John		June	Phil	Mark	Lynn
3	Mike B.	Donna	Steve	Diane		Lonnie
2	Evelyn	Bill		Dan	Judy R.	Roy
1	Martha	Carol	Debbie	Jean	Terry	Bob
0	Peter	Mary	Stu	Phyllis	Ginny	Mike W.
	0	1	2	3	4	5

→ R

Discuss some of the disadvantages of numbering the seats from one to thirty-six. For example, the fact that Diane sits in seat number 21 does not pinpoint her seat quickly.

The speed with which one is able to locate a point is a measure of the efficiency of the coordinate system. Use the notation (R,S). Tell Peter that his number is (0,0). Ask Stu what he thinks his number is. He might answer (0,2). Tell him he is close and see if he changes it to (2,0).

Have the class consider the alternate notation (S,R). They should readily see that some convention as to order is needed.

Define the terms row coordinate, seat coordinate, row axis, seat axis, and origin. These terms will only be used in a general introductory way, so do not over-emphasize them.

ANSWERS TO EXERCISES

1. Seat, 2. (0,0), 3. Zero, 4. Zero, 5. (2,3), 6. Dan, 7. No.

8. We use the symbol (R,S) for the coordinates of a place on the seating chart. R is a number along the <u>row axis</u> and S is a number along the <u>seat axis</u>.

a. Give the names of those students sitting where R=5.

(5,0) _____ (5,3) _____

(5,1) _____ (5,4) _____

(5,2) _____ (5,5) _____

b. Give the names of those students sitting where R=S.

_____ _____

_____ _____

_____ _____

c. Give the names of those students sitting where R + S = 4.

_____ _____ _____

_____ _____

d. Give the names of those students sitting where R + S = 7.

_____ _____ _____

_____ _____

ANSWERS CONTINUED

8. a.	(5,0) Mike W.	(5,3) Lonnie
	(5,1) Bob	(5,4) Lynn
	(5,2) Roy	(5,5) Harry
b.	(0,0) Peter	(3,3) Diane
	(1,1) Carol	(4,4) Mark
	(2,2) (empty seat)	(5,5) Harry
c.	(0,4) John	(3,1) Jean
	(1,3) Donna	(4,0) Ginny
	(2,2) (empty seat)	
d.	(2,5) Al	(5,2) Roy
	(3,4) Phil	
	(4,3) (empty seat)	

Indicate the front row of seats as the row axis. These seats are numbered from left to right and each number is a row coordinate.

Indicate the extreme left-hand row of seats as the seat axis. The seats are numbered from front to back and each number is a seat coordinate.

The axes intersect at the origin. The origin and the axis may be relocated.

Vary the activity by having certain blocks of students stand. Have the class specify those standing by giving their seat coordinates.

If the class is doing well and further exploration is in order, vary the position of the origin and see if they can catch on to their new seat coordinates.

Following the discussion of the seating chart coordinates, have the students start their assignment.

THE GLOBAL COORDINATE SYSTEM

A ship in distress out in the Atlantic Ocean radios its position to potential rescuers.

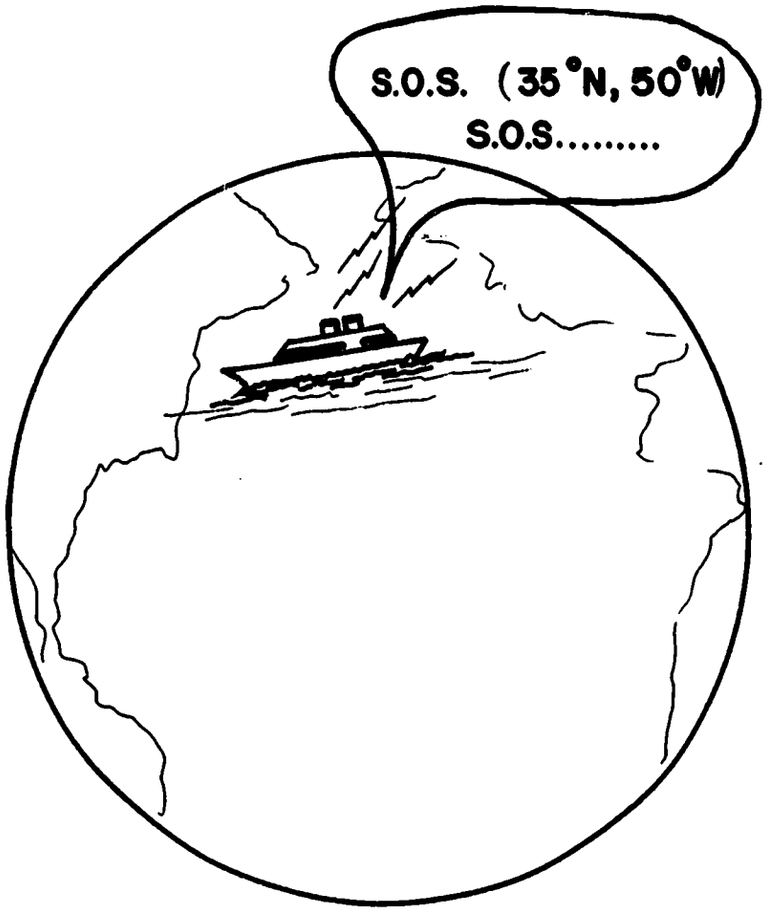


Figure 1

How will the rescuing ships or planes use the location information?

THE GLOBAL COORDINATE SYSTEM

OBJECTIVES

1. To provide a coordinate scheme which is in contrast to those presented in Lessons 1 and 2.

2. The student is able to use the global coordinate system to locate a given position on the globe (or flat map).

   a. He can point to (30° N, 40° W), for example.

   b. Given a specific place on the globe (or flat map) he can describe its coordinates to the nearest degree.

NOTE It is assumed that the student can read a protractor to the nearest degree; knows that there are 60 seconds in a minute, 60 minutes in a degree and 360 degrees in a circle. The student should have prior experience measuring central angles of a circle.

The mastery of longitude and latitude is not essential to the achievement of the objectives for this Booklet so judge the time you spend on this lesson accordingly.

Lines on the globe form a sort of grid. They help us locate the position of an object anywhere on the surface of the earth.

The lines running from the North Pole to the South Pole are called lines of longitude. Lines running around the globe, parallel to the Equator, are called lines of latitude.

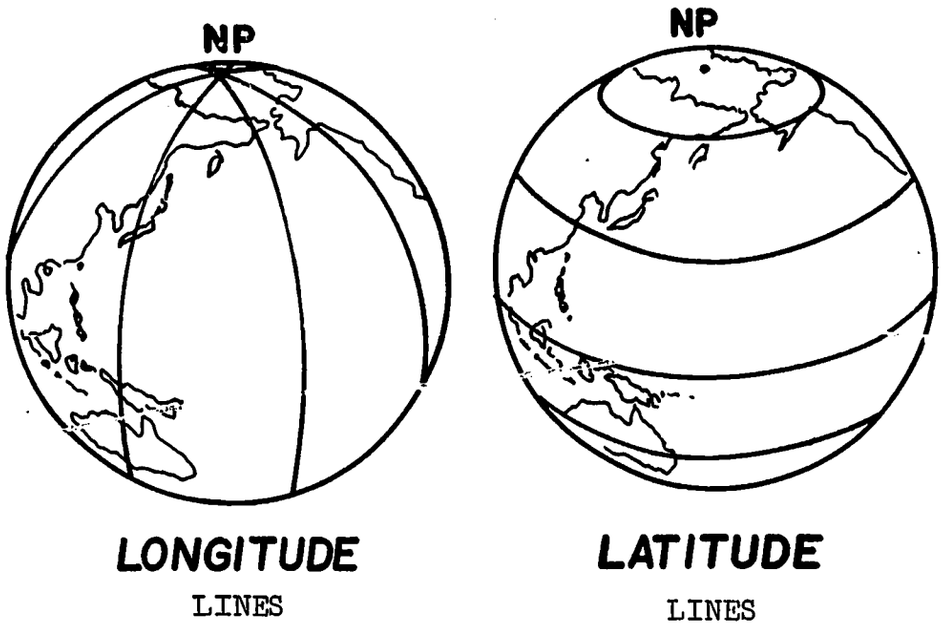


Figure 2

EQUIPMENT AND TEACHING AIDS

It is desirable to have several globes in the classroom for this lesson. A flat map has the longitude and latitude lines, but it is not as effective. An atlas, almanac or other reference is helpful in obtaining geographic coordinates.

TRANSPARENCIES

3-1. Two globes showing latitude and longitude lines with overlays showing reference lines and degree markings.

3-2. Two globes showing specific angles of longitude and latitude (slices taken out of the globe).

3-3. A realistic illustration of a globe showing the coordinates of several key cities. (See Appendix B)

PURPOSE

1. To help illustrate the global coordinate system.

A rubber ball can be cut to show an angle of longitude or an angle of latitude.

Latitude is measured, in degrees, north and south from the Equator. The Equator is 0° latitude. The Equator divides the globe into two halves, north latitude and south latitude.

To see an angle of latitude you have to imagine the globe with a slice taken out.

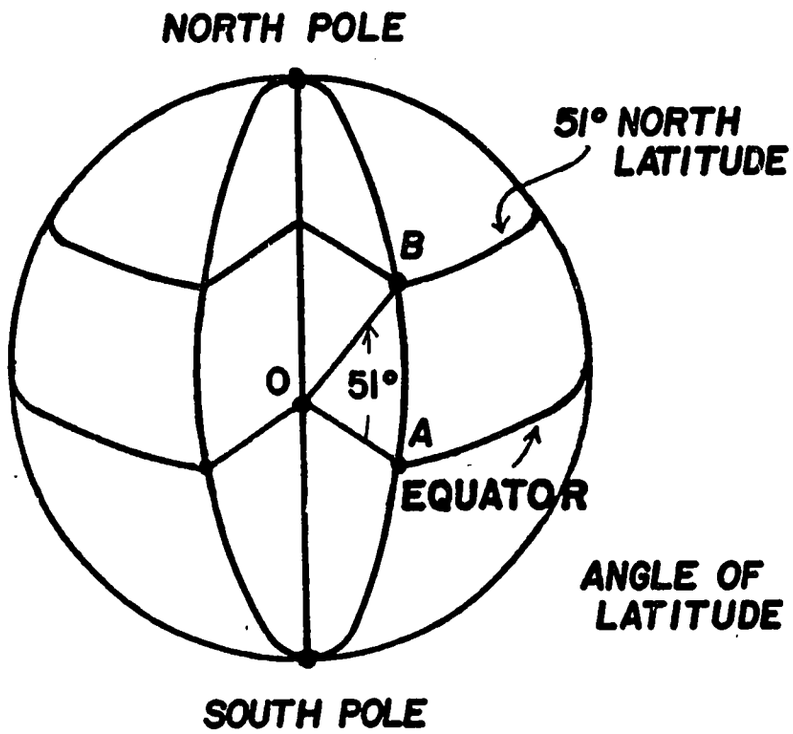


Figure 3

Point A, in Figure 3, is on the Equator. Point O is the center of the Earth. The measure of angle AOB is 51°. Every point on the 51° north latitude circle is 51° above the Equator.

The North Pole is at 90° north latitude. Where is the South Pole located?

The <u>longitude lines</u> are called <u>great circles</u> because the center of each circle of longitude is also the center of the Earth.

CONTENT AND APPROACH

Globe location provides a contrast for plane coordinates. A knowledge of angle measure in degrees and minutes is needed. by the student.

The imaginary lines of latitude and longitude form a unique coordinate system for the Earth. Any object on Earth can be "fixed" by obtaining the coordinates of latitude and longitude.

Thus, the ship shown in Figure 1 on the students' lesson is radioing its position-thirty-five degrees north latitude: (that is, 35° above the Equator) and fifty degrees west longitude (that is, 50° west of the Greenwich line).

A goal of this first unit in coordinate geometry is to provide the student with some experiences from which he will be able to draw a generalization. These experiences are related to coordinatization of various geometric regions. The generalization centers around the variety of ways one can design a coordinate scheme. This lesson provides one such experience.

To see the relationship between an angle of latitude and a circle of latitude, bend a piece of wire to form an acute angle. The vertex of the angle will represent the center of the Earth. Rotate the wire angle so that the locus of one endpoint represents the Equator and the locus of the other endpoint represents a circle of latitude. Use transparency 3-2 to show an angle of latitude. Sketch in the circle of latitude, 30° N.

Great circles are important because the shortest distance between two points on the globe is along an arc of a great circle.

If you slice the globe along any great circle, you will cut the globe in half.

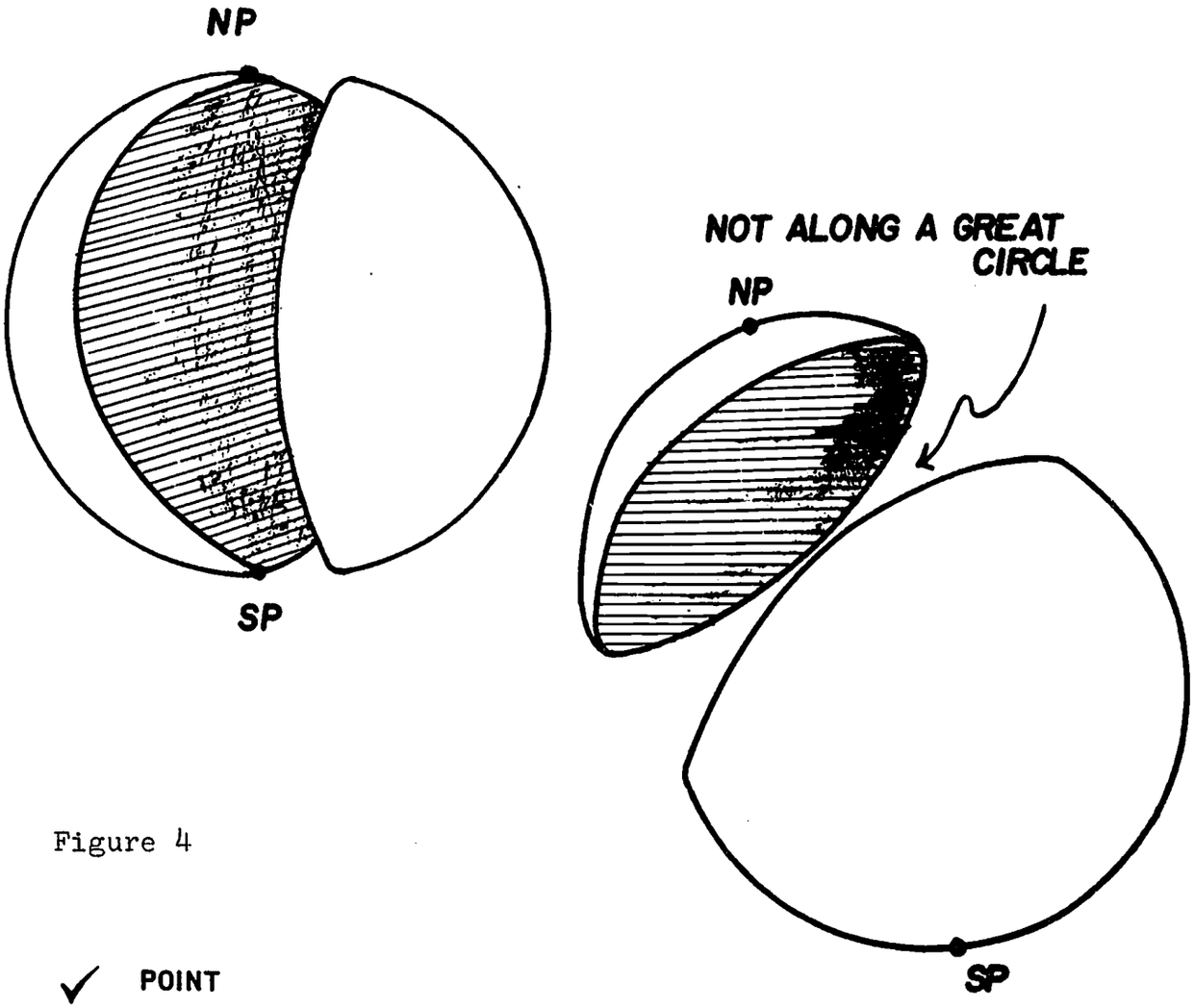


Figure 4

✓ **POINT**

1. There is only one line of latitude which is a great circle. What is its name?
2. Which lines are parallel to the Equator?
3. The latitude of the Equator is _?_ degrees.

✓ POINT ANSWERS

1. Equator
2. Circles of latitude
3. Zero

All lines of latitude do not have the same circumference. Have the students see the circles growing smaller from the Equator to one of the poles.

Navigators talk of taking the "great circle route". Use a piece of string on the globe to illustrate a great circle route. The great circle route is the shortest distance between two points on the globe.

Ask the students to imagine slicing a sphere with a plane. The slice which passes through the center of the sphere contains the circle of maximum radius.

The Earth is quite spherical. (It is slightly "fatter" around the Equator, but the degree of out-of-roundness is so slight that, for all practical purposes, it is a sphere.)

An important line of longitude runs through Greenwich, England. Greenwich is just outside London.

Actually, this important circle of longitude has two parts. The half circle that runs through Greenwich is often called the Prime Meridian. The other half of this circle is called the International Dateline.

This circle made up of the Prime Meridian and the International Dateline divides the globe into two halves, east longitude and west longitude.

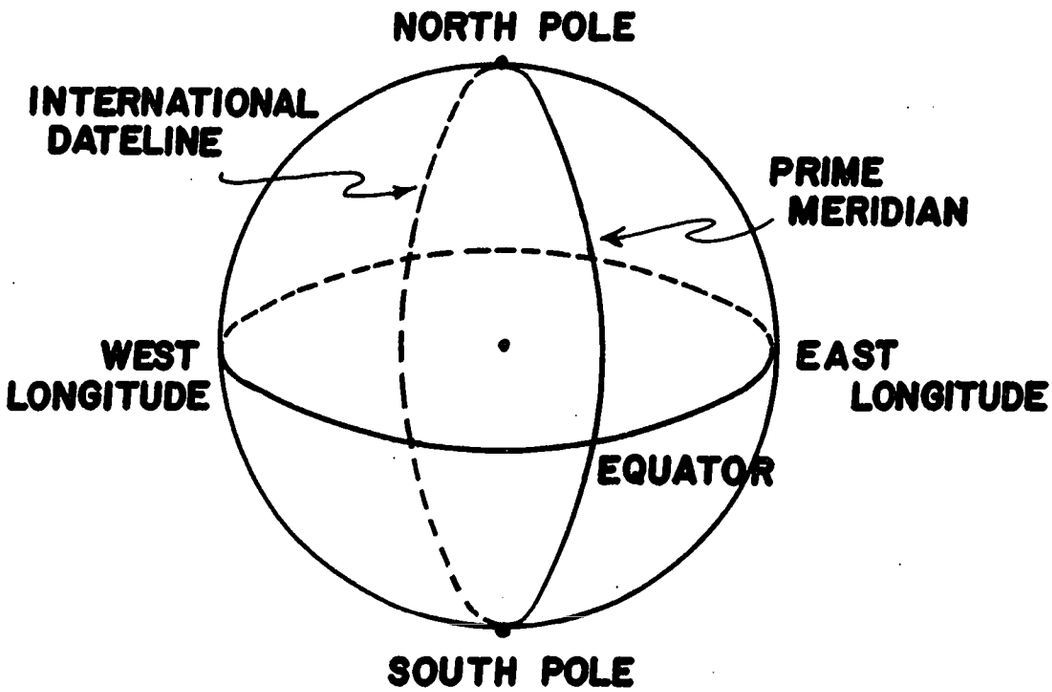
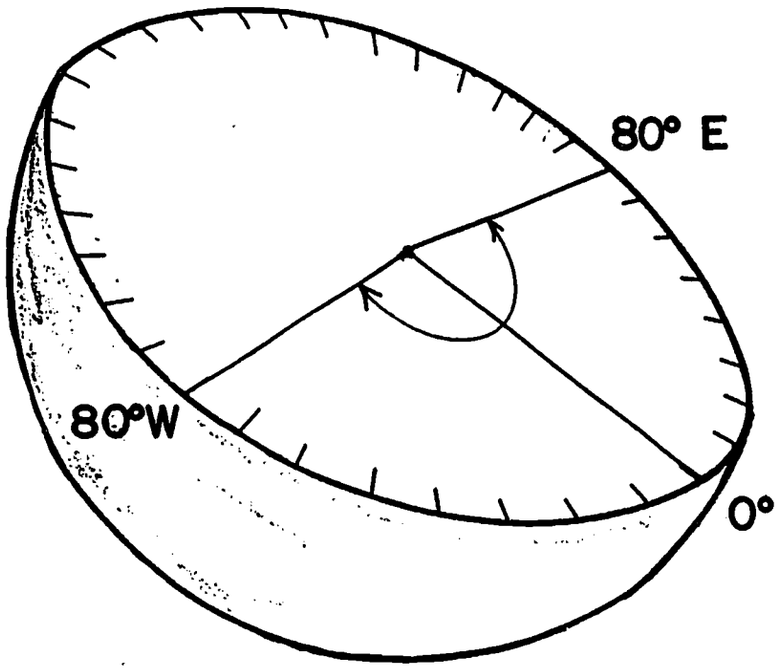


Figure 5

Each line of longitude "cuts" the Equator. Beginning with the Prime Meridian, lines of longitude are numbered from 0° to 180° in both the east and west directions.

Use transparency 3-2 to show an angle of longitude.

Have the students imagine the Earth cut in half along the Equatorial plane. Place a full circle protractor on this "half-Earth" to illustrate how the angles of longitude are marked off on the equator on the globe.



The Prime Meridian is 0° longitude. The International Dateline is 180° longitude.

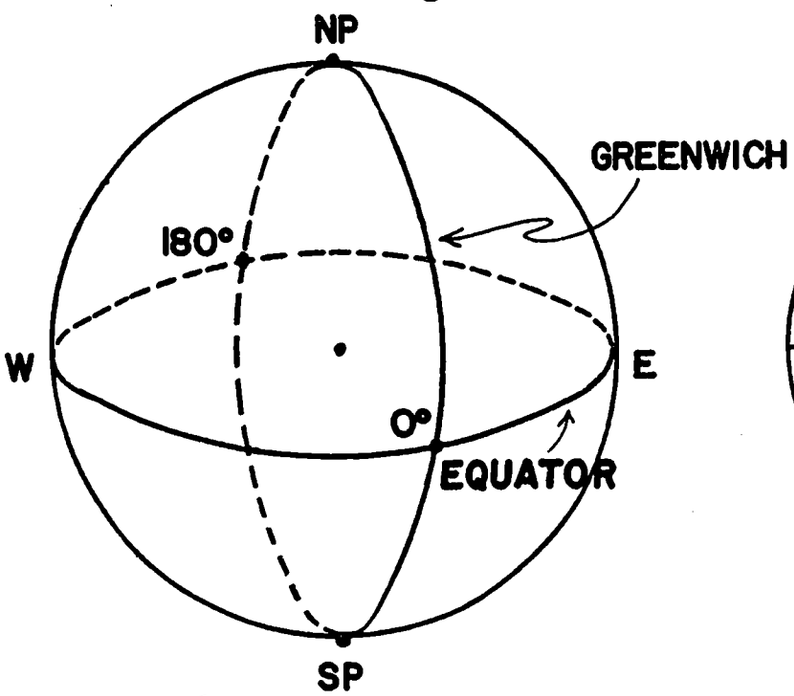


Figure 6

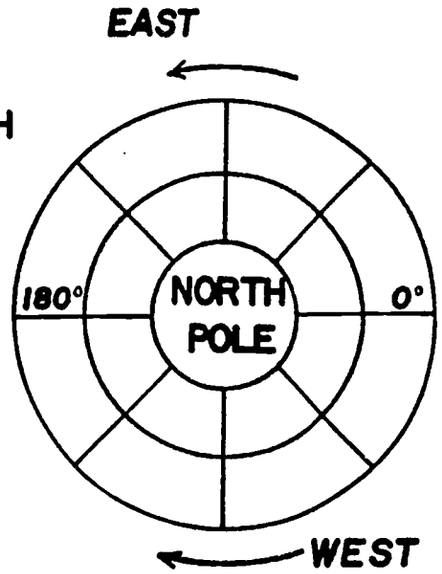


Figure 7

To see an angle of longitude you must again imagine the globe with a slice taken out.

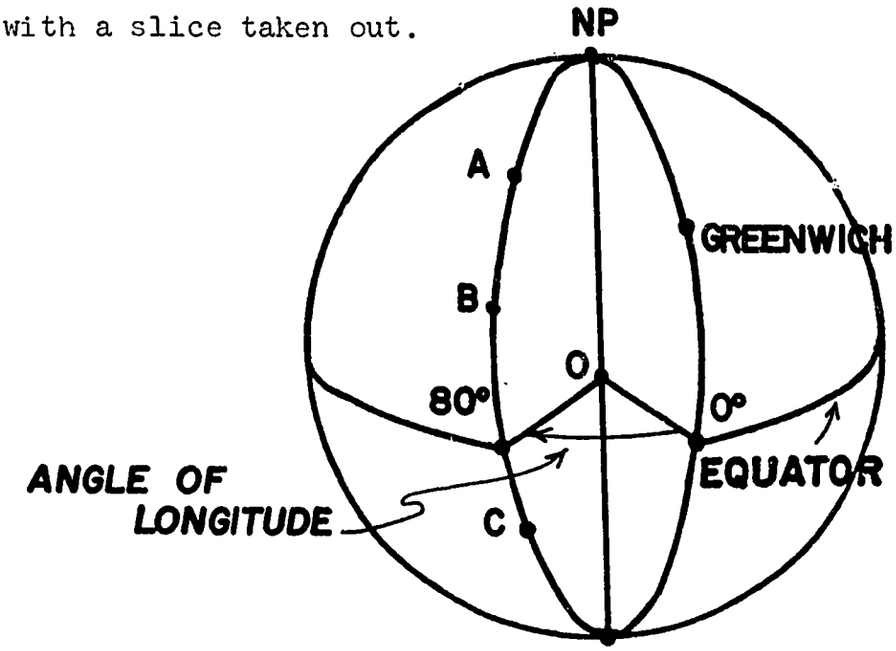


Figure 8

Points A, B, and C in Figure 8 all lie on the 80° west longitude line.

Focus the students' attention on the locus of all points having the same longitude. For example, all points having the longitude 60° W lie on the same half-circle (line of longitude).

The difficulty in seeing this, for some students, is that longitude is measured east and west, but the lines of longitude run north-south.

Suppose you were to travel around the Earth on a circle which passes through both Poles.

Using Figure 8, to be specific, suppose you start your trip at the South Pole and travel north along the 80° west longitude line.

As you travel along this arc, passing through points C, B and A you are always 80° west of the Prime Meridian.

However, when you cross over the North Pole heading south you are on a different longitude line. How many degrees <u>east</u> of the Prime Meridian are you now?

✓ **POINT**

1. The circle formed by the Prime Meridian and the International Dateline divides the globe into two halves, _?_ longitude and _?_ longitude.
2. Longitude is measured in degrees east and west from the _____ _____.
3. All lines of longitude pass through which points on the globe?

Latitude and longitude lines form a set of geographic coordinates. Any position on Earth can be located by giving two angle measures: The number of degrees latitude (stating whether north or south) and the number of degrees longitude (stating whether east or west). By agreement, latitude is always given first.

Show the students how a circle of longitude has two parts. For example, the half circle which represents the 80° W longitude line is part of a circle where the other half represents the 100° E longitude line. Have them discover that the sum of the two longitudes which are part of the same great circle is 180°.

✓ POINT ANSWERS

1. East and West.
2. Prime Meridian or Greenwich Line.
3. North and South Poles.

Two great circles act as the reference lines or axes of the geographic coordinate system.

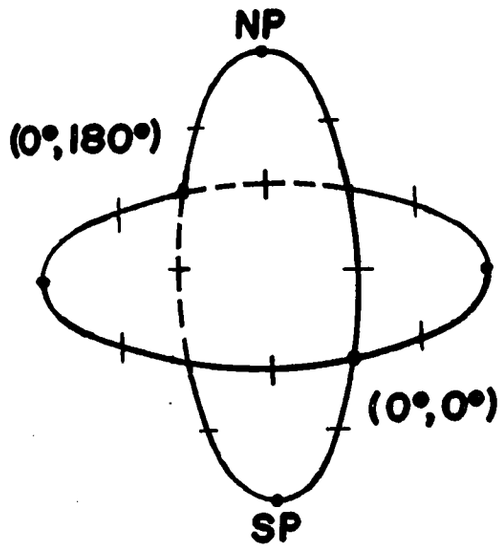


Figure 9

The point where the Prime Meridian and the Equator intersect is called the origin of the global coordinate system.

Degrees of latitude are marked off on the Greenwich Line. starting at the origin and proceeding to the North Pole or to the South Pole.

Degrees of longitude are marked off on the Equator starting at the origin and proceeding east or west.

EXAMPLE:

Mexico City is located approximately at (19° N, 99° W). Find this on your globe.

Geographic coordinates for the two Poles need a special agreement. Since all lines of longitude pass through both the North and South Poles we have to agree on what to use for their second coordinate.

Have the students see the relationship between the angles formed with vertices at the center of the Earth and how the angle measures are labeled on the globe (or flat map).

Since each Pole is situated on all lines of longitude, it is necessary to make a special agreement as to what to use as the longitude coordinate. We have designated zero degrees for this purpose.

Make sure that the students understand that giving latitude first in geographic coordinates is simply an agreement. Order is important, but for most coordinate systems it is an arbitrarily agreed upon order.

Compare locating points on the globe with locating points on the map in Lesson 1. The axes in the former case are the Prime Meridian and the Equator.

We will agree to use 0° for the longitude coordinate of the poles.

	Geographic Coordinates
Scuth Pole	$(90^{\circ}$ S, $0^{\circ})$
North Pole	$(90^{\circ}$ N, $0^{\circ})$

✓ **POINT**

1. What are the coordinates of the origin? Are the abbreviations N, S, E or W necessary?
2. What are the coordinates of the point where the International Dateline and the Equator intersect?
3. Explain how the Equator and the circle formed by the Prime Meridian and the International Dateline are like the axes used in Lessons 1 and 2.

✓ POINT ANSWERS

1. $(0^\circ, 0^\circ)$. Not really.
2. $(0^\circ, 180^\circ)$.
3. They are reference lines along which the unit, in this case an angle measure, is counted off. Like the axes in Lessons 1 and 2, these circles intersect perpendicularly at the origin. There is, however, a second point of intersection, $(0^\circ, 180^\circ)$.

EXERCISES

In which half do the following countries lie?

COUNTRY	EAST OR WEST LONGITUDE
1. Brazil	_____
2. Japan	_____
3. United States	_____
4. Australia	_____
5. France	_____

COUNTRY	NORTH OR SOUTH LATITUDE
6. Canada	_____
7. India	_____
8. Australia	_____
9. Argentina	_____
10. Sweden	_____

11. Which line of latitude is a great circle?

12. Navigators talk about traveling the "great circle route". What does that mean?

13. The 0° longitude line is called the _____ line.

14. Can the circumference of the earth be measured along any circle of latitude?

ANSWERS CONTINUED

1. West
2. East
3. West
4. East
5. A little in each half
6. North
7. North
8. South
9. South
10. North
11. Equator
12. Taking the route of shortest distance from one point on the Earth to another. The center of a great circle is also the center of the Earth.
13. Greenwich. It is also called the Prime Meridian.
14. No. The circumference of various circles of latitude change with the location of the circle.

15-18. Give the coordinates (to the nearest degree) of each city given below.

CITY	LOCATION
15. Paris, France	_____
16. New York City	_____
17. Pontiac, Michigan	_____
18. Greenwich, England	_____

19. The center of which country is located approximately at (25° S, 135° E)?

20. Find the origin on the globe Describe its geographic location.

21. Imagine a line running from a point on the globe, through the center of the globe and exiting on the "opposite" side of the globe.

    Give the coordinates of the point located on the "opposite" side of the globe from the given point.

a. (0°, 50° W) ... _____

b. (30° N, 0°) ... _____

c. (45° N, 180°). .. _____

d. (30° N, 80°W) .. _____

22. Find the locations of Boulder, Colorado and Philadelphia, Pennsylvania. Subtract their longitude coordinates to obtain the number of degrees longitude between them.

ANSWERS CONTINUED

15. (48° N, 2° E)

16. (40° N, 74° W)

17. (43° N, 83° W)

These should be approximations.

18. (51° N, 0°). Discuss why you do not need an east or west designation.

19. Australia.

20. At the intersection of the Greenwich Line and the Equator. A point approximately 330 miles south of Ghana and 660 miles west of Gabon.

21. a. (0°, 130° E)
    b. (30° S, 180°)
    c. (45° S, 0°)
    d. (30° S, 100° E)

22. (40° N, 105° W), (40° N, 75° W). 30° difference in longitude. As a special project you may wish to have a few students determine the distance between these two cities based on the 30° difference.

EXAMPLE:

$$\frac{(7{,}918 \text{ mi.})\ \pi\ (\cos 40^\circ)}{360^\circ}. \quad 30^\circ \approx 1{,}608 \text{ mi.}$$

POWER QUESTIONS

1. Each rectangular section on the coordinate system used on the map (See Lesson 1) was the same size. Explain why this is not so on the globe.

2. A hunter set out from his camp one day. He walked one mile south and then two miles east. He then shot a bear and towed it one mile north and arrived at his camp. Where was the hunter's camp?

FOR YOU TO DO

Look up the coordinates of the geographic center of the United States.

ANSWERS TO POWER QUESTIONS

1. Circles of latitude have smaller circumferences as one moves from the Equator to either pole.

2 (This exercise can be used to provoke an interesting discussion on problem solving.) North Pole. The hunter's camp could also be anywhere on a circle of approximately 89° 58' 51" south latitude. The hunter walks one mile south from his camp which places him on a circle of latitude with an approximate circumference of two miles. Thus, a two mile walk east brings him completely around the circle and back to the point one mile south of his camp.

Also, consider a circle of latitude such that its circumference C is an exact divisor of 2 (2 = NC). The hunter's camp could be any place on the circle of latitude located one mile north from this circle.

FOR YOU TO DO : The geographic center of the forty-eight conterminous states was determined by the Coast and Geodetic Survey in 1918. The center-of gravity method was used. This geographic center was approximately at (39° 50'N, 98° 35'W) near Lebanon, Kansas. When Alaska became our 49th state the geographic center shifted to (44° 59'N, 103° 38'W) near Castle Rock, Butte County, South Dakota. Later when Hawaii was admitted to the Union, the geographic center moved to (44° 58'N, 103° 46'W).

RULER AND PROTRACTOR COORDINATES

Part I The Origin at Lansing

Fix the full circle protractor on the map so that the center is on the "star" 

and the 0-180 degree line runs north-south. See the picture below.

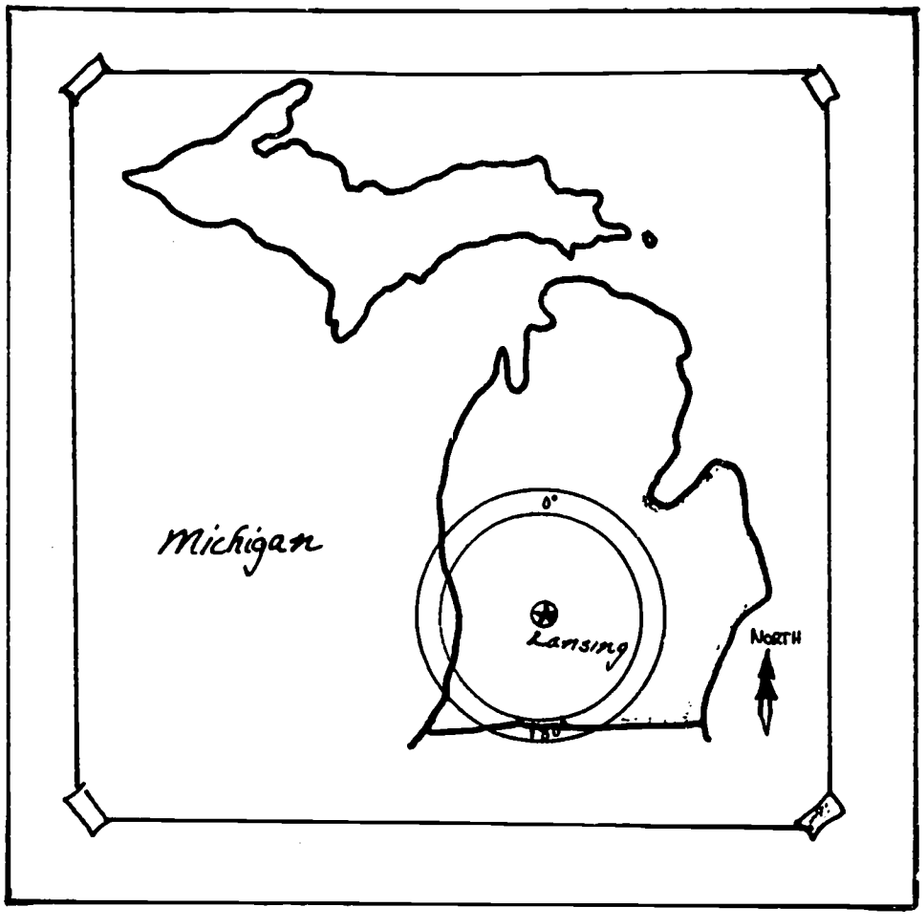


Figure 1

U.S. Highway 27 runs north-south out of Lansing, just below St. Johns. You can use it to line up your protractor.

RULER AND PROTRACTOR COORDINATES

OBJECTIVES

1. To introduce the student to polar coordinates.
2. To extend the student's concept of coordinatization.
3. To provide a contrast to rectangular coordinates.
4. The student will be able to set up a type of polar coordinate scheme on a map and utilize the scheme to locate positions and obtain coordinates.

NOTE: It is not necessary to use the term polar coordinate with the students.

EQUIPMENT AND TEACHING AIDS

Give each small group of students a highway map of Michigan*, a full circle protractor, and a foot ruler (transparent plastic if possible). See Appendix B.

NOTE: The activity discussed below uses this equipment. However, many variations are possible so be creative if you do not have the recommended material at hand.

TRANSPARENCIES

4-1. An outline map of Michigan. Several key cities are indicated. A 10 inch ruler (unit: one-eighth inch) and a full circle protractor (unit: 5 degrees). Both measuring instruments are transparent and movable. (See Appendix B)

PURPOSE

1. To illustrate a polar coordinate scheme in locating cities in Michigan.

***NOTE:** The map used to determine the coordinates in this lesson is the Michigan Highway Map published by Tempo Designs, Lincolnwood, Illinois for Standard Oil Division American Oil Company.

The direction of an object on the map is obtained by measuring the angle between the north line and the line joining the center of the protractor to the object.

The direction angle is measured in degrees from north in a clockwise direction. This angle is called the bearing of the object from the center of the protractor. Thus a bearing of 90° would be due East.

A bearing and a distance can be used to locate any place on the map. When we give the location of a place on the map we give its coordinates.

(distance, bearing)

By agreement, the distance is listed first.

EXAMPLE:
The coordinates of Cadillac are $(5\frac{7}{8}'', 338^{\circ})$.

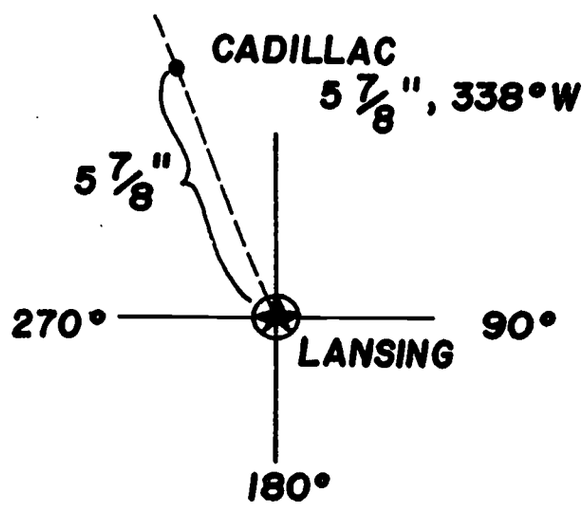


Figure 2

The distance coordinate for Lansing is zero. What is the bearing for Lansing?

Since there is no angle between Lansing and itself we will agree to call the bearing of Lansing 0°.

CONTENT AND APPROACH

Present the students with the problem of coordinatizing the map of Michigan using a ruler and a full circle protractor.

Use the analogy of the pilot flying his plane on a certain bearing for a certain distance. Or use the picture of the hunter in the woods finding his way with a magnetic compass.

Help the students position their protractors and line up the zero-degree mark toward north. Working in small groups, of from three to four students each, the students should be able to answer most of their own questions.

After the students have completed Exercises 1-8 (Part 1) in their small groups, draw them together with a class discussion.

THINGS TO DISCUSS

The follow-up discussion should take up these points:

1. The arbitrariness of:
   a. The placement of the center of the protractor.

      Move the origin around so that the students see that the coordinates of a position change relative to the position of the origin.

✓ **POINT**

Explain

1. Where the origin of this coordinate system is.
2. What are the coordinates of the origin?

EXERCISES

1. Give the name of the city located by the given coordinates. (The measurements have been rounded off to the nearest $\frac{1}{8}$ inch and nearest 1 degree.)

COORDINATES	CITY
a. $(5\frac{5}{8}'', 80^\circ)$	a. _____
b. $(4'', 225^\circ)$	b. _____
c. $(10\frac{3}{8}'', 2^\circ)$	c. _____
d. $(1\frac{6}{8}'', 167^\circ)$	d. _____
e. $(0'', 0^\circ)$	e. _____
f. $(15\frac{4}{8}'', 332^\circ)$	f. _____

2. Determine the coordinates of the following cities. Measure to the center of the circle used to indicate cities on the map.

CITY	COORDINATES
a. Muskegon	a _____
b. Midland	b. _____
c. Kalamazoo	c. _____

b. The way in which the angle measure is reported.

Some protractors are labeled 0°-360° in a counterclockwise direction. The 0° - 180° line could be placed to run east-west instead of north-south. Positive and negative or east, west, north and south designations could be used along with the angle measure in degrees.

The students should see the importance of agreeing to some convention.

✓ POINT ANSWERS

1. Lansing
2. $(0'', 0^\circ)$

ANSWERS TO EXERCISES

PART I

1. a. Port Huron
   b. Three Rivers
   c. Cheboygan
   d. Jackson
   e. Lansing
   f. Marquette

2. a. $(4\frac{6}{8}'', 292^\circ)$
   b. $(3\frac{2}{8}'', 13^\circ)$
   c. $(3\frac{1}{8}'', 240^\circ)$

d. Traverse City d. _____

e. Pontiac e. _____

f. Lansing f. _____

3. What is the distance coordinate of Wyandotte?

4. What is the angle coordinate for Detroit?

5. Could the order of the coordinates be changed? That is, could the angle measure be the first coordinate? Why?

Use the map of Michigan on the following page to do these exercises.

6. Place the center of your protractor on the dot labeled O. Point 0° north. Plot the following points.

A. $(1\frac{7}{8}'', 208^{\circ})$ C. $(1\frac{3}{4}'', 333^{\circ})$

B. $(1\frac{3}{8}'', 130^{\circ})$ D. $(3'', 348^{\circ})$

7. Give the names of the cities that you think are located at points A, B, C, D and O.

8. What are the coordinates of point O?

ANSWERS CONTINUED

d. $(7\frac{6}{8}'', 338^\circ)$

e. $(3\frac{3}{8}'', 95^\circ)$

f. $(0'', 0^\circ)$

3. $4\frac{1}{8}''$ 4. 109°

5. Yes. The order of listing the coordinates is arbitrary. The students, however, should agree on the need for a convention. Polar coordinates have the distance coordinate listed first.

6. See the map on page T 33.

7. A. Kalamazoo; B. Flint; C. Traverse City
   D. St. James, Beaver Isle. O. Mt. Pleasant

8. $(0'', 0^\circ)$

THINGS TO DISCUSS CONTINUED

2. The idea of order. We agree to give the distance first. It is not as easy to mix the coordinates $(6'', 23^\circ)$ with $(23^\circ, 6'')$ as it is to mix coordinates like (5,10) and (10, 5). Note: the symbol " "for inches should not be confused with the same symbol used for seconds on angle measure.

3. The comparison of rectangular coordinates to polar coordinates.

   a. Both the rectangular grid (Lesson 1) and this lesson use an ordered pair for coordinates. There is a first coordinate which tells you one thing about the location (but not enough); and a second coordinate which, with the first, "locks in" the position.


0°
270°
90°
180°
MICHIGAN


0°
270°
90°
180°
D
C
MICHIGAN
B
A

THINGS TO DISCUSS CONTINUED

b. Lesson 1 showed two reference lines (axes). A location was given in terms of its perpendicular distance from each of these reference lines. Lesson 4 shows essentially two rays. To obtain a pair of coordinates one measures the distance along one of the two rays. The fixed ray, in this case, is the one which points north.

4. The fact that error was involved in the measurements. Compare the coordinates given for the same city by different students.

5. Non-uniqueness. Show how two or more different pairs of coordinates can locate the same point.

EXAMPLE: $(3'', 70^\circ)$, $(3'', 430^\circ)$

$(3'', 790^\circ)$,..., $(3'', 70^\circ + K \cdot 360^\circ)$, ...

Contrast this situation with Lesson two's seat coordinates where if you change a coordinate you change the point being located.

Part 2 The Tornado Plotting Map

A distance-angle coordinate system is used by the weather bureau to keep track of tornados or other severe weather.

The circles shown on the map all have a common center (they are concentric circles) located at the Detroit radar equipment site. This is the origin of this coordinate system.

The circles are called range marks because they help determine the distance from the origin to some point on the map.

The 100 mile (nautical miles are used) range mark is divided up like a protractor. This helps determine the bearing of a point on the map.

Notice that lines of latitude and longitude are also shown on the Tornado Plotting Map.

When a tornado is reported, its range (distance from the origin) and bearing are used to give its coordinates.

EXAMPLE:

A tornado is spotted near London, Ontario (top right section on the map). Its range is approximately 109 miles and its bearing is approximately 63°.

Thus, its coordinates are (103 mi., 63°).

Part 2 The Tornado Plotting Map

The Tornado Plotting Map provides a useful application of ruler-protractor coordinates.

The direction of an object from a fixed point is obtained by measuring the angle between the north line and the line joining the fixed point to the object. The direction angle is measured in degrees from north in a clockwise direction. This angle is called the bearing of the object from that particular point.

This section can be treated as optional. You may wish to assign it for individual work.

NOTE: See Item 1 in Appendix C for a supplementary activity.

EXERCISES (All questions refer to the Tornado Plotting Map)

1. Determine the range (approximately) of each of the following cities:

   a. Port Huron
   b. Pontiac
   c. Lansing
   d. Cleveland, Ohio
   e. Fort Wayne, Indiana
   f. Canton, Ohio

2. Determine the bearing (approximately) of each of the cities, a through f in Exercise 1.

3. A tornado is spotted at (100 mi., 130°) heading north. What large city is in danger?

4. A thunderstorm is reported at Lansing. What are its coordinates (approximately)?

5. What are the coordinates of Pontiac?

6. What city is located at (100 mi., 175°)? In what state?

7. The geographic coordinates of St. Johns, Mich. are (43° N, 84° 33' W). Find this point on your map and give its range and bearing.

8. What city is located at (41° 5' N, 85° 8' W)?

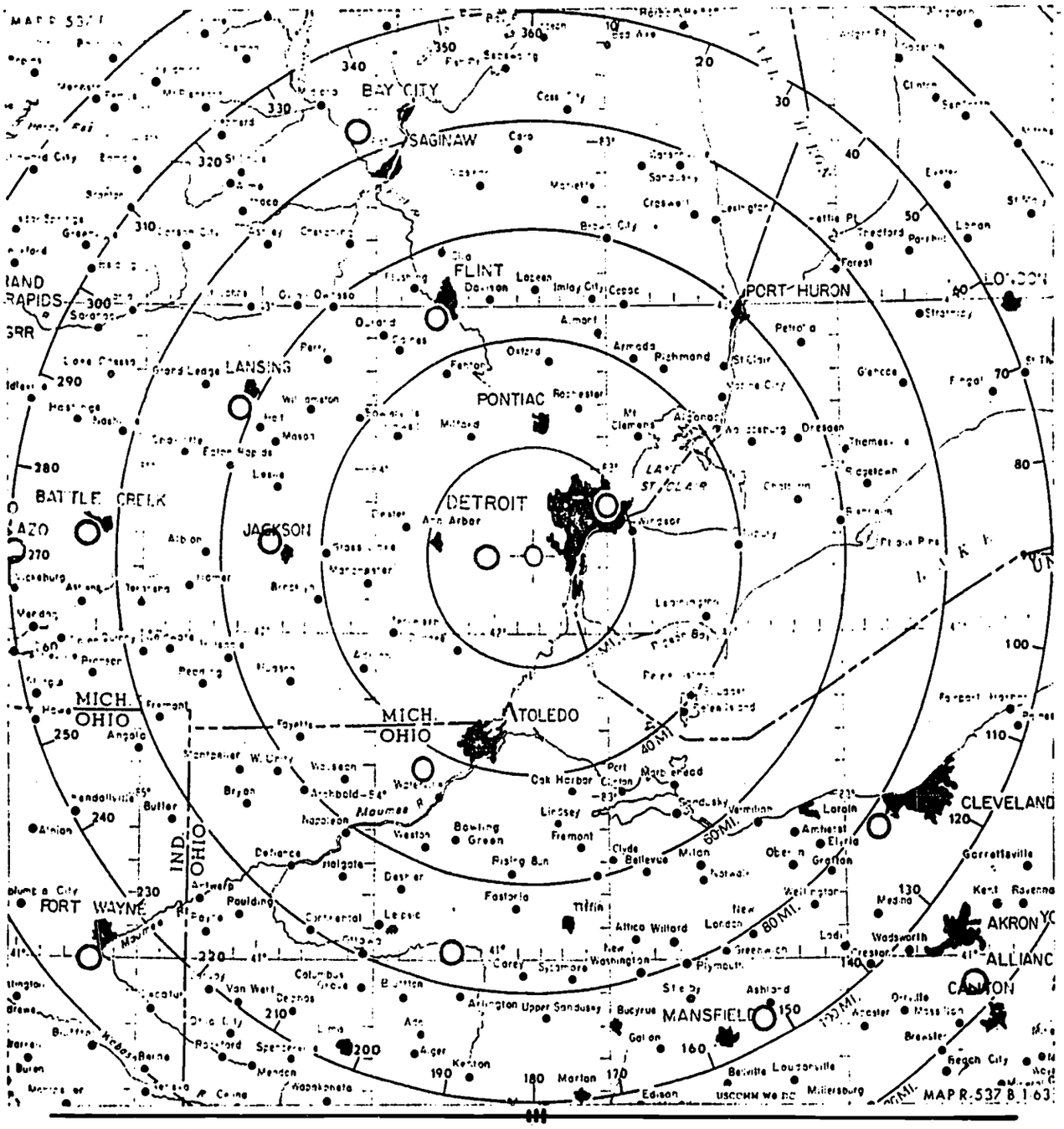
9. Perry, Mich. is at (54 mi., 313°) and Edmore, Mich. is at (104 mi., 313°). How far, in nautical miles, is Edmore from Perry?

ANSWERS TO EXERCISES

1. a. 60 mi.
   b. 25 mi.
   c. 62 mi.
   d. 89 mi.
   e. 107 mi.
   f. 203 mi.

2. a. 40°
   b. 4°
   c. 301°
   d. 120°
   e. 230°
   f. 133°

3. Cleveland, Ohio
4. (62 mi., 301°)
5. (25 mi., 4°)
6. Marion, Ohio
7. (70 mi., 311°)
8. Fort Wayne, Ind.
9. 50 mi.



TORNADO PLOTTING MAP — An actual work sheet used by the weather bureau radar meteorologist to track severe weather, this map has as its center the location of the Detroit radar equipment. The map provides those listening to the radio or television a quick reference when severe weather occurs, giving both distance in nautical miles and direction in degrees from Detroit. The prominent range marks are at 20-mile intervals.

SECRET CODE TABLE

Each ordered pair of numbers will lead you to a letter of the alphabet. Use the Secret Code Table.

Secret Code Table

4	J	S	E	P	H
3	U	C	O	B	Y
2	I	L	F	X	N
1	G	R	V	D	W
0	A	M	Q	T	K
	0	1	2	3	4

1. Decode the following secret message:

   (4,3) (2,3) (0,3) – (0,0) (1,1) (2,4) – (1,3) (1,2) (2,4) (2,1) (2,4) (1,1) .

2. Which letter of the alphabet is not in the table? Think of a pair of coordinates that could be used to code this letter.

3. Which letter is at the <u>origin</u> in the Secret Code Table?

SECRET CODE EXERCISE

The main purpose of this lesson is to provide some review and enrichment.

After two lessons dealing with angle measure, it focuses the students' attention on a system more aligned with the familiar rectangular coordinate system in the plane.

You may wish to have your students do some additional work with operations with the integers by having them perform various transformations with the secret code coordinates.

ANSWERS TO EXERCISES

1. You are clever.
2. The letter Z is missing. Student answers will vary as to an appropriate pair of coordinates for Z. The pair ($^{-}1$, $^{-}1$) is a good candidate.
3. A

4. A message can be coded by altering the coordinates.

WORD	COORDINATES	CODE	CODED WORD
C	(1,3)	(1,2)	L
O	(2,3)	(2,2)	F
P	(3,4)	(3,3)	B
Y	(4,3)	(4,2)	N

What was done to change the coordinates in Column 2 to the coordinates in Column 3?

5. Another way to code a word is to exchange the first and second coordinates.

EXAMPLE: (3,1) becomes (1,3). Give the coded word for the word **MAN**.

6. Something has been done to make this next message harder to decode. You will have to think a little before you can spell out this secret message:

(3,2) (1,2) ($^-$1,2) – (0,3) (3,3) (1,2) ($^-$1,2) (0,1) (2,0) – ($^-$1,0) (1,3) (2,$^-$1) – ($^-$1,$^-$1) (3,1) – ($^-$1,$^-$1) – (2,$^-$1) (1,2) (2,0) ($^-$1,$^-$1) (3,2) .

7. Use the Secret Code Table to code your first name. Do something to the coordinates to make your code hard (but not impossible) to figure out. Give your coded name to a friend and see if he can break the code.

4. One was subtracted from each of the second coordinates.

5. MAN becomes GAE.

6. One has been subtracted from each coordinate. The message is: You should get an A today.

SPACE COORDINATES

Julie Austin was watching a fly buzz around the classroom. She wondered if there was a way to keep track of the fly's position using mathematics. As the fly flew near a corner of the room, Julie thought of using space coordinates.

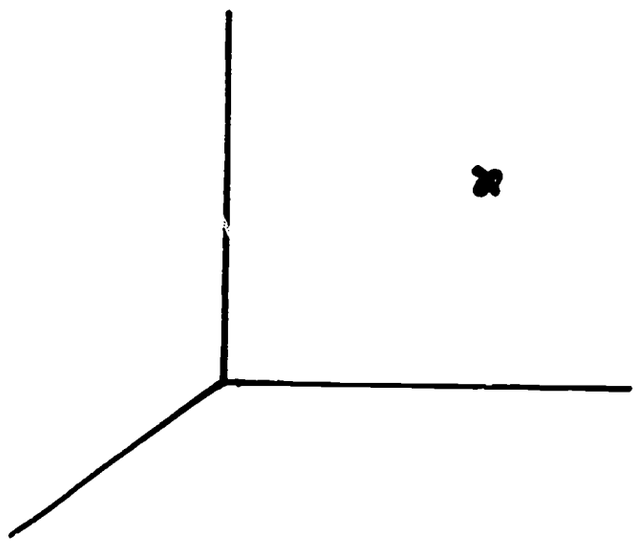


Figure 1

The lines where the walls and floor meet are perpendicular to each other. These lines could be thought of as axes. The walls and the floor can be thought of as planes. Each pair of axes lie in a plane.

With a way of counting off distance on each axis the position of the fly at any one instant in time could be determined.

LESSON 5

SPACE COORDINATES

OBJECTIVES

1. To extend the concept of coordinates to three dimensions. (Using the first octant only.)

   a. To provide a contrasting situation to plane coordinates.

   b. To review the terms coordinate(s), axis, and origin.

2. To associate the ordered triple of whole numbers with the three mutually perpendicular axes and the origin.

   a. The student is able to locate, by pointing, the point on the lattice cube which corresponds to, say, the coordinates (1,2,3).

   b. Given a point on the lattice cube the student can obtain the coordinates of that point (knowing, of course, the location of the origin).

   c. The student can associate each coordinate with its corresponding axis and corresponding plane. That is, given the ordered triple (5,3,8), for example, the student is able to describe the coordinate 3 as 3 units, measured along the y-axis, from the xz plane.

In Figure 2, a scale has been placed on each axis. The units on each scale are the same. The axes are labeled x, y and z so that we can keep track of them. The point where the axes intersect is called the <u>origin</u>.

It is very difficult to draw the axes for space coordinates on paper. That y-axis is really supposed to be sticking out of the paper right at you.

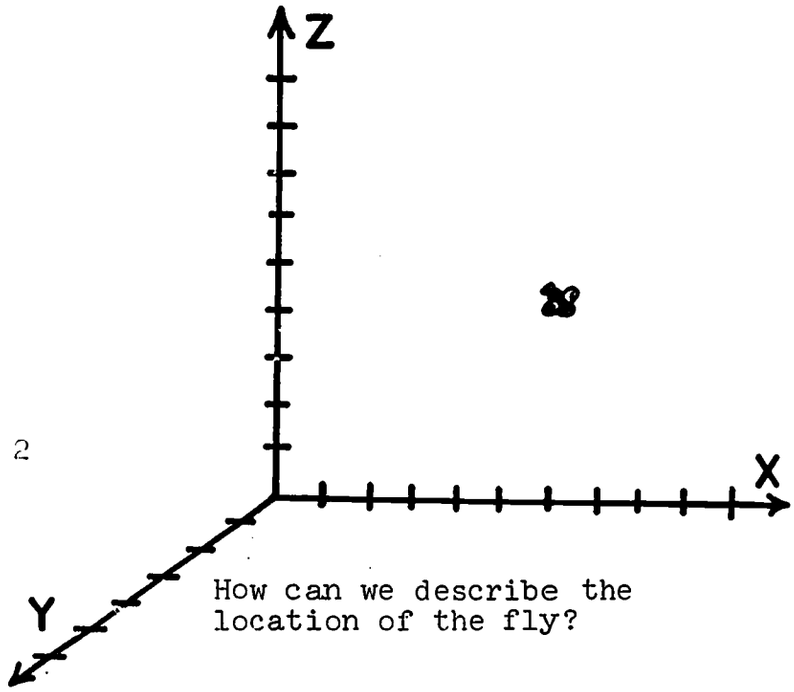


Figure 2

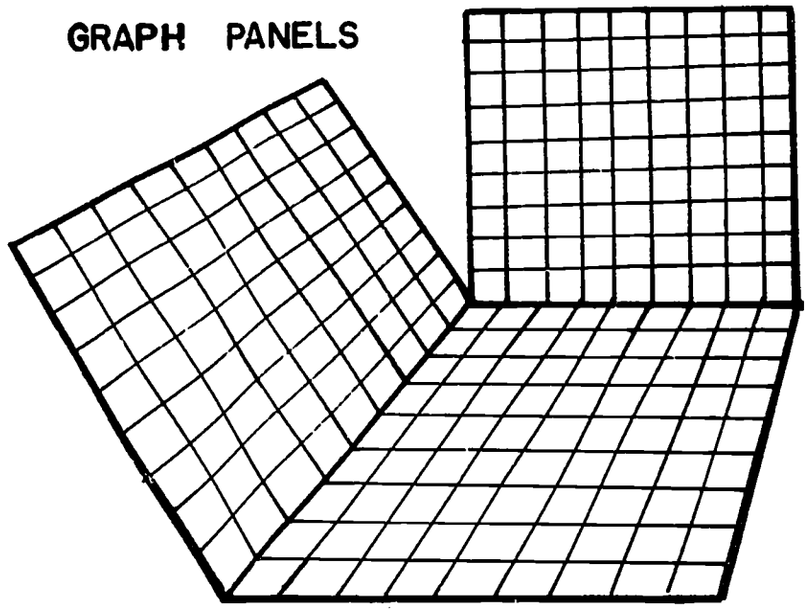
The fly is located at a certain distance from each plane. To describe his coordinates we must determine these distances.

Recall that two intersecting lines determine a plane. In describing the fly's coordinates we shall refer to the xy plane, the yz plane, and xz plane. Look at a corner in your classroom to visualize these three planes.

EQUIPMENT AND TEACHING AIDS

It is difficult for students to picture coordinates in three dimensions when the three axes are shown in two dimensions. Thus it is essential that some appropriate physical model be used in the discussion of this lesson. Some suggestions for models of the first Octant of the three-dimensional rectangular coordinate system are:

1. Paste sheets of graph paper onto each of three cardboard panels. Assemble the three panels to form the three coordinate planes in the first Octant.



2. A corner of the room or a corner of a cardboard box. Painted black, the sides of the box can be written on with chalk in order to show the axes and the scale. Segments of coat hanger wire can be used to puncture the sides of the cardboard box to locate points.

NOTE: See Items 5 and 6 in Appendix C for supplementary activities.

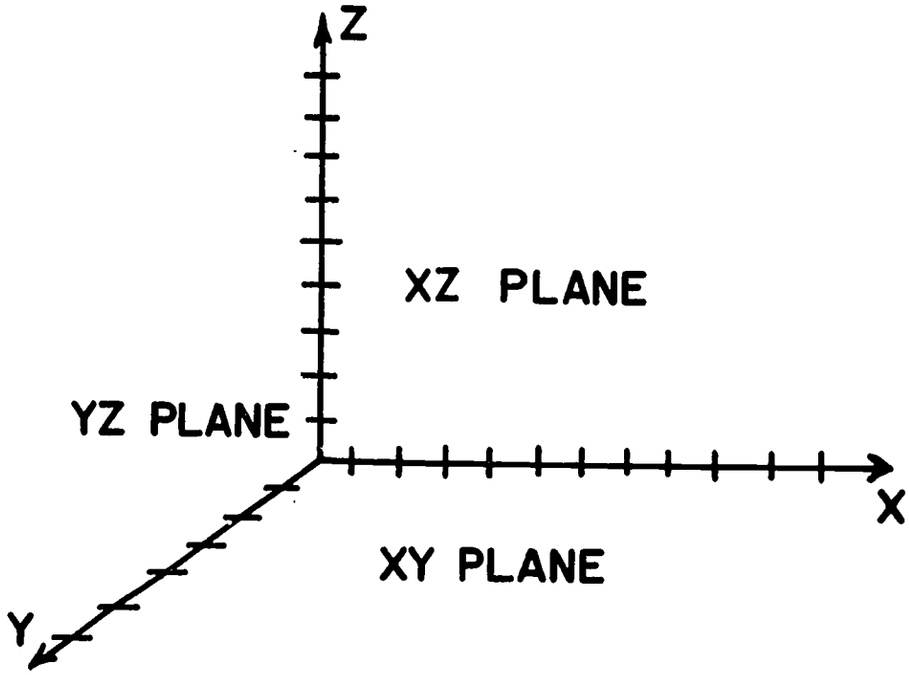
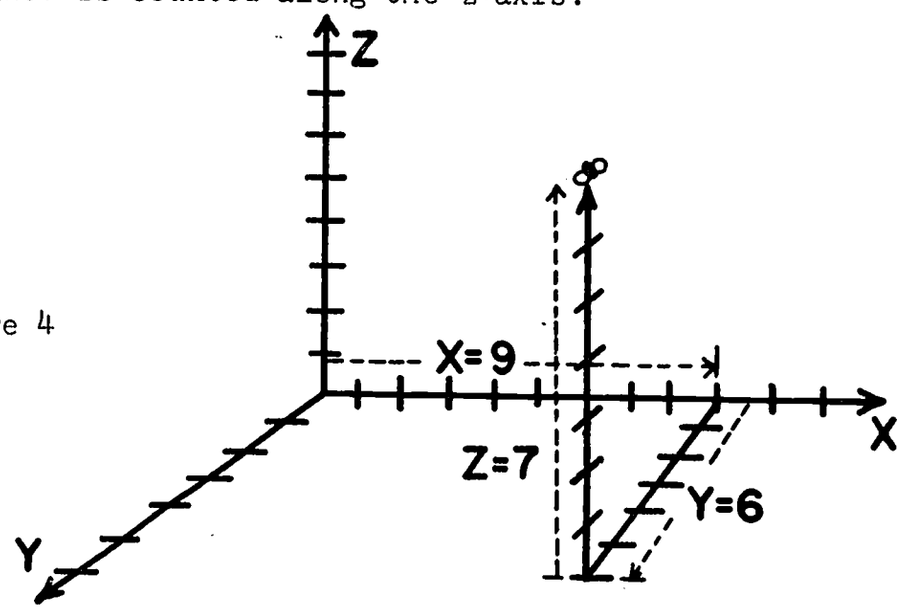


Figure 3

In Figure 4, the fly is 7 units from the xy plane. This distance is counted along the z-axis.

Figure 4



The coordinates of the fly's position at the instant shown in Figure 4 are determined by three numbers. The symbol (x, y, z) is used to give the space coordinates of a point.

TRANSPARENCY

5-1. A cubical box shown in the first Octant.

PURPOSES

1. To illustrate locating a point in three-dimensions.
2. To show the connection between the coordinates and the unit distances from each plane.
3. The student will scale the axes and describe the coordinates of the vertices.
4. Each face of the box will be described in terms of the common properties shared by the coordinates of the points on that face.

CONTENT AND APPROACH

If your classroom has a relatively unobstructed corner, you may wish to start this lesson by using your classroom as a model of the first octant. The walls can represent the coordinate planes and the axes can be represented by the lines where the floor and walls meet.

An object in the room can thus be located by giving its distance from the walls and up from the floor.

Have the students assemble their graph panels as shown on page T-41. Discuss how the axes determine the planes and how distances are measured perpendicularly from a plane to a point not in the plane.

Have the students indicate a point in space by placing a standard (or some sort of tower) on the xy plane of the cardboard model. Relate each coordinate to its corresponding axis and plane.

✓ POINT

1. The symbol (x,y,z) is called an ordered triple. Explain why.

2. Explain what the number used for y tells us in the ordered triple (x,y,z). Explain what z tells us.

3. Which two axes lie in the xz plane?

EXERCISES

1. Give the coordinates of the fly's position as shown in Figure 4.

2. Which plane represents the floor, the xy plane, the yz plane, or the xz plane?

3. In the ordered triple (x,y,z), the number for y gives the distance of the point from which plane?

4. All points in the xy plane have a value of _?_ for the z coordinate.

5. The point (5,18,3) is located how many units from the xy plane?

6. What are the coordinates of the origin?

7. The point (2,4,0) lies in which plane?

8. The point (5,3,1) is located how many units from the xz plane?

9. The point (a,b,c,) lies in the xy plane. What is the value of c?

10. The point (r,s,t) is located 5 units from the xy plane, 7 units from the xz plane, and 4 units from the yz plane. Give the values for r, s, and t.

Let the students help each other, in small groups, learn to describe the coordinates of a given point and to determine the location of a point given its coordinates.

Discuss the importance of order in the term ordered triple.

✓ POINT ANSWERS

1. There are three numbers and the order in which they are given is important. Point (2,3,4) is not the same as point (4,3,2).

2. y tells the number of units that the point is located from the xz plane. z tells the number of units that the point is located from the xy plane.

3. The x and the z axes.

ANSWERS TO EXERCISES

1. (9, 6, 7)	5. 3 units	8. 3 units
2. xy plane	6. (0, 0, 0)	9. zero
3. xz plane	7. xy plane	10. r = 4, = 7, t = 5
4. zero		

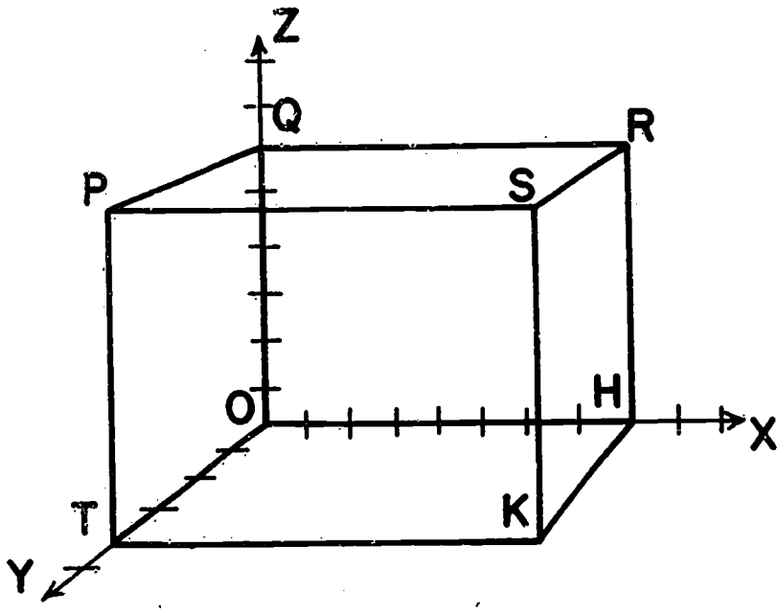


Figure 5

11. Give the coordinates of the following points.

T	_____	O	_____
P	_____	H	_____
Q	_____	K	_____

12. There is one coordinate which is the same for all of the points T, P, Q and O. Which coordinate has the same value for all of these points?

13. The ___?___ coordinate of the points T, K, and H, has the same value for each point.

14. The x-coordinate of the points R, S, K and H will have the same value for each point. What will x equal for these points?

ANSWERS CONTINUED

11. T : (0, 4, 0) O : (0, 0, 0)
    P : (0, 4, 6) H : (8, 0, 0)
    Q : (0, 0, 6) K : (8, 4, 0)

12. X = 0

13. Z

14. X = 8

A useful way for the students to see that order counts in giving coordinates is to have them consider all of the points which could have coordinates consisting of the numbers 5, 3 and 1.

ANSWER: (5, 3, 1) , (5, 1, 3) , (3, 5, 1),
(3, 1, 5) , (1, 5, 3) , and (1, 3, 5)

Many students have difficulty describing the coordinates which lie on an axis or in a coordinate plane. Give additional practice determining the coordinates of such points.

15. The z coordinate will have a value of 6 for which vertices in Figure 5, page 44?

16. The side of the box, in Figure 5, labeled OTKH is in the xy plane. Which coordinate, x, y or z is the same for all points in this plane?

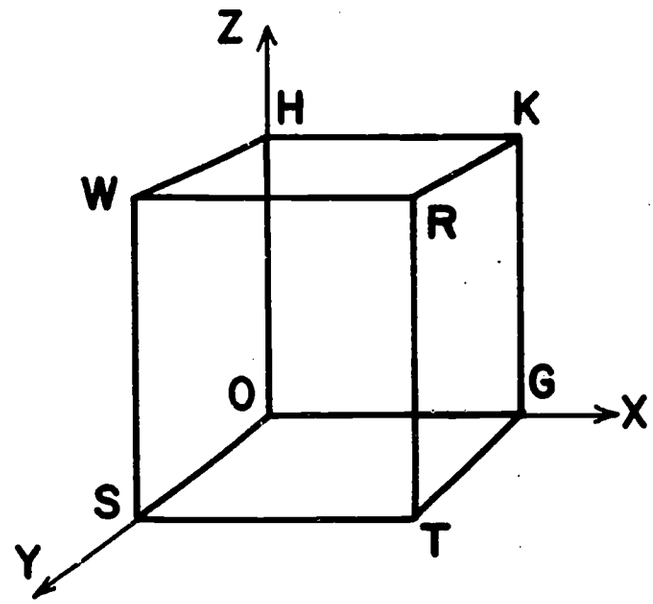


Figure 6

17. The box shown in Figure 6 is a cube.

a. Give the coordinates of each vertex. Use the length of each side of the cube as the unit distance on the scale.

O	_____	T	_____
H	_____	G	_____
W	_____	K	_____
S	_____	R	_____

b. There is a rectangular region determined by points O, H, R and T. What is true about the x and y coordinates of any point on this region?

ANSWERS CONTINUED

15. Q, R, S, and P

16. Z = 0

17. a.

O : (0, 0, 0)	T : (1, 1, 0)
H : (0, 0, 1)	G : (1, 0, 0)
W : (0, 1, 1)	K : (1, 0, 1)
S : (0, 1, 0)	R : (1, 1, 1)

b. x = y for every (x,y,z) in this region.

Use the transparency 5-1 to indicate how the axes can be scaled with an arbitrary unit distance. Scale and unit distance will be covered in more detail in later lessons. For the present it will suffice to mark off each axis with a selected unit distance and then relate this procedure to its influence on the coordinates of a given point.

You may have to explain the terminology used in Exercise number 17.

POWER QUESTIONS

1. Stuart coordinatized the vertices of a cube using stick-on labels. Unfortunately he dropped the cube and all but three of the labels fell off. (HINT: Get a cube and label the vertices as shown in Figure 7.) Which vertex did Stuart use as the origin?

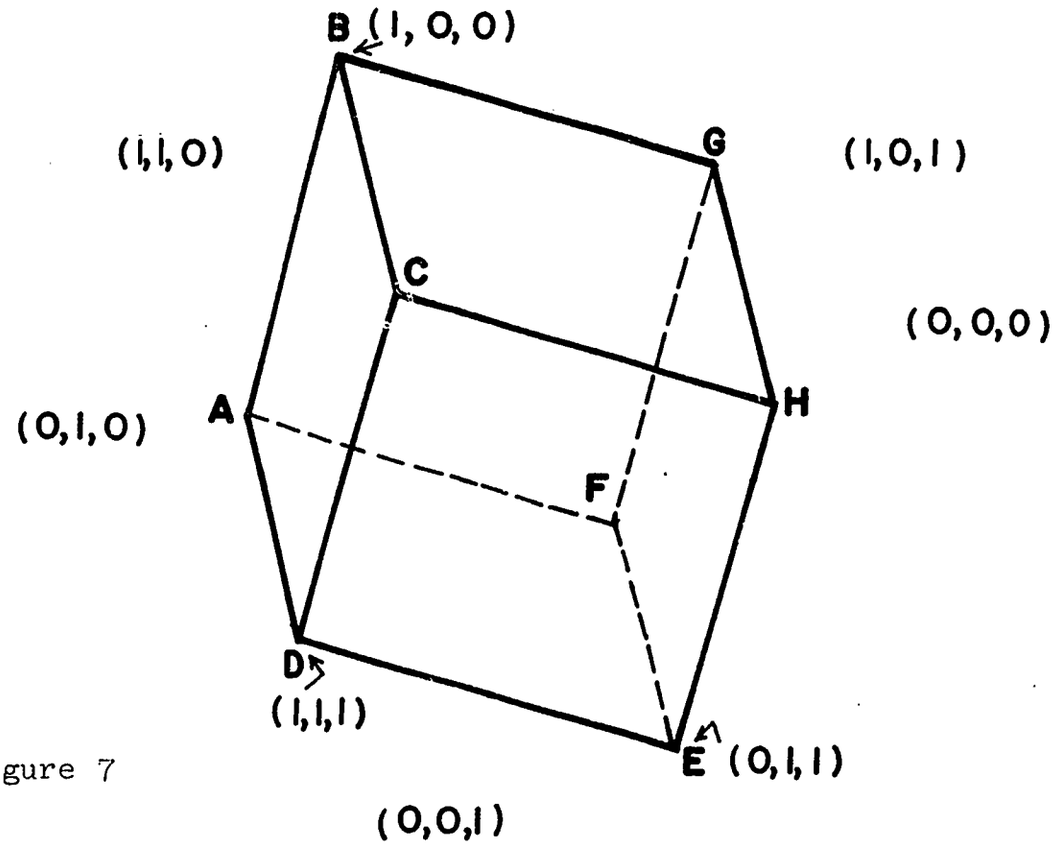


Figure 7

2. Explain where the point $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ is in Stuart's cube.

3. Which segment (edge of the cube) lies along the x-axis?

4. What are the coordinates of vertex F?

POWER QUESTIONS ANSWERS

1. Vertex G
2. At the center of the cube
3. G B
4. (0, 0, 1)

Note: The origin, G, must be on the opposite end of the diagonal from point D. Discuss this with your students.

THINGS TO DISCUSS

FOLLOW-UP DISCUSSION

1. How many axes are there? How many planes are determined?
2. Discuss the construction of the demonstration model. How were the axes scaled? (The same unit distance was used on all three axes.)
3. Locate the point whose coordinates are (5, 2, 8). Discuss the role played by each coordinate. For example, the x coordinate tells us how far the point is from the yz plane. Emphasize that this is a perpendicular distance.
4. (x, y, 2) stands for all the points where the last coordinate is two and the first two coordinates can be any numbers. Where are all of these points located?
5. Discuss Exercise 17. It is a difficult question.
6. Compare this coordinate scheme to those presented in Lessons 2 and 3. What is the same? What is different?
7. Discuss points having zero for one or more of their coordinates.
8. Discuss such questions as: The segment determined by (6, 4, 3) and (2, 8, 3) is parallel to what coordinate plane?

A PUZZLING LINE

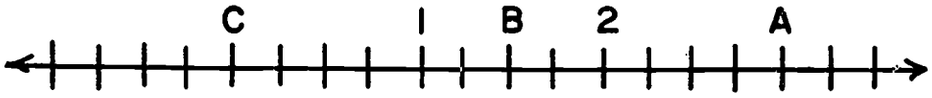
In an investigation, a detective only needs to find one or two clues to solve a mystery.

Use the clues given in the following questions to solve the puzzles on the number line.

EXERCISES

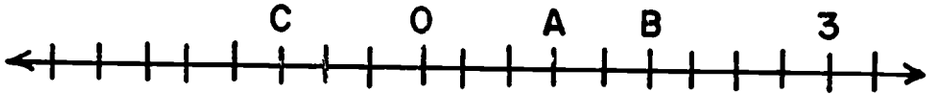
1. Fill in the missing numbers.

a)



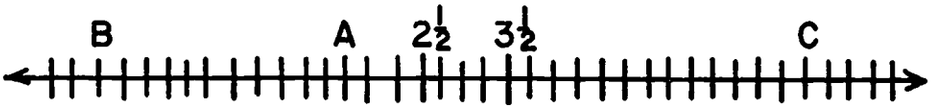
A = _____ B = _____ C = _____

b)



A = _____ B = _____ C = _____

c)



A = _____ B = _____ C = _____

A PUZZLING LINE

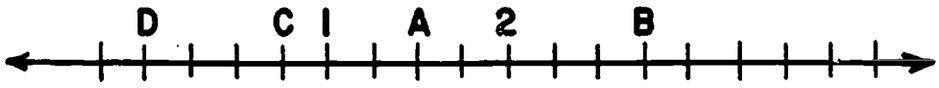
OBJECTIVES

The student will be able to:

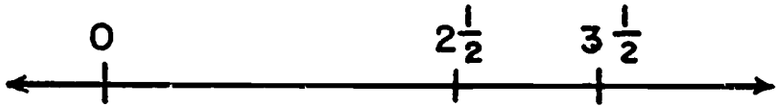
1. Indicate that positive numbers are located to the right of zero and that negative numbers are located to the left. (He realizes that this is by convention.)

2. Indicate that the larger number lies to the right of the smaller number. (under the usual convention).

3. Give the missing numbers (using the indicated scale).



4. Indicate that he understands the concept of unit distance by locating the number one on the given number line.



5. Indicate that he understands the idea of a constant unit in constructing a scale.

EQUIPMENT AND TEACHING AIDS

Each student should have a ruler or compass to lay off distances on the number line.

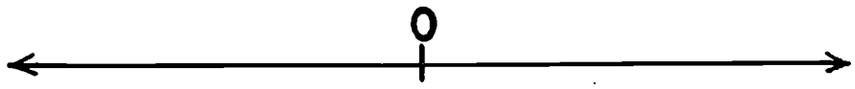
ANSWERS TO EXERCISES

1.	a.	$A = 3$	$B = 1\frac{1}{2}$	$C = 0$
	b.	$A = 1$	$B = 1\frac{2}{3}$	$C = {}^{-}1$
	c.	$A = 1\frac{3}{4}$	$B = {}^{-}1$	$C = 6\frac{3}{4}$

2. Positive numbers are greater than zero. Thus if, b is a positive number, this can be written as

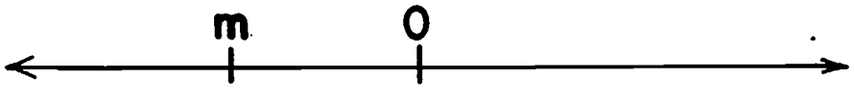
$$b > 0.$$

On which side of zero, on the number line, is b located if $b > 0$?



(answer by placing b on the correct side of zero)

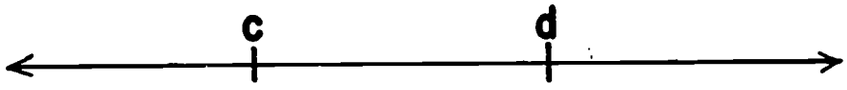
3. The number m is shown on the number line.



Is m a positive or negative number?

4. In question 3, what about the numbers between m and zero? Are they all positive; all negative; or some of each?

5. The numbers c and d are shown on the number line.



c is a negative number, d is a positive number. Make a mark on the number line where you think the number zero might be located.

ANSWERS CONTINUED

2. Anyplace to the right of zero.

3. Negative.

4. All negative.

5. Between c and d.

CONTENT AND APPROACH

This lesson offers a short review of the number line emphasizing the ordering of the rational numbers.

Review the number line concept with your students. A number line is constructed by:

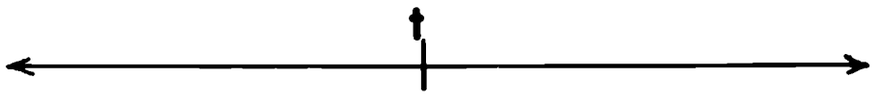
1. Drawing a line (not a line segment) and arbitrarily selecting a point on it to represent zero.

2. Selecting a unit distance and using this distance to mark off the scale. Emphasize the arbitrariness of the length of the unit line segment.

Essentially, the above procedure coordinatizes the given line. The numeral eight, for example, is the coordinate of the point located eight units to the right of the point whose coordinate is zero. The formal language of line coordinates appears in Lesson 11, Part 1.

Emphasize that the unit distance is constant throughout the scale. Show a contrasting situation where, say, the distance between 1 and 2 is different than the distance between 2 and 3 or 3 and 4, etc. (Note: Some scales are designed such that a non-constant unit scale is used. The logarithmic scale, for example.)

6. The number t is shown on the number line.



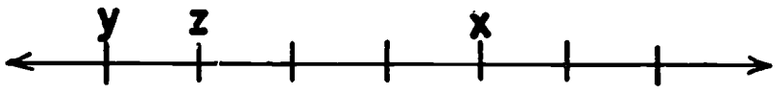
s is a number which is greater than t.

$$s > t$$

Place s on the number line so that it is on the correct side of t.

7. Do the statements about the numbers agree with their position on the number line?

Answer True, False, or Cannot tell.



Circle your answer.

a)	$x > y$	T	F	?
b)	$x > 0$	T	F	?
c)	$z < x$	T	F	?
d)	x is twice as large as y.	T	F	?
e)	$y < 0$	T	F	?

ANSWERS CONTINUED

6. On the right side of t

7. a. T, b. ? c. T, d. ?, e. ?

Negative integers, fractional number, decimals, etc., should be specifically indicated as the discussion progresses. Irrational numbers, however, are not specifically dealt with in this ninth grade program. The students will not, most likely, notice any "gaps" in the number line at this time.

THINGS TO DISCUSS

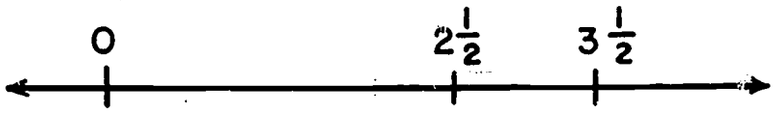
1. Review the steps in the construction of a number line.

2. What is the minimum that has to be known before the number, say $2\frac{1}{2}$, can be located on a number line? (Answer. Where zero is located, the unit distance, and which direction is positive or, where zero and one are located.)

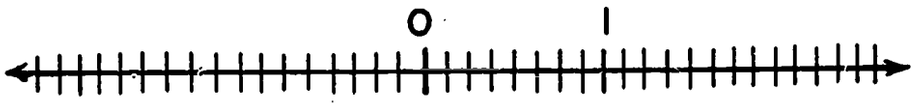
3. Emphasize that the unit distance is constant throughout the scale.

4. In Exercise 7, page 49, what effect would labeling the first mark to the right of z zero have on the answers a through e? What would result if one also labeled the mark just to the left of x one?

8. Locate, by some means other than guessing, the number one on this number line.



9. Place each given number on the number line.



a) $1\frac{7}{8}$ c) $^-2$ e) .75 g) $^-2.25$

b) $2\frac{1}{2}$ d) $^-1\frac{1}{3}$ f) 1.6 h) $\frac{1}{8}$

10. I am thinking of a number. It is not negative and it is not positive. What number am I thinking about?

POWER QUESTIONS

1. Tell what is wrong with this number line.

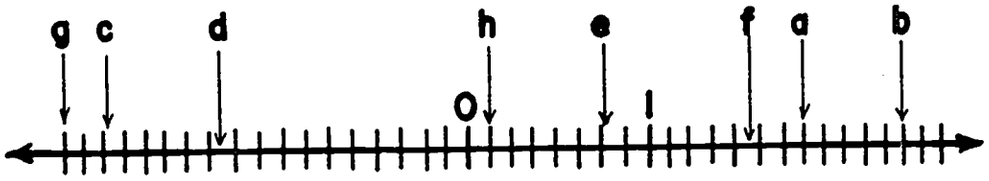
2. Construct your own number line. Make room enough to label the line segment with the integers from $^-5$ to 5. This line segment will be your <u>unit distance</u>. You can

use a ruler or a compass to make the number line.

ANSWERS CONTINUED

8. (Use a compass or a ruler.)

9.



10. Zero

ANSWERS TO POWER QUESTIONS

1. The unit distance is not constant.
2. Check to see that the unit distance is used.

THE LATTICE PLANE

Choose a point near the center of your lattice plane. Call this point O for origin.

Draw a vertical and a horizontal line through point O.

These two lines are called axes. Label the vertical axis y and label the horizontal axis x. Most mathematics textbooks call these axes the x-axis and the y-axis.

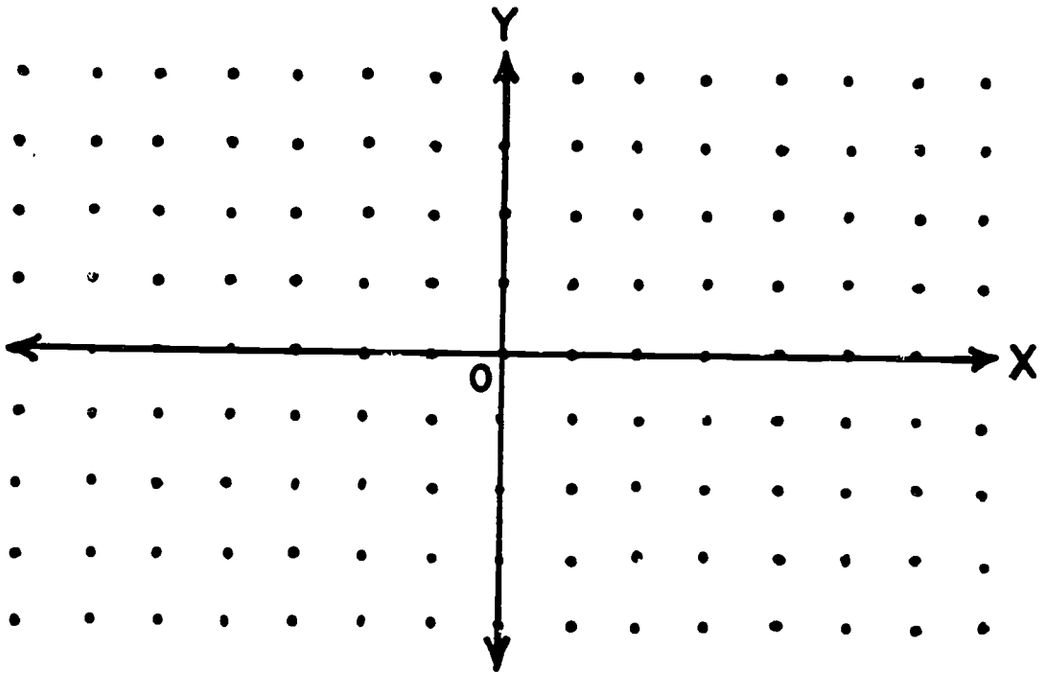


Figure 1

Label the lattice point, on the x-axis (on your paper) just to the right of the origin with a 1.

EXERCISES

1. Label the lattice point just to the left of the origin on the x-axis.

THE LATTICE PLANE

OBJECTIVES

1. To introduce the lattice plane.

2. The student will be able to plot points on the lattice plane using ordered pairs of integers.

3. Given the lattice point (i,j) the student will be able to identify the x and y-coordinates of this point.

EQUIPMENT AND TEACHING AIDS

Each student should have two or three sheets of lattice paper (rectangular coordinate paper is an acceptable substitute) and a straightedge. Use transparency 7-1 as a master to produce ditto sheets of lattice plane paper.

TRANSPARENCY

7-1. Lattice plane. The dots are in a square array—four squares per inch.

PURPOSES

1. To illustrate the plotting of points on the lattice plane using ordered pairs of integers.

NOTE: Do Exercises 1-6 with the class while discussing the plotting of points on lattice paper using the rectangular coordinate system in the plane. See pages T 53 through T 56. Then have the students work Exercises 7-23 individually.

ANSWERS TO EXERCISES

1. The point should be labeled $^{-}1$.

2. Label all of the lattice points on the x-axis with integers.

3. Label all of the lattice points on the y-axis with integers.

4. The coordinates of point A, figure 2, are (5,3). What are the coordinates of point B?

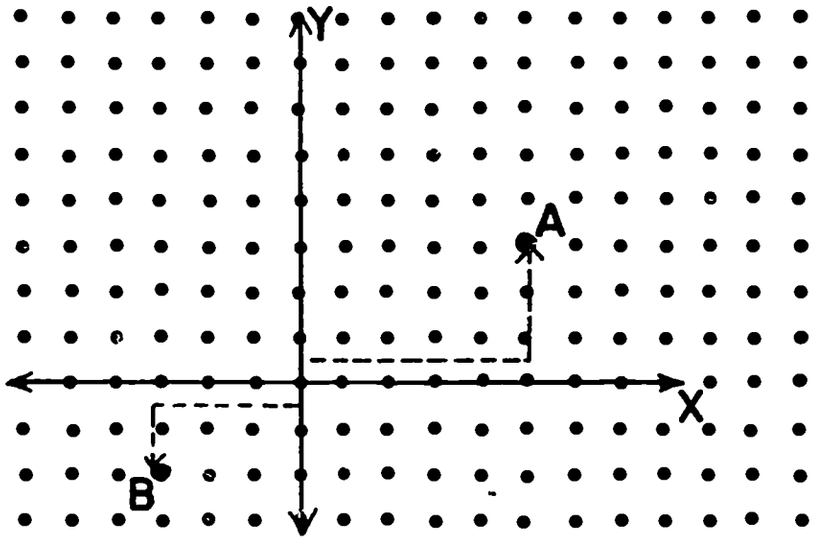


Figure 2

5. Start at the origin (on your paper) and follow these directions:

   (1) Count four lattice points to the right (the positive direction) on the x-axis. Mark this point. (Circle it lightly.)

   (2) Draw a vertical line through this point. The line should be parallel to the y-axis.

   (3) Starting at the origin again, count three lattice points up (the positive direction) on the y-axis. Mark this point also.

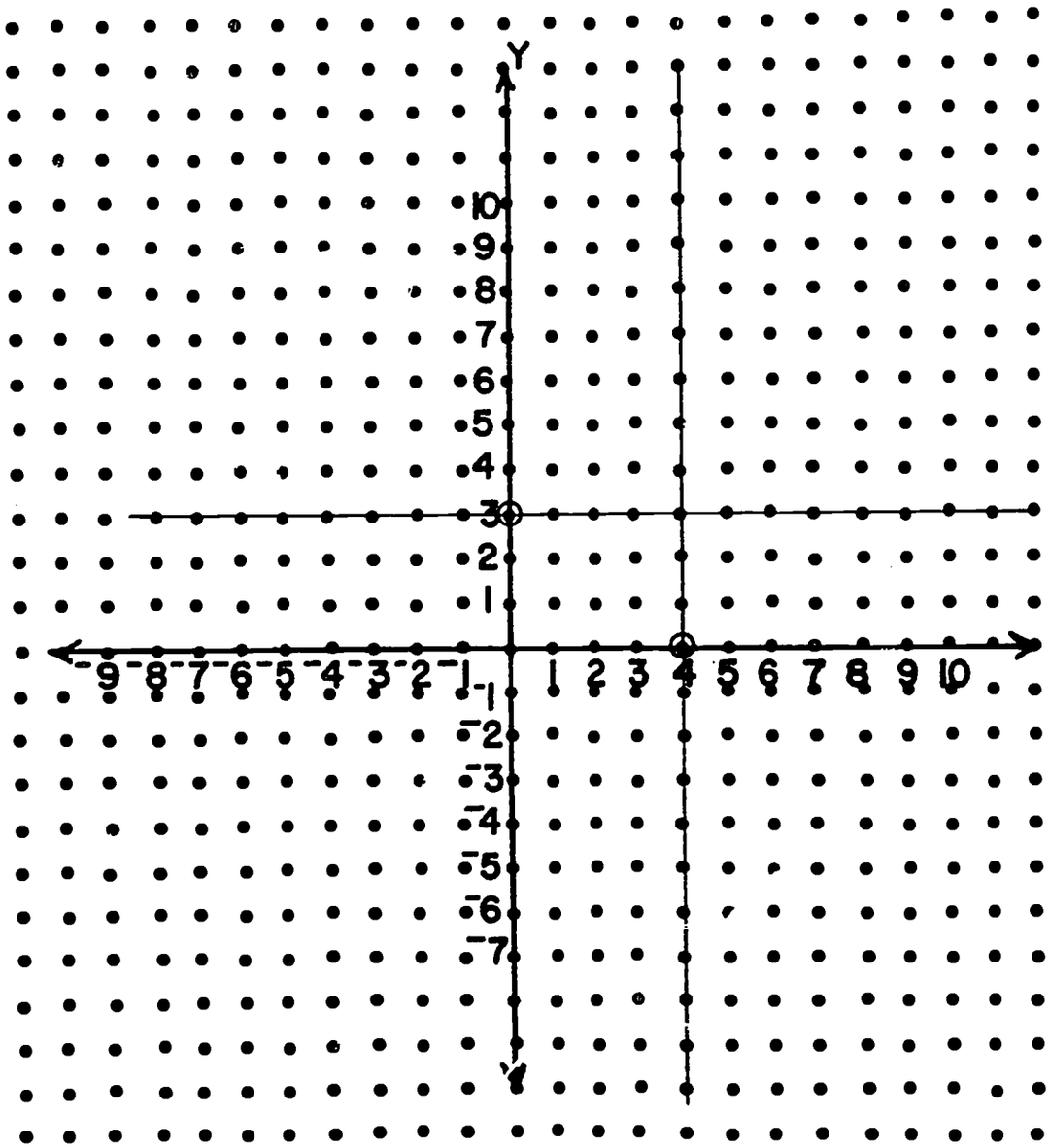
   (4) Draw a horizontal line through this point. The line should be parallel to the x-axis.

ANSWERS CONTINUED

2. and 3. Draw a pair of axes and label them on transparency 7-1.

4. ($^-3$, $^-2$)

5. (1) - (4)



6. The two lines you drew should intersect at a lattice point. Give the coordinates of this lattice point.

Figure 3 shows some lattice points labeled A, B, C, D, E, F, G, H, I.

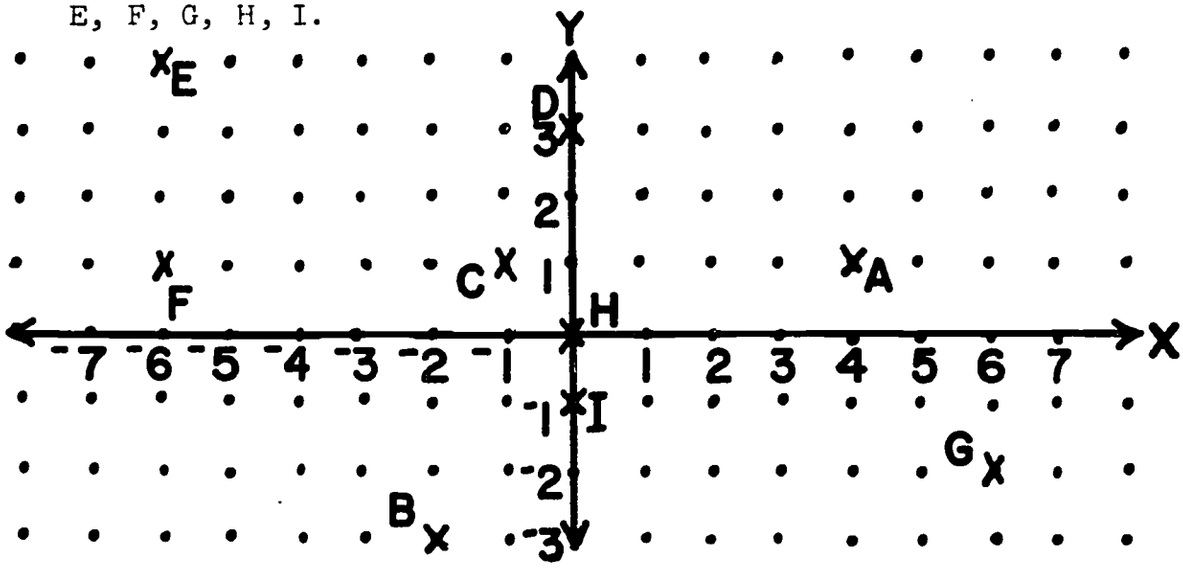


Figure 3

7. What is the x-coordinate of point A?

8. What is the y-coordinate of point B?

9. What are the coordinates of point E?

10. What are the coordinates of point G?

11. Some of the lettered lattice points have the same y-coordinate. Which ones are these?

12. List the lettered lattice points which have zero as their first coordinate.

13. Lattice points which have zero as their first coordinate are all on which axis?

14. The points whose coordinates are (1,0), (2,0), (3,0) (6,0), ($^{-}$4,0), and (0,0) are all on which axis?

ANSWERS CONTINUED

6. (4, 3)

7. 4 8. $^{-}3$ 9. ($^{-}6$, 4) 10. (6, $^{-}2$)

11. A, C, and F have y = 1.

12. D, H, and I

13. y-axis

14. x-axis

CONTENT AND APPROACH

By lattice plane we mean a plane of discrete points arranged in a rectangular array. A constant unit distance is used which is the same for both vertical and horizontal axes. The lattice points are usually described with integral coordinates. However, fractions could be used.

This lesson should help make the transition from a region - ordered pair correspondence (Lesson 1) to a point - ordered pair correspondence.

Much of the first part of this lesson should be handled by the teacher using the lattice plane transparency. Use the terms: axis, first coordinate, second coordinate, lattice points (as differentiated from the points which could be interpolated elsewhere in the plane), ordered pair of integers, positive direction, negative direction, parallel and perpendicular. Establish the conventional order, i.e., (x,y).

15. Lattice points which have zero as their second coordinate are all on which axis?

16. The points whose coordinates are (0,8), (0,⁻2), (0,⁻3), (0,2) and (0,0) are on which axis?

17. Give the coordinates of the point which lies on both axes. What is this point called?

18. Draw a line through those lattice points whose x-coordinate is 6. (Draw the line on your paper.)

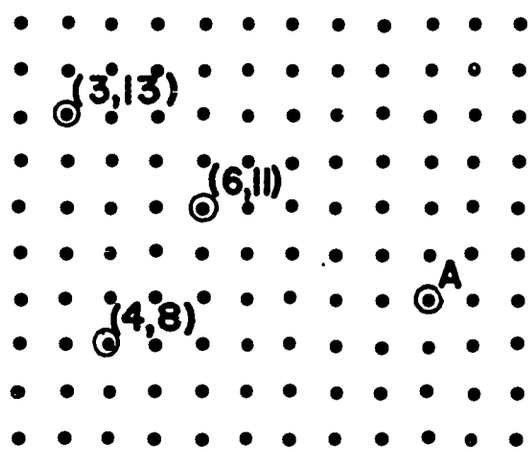
19. Draw a line through those lattice points whose y-coordinate is ⁻2.

20. The point of intersection of the lines is _____. (Give the coordinates of this point.)

21. All of the lattice points which lie on the x-axis have _____ for their second coordinate.

22. Some lattice points have x and y-coordinates that are the same. Draw a line through these points.

23. Part of the lattice plane is shown below. Use the given information to determine the coordinates of point A.



ANSWERS CONTINUED

15. x-axis

16. y-axis

17. (0,0); origin

18. The student should draw a vertical line through the point (6, 0).

19. The student should draw a horizontal line through the point (0, $^-2$)

20. (6, $^-2$)

21. zero

22. The student should draw the line whose equation is $y = x$.

23. (11, 9). Point out that only two of the three given points are necessary to obtain the answer.

THINGS TO DISCUSS

1. Select a lattice point near the center of the paper and call this the origin. The students should see the arbitrariness of the location of the origin.

2. Indicate how the two axes are like two number lines. Discuss scale, labeling with integers, and the constant unit. Review some of Lesson 6 if necessary.

3. Discuss the terminology. The students should view the term rectangular coordinate system in terms of "the axes are perpendicular thus forming right angles." The students should recognize the referents for the terms origin, axes, and coordinates. There is no need for them to master any verbal definitions.

POWER QUESTIONS

1. Several incomplete coordinates are given on the lattice plane below. Find the point which is the origin of the indicated rectangular coordinate system. Circle your answer.

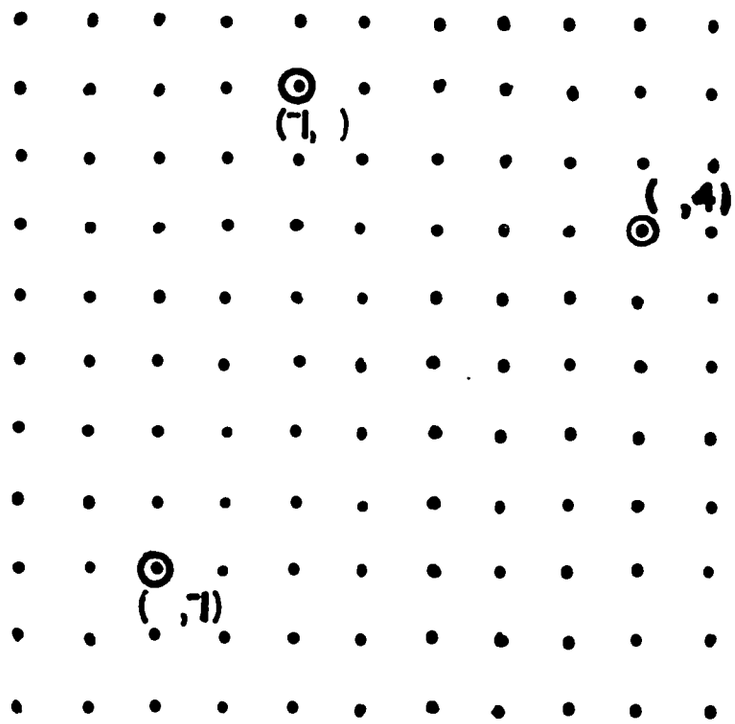


Figure 4

THINGS TO DISCUSS CONTINUED

4. Establish what each coordinate means in terms of its relationship to the axes.

   For example: How far is the point $(5, 3)$ from the x-axis? From the y-axis?

   Which point is nearer the y-axis(x-axis): $(5, 2)$ or $(7, 4)$? ; $(^-2, 8)$ or $(3, ^-5)$?

5. Focus on sets of points which lie on lines parallel to the axes.

   For example: What do the points: $(6, 2)$, $(6, ^-5)$, $(6, 8)$, $(6, 0)$,... have in common? Where do these points lie on the lattice plane?

6. Focus on the coordinates of the points which lie on the axes. Students should view the points on the x-axis, for example, not in terms of ... $^-3$, $^-2$, $^-1$, 0, 1, 2, 3,..., but in terms of ..., $(^-3, 0)$, $(^-2, 0)$, $(^-1, 0)$, $(0, 0)$, $(1, 0)$, $(2, 0)$, $(3, 0)$,... .

ANSWERS TO POWER QUESTIONS

1. The students should be able to check their own answer by counting from their choice of origin to the given points.

2. Draw an x-axis and a y-axis on a new lattice plane. Make the x-axis run the length of the paper. For each exercise plot the lattice points and connect them with a line in order.

   a) $(^-1, 5)$, $(^-1, 2)$, $(1, 2)$.

   b) $(^-9, 7)$, $(^-8, 4)$, $(^-7, 5)$, $(^-6, 4)$, $(^-5, 7)$.

   c) $(^-3, ^-3)$, $(^-4, ^-2)$, $(^-3, ^-1)$, $(^-2, ^-2)$, $(^-3, ^-3)$.

   d) $(6, ^-1)$, $(4, ^-1)$, $(4, ^-2)$, $(5, ^-2)$, $(4, ^-2)$, $(4, ^-3)$, $(6, ^-3)$

   e) $(^-2, 4)$, $(^-4, 4)$, $(^-4, 5)$, $(^-3, 5)$, $(^-4, 5)$, $(^-4, 6)$, $(^-2, 6)$

   f) $(^-6, ^-2)$, $(^-7, ^-1)$, $(^-8, ^-1)$, $(^-8, ^-3)$, $(^-7, ^-3)$, $(^-6, ^-2)$

   g) $(2, 3)$, $(2, 0)$, $(4, 0)$.

   h) There is a missing letter. Do you know what it should be? Describe the missing letter by giving the coordinates which could be used to draw this letter.

THINGS TO DISCUSS CONTINUED

Plot Points A (3, 2) and B ($^{-}4$, 5) on transparency 7-1 and ask questions about the coordinates.

Examples of some desired student responses:

1. Student: "Point A has coordinates (3,2). This means it is located three units to the right of the y-axis and two units above the x-axis."

2. Student: "Point B has coordinates (-4, 5). This means that the x-coordinate is -4 and the y-coordinate is 5."

3. Student: "All points whose x-coordinate is negative lie to the left of the y-axis."

Start the students on exercises 7-23.

ANSWERS TO POWER QUESTIONS CONTINUED

2. The points and connecting segments form the words WELL DONE.
   a. L b. W c. O d. E e. E f. D
   g. L h. The missing letter is N. A possible answer is: (0, $^{-}3$), (0, $^{-}1$), (2, $^{-}3$), (2, $^{-}1$).

NOTE: See Item 6 in Appendix C for a supplementary activity.

PICTURE PLOTTING

EXERCISES

1. In the first table of ordered pairs given in Exercise 2 below, the largest value of all of the first coordinates is 24. The smallest value for all of the first coordinates is ⁻22.

   a. The largest value for all of the second coordinates is ___?___.

   b. The smallest value for all of the second coordinates is ___?___.

   In numbering the x-axis you will have to go from ⁻22 to 24. This means using the unit distance to mark off 22 units to the left and 24 units to the right on the x-axis.

   c. How many total units are necessary on the x-axis?

   d. How many total units are necessary on the y-axis?

   e. How can you use the above information to make the points fit on your graph paper?

2. Plot the following points and connect them with segments in the order that you plot them.

a.	(⁻22,9)	j.	(⁻4,14)
b.	(⁻21,8)	k.	(0,10)
c.	(⁻17,⁻6)	l.	(⁻3,10)
d.	(15,⁻6)	m.	(⁻4,14)
e.	(24,6)	n.	(⁻9,14)
f.	(11,6)	o.	(⁻8,10)
g.	(9,10)	p.	(⁻12,10)
h.	(5,10)	q.	(⁻14,8)
i.	(4,14)	r.	(⁻21,8)

PICTURE PLOTTING

OBJECTIVES

1. To introduce rational coordinates.
2. The student will be able to plot points with rational coordinates on a lattice plane.
   a. He will be able to construct and label the axes with integers.
   b. He will be able to use an appropriate scale.
3. The students will be able to connect points with line segments, in the order specified, to form a picture.

EQUIPMENT AND TEACHING AIDS

Each student will need a straightedge and four to six sheets of graph paper (5 squares per inch is satisfactory).

You may use transparencies 9-1 or 7-1 with this lesson. Transparencies can be made from the picture graphs, pages T 59 through T 61d, to show the pictures resulting from plotting the points in the Exercises.

CONTENT AND APPROACH

The students are to construct pictures by plotting points with rational coordinates and connecting the points with line segments.

Students should pay attention to the scale appropriate for the axes for each graph.

NOTE: The question of using a different scale on each axis will be treated in Lesson number 9. It may be wise not to discuss distortion due to different scales at the present time.

ANSWERS TO EXERCISES

1. a. 14, b. $^-6$, c. 46, d. 20, e. Let the x-axis run up and down on the graph paper instead of across.
2. Boat, see page T 59.

3. The flag is missing from the pole in Number 2 (see $(^-22,9)$). Design your own flag and give the coordinates of its points.

4. For the table of ordered pairs below determine the total number of units needed on the x-axis and the total number needed on the y-axis.

a.	$(^-3\frac{1}{2}, ^-10)$	l.	$(1\frac{1}{2}, 12\frac{1}{2})$
b.	$(^-3\frac{1}{2}, ^-15)$	m.	$(2, 3\frac{1}{2})$
c.	$(^-5\frac{2}{3}, ^-17)$	n.	$(2\frac{1}{2}, 2\frac{1}{2})$
d.	$(^-5\frac{2}{3}, ^-12)$	o.	$(2\frac{3}{4}, ^-5)$
e.	$(^-3\frac{1}{2}, ^-10)$	p.	$(3\frac{1}{2}, ^-6)$
f.	$(^-3\frac{1}{2}, ^-6)$	q.	$(3\frac{1}{2}, ^-15)$
g.	$(^-2\frac{3}{4}, ^-5)$	r.	$(5\frac{2}{3}, ^-17)$
h.	$(^-2\frac{1}{2}, 2\frac{1}{2})$	s.	$(5\frac{2}{3}, ^-12)$
i.	$(^-2, 3\frac{1}{2})$	t.	$(3\frac{1}{2}, ^-10)$
j.	$(^-1\frac{1}{2}, 12\frac{1}{2})$	u.	$(3\frac{1}{2}, ^-15)$
k.	$(0, 14\frac{1}{3})$	v.	$(^-3\frac{1}{2}, ^-15)$

Plot the points and connect them with segments in the order that you plot them.

CONTENT AND APPROACH CONTINUED

The student will be able to check his work against the consistency of the pattern in the picture. However, he will have to be able to estimate the location of some of the points, especially those with fractional coordinates. You may wish to briefly review Lesson number six.

There is a question as to how many graphs a student will be able to complete in a class period. You may wish to have the students do only a selected number of the seven possible graphs. You may wish to have your students develop their own picture graph.

THINGS TO DISCUSS

Help start the students out on picture number one. They may need to be reminded of the following:

a. Scan the table of coordinates to find the minimum and maximum values for the first and second coordinates. This information is then used to determine the placement of the axes and an appropriate scale.

b. Once a scale is selected, (e.g., 1,2,3,4,...; 2,4,6,8,...; 5,10,15,20,...) it should be used on both axes. See NOTE page T 57.

ANSWERS TO EXERCISES CONTINUED

3. Answers will vary from student to student.

4. Units on x-axis $11\frac{1}{3}$

   Units on y-axis $29\frac{1}{2}$

   Rocket, see page T 60

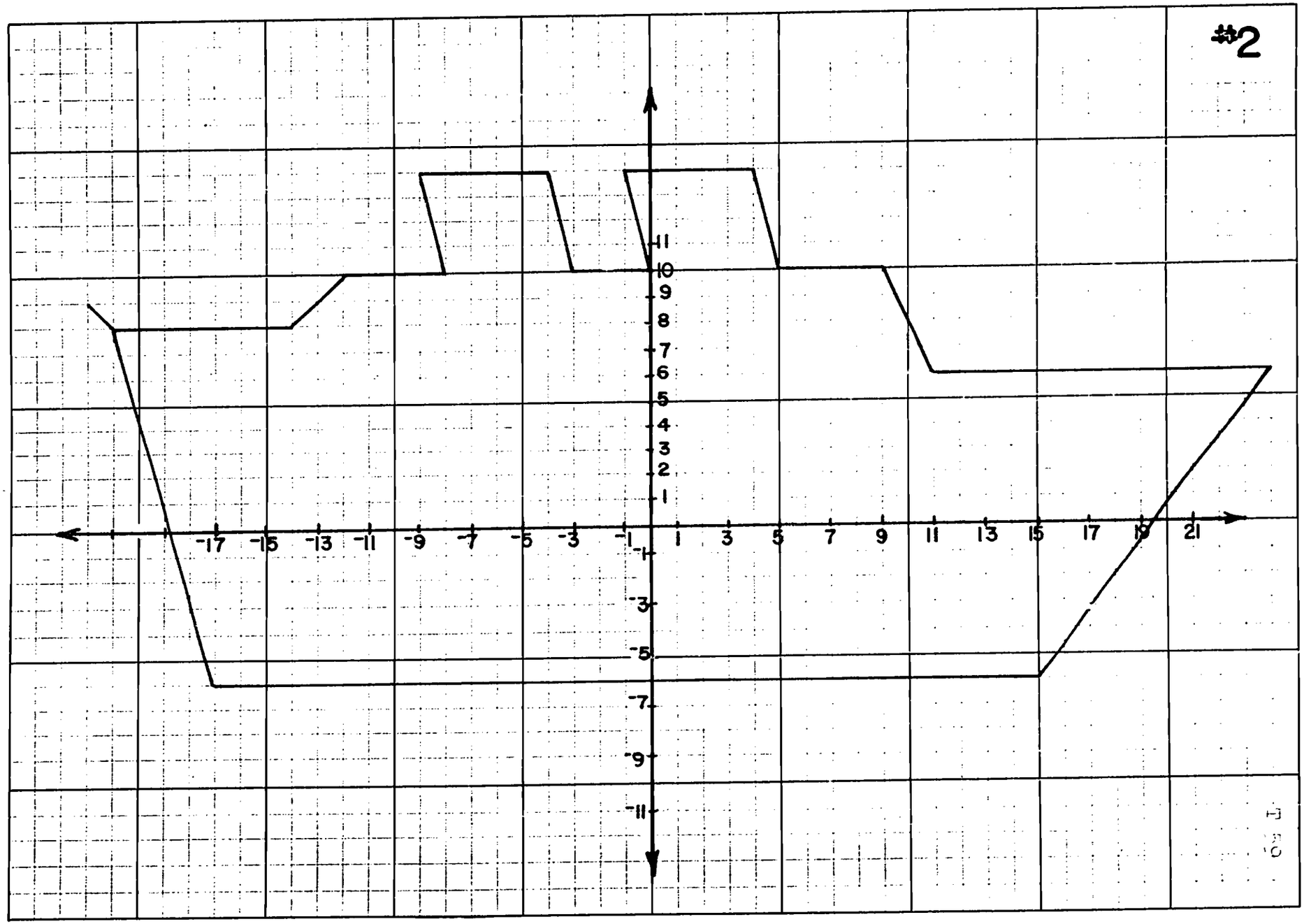
NOTE: Answers to Exercises 5-9 are on pages T 59 - T 61d.

5.

a.	(15,⁻20)	n.	(⁻14,20)
b.	(⁻2,⁻25)	o.	(⁻15,14)
c.	(⁻20,⁻42)	p.	(⁻19,14)
d.	(0,⁻32)	q.	(⁻14,12)
e.	(⁻5,⁻30)	r.	(⁻15,11)
f.	(15,⁻25)	s.	(⁻15,13)
g.	(20,20)	t.	(⁻27,⁻1)
h.	(15,30)	u.	(⁻25,⁻2)
i.	(⁻5,30)	v.	(⁻13,⁻1)
j.	(⁻14,20)	w.	(⁻11,⁻7)
k.	(⁻6,21)	x.	(⁻2,⁻5)
l.	(⁻4,19)	y.	(⁻8,⁻13)
m.	(⁻6,20)	z.	(0,⁻15)
		aa.	(⁻5,⁻30)

6.

a.	(⁻14,14)	n.	(⁻11,⁻15)
b.	(⁻15,11)	o.	(⁻2,⁻9)
c.	(⁻17,10)	p.	(⁻1,0)
d.	(⁻11,8)	q.	(⁻5,0)
e.	(2,13)	r.	(⁻5,⁻4)
f.	(17,8)	s.	(⁻4,⁻7)
g.	(26,8)	t.	(⁻6,⁻5)
h.	(37,15)	u.	(⁻7,0)
i.	(30,6)	v.	(⁻10,0)
j.	(12,4)	w.	(⁻18,6)
k.	(1,⁻4)	x.	(⁻21,4)
l.	(0,⁻10)	y.	(⁻22,5)
m.	(⁻11,⁻17)	z.	(⁻20,9)
		aa.	(⁻14,14)


#2
11
10
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2
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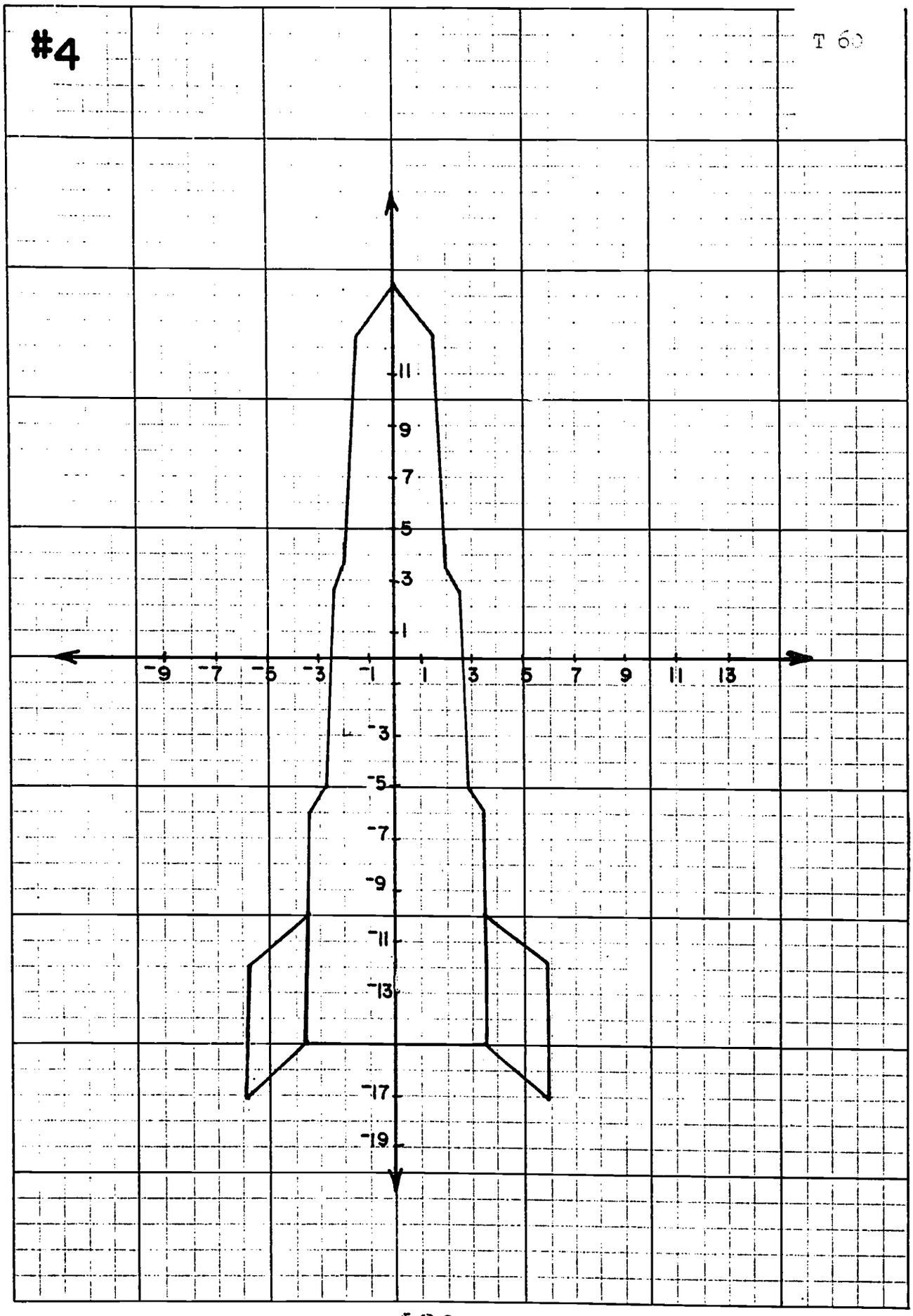
-11
-9
-7
-5
-3
-1
1
3
5
7
9
11
13
15
17
19
21

7.
a. $(^-5, ^-12)$
b. $(^-4, ^-13)$
c. $(^-3, ^-12)$
d. $(^-3,4)$
e. $(1,4)$
f. $(1,2)$
g. $(^-1,2)$
h. $(^-3,4)$
STOP (Do not draw a line from h to i.)
i. $(1,2)$
j. $(1, ^-12)$
STOP
k. $(^-1,2)$
l. $(^-1, ^-12)$
m. $(0, ^-13)$
n. $(1, ^-12)$
o. $(0, ^-11)$
p. $(^-1, ^-12)$
STOP
q. $(3,7)$
r. $(5,5)$
s. $(5, ^-12)$
t. $(4, ^-13)$
u. $(3, ^-12)$
v. $(4, ^-11)$
w. $(5, ^-12)$
STOP
x. $(^-3, ^-12)$
y. $(^-4, ^-11)$
z. $(^-5, ^-12)$
aa. $(^-5,7)$
bb. $(3,7)$
cc. $(3, ^-12)$

8.
a. $(8,2)$
b. $(6,0)$
c. $(^-8,3)$
d. $(^-8, ^-2)$
e. $(10, ^-5)$
f. $(8, ^-7)$
g. $(^-10, ^-4)$
h. $(^-10,5)$
i. $(8,2)$
j. $(8, ^-3)$
k. $(^-6,0)$
l. $(^-6, 2\frac{1}{2})$
STOP
m. $(^-6,0)$
n. $(^-8, ^-2)$
STOP
o. $(6,0)$
p. $(6, ^-2\frac{1}{2})$
STOP
q. $(^-10,5)$
r. $(^-8,7)$
s. $(10,4)$
t. $(10, ^-5)$
STOP

#4

T 60



9.

a. $(1, 11\frac{1}{3})$

b. $(3\frac{1}{2}, 11)$

c. $(4, 10)$

d. $(3\frac{2}{3}, 9\frac{2}{3})$

e. $(2\frac{1}{2}, 10)$

f. $(^{-}\frac{1}{2}, 10)$

g. $(^{-}\frac{3}{4}, 12)$

h. $(^{-}1, 12\frac{1}{2})$

i. $(^{-}1\frac{1}{2}, 14\frac{1}{2})$

j. $(^{-}2\frac{1}{3}, 15)$

k. $(^{-}2, 15\frac{1}{2})$

l. $(^{-}3, 15\frac{1}{3})$

m. $(^{-}2\frac{2}{3}, 15)$

n. $(^{-}3, 15)$

o. $(^{-}4, 14)$

p. $(^{-}3\frac{1}{2}, 12\frac{1}{2})$

q. $(^{-}3, 12)$

(STOP

r. $(8, 10)$

s. $(9\frac{1}{2}, 10)$

t. $(11\frac{1}{2}, ^{-}\frac{1}{2})$

u. $(8\frac{1}{2}, ^{-}1)$

v. $(8\frac{1}{2}, ^{-}7)$

w. $(^{-}6\frac{1}{2}, ^{-}7)$

x. $(^{-}6\frac{1}{2}, ^{-}1)$

y. $(^{-}9\frac{1}{2}, ^{-}\frac{1}{2})$

z. $(^{-}7\frac{1}{2}, 10)$

aa. $(^{-}3\frac{1}{2}, 10)$

bb. $(^{-}4\frac{1}{3}, 11)$

cc. $(^{-}3\frac{1}{2}, 12\frac{1}{2})$

STOP

dd. $(4, 10)$

ee. $(8, 10)$

ff. $(7\frac{3}{4}, 12\frac{3}{4})$

gg. $(6\frac{3}{4}, 13\frac{1}{2})$

hh. $(6, 12\frac{2}{3})$

ii. $(6\frac{3}{4}, 11)$

jj. $(5\frac{1}{3}, 11)$

kk. $(4\frac{2}{3}, 11\frac{3}{4})$

ll. $(3\frac{2}{3}, 13)$

mm. $(2\frac{1}{3}, 13\frac{2}{3})$

nn. $(1\frac{1}{2}, 13\frac{1}{2})$

oo. $(0, 12)$

pp. $(^{-}\frac{3}{4}, 12)$

qq. $(^{-}\frac{1}{2}, 10)$

rr. $(^{-}1\frac{3}{4}, 10)$

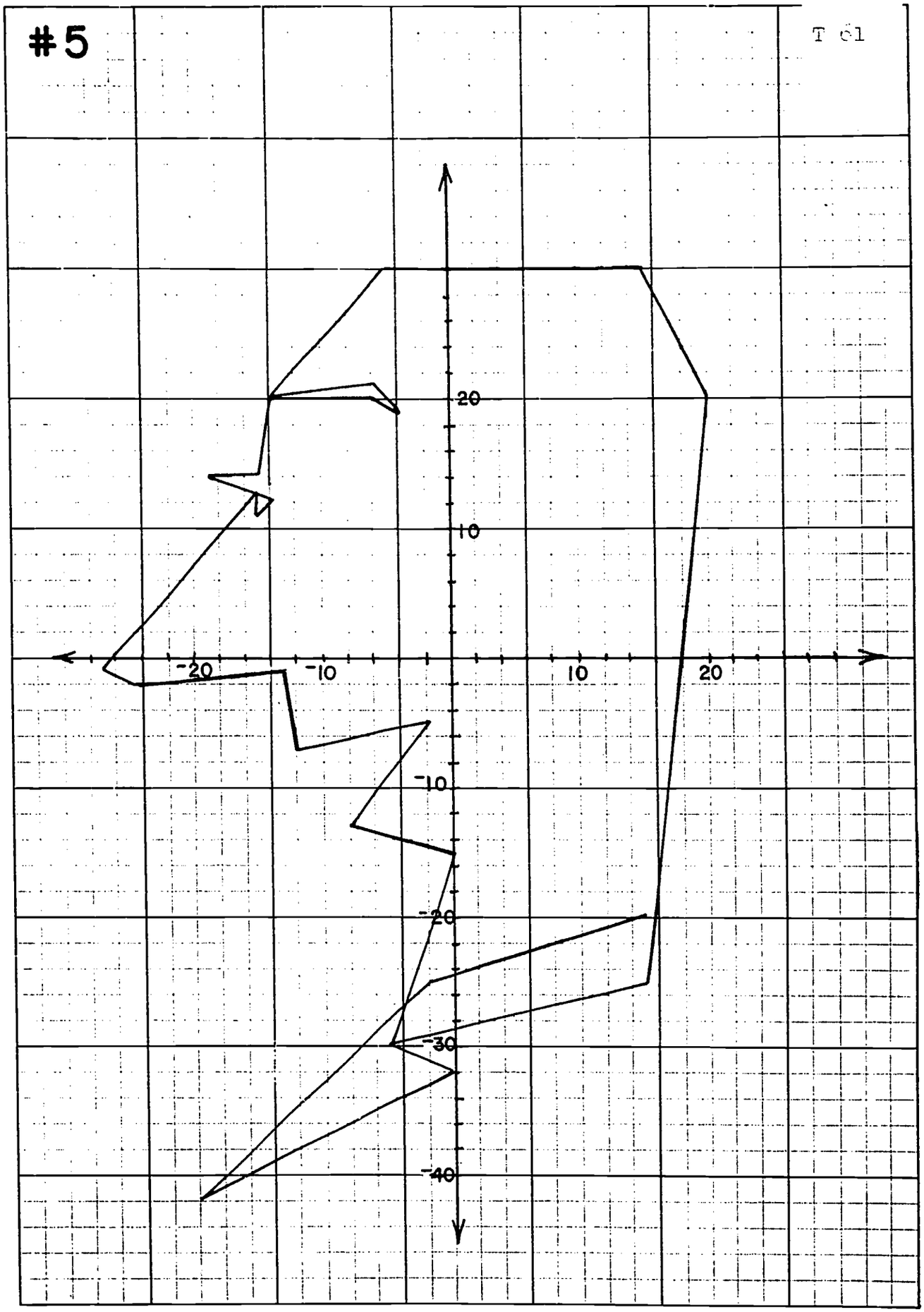
ss. $(^{-}2\frac{1}{3}, 7\frac{1}{2})$

tt. $(^{-}2\frac{1}{2}, 7)$

uu. $(^{-}3, 7\frac{1}{2})$

vv. $(^{-}3, 10)$

ww. $(^{-}3\frac{1}{2}, 10)$


#5
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-20
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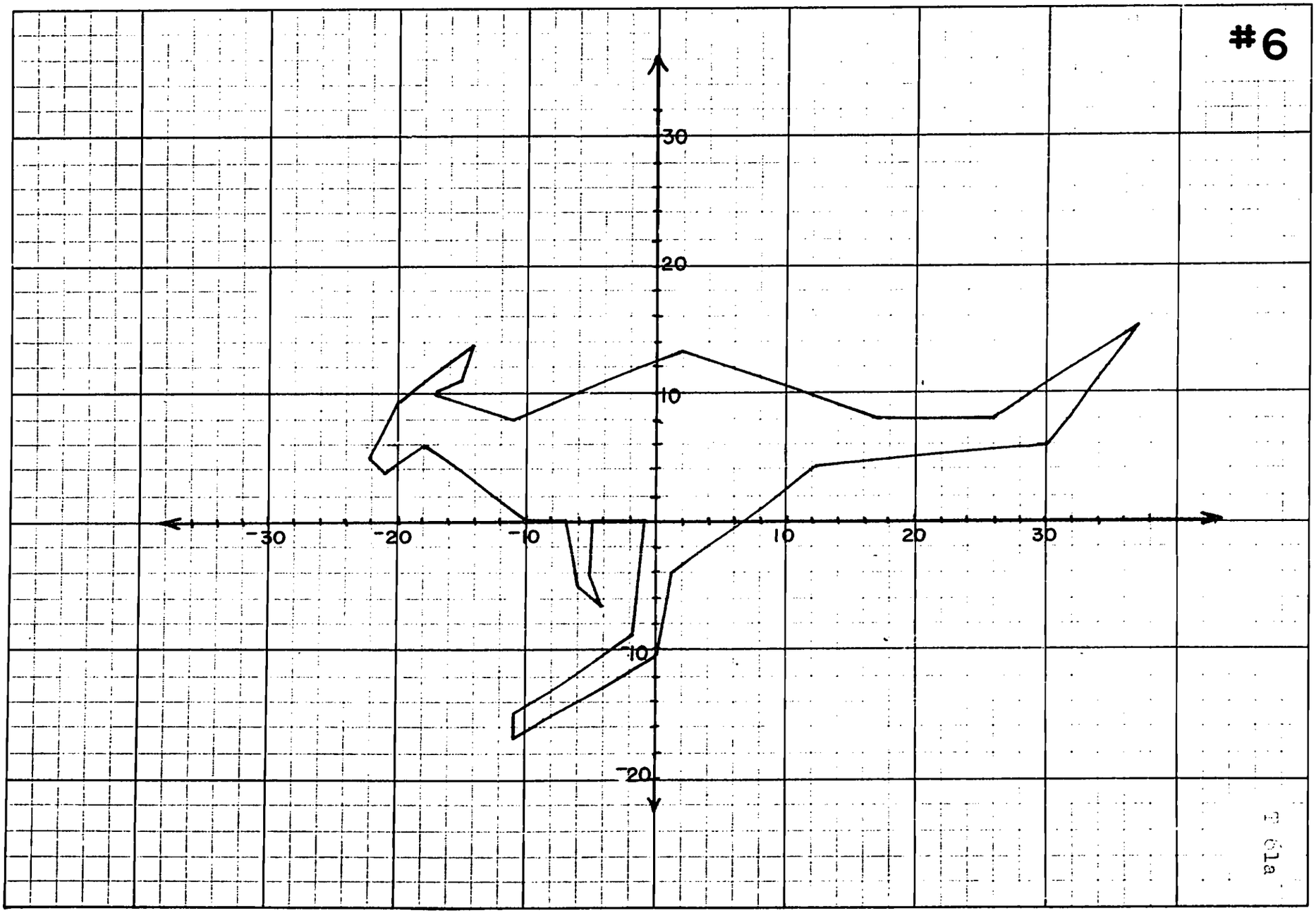
-30
-40


#6
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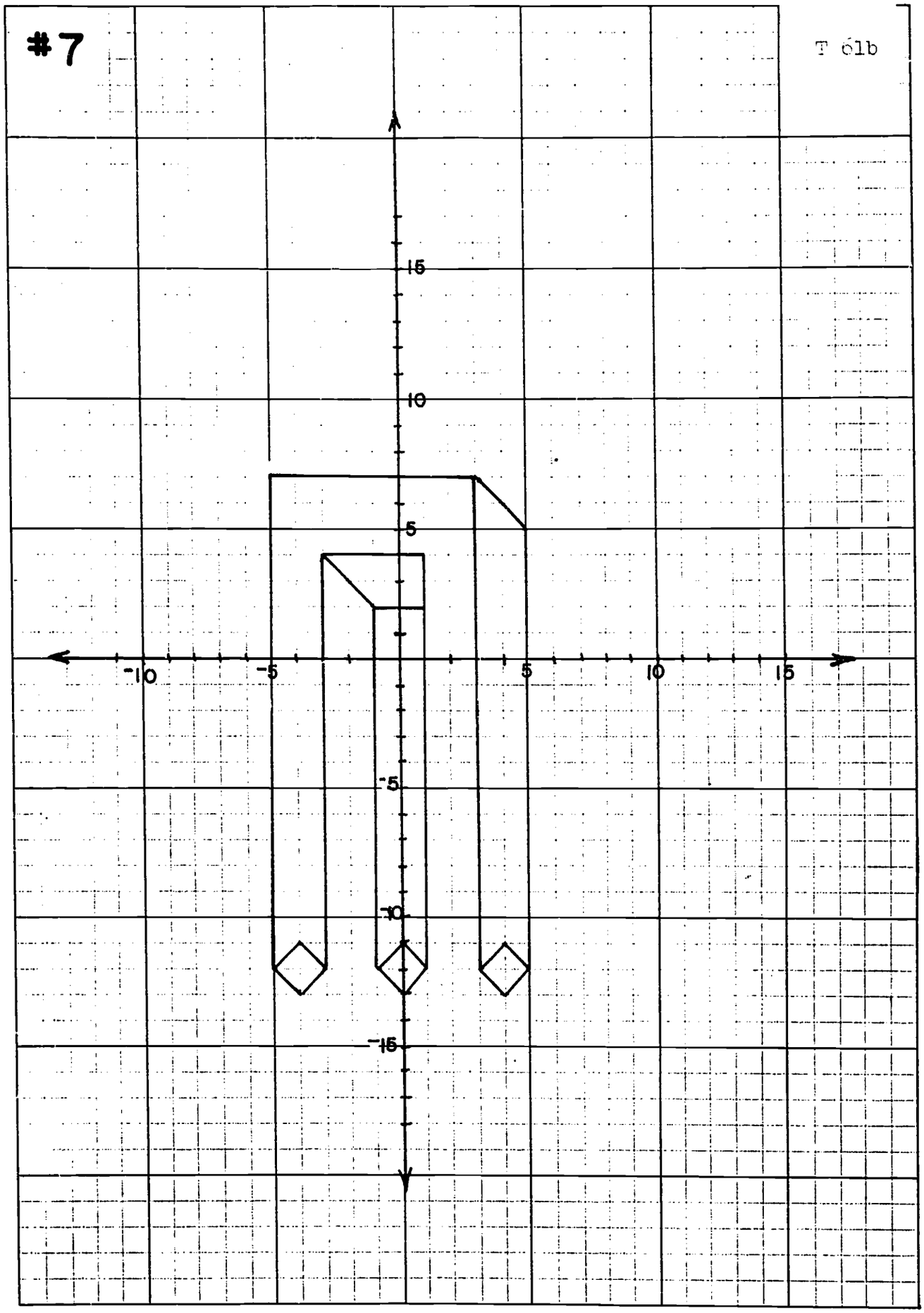
-10
-20
-30
-20
-10
10
20
30

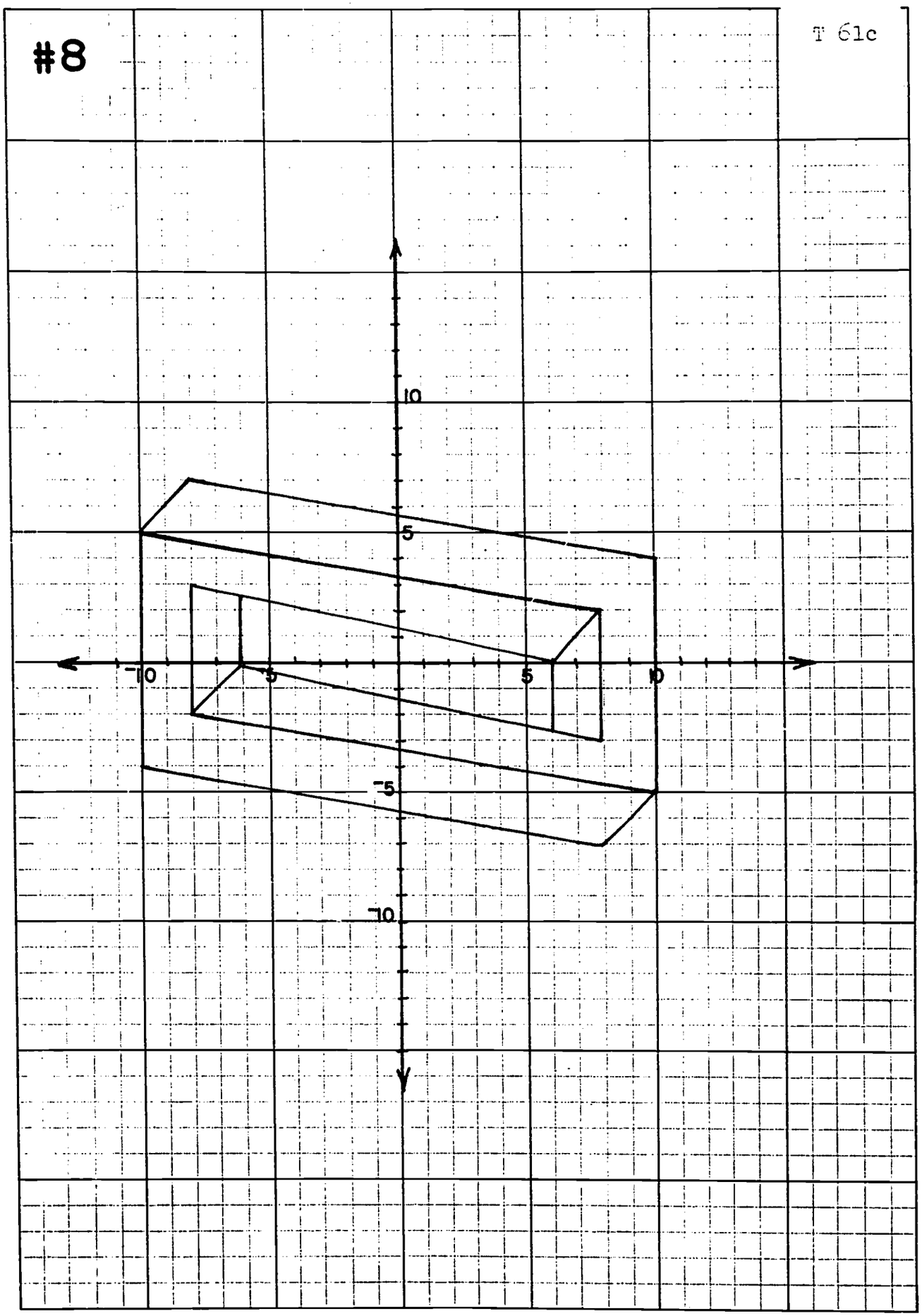
#7

T 61b



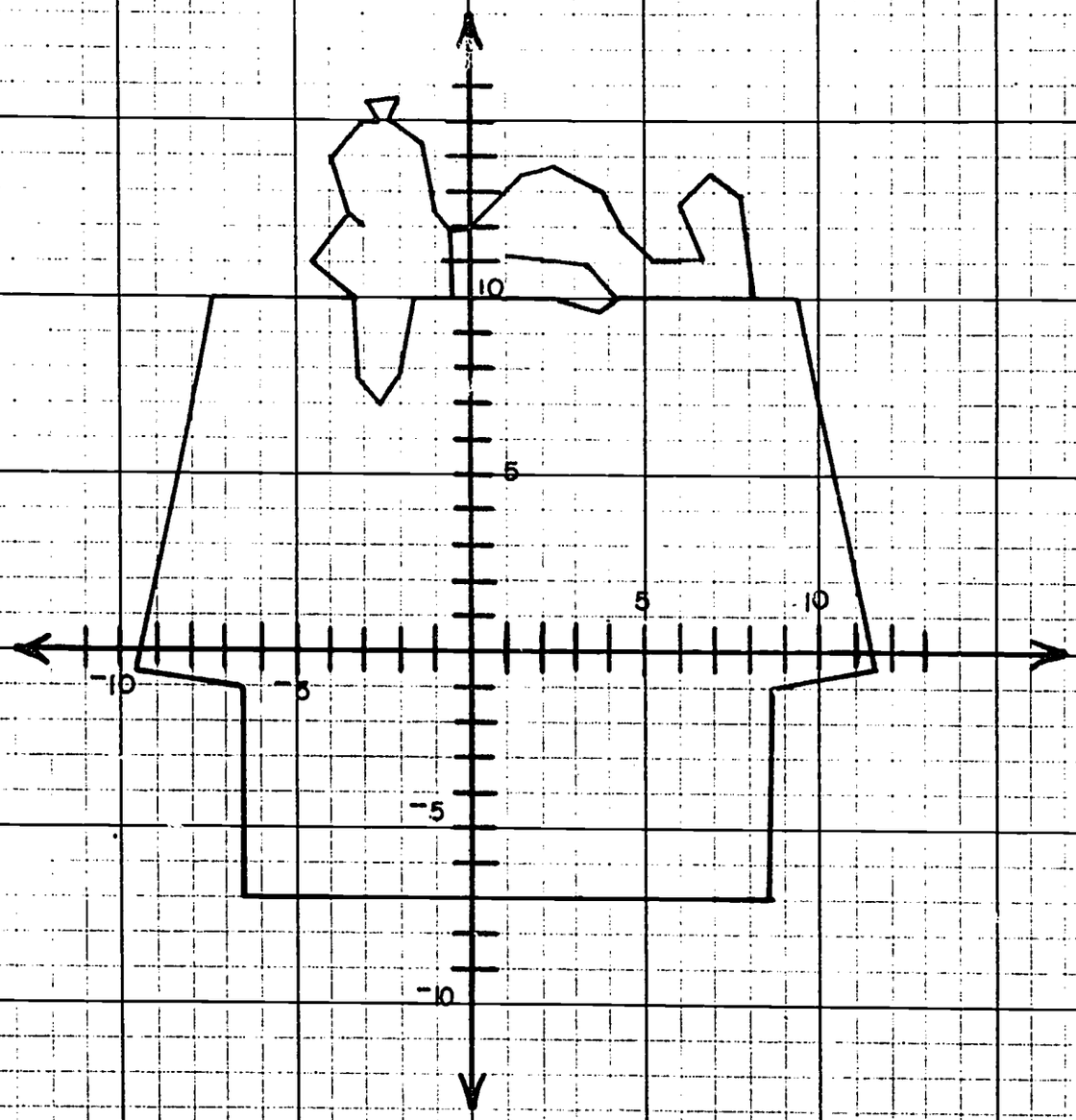
#8

T 61c



#9

SNOOPY ON HIS DOGHOUSE



PICTURE DISTORTION

In most cases a rectangular coordinate system is laid out so that the unit distance is the same on each axis.

Figure 1 shows a graph of a five-pointed star. The coordinates are given in the table.

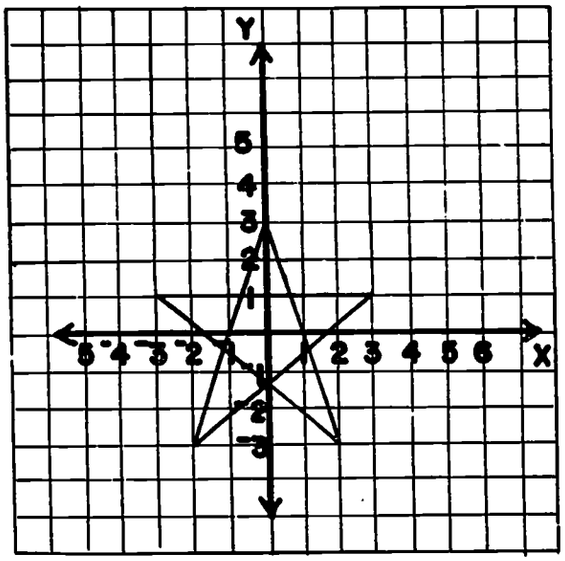


Figure 1

Table of coordinates for the five-pointed star

X	Y
-3	1
3	1
-2	-3
0	3
2	-3

Notice that the scale is marked off the same on each axis.

PICTURE DISTORTION

OBJECTIVES

1. The student will be able to scale the axes of a coordinate system.
   a. He understands the term unit distance. (See Lesson 6.)
   b. He recognizes the desirability of a constant unit distance.
   c. He recognizes that distortion occurs on a graph where the unit distances do not agree on each axis.
2. The student realizes that perpendicularity between the axes is a convention.
   a. He realizes that we are more familiar with rectangular coordinate systems because many things in everyday life are laid out rectangularly. (e.g., rooms, football fields, writing paper, etc.)
   b. He can plot points on a coordinate system with oblique axes.
   c. He realizes that distortion occurs on a graph when the angle between the axes is changed.

EQUIPMENT AND TEACHING AIDS

Each student should have 3-4 sheets of graph paper lined off in 5 squares per inch or 4 squares per inch, a ruler, a compass, and 2-3 sheets of unlined paper.

TRANSPARENCIES

9-1. Rectangular grid, 5 lines per inch.

PURPOSE

1. To illustrate graphs where the scale is not the same on each axis.

9-2. Use transparency 7-1, lattice plane.

PURPOSE

1. To illustrate oblique axes.

The five-pointed star in Figure 2 has the same coordinates as the star shown in Figure 1. However, the two stars do not look the same.

Marking off the scale differently on each axis caused <u>distortion.</u>

The scale on the x-axis in Figure 2 does not have the same unit distance as the scale on the y-axis.

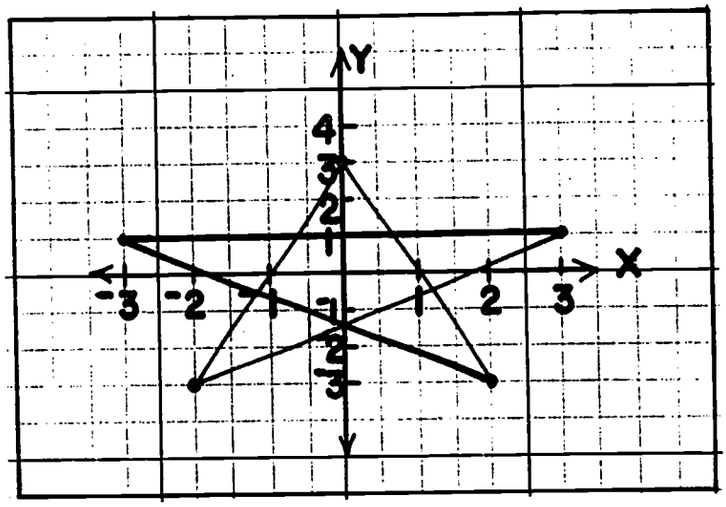


Figure 2

On the graph paper shown in Figures 2 and 3 there are 5 grid lines per inch. The grid lines are the vertical and horizontal lines that make up the graph paper.

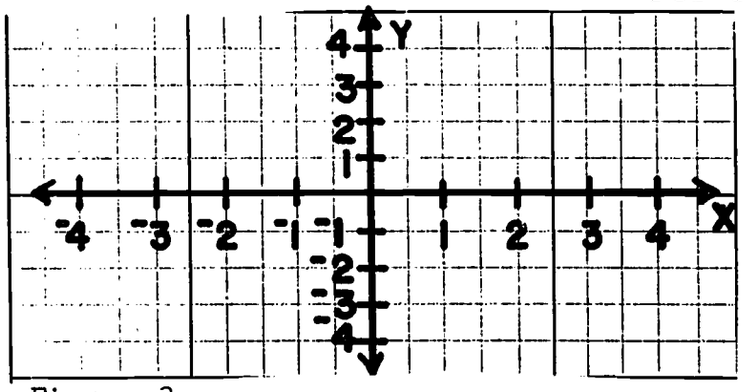


Figure 3

For every 2 grid lines cutting the x-axis there is one grid line cutting the y-axis.

$$\frac{\text{x-axis unit distance}}{\text{y-axis unit distance}} = \frac{\text{2 grid lines}}{\text{1 grid line}}$$

CONTENT AND APPROACH

The main purpose of this lesson is to further acquaint the student with the rectangular coordinate system by exposing him to deviations caused by altering the scale on the axes or the angle between them.

In this lesson, geometric figures are plotted in the familiar rectangular coordinate system. Then either the identity of scales restriction or the perpendicularity of axes restriction is altered. The "same" figure (i.e., the same coordinates) are then plotted in this new system. The two figures are compared.

The exercises for this lesson need considerable class discussion. The teacher is encouraged to explore and extend the discussion to challenge the students' abilities. For example, the students could determine what geometric properties remain invariant after a change in scale or angle between axes.

The grid lines on the graph paper provide a convenient means of comparing the unit distances used on the axes. In Figure 3, page 63, the unit distance on the x-axis is "2 grid lines long" and the unit distance on the y-axis is "1 grid line long". Thus, the ratio of the two unit distances is 2 to 1.

Although the same coordinates were used, the star in Figure 2 is twice as wide, but the same height as the star in Figure 1. To explain this we compare the unit distances used in the two graphs.

EXERCISES

1. Number the y-axis 1, 2, 3, and so on, using every third grid line on your graph paper. Number the x-axis using each grid line in turn. Number the negative part of each axis in the same manner.

   a. Thus, for every _____ grid line(s) on the y-axis there is one grid line on the x-axis.

   b. The ratio of the x-axis unit distance to the y-axis unit distance is ____?____ .

   c. Plot the coordinates of the five-pointed star. (Use the number pairs given in Figure 1.) Draw the star and compare its shape to those in Figures 1 and 2.

2. When the axes of a rectangular coordinate system are scaled so that the unit distances are the same; the coordinates

   (0,0), (2,0), (2,2), and (0,2)

   are vertices of a square. (See Figure 4.)

   a. Number the axis on your graph paper so that the unit distance on the x-axis is 5 grid marks long and the unit distance on the y-axis is 2 grid marks long.

   b. What is the ratio of the x-axis unit distance to the y-axis unit distance?

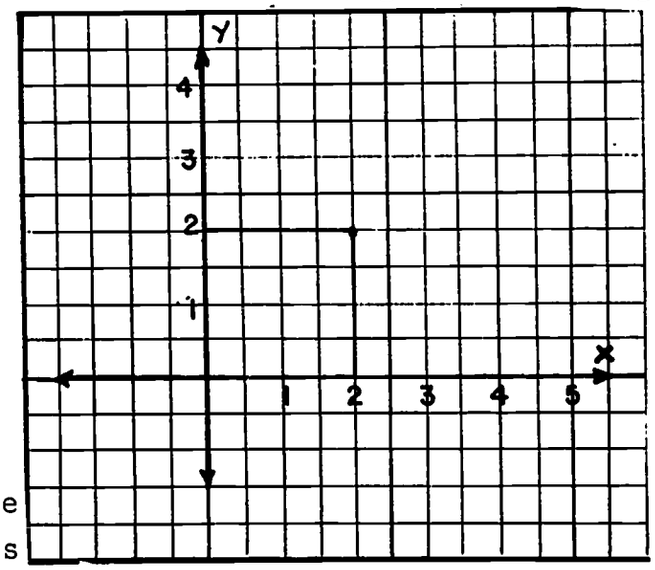


Figure 4

THINGS TO DISCUSS

The key points are:

1. There are times when it is desirable to have a different scale on one axis than the other. The x-coordinates, for example, may range over very large numbers. However, the use of different scales causes distortion.

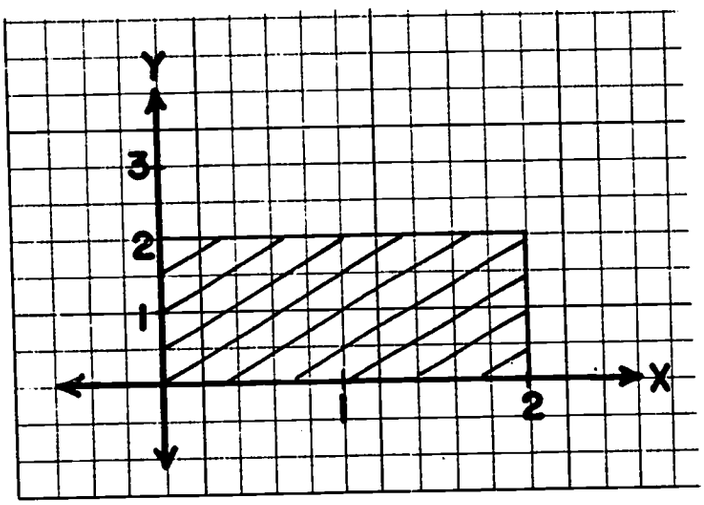
2. A unit distance is the length of the line segment between zero and one on the scale. It should be clear that the unit distance should be constant on an axis: (Refer to Lesson 6.) A slide rule is an example of a scale with no constant unit distance.

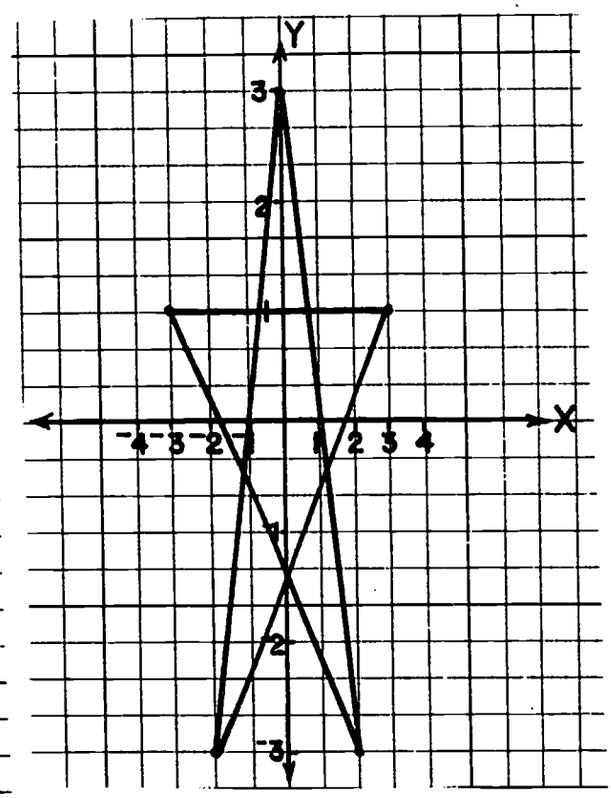
ANSWERS TO EXERCISES

1. a. 3, b. $\frac{1}{3}$, c.

2. b. $\frac{5}{2}$

2. c.





c. Plot the coordinates (0,0), (2,0), (2,2), and (0,2). Connect the points in the order given. Is this figure a square? Each side is two units long. What makes the difference?

3. How are axes scaled in Figure 5?

a. Number the axes on your graph paper so that for every 4 grid marks on the y-axis there is one grid mark on the x-axis.

b. What is the ratio of the x-axis unit distance to the y-axis unit distance.

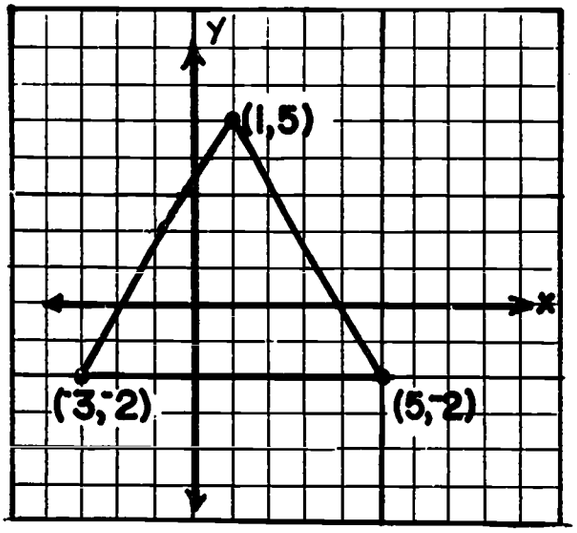


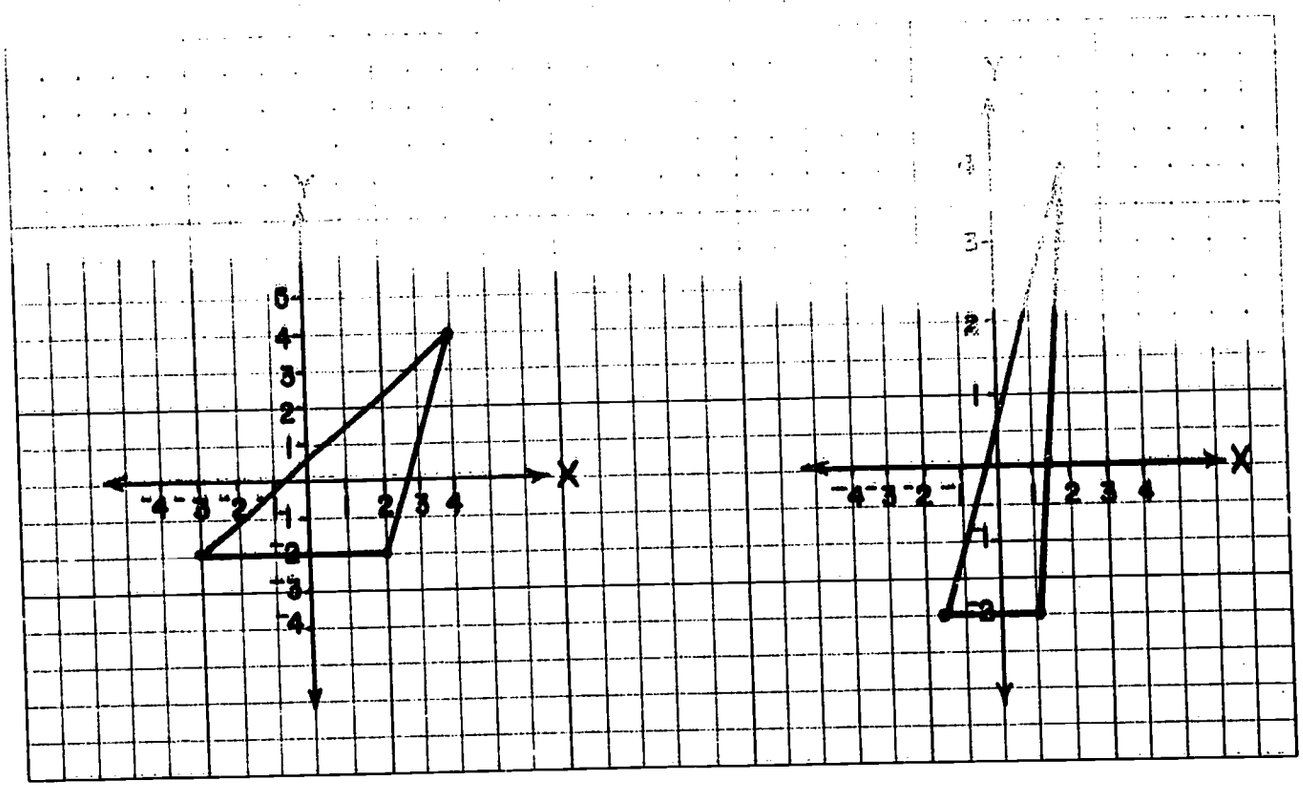
Figure 5

c. Plot the points (1,5), (5, ⁻2), and (⁻3,⁻2) and draw in the triangle. Does it appear to have the same shape as the triangle shown above?

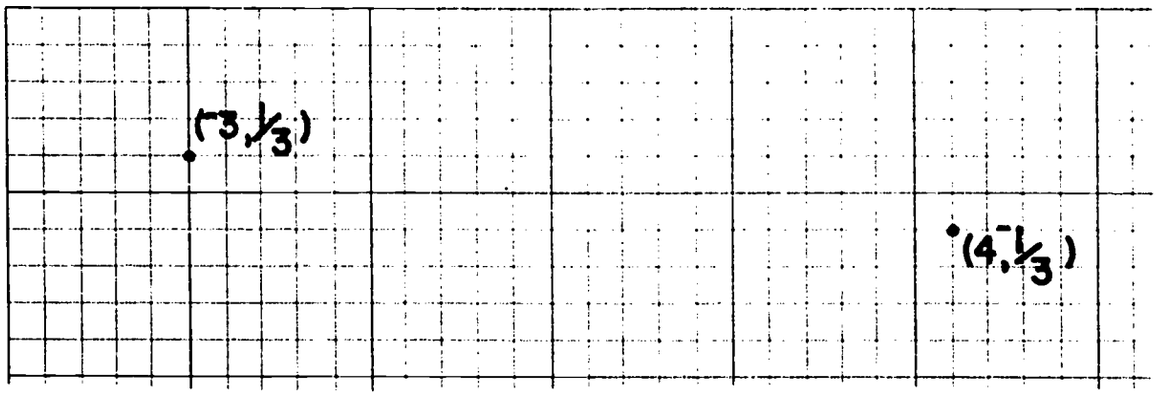
ANSWERS CONTINUED

2c. See graph on page T 64. In one sense the figure is a square because it is two units on a side. However, the unit distance on each axis are not equal so the figure is not a square.

THINGS TO DISCUSS CONTINUED



4. The coordinates of the points shown below are correct. Draw in the proper x-axis and y-axis unit distances.



A SECOND WAY TO DISTORT A GRAPH IS TO CHANGE THE ANGLE BETWEEN THE AXES.

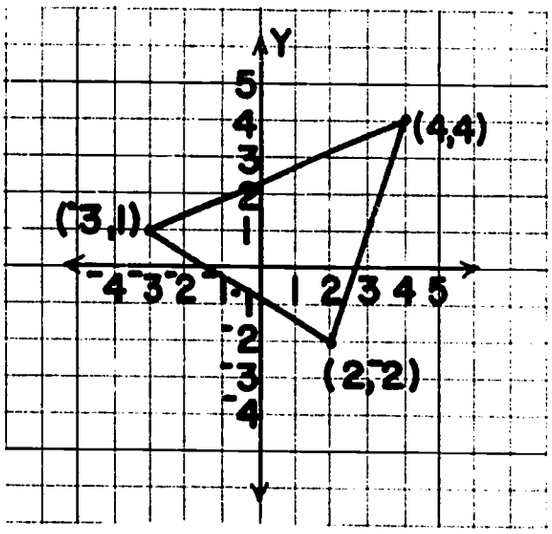


Figure 7a

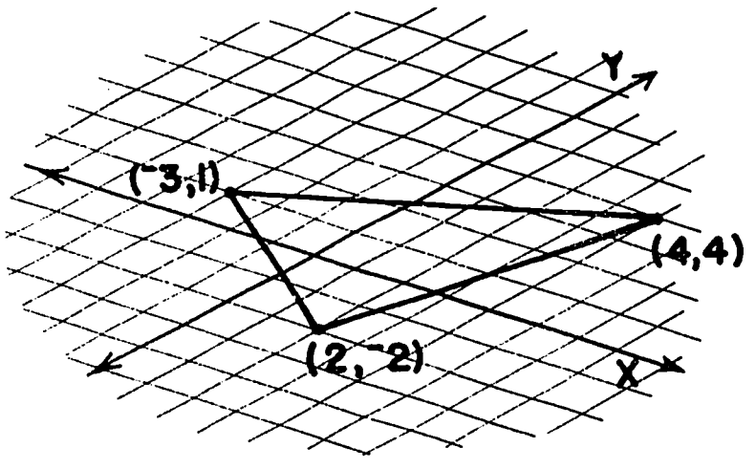
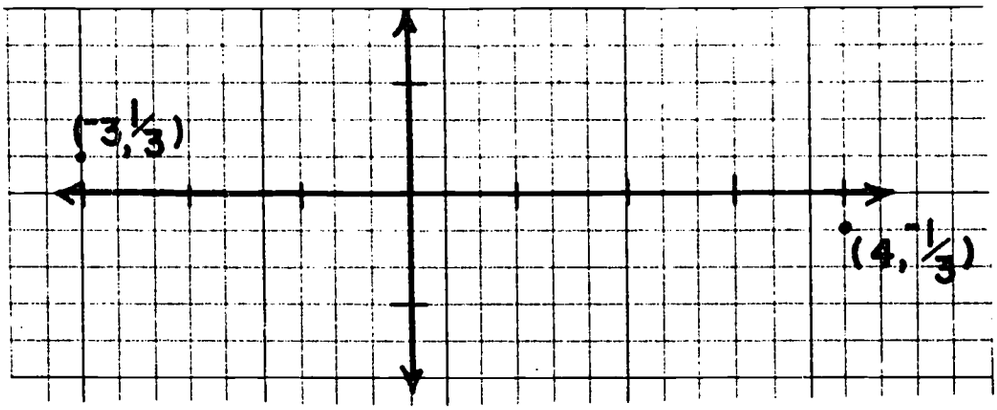


Figure 7b

Figure 7a shows a triangle graphed on the familiar rectangular coordinate system. Figure 7b shows the "same" triangle (the same coordinates that is) plotted on a coordinate system where the axes are not perpendicular.

ANSWERS CONTINUED

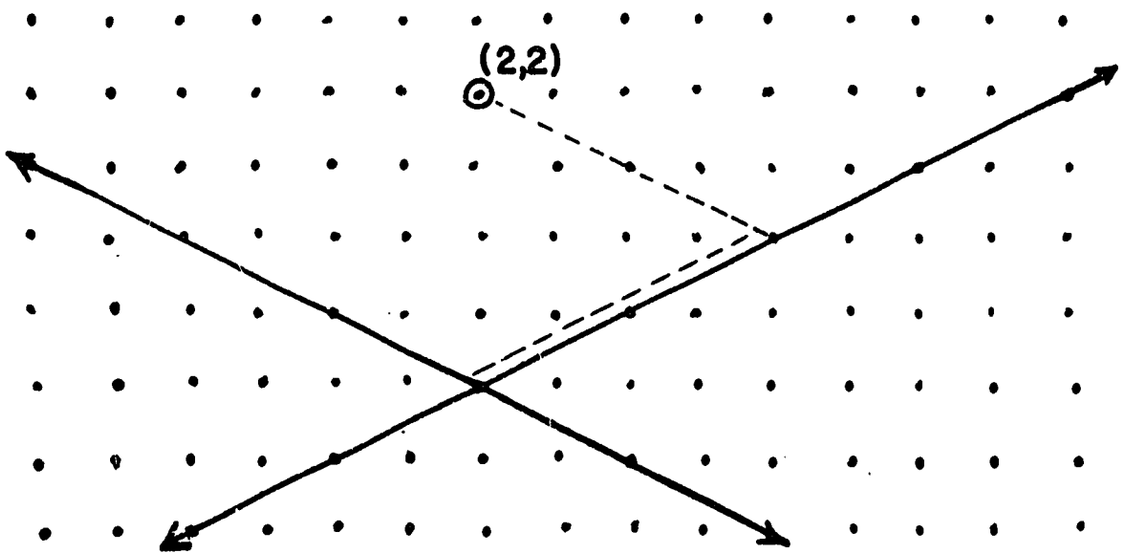
4.



THINGS TO DISCUSS CONTINUED

4. The axes need not be perpendicular. We are used to perpendicular reference lines in everyday life. Also, the mathematics of determining the distance between two points is "nicer" when the axes are perpendicular.

5. When plotting points on a coordinate system where the axes are oblique, it is important to stress that one moves <u>parallel</u> to an axis in counting off the units.



6. The coordinates of the points shown below are correct. Draw in the proper x- and y-axis.

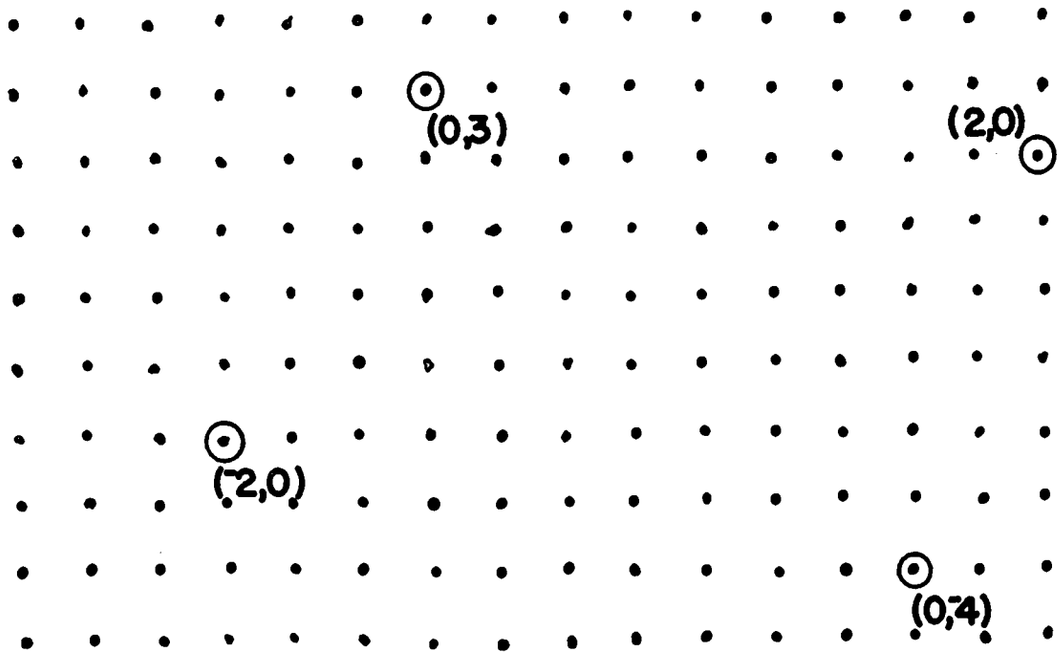


Figure 8

7. When plotting points in a coordinate system where the axes are not perpendicular, it is important that you move parallel to an axis in counting off the units. What are the coordinates of point A below?

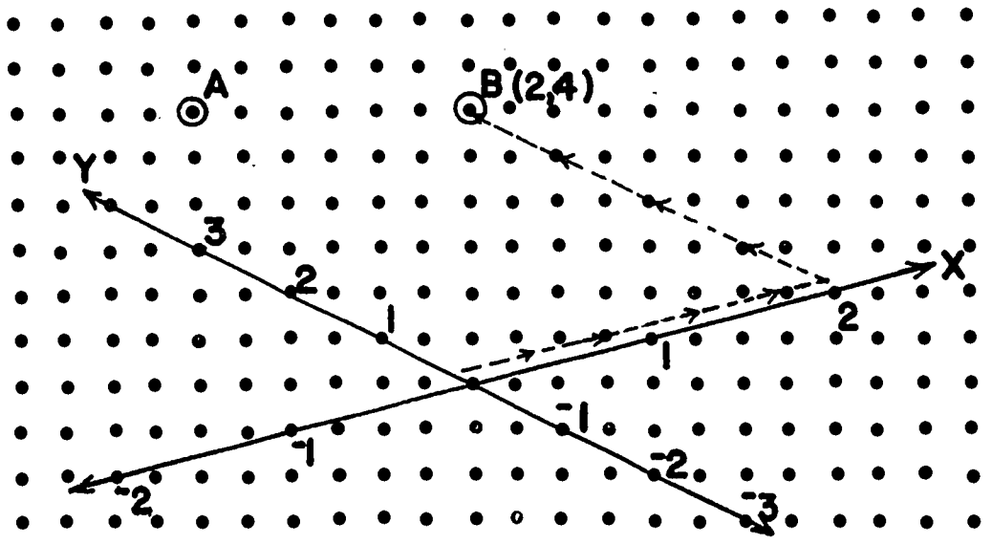
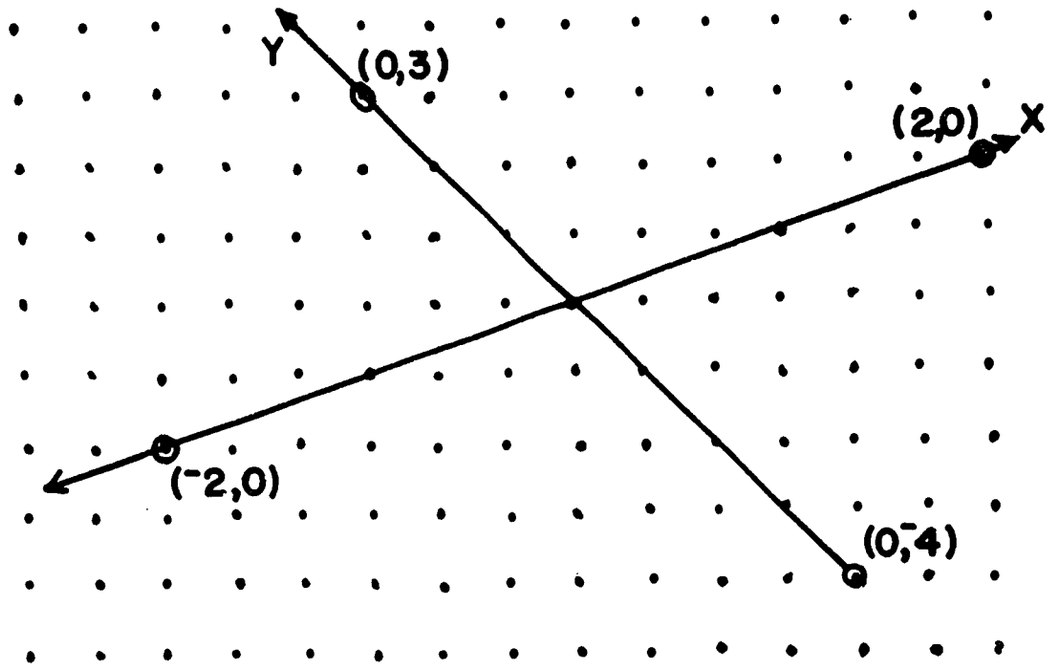


Figure 9

ANSWERS CONTINUED

6.



7. (1,5)

THINGS TO DISCUSS CONTINUED

6. Considering the ways in which the scales or the angle between the axes can be changed so that similar figures will be formed can lead to an interesting discussion.

   If the angle between the axes remains constant, then a change in scale proportional to the original scale will result in a graph of a similar figure.

POWER QUESTIONS

1. Use a piece of unlined paper to construct the following graph. (Use a straightedge and a compass.)

   a. Place three noncollinear points (points that are not all on the same line) in a cluster near the center of your paper.

   b. Select one of the three points to be the origin. Draw two lines; each line is determined by the origin and one of the other two points.

   c. Each line is an axis. Let the segment formed by the origin and the other point on the axis determine the unit distance for that axis.

   d. Use a compass to mark off each axis with its unit distance. Number the axes.

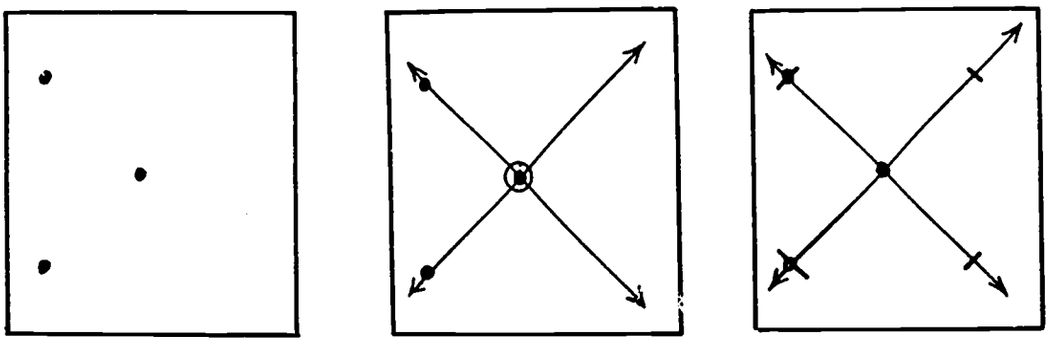
2. Plot the coordinates given in Figure 1 and draw the five-pointed star on your coordinate system.

THINGS TO DISCUSS CONTINUED

7. When discussing the first Power Question on page 68, point out the power of three noncollinear points in determining a coordinate system. The three noncollinear points determine the axes, the origin and the unit distance for each axis.

ANSWERS TO POWER QUESTIONS

1-2. The graph will vary from student to student. Utilize this variability to emphasize objectives 1c and 2c.



Step 1 Step 2 Step 3

NOTE: See Item 2 in Appendix C for a supplementary activity.

REGIONS IN THE PLANE

Part I The Quadrants

The perpendicular axes of the familiar rectangular coordinate system are shown in Figure 1.

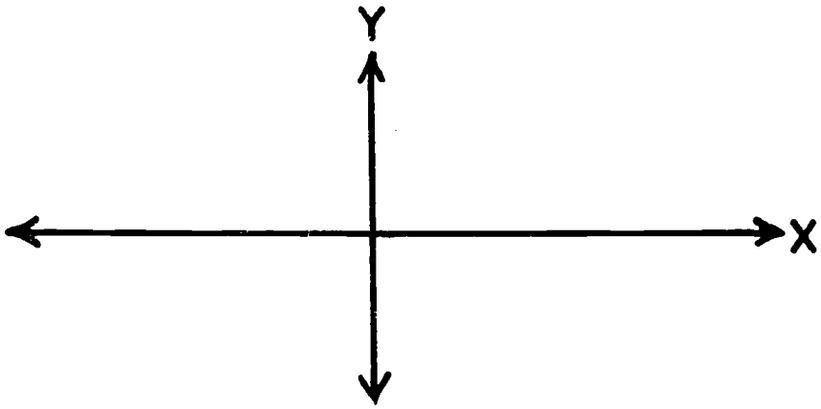


Figure 1

On which side of the y-axis are the points whose x-coordinate is positive?

Imagine the two axes in Figure 1 extended in the plane indefinitely in the four directions. The two axes divide the plane into four regions.

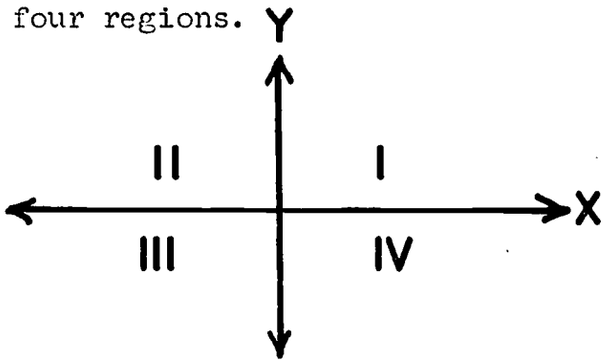


Figure 2

The four regions shown in Figure 2 are called Quadrants. The Quadrants are always numbered I, II, III, and IV in a counterclockwise direction. This way, mathematicians always agree on where, say, the third Quadrant is located.

REGIONS IN THE PLANE

OBJECTIVES

1. To provide groundwork for later development of graphing simple inequality relations in two variables in the rectangular coordinate system.

2. To review and extend the notion of coordinates. For example, the student is asked, "Which coordinate remains unchanged for every point on a vertical line?"

3. The student will be able to determine the region on the rectangular plane which corresponds to a description of the coordinates of the points in that region. For example, given a description like: All points (x,y) where $x < 3$ and $x > -1$ any $y \geq -2$ and $y \leq 4$; the student will respond by shading in the proper region.

4. The student will be able to identify, by number (I, II, III, or IV) each Quadrant and describe the coordinates of the points common to each Quadrant.

EQUIPMENT AND TEACHING AIDS

Several sheets of rectangular coordinate paper and a straightedge.

TRANSPARENCIES

10-1. Rectangular coordinate axes in the plane. The first overlay colors quadrants II and III blue and leaves quadrants I and IV clear. The second overlay colors quadrants III and IV red and leaves quadrants I and II clear.

PURPOSE

1. To illustrate the four quadrants.

10-2. Use transparency 9-1 to graph some of the regions in the plane.

The axes are not included as part of any of the four Quadrants.

The coordinates of the points in each Quadrant share a common property. For example, each point in Quadrant I has a positive x-coordinate and a positive y-coordinate.

The y-axis divides the plane into three sets of points:

All points whose x-coordinates are positive make up one set of points; all points whose x-coordinates are negative make up a second set; and all points whose x-coordinate is zero make up a third set.

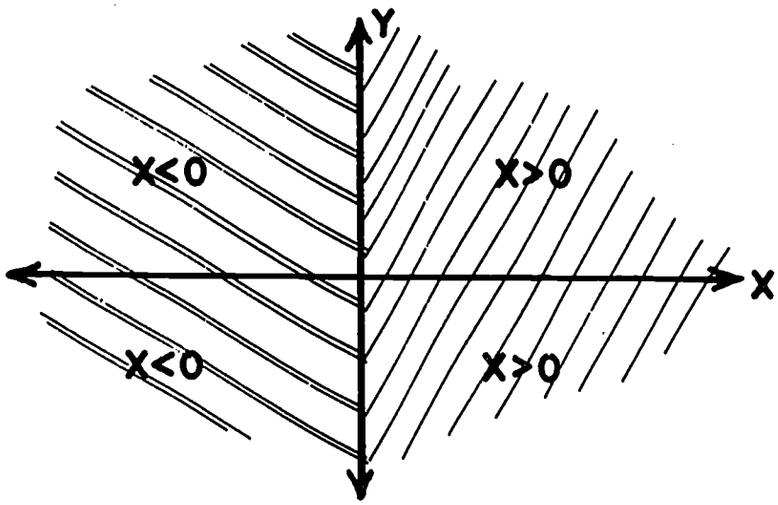


Figure 3

The x-axis also divides the plane into three sets of points:

All points whose y-coordinates are positive; all points whose y-coordinates are negative; and all points whose y-coordinates are zero.

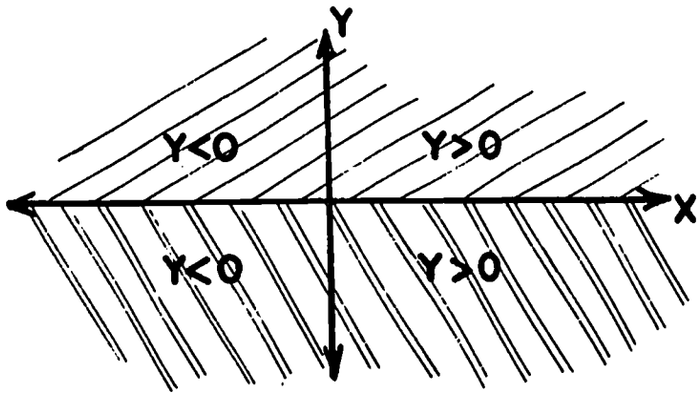


Figure 4

CONTENT AND APPROACH

The student learns to associate a region on the coordinate plane with a description of the common properites of the coordinates of the points located in that region.

The general purpose of this lesson is to prepare the student for later work in graphing linear inequalities. The student is to become familiar with the notion that all of the points in a given region, or on a given line, share a common property. Their coordinates can be described, for example, by using statements like: the x-coordinate is positive and the y-coordinate is less than 3.

An extension to this lesson can be made by giving the students a shaded region on the graph and asking them to describe the coordinates of all the points in the region.

THINGS TO DISCUSS

1. The axes do not belong to any Quadrant. Zero is neither a positive nor a negative number. Thus, points with either coordinate equal to zero do not belong to any Quadrant.

2. The x-coordinate tells how far a point is located from the y-axis. Thus, a vertical line consists of all points whose x-coordinates are the same. Horizontal lines consist of points whose y-coordinates remain constant.

3. "Open" and "closed" regions. These terms are not used in the exercises, but you may wish to introduce them. Stress the difference between the statement $x > a$ and the statement $x \geq a$.

4. In discussing descriptions like, all points (x,y) where $x > 2$ and $x < 4$ emphasize the meaning of "and". That is, the point $(6,3)$ is not in this region because, although $6 > 2$, 6 is not less than 4. The term "and", in the above sense, is compared with the term "intersection".

EXERCISES

1. In which Quadrants are the points whose x-coordinate is negative?

2. In which Quadrants does the y-axis lie?

3. Draw an x- and y-axis on your graph paper. Label the four Quadrants.

4. In which Quadrant does the origin lie?

5. Complete the following table:

QUADRANT	The common property of all of the points, (x, y), in the Quadrant
I	$x > 0$ and _____
II	_____ and $y > 0$
III	_____ and _____
IV	_____ ___ _____

6. All of the points on the ___ axis have the property that the x-coordinate is zero.

7. Draw a vertical line through the point (3, 0). Every point on this line has the property that its x-coordinate has what value?

ANSWERS TO EXERCISES

PART I

1. II and III
2. None. The axes are not part of any Quadrant.
3. Quadrants should be labeled as shown in Figure 2, Page 69.

4. None.

5. $x > 0$ and $y > 0$
   $x < 0$ and $y > 0$
   $x < 0$ and $y < 0$
   $x > 0$ and $y < 0$

6. y
7. 3

NOTE: See Item 7 in Appendix D for a supplementary activity.

8. Every point on a horizontal line through the point (0, 5) has what property?

9. A vertical line is drawn through the point (⁻3, 0). Consider the points on this line which are also in Quadrant II. Describe the x- and y-coordinates of each of these points.

   Every x-coordinate _____ Every y-coordinate _____

10. Which coordinate remains unchanged for every point on a horizontal line?

11. Which coordinate tells you how far a point is from the x-axis?

12. Draw a line through the points (2, 2), (⁻3, ⁻3) and (0, 0). Describe the coordinates of all the points on this line.

POWER QUESTIONS

1. The coordinates of point W are (a, b) and the coordinates of point H are (c, d). Here are some facts about these coordinates:

   (1) $a < 0$ and $b > 0$.

   (2) $c > 0$ and $d < 0$.

   Describe the positions of W and H as completely as you can.

ANSWERS CONTINUED

8. The y-coordinate is equal to 5.

9. Every x-coordinate is equal to $^{-}3$.
   Every y-coordinate is positive.

10. The y-coordinate remains constant for every point on a horizontal line.

11. The y-coordinate.

12. The x-coordinate equals the y-coordinate for each point on this line.

ANSWERS TO POWER QUESTIONS

1. Point W is in the second quadrant. Point H is in the fourth quadrant.

NOTE: Emphasize Exercises 6,7,8,10, and 11. If your students are not successful with these exercises, then you might try reviewing Lesson 7.

Part 2 Graphing Inequalities

Figure 5 shows a vertical line drawn through the point (4, 0). This line is the graph of the sentence $x = 4$. Every point on this line has the property that the x-coordinate has a value of 4.

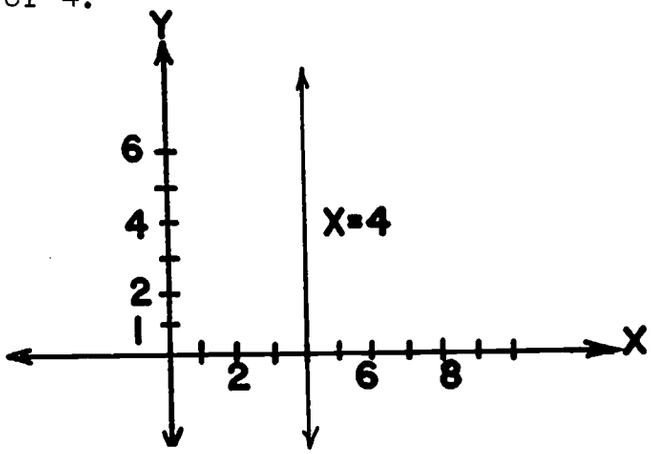
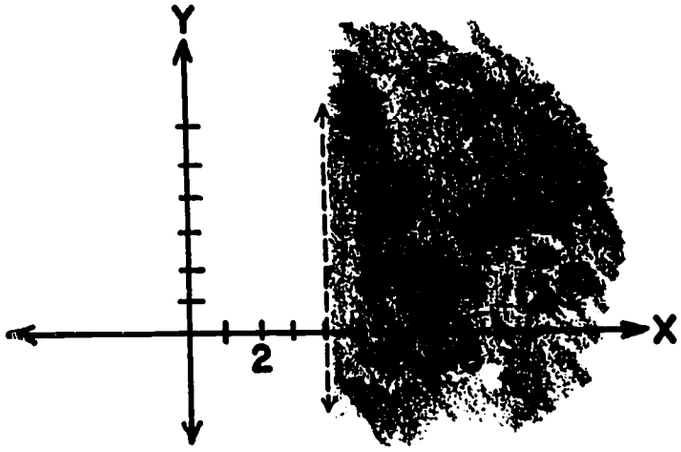


Figure 5

What about the points in the region to the right of the line $x = 4$?

The region to the right of the line $x = 4$ can be described by using an inequality sentence.

All points (x, y) where $x > 4$ are in the shaded region shown in Figure 6.



A "dash line" is used when the line is <u>not</u> to be included as part of of the shaded region.

Figure 6

Notice that y can be any number. It is only the x-coordinate that is restricted.

The dash line is used to distinguish an open half-plane from a closed half-plane.

While referring to Figure 6, have the students notice that the value of y does not matter. The complete statement which is associated with the shaded region in Figure 6 is: All points (x,y) where $x > 4$ and y can be any number.

If the line, $x = 4$, is to be included in the shaded region, then we say

$x \geq 4$ which is read
"x is greater than or equal to 4"

The symbol $>$ means strictly "greater than" and does not include "equal to".

EXERCISES

For each exercise below draw an x-axis and a y-axis on your graph paper. Do not use a whole sheet of graph paper for each exercise--a quarter section will do. Shade in the region described.

1. The x-coordinate is less than 2.

2. The y-coordinate is greater than $^-3$.

3. All points (x, y) where $x < 0$ and $y > 0$. Which Quadrant is this?

4. All points (x, y) where $x \geq 1$ and $y \geq 2$.

5. All points (x, y) where $x > 3$ and $x < 5$. y can be any number.

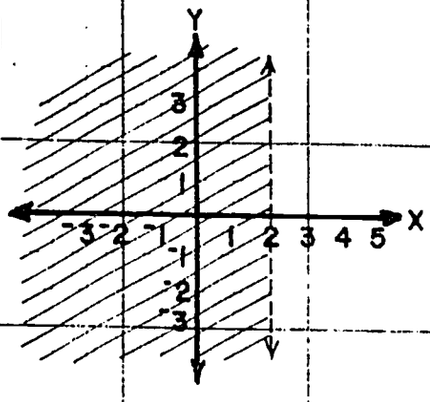
6. All points (x, y) where $y \geq 1$ and $y < 5$ and $x > 0$.

7. All points (x, y) where $y < 3$ and $y > {}^-1$ and $x > {}^-2$ and $x < 5$.

POWER QUESTION

1. Watch out for this one. All points (x, y) where $x < 5$ and $y \geq 3$ and $x \geq y$.

*1



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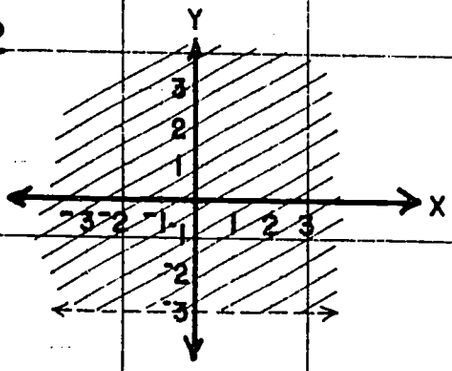

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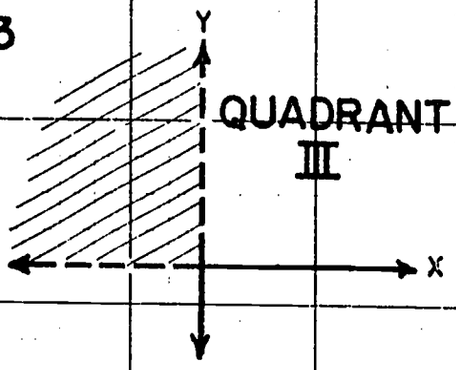

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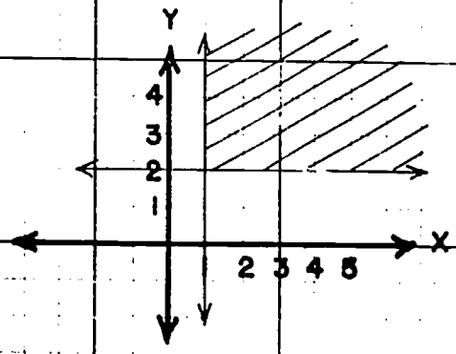

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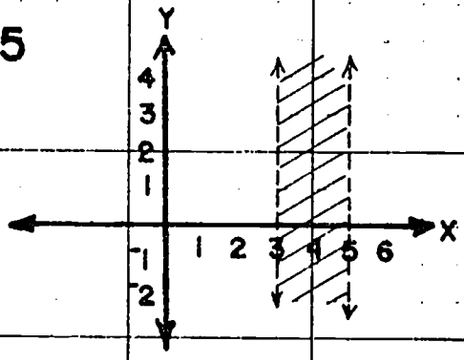

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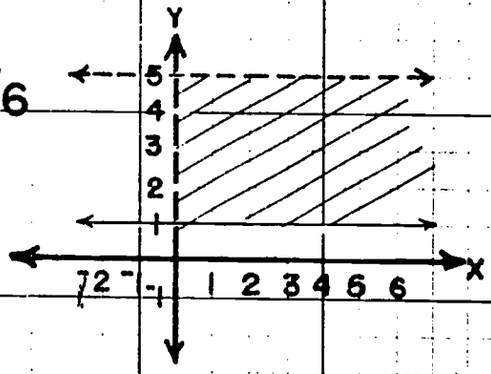

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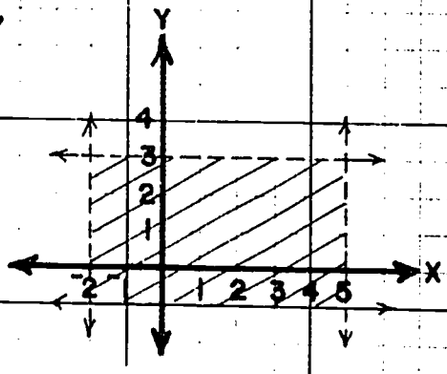
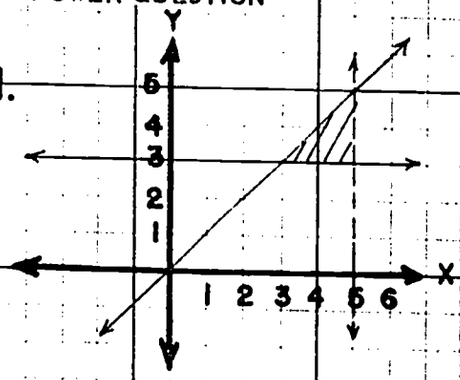


POWER QUESTION

1.



WHAT IS A COORDINATE SYSTEM?

There are many different coordinate systems. Which one you use would depend on several things. Do you wish to coordinatize a plane, the surface region of a geometric solid, all of space or perhaps a line? You may wish to use angle measures for some of the coordinates. If you use axes, then you may want them to be oblique instead of perpendicular.

The main purpose of this lesson is to have you draw together and examine some of the things that you have learned about coordinate systems.

Part I Coordinate Systems Without Angles

In Lesson 5, only points with non-negative coordinates were considered. Figure 1, below, shows the three axes extended in their negative directions.

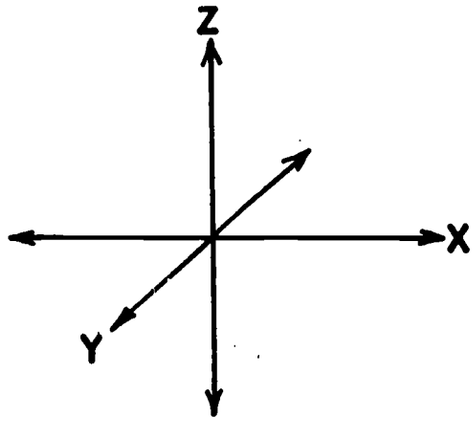


Figure 1

The negative y-axis points into the paper and the negative z-axis points down.

The way in which axes are labeled varies from book to book. There is no "always do it this way" rule. It is wise however, to select one way and stick to it to avoid confusion.

WHAT IS A COORDINATE SYSTEM?

OBJECTIVES

1. To review the basic notions of coordinatization.

   a. The student will be able to answer the question: "What is a coordinate system"? in terminology similar to that used in the Student's Preface.

   b. The student will be able to make proper use of such terms as: origin, axes, scale and coordinates.

2. The student will be able to relate the number of coordinates used in a system to the number of dimensions in the system.

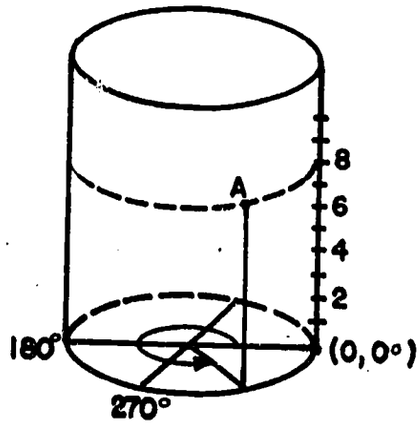
3. The student will be able to coordinatize a line.

4. The student will be able to describe, orally using a physical model, some coordinate scheme appropriate for the surface region of a regular solid.

EXAMPLES :

a. Given a cube, the student may decide to paint each face a separate color. Then each face could be coordinatized using rectangular coordinates. Thus, some point on the surface region of the cube would have the coordinates, say, (black, 2,8).

b.



$A(8, 300^{\circ})$

The location of any point on the surface region of this cylinder can be described by using a combination of angle measure and height.

The three axes form three planes. In space these three planes can be used to locate a point by giving its directed distance from each of the planes.

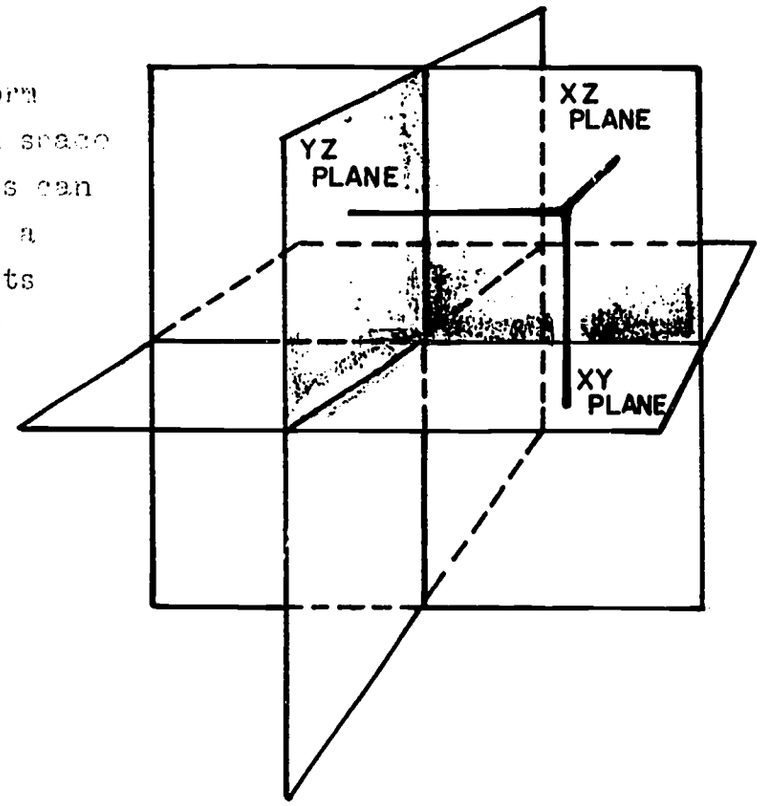


Figure 2

Each directed distance is a coordinate of the point's position. The coordinates of a point in space are given in the form of an ordered triple of numbers.

If the three axes are mutually perpendicular, then the coordinates are called <u>rectangular coordinates</u>. The word rectangular comes from right-angled.

The three coordinate planes (the xy-plane, the xz-plane and the yz-plane) divide space up into compartments. Each compartment is called an <u>Octant</u>. The axes are not included in any Octant.

DISCUSSION QUESTIONS

1. Why is the term Octant used to name the compartments created by the intersecting coordinate planes?

2. Explain why three coordinates are necessary to describe a point's location in space.

EQUIPMENT AND TEACHING AIDS

Physical models for demonstration purposes, e.g., cubes, cylinders, cones and spheres. Flexible rulers (plastic or cloth tapes), full circle protractors, and rectangular coordinate paper. Construction paper, tape, scissors, etc., depending on how much display work is to be done. Enough for each small group of students.

CONTENT AND APPROACH

This lesson is divided into three parts and it will take two or three days to complete. Part 1 is meant to be handled largely via class discussion. Use transparencies from earlier lessons for review.

The main purpose of Part 1 is for the student to see the relationship between coordinatizing space, the plane, and the line and the number of coordinates needed for each system.

In Part 2, the student works exercises related to spherical coordinates. These exercises extend naturally from the student's earlier work in Lesson 3.

ANSWERS TO DISCUSSION QUESTIONS

1. There are 8 compartments. Oct is the Latin root for 8.
2. Space has three dimensions. There has to be a coordinate for each dimension.

EXERCISES

1. The Octant which is usually number one is the one where each coordinate is positive. This is the Octant we used in Lesson 5. The other Octants can be numbered as follows:

   The Octants II, III and IV are numbered counterclockwise around the positive z-axis Octant number V is directly below Octant I. Then Octants VI, VII and VIII are numbered counterclockwise around the negative z-axis.

   Complete the following table:

	OCTANT	The common property of all The points (x,y,z) in the Octant
	I	$x > 0$, $y > 0$, $z > 0$
a	II	
b	III	
c	IV	
d	V	
e	VI	
f	VII	
g	VIII	

2. Give the Octant number for each of the following points:

   a. $(3, {}^{-}2, 1)$ c. $(0, 0, 0)$

   b. $({}^{-}4, {}^{-}2, 3)$ d. $(3, {}^{-}2, {}^{-}4)$

3. The segment connecting points (3,5,2) and $({}^{-}4,3,8)$ will pass through which coordinate plane?

ANSWERS TO EXERCISES

1. a. $x > 0$, $y < 0$, $z > 0$

   b. $x < 0$, $y < 0$, $z > 0$

   c. $x < 0$, $y > 0$, $z > 0$

   d. $x > 0$, $y > 0$, $z < 0$

   e. $x > 0$, $y < 0$, $z < 0$

   f. $x < 0$, $y < 0$, $z < 0$

   g. $x < 0$, $y > 0$, $z < 0$

2. a. II, b. III, c. Not in an octant

   d. VI

3. The yz plane

NOTE: It is not necessary that the students memorize the numbering of the Octants. It is intended that they reason out the Octant location of a point through examining the coordinates.

In this day of modern technology, computers often control certain machine operations. Such a setup is referred to as "numerical control".

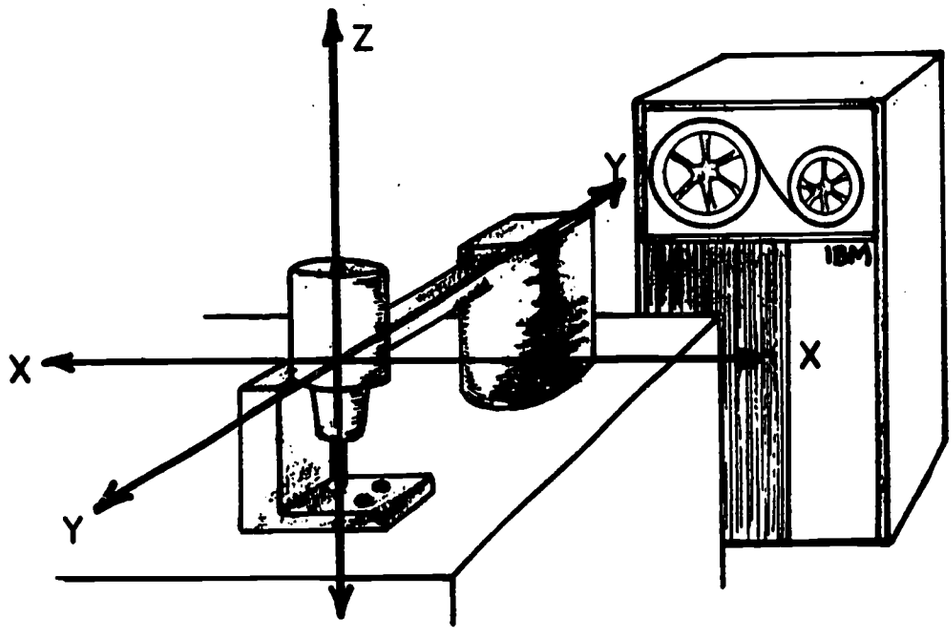


Figure 3

4. The drill shown in Figure 3 began drilling at the point (3 in., 5.2 in., 6.4 in.) and drilled to the point (3 in., 5.2 in., 2.1 in.). How far did the drill go?

5. The drill returned to the point (3 in., 5.2 in., 6.4 in.) and then moved $^{+}$.75 in. along the y-axis. What are the coordinates of the drill now?

The basis for measuring for numerically controlled machine tools is the rectangular coordinate system of three dimensions. The machine tool can be programmed to move from point-to-point or to move in a continuous manner.

In drilling, for example, the drill spindle is positioned at a single specific point. After the drill performs its operation at that point it is advanced to the next workpoint.

ANSWERS CONTINUED

4. 4.3 inches
5. (3 in., 5.95 in., 6.4 in.)

6. (Optional)

The Acme Warehouse stores electronic parts. There are thousands of different types of parts (tubes, switches, relays, wire, transistors, etc,) and it is quite a job deeping track of them.

The parts are stored on shelves. Each section of shelves contains 40 storage boxes (called bins).

One section of 40 bins.

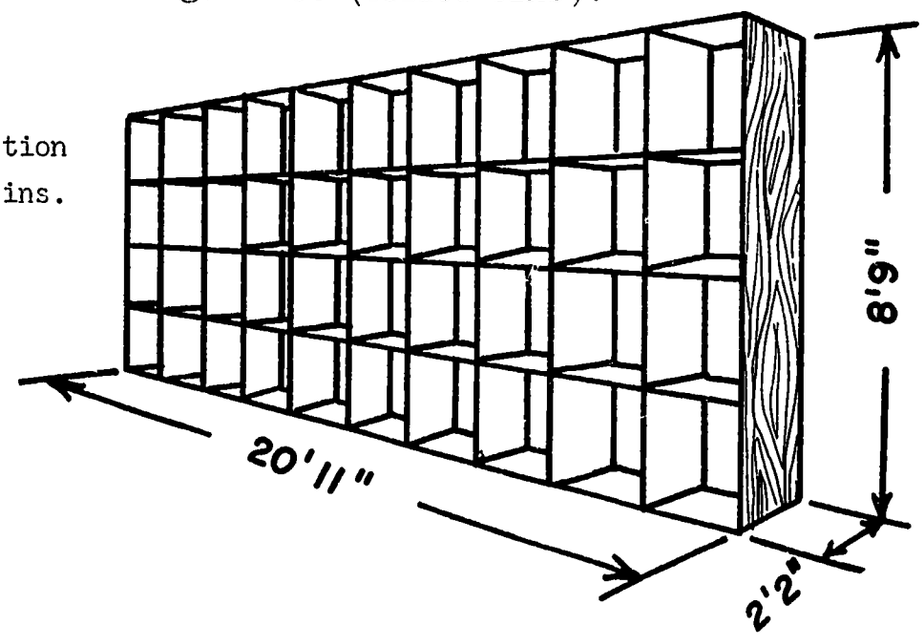


Figure 4

Each bin contains a different type of part.

The warehouse has 6 floors and each floor contains 48 sections of bins.

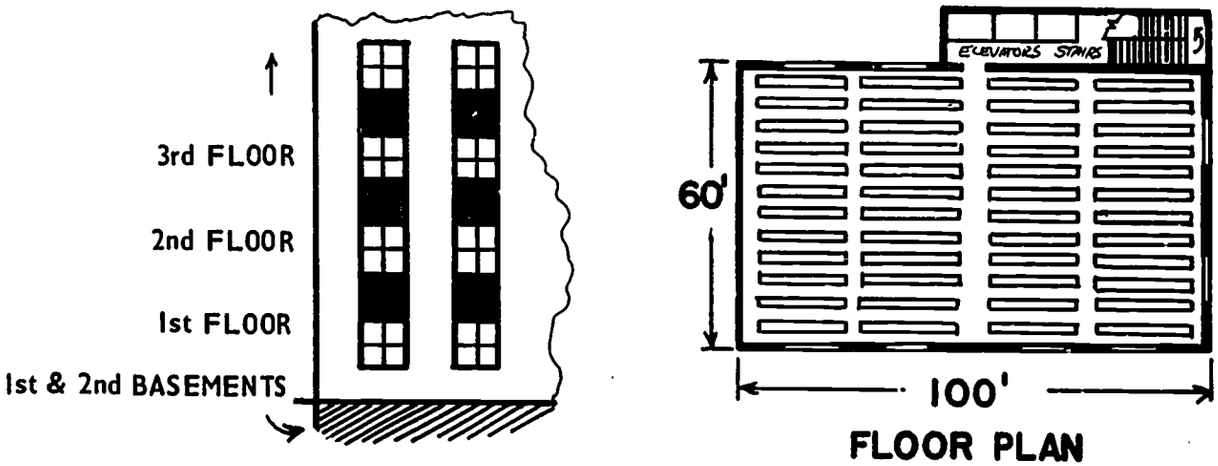


Figure 5

ANSWERS CONTINUED

6. This exercise is optional. Some of the better students may be challenged by it if they are encouraged to use their imagination.

   The coordinate system that is suggested by the layout of the building is the basic three-dimensional rectangular system with some modifications.

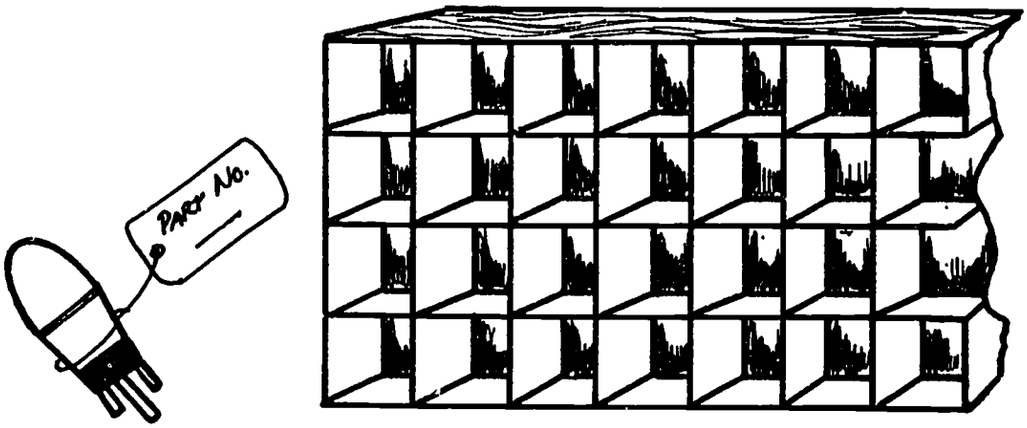
   POSSIBLE ANSWER: First coordinate - Floor numbers, using negatives for basement floors.

   Second coordinate - row number
   Third coordinate - column number

   These three coordinates would locate a specific rack of bins, but not a particular bin in the rack.

   A modification might be to use a "pseudo decimal point". For example the coordinates (3.4, 6.4, 9) would fix the bin which is in the rack on the third floor, in row 6, column 9, fourth bin from floor (i.e., 3.4) and fourth bin from the left.

The manager of the warehouse would like to invent a coordinate system to help keep track of the parts. He would like to assign a label to each part so that the label can be used to locate where the part is stored in the warehouse.



THE PART NO. SHOULD TELL WHERE THE PART IS LOCATED.

Figure 6

Help the manager invent a coordinate system so that each part has a different set of coordinates and each set of coordinates locates a different part in the warehouse.

LINE COORDINATES

When we move from space to the plane we lose a coordinate. A plane has only two dimensions. Space has three dimensions. In Lessons 7, 8, 9 and 10 you worked with coordinates of points in the plane so we won't spend extra time on them here.

We didn't, however, discuss coordinates of a point on a line. How do you coordinatize a line?

While discussing line coordinates, have the students focus on the commonalities of three coordinate systems (Rectangular coordinates in space, in the plane and line coordinates).

Each one has an axis for each dimension; an origin, a unit distance on each axis, a positive and negative direction on each axis and a coordinate for each dimension.

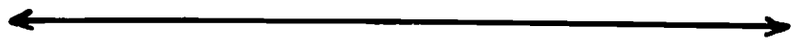


Figure 7

The line shown in Figure 7 resembles our familiar number line. However, it has not been coordinatized yet.

First, a point must be selected for the origin. The coordinate of this point will be zero.

Second, we select a point whose coordinate will be one. These selections are arbitrary.

The distance between the point whose coordinate is zero and the point whose coordinate is one is the <u>unit distance.</u>

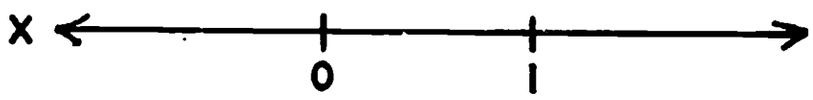


Figure 8

Once the unit distance is known, the coordinate of any point on the line can be determined. Can you explain why the point whose coordinate is 1 could have been selected to the <u>left</u> of the origin?

EXERCISES CONTINUED

7. What is the coordinate of the point located five units to the right of zero on the line?

8. Give the coordinate of the point located two and one-half units to the left of zero.

The point whose coordinate is 1 could have been selected to the left of zero. Positive numbers would be to the left of zero and negative numbers to the right. This is contrary to the order used, but it doesn't contradict the way things have to be.

ANSWERS TO EXERCISES CONTINUED

7. 5
8. $-2\frac{1}{2}$

9. Complete the following table:

Region to be Coordinatized	Number of Axes	Number of Coordinates	Coordinates of the origin
a) Space			
b) Plane			
c) Line			

10. The letter x is used to label the horizontal axis in rectangular coordinate systems for space, the plane, and the line. The graph of $x = 3$, on the line, is a dot located three units to the right of zero. Describe the graph of $x = 3$ (a) in the plane and (b) in space.

Part 2 **Space Coordinates With Angles (optional)**

In Lesson 5, an ordered triple of numbers was used to describe the coordinates of a point in space.

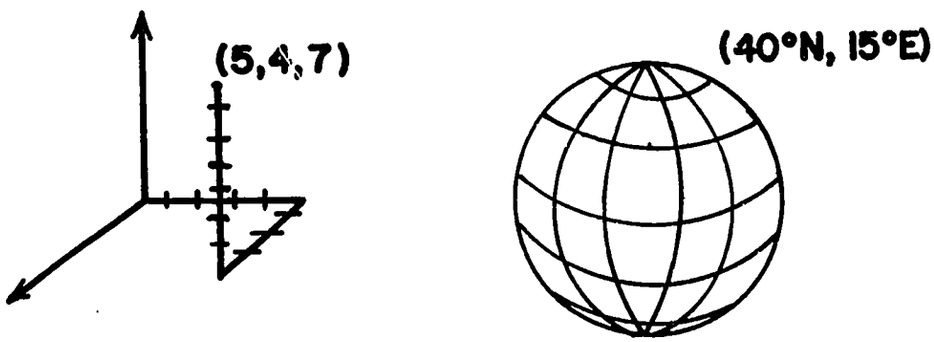


Figure 9

In Lesson 3, points on the globe were determined by an ordered pair of angle measures. Figure 10, on page 83, suggests a way in which these ideas could be combined to obtain space coordinates.

ANSWERS TO EXERCISES CONTINUED

9.	a.	3	3	(0,0,0)
	b.	2	2	(0,0)
	c.	1	1	0

10. a. A vertical line such that every x coordinate has the value 3.

    b. A vertical plane, parallel to the yz plane, such that every point on this plane has the coordinates (3,y,z).

PART 2 SPACE COORDINATES WITH ANGLES (OPTIONAL)

The purpose of this section is to show an alternate way to coordinatize space.

The coordinate scheme presented here is not the traditional spherical coordinate system, but an extension of the geographic coordinates studied in Lesson 3.

You may omit this section if there is no time for it. The completion of Part 3 is more desirable for the overall purposes of this booklet.

Let P be a point somewhere out in space. A segment, OP, connects the center of the Earth, O, with point P.

The first space coordinate of point P will be the distance OP. The other two coordinates will be the latitude and longitude of the point where OP cuts the Earth's surface.

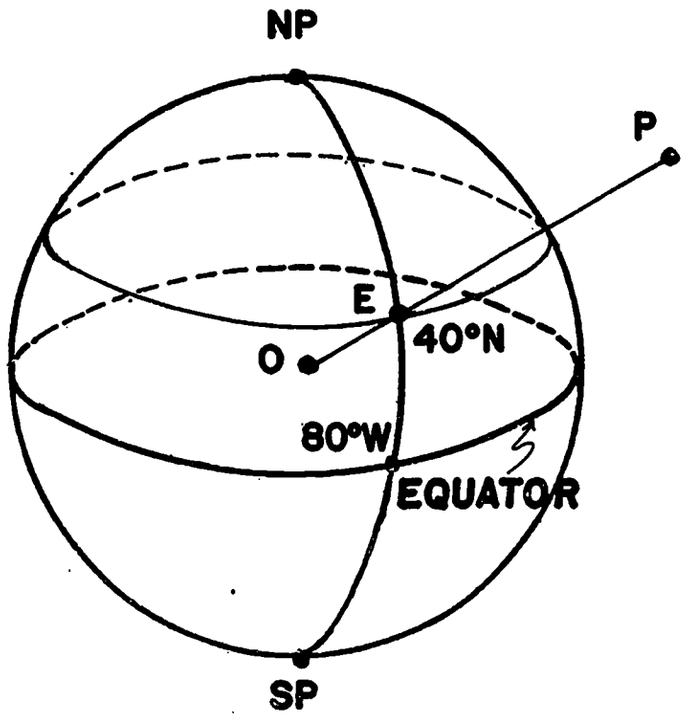


Figure 10

EXAMPLE: Point P, in Figure 10, is 423 miles directly over point E on the Earth's surface. Point E is 3,959 miles from the center of the Earth. The geographic coordinates of point E are (40° N, 80° W), so the space coordinates of point P are

(4,382 mi., 40° N, 80° W)

Have the students imagine segment OP cutting the Earth at point E. The latitude and longitude of point E are determined just as in Lesson 3.

Discuss the point that the angles of latitude and longitude do not change as one imagines the size of the sphere increasing from a radius of 3,959 miles to a radius of 4,382 miles.

As in Lesson 3, the two poles lie on all circles of longitude so we have to make a special agreement. We will use 0° for their third coordinate.

	SPACE COORDINATES
North Pole	(3,959 mi., 90° N, 0°)
South Pole	(3,959 mi., 90° S, 0°)

We are assuming that the Earth is a perfect sphere and that all points on the Earth's surface are 3,959 miles from the center.

The origin of this space coordinate system is the center of the Earth. Since any angle of latitude or longitude would be correct for the Earth's center, we will agree to use 0° for both the second and third coordinates of the origin.

EXERCISES

1. A stationary communications satellite is 138 miles directly overhead from a point on Earth whose coordinates are (57° N, 36° W). Give the space coordinates of the satellite.

2. What are the coordinates of the origin of this space coordinate system?

3. Take a point on a segment half-way between the center of the Earth and the South Pole. What are the coordinates of this point?

Discuss the fact that the Earth is not a perfect sphere.

Note that as in Lesson 4 an agreement has to be made regarding the origin.

ANSWERS TO EXERCISES

1. (4,097 mi., 57° N, 36° W)
2. (0 mi., 0°, 0°)
3. (1,979.5 mi., 90°S, 0°)

4. Describe where the point (3,959 mi., 0°, 90° E) is.

5. A plane is flying over Columbus, Ohio (geographic coordinates (40° N, 83° W)) at an altitude of 35,000 feet. Give the coordinates of the plane. (Round off to the nearest tenth of a mile.)

6. At one instant in its flight, Apollo 11's space coordinates were (4,073 mi., 26° N, 80° W). What city was it flying over and how high, from the Earth's surface was it?

7. A satellite is in a circular orbit about the Earth. Its coordinates at this instant are (4,307 mi., 70° N, 110° W). In less than an hour the satellite will be directly opposite the Earth from where it is now. What will be its coordinates then?

Part 3 Invent Your Own Coordinate System

This unit in coordinate geometry has been concerned with one central idea: the location of points or regions using various methods of <u>coordinatization</u>.

You will be given several geometric solids - cylinders, cones, spheres, cubes, etc.

Take one of the geometric solids and examine its surface region. Your assignment is this:

1. Think of a way to give coordinates to each point on the surface of your solid. Apply one of the methods you have studied or invent your own.

ANSWERS TO EXERCISES CONTINUED

4. On the Equator in Kenya
5. (3,965.6 mi., 40° N, 83° W)
6. Ft. Lauderdale, 119 miles
7. (4,307 mi., 70°S, 70° E)

PART 3 INVENT YOUR OWN COORDINATE SYSTEM

Part 3 is designed to be an open ended laboratory session. The students are encouraged to be creative in inventing their own coordinate system.

Have the students work in groups of from 3-4 students each. Their task is to work cooperatively in designing various coordinate schemes and applying these to the various geometric regions they have to work with.

THINGS TO DISCUSS

The discussion should be a review of Lessons 1-11. There is a wide variety of methods for locating points both on surface regions and in space. The discussion of these methods should bring out such basic notions as:

a. The correspondence between a symbol and a region or a point.

b. The ways in which coordinates are related to referents. (both axes and angles)

c. The arbitrariness in the various conventions that are used such as order, uniformity of scale and perpendicularity of axes.

d. The fact that angle measures are used as well as linear distances for coordinates.

e. Distortion caused by altering the scale or the angle between the axes.

Have the students suggest applications for various coordinate systems. You may receive responses such as:

1. Map coordinates
2. Navigation in space
3. Global zip code. (A letter addressed to Mr. James Perry, (51°24' N, 0°28' W) might stand a fair chance of being delivered.)
4. Surveying
5. Graphing functions in algebra

2. Display your coordinates scheme so that another student can see how yours works. Use the materials available to make your method of locating points understandable.

Your second assignment is similar to the first. A spacecraft is on its way to the moon.

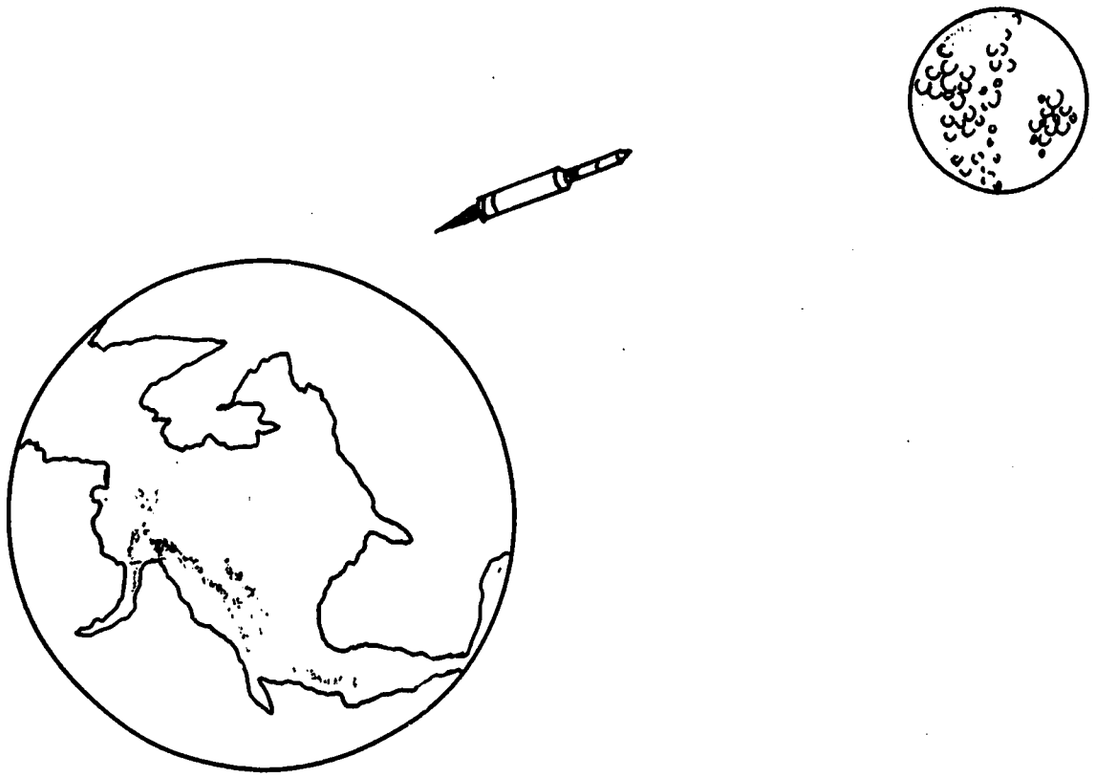


Figure 11

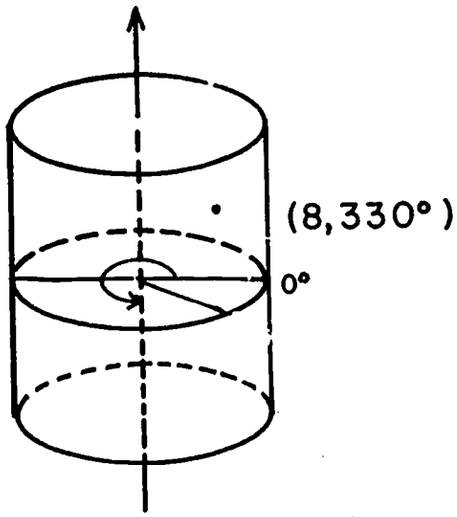
3. Assume that the spacecraft is at some fixed point in space. Think of a way to describe its location using coordinates. You may apply one of the methods you have studied or invent one of your own.

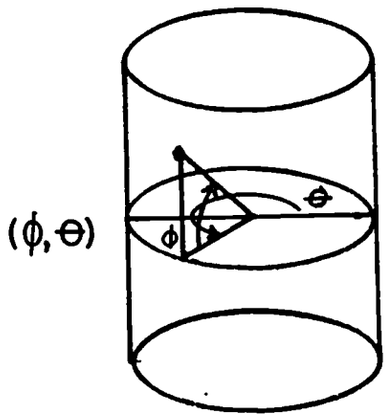
4. Explain your method to the other students in your group.

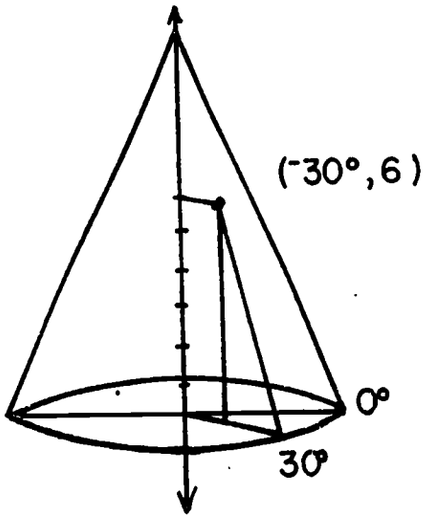
It would be a good idea to have the students fix up their models for display. Each display would show the representation of a point, its coordinates and a brief description of how the coordinate scheme works.

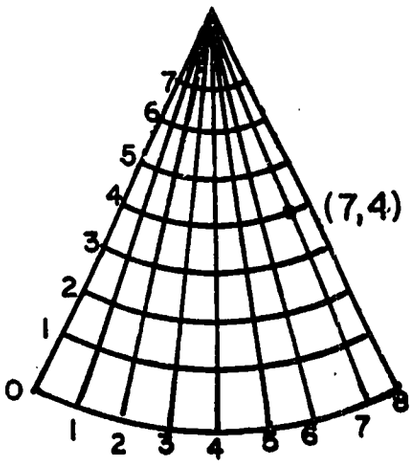
Summarize the laboratory work by relating some of the student inventions to the ideas developed in previous lessons in this unit.

Possible coordinate schemes.









APPENDIX A
Map of Central Washington D.C.

In case the Official Highway Map of Oakland County is unavailable, you may wish to utilize the enclosed map. A list of "points of interest" is included.

Reproduce the map, one copy per student. The students can label the axes and you can provide exercises similar to those in Lesson 1, Part 1.

PLACES OF INTEREST ON INSET MAP OF CENTRAL WASHINGTON

1. American Red Cross District Headquarters
4. Auditor's Building
5. Blair House
5a Bureau of Employment Security
6. Bureau of Engraving and Printing
7. Bureau of Engraving and Printing Annex
8. Bureau of Internal Revenue
9. Capitol, The
11. Capitol Boiler Plant
12. Central Heating Plant
13. City Post Office
14. Civil Service Commission
15. Coast Guard Building
16. Court of Claims
17. Dept. of Agriculture
18. Dept. of Agriculture Annex
19. Dept. of Commerce
21. Dept. of Health, Education and Welfare
22. Dept. of Health, Education and Welfare (South Building)
23. Dept. of Justice (FBI)
24. Dept. of Labor and Departmental Auditorium
25. Dept. of State
26. Dept. of the Interior
27. Dept. of the Navy
28. Dept. of the Navy (Potomac Annex)
29. Dept. of the Treasury
30. Dept. of the Treasury Annex
30a Executive Office Building (New)
31. Executive Office Building (Old State Building)
31a Federal Aviation Agency
32. Federal Building No. 1
32a Federal Building No. 6
33a Federal Court Building

34. Federal Deposit Insurance Corp.

35. Federal Home Loan Bank Board

36. Federal Housing Administration

37. Federal Reserve Building

38. Federal Trade Commission

40. Food and Drug Administration

41. General Accounting Office Building

42. General Services Administration

43. General Services Regional Office

44. Government Printing Office (Closed to Visitors)

45. Government Printing Office Annex

46. House of Representatives Office Building

47. House of Representatives Rayburn House Office Building

48. House of Representatives Office Building (New)

49. Information Service

50. Interstate Commerce Commission

51. Lafayette Building

52. Library of Congress

52a Medical Museum of the Armed Forces Institute of Pathology

53. Munitions Building

53a National Academy of Sciences

53b National Aeronautics and Space Administration

54. National Archives

54a National Science Foundation

55. Pension Building

56. Post Office Dept. (New Building)

57. Post Office Dept. (Old Building)

58. Securities and Exchange Building

59. Selective Service System HQ

61. Senate Office Building

62. Senate Office Building (New)

62a Simon Bolivar Statue and Plaza

63. Supreme Court Building

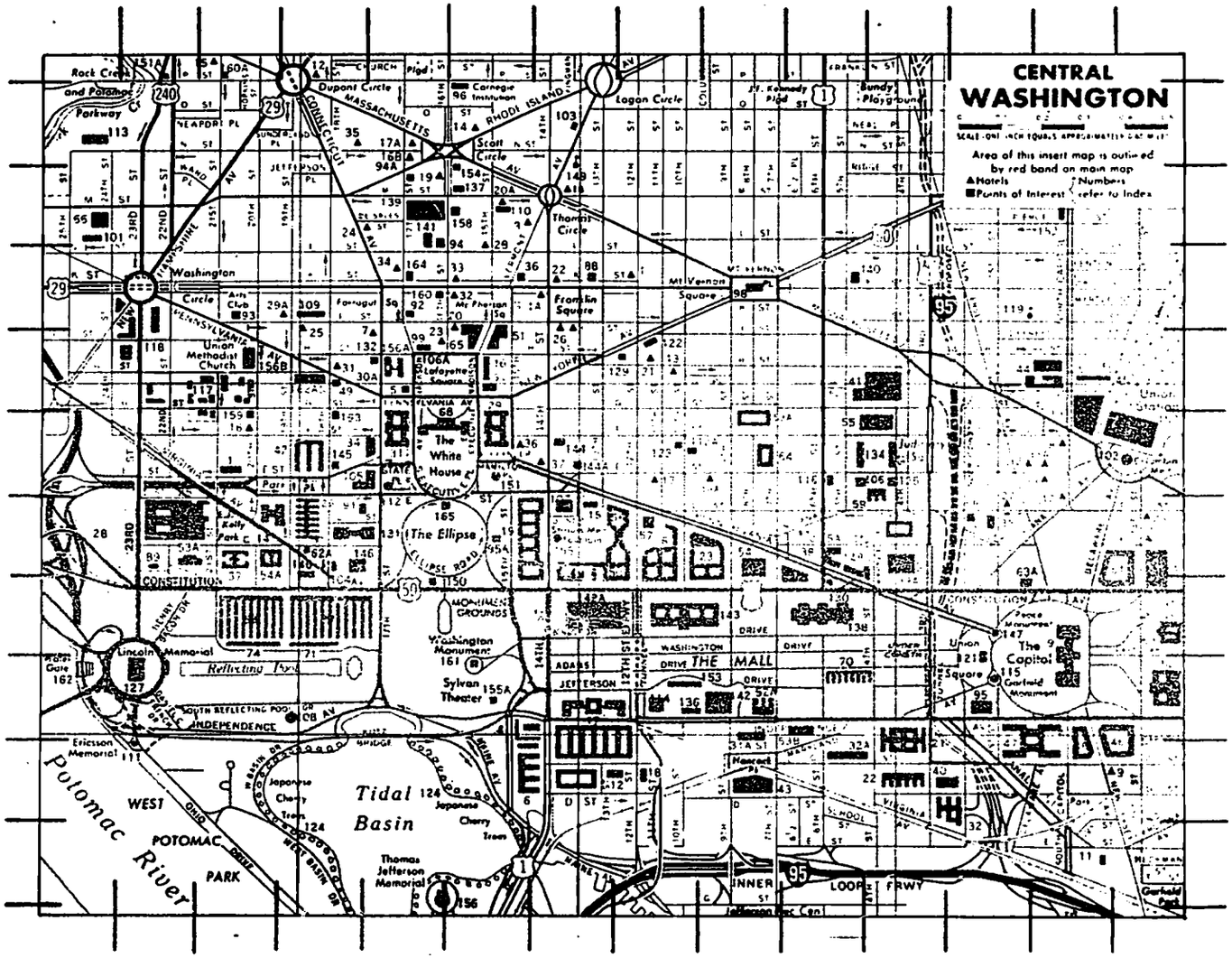
63a Taft Memorial, Robert A.

64. Tariff Commission

65. Veterans Administration

66. Weather Bureau

68. White House

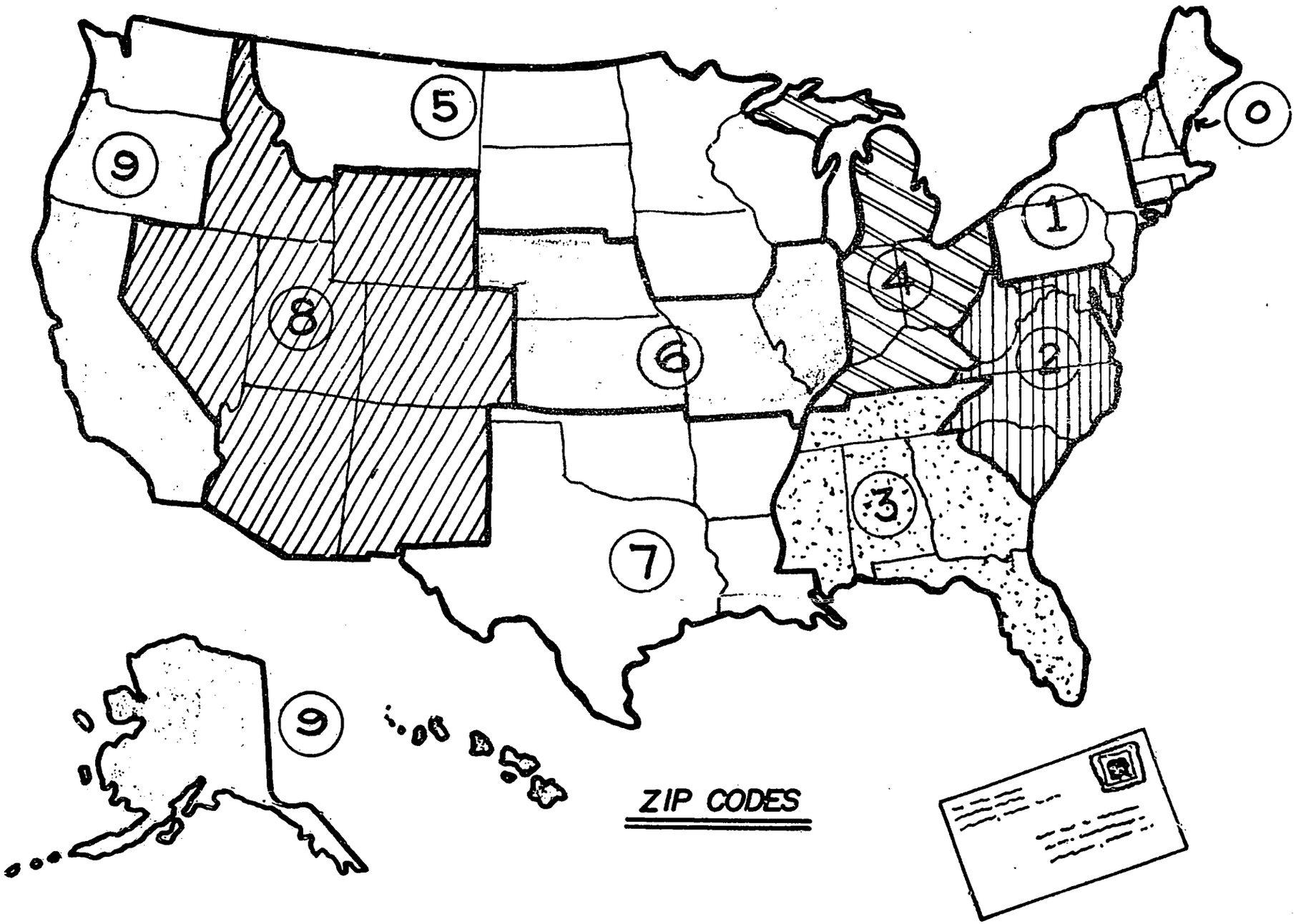

CENTRAL WASHINGTON
Area of this insert map is outlined by red band on main map
▲Hotels
■Points of Interest
Numbers refer to Index
Dupont Circle
Logan Circle
Scott Circle
Thomas Circle
Washington Circle
Mt Vernon Square
Franklin Square
The White House
The Ellipse
Lincoln Memorial
Reflecting Pool
Washington Monument
Sylvan Theater
THE MALL
The Capitol
Tidal Basin
Thomas Jefferson Memorial
Potomac River
WEST POTOMAC PARK
INNER LOOP FRWY
Rock Creek and Potomac Parkway
Carnegie Institution
Bundy Playground
Union Methodist Church
Ericsson Memorial
Japanese Cherry Trees
Garfield Monument

APPENDIX B

TRANSPARENCY MASTERS

(For those transparencies which are recommended but not provided by Oakland Schools)

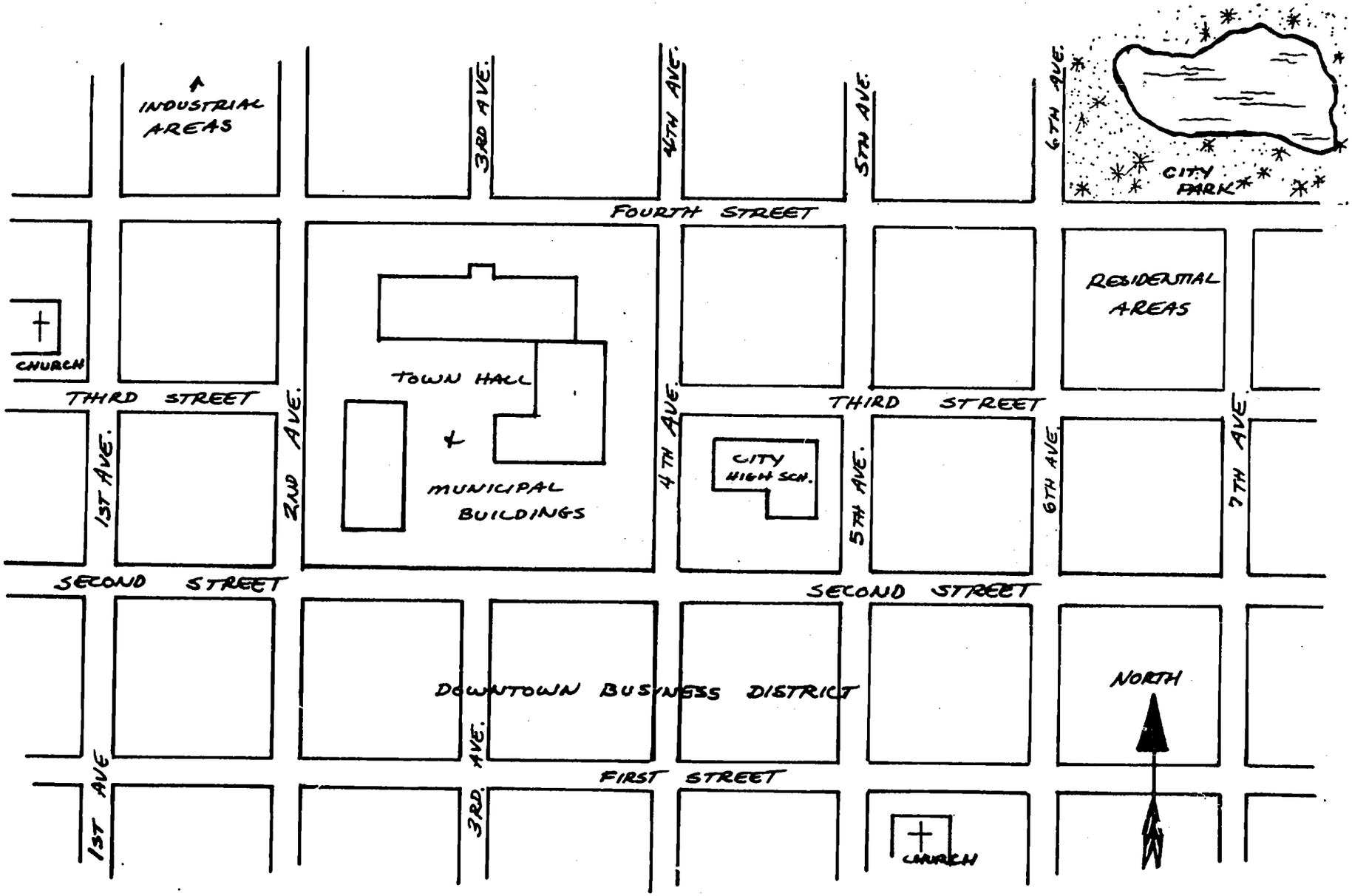
2-1



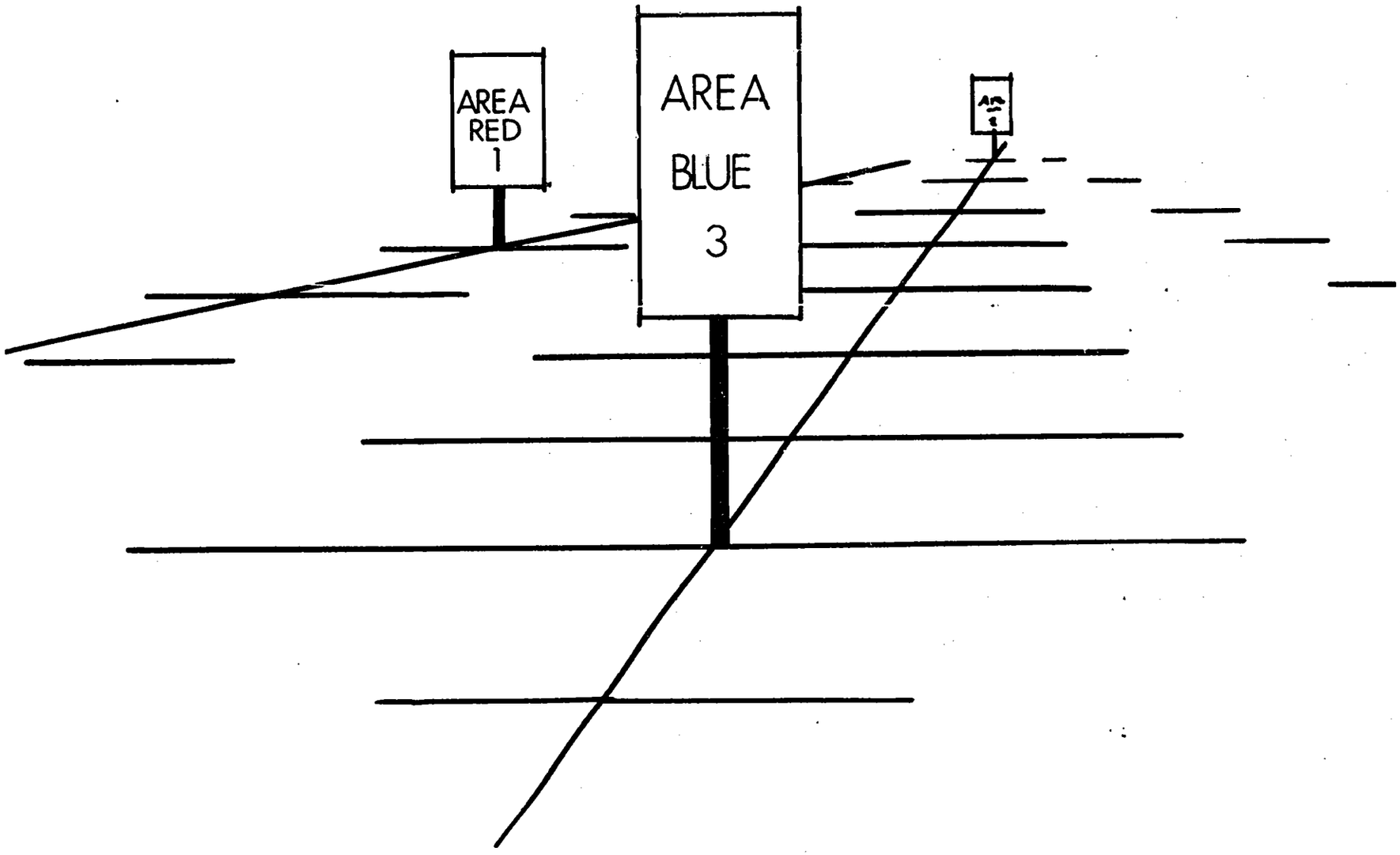
ZIP CODES

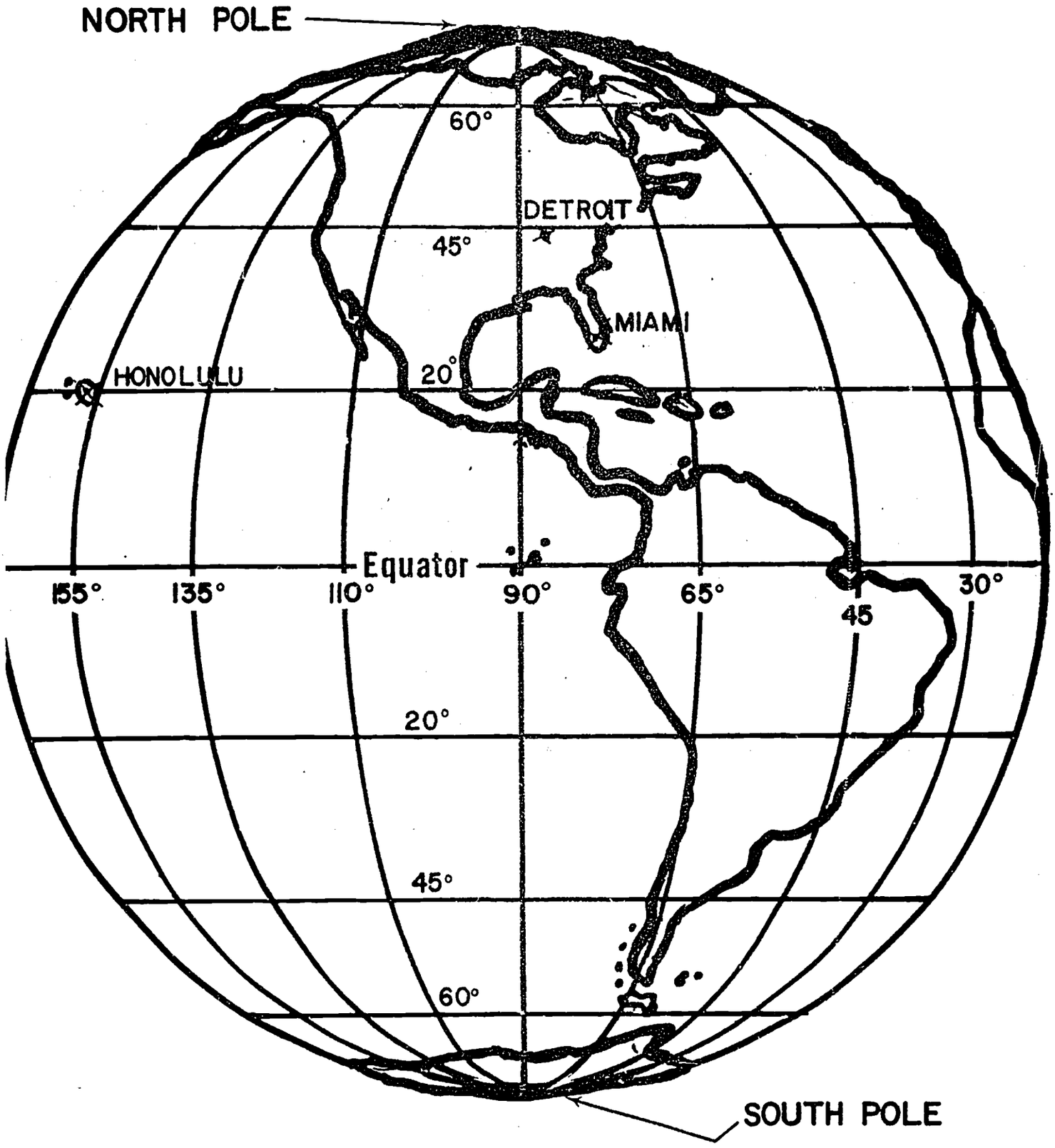
CENTERVILLE CITY

2-3

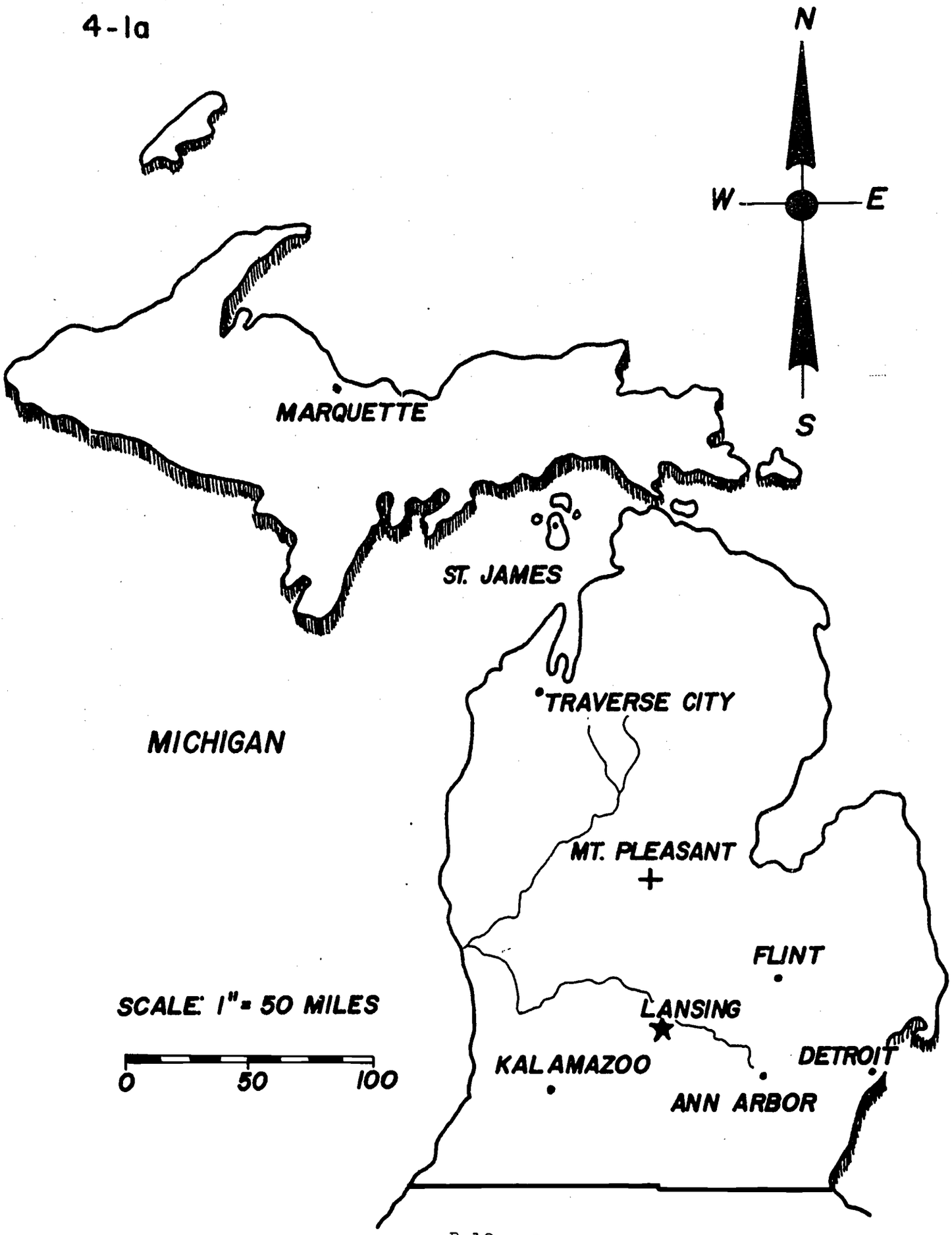


2-4

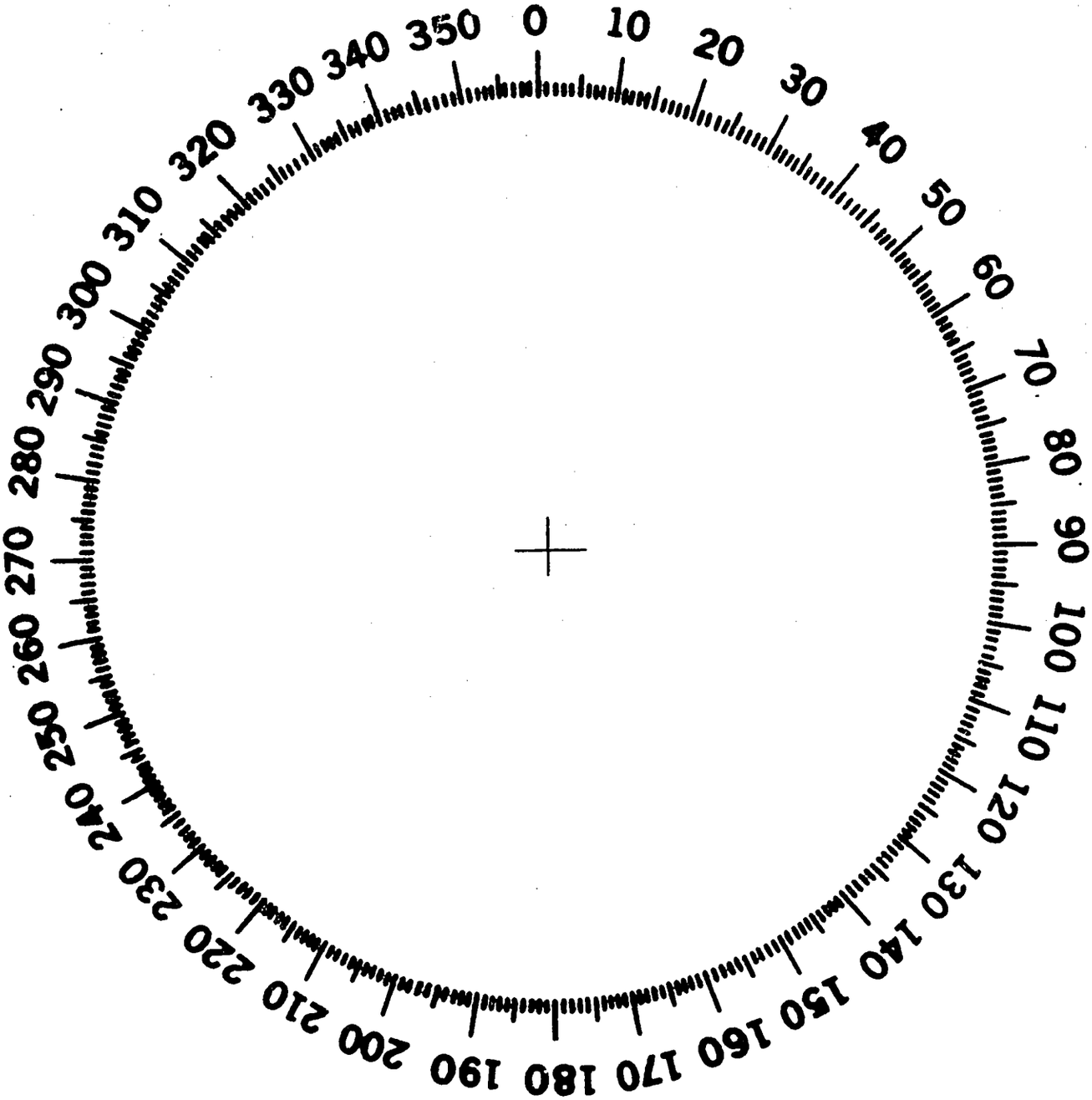



NORTH POLE
60°
DETROIT
45°
MIAMI
HONOLULU
20°
Equator
155°
135°
110°
90°
65°
45
30°
20°
45°
60°
SOUTH POLE

4-1a


N
W
E
S
MARQUETTE
ST. JAMES
TRAVERSE CITY
MICHIGAN
MT. PLEASANT
FLINT
LANSING
KALAMAZOO
DETROIT
ANN ARBOR
SCALE: 1" = 50 MILES
0
50
100

4-lb


0
10
20
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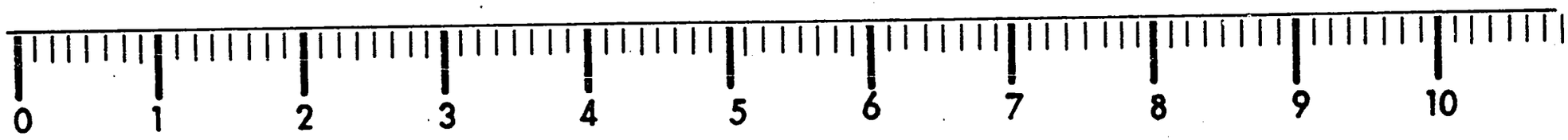
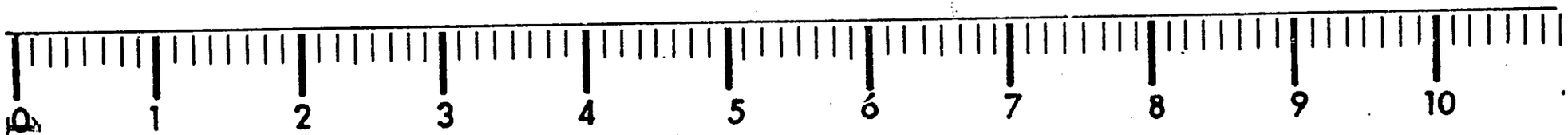
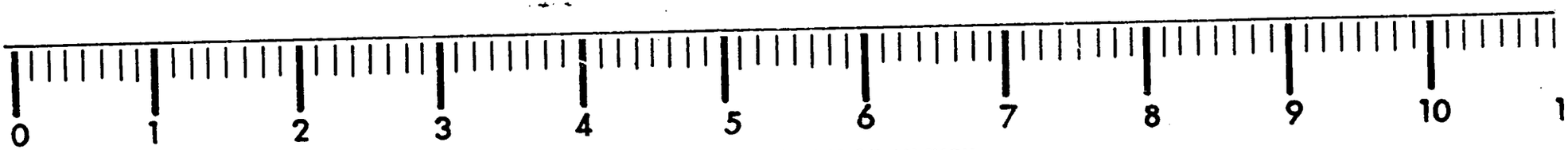
350



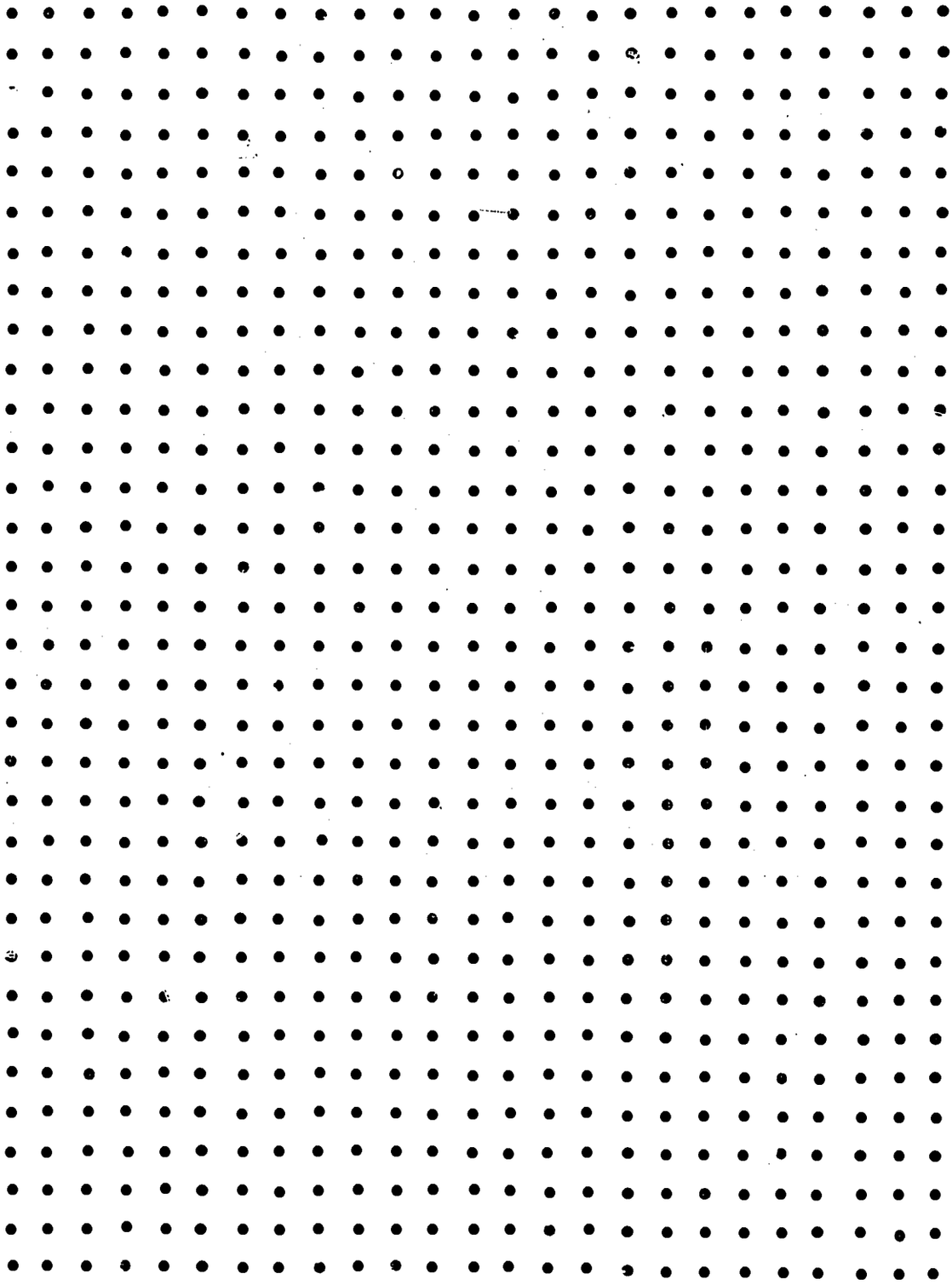
EIGHTHS OF AN INCH



EIGHTHS OF AN INCH



EIGHTHS OF AN INCH



9-1

APPENDIX C

Supplementary Activities

Circle Coordinates

A Supplementary Activity

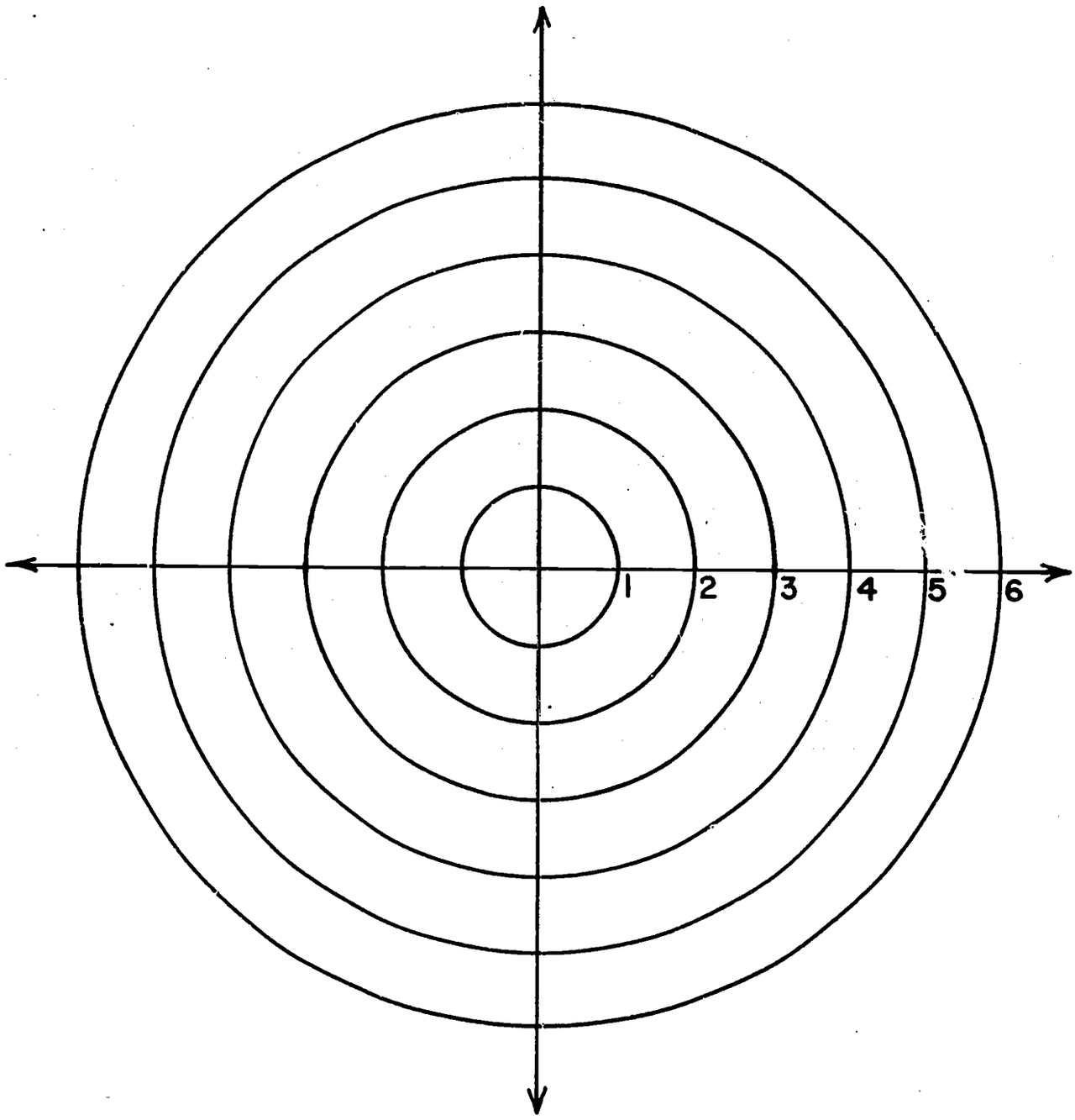
PURPOSES

1. To have the students "discuss", via verbal cues supplied by the teacher, how this particular coordinate system works.

2. To investigate a coordinate system which is "many-to-one". That is, a point is represented by more than one pair of coordinates.

3. To review some aspects of angle measure. For example, an angle swept out by $\frac{1}{8}$ of a turn has a measure of 45°. Circle Coordinates are similar to the Ruler and Protractor coordinates used in Lesson 4.

4. To work with rational numbers within the framework of a "What's My Rule" type game. Students may work with equivalent fractions; converting from improper fractions to mixed numbers and vice versa; adding and subtracting directed numbers; changing direction with directed numbers, etc..


1
2
3
4
5
6

PROCEDURE

You may wish to produce a transparency of the circle coordinate grid on page C-3 , or you may draw a representation of it on the blackboard.

There are many ways to plot points on this grid. One way would be to select a direction of rotation to associate with positive numbers, say counterclockwise.

The point $(3, \frac{1}{8})$ is then plotted by moving horizontally to the third circle, then around this circle $\frac{1}{8}$ of a turn in a counterclockwise (c.c.) direction.

The vertical line on the grid is useful only for estimating amounts of turn.

Other Points:

a. $(\frac{1}{2}, \frac{1}{2})$. Estimate the position of the circle with a radius of $\frac{1}{2}$. Then move $\frac{1}{2}$ of a turn in a c.c. direction.

b. $(3.25, 4)$. Estimate the position of the circle whose radius is 3.25, then move around this circle 4 times. Note that it is the same point as located by $(3.25, 0)$.

c. $(3, {}^{-}\frac{1}{4})$. A $\frac{1}{4}$ turn in the clockwise direction. Note that $(3, {}^{-}\frac{1}{4})$ represents the same point as $(3, \frac{3}{4})$.

d. $({}^{-}3, \frac{3}{8})$. Count three circles to the left. This is the same as circle 3, but you have a different starting point for the rotation. $({}^{-}3, \frac{3}{8})$ is the same point as $(3, \frac{-1}{8})$ and $(3, \frac{7}{8})$. It is also "opposite" the point $(3, \frac{3}{8})$.

e. $(0,\frac{1}{2})$. We can't plot points on the circle of zero radius. We could locate all points (0, N) where the vertical and horizontal lines intersect and call this point (0, 0), the origin.

Have the students suggest rules in their own words, concerning the location of points like a-d above. Guide them to generalize rules such as:

(1) The negative of the second coordinate reverses direction.

(2) The point (a,b) and the point ($^{-}$a,b) lie "diametrically" opposite each other on the same circle (circle: r = b).

(3) Point (a, b) and point (a, b + N) where N is a whole number represent the same point.

As long as the discussion centering on the Circle Coordinates utilizes rational numbers this is an appropriate place to work on some of the operations such as addition and subtraction.

2. **Fixing a Point**

PURPOSE

The following is an activity which can help students gain a better understanding of how a coordinate system "fixes" or "locks in" the position of a point.

PROCEDURE

Each pair of students receive two unlined 3 x 5 inch cards. Student A places a dot somewhere on his card. His goal is to communicate the position of this point to student B so that student B can place a dot on his card in the same position as student A's dot.

Student A may use a ruler to establish the position of his dot. He can measure the distances from the left-hand corner and from the bottom of the card. Student B then has to receive the coordinates, how the coordinate system operates and the ruler.

Student A could use a ruler and a protractor to report the dot's coordinates similar to Lesson 4. The bottom left-hand corner of the card can serve as the origin.

If student A places three noncollinear dots on his card as was done in Lesson 9, page 68, he will be able to "fix" the position of the dot. The three noncollinear points determine the origin, axes and unit distances for a two dimensional coordinate system. Student A will have difficulty, however, in communicating the position of his dot because student B will not have the necessary reference lines.

Capitalize on the above situation since it underlines the nature of the connection between a set of coordinates and their referents.

3. **Tic-Tac-Toe**

PURPOSE

To provide additional practice in plotting points.

PROCEDURE

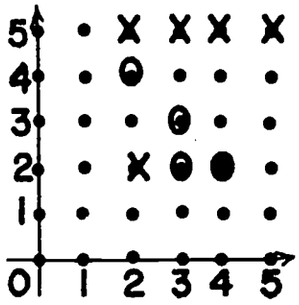
The basic Tic-Tac-Toe game is played on a 3 x 3 grid with X's and O's.

	O	X
O	X	O
X		

The player with three of his markers in a line wins. This game is easily adapted to a coordinate system.

You may use either the grid cells or the lattice points. Four in a row, instead of three in a row, brings a little more playing strategy into the game. Teams alternate turns. If a pair of coordinates is called out twice, the offending team loses their turn.

EXAMPLE:



TEAM X	TEAM O
1. (2, 2)	1. (3, 2)
2. (5, 5)	2. (3, 3)
3. (3, 5)	3. (4, 2)
4. (2, 5)	4. (2, 4)
5. (4, 5) WIN!	

4. **BATTLESHIP**

PURPOSE

To provide additional pratice in plotting points.

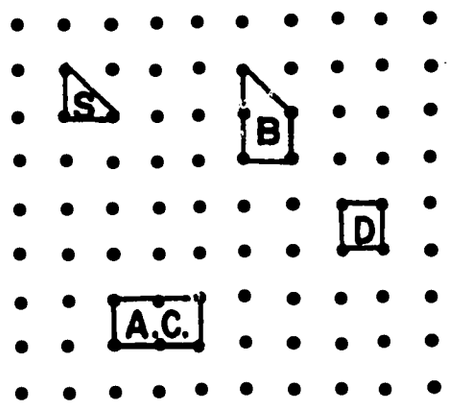
PROCEDURE

Each team has a navy with 4 ships. Team A tries to sink all of team B's ships. The four ships are deployed on a 10 x 10 coordinate grid. As in Tic-Tac-Toe you may use either the grid cells or the lattice points. Each team places its own ships in its own way. The number of points (cells) allocated for each ship is as follows:

Aircraft carrier	...	6 points (cells)
Battleship	...	5 points (cells)
Destroyer	...	4 points (cells)
Submarine	...	3 points (cells)

Team A calls out a shot (a pair of coordinates). If a ship of Team B's is hit, Team B must say "hit" and name the type of ship hit. Teams alternate turns. Since Aircraft Carriers and Battleships are so big, it takes two separate hits to sink them. Only one hit, however, to sink the Destroyers and Submarines.

EXAMPLE:



TEAM A'S DEPLOYMENT

Each team will find it wise to keep a record of their tries so that they will not waste any shots. The team that loses all of its ships loses the war.

5. **QUBIC**

PURPOSE

To provide additional practice in plotting points in a three-dimensional rectangular coordinate system.

7. **Coordinate Baseball**

PURPOSE

To provide additional practice in plotting points in a two dimensional rectangular coordinate system. This activity is also useful in extending the student's algebraic skills by providing an opportunity to supply the linear equation which "fits": a set of coordinates of points which lie in a line. This follows nicely from Lesson 7.

PROCEDURE

Ten students are divided into two teams of five players each. Team A is up to bat and Team B is in the field. The pitcher who is on Team B, calls out an ordered pair, say $(^-3,2)$. The first batter on Team A responds with <u>any</u> other ordered pair, say $(0,^-1)$ and is awarded first base. The second batter of Team A, however, must give the coordinates of a point which will lie on the line determined by $(^-3, 2)$ and $(0, ^-1)$. If he does, he is awarded first base and the other runner moves to second base. If not, he is <u>out</u>.

Let us assume the second batter of Team A said $(^-1, 0)$ and reached first base. The third batter must give a pair of coordinates of a point which lies on this line to reach first base and thus advance the other two batters. If he doesn't, then he is out and the fourth batter comes up, etc.. After the line has been determined by the pitcher and the first batter, any succeeding batter (provided there are not two outs) can hit a <u>home run</u> by giving the rule which determines the line (i.e., its equation).

In the case described above the home run would be hit by the batter who gives the equation $x + y = ^-1$. This equation may be given verbally as in saying: "The sum of the two coordinates is negative one".

PROCEDURE

Qubic is a commercial game by Parker Brothers, Incorporated. Essentially it is a three-dimensional Tic-Tac-Toe game. Four planes with four by four cells on each plane. The moves can be recorded by means of an ordered triple.

5. **A 3-Dimensional Tinker Toy Lattice**

PURPOSE

To provide additional practice in plotting points in a three-dimensional rectangular coordinate system.

PROCEDURE

A tinker toy construction. Each connector of the tinker toy lattice can be made to represent an ordered triple. The connecting sticks which represent the axes can be colored so as to contrast with the other sticks.

As a side benefit the students can calculate the number of necessary sticks and connectors needed. They also could benefit from taking part in the actual construction of the lattice.

If the team in the field discovers the rule for the line before the batter does, then the team at bat is retired automatically and the two teams switch positions.

A referee will be needed to determine the outs, hits, walks, and home runs.

If the team in the field announces an incorrect rule, then the batter is awarded a walk to first base.

To speed the game up it would be wise to allow only two outs per side per inning. But if the team in the field gives the rule, then the two teams change places automatically.

EXAMPLE:

Team A at bat and the pitcher says ($^-3$, 2). The 1st batter says (0, $^-1$) and takes 1st base. The 2nd batter is up and says ($^-1$,0). "Correct", says the referee, "take 1st base". Now there are two runners on and no outs. Batter number 3 says ($^-2$, 0). "OUT!", says the referee. Batter 4 says $x + y = {}^-1$. "Home run!", says the referee. The score is 3 to 0 and there is still one out.

Batter 5 comes up, the pitcher says (8, 4) and the batter swings with (6, 6) and takes first base. Some sharp fielder on Team B says $x + y = 12$. "Side out!" cries the referee and Team B comes to bat with score 3 to 0 in favor of A. Play continues for as many innings as deemed appropriate.