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AUTHOR	Choate, Stuart A.
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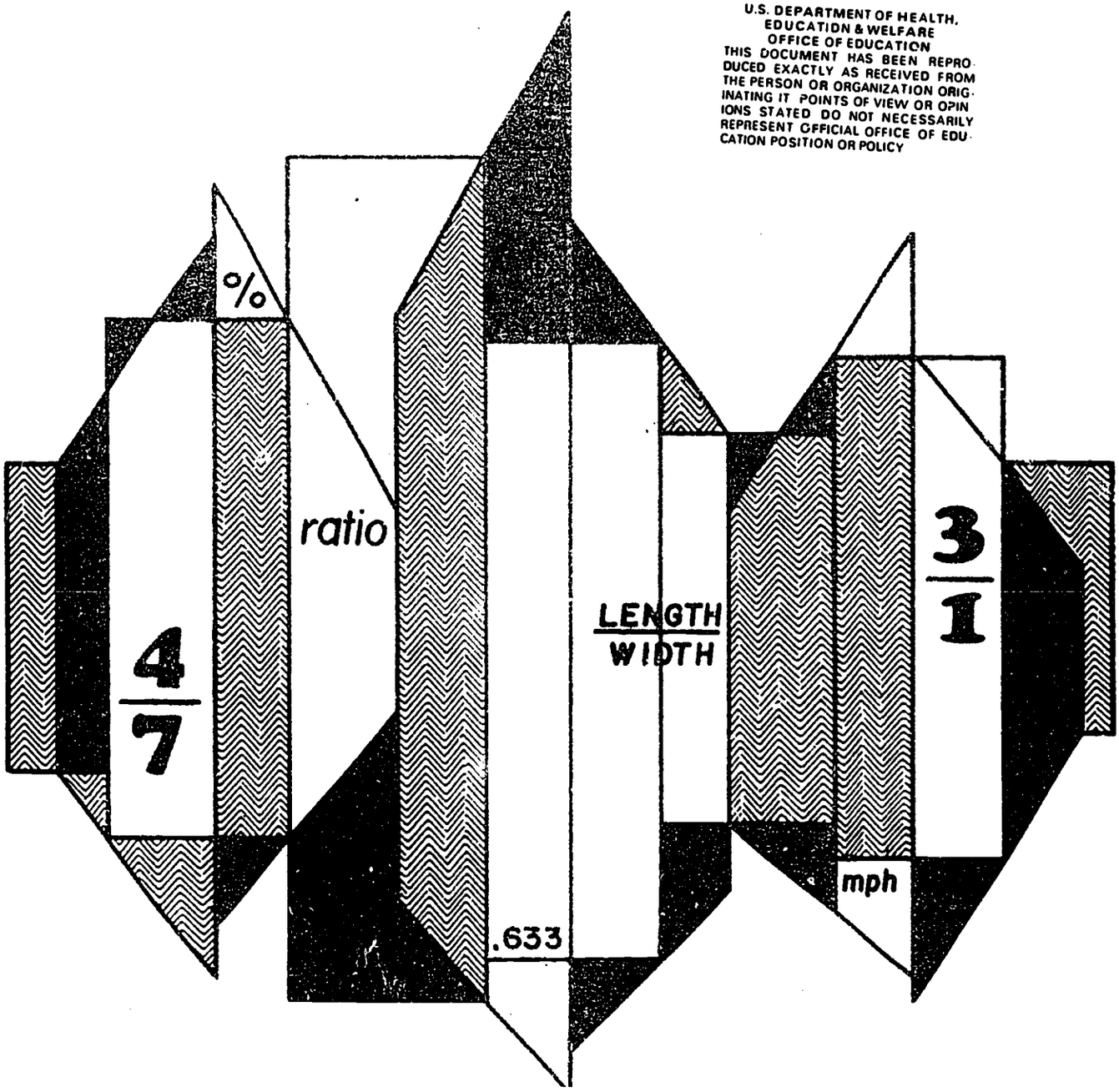
ABSTRACT

This guide to accompany "Activities with Ratio and Proportion," contains all of the student materials in SE 015 336 plus supplemental teacher materials. It includes a listing of terminal objectives, necessary equipment and teaching aids, and resource materials. Answers are given to all problems and suggested approaches and activities are presented for each section. Related documents are SE 015 334 through SE 015 336 and SE 015 338 through SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

ACTIVITIES WITH RATIO & PROPORTION



TEACHERS GUIDE

OAKLAND COUNTY MATHEMATICS PROJECT STAFF

Dr. Albert P. Shulte, Project Director
Mr. Terrence G. Coburn, Assistant Director and Project Writer
Dr. David W. Wells, Project Coordinator
Mr. Stuart A. Choate, Project Writer
Mr. Philip L. Cox, Project Writer
Mr. Daniel L. Herman, Project Writer
Mr. Robert E. Peterson, Project Writer
Miss Diane M. Ptak, Project Writer
Miss Joyce Sweet, Project Writer
Mrs. Kathleen Danielson, Special Project Writer
Mrs. Lawanda Beane, Secretary
Miss Shirley Biggs, Secretary
Miss Deborah J. Davis, Secretary
Mrs. Judy Orr, Secretary
Miss Linda Tee, Secretary

ACTIVITIES WITH RATIO AND PROPORTION

OAKLAND COUNTY MATHEMATICS PROJECT

All units have benefited from the combined attention of the entire project staff. The major writing responsibility for this unit was handled by

STUART A. CHOATE

ILLUSTRATIONS BY KATHLEEN DANIELSON

Oakland Schools
2100 Pontiac Lake Road
Pontiac, Michigan 48054

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PREFACE

BASIC ASSUMPTIONS

The assumption is made that the [illegible] learner [illegible], can not achieved, the objectives of this booklet. The [illegible] some experience with fractions, ratios, and per cents. [illegible]petence in these areas is not assumed, yet a complete fundamental approach is not undertaken. The attempt has been made to start at a point where all students can succeed and move forward from there. Whole number computation and some elementary measurement skills are all that the student needs.

OBJECTIVES

A summary of the terminal objectives appears just after the PREFACE. The Teacher's Guide for each lesson contains the objectives for that particular lesson. The objectives that are listed should not limit the scope of the teaching process, but should be used as a basis for evaluating student progress.

OVERVIEW

This booklet focuses on writing ratios and proportions [illegible] by problem situations, the solution of problems using proportions, and determining whole number per cents of a given number.

The understandings and skills gained while working here can be applied in most of the remaining booklets in this sequence.

PREFACE

Ratios and proportions are useful tools for problem solving. They can be used in many situations involving numerical relations. Some examples are comparing winning records of baseball teams, predicting the weight of an object suspended on a spring, and increasing the size of a recipe to feed a large number of people. It is because of the problem-solving power of ratio and proportion that this booklet was written.

If a technician working for a manufacturing firm knows the weight and volume of a metal object, he can tell what it is made of. This identification process (known as finding specific gravity) can be used by a detective (What is this piece of metal?), a dairy worker (Does the milk meet the state butterfat content requirement?), a service station attendant (Is there enough antifreeze in the radiator? Is this battery fully charged?), as well as in many other areas.

Each lesson centers on one or two main ideas. This booklet contains a variety of activities to help you develop and apply the ideas of ratio and proportion. Read, listen, and above all, participate in the discussions and class activities.

After actively <u>participating</u> in the activities of this booklet, you should be able to:

1. Write ratios and draw diagrams which represent statements like "7 out of 9 tires are whitewalls" and "For every 3 squares there are 5 triangles."

(LESSON 1)

Student involvement in activities and estimation are important among the teaching strategies intended in the booklet.

BOOKLET ORGANIZATION

Student Booklet:

Each section of the booklet lends its own contribution to the progress of the student.

Exercises are intended mainly for individual supervised study. It is not anticipated that much homework will be assigned, although brief sections of exercises may well serve that function.

Class activity sections are problem situations to be handled on a cooperative or group basis. They are to be in-class activities and are not appropriate homework exercises. The role of the teacher is that of a resource person and a director of learning. Students should be allowed to gather information and make discoveries on their own.

Discussion Questions are to be used for in-class discussion rather than homework. They serve as an aid in clarifying items which need added discussion.

The student has an opportunity to check his own progress in those sections labeled ✓Point. The intent is for the student to use them to evaluate his own progress rather than a quiz section to be given by the teacher.

2. Write ratios indicated by written problems.

(LESSON 2)

3. Determine whether or not two ratios are equivalent using paper folding or drawing diagrams.

(LESSON 3)

4. Write a ratio when a problem situation is read aloud to you.

(LESSON 4)

5. Simplify a given ratio.

(LESSON 5)

6. Determine if a proportion is true or false.

7. Solve a proportion for the missing member.

8. Write and solve a proportion suggested by a given problem situation.

(LESSON 7)

Teacher's Guide:

The Teacher's Guide is organized in this way for each lesson: Objectives, Equipment and Teaching Aids, Content and Approach, Things to Discuss and Answers.

The information appearing under Content and Approach is intended to assist the teacher in interpreting the emphasis of the lesson as well as give direction. The questions and comments under Things to Discuss suggest a class discussion of those ideas which can be further developed to aid the student's understanding.

Answers are printed in proximity to the exercises.

A complete list of Equipment and Teaching Aids appears on pages ix, x, and xi. The needs of each lesson are indicated in this list. Also in each lesson, the equipment and teaching aids needed for that particular lesson are listed.

It is important to the smooth operation of your classroom that the equipment needs are anticipated far enough in advance so they can easily be secured.

9. Use data resulting from an experimental activity to predict the outcome of a future trial of the experiment.

(LESSONS 6, 8, 10, 11, 17)

10. Use the cross-product method to determine whether or not two ratios are equivalent.

(LESSON 9)

11. Use proportions and per cent to solve problems like the following.

a. Use a proportion and 100 as a basis to compare costs and athletic performances.

b. Determine the amount of medicine required to mix 40 oz. of a 35% solution.

(LESSON 12)

12. Estimate a whole number per cent of a given line segment.

13. Write a per cent in its various forms, such as 40% , $\frac{40}{100}$, .40 , and $\frac{2}{5}$.

(LESSON 13)

SUMMARY OF TERMINAL OBJECTIVES

1. Given the phrases "A out of B" and "For every A there are B", the student shall be able to write ratios representing the phrases.

2. The student shall be able to draw diagrams representing the phrases, "A out of B", and "For every A there are B".

3. Given combinations of written and pictorial information, the student shall be able to write the indicated ratios.

4. The student shall be able to demonstrate whether or not two ratios are equivalent. Methods to be used are:
   a. Paper folding or drawing diagrams.
   b. Multiplication, division, or factoring methods.
   c. Cross-product method.

5. Given data from a problem situation, the student shall be able to write an indicated proportion.

6. Given sufficient data resulting from an experimental activity or a problem situation, the student shall be able to use proportions to predict the outcome of future trials of the experiment.

7. Given a ratio, the student shall be able to write it in simplified form.

8. Given a proportion, the student shall determine whether it is true or false.

9. The student shall be able to find the truth set of an open proportion.

14. Estimate a whole number per cent of a given number.

(LESSON 14)

15. Use proportions to calculate a whole number per cent of a given number.

(LESSON 15)

16. Recognize whether or not a problem situation requires an inverse proportion.

17. Solve problems requiring inverse proportions.

(LESSON 16)

You should be able to apply what you have learned about ratio and proportion to other booklets this year as well as in your future mathematics and science classes.

10. The student shall be able to solve an open proportion using the cross-product method.

11. The student shall be able to use proportions to solve problems such as:

    a. Comparing the costs of two items using 100 as a basis for comparison.
    b. Determining the amount of medicine required to mix 40 oz. of a 35% solution.
    c. Comparing the performances of two athletes, or two automobiles.

12. The student shall be able to estimate a whole number per cent of a given line segment.

13. The student shall be able to write a per cent in its various forms such as 40%, $\frac{40}{100}$, .40, $\frac{2}{5}$.

14. The student shall be able to estimate a whole number per cent of a given number.

15. The student shall be able to use proportions to calculate a whole number per cent of a given number.

16. The student shall be able to recognize whether or not a problem situation requires an inverse proportion.

17. The student shall be able to solve problems requiring inverse proportions.

TABLE OF CONTENTS

TABLE OF CONTENTS

TITLE	PAGE

EQUIPMENT AND TEACHING AIDS

A. STUDENT		LESSON
*1.	Several sets of 5" x 8" printed cards containing pictorial and written information about ratios.	2
*2.	Paper folding sheets: P_1 illustrates equivalence of $\frac{1}{4}$, $\frac{2}{8}$, $\frac{4}{16}$, $\frac{8}{32}$; P_2 illustrates equivalence of $\frac{3}{5}$, $\frac{6}{10}$, $\frac{12}{20}$.	3
*3.	One ruler graduated in half-centimeters (Cut out from student page 44.)	6
#4.	One ball: golf ball, tennis ball or super ball.	8
#5.	One yardstick.	8
#6.	One meter stick.	10, 11, 17
#7.	Seven feet brown wrapping paper, 8 inches wide.	10
#8.	Ten inches of masking tape.	10
#9.	One spring.	11
#10.	One hook.	11

STUDENT (CONT'D.)		LESSON
#11.	One cardboard pointer.	11
#12.	Three known weights.	11, 17
#13.	Two unknown weights.	11
#14.	One weight, x, for tension.	11
#15.	One nut and bolt.	11
*16.	One block of wood with holes, A,B,C	11
*17.	One strip of tagboard, one inch long.	11
#18.	One-half rubber band, Size 31: $\frac{1}{8}''$ x $2\frac{1}{2}''$	13
#19.	One egg beater or one hand speed drill.	17
#20.	Two malted milk cartons.	17
#21.	One $2\frac{1}{2}''$ metal paper clamp.	17
#22.	One wire rod ($\frac{1}{8}''$ diameter, 6" long).	17
#23.	Two metal hooks.	17

* Materials provided with the booklets.

\# Quantity needed to supply one activity station.

B. TEACHER	LESSON
1. Overhead projector.	1,2,3,4,5,7,9,11, 12,17
2. Projection screen.	1,2,3,4,5,7,9,11, 12,17
*3. Five packages of acetate geometric figures, 10 figures per package.	1
*4. Prepared overhead transparencies.	1,2,3,4,5,7,9,11, 12,17
5. One small candle with thumbtack and cardboard candle holder.	4
6. One each of the following glass jars: $\frac{1}{2}$ pint, 1 pint, 1 quart, $\frac{1}{2}$ gallon, 1 gallon	4
7. One tin can with viewing window.	4
8. One book of matches.	4
9. One timer - watch with second hand.	4

*Materials provided with the booklets.

NOTE: The Teacher's Guide for each lesson lists the equipment and teaching aids needed for that lesson.

THREE OUT OF EVERY FOUR. . .

THREE OUT OF EVERY FOUR. . .

OBJECTIVES

1. To review with the students how ratios are written given a phrase or diagram indicating a particular relationship.

2. The student shall be able to:

   a. Write a ratio given

      1. a phrase such as "A out of B."

      2. a diagram such as ●○●○○.

      3. a phrase such as "For every A there are B."

      4. a diagram such as ○○○→△△△△.

   b. Draw diagrams representing the phrases "A out of B" and "For every A there are B."

   c. Draw diagrams representing a given ratio.

EQUIPMENT AND TEACHING AIDS

TEACHER

1. Overhead projector

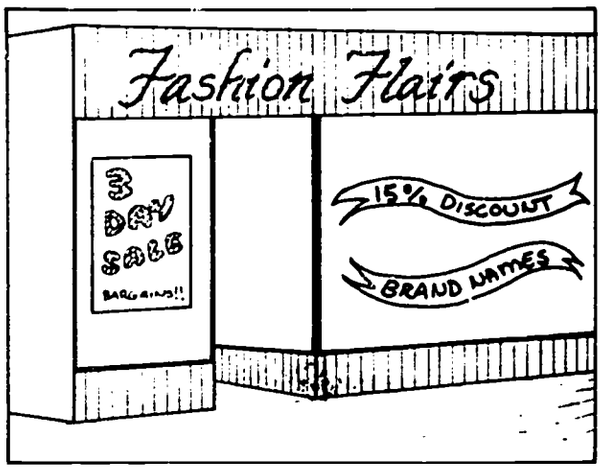
2. Projection screen

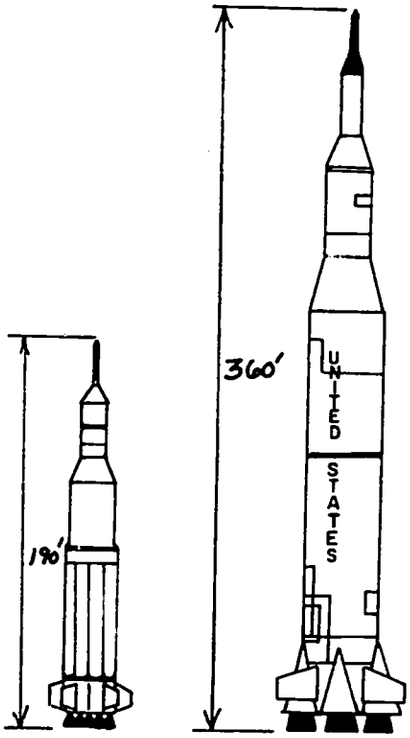
* 3. 5 packages of acetate geometric figures 10 figures per package

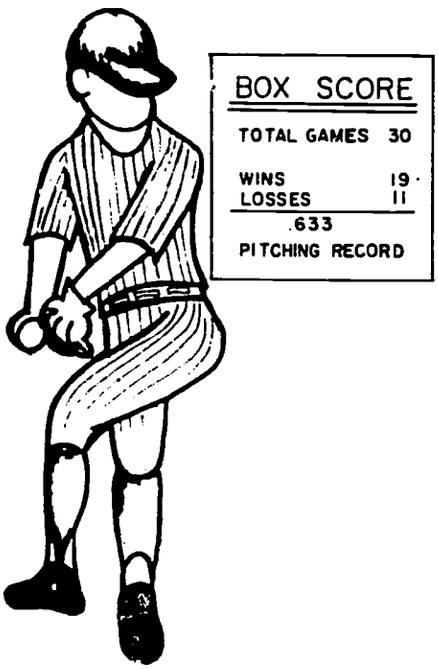
* 4. Transparencies: ARP 1A and ARP 1B

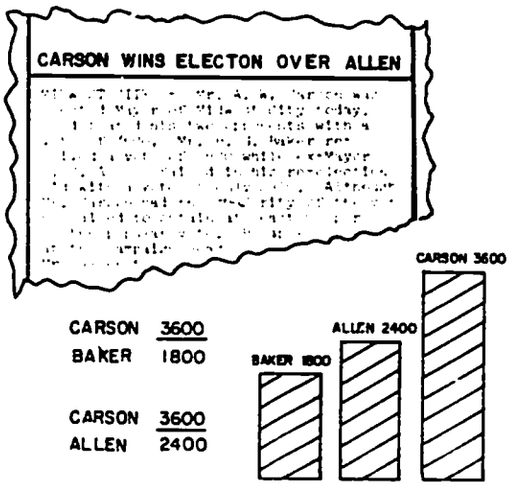
   ARP 1A contains several sets of geometric figures having their interiors shaded to illustrate "A out of B are shaded."

   ARP 1B contains several sets of geometric figures to illustrate "For every A there are B." ARP 1B is masked so various combinations can be shown in sequence.


Fashion Flairs
3 DAY SALE
BARGAINS!!
15% DISCOUNT
BRAND NAMES


190'
360'
UNITED STATES
SATURN I
14 ENGINES
SATURN V
11 ENGINES


BOX SCORE
TOTAL GAMES 30
WINS 19
LOSSES 11
.633
PITCHING RECORD


CARSON WINS ELECTON OVER ALLEN
CARSON 3600
BAKER 1800
CARSON 3600
ALLEN 2400
CARSON 3600
ALLEN 2400
BAKER 1800

CONTENT AND APPROACH

The pictures and commentary on pages 2 and 3 of the student booklet illustrate several kinds of situations which use ratio and proportion. These pages are to be discussed briefly, for the students are not expected to answer the questions now.

PART I

Being able to write ratios is an essential skill in problem solving. The intent of Lesson 1 is to provide opportunity for students to write ratios so that the class will come to a common understanding of writing ratios.

Preparatory to having the students work the exercises, a teacher-led discussion using the overhead projector and the geometric figures is proposed. A possible discussion sequence follows.

The sequence uses specific numbers. This is done because the discussion is easier to write this way. Use the figures in combinations you think appropriate.

1. Place 3 squares and 5 triangles on the overhead stage. Ask these questions. How many figures are there in all? How many figures are squares? How many are triangles?

   Tell them that 3 out of 8 figures are squares. Ask them (or tell them if necessary) how "3 out of 8" can be written as a ratio, $\frac{3}{8}$.

2. Place some hexagons and rectangles on the stage. Ask how many are hexagons, how many are rectangles, and how many are in the total group.

A baseball pitcher's record can be written as a ratio. The ratio is $\frac{19}{30}$ where the numerator is the number of wins and the denominator is the total of wins and losses. The decimal .633 is found by dividing 19 by 30. This is the way a pitcher's record is reported in the newspaper.

Stores offer sales to clear out goods they have had on hand in order to make room for new merchandise. The amount items are reduced is usually figured by using per cent. What does it mean when a store advertises 15% off? How are per cents written as ratios?

The results of an election are reported using the number of votes each candidate receives. The person receiving the highest total wins the election. Who was the winning candidate? By how many votes did he beat his nearest rival? Ratios can be used to compare votes. The ratio $\frac{3600}{1800}$ shows that people voted 2 to 1 in favor of Mr. Carson over Mr. Baker. What ratio indicates the comparison of votes for Mr. Carson to votes for Mr. Allen? What per cent of the vote did Mr. Carson get?

Saturn I was one test rocket used to explore space before an actual moon flight was attempted using the Saturn V. What is the ratio of the height of Saturn V to the height of Saturn I? About how many times longer is Saturn V? What is a ratio that compares the number of engines? Does it seem reasonable that a longer space rocket uses fewer engines?

Write on the overhead stage or the blackboard: "____ out of ____ figures are rectangles." Ask them to name the indicated ratio.

Have them fill in: "____ out of ____ figures are not rectangles." Write as a ratio.

3. Place some blue triangles, yellow triangles, and squares on the stage.

   The students should tell the number of blue triangles, yellow triangles, squares, total number of figures.

   Complete: ____ out of ____ figures are blue triangles.
   Write as a ratio.
   ____ out of ____ figures are squares.
   Write as a ratio.
   ____ out of ____ figures are yellow triangles.
   Write as a ratio.
   ____ out of ____ figures are triangles.
   Write as a ratio.

4. Present the statement: "4 out of 9 figures are hexagons." Ask: "What figures should be put on the stage?"

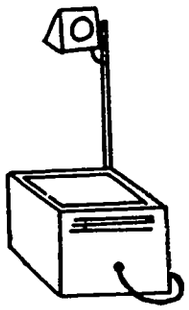
   A variety of answers may be suggested including, "Put 9 hexagons on the stage." Do it. Then some class members will offer to correct the picture.

5. Place some rectangles, yellow triangles, and squares on the stage. Ask them what kinds of statements and ratios could be given to describe the situation.

Ratios are used in many situations, some of which are indicated on the previous two pages. Performances of cars, athletes, students, and home appliances can be compared using ratios.

PART I

The first activity of this unit contains a review of the meaning of ratio. Your teacher will use the overhead projector to illustrate several ways ratios can be interpreted.



Watch carefully and participate in the discussion. If you have questions during the discussion, ask them.

PART 2

There are several kinds of statements indicating that a ratio could be written. In this part, you will work with one kind of statement.

PART 2

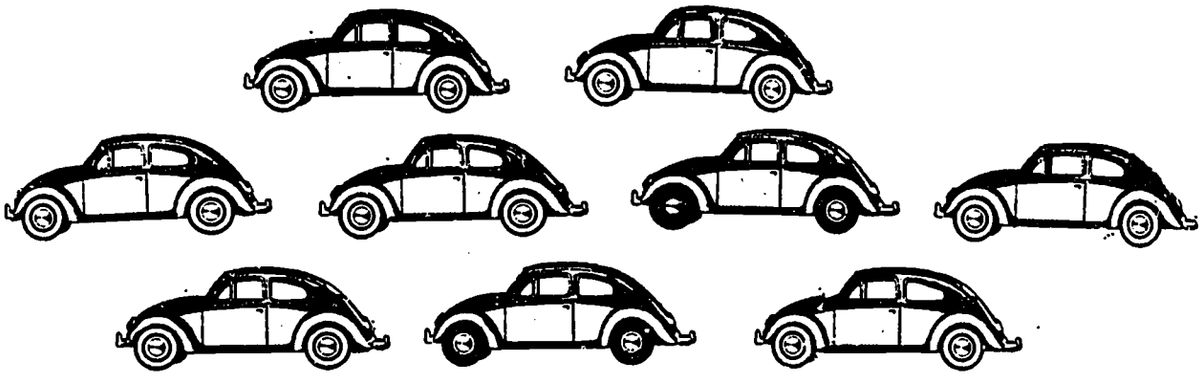
Use Transparency ARP 1A to make the transfer from manipulating geometric figures to the shading of a set of figures. ARP 1A is an example of the exercises in Part 2.

For each of the five sets of figures (or as many sets as you think necessary) ask questions like this:

How many circular regions are in the set? How many circular regions are shaded? What is the ratio of shaded circular regions out of unshaded circular regions?

After discussing the transparency, the students should be ready to do the exercises on pages 5-10.

EXAMPLE 1:



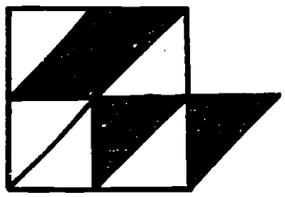
7 out of the 9 cars have white-walled tires.

$\frac{7}{9}$ expresses the statement in ratio form.

The denominator of the ratio represents the entire set.

The numerator represents the part of the set referred to.

EXAMPLE 2: 4 out of 9 triangular regions are shaded.



4 out of 9 is represented by $\frac{4}{9}$.

The denominator of the ratio is 9, the numerator is 4.

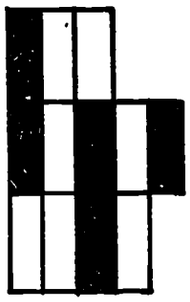
EXERCISES

1. 2 out of 5 triangular regions are shaded. Write the ratio represented by the statement.

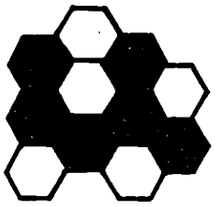
ANSWERS TO EXERCISES I

1. $\frac{2}{5}$

2. 4 out of 7 square regions are shaded. Write the ratio.

3. 

Of the 12 rectangular regions, 5 are shaded. Write the ratio indicated.

4. 

a. 6 out of the 11 hexagonal regions are shaded. Write the indicated ratio.

b. 5 out of the 11 hexagonal regions are <u>not</u> shaded. What ratio represents this statement?

5. 

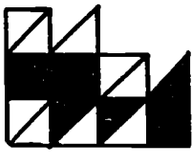
a. ____ out of 6 circular regions are shaded.

b. Write the ratio.

6. 

a. 4 out of ____ squares are circled.

b. Write the ratio.

7. 

a. ____ out of _____ triangular regions are shaded.

b. Write the indicated ratio.

ANSWERS (CON T'D.)

2. $\frac{4}{7}$

3. $\frac{5}{12}$

4. a. $\frac{6}{11}$

   b. $\frac{5}{11}$

5. a. 5

   b. $\frac{5}{6}$

6. a. 9

   b. $\frac{4}{9}$

7. a. 9 out of 18

   b. $\frac{9}{18}$

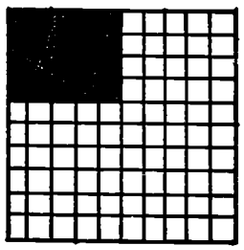
8. 

a. Circle the correct number of triangles to illustrate: 7 out of 9 triangles are circled.

b. Write the indicated ratio.

✓ **POINT**

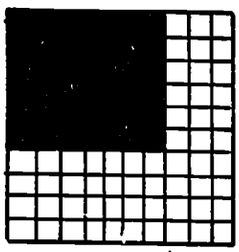
How do you know which number is the numerator and which is the denominator?

9. 

The picture contains 100 squares like this:□

a. ____ out of the 100 square regions are shaded.

b. What ratio is indicated?

10. 

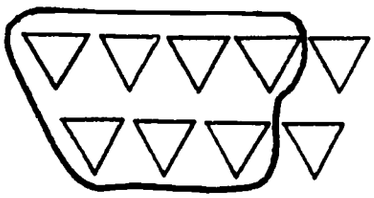
a. ____ out of 100 regions are shaded.

b. Write the ratio indicated.

c. ____ out of ____ regions are <u>not</u> shaded.

d. What ratio is represented now?

ANSWERS (CONT'D.)

8. a. 

The actual triangles circled may vary, but 7 of the 9 are to be circled.

b. $\frac{7}{9}$

ANSWER TO ✓POINT The denominator is the total number of members of the set. The numerator is that part of the total which is circled, discussed, shaded, etc.

9. a. 20

b. $\frac{20}{100}$

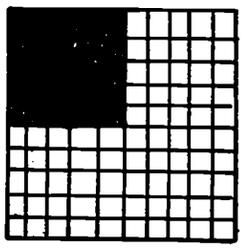
10. a. 42

b. $\frac{42}{100}$

c. 58 out of 100

d. $\frac{58}{100}$

25 out of 100 regions are shaded. $\frac{25}{100}$ is the indicated ratio. Ratios having a denominator of 100 are given the special name PER CENT. Per cent means "out of 100" or "for each 100".



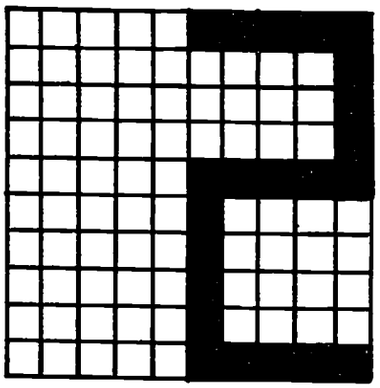
25 per cent and $\frac{25}{100}$ represent the same idea.

11. a. Write 35 per cent as a ratio.

    b. $\frac{75}{100}$ can be written as ____ per cent.

    c. If 40 out of 100 regions are shaded, the ratio indicated is ___ . Write the ratio using its special name.

12.



a. In the picture, ____ out of 100 regions are shaded.

b. The ratio indicated is ____.

c. Express the ratio as a per cent.

ANSWERS (CONT'D.)

The student has been exposed to per cent in the past. At this point no extensive discussions are carried out. These exercises are to review the relationship between "per cent" and ratios.

11. a. $\frac{35}{100}$

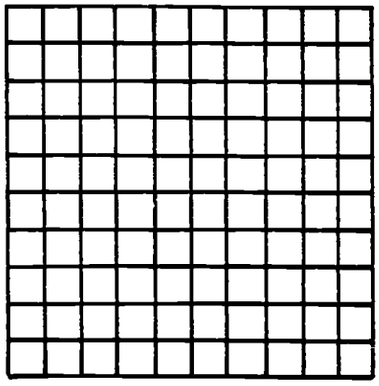
b. 75

c. $\frac{40}{100}$, 40 per cent, 40%

12. a. 22

b. $\frac{22}{100}$

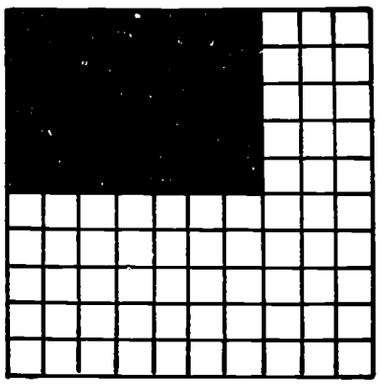
c. 22%

13. 

a. Shade the small square regions to show that 15 out of 100 regions are shaded.

b. What ratio is represented by the picture?

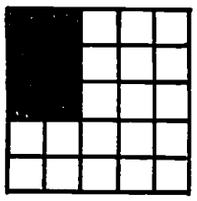
c. Write the ratio as a per cent.

14. 

35 per cent of the regions are shaded.

Circle below the ratio representing the statement:

$\frac{35}{65}$ $\frac{65}{100}$ $\frac{100}{35}$ $\frac{35}{100}$

15. 

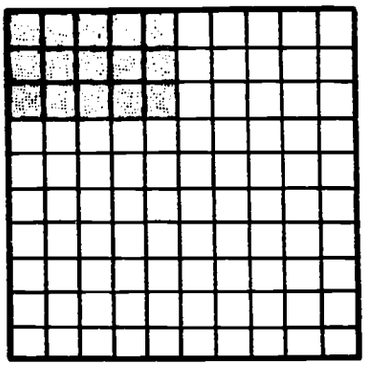
a. ___ out of ____ regions are shaded.

b. The indicated ratio is ____.

c. At the same rate, how many regions would be shaded out of a total of 50 regions?

ANSWERS (CONT'D.)

13. a.



Any combination of 15 shaded regions is correct.

b. $\frac{15}{100}$

c. 15 per cent or 15%

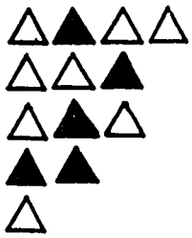
A student may say that 85 out of the 100 regions are unshaded and write the ratio $\frac{85}{100}$. This is an acceptable response.

14. $\frac{35}{65}$ $\frac{65}{100}$ $\frac{100}{35}$ $\frac{35}{100}$

15. a. 6 out of 25

b. $\frac{6}{25}$

c. 12

16. 

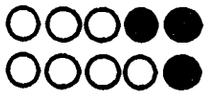
a. ____ out of ____ regions are <u>not</u> shaded.

b. Write the indicated ratio.

17. 4 out of 10 triangular regions are to be shaded.

a. Complete the picture: 

b. Write the ratio.

18. a. Complete the picture to represent $\frac{6}{10}$: 

b. ____ out of ____ circular regions are shaded.

19. In a picture using circles to represent $\frac{4}{11}$,

a. How many circles would be in the entire picture?

b. How many circular regions would you shade?

c. Draw a picture to represent the ratio $\frac{4}{11}$.

20. Which of the ratios represents the statement, 18 out of 53 triangular regions are shaded?

$\frac{53}{18}$ $\frac{18}{100}$ $\frac{18}{53}$ $\frac{53}{100}$

ANSWERS (CONT'D.)

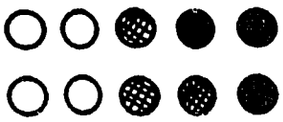
16. a. 8 out of 13

    b. $\frac{8}{13}$

17. a. 

    Any 4 triangular regions may be shaded.

    b. $\frac{4}{10}$

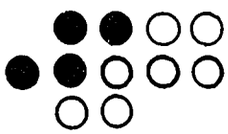
18. a. 

    Any 3 additional circular regions may be shaded.

    b. 6 out of 10

19. a. 11

    b. 4

    c. 

    The student might decide to shade 7 circles and claim the drawing represents, "4 out of 11 circles are not shaded." This would be acceptable. Then the drawing in 19c would be 

20. $\frac{53}{18}$ $\frac{18}{100}$ $\frac{18}{53}$ $\frac{53}{100}$

PART 3

The ratio $\frac{5}{8}$ can be used to express the statement "For every 5 there are 8." This statement differs from the first type we wrote because it indicates that two different sets of objects are being compared. The first type of ratio was a comparison of part of a set to the whole set.

Look at the two types of ratios to note their differences:

5 out of 8 triangles are circled.

$\frac{5}{8}$

For every 5 triangles there are 8 squares.

$\frac{5}{8}$

EXAMPLE: Draw the picture and write the ratio representing: For every 11 * there are 6 #.

Answer:

Picture	Ratio
* * * * * * * * * → # # # # # #	$\frac{11}{6}$

✓ **POINT**

How do the ratio statements and pictures introduced in Part 3 differ from those you worked with in Part 2 ?

PART 3

While Part 2 discussed ratios related to subsets of sets of figures, Part 3 deals with comparison of two distinct sets.

The statement "for every 3 triangles there are 4 squares" is pictured as △△△→□□□□ and is represented by the ratio $\frac{3}{4}$.

On the stage of the overhead you could place three sets of three triangles and three sets of four squares. The statement "For every 3△ there are 4□" is used to express the picture. "For every" in this situation takes on meaning and will be less likely to cause confusion than when "for every" is used with a single correspondence.

1. Placing 4 rectangles and 7 squares on the stage in this way,

the students should complete the statement: "For every 4 rectangles there are ____ squares. Draw the arrow to complete the picture.

The ratio representing the picture is $\frac{4}{7}$.

The direction of the arrow indicates the order in which the comparison is to be made.

ANSWER TO ✓POINT

In Part 2 a single set of objects was being dealt with, and part of the set was being compared to the entire set. In Part 3 two sets are considered, and the number of members in one set is being compared to the number of members in the other set.

EXERCISES

1. For every 2 △ there are 5 □ .

   a. Complete the picture: △△ →

   b. Write the ratio which represents the statement.

2. For every 3 △ there are 7 □ .

   a. Complete the picture: → □□□□□□□

   b. Write the ratio representing the statement.

3. For every 4 △ there are 5 □ .

   a. Draw a picture representing the statement.

   →

   b. Write the ratio.

4. Use the picture to fill in the blanks: For every _____ there are _____ .

   △△△△ → □□□□□□□

5. Consider a set of 5 ○ and a set of 6 ⬡.

   a. Write a ratio indicating the number of ○ compared to the number of ⬡.

   b. Draw a picture which represents the ratio.

   →

   c. Fill in: For every _____ there are _____.

2. Place some hexagons and triangles on the stage. Have the students complete the statement: "For every ____ there are _____." Draw in the arrow (in the appropriate direction) and write the indicated ratio.

3. Present the statement, "For every 3 squares there are 5 hexagons." Ask how many of each kind of figure would be used (put them on the overhead stage); how the arrow should be drawn; and what ratio represents the statement.

4. Use Transparency ARP 1B which brings together the diagram, the picture and the ratio.

   B. This section points out that the numerator is not always the smaller number.

   C. In this case the direction of the arrow is reversed. The arrow depends upon the statement and the order in which the figures are drawn.

ANSWERS TO EXERCISES 2

1. a. □□□□□

   b. $\frac{2}{5}$

2. a. △△△

   b. $\frac{3}{7}$

3. a. △△△△ → □□□ □□

   b. $\frac{4}{5}$

4. 4△, 7□

5. a. $\frac{5}{6}$

   b. ○○○○○ → ⬡⬡⬡ ⬡⬡⬡

   c. 5○, 6⬡

The statement is intended to allow the student to write either $\frac{5}{6}$ or $\frac{6}{5}$. The important thing is that the students are consistent throughout the exercise.

6. For every 3 ◯ there are 7 ⬡.

   Circle the ratio which represents the above statement. $\frac{3}{7}$ $\frac{7}{3}$

7. Given 4 ⬡ and 9 ◯ and the ratio $\frac{9}{4}$, fill in the blanks:

   For every _____ there are _____ .

8. Draw a picture representing the statement: For every 6 □ there are 11 △ .

   a. →

   b. What ratio represents the statement?

9. For every 7 △ there are 4 □ .

   a. Complete the picture: → □□□□

   b. Write the ratio.

10. Consider the ratio, $\frac{3}{5}$.

    a. Draw a picture representing the ratio.

    b. For every _____ there are _____.

ANSWERS (CONT'D.)

6. $\frac{3}{7}$ $\frac{7}{3}$

7. 9○, 4⬡

8. a. □□□□□□ → △△△△△△△△△△△

b. $\frac{6}{11}$

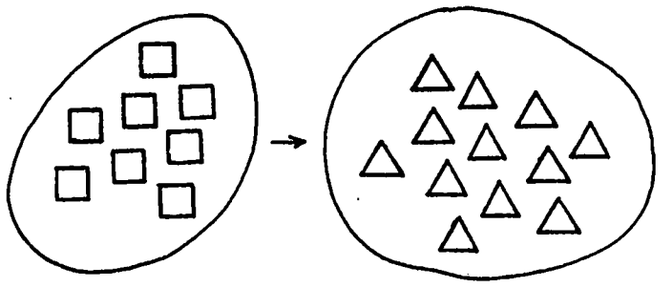
9. a. △△△△△△△

b. $\frac{7}{4}$

10. a. # # # → & & & & &

b. 3# , 5&

11. Look at these two sets:



a. For every 8 □ there are _____.

b. Write a ratio comparing □ to △ .

12. For every 8 △ there are 10 ○.

a. Complete the picture

△△△△△△△△ →

b. Write a ratio which represents the picture.

13. Write the ratio represented by

□□□□ → △△△△△△△

14. For every 6 ○ there are 9 □.

a. Complete the picture: → □□□□□ □□□□

b. Write a ratio which represents the picture.

c. This exercise compares the number of _____ to the number of _____.

ANSWERS (CONT'D.)

11. a. 12△

b. $\frac{8}{12}$ The simplified ratio, $\frac{2}{3}$, would be acceptable.

12. a. ○○○○○○○○○○

b. $\frac{8}{10}$ The ratio, $\frac{4}{5}$, would be acceptable. A student giving such a response should be asked to demonstrate how he knows it is true.

13. $\frac{4}{7}$

14. a. ○○○○○○

b. $\frac{6}{9}$ See note for Exercise 12b.

c. ○'s □'s

For every 2△ there are 3⬡ is expressed by the ratio $\frac{2}{3}$.

△△ → ⬡⬡⬡

Another statement can be written for the picture. It is "2 triangles compared to 3 hexagons." The ratio $\frac{2}{3}$ also represents this.

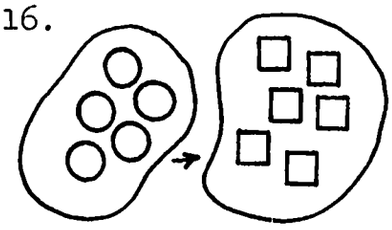
Example: There are 28 days in February compared to 31 days in March. The ratio representing this comparison is $\frac{28}{31}$.

15. There are 4 triangles compared to 7 circles.

    a. Complete the picture: △△△△ →

    b. Write the indicated ratio.

16.



    a. 5 circles are compared to ____ squares.

    b. The ratio is _____.

17. There are 7 * compared to 11 #

    a. Draw a picture representing the comparison. →

    b. Write the ratio.

    c. Complete the statement: For every ____ there are ___ .

ANSWERS (CONT'D.)

15. a. ○○○○○○○

    b. $\frac{4}{7}$

16. a. 6

    b. $\frac{5}{6}$

17. a. * * * * * * * → # # # # # # # # # # #

    b. $\frac{7}{11}$

    c. 7*, 11#

18. 5 out of 9 triangles are shaded.

    a. Write the ratio indicated by the statement.

    b. Draw a picture representing the ratio.

19. Write the <u>ratio</u> and <u>draw</u> a picture representing each of the following:

    a. For every 9 squares there are 6 triangles.

    b. 6 out of 9 squares are shaded.

    c. There are 9 triangles compared to 6 squares.

20. Circle the ratio which represents each of the following:

    a. There are 16 squares compared to 11 triangles.

    $\frac{11}{16}$ $\frac{11}{27}$ $\frac{16}{11}$ $\frac{27}{16}$

    b. For every 8 shaded triangles there are 15 circles.

    $\frac{15}{8}$ $\frac{8}{15}$ $\frac{8}{23}$ $\frac{23}{15}$

    c. 9 out of the 17 squares are shaded.

    $\frac{17}{9}$ $\frac{9}{17}$ $\frac{9}{26}$ $\frac{8}{26}$

ANSWERS (CONT'D.)

18. a. $\frac{5}{9}$

b. ▲▲▲▲▲△△△△

19. a. $\frac{9}{6}$ □□□□□□□□□→△△△△△△

b. $\frac{6}{9}$ ■■■■■■□□□

c. $\frac{9}{6}$ △△△△△△△△△→□□□□□□

20. a. $\frac{11}{16}$ $\frac{11}{27}$ $\frac{16}{11}$ $\frac{27}{16}$

b. $\frac{15}{8}$ $\frac{8}{15}$ $\frac{8}{23}$ $\frac{23}{15}$

c. $\frac{17}{9}$ $\frac{9}{17}$ $\frac{9}{26}$ $\frac{8}{26}$

DISCUSSION QUESTIONS

1. Look at the two pictures: A □□□ → △△△△

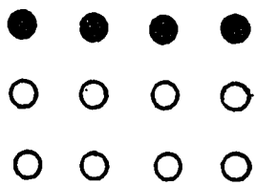
   B □□□ ← △△△△

   a. How do they differ?

   b. What ratios do they represent?

   c. Are both pictures correct? Explain.

2. Look at the drawing.



   a. Write all of the different ratios and sentences that could describe the drawing.

   b. Was it possible to use a statement like

      For every ____ there are ____ , or ____ out of ____.

      to describe any of your ratios? Explain your answer.

3. a. If you were to close your eyes and imagine a picture for each of the statements, what would you see?

      2 out of 3 cars are red.

      For every 2 red cars there are 3 blue cars.

   b. Are the ratios different?

   c. Tell how the statements describe different situations.

ANSWERS TO DISCUSSION QUESTIONS

1. a. Picture A compares the number of squares to the number of triangles as indicated by the arrow pointing to the right. In picture B, the arrow indicates that the number of triangles is being compared to the number of squares.

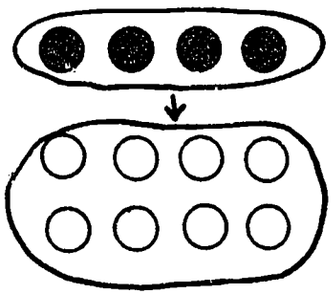
   b. Picture A represents the ratio $\frac{3}{4}$ while picture B represents $\frac{4}{3}$.

   c. Both pictures are correct. They just represent different ratios. The person must interpret the picture carefully so that he does not overlook the direction of the arrow which tells in which direction the comparison is being made.

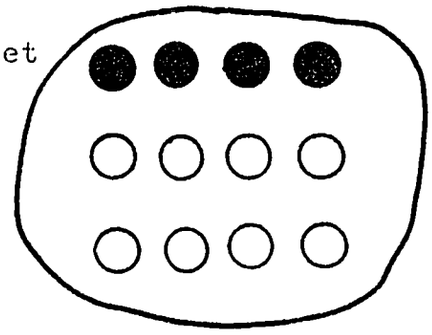
2. a. $\frac{4}{8}$, $\frac{2}{4}$, $\frac{1}{2}$ This is from a "For every" statement.

   $\frac{4}{12}$, $\frac{2}{6}$, $\frac{1}{3}$ This is from an "Out of" statement.

   b. Both kinds of statements can be used depending on whether you wish to consider the drawing as two sets



or one set



ANSWERS (CONT'D.)

3. a. "2 out of 3 cars are red."

There are three cars, two of them are red and the other one is not red.

"For every 2 red cars there are 3 blue cars."

There are five cars, two are red and three are blue.

b. The ratios (the ideas) are different but the symbols (fractions) are not different.

(Teacher: Do not press the number-numeral idea. Just ask them to explain their response.)

c. This refers to a previous ✓Point question (pg. 11). "Out of" deals with one set and a subset thereof. "For every" deals with two disjoint sets.

(NOTE: Student page 18 and Teacher page T18 are blank.)

CARDS, CARS, AND RATIOS

Ratios express relationships in a variety of situations. In this lesson you will be writing more ratios. These ratios come from many topics that you might find interesting.

EXERCISES I

You will receive cards containing some facts and questions about these facts. An objective of this exercise is for you to be able to write ratios which represent certain data given to you.

There are twelve cards in the set. Your teacher will explain how you are to do the exercises. Write your answers below.

CARD NUMBER	ANSWERS TO EXERCISES

CARDS, CARS, AND RATIOS

OBJECTIVES

The student shall write the indicated ratios given

1. Combinations of written and pictorial information.
2. Statements in written form only.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

*#Several sets of 5" x 8" printed cards containing pictorial and written information about ratios. The Cards are reproduced on pages T 19b - T 19j.

B. TEACHER

1. Overhead Projector
2. Projection screen

*3. Transparency: ARP 2
Transparency ARP 2 illustrates the graphs on cards 1 and 2. It might be useful in explaining how the graphs are read.

CONTENT AND APPROACH

In Exercises 1, the students are to be given 5" x 8" cards containing pictorial and written facts about particular topics. In your equipment file there are some sets of twelve cards. There are several ways the activity can be handled.

1. All students could remain in their seats and each one given a couple of cards. Then they would read each card and answer the questions. When finished, the students would pass the card to a neighbor for him to read and answer. The cards should be circulated until each student has worked at least six cards.

2. The class could be broken up into small groups with each group having a set of cards. Within the group, they would discuss the set of cards and record their responses. Total class discussion could resolve any questions that were unanswered or caused debate.

3. The cards could be placed around the room in piles and the students could circulate about the room answering the questions. This mode of operation would be preparatory to students doing experimental activities where station-to-station movement is required.

THINGS TO DISCUSS

In both Exercises 1 and 2 there is a reference to per cent. The development of per cent does not occur until Lesson 12; however, the students' previous exposure to the topic cannot be ignored. In each set of exercises there is a short paragraph which reviews writing per cents using ratios and using the % symbol. There may be a need to discuss this idea with the students as they work the exercises.

Each set of exercises also contains items about miles per hour, feet per second, cost per day, etc. A statement is made concerning their meaning (60 mph is 60 miles per 1 hour). Some discussion might be helpful to the students.

As the lesson is concluded, both of the ideas mentioned above should be sufficiently discussed so that the student can write these kinds of ratios.

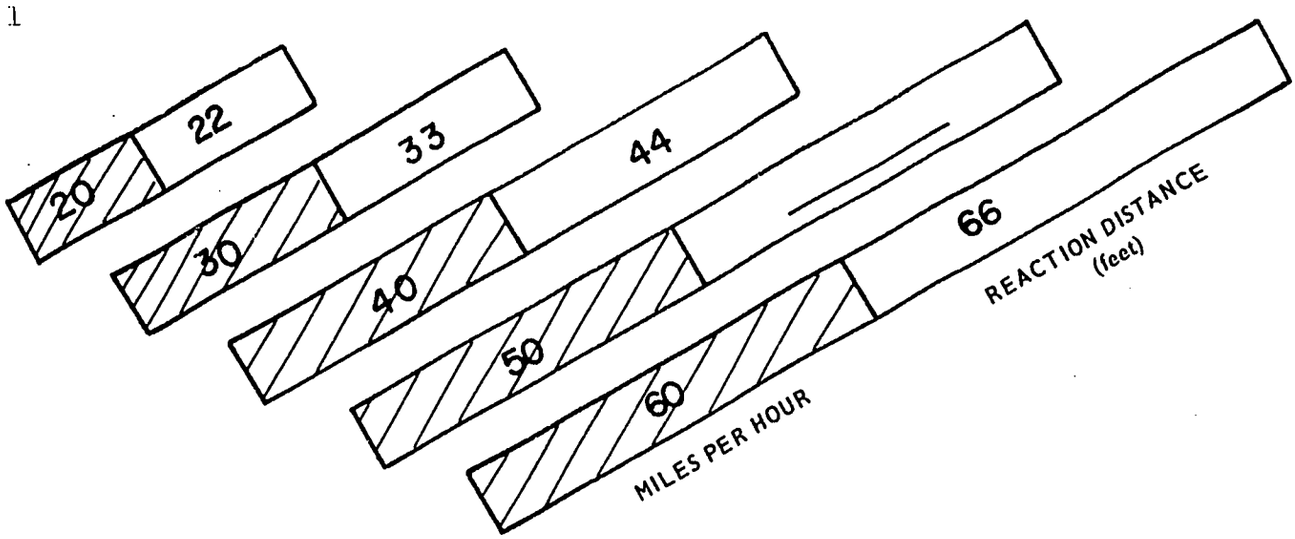
CARD 1

AUTOMOBILE DRIVING

The faster you drive, the farther you travel from the time you realize you must stop until you apply the brakes. (This is called Reaction Distance.)

(Answer the questions on the other side.)

1



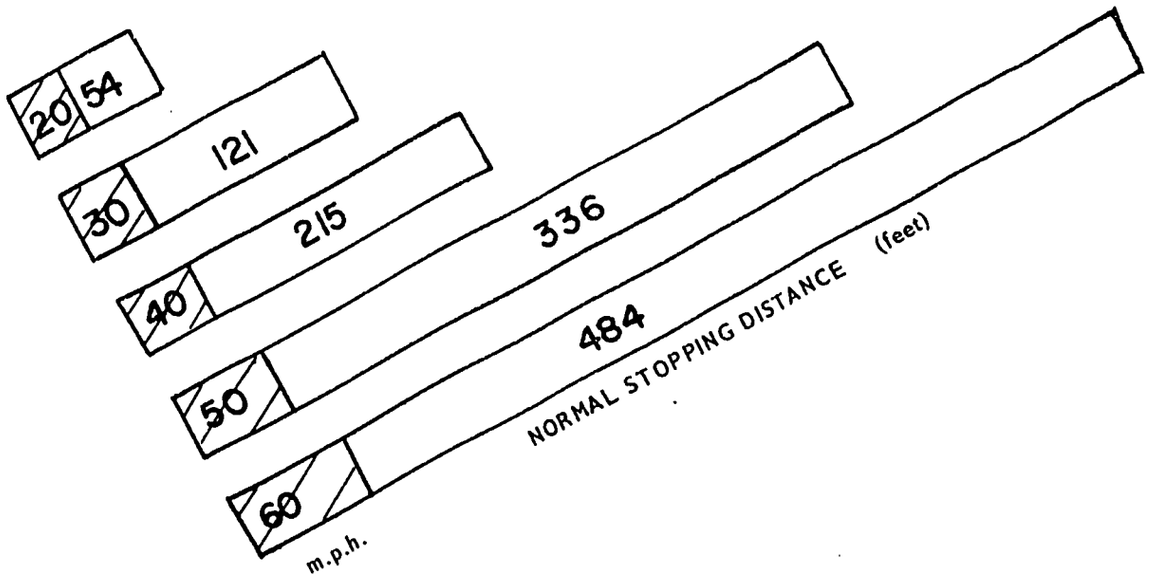
1. Write a ratio which compares reaction distance for 20 mph to reaction distance for 40 mph.
2. Write a ratio which compares reaction distances of 20 mph to 60 mph.
3. Write a ratio to compare reaction distances for 30 mph and 60 mph.
4. The speeds increase by 10 mph. How do the reaction distances change?
5. What is the reaction distance for 50 mph?

CARD 2

AUTOMOBILE DRIVING

The faster you drive, the longer the stopping distance.

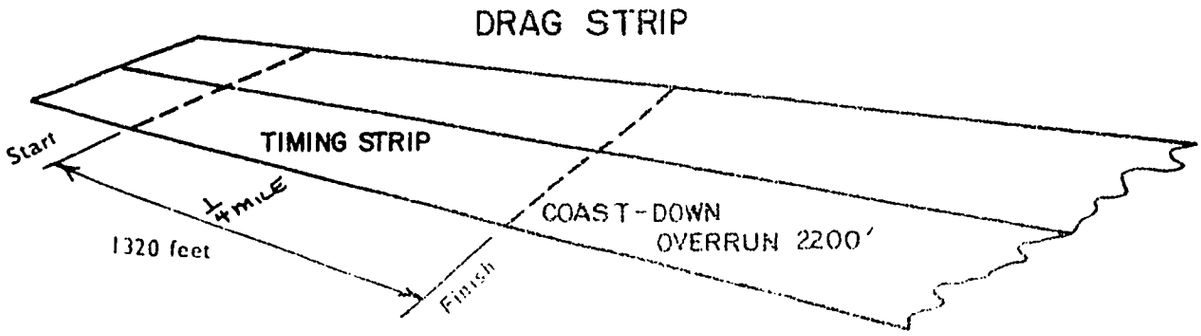
2



1. What ratio compares stopping distance of 20 mph to stopping distance of 40 mph? of 40 mph to 60 mph? of 30 mph to 60 mph?

2. If the speed (mph) doubles, how does the stopping distance change?

CARD 3

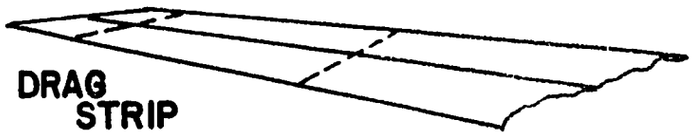


1. Which is longer, the timing strip or the overrun?

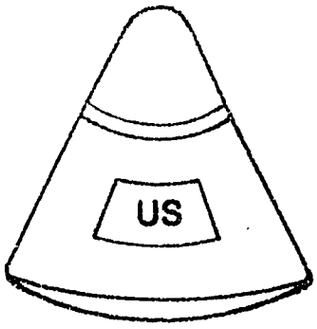
2. Write as a ratio, the comparison of the length of the timing strip to the overrun.

3. Is the overrun closer in length to $\frac{1}{4}$ mile or $\frac{1}{2}$ mile?

CARD 4



The driver takes a deceleration shock of 4.5 G's when stopping his dragster.



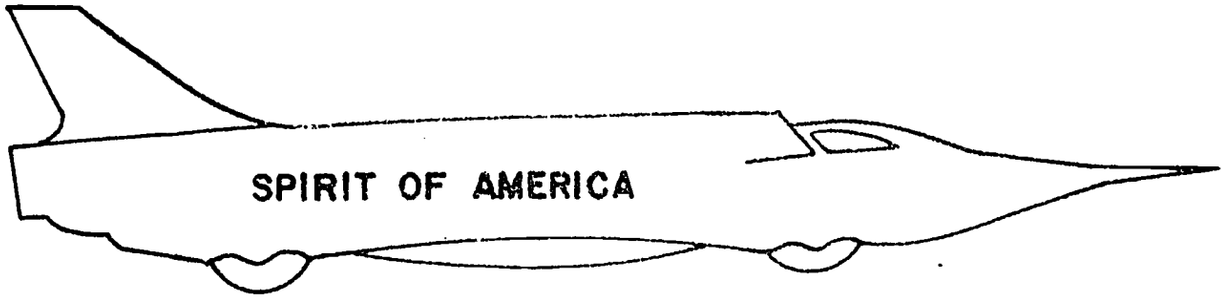
APOLLO COMMAND MODULE

The astronaut takes a deceleration shock of about 7 G's when coming back to earth.

1. Write a ratio comparing the G's taken by a dragster driver to that taken by an astronaut.

2. What geometric figure does the Apollo command module look like?

Mach I

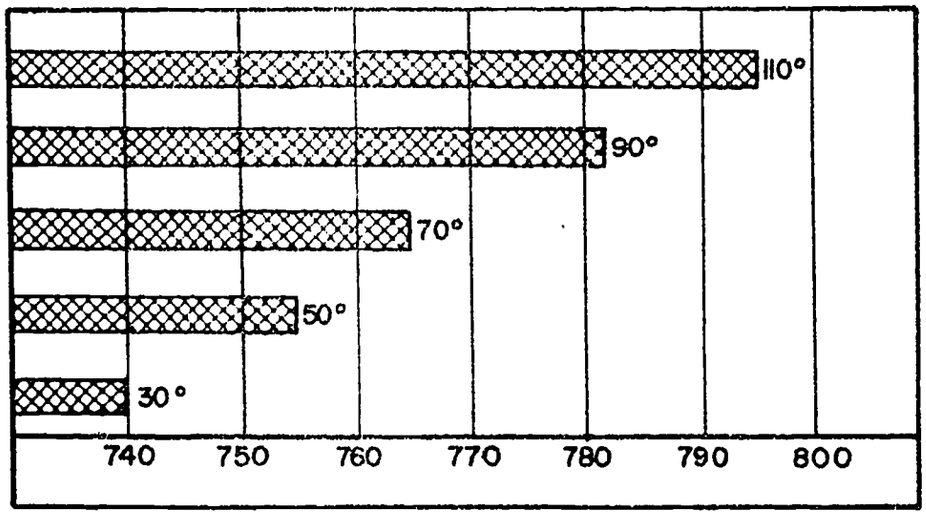


Land speed records are attempted at the Bonneville Salt Flats.

They are attempting to beat the speed of sound.

The speed of sound increases as temperature increases.

The speed of sound is called Mach I.



1. How fast must the car go to reach Mach I at 30° outside temperature?

2. If the outside temperature is 50°, how fast is Mach I?

3. What ratio compares the speeds for Mach I at temperatures of 30° and 90°?

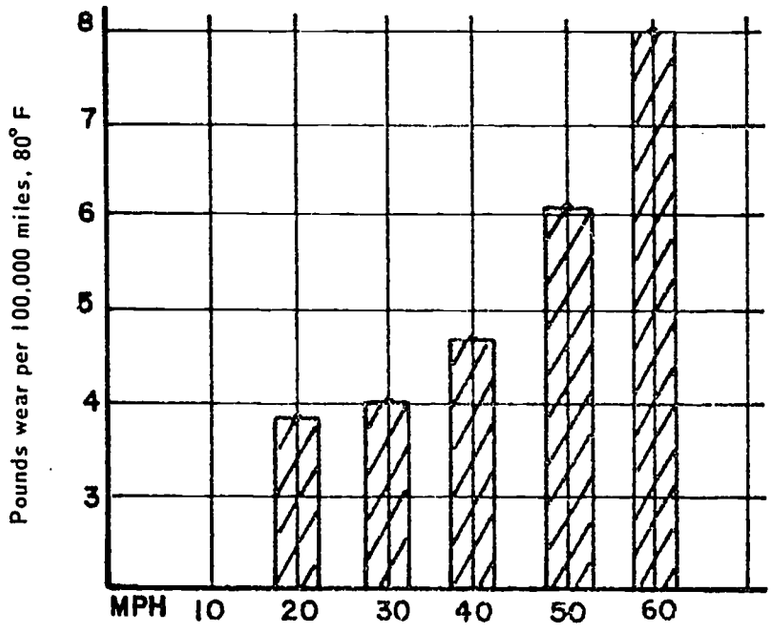
4. At Mach I and a temperature of 50°, the car temperature is 125°. When it is 90° outside, the car temperature is 200°. Write a ratio comparing the car temperatures.

CARD 6

Temperature and inflation [illegible] tire wear.

A tire will wear almost [illegible] as when operated at 60°F.

6



1. Use a ratio to show the comparison: tire wear at 60 mph to tire wear at 30 mph.

2. Tire wear at 55 mph is 7. Write a ratio comparing wear at 30 mph to wear at 55 mph.

3. How does wear at 50 mph compare to wear at 30 mph?

CARD 7

SHARKS

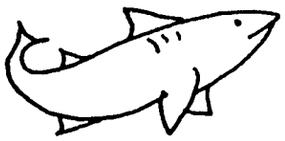
There are about 250 species of sharks, but only five are dangerous to humans.

The largest shark is called the Whale Shark.

Two types of sharks are described on the back.

7

SHARKS



Whale Shark
length: 60 feet
weight: 25000 pounds

Basking Shark
length: 30 feet
weight: 8600 pounds

1. What is the ratio: length of Whale shark to length of Basking shark?
2. Compare the weights using a ratio: Whale shark to Basking shark.
3. The weight of the Whale shark is how many times the weight of the Basking shark?
4. How might you explain why the Whale shark weighs so much more than the Basking shark when it is only twice as long?

CARD 5

GASOLINE AND OCTANE

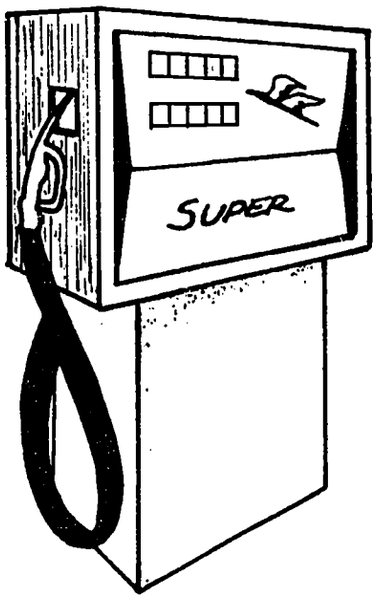
Antiknock quality of gasoline is expressed using octane number.

Performance of gasoline is compared to pure iso-octane fuel.

If gasoline performs like pure (100%) iso-octane, its octane number is 100.

If gasoline performs like a fuel having 80% iso-octane, its octane number is 80.

8



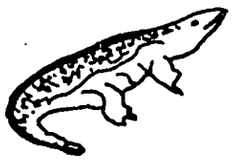
1. What ratio would express the comparison of a fuel which is 93 octane to the standard, iso-octane?

2. Write the ratio comparing a fuel rated at 110 octane to the standard, iso-octane.

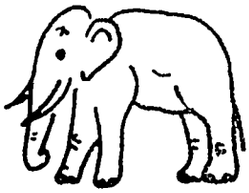
3. Write the ratio which compares 93 octane gas to 110 octane gas.

CARD 9

RECORDS OF AGE (YEARS)



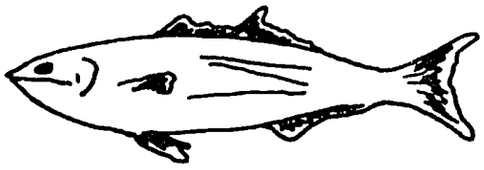
ALLIGATOR 50



ELEPHANT 100

TORTOISE 200

MAN 145



CARP 150

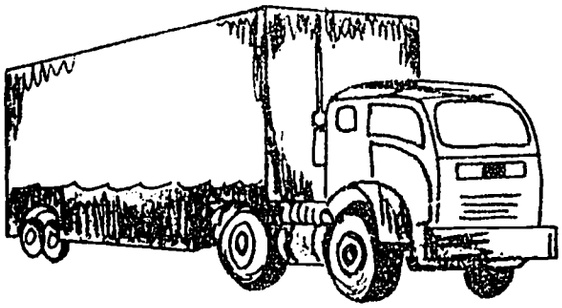
EAGLE 115

CATFISH 60

Write as ratios the comparisons of the record life spans of the following animals:

1. Tortoise to Carp
2. Alligator to Elephant
3. Man to Eagle
4. Carp to Tortoise

CARD 10



A nationwide moving company requires a down payment of $10 for each $100 the job costs.

1. Write a ratio comparing the down payment to the total cost.

2. If the total cost was $500, how much down payment is required?

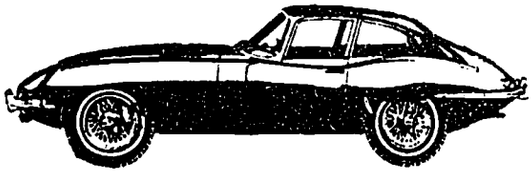
Ratios having denominators of 100 are called per cents and are written this way: 10%

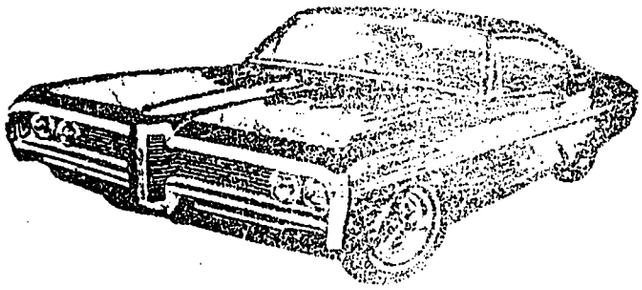
$$10\% = \frac{10}{100} \quad (\% \text{ means for each hundred})$$

3. If a down payment of 10% is required, the larger the moving costs, the_____ the down payment.

4. Another company requires a 15% down payment. Write this as a ratio.

CARD 11





Imported cars account for about [illegible] of total [illegible]
This means that out of every 100 new cars sold, [illegible]

1. Write 10% as a ratio.

Volkswagen accounts for nearly 60% of the total [illegible]
cars sold in the U.S. (This is 60 out of every [illegible]

2. Write 60% as a ratio.

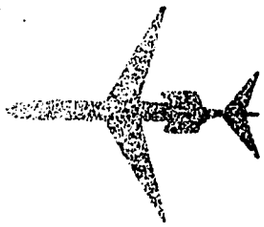
The Opel claims over 9% of total import sales.

3. Write 9% as a ratio.

CARD 12

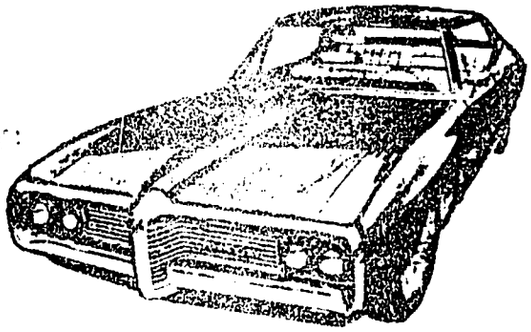
Modern jet aircraft of the world burn about [illegible] million
of fuel per day. (They burn 50,000,000 gallons for [illegible]

1. Write a ratio representing this rate.



Statements using terms like: per day, per hour, per
etc. mean for 1 day, for 1 hour, for 1 gallon, etc.

Two numbers are needed to write a ratio. Statements
above type involve the "hidden" number 1, and it is [illegible]
the denominator of the ratio.



2. Write ratios which represent the following:
   a) 50 miles per hour
   b) 18 miles per gallon
   c) 4900 r.p.m. (revolutions per minute)

ANSWERS TO EXERCISES I (Students are not expected to simplify the ratios, but it is [illegible].)

CARD NUMBER	ANSWERS TO EXERCISES
1	1. $\frac{22}{44}$ 2. $\frac{22}{66}$ 3. $\frac{33}{[illegible]}$
	4. The reaction distances increase by multiples of 11.
	5. 55 feet
2	1. $\frac{54}{215}$, $\frac{215}{484}$, $\frac{121}{[illegible]4}$
	2. It more than doubles.
	The ratios are not equivalent or they do not increase regularly. Various answers suggesting this idea should be accepted.
3	1. The overrun. 2. $\frac{1320}{2200}$ 3. $\frac{1}{2}$ mile
4	1. $\frac{4.5}{7}$ This form is not preferred, but if students are not proficient with equivalent ratios, this is what will be given. Of course $\frac{9}{14}$ is acceptable.
	2. A cone.
5	1. 740 mph 2. [illegible] mph
	3. $\frac{740}{782}$ 4. $\frac{125}{200}$

ANSWERS (CONT'D.)

6 1. $\frac{8}{4}$ 2. $\frac{4}{7}$ 3. $\frac{6}{4}$

7 1. $\frac{60}{30}$ 2. $\frac{25000}{8600}$ 3. About 3 times as many.

4. Difference in body structure.

8 1. $\frac{93}{100}$ 2. $\frac{110}{100}$ 3. $\frac{93}{110}$

9 1. $\frac{200}{150}$ 2. $\frac{50}{100}$ 3. $\frac{145}{115}$ 4. $\frac{150}{200}$

10 1. $\frac{10}{100}$ 2. $50 3. larger 4. $\frac{15}{100}$

11 1. $\frac{10}{100}$ 2. $\frac{60}{100}$ 3. $\frac{9}{100}$

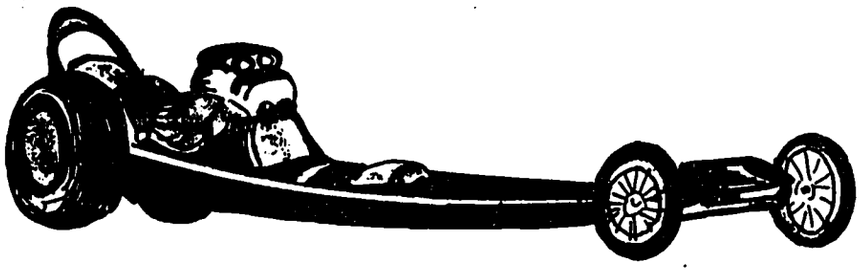
12 1. $\frac{50,000,000}{1}$ 2. a. $\frac{50}{1}$ b. $\frac{18}{1}$

c. $\frac{4900}{1}$

EXERCISES 2

Do the following exercises using ratios:

1.



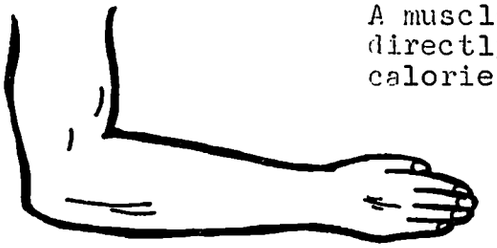
A dragster uses about 3 gallons of fuel for each run at a cost of $21.00.

a. Write a ratio that tells total cost per gallons of fuel for one run.

b. What does one gallon of fuel cost?

c. Write as a ratio the cost per gallon of fuel.

2.



A muscle in the human body is about as efficient as a good automobile engine.

A muscle converts about 30 calories directly into energy out of every 100 calories used.

a. What ratio represents this comparison?

b. What % of the calories are used directly for energy?

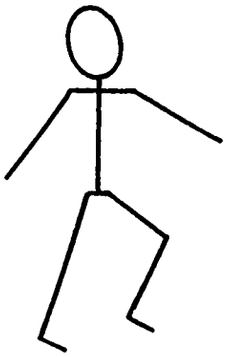
3. During one minute of vigorous activity your lungs take in about 100 quarts of air. At rest your lungs take in about 8 quarts of air. Write a ratio comparing intake at rest to intake during activity.

ANSWERS TO EXERCISES 2

1. a. $\frac{21}{3}$ b. \$7 c. $\frac{7}{1}$

2. a. $\frac{30}{100}$ b. 30%

3. $\frac{8}{100}$

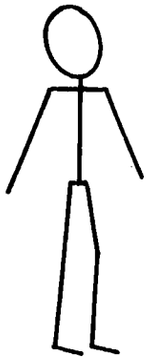
4. 

679 CALORIES

Walking uses 334 calories per hour. Jogging uses 679 cal/hr.

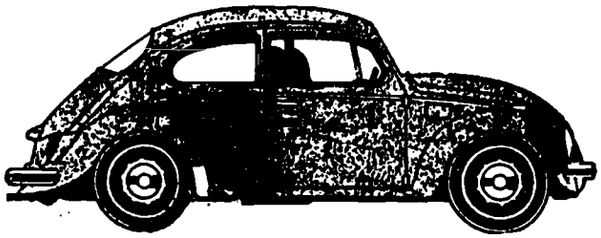
a. What is the ratio of calories used for jogging compared to calories used for walking?

b. The number of calories used for jogging is about how many times those used for walking?



334 CALORIES

5.



Today a gallon of gasoline can move a 2000 pound car at 40 mph nearly 40 miles.

35 years ago the same amount of gas would move the car about 30 miles.

Write a ratio comparing the miles traveled now to the miles traveled 35 years ago on a gallon of gas.

6. Most automobile trips are 10 miles or less.

It takes almost 7 miles to warm up an engine.

a. What fraction of a 10-mile trip is used for warm-up?

b. What is the ratio of warm-up distance to trip distance for a 10-mile trip?

The next exercises involve writing per cent as a ratio. Since % means "for each hundred" or "out of one hundred", 30% means 30 for each 100 (30 out of 100) and is written as a ratio like this: $\frac{30}{100}$. 95% is the ratio $\frac{95}{100}$.

ANSWERS (CONT'D.)

4. a. $\frac{679}{334}$ b. about 2 times

5. $\frac{40}{30}$

6. a. $\frac{7}{10}$ b. $\frac{7}{10}$

7.

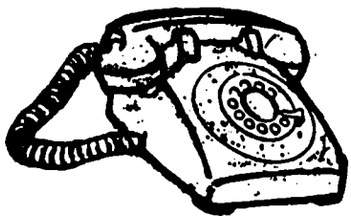
A maker of men's trousers stated that the material was 50% Dacron, 25% Orlon, 25% Rayon.

Write ratios that express these per cents:

Dacron: Orlon: Rayon:

8. A telephone company advertised that by dialing long distance direct you could save 40% on a three-minute call.

What ratio expresses this savings?



9.

A newspaper ad about tires stated that the supertread tire was 23% stronger than the average tire.

Express 23% as a ratio.

10. The newspaper ad also stated that this tire stops 25% more quickly than the average tire.

Write 25% as a ratio.

ANSWERS (CONT'D.)

7. Dacron: $\frac{50}{100}$ Orlon: $\frac{25}{100}$ Rayon: $\frac{25}{100}$

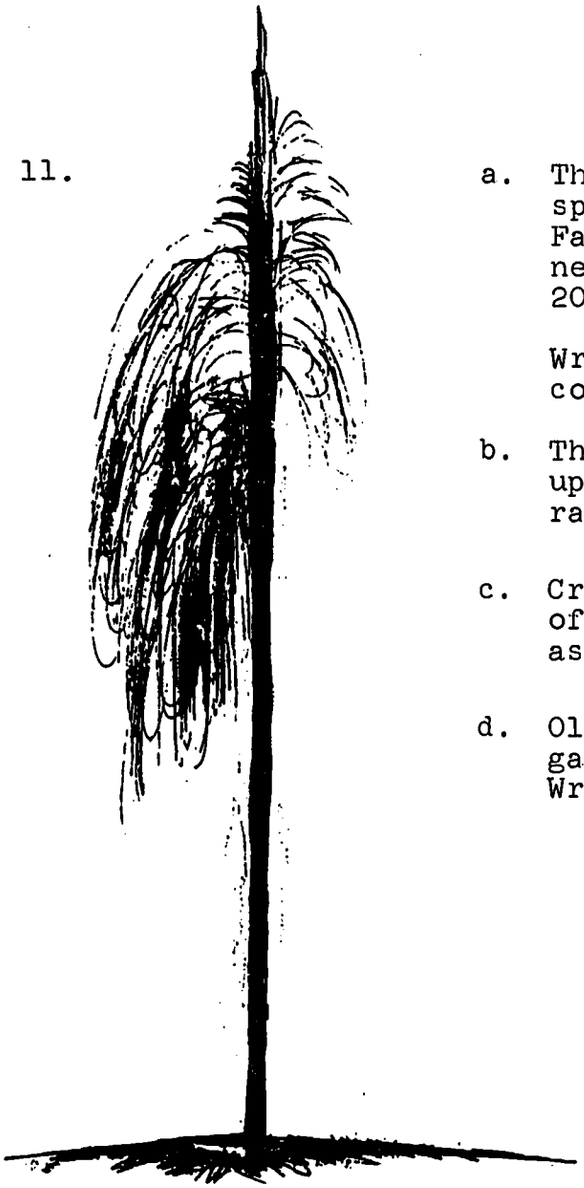
8. $\frac{40}{100}$

9. $\frac{23}{100}$

10. $\frac{25}{100}$

A car traveling 60 miles per hour is going 60 miles for each 1 hour of time. The ratio representing this is $\frac{60 \text{ miles}}{1 \text{ hour}}$. Another way to express this speed is 88 feet per second. Per second means "for each 1 second". The ratio would be written as $\frac{88 \text{ feet}}{1 \text{ second}}$.

11.



a. The Crump Geyser, in Oregon, is spraying water higher than Old Faithful's 160 foot spray. The new geyser sprays water and steam 200 feet in the air.

   Write a ratio of Crump's height compared to Old Faithful's height.

b. The Crump Geyser sprays at a speed up to 80 feet per second. What ratio represents this?

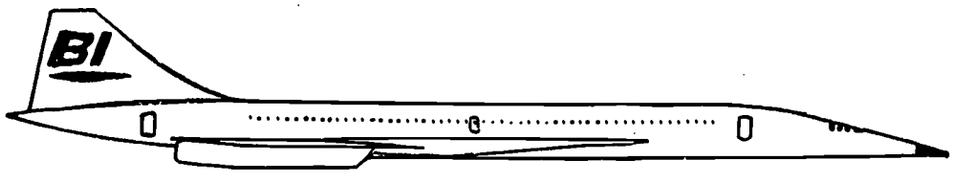
c. Crump also sprays up to 600 gallons of water per minute. Write this as a ratio.

d. Old Faithful releases up to 2500 gallons of water per minute. Write the ratio.

ANSWERS (CONT'D.)

11. a. $\frac{200}{160}$ b. $\frac{80 \text{ feet}}{1 \text{ second}}$ c. $\frac{600 \text{ gallons}}{1 \text{ minute}}$

d. $\frac{2500 \text{ gallons}}{1 \text{ minute}}$



12. Today a jet going from New York to London travels about 600 miles per hour. Write this as a ratio.

13. The new supersonic jets will travel at a speed of nearly 1800 mph. What ratio represents this?

14. Today's jet going from New York to London uses a fuel at the rate of 13,000 gallons per trip. The new supersonic jets will use nearly 30,000 gallons per trip between these two cities. Write ratios to represent these ideas.

15.  Write as a ratio: There are 5280 ft. per mile.

16. Almost 8% of the earth's crust is aluminum. Write this as a ratio.

17. Write a ratio which represents that one acre of soil contains nearly 4 million insects.

ANSWERS (CONT'D.)

12. $\frac{600 \text{ miles}}{1 \text{ hour}}$

13. $\frac{1800 \text{ miles}}{1 \text{ hour}}$

14. $\frac{13{,}000 \text{ gallons}}{1 \text{ trip}}$, $\frac{30{,}000 \text{ gallons}}{1 \text{ trip}}$

15. $\frac{5280 \text{ feet}}{1 \text{ mile}}$

16. $\frac{8}{100}$

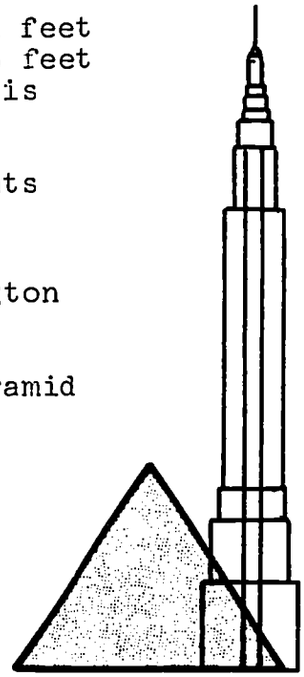
17. $\frac{4 \text{ million insects}}{1 \text{ acre}}$ or $\frac{1 \text{ acre}}{4 \text{ million insects}}$

18. The Egyptian pyramid (Cheops) is 481 feet tall; the Washington Monument is 555 feet high; and the Empire State Building is 1472 feet high.

Write ratios which compare the heights of the:

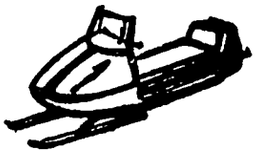
a. Pyramid of Cheops to the Washington Monument.

b. Empire State Building to the Pyramid of Cheops.



19. About how many times taller is the Empire State Building than the Egyptian Pyramid?

20. The oceans contain 97.3 per cent of the earth's water supply. Write this as a ratio.

21. 

Two-cycle engines require a mixture of oil and gasoline. Some engines require 2 quarts of oil for every 5 gallons of gasoline.

Write a ratio representing the oil-gas mixture.

ANSWERS (CONT'D.)

18. a. $\frac{481}{555}$ b. $\frac{1472}{481}$

19. About 3 times

20. $\frac{97.3}{100}$

21. $\frac{2 \text{ quarts}}{5 \text{ gallons}}$

Since 5 gallons = 20 quarts, the ratio could be $\frac{2}{20}$ or $\frac{1}{10}$

22. The table below shows how different tire aids improve stopping distances when a car is traveling 20 mph on glare ice.

Complete the table by writing each per cent as a ratio.

Tire aid for stopping	% Better stopping distance than with regular tires	Ratio
a. Studded Snow Tires (rear, new)	19%	
b. Studded Snow Tires (front, rear, new)	31%	
c. Tire chains	50%	

23.

A Sea Diver faces the problem of changing pressure on his body. To avoid danger, he must adjust the kind of air mixture he breathes and the speed he rises to the surface.

a. Air pressure at sea level is 14.7 pounds per square inch. Write this as a ratio.

b. For every 33 feet he dives, the pressure is increased 14.7 pounds per square inch (psi). What ratio represents this?

ANSWERS (CONT'D.)

22.

Tire aid for stopping	% Better stopping distance than with regular tires	Ratio
a. Studded Snow Tires (rear, new)	19%	$\frac{19}{100}$
b. Studded Snow Tires (front, rear, new)	31%	$\frac{31}{100}$
c. Tire chains	50%	$\frac{50}{100}$

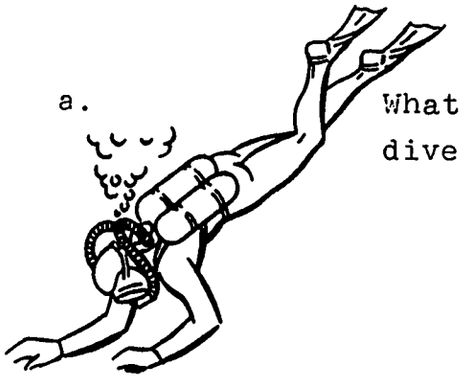
23. a. $\frac{14.7 \text{ pounds}}{1 \text{ square inch}}$ b. $\frac{33 \text{ feet}}{14.7 \text{ psi}}$

c. At a depth of 33 feet, the pressure would be 29.4 psi. (14.7 psi at 0 feet + 14.7 psi for the depth of 33 ft. = 29.4 psi)

What would be the pressure at 66 ft. below sea level?

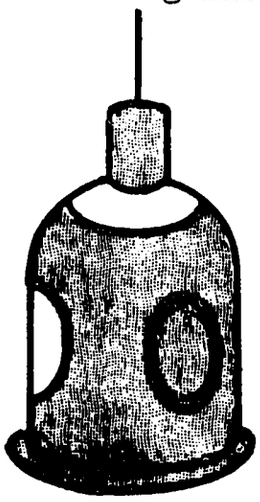
24.

Depth (ft.)	Event
0	Sea level
150	Normal skin diving limit
275	Deepest hard-hat salvage of a ship
300	Deepest skin dive breathing air
475	Deepest diving chamber salvage of a ship
4000+	Nuclear submarine depth
35,802	Deepest spot in the ocean



a. What is the ratio of the deepest skin dive to the normal skin diving limit?

b. What is the ratio of the deepest chamber salvage to the deepest hard-hat salvage?



ANSWERS (CON T'D.)

23. c. 44.1 psi

24. a. $\frac{300}{150}$

b. $\frac{475}{275}$

A PICTURE OF EQUIVALENCE

EQUIVALENT RATIOS

There are many ratios which can represent the same picture. These ratios are said to be equivalent. Two questions that might be asked are: (1) How do you know when two ratios are equivalent? (2) How do you know if two ratios represent the same picture?

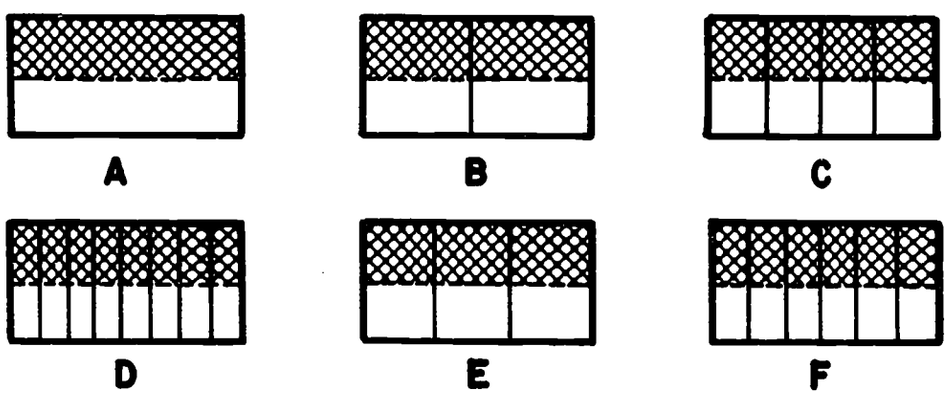
CLASS ACTIVITY

Your teacher will pass out some sheets of paper and discuss the activity. The purpose of the activity is to fold paper to illustrate equivalent ratios.

EXERCISES

The rectangles drawn below are like the rectangles you just worked with while paper-folding.

The rectangles are the same size overall, and the shaded regions are of equal size.



A PICTURE OF EQUIVALENCE

OBJECTIVES

Given a region, partially shaded, the student shall demonstrate the equivalence of a set of ratios.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

*1. Sheets of paper for folding to illustrate equivalent ratios, P1 and P2.

P1 illustrates the equivalence of $\frac{1}{4}$, $\frac{2}{8}$, $\frac{4}{16}$ and $\frac{8}{32}$.

P2 illustrates the equivalence of $\frac{3}{5}$, $\frac{6}{10}$ and $\frac{12}{20}$.

B. TEACHER

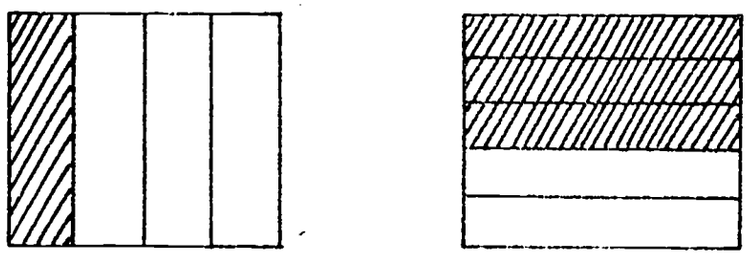
1. Overhead projector
2. Projection screen

*3. Transparency: ARP 3
ARP 3 contains several overlays which show how line segments can be drawn to illustrate equivalent ratios. ARP 3 is an example of the kinds of exercises the student will be working.

CONTENT AND APPROACH

The first part of the lesson [illegible] demonstrate the equivalence of [illegible] ratios.

A class activity of paper-folding [illegible] illustrate examples of two sets of equivalent ratios. Following is a diagram of the two sheets of paper used in this activity.



P1 P2

P1 Illustrates the equivalence of $\frac{1}{4}$, $\frac{2}{8}$, $\frac{4}{16}$, $\frac{8}{32}$.

After passing out P1 to each student, questions like the following may be asked. Into how many regions is the paper divided? How many regions are shaded? ____ out of ____ regions are shaded. What ratio is indicated?

Fold the paper in half so that the crease passes through the shaded region. What has happened to the total size of the paper? (Nothing) What has happened to the size of the large shaded region? (Nothing) Thinking of the crease as a line segment, how many regions, of smaller size, are there altogether? (4) How many of the smaller shaded regions are there? (2) Considering the smaller regions, what is the ratio of shaded regions to the total number of regions? ($\frac{2}{8}$)

P1 should be folded once again with a similar set of questions asked. Repeat the folding once again.

Sheet P2 illustrates $\frac{2}{3} = \frac{4}{6} = \frac{12}{18}$. An exercise similar to what was carried out with 11 may be done or the students may be asked to fold and then tell which ratios are equivalent.

Transparency ARP 3 is used as an intermediate step between folding paper and the Exercises.

It is like diagrams A-D at the bottom of student page 28 in that as each overlay is used, the sequence progresses like diagrams A to B to C to D. Questions similar to those asked during paper-folding should be used. Transparency ARP 3 is an example for exercises 1-5 on student page 29.

1. Figure A is divided into two rectangular regions. One of them is shaded.

   a. ____ out of ____ regions is shaded.

   b. Write the ratio.

2. Consider figure B.

   a. Into how many regions is figure B separated?

   b. How many of the smaller regions are shaded?

   c. Write the ratio that represents the situation.

3. Consider figure C.

   a. The number of small regions is ____ .

   b. ____ of the small square regions are shaded.

   c. What ratio represents the situation in figure C?

4. What ratio represents the situation in:

   a. figure D?

   b. figure E?

   c. figure F?

Each of the six pictures has the same area shaded, but different ratios have been used to describe the comparisons.

Different ratios describing the same situation (each rectangle had the same total area and the same amount of shading) are called EQUIVALENT RATIOS.

5. Write the six equivalent ratios described by pictures A, B, C, D, E, F.

ANSWERS TO EXERCISES

1. a. 1 out of 2 b. $\frac{1}{2}$

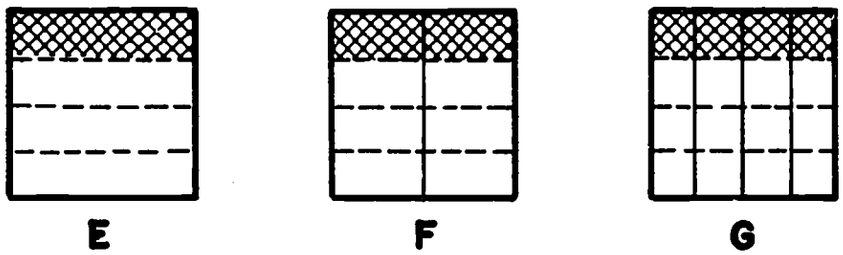
2. a. 4 b. 2 c. $\frac{2}{4}$

3. a. 8 b. 4 c. $\frac{4}{8}$

4. a. $\frac{8}{16}$ b. $\frac{3}{6}$ c. $\frac{6}{12}$

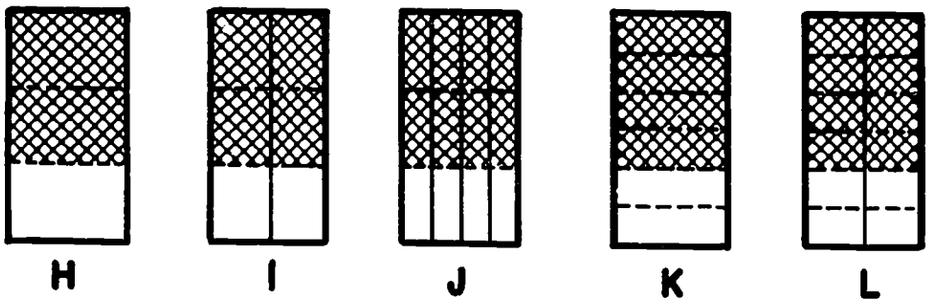
5. $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, $\frac{8}{16}$, $\frac{3}{6}$, $\frac{6}{12}$

6. a. Write the ratio representing the amount of shaded area to total area for each picture below:



b. Since each of the ratios you wrote in part a represent the same size regions, they are called _____ ratios.

7.



a. Picture H is represented by the ratio _____.

b. How do the sizes of the large rectangle compare?

c. How do the sizes of the shaded portions compare?

d. What term applies to the set of ratios representing pictures H - L?

e. Write the ratios representing I: J: K: L:

ANSWERS (CONT'D.)

6. a. E: $\frac{1}{4}$ F: $\frac{2}{8}$ G: $\frac{4}{16}$

b. equivalent

7. a. $\frac{2}{3}$ b. same area c. same area

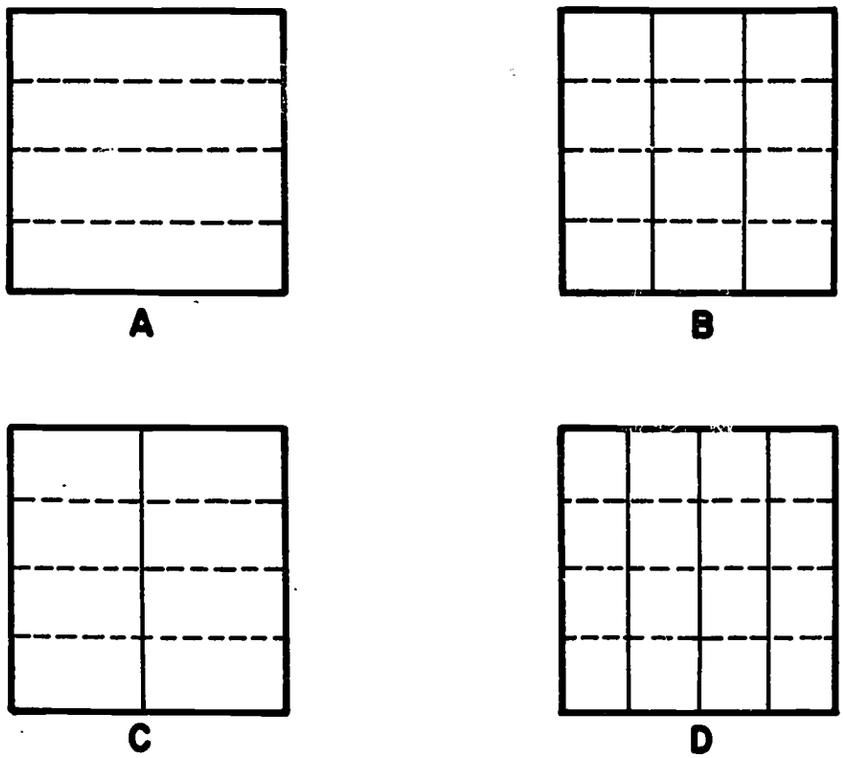
d. equivalent ratios e. I: $\frac{4}{6}$ J: $\frac{8}{12}$ K: $\frac{4}{6}$ L: $\frac{8}{12}$

✓ **POINT**

Suppose a picture having a shaded region is subdivided several times. As you further subdivide, what happens to the total area of the figure and to the area of the shaded region?

EXERCISES , CONT'D.

8.



a. In picture A, represent the ratio $\frac{1}{4}$ by shading.

b. Shade the remaining diagrams according to the table to show that the ratios are equivalent.

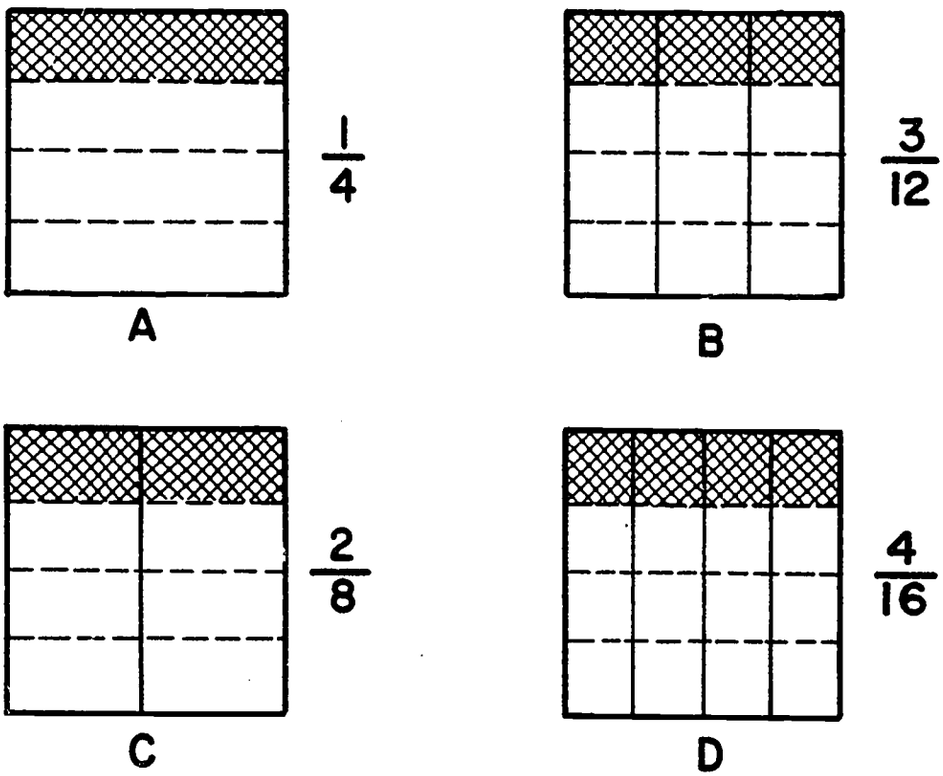
Diagram	A	B	C	D
Ratio	$\frac{1}{4}$	$\frac{3}{12}$	$\frac{2}{8}$	$\frac{4}{16}$

ANSWER TO ✓ POINT The total area of the figure as well as the area of the shaded region remains unchanged.

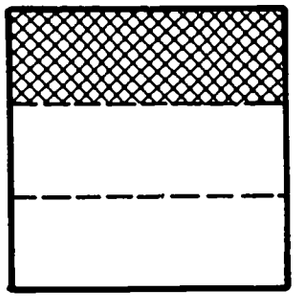
Exercise 8 differs from the previous exercises in that the figures are given along with the subdivision line segments The student, then, must provide the shading.

ANSWERS (CONT'D.)

8.



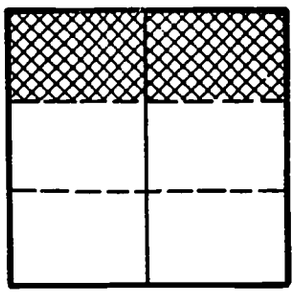
Example:



M

Figure M has been separated into 3 regions and one of them is shaded.

The ratio represented here is $\frac{1}{3}$.



N

Figure N has an additional line segment.

The total number of regions has been doubled, 6 in all.

Also the number of shaded regions has been doubled, 2.

The ratio represented here is $\frac{2}{6}$. $\frac{2}{6}$ is equivalent to $\frac{1}{3}$ because the overall figures and the shaded portions are equal in area.

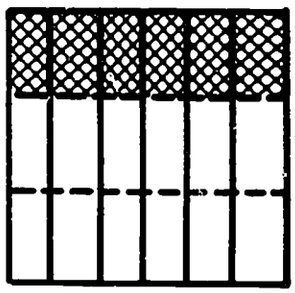
Exercise 9 requires the student to provide the subdivision line segments to illustrate equivalent ratios. The example given on pages 32-33 provides background for the exercise.

With the completion of exercise 9, the student has demonstrated equivalent ratios given:

1. The entire set of figures, shaded and subdivided, requiring the regions to be counted.

2. The set of figures with the subdivision line segments requiring the shading to be done.

3. The set of figures, shaded, requiring the drawing of subdivision line segments.



R

Figure R has 6 times the total number of regions as does figure M, 18.

Figure R also has 6 times as many shaded regions as figure M, 6.

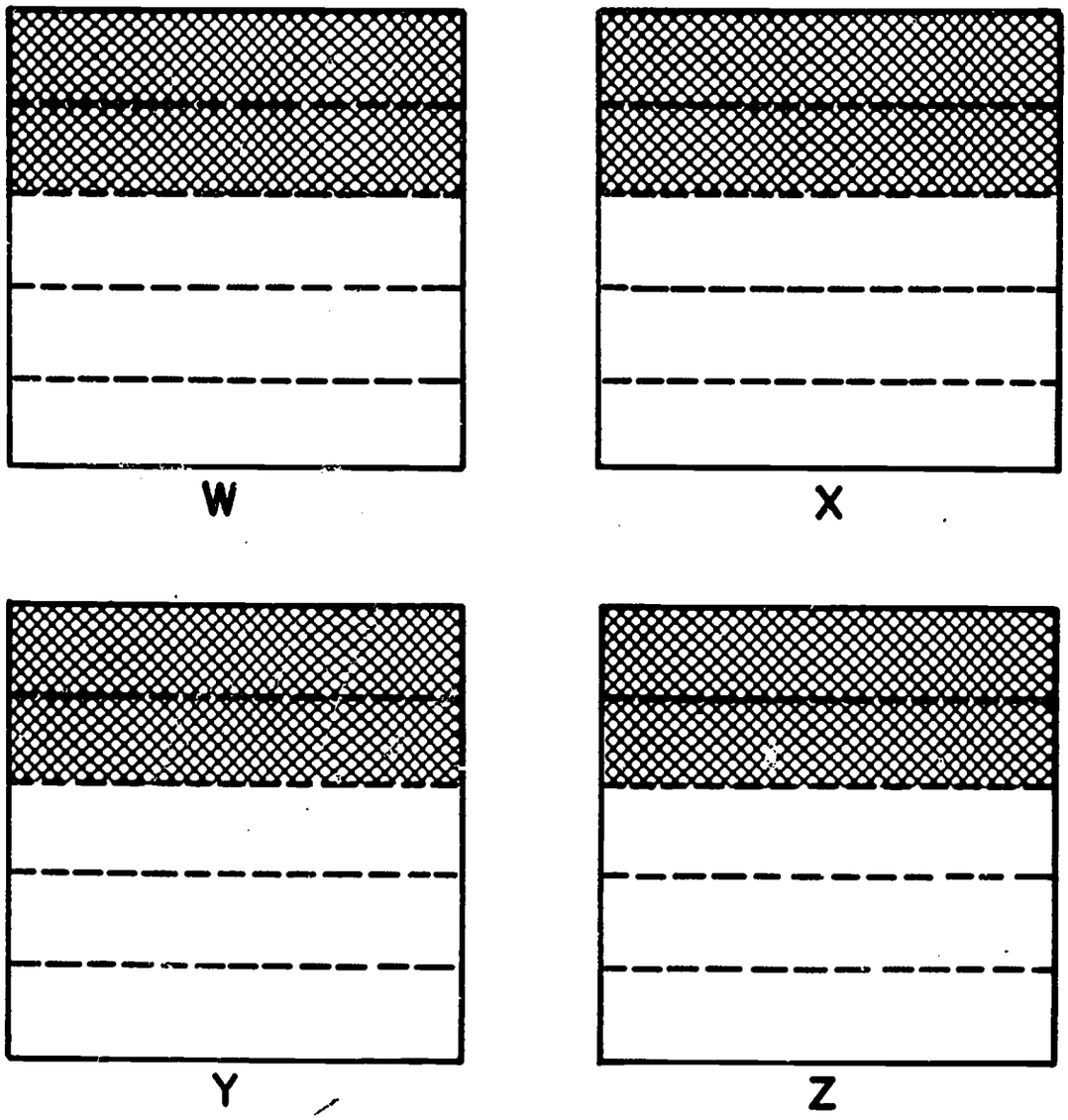
The ratio represented by R is $\frac{6}{18}$.

The example shows that $\frac{1}{3}$ is equivalent to $\frac{2}{6}$ and to $\frac{6}{18}$. That is, $\frac{1}{3} = \frac{2}{6}$ and $\frac{1}{3} = \frac{6}{18}$.

N and R have the same areas (total and shaded) so $\frac{2}{6} = \frac{6}{18}$.

The total number of regions in R is 3 times the number in N. Also the number of shaded regions in R is 3 times the number in N.

9.



In the pictures above, 2 out of 5 regions have been shaded, representing the ratio $\frac{2}{5}$.

Pictures X, Y, Z are to be changed to represent ratios equivalent to $\frac{2}{5}$ (picture W.)

a. Draw a segment in picture X so that the regions will be equal in area and represent $\frac{4}{10}$, a ratio equivalent to $\frac{2}{5}$.

b. What is the total number of regions in X now?

c. How many shaded regions are there in X?

d. Write the ratio represented by X.

e. Draw four vertical line segments equally spaced in picture Y. The total number of regions should be 25. How many shaded regions are there?

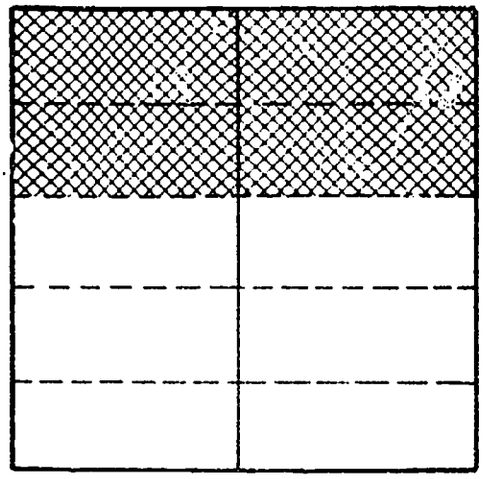
f. What ratio does Y represent now?

g. Draw vertical lines in picture Z to represent the ratio $\frac{6}{15}$.

h. Write the four equivalent ratios discussed in the exercise.

ANSWERS (CONT'D.)

9. a.



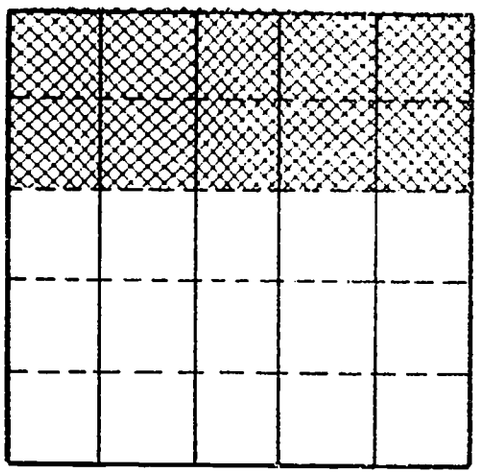
X

b. 10

c. [illegible]

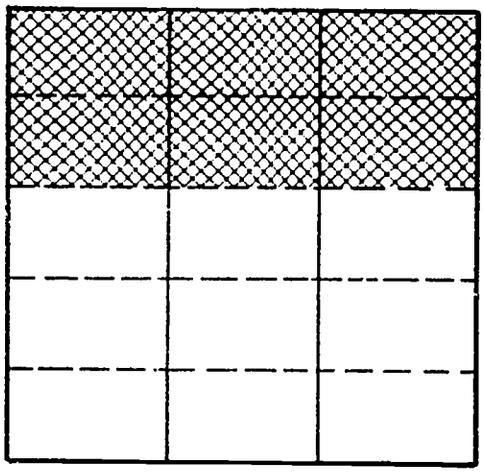
d. $\frac{4}{10}$

e. 10 shaded regions



Y

f.



Z

f. $\frac{10}{25}$

h. $\frac{2}{5}$, $\frac{4}{10}$, $\frac{10}{25}$, $\frac{6}{15}$

✓ **POINT**

1. Write the ratios indicated by the following:

   a. Four out of five girls wear rings.

   b. A sign read: "Shoe Sale, 15 per cent off".

   c. For every 6 boys there are 8 girls.

   d. & & & ⟶ # # # # # # #

   e. My test score was 85 per cent.

   f. An iceberg was estimated to be 200 ft. high above the water and 400 ft. below the surface. What ratio compares the distance under water to the distance above water?

   g. ●○●○●●

2. How do you know if two ratios are equivalent or not?

3. Show that $\frac{2}{3}$ and $\frac{8}{12}$ are equivalent.

4. Complete the following to represent the ratio $\frac{4}{9}$.

   a. △△△△→

   b. ____ out of ____ are green.

   c. Shade: △△△△△△△△△

   d. For every____ there are ____ .

ANSWERS TO ✓POINT

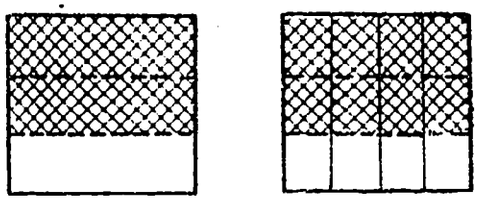
1. a. $\frac{4}{5}$ b. $\frac{15}{100}$ c. $\frac{1}{7}$

   d. $\frac{3}{7}$ e. $\frac{75}{100}$ f. $\frac{[illegible]}{[illegible]}$

   g. $\frac{4}{6}$

2. Possible response: They can be represented by two figures, partially shaded. The figures are equal in area as are the shaded portions. (Accept any correct response, even if it has not been discussed in the booklet.)

3. Possible response:



4. a. ○○○○○○○○○

   b. 4 out of 9

   c. △△△△△▲▲▲▲

   d. 4, 9

CANDLES AND RATIOS

PART I

Ratio and proportion is used a great deal when predictions need to be made. Many experiments involve proportional relationships. In later lessons you will be doing activities that use proportions.

Your teacher will demonstrate the activity, CANDLES AND JARS, as part of this lesson. The important things for you to think about are:

1. An equipment inventory should be made.
2. The entire activity should be read over so that the purpose and procedure are understood.
3. Do the activity carefully.
4. Follow the directions and make predictions.
5. Think about the results:
   a. Are they reasonable?
   b. What accounts for differences in the experimental results and the predicted results?
   c. Are these differences acceptable?
   d. Would there be any advantage in repeating the experiment several times?

PART 2

EXERCISES

This is an exercise in listening and writing. Your teacher will read aloud some information and then ask you to write ratios about it.

CANDLES AND RATIOS

OBJECTIVES

1. To introduce students to experimental activities in which students apply the idea of ratio in prediction. (PART 1)

2. After listening to information read aloud, the student shall write the indicated ratios. (PART 2)

PART I - EQUIPMENT AND TEACHING AIDS

TEACHER

1. Overhead projector
2. Projection screen

*3. Transparency: ARP 4

ARP 4 is a table for recording the data obtained when performing the activity, and a table of values obtained from in-service groups which the author directed.

4. 1 small candle with thumbtack and cardboard candle holder.

5.Δ 1 Each of the following glass jars: $\frac{1}{2}$ pint, 1 pint, 1 quart, $\frac{1}{2}$ gallon, 1 gallon.

6.± 1 tin can with a viewing window in the side, a large peanut butter jar, or some large odd-shaped glass jar.

7. 1 Book of matches.

8. 1 timer - a watch with a second hand.

Δ± see top of page T 37a

[Δ]All of the jars except the gallon jar can be standard "Mason" canning jars.

The gallon jar might be obtained from the school cafeteria, since they are restaurant-size food containers.

[‡]The tin can should be a 1 or 2-pound coffee can with a rectangular viewing hole cut in the side with the cheapest of tin snips. The jagged edges should be covered with masking tape and an acetate window taped on with masking tape. (The acetate can be a piece of overhead projection acetate.)

The window should not be cut in the bottom of the can because when the can is inverted over the candle, the acetate will melt.

EXPERIMENTS IN THE CLASSROOM - GENERAL

Your students are not likely to be experienced in lab-activity oriented procedures, especially in a mathematics classroom. Based on this assumption this booklet presents a developmental approach to experimental activities. You will note that the development has the following sequence:

1. Teacher demonstration of one activity.

2. Total class performing a single activity, individually.

3. A small group of students demonstrating one activity before the whole class.

4. Total class performing a single activity, in small groups.

5. Total class performing two successive experiments at stations and in small groups.

The Preparation --

Prior to the day of an activity, several considerations must be made. First, what equipment (and how much) will be required for the class? This determination must be made sufficiently in advance so that equipment can be acquired. The equipment might be obtained within the department or elsewhere in the building, constructed by the teacher, or by purchase if necessary. At any rate planning must be done to avoid a last minute rush.

Students need to be prepared for activity sessions, also. They may not be experienced in activity-oriented procedures, especially in a mathematics classroom. Hence, some groundwork must be done to obtain maximum results. The preparation should stress:

1. An indication of the nature of the experiment.

2. That students will be involved in group activity; that they will be moving about the room; and what expectations for accomplishment the teacher has for them.

3. The importance of following directions.

4. They will report their results to the whole class (they should be held accountable for their results.)

If predicting outcomes of additional trials of the activity is a goal, this should be explained and stressed. Provision should be made for students to verify their predictions. An important point to express is that predictions often result in specific answers (when using proportions to predict, for example.) The prediction should be interpreted in the experimental sense. "It should be approximately equal to..." rather than "it will be equal to..." Then students will be able to interpret their results in perspective.

Example: Suppose a weight of 210 grams stretches a spring 8 inches. An unknown weight, W, stretches the string 14 inches. If W is predicted by the proportion $\frac{210}{8} = \frac{W}{14}$ and is calculated to be 357.5 grams, then the interpretation should be that W weighs about 360 grams. If upon weighing W it is found to be 340 grams, the student should be helped to accept the validity of the activity. (Error is 5%.)

The On-Going Activity --

While activities are in progress, the job of the teacher is that of a resource person and a director of learning. Students should be allowed to gather information and make discoveries on their own. However, the teacher should be available to help those having difficulty, either conceptually or with the equipment involved.

The group activity session provides the teacher with a prime opportunity to talk with and observe both individuals and small groups. The teacher should focus on areas where students are having conceptual problems, computational difficulties, as well as to help improve interpersonal relationships and attitudes. The personal attention received by individuals and small groups could do much for the class as a whole.

The Summarization --

An often overlooked but vital part of classroom group activity is the discussion and summarization that should take place immediately following the activity session.

The results obtained by the small groups should be presented to the entire class and discussed. Depending upon the ability of your students, the members of a group might present their results and lead the discussion.

The data should be tabulated in such a way that the entire class can see the findings of all the groups. Transparencies in table form provide an excellent method of presentation.

Following are some questions that might be asked.

1. How do the results of the various groups compare?
2. What are the average results?
3. What predictions did you make?
4. How did you make the predictions?
5. How do your predictions compare with those of the class and with the average experimental results?
6. What might account for differences between the predictions and the results? Between your results and that of another group?
7. How do "errors" creep into an experimental activity?

Summarizing provides students with the opportunity to compare and discuss their results with others. If there is no discussion of the activities, then why do them?

CONTENT AND APPROACH

In this lesson the experiment is performed as a demonstration by the teacher in front of the whole class. In this situation, error is kept at a minimum and identical data is observed by the students. Thus the conclusions based upon that data are similar and controlled.

The object of the experiment is to determine the time it takes to extinguish a candle after a glass jar has been placed over it. (Theoretically, the times are proportional to the size of the jar and hence a graph of the data would be a straight line.)

Procedure:

1. Place the gallon jar over the lighted candle and find the time (in seconds) that the candle stays lighted. Record the time on the data sheet, transparency ARP 4.

2. Use the $\frac{1}{2}$ gallon jar and repeat step 1.

3. Fill in the number of half-pints each jar is equivalent to.

4. Based on the information obtained thus far, use proportions to predict the time to extinguish the flame for the remaining jars. Record the information in the table.

5. Perform the experiment to verify the predictions. Record the times in the table, ARP 4.

THINGS TO DISCUSS

A discussion of experimental error and discrepancies between observed and calculated results may result in these kinds of responses.

1. Careless handling of equipment can cause inconsistent results.

   a. If a person holds the jar inverted while waiting for the timer's "go" signal, the air inside the jar may be affected.

   b. If there is not a seal between the lip of the jar and the surface it rests upon, the burning time may be affected.

2. "Error" may result because an attempt is made to force a physical situation to fit a precise mathematical model. A mathematical model (in this case, proportionality) is used as an attempt to describe the behavior of a variety of experimental observations. Persons may try to make the actual time match the predicted time.

3. The volume of a "quart" jar exceeds 1 quart when the volume is considered all the way to the lip of the jar. The "excess" volume varies as the size and shape of the jar varies.

4. The candle displaces some air in the jar, affecting the burning time slightly.

5. After a jar has been used once it contains the remnants of burning, and filling the jar with "good" air may not be as easy as it appears.

Other factors could probably be suggested, but these are sufficient to explain discrepancies in data.

Since it is a possibility that your results might look inconsistent, the following table of times is included. (It is also provided in transparency A [illegible] 4.)

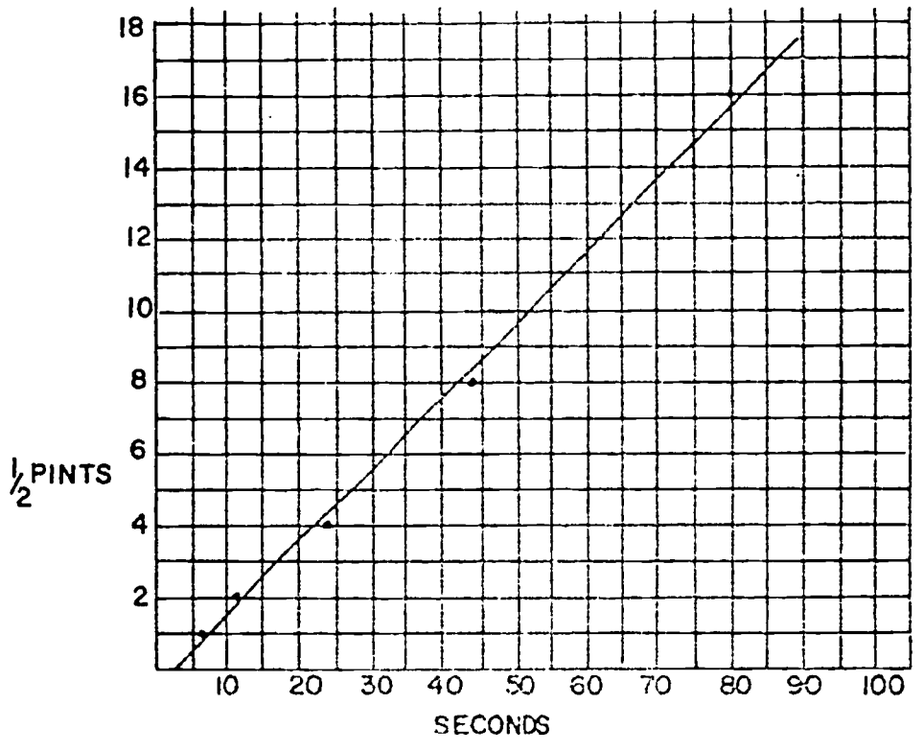
REPORT FORM												
JAR	TIME (SECONDS)											
GALLON	94	128	64	55	74	[illegible]	70	95	52	100	90	70
$\frac{1}{2}$ GALLON	[illegible]	[illegible]	35	35	34	[illegible]	50	50	33	46	48	42
QUART	[illegible]	[illegible]	[illegible]	20	24	[illegible]	18	29	16	28	25	21
PINT	[illegible]	[illegible]	[illegible]	10	11	13	15	11	8	14	13	13
$\frac{1}{2}$ PINT	7	[illegible]	[illegible]	5	5	7	8	9	5	9	9	8

The times in [illegible] table were obtained from teacher in-service and [illegible] sessions.

Although the [illegible] appear to be consistent, the averages produce surprising [illegible] as shown below.

JAR	[illegible]	[illegible] gallon	1 qt.	1 pt.	$\frac{1}{2}$ pt.
TIME (sec.)		[illegible]	[illegible]	12	7

Add your results to the table using transparency ARF 4, find the average times and plot the times vs. volume on the graph provided below. (This is for your own benefit.) The graph as printed is that of the average times which appear at the bottom of page T 37g.



If you save your results along with the table of times which was provided, you can build a good data file for future use.

Now predict (and verify) how many half-pints the tin can will hold by observing the burning time of the candle through the acetate window.

PART 2 - EXERCISES AND ANSWERS

An objective of this set of exercises is to have the student listen to and <u>hear</u> information which suggests ratios.

Listening is an important problem in communication. This set of exercises is intended to give this area some attention.

CONTENT AND APPROACH

Select exercises that are appropriate to your students and then read the situations to them carefully and clearly. The student is to have every possible opportunity to hear the information. Re-reading is encouraged.

1. In the U.S. the average life-span has increased to about [illegible] years. It was [illegible] years about a century ago. Write a ratio comparing life expectancy now to life expectancy a century ago. $\left[\frac{7}{4}\right]$

2. The Ground Shark stays close to shore and grows to about 10 feet in length. Another shark lives in deep water and measures just 6 inches when full grown. Write a ratio comparing the lengths of Ground Shark to the other shark. $\left[\frac{10}{\frac{1}{2}}\text{, units are different; } \frac{1}{[illegible]}\right]$

3. At rest, human muscles use about [illegible] milliliters of oxygen per minute. Write this as a ratio. $\left[\frac{[illegible]}{1}\right]$

4. During exercise, the muscles use about [illegible] milliliters of oxygen for each minute. Write ratio representing this. $\left[\frac{3[illegible]}{1}\right]$

5. Write a ratio comparing the amount of oxygen used each minute during rest, to the amount used each minute during exercise. $\left[\frac{[illegible]}{3[illegible]}\right]$

6. Regular exercise can increase your breathing capacity by 20%. Write 20% as a ratio. $\left[\frac{[illegible]}{1[illegible]}\right]$

7. A road test of a new car showed that at [illegible] mph, the car had a gasoline mileage of [illegible] miles per gallon. Write the mileage as a ratio. $\left[\frac{2[illegible]}{1}\right]$

7. An owner's questionnaire about the new car mentioned before showed that 90% of the owners liked the car. Write this per cent as a ratio. $\left[\frac{90}{100}\right]$

9. Electrical circuits in most homes can handle 15 amperes of electricity. If too many appliances are run on a circuit, the fuse will blow. For instance, an iron uses 10 amps and a toaster uses 7 amps. If they were on the same circuit at the same time, a fuse would blow.

   Now write a ratio which compares the amps used by an iron to the amps used by a vacuum cleaner. An iron uses 10 amps and a vacuum cleaner uses 3 amps. $\left[\frac{10}{3}\right]$

10. Because of differences in gravity, a person's weight will vary on different planets. If you weigh 110 pounds on Earth, you would weigh only 18 pounds on the Moon. Write a ratio comparing your weight on Earth to your weight on the Moon. $\left[\frac{110}{18}\right]$

11. A man weighing 200 pounds on Earth would weigh only 32 pounds on the moon. Write a ratio comparing these weights, weight on Earth to weight on the moon. $\left[\frac{200}{32}\right]$

12. On Jupiter the same 200 pound man would weigh 528 pounds. Compare the weights, Earth to Jupiter, using a ratio. $\left[\frac{200}{528}\right]$

13. Human blood is divided into four groups. O, A, B, AB. 45 out of 100 people will have O type. Write this as a ratio. $\left[\frac{45}{100}\right]$

14. Only 5 out of 100 people will have AB type. Write this as a ratio. Now write this as a per cent. $\left[\frac{5}{100}, 5\%\right]$

15. Write as a ratio the number of people having O type compared to AB type. $\left[\frac{45}{5}\right]$

16. How many times as many people have O type as have AB type? Write this as a ratio. $\left[9 \text{ times as many}, \frac{9}{1}\right]$

MORE ABOUT EQUIVALENCE

SIMPLIFYING RATIOS

Simplified ratios are used mainly because they help people communicate. Gear, pulley, recipe, oil-gas, and other ratios are written in simplest form. Otherwise any number of equivalent ratios could be used and a person might not recognize them as equivalent.

DIVISION METHOD

This method depends upon the idea that division by 1 does not change the value of a number; and that a number divided by itself (except 0) is 1.

For instance: $15 \div 1 = 15$

$\frac{5}{5} = 1$

Example 1:

Simplify $\frac{12}{18}$

$\frac{12}{18} \div 1 = \frac{12}{18} \div \frac{6}{6}$

(Since 12 and 18 are both divisible by 6 and $\frac{6}{6} = 1$)

$= \frac{12 \div 6}{18 \div 6}$

$= \frac{2}{3}$

Therefore $\frac{12}{18} = \frac{2}{3}$

FACTORING METHOD

The factoring method depends upon your ability to factor numbers into primes and your understanding of multiplying fractional numbers.

For instance, if $\frac{4}{5} \cdot \frac{3}{3} = \frac{4 \cdot 3}{5 \cdot 3}$,

then we must agree that

$\frac{4 \cdot 3}{5 \cdot 3} = \frac{4}{5} \cdot \frac{3}{3}$.

Example 1:

Simplify $\frac{12}{18}$.

by factoring: $\frac{12}{18} = \frac{2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 3}$

$\frac{2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 3} = \frac{2 \cdot 3 \cdot 2}{2 \cdot 3 \cdot 3}$

$= \frac{2 \cdot 3}{2 \cdot 3} \cdot \frac{2}{3}$

$= 1 \cdot \frac{2}{3}$

$= \frac{2}{3}$

Therefore $\frac{12}{18} = \frac{2}{3}$.

MORE ABOUT EQUIVALENCE

OBJECTIVES

1. Given a ratio, the student shall be able to simplify it by using either the factoring method or the division method.

2. Given a pair of ratios, the student shall determine whether they are equivalent using the multiplication method, factoring method, or division method.

EQUIPMENT AND TEACHING AIDS

TEACHER

1. Overhead projector
2. Projection screen

*3. Transparency: ARP 3 (used in Lesson 3)

CONTENT AND APPROACH

The lesson suggests two approaches to simplifying ratios and three approaches to testing for equivalence of ratios.

It is not suggested that the teacher discuss each approach and demand proficiency in each method. Rather, the way in which simplifying of and equivalence of ratios are presented should be the way which most easily fits the students' background.

Simplifying Ratios - Division Method

The division method can be tied in with Lesson 3 which used paper-folding to illustrate equivalent ratios. As the paper was folded, the total number of regions was doubled, as were the number of shaded regions. Each time a fold was made, the number of regions (total and shaded) was doubled. The ratios were these: $\frac{1}{4}$, $\frac{2}{8}$, $\frac{4}{16}$, $\frac{8}{32}$.

It will be noted that simplifying ratios using the paper-folding model is the reverse of the above. Ironing out the creases of the folded paper would amount to dividing the numerator and denominator by two in successive steps: $\frac{8}{32}$, $\frac{4}{16}$, $\frac{2}{8}$, $\frac{1}{4}$.

Transparency ARP 3 could be used in this way. Place the entire transparency on the overhead stage and remove the overlays one at a time. Each time an overlay is removed, it would amount to erasing one line segment in the diagram (or ironing out a crease.) Erasing a segment or ironing a crease corresponds to dividing the numerator and denominator.

Example 1: Simplify $\frac{12}{18}$

$\frac{12}{18} = x$

$\frac{12}{18} \div 1 = x \div 1$ and $x \div 1 = x$,

$\frac{12}{18} \div 1 = x$. Now 12 and 18 are both divisible by 6, and $\frac{6}{6} = 1$, then

$\frac{12}{18} \div 1 = \frac{12}{18} \div \frac{6}{6}$

It should be pointed out that 6 was obtained by insight and results from experience. The student should not be led to think that the 6 was automatic and that it should be immediately obvious to them.

$\frac{12}{18} \div \frac{6}{6} = \frac{12 \div 6}{18 \div 6}$ and $\frac{12 \div 6}{18 \div 6} = \frac{2}{3}$; then $\frac{2}{3} = x$.

DIV. METHOD (cont'd.)

Example 2:

Simplify $\frac{36}{120}$

$$\frac{36}{120} \div 1 = \frac{36}{120} \div \frac{12}{12}$$

$$= \frac{36 \div 12}{120 \div 12}$$

$$= \frac{3}{10}$$

Therefore $\frac{36}{120} = \frac{3}{10}$

FACT. METHOD (cont'd.)

Example 2:

Simplify $\frac{36}{120}$

factoring: $\frac{36}{120} = \frac{2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5}$

$$\frac{2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5} = \frac{2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 2 \cdot 5}$$

$$= \frac{2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3} \cdot \frac{3}{2 \cdot 5}$$

$$= 1 \cdot \frac{3}{10}$$

$$= \frac{3}{10}$$

Therefore $\frac{36}{120} = \frac{3}{10}$.

EXERCISES I

Simplify each of the following ratios.

1. $\frac{6}{8}$ 2. $\frac{6}{9}$ 3. $\frac{8}{10}$ 4. $\frac{15}{21}$ 5. $\frac{8}{12}$

6. $\frac{24}{30}$ 7. $\frac{8}{9}$ 8. $\frac{14}{28}$ 9. $\frac{18}{12}$ 10. $\frac{20}{24}$

11. $\frac{12}{16}$ 12. $\frac{16}{24}$ 13. $\frac{10}{15}$ 14. $\frac{8}{15}$ 15. $\frac{16}{10}$

16. $\frac{9}{21}$ 17. $\frac{21}{28}$ 18. $\frac{18}{34}$ 19. $\frac{35}{20}$ 20. $\frac{24}{16}$

Simplifying Ratios - Factoring Method

The factoring method is dependent upon prime factorization. This method is more closely related to algebraic simplification than is the division method. For this reason some teachers prefer it.

A point of difficulty can be mentioned. One step in the factoring method is $\frac{2\cdot3\cdot2}{2\cdot3\cdot3} = \frac{2\cdot3}{2\cdot3} \cdot \frac{2}{3}$ and many students have difficulty in connecting this step with multiplying fractions. Time should be spent to help the student overcome this problem.

Simplifying in General: In most instances gear ratios are expressed as simplified ratios, mainly for ease of communication. So the ratios $\frac{20}{4}$ and $\frac{10}{2}$ are not the forms used to express gear ratios. Rather the ratio $\frac{5}{1}$ would be used, since $\frac{20}{4}$, $\frac{10}{2}$, and $\frac{5}{1}$ are all equivalent.

ANSWERS TO EXERCISES

1. $\frac{3}{4}$	2. $\frac{2}{3}$	3. $\frac{4}{5}$	4. $\frac{5}{7}$	5. $\frac{2}{3}$
6. $\frac{4}{5}$	7. $\frac{8}{9}$	8. $\frac{1}{2}$	9. $\frac{3}{2}$	10. $\frac{5}{6}$
11. $\frac{3}{4}$	12. $\frac{2}{3}$	13. $\frac{2}{3}$	14. $\frac{8}{15}$	15. $\frac{8}{5}$
16. $\frac{3}{7}$	17. $\frac{3}{4}$	18. $\frac{2}{17}$	19. $\frac{7}{4}$	20. $\frac{3}{2}$

EQUIVALENT RATIOS

Ratios can be shown equivalent without using pictures.

MULTIPLICATION METHOD

If both the numerator and denominator are multiplied by the same number, the result is an equivalent ratio.

Example 1:

$$\frac{1}{3}\cdot\frac{2}{2} = \frac{1\cdot 2}{3\cdot 2} = \frac{2}{6},$$

Therefore $\frac{1}{3} = \frac{2}{6}$.

$\frac{1}{3}\cdot\frac{2}{2}$ is equivalent to $\frac{1}{3}\cdot 1$ (since $\frac{2}{2} = 1$). Multiplying a number by 1 does not change the value so an equivalent ratio is obtained.

FACTORING METHOD

If we agree that $\frac{4}{5}\cdot\frac{3}{3} = \frac{4\cdot 3}{5\cdot 3}$, then this must be true:

$$\frac{4\cdot 3}{5\cdot 3} = \frac{4}{5}\cdot\frac{3}{3}.$$

Example 1:

$$\frac{2}{3}\cdot 1 = \frac{2}{3}\cdot\frac{5}{5}$$

$$= \frac{2\cdot 5}{3\cdot 5}$$

$$= \frac{10}{15}$$

This example can be worked "backwards":

$$\frac{10}{15} = \frac{5\cdot 2}{5\cdot 3} \text{ (by factoring)}$$

$$= \frac{5}{5}\cdot\frac{2}{3} \text{ (using the definition of multiplication)}$$

$$= 1\cdot\frac{2}{3}$$

$$= \frac{2}{3}$$

Therefore $\frac{10}{15} = \frac{2}{3}$.

EQUIVALENT RATIOS

This section discusses several methods to determine whether or not two ratios are equivalent. Similar to the previous section, the students are not required to know all of the methods. Instead they should use the method that does the job for them.

It is necessary that the students be given a sufficient number of examples and that explanations are made carefully and thoroughly. The examples in this lesson were not intended to stand alone but are to be an aid in discussion.

Select appropriate ratios from common everyday experiences to illustrate the point being discussed.

For example, if orange juice is priced 3 cans for 98¢, then 6 cans would cost $1.96. The ratios $\frac{3}{98}$ and $\frac{6}{196}$ are equivalent.

Another example could be taken from the Candle and Jars experiment to illustrate equivalence.

MULT. METHOD (cont'd.)

Example 2:

$$\frac{1}{3} \cdot \frac{6}{6} = \frac{1 \cdot 6}{3 \cdot 6} = \frac{6}{18}$$

Then $\frac{1}{3} = \frac{6}{18}$

Look at examples 1 and 2. If both $\frac{2}{6}$ and $\frac{6}{18}$ are equivalent to $\frac{1}{3}$, then $\frac{2}{6} = \frac{6}{18}$.

Example 3:

$$\frac{4}{6} \stackrel{?}{=} \frac{12}{18}$$

($\stackrel{?}{=}$ means "Are they equal?")

Since $12 \div 4 = 3$,

multiply $\frac{4}{6} \cdot \frac{3}{3} = \frac{12}{18}$

Therefore $\frac{4}{6} = \frac{12}{18}$.

FACT. METHOD (cont'd.)

Example 2:

$$\frac{8}{10} \stackrel{?}{=} \frac{4}{5}$$

$$\frac{8}{10} = \frac{2 \cdot 2 \cdot 2}{2 \cdot 5}$$

$$= \frac{2}{2} \cdot \frac{2 \cdot 2}{5}$$

$$= 1 \cdot \frac{4}{5}$$

$$= \frac{4}{5}$$

Therefore $\frac{8}{10} = \frac{4}{5}$.

Example 3:

$$\frac{12}{18} \stackrel{?}{=} \frac{9}{12}$$

$$\frac{12}{18} = \frac{2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 3} \text{ and } \frac{9}{12} = \frac{3 \cdot 3}{2 \cdot 2 \cdot 3}$$

$$\frac{2 \cdot 3}{2 \cdot 3} \cdot \frac{2}{3} \stackrel{?}{=} \frac{3}{3} \cdot \frac{3}{2 \cdot 2}$$

$$1 \cdot \frac{2}{3} \stackrel{?}{=} 1 \cdot \frac{3}{4}$$

$$\frac{2}{3} \neq \frac{3}{4}$$

DIVISION METHOD

Dividing a number by 1 does not change its value.

Example 1:

$$\frac{10}{15} \stackrel{?}{=} \frac{2}{3}$$

$$\frac{10}{15} = \frac{10}{15} \div 1$$

$$= \frac{10}{15} \div \frac{5}{5}$$

(since 10 and 15 can both be divided by 5)

$$= \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$$

Therefore $\frac{10}{15} = \frac{2}{3}$.

Example 2:

$$\frac{27}{36} \stackrel{?}{=} \frac{3}{4}$$

(27 and 36 can both be divided by 9)

$$\frac{27}{36} \div \frac{9}{9} = \frac{27 \div 9}{36 \div 9}$$

$$= \frac{3}{4}$$

Therefore $\frac{27}{36} = \frac{3}{4}$.

FACT. METHOD (cont'd.)

Example 4:

$$\frac{48}{72} \stackrel{?}{=} \frac{60}{90}$$

$$\frac{48}{72} = \frac{2\cdot2\cdot2\cdot2\cdot3}{2\cdot2\cdot2\cdot3\cdot3}$$

and

$$\frac{60}{90} = \frac{2\cdot2\cdot3\cdot5}{2\cdot3\cdot3\cdot5}$$

$$\frac{2\cdot2\cdot2\cdot3}{2\cdot2\cdot2\cdot3} \cdot \frac{2}{3} \stackrel{?}{=} \frac{2\cdot3\cdot5}{2\cdot3\cdot5} \cdot \frac{2}{3}$$

$$1 \cdot \frac{2}{3} \stackrel{?}{=} 1 \cdot \frac{2}{3}$$

$$\frac{2}{3} = \frac{2}{3}$$

Therefore $\frac{48}{72} = \frac{60}{90}$.

EXERCISES 2

Choose the method you want to use and show which of the following pairs of ratios are equivalent.

Circle the pairs of ratios that are equivalent.

1. $\frac{6}{9} \stackrel{?}{=} \frac{12}{18}$

2. $\frac{6}{8} \stackrel{?}{=} \frac{18}{24}$

3. $\frac{24}{30} \stackrel{?}{=} \frac{12}{15}$

4. $\frac{12}{18} \stackrel{?}{=} \frac{2}{3}$

5. $\frac{2}{5} \stackrel{?}{=} \frac{8}{20}$

6. $\frac{15}{18} \stackrel{?}{=} \frac{5}{6}$

7. $\frac{9}{12} \stackrel{?}{=} \frac{36}{46}$

8. $\frac{24}{30} \stackrel{?}{=} \frac{8}{10}$

9. $\frac{7}{4} \stackrel{?}{=} \frac{36}{46}$

10. $\frac{8}{12} \stackrel{?}{=} \frac{16}{24}$

11. $\frac{30}{36} \stackrel{?}{=} \frac{10}{13}$

12. $\frac{18}{10} \stackrel{?}{=} \frac{9}{6}$

13. $\frac{4}{9} \stackrel{?}{=} \frac{20}{45}$

14. $\frac{25}{10} \stackrel{?}{=} \frac{5}{2}$

15. $\frac{18}{20} \stackrel{?}{=} \frac{6}{7}$

16. $\frac{28}{20} \stackrel{?}{=} \frac{7}{5}$

ANSWERS TO EXERCISES 2

1. $\frac{6}{9} \stackrel{?}{=} \frac{12}{18}$
2. $\frac{6}{8} \stackrel{?}{=} \frac{18}{24}$
3. $\frac{24}{30} \stackrel{?}{=} \frac{12}{15}$
4. $\frac{12}{18} \stackrel{?}{=} \frac{2}{3}$
5. $\frac{2}{5} \stackrel{?}{=} \frac{8}{20}$
6. $\frac{15}{18} \stackrel{?}{=} \frac{5}{6}$
7. $\frac{9}{12} \stackrel{?}{=} \frac{36}{46}$
8. $\frac{24}{30} \stackrel{?}{=} \frac{8}{10}$
9. $\frac{7}{4} \stackrel{?}{=} \frac{36}{46}$
10. $\frac{8}{12} \stackrel{?}{=} \frac{16}{24}$
11. $\frac{30}{36} \stackrel{?}{=} \frac{10}{13}$
12. $\frac{18}{10} \stackrel{?}{=} \frac{9}{6}$
13. $\frac{4}{9} \stackrel{?}{=} \frac{20}{45}$
14. $\frac{25}{10} \stackrel{?}{=} \frac{5}{2}$
15. $\frac{18}{20} \stackrel{?}{=} \frac{6}{7}$
16. $\frac{28}{20} \stackrel{?}{=} \frac{7}{5}$

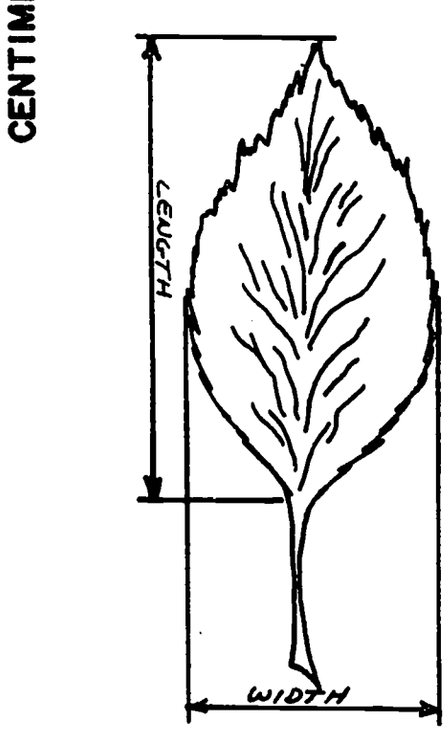
MEASURING LEAVES

CLASS ACTIVITY

For this activity you will be working by yourself.

Read over the activity before beginning to do any measuring.

Equipment Inventory: 1 ruler graduated in half-centimeters (cm.), at least 8 leaves from the same plant.



Activity:

1. Measure the length and width of 4 leaves.
   a. If you do not have real leaves, use those on the next page.
   b. Measure to the nearest half centimeter. Cut the ruler from the left margin of this page.
   c. Record your measurements and complete the table at the bottom of the next page.

MEASURING LEAVES

OBJECTIVES

1. To involve the student in experimental activities in which students apply the idea of ratio in prediction.

2. Given the ratio of length to width of a particular type of leaf, the student shall be able to predict the width of a leaf upon measuring the length.

EQUIPMENT AND TEACHING AIDS

STUDENT

1. If it is the teacher's desire to use actual leaves rather than the reproductions in the booklet, approximately 10 leaves the style of elm leaves as opposed to maple leaves will be needed.

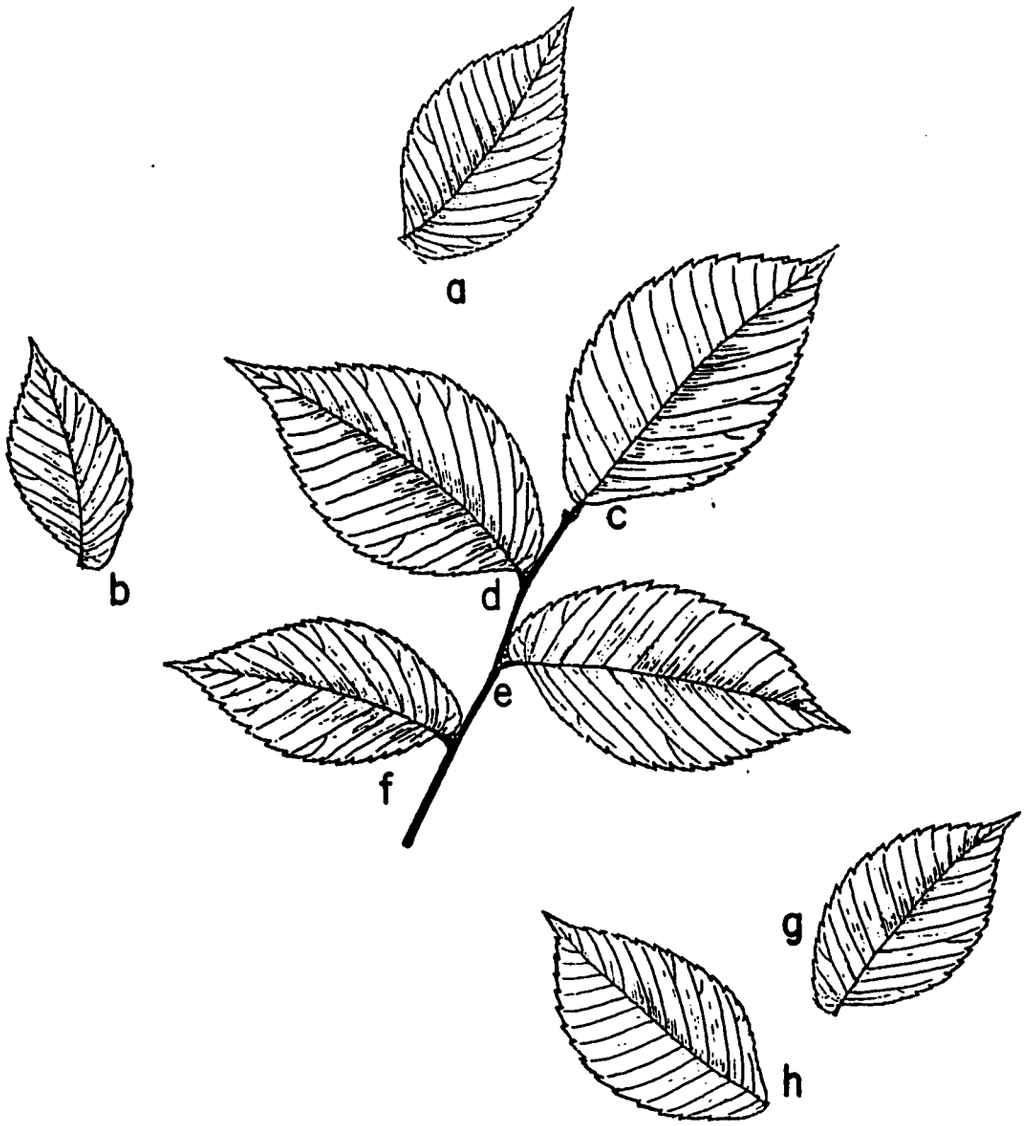
*2. 1 ruler graduated in half-centimeters.

CONTENT AND APPROACH

Refer to the discussion on the laboratory-activity approach beginning on page T 37a.

The students are to measure the length and width of the leaves to the nearest half-centimeter.

One of the important aspects of the activity is the predicting that the students are to perform. Often a student will measure all of the lengths and widths in order to complete the answer spaces rather than do the predicting and verifying requested. The object of the activity is to use ratios to predict. As a result you should promote the spirit of the lesson to ensure its success.



AMERICAN ELM LEAVES

Leaf	a	b	c	d
Length (cm.)				
Width (cm.)				
Ratio: $\frac{\text{Length}}{\text{Width}}$				

ANSWERS TO EXERCISES

Leaf	a	b	c	d
Length (cm.)	4	3	5	5
Width (cm.)	2	$1\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{2}$
Ratio: $\frac{\text{Length}}{\text{Width}}$	$\frac{4}{2}$	$\frac{3}{1\frac{1}{2}}$	$\frac{5}{2\frac{1}{2}}$	$\frac{5}{2\frac{1}{2}}$

2. Find the average of the four ratios in this way:
   a. Change each ratio to decimal form by dividing the denominator into the numerator.

      Example:

      $\frac{2}{4}$ can be written in decimal form by dividing

$$\begin{array}{r} 1.25 \\ 4\overline{)5.00} \\ \underline{4} \\ 10 \\ \underline{8} \\ 20 \end{array}$$

Leaf	a	b	c	d
Ratio				
Decimal				

   b. Add the 4 decimal values = _____

   c. Average: Sum ÷ 4 = _____

3. Measure the widths of 4 other leaves from the same plant used in part 1.

Leaf	e	f	g	h
Width (cm.)				

4. Predict the lengths of the 4 leaves using the "average ratio."

   Length = width × average ratio.

Leaf	e	f	g	h
Predicted Length				
Measured Length				

ANSWERS (CONT'D.)

2.

Leaf	a	b	c	d
Ratio	$\frac{4}{2}$	$\frac{3}{1\frac{1}{2}}$	$\frac{5}{2\frac{1}{2}}$	$\frac{5}{2\frac{1}{2}}$
Decimal	2	2	2	2

b. Add the 4 decimal values = 8

c. Average: Sum ÷ 4 = 2

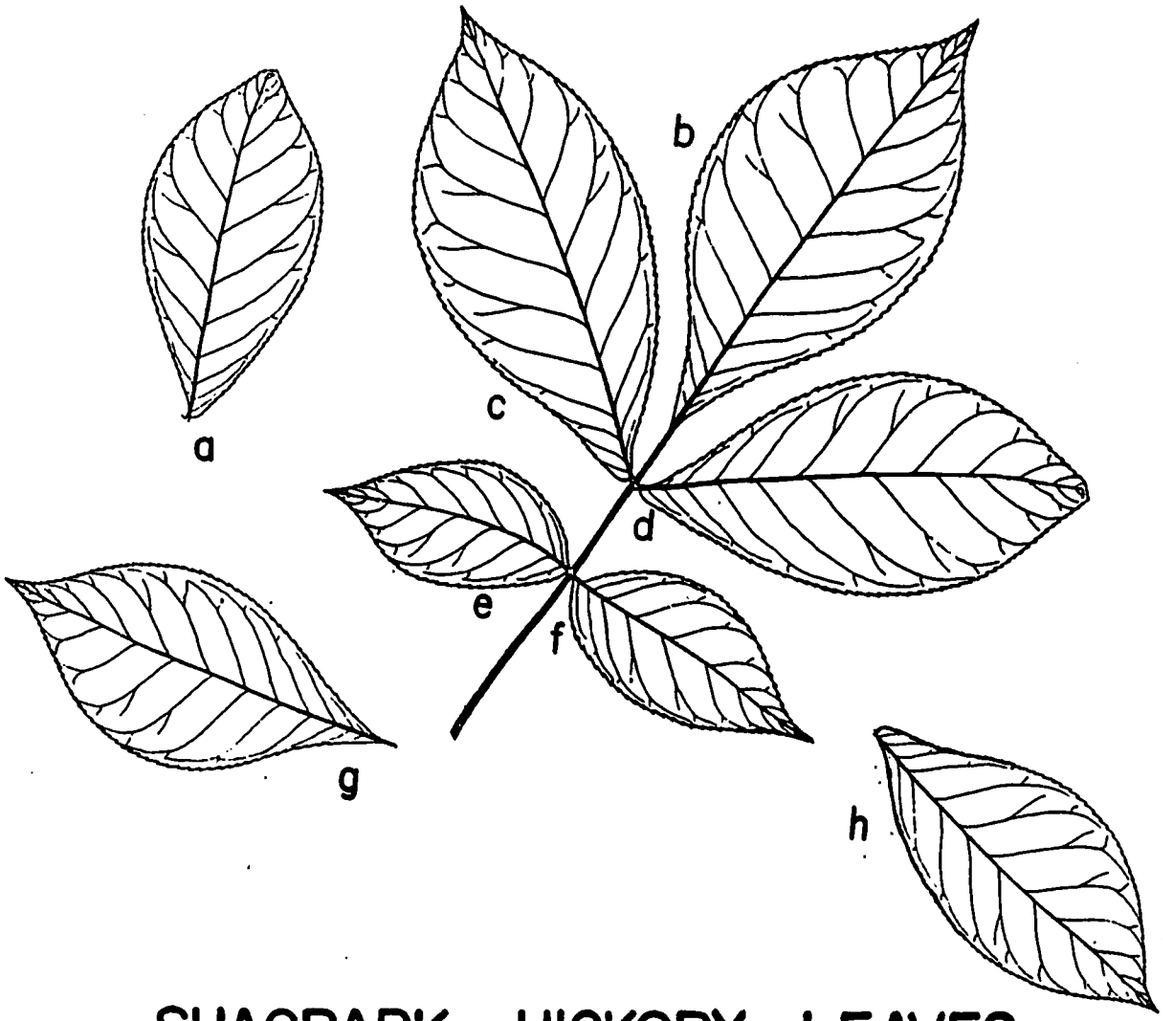
3.

Leaf	e	f	g	h
Width (cm.)	$2\frac{1}{2}$	2	2	2

4.

Leaf	e	f	g	h
Predicted Length	5	4	4	4
Measured Length	5	4	4	4

6. Repeat parts 1-4 using the leaves below.



SHAGBARK HICKORY LEAVES

Leaf	a	b	c	d
Length (cm.)				
Width (cm.)				
Ratio: Length / Width				
Decimal				

Leaf	e	f	g	h
Measured Width				
Predicted Length				
Measured Length				

ANSWERS (CONT'D.)

6.

Leaf	a	b	c	d
Length (cm.)	5	7	7	6
Width (cm.)	$2\frac{1}{2}$	$3\frac{1}{2}$	3	3
Ratio: $\frac{\text{Length}}{\text{Width}}$	$\frac{5}{2\frac{1}{2}}$	$\frac{7}{3\frac{1}{2}}$	$\frac{7}{3}$	$\frac{6}{3}$
Decimal	2	2	2.3	2

Average Ratio $8.3 \div 4 \approx 2.1$

Leaf	e	f	g	h
Measured Width	2	2	$2\frac{1}{2}$	$2\frac{1}{2}$
Predicted Length	4.1	4.1	5.25	5.25
Measured Length	$3\frac{1}{2}$	4	6	$5\frac{1}{2}$

(NOTE: Student page 48 and Teacher page T 48 are blank.)

ROAST BEEF AND THE WEATHER

PROPORTION

A useful tool in solving problems is equivalent ratios. Equivalent ratios written like this, $\frac{2}{3} = \frac{6}{9}$, have another name. The whole expression, $\frac{2}{3} = \frac{6}{9}$, is called a proportion.

A PROPORTION IS A STATEMENT THAT TWO RATIOS ARE EQUIVALENT.

Other examples of proportions are: $\frac{3}{4} = \frac{6}{8}$, $\frac{5}{10} = \frac{10}{20}$, $\frac{3}{1} = \frac{9}{3}$.

Each of the above are true proportions because each is a pair of equivalent ratios.

A <u>false proportion</u> is one where the ratios are not equivalent. Examples are: $\frac{1}{3} = \frac{1}{2}$, $\frac{2}{3} = \frac{4}{8}$, $\frac{3}{5} = \frac{5}{3}$.

✓ POINT

How can you tell whether or not a proportion is a true statement?

EXERCISES I

Circle each true proportion.

1. $\frac{2}{3} = \frac{4}{6}$
2. $\frac{3}{7} = \frac{9}{21}$
3. $\frac{2}{5} = \frac{4}{8}$
4. $\frac{18}{33} = \frac{6}{11}$
5. $\frac{1}{3} = \frac{5}{14}$
6. $\frac{5}{3} = \frac{25}{15}$
7. $\frac{4}{5} = \frac{16}{20}$
8. $\frac{6}{8} = \frac{9}{12}$

ROAST BEEF AND THE WEATHER

OBJECTIVES

1. Given a proportion, the student shall
   a. Determine if it is true or false.
   b. Solve the proportion for the missing member.
2. Given a problem situation, the student shall write a proportion that could be used to solve the problem.

EQUIPMENT AND TEACHING AIDS

TEACHER

1. Overhead projector
2. Projection screen
*3. Transparency: ARP 7

   ARP 7 illustrates example 3 on page 51.

ANSWER TO ✓POINT Use paper folding, sketches of rectangles, multiplication method, division method, or factoring method.

ANSWERS TO EXERCISES I

1. $\frac{2}{3} = \frac{4}{6}$ 2. $\frac{3}{7} = \frac{9}{21}$ 3. $\frac{2}{5} = \frac{4}{8}$ 4. $\frac{18}{33} = \frac{6}{11}$

5. $\frac{1}{3} = \frac{5}{14}$ 6. $\frac{5}{3} = \frac{25}{15}$ 7. $\frac{4}{5} = \frac{16}{20}$ 8. $\frac{6}{8} = \frac{9}{12}$

9. $\frac{4}{12} = \frac{16}{44}$ 10. $\frac{8}{6} = \frac{12}{9}$ 11. $\frac{18}{26} = \frac{27}{39}$ 12. $\frac{10}{18} = \frac{18}{10}$

13. $\frac{8}{14} = \frac{12}{21}$ 14. $\frac{14}{8} = \frac{7}{3}$ 15. $\frac{15}{20} = \frac{10}{18}$ 16. $\frac{6}{14} = \frac{9}{20}$

Proportions are useful in solving problems and making predictions in experiments. If three members of a proportion are known, the fourth can be found by using equivalent ratios.

Example 1: Find N in the proportion $\frac{2}{3} = \frac{N}{12}$ so that it makes a true proportion.

Remember, a proportion is a statement that two ratios are equivalent.

$$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}, \qquad \text{so } N = 8.$$

Multiplying a number by 1 does not change its value. A new form is obtained called an equivalent ratio.

$$\text{In this case, } \frac{2}{3} \times 1 = \frac{2}{3} \times \frac{\square}{\square} = \frac{2\times \square}{3\times \square} = \frac{N}{12}.$$

Since $3 \times \square = 12$, the number we want is the solution of $12 \div 3 = \square$ or 4.

ANSWERS (CONT'D.)

9. $\frac{4}{12} = \frac{16}{44}$ 10. $\frac{8}{6} = \frac{12}{9}$ 11. $\frac{18}{26} = \frac{27}{39}$ 12. $\frac{10}{18} = \frac{18}{10}$

13. $\frac{8}{14} = \frac{12}{21}$ 14. $\frac{14}{8} = \frac{7}{3}$ 15. $\frac{15}{20} = \frac{10}{18}$ 16. $\frac{6}{14} = \frac{9}{20}$

CONTENT AND APPROACH

PROPORTION

Proportions are used in making predictions and some of the students will apply proportionality automatically. Take the opportunity provided by their intuitive knowledge to define proportion. Then use proportion to solve some of the questions posed by the experiments. The method of solution of proportions here is equivalent ratios (The cross-product method will be discussed in Lesson 9). Use specific examples.

In this lesson proportions where all four terms are known are referred to as true or false proportions (comparable to equations).

If one term of the proportion is a variable, it is thought of as an open proportion. When the variable is replaced by a number, then the truth or falsity of the proportion can be determined.

In Exercises 1, the student determines if the proportion is true or false in the same way as he determined whether two ratios were equivalent.

$$12 \div 3 = 4, \quad \text{so } \frac{2}{3} \times 1 = \frac{2}{3} \times \frac{\boxed{4}}{\boxed{4}} = \frac{2 \times \boxed{4}}{3 \times \boxed{4}} = \frac{8}{12}.$$

Therefore, N = 8.

Example 2: $\frac{4}{7} = \frac{M}{21}$. Since $21 \div 7 = 3$, multiply $\frac{4}{7}$ by $\frac{3}{3}$.

Then $\frac{4}{7} \times \frac{3}{3} = \frac{12}{21}$, so M = 12.

Example 3: If a weight of 5 ounces stretches a spring 3 centimeters, and some unknown weight W stretches the spring 12 cm., how much is W?

A proportion expressing this problem is $\frac{5}{3} = \frac{W}{12}$.

Here 1 is expressed by $\frac{4}{4}$, and W is 20.

Example 4: A weight of 20 oz. stretches a spring 15 cm. What weight W stretches the same spring 3 cm.?

$\frac{20}{15} = \frac{W}{3}$ is the proportion to be solved.

In this case division by 1 can be used.

$$\frac{20}{15} \div \frac{5}{5} = \frac{4}{3}$$

Since $15 \div 5$ is 3 and dividing by 1 does not change the value of a number, we must divide 20 by 5 and find out that W = 4.

THINGS TO DISCUSS

1. $\frac{a}{b} = \frac{c}{d}$ is a stated proportion, based on the definition.

2. We say that a, b and c, d are proportional. If m, n and x, y are proportional, then $\frac{m}{n} = \frac{x}{y}$.

3. If $\frac{a}{b} = \frac{c}{d}$ is true, then $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent ratios.

4. To test a stated proportion to see if it is true, determine if $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent ratios.

5. If three members of a proportion are known, then the fourth member can be found, based on our knowledge of equivalent ratios.

6. Proportions are useful in making predictions like those asked for in the experiment. It will have to be pointed out that the situation dictates how the proportion is to be set up.

7. A <u>key point</u> here is that the students need help in setting up proportions. They may be able to write ratios properly, but if the members are not compared in the <u>proper</u> order, the proportionality will be incorrect. It is hoped class discussion will help them, and that positive results are manifested in this lesson.

8. Use the experiments to illustrate how proportions are used in making predictions.

EXERCISES 2

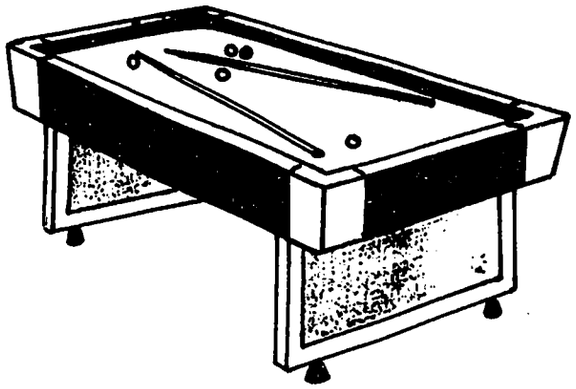
Solve each proportion for the missing member.

1. $\frac{3}{8} = \frac{N}{24}$
2. $\frac{4}{5} = \frac{M}{15}$
3. $\frac{24}{36} = \frac{H}{12}$
4. $\frac{15}{20} = \frac{30}{E}$
5. $\frac{20}{x} = \frac{5}{3}$
6. $\frac{R}{10} = \frac{32}{40}$
7. $\frac{7}{9} = \frac{W}{27}$
8. $\frac{8}{6} = \frac{32}{T}$
9. $\frac{5}{7} = \frac{N}{35}$
10. $\frac{12}{20} = \frac{N}{5}$
11. $\frac{N}{8} = \frac{25}{40}$
12. $\frac{9}{6} = \frac{N}{36}$

PROPORTIONS AND POOL TABLES

The length and width of a regulation pool table are in the ratio of $\frac{2}{1}$.

A regulation table that is 88" long is 44" wide.



ANSWERS TO EXERCISES 2

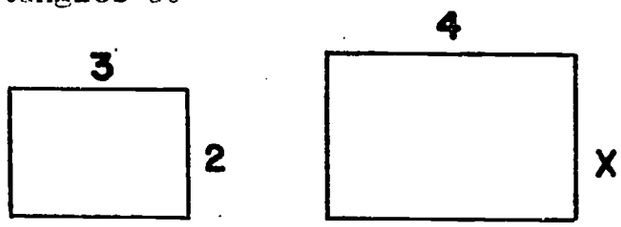
1. N = 9	2. M = 12	3. H = 8	4. E = 40
5. X = 12	6. R = 8	7. W = 21	8. T = 24
9. N = 25	10. N = 3	11. N = 5	12. N = 54

CONTENT AND APPROACH

PROPORTIONS AND POOL TABLES

The objective of this portion of the lesson is to illustrate the ways in which a proportion can be set up correctly to solve a particular problem. The pool table is used as a tool for getting at this notion.

This objective is illustrated by the following example as to how proportions can be written for the two rectangles at the right.



1. $\frac{3}{4} = \frac{2}{x}$ or $\frac{4}{3} = \frac{x}{2}$

2. $\frac{3}{2} = \frac{4}{x}$ or $\frac{2}{3} = \frac{x}{4}$

EXERCISES 3

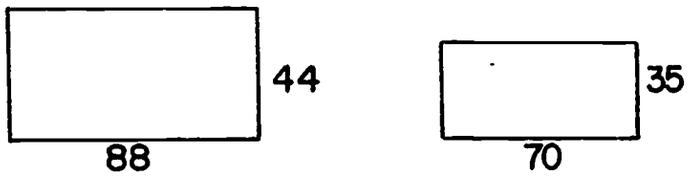
Some measures of other regulation pool tables are given. Find the missing measures.

Table	Length	Width
1	76	
2		41
3	70	

SEVERAL PROPORTIONS

When proportions are used to determine unknown values, there is more than one way to set up the proportion. Not all of them are correct ways.

For example, let's think about pool tables. Since all regulation tables have lengths and widths in the ratio of $\frac{2}{1}$, regulation pool tables are proportional.



A. One way the proportion can be written is

$$\frac{88}{44} = \frac{70}{35}, \qquad \frac{\text{length}}{\text{width}} = \frac{\text{length}}{\text{width}}.$$

B. Another way to write the proportion is

$$\frac{88}{70} = \frac{44}{35}, \qquad \frac{\text{length}}{\text{length}} = \frac{\text{width}}{\text{width}}.$$

ANSWERS TO EXERCISES 3

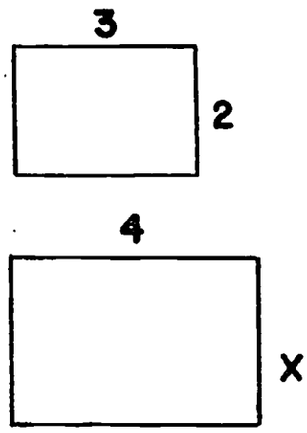
Table	Length	Width
1	76	38
2	82	41
3	70	35

SEVERAL PROPORTIONS

Referring to the previous example:

It is important that the student understands the alternate ways to write the proportion so the incorrect representations such as

$\frac{3}{4} = \frac{x}{2}$ or $\frac{3}{2} = \frac{x}{4}$ are not used.



EXERCISES 4

Each exercise of the table below contains the measures of two items. They are proportional. One ratio is given and you are to write the second ratio to form a proportion.

Exercise	Item A	Item B	First Ratio	Second Ratio
1	35 miles, 3 gal.	52.5 miles, 4.5 gal.	$\frac{35}{3}$	
2	5 degrees, 30 min.	12 min., 2 degrees	$\frac{5}{2}$	
3	7.5 zips, 4.5 zaps	5 zips, 3 zaps	$\frac{5}{7.5}$	
4	6 ft.long,4 ft. wide	10 ft.wide,15 ft.long	$\frac{4}{6}$	
5	17x, 23y	4.25x, 5.75y	$\frac{5.75}{23}$	
6	30 days, 6 jobs	10 days, 2 jobs	$\frac{2}{10}$	

DISCUSSION QUESTIONS

1. How are the numerator and denominator related for each ratio in proportion A? (Bottom of page 53.)

2. In proportion A, how do the numerators compare? The denominators?

3. How are the numerator and denominator related for each ratio in proportion B? (Bottom of page 53.)

4. In proportion B, how do the numerators compare? The denominators?

5. What is the most important thing you must consider when setting up a proportion?

ANSWERS TO EXERCISES 4

1. $\frac{52.5}{4.5}$
2. $\frac{30}{12}$
3. $\frac{3}{4.5}$
4. $\frac{10}{15}$
5. $\frac{4.25}{17}$
6. $\frac{6}{30}$

ANSWERS TO DISCUSSION QUESTIONS

1. The numerator and denominator of each ratio contain the measures of one table. Each ratio is related to a single table.
2. Both numerators are measures of length. Both denominators are measures of width. The important idea is that both denominators (or both numerators) contain measures of the same kind (length or width).
3. The numerator and denominator of a ratio contain measures of the same kind (length or width).
4. The numerators contain measures from the same object (length and width).
5. If the first ratio contained the measures from one object, then the second ratio would contain measures from the second object. The numerators (and denominators) would contain the corresponding measures of the two objects. If the first ratio contained the corresponding measures from the two objects, then the second ratio would contain the other corresponding measures in which we are interested. Both numerators (and denominators) would contain measures from the same object.

PROBLEM SOLVING USING PROPORTIONS

Using proportions to solve problems is a valuable tool, and the following exercises will give you practice in setting up proportions.

Before writing proportions, you should ask yourself:

(1) What quantities are involved?

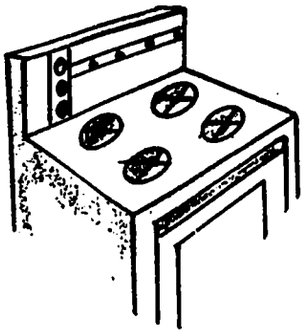
(2) What is the unknown quantity?

EXERCISES 5

Read each exercise carefully and write <u>only</u> the proportion that you would use to solve the problem. Do not take the time to find the actual answer to the problem.

1. A car travels 96 miles on 6 gallons of gasoline. How far could it go on 24 gal.?

2. Roast beef requires a cooking time of 20 minutes per pound. How many minutes would be required to cook a $5\frac{1}{2}$ pound roast?



3. One cubic foot of water equals 7.48 gallons and weighs 62.5 pounds. How much does 1 gallon of water weigh?

PROBLEM SOLVING USING PROPORTIONS

Students need help and practice in setting up a proportion which is to be used in solving a problem. For this reason the student is asked to set up the proportions only. In a later set of exercises the students come back here to actually calculate the solutions.

It is important that either the students or the teacher retain this set of exercises for future work.

ANSWERS TO EXERCISES 5

1. $\frac{96}{6} = \frac{x}{24}$

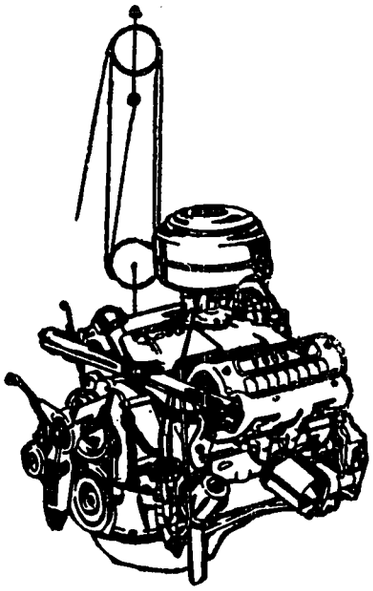
2. $\frac{20}{1} = \frac{x}{5.5}$

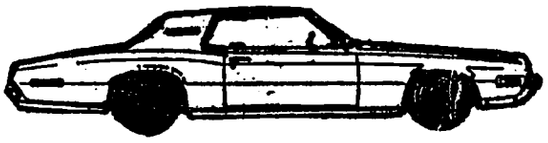
3. $\frac{7.48}{62.5} = \frac{1}{x}$

4. Garden hose varies in size according to the diameter of the opening. A $\frac{3}{4}$-inch hose will deliver 31 gallons per minute. A lawn needs 744 gal. of water for each 1000 sq. ft. of lawn. How many minutes would it take to squirt 744 gal. of water?

5. A $\frac{1}{2}$-in. hose will deliver only 10.5 gal. per minute. How long would this hose take to produce 744 gallons of water?

6. Pulleys are used to help people lift heavy loads. The pulley system shown works like this: To lift a load of 24 lbs., you would have to exert a 6 lb. pull. How many pounds pull would you have to exert to lift a 600 lb. car engine?





7. A car traveling 60 mph is going at the rate of 88 feet per second. Find the rate of a car in feet per second when it travels 50 mph.

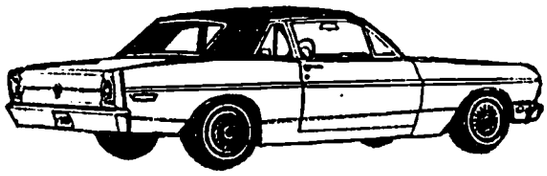
ANSWERS (CONT'D.)

4. $\frac{31}{1} = \frac{744}{x}$

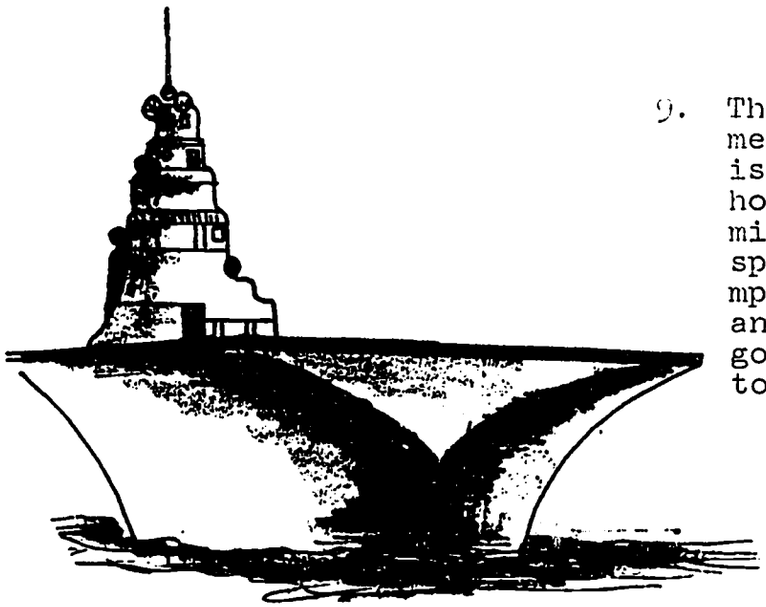
5. $\frac{10.5}{1} = \frac{744}{x}$

6. $\frac{24}{6} = \frac{600}{x}$

7. $\frac{60}{88} = \frac{50}{x}$



8. Most countries use the metric system for measuring. This means they measure speeds in kilometers per hour. A car traveling 50 mph is going at the rate of 80 km. per hour. What speed in mph would a car be going if it traveled at 120 km. per hour?



9. The speed of ships is measured in knots. A knot is one nautical mile per hour. Nautical and land miles are different. A speed of 20 knots is 23 mph. How many mph will an aircraft carrier be going if it travels at top speed, 30 knots?

10. The marine weather bureau gives wind speed in knots. If they forecast wind speeds up to 55 knots, how many mph would this be?

DISCUSSION QUESTION

Jim Ryan can run the mile in 4 minutes. How long will it take him to run 7 miles? What do you think of the reasonableness of this question?

ANSWERS (CONT'D.)

8. $\frac{50}{80} = \frac{x}{120}$

9. $\frac{20}{23} = \frac{30}{x}$

10. $\frac{20}{23} = \frac{55}{x}$

ANSWER TO DISCUSSION QUESTION

It will take longer than the proportion suggests. Some situations which on the surface appear to be proportional are not. The purpose of this question is to point this out.

✓POINT

1. Write the proportion that would be used to solve each problem.

   a. The Imperial gallon is used in Canada. It is larger than the U.S. gallon. For every 5 Imperial gallons, there are 6 U.S. gallons. 17 gallons of gasoline purchased in Canada would be equal to how many gallons in the U.S.?

   b. United States Nickels are made of copper and nickel. For every 3 oz. of copper, 1 oz. of nickel is used. If 14 pounds of copper are to be used for making Nickels, how much nickel is required?

2. Circle the pairs of ratios that are equivalent.

   a. $\frac{4}{9} = \frac{20}{45}$ b. $\frac{15}{10} = \frac{18}{12}$ c. $\frac{14}{18} = \frac{35}{45}$

3. Circle the proportions that are true.

   a. $\frac{9}{6} = \frac{12}{10}$ b. $\frac{12}{18} = \frac{8}{12}$ c. $\frac{7}{11} = \frac{28}{42}$

4. Simplify the following ratios:

   a. $\frac{10}{18}$ b. $\frac{24}{16}$ c. $\frac{28}{36}$

5. Solve these proportions for N.

   a. $\frac{5}{8} = \frac{N}{40}$ b. $\frac{6}{10} = \frac{24}{N}$ c. $\frac{12}{N} = \frac{9}{6}$

ANSWER TO ✓POINT

1. a. $\frac{5}{6} = \frac{17}{x}$

   b. $\frac{3}{1} = \frac{14}{x}$

2. a. $\frac{4}{9} = \frac{20}{45}$ b. $\frac{15}{10} = \frac{18}{12}$ c. $\frac{14}{18} = \frac{35}{45}$

3. a. $\frac{2}{6} = \frac{12}{10}$ b. $\frac{12}{18} = \frac{8}{12}$ c. $\frac{7}{11} = \frac{35}{42}$

4. a. $\frac{5}{9}$ b. $\frac{3}{2}$ c. $\frac{7}{9}$

5. a. N = 25 b. N = 40 c. N = 8

THE BOUNCER

Several students will demonstrate this activity for the whole class. If you are one of those who perform this activity, prepare yourself and do a good job. If you are watching the demonstration, think about how an activity should be prepared for, how it should be carried out, and be ready to participate in the summary of the activity.

CLASS ACTIVITY

Read over the activity before beginning the demonstration.

Tell the rest of the students what the activity is about.

The purpose of this activity is to determine the height a ball will bounce when dropped from various distances from the floor. Use this information, in a proportion to predict the height the ball will bounce when dropped from other positions.

Equipment Inventory: 1 ball (golf ball, tennis ball, super ball, etc.)

1 yardstick

THE BOUNCER

OBJECTIVES

1. Given an experimental situation, the student shall use ratios, equivalent ratios, proportions, and the solution of proportions to make predictions of additional trials of the experiment.

2. Given the components of a proportion which are based on a problem situation, the student shall form a valid proportion.

3. If a group of students are presented an activity situation, they will demonstrate and discuss the activity before the remainder of the class.

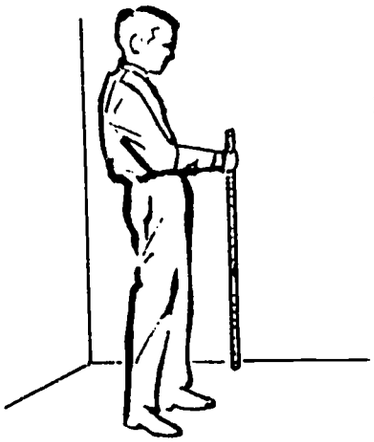
EQUIPMENT AND TEACHING AIDS

#1. 1 ball - a golf ball, tennis ball or a super ball.

#2. 1 yardstick

CONTENT AND APPROACH

The activity involves students working as a group in a controlled situation. Three or four students should be selected to perform the activity as a demonstration. This activity moves the class a step closer to group work and its shifts the responsibility for the activity from teacher to student.



Activity:

1. Hold the yardstick against the wall perpendicular to the floor with the 0-inch mark at the floor.

2. Drop the ball from a Height (H) of 40 inches. How far did it rise on the first bounce? _____

3. Drop the ball from 40 inches several times and record your best judgment of the height (h) it bounces. _____

4. Write the ratio, $\frac{H}{h}$ $\left(\frac{\text{Height ball dropped from}}{\text{height ball bounces}}\right)$: _____

5. Use proportions to predict the missing values in the table.

Height dropped from, H inches	20	50	72
Predicted h after first bounce			

6. Check your results by performing the experiment.

Height, H inches	20	50	72
Experimental results, h			

Since results will vary from experiment to experiment, no results are provided.

7. Use proportions to predict the missing values in the table. Note: In this part you are given the height of the bounce, and you are predicting where the ball was dropped from.

height after first bounce	30	45	60
Predicted H			

8. Check your results by performing the experiment.

height h, inches	30	45	60
Experimental results, H			

9. What effect might a different kind of flooring or different ball have on the amount of bounce?

ANSWER

9. Different materials cause different reactions, but for a given ball and a given floor the results are linear.

THE CROSS-PRODUCT

The Cross-Product is used to solve proportions when one number is unknown. It is also used to test whether or not a proportion is true.

Some proportions do not have easy numbers to work with, like $\frac{6}{9} \stackrel{?}{=} \frac{14}{21}$. It is not an easy job to find out what number multiplied by 9 equals 21.

So we will look at another way to test proportions, the Cross-Product.

COMPARING RATIOS

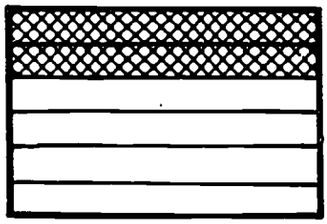
Example 1:

Is the proportion $\frac{2}{6} = \frac{3}{9}$ true?

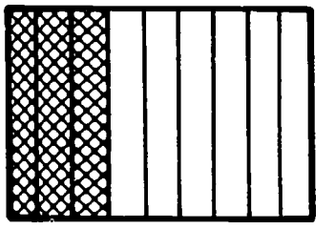
The description of the model given below along with your teacher's explanation should allow you to see how the model works.

1. Shade two rectangles; each to represent one of the ratios. (The rectangles are the same size.)

$\frac{2}{6}$



$\frac{3}{9}$



THE CROSS-PRODUCT

OBJECTIVES

Given a proportion, the student shall

a. Determine if it is true or false using the cross-product method.

b. Solve the proportion for the missing member using the cross-product method.

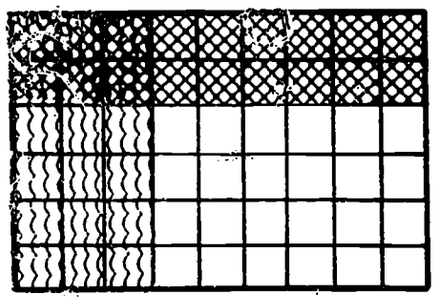
EQUIPMENT AND TEACHING AIDS

TEACHER

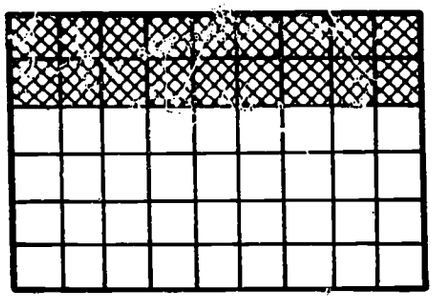
1. Overhead projector
2. Projection screen
*3. Transparency: ARP 9

ARP 9 illustrates the cross-product model, similar to the examples on pages 62-65, and contains <u>two</u> examples. The static acetate sheet shows the rectangles that represent the ratios before they are superimposed. When the overlay is put in place, the rectangles contain both horizontal and vertical lines which allow the small shaded regions to be counted. Then it can be determined whether or not the ratios are equivalent.

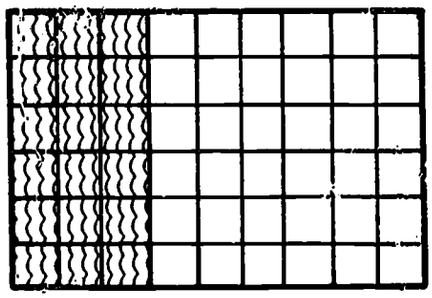
2. Slide the two rectangles together so they coincide. The lines from the first rectangle are to be transferred to the second, and the lines from the second to the first.



3. Separate the two rectangles and write the ratio each represents now.



$\frac{18}{54}$



$\frac{18}{54}$

4. The rectangles are now cut into the same number of regions, 54. The shaded areas have different shapes, but each is cut up into 18 small regions of equal size.

Each rectangle represents $\frac{18}{54}$. Since the rectangles are the same as at the beginning except for the number of regions drawn, $\frac{2}{6} = \frac{3}{9}$.

NOTE: Through the General Learning Corporation, the National Council of Teachers of Mathematics has produced a film (approximately 10 minutes) illustrating the ideas in Lesson 9. The film, Comparing Rational Numbers (The Genie and the Clam) was produced for elementary school students, but it is a cartoon-type film which might go well in the junior high school. It is available at the Oakland Schools film library, film number 1080.

CONTENT AND APPROACH

The cross-product application to solution of proportions is justified by the model used in the student text.

Since the cross-products of all true proportions yield equal answers, and of all false proportions yield unequal answers, the cross-product method is also of value when comparing ratios.

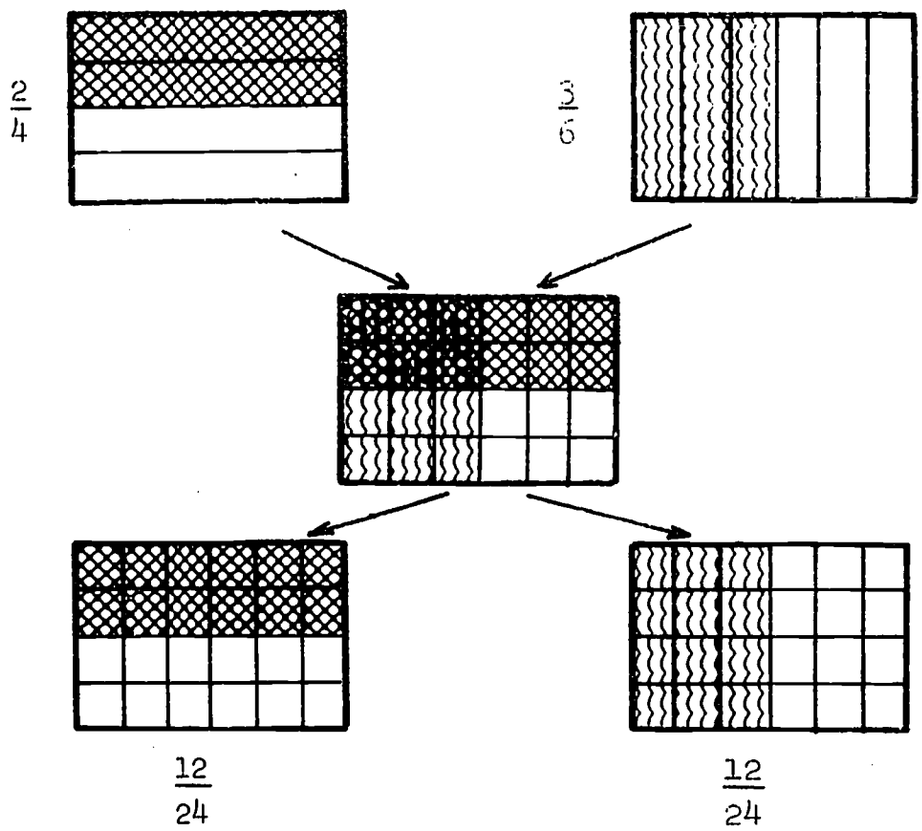
COMPARING RATIOS

The number of small rectangles in each figure is the vehicle used to point out the equivalence (or non-equivalence) of two given ratios.

Example 2:

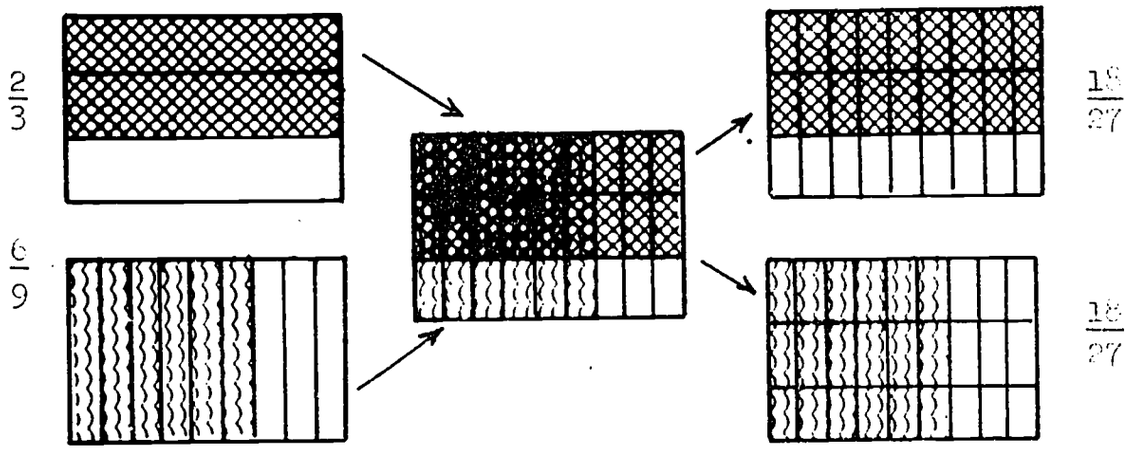
$$\frac{2}{4} \stackrel{?}{=} \frac{3}{6}$$

Draw rectangles to represent each of the ratios, slide them together, and pull them apart with each rectangle containing the horizontal and vertical lines.



Then $\frac{2}{4} = \frac{3}{6}$

Example: $\frac{2}{3} = \frac{6}{9}$ (The ratios are assumed equivalent at the outset.)



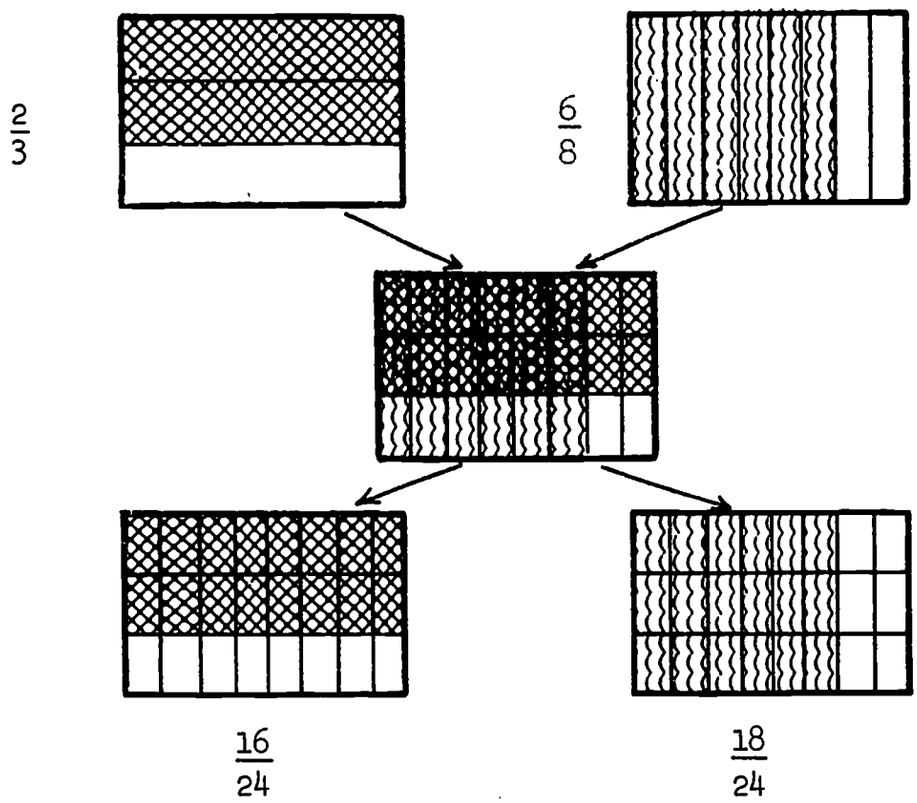
Two rectangles are used, one for each ratio. One rectangle is lined horizontally and shaded to represent one ratio, and the other rectangle is lined vertically and shaded to represent the other ratio.

The two rectangles are superimposed and the line segments are "transferred" to both rectangles so that each is divided into the same number of equal-sized regions. This corresponds to the common denominator of the two ratios.

The number of shaded regions in each rectangle are counted. If the number of small regions is the same for each rectangle, the ratios are equivalent. If the number of regions are unequal, the ratios are not equivalent.

Example 3:

$$\frac{2}{3} \stackrel{?}{=} \frac{6}{8}$$



Since $\frac{16}{24} \neq \frac{18}{24}$, the proportion $\frac{2}{3} = \frac{6}{8}$ is false.

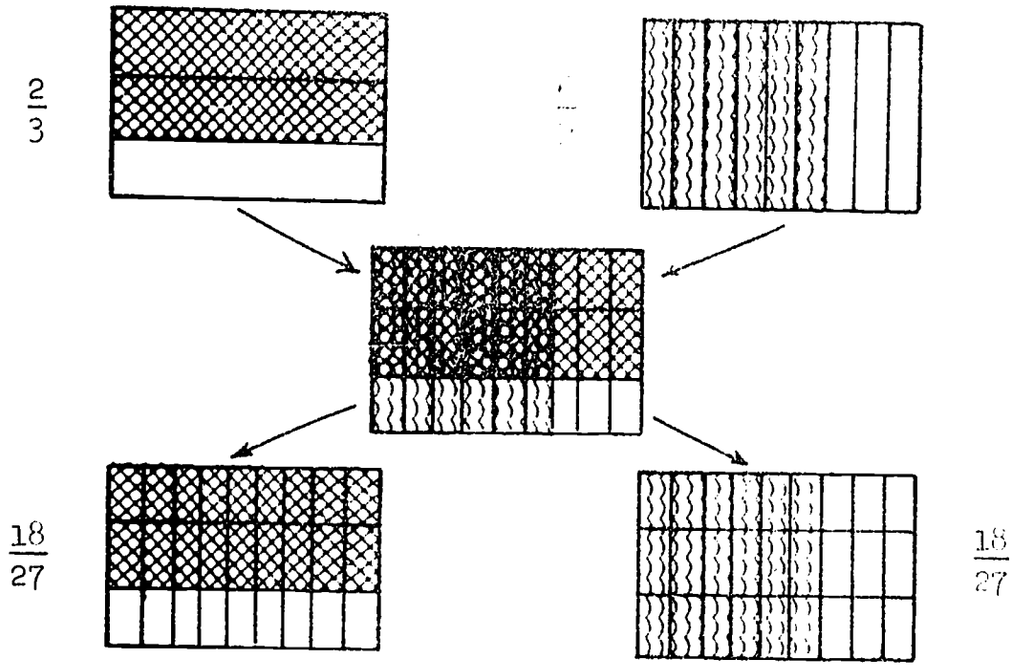
EXERCISES I

Draw diagrams like those above to determine whether the proportions are true or false. A worksheet can be found on page 129 for you to show your work.

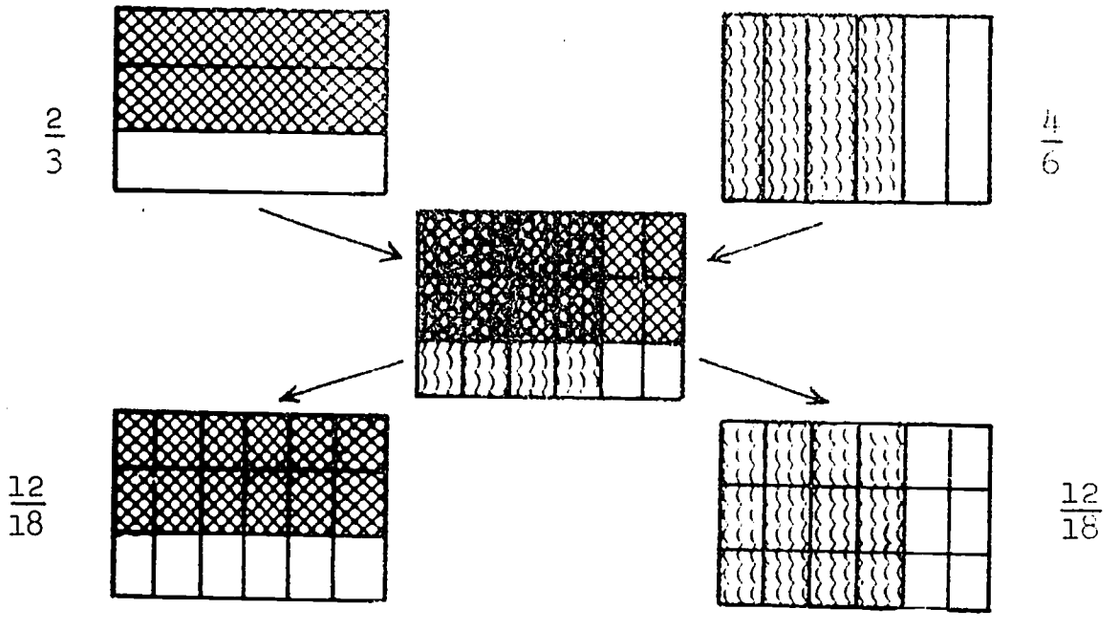
1. $\frac{2}{3} = \frac{6}{9}$
2. $\frac{2}{3} = \frac{4}{6}$
3. $\frac{4}{6} = \frac{6}{8}$
4. $\frac{3}{4} = \frac{6}{8}$
5. $\frac{3}{6} = \frac{4}{8}$

ANSWERS TO EXERCISES I

1.



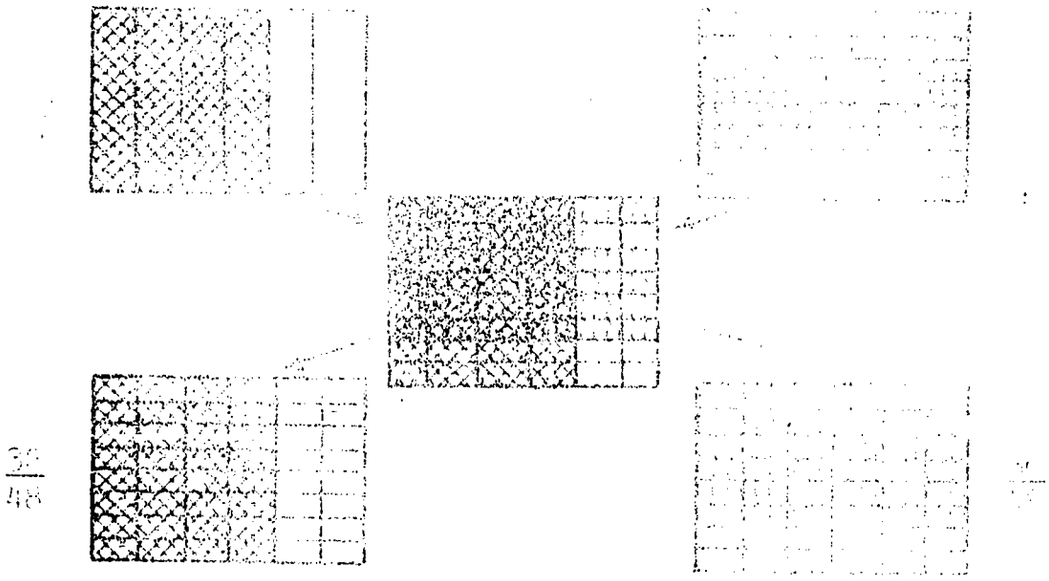
2.



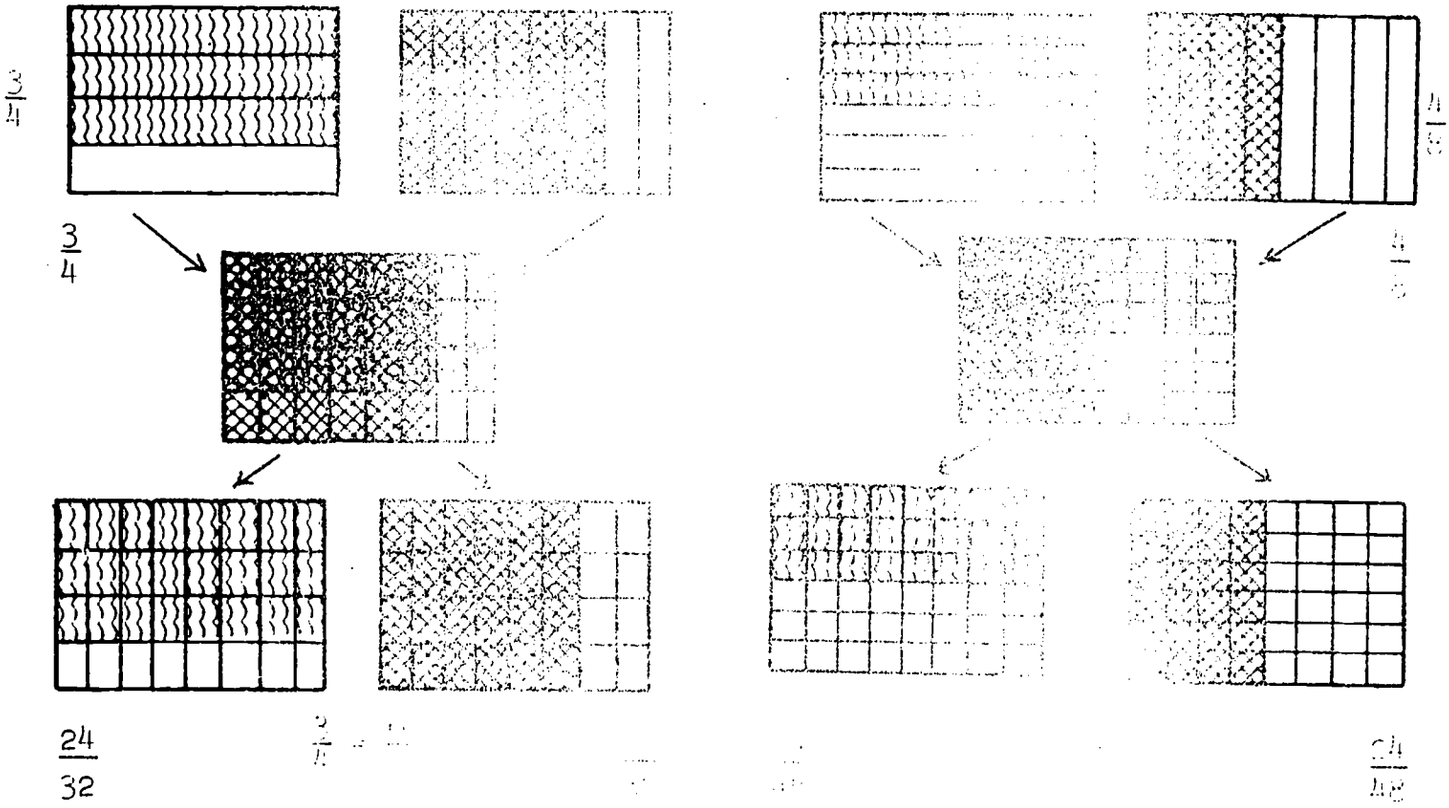
$\frac{2}{3} = \frac{4}{6}$

ANSWERS (CONT'D.)

3.



4.

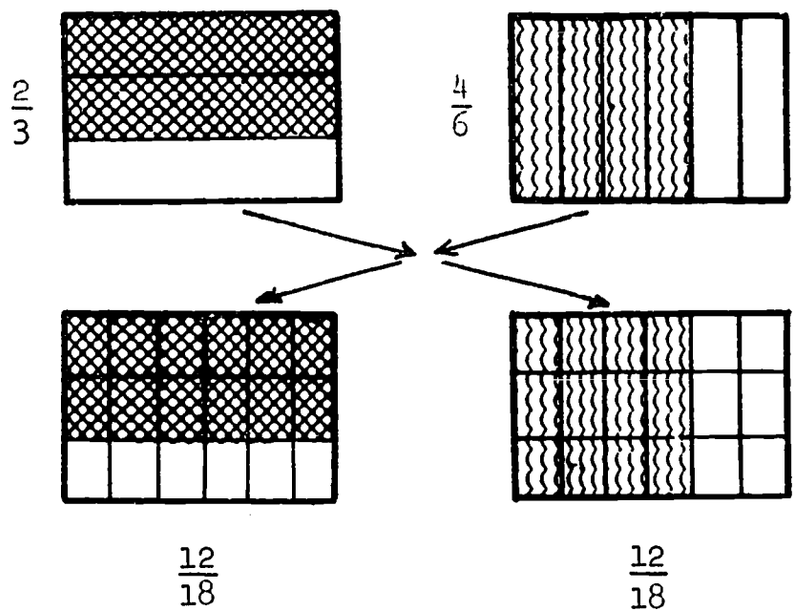


THE CROSS-PRODUCT MODEL

The rectangle model can be used to illustrate the idea of cross-product. The cross-product is a quick way to calculate whether or not a proportion is true.

Example 1:

You found out in Exercises 1 that the proportion $\frac{2}{3} = \frac{4}{6}$ is true. The rectangles looked like this.



The shaded portion of the left hand rectangle is 2 units by 6 units (2×6). The shaded portion of the right hand rectangle is 3 units by 4 units (3×4).

Notice that $2 \times 6 = 12$ and $3 \times 4 = 12$. Twelve is the numerator of both ratios above.

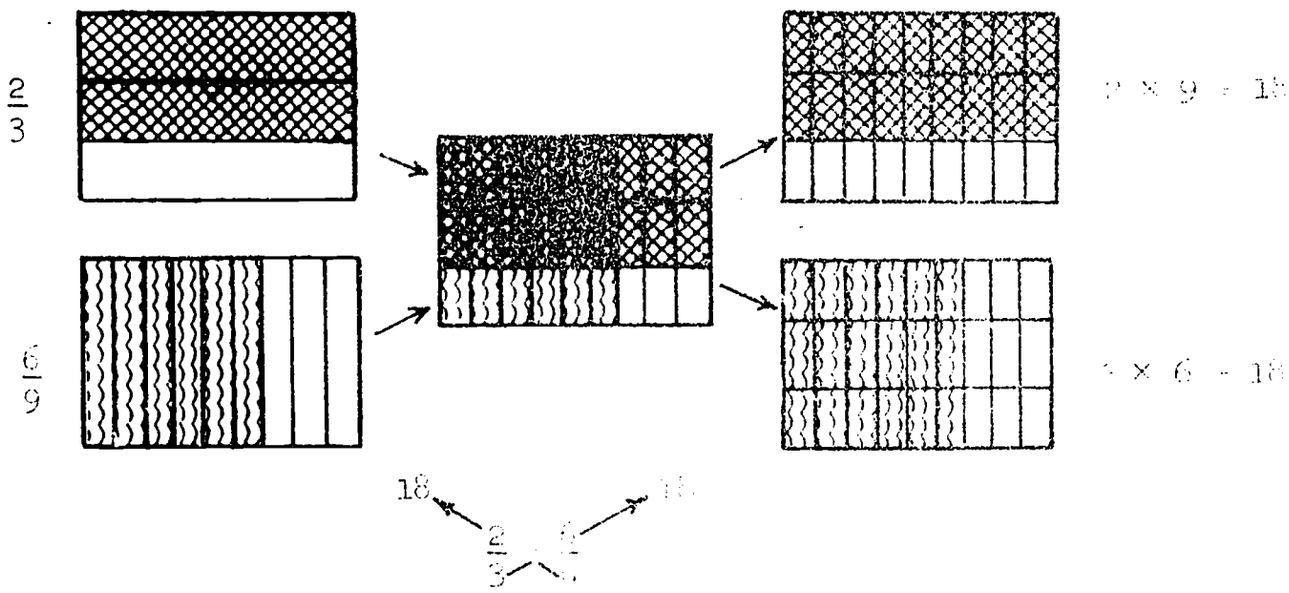
Also notice

$$12 \leftarrow \frac{2}{3} = \frac{4}{6} \rightarrow 12$$

The length and width of the shaded portion of one rectangle. $\frac{2}{3} = \frac{4}{6}$ The length and width of the shaded portion of one rectangle.

THE CROSS-PRODUCT MODEL

After the rectangles have been superimposed and separated again, the number of small regions in each rectangle can be found by multiplying the number of columns by the number of rows.



It will be noted that factors used to find the shaded areas are the same factors used in the cross-product.

Multiplying as shown by the arrows is a quick way to find the numerators of the ratios, which are both $\frac{12}{18}$.

Example 2:

Look at example 1 on page 62. $\frac{2}{6} \stackrel{?}{=} \frac{3}{9}$

$$18 \leftarrow \frac{2}{6} = \frac{3}{9} \rightarrow 18$$

Since both products are 18, the ratios are equivalent and the proportion is true.

18 is the numerator of the ratio $\frac{18}{54}$.

Example 3:

Look at example 3 on page 65. $\frac{2}{3} \stackrel{?}{=} \frac{6}{8}$

$$16 \leftarrow \frac{2}{3} = \frac{6}{8} \rightarrow 18$$

Since the products are unlike, 16 and 18, $\frac{2}{3} \neq \frac{6}{8}$.

16 and 18 are the numerators of $\frac{16}{24}$ and $\frac{18}{24}$.

GIVEN A PROPORTION, IF THE CROSS-PRODUCTS ARE EQUAL, THEN IT IS A TRUE PROPORTION. IF THE CROSS-PRODUCTS ARE NOT EQUAL, THEN THE PROPORTION IS FALSE.

The "cross-product" method can be used to test whether or not a proportion is true. (This means that you can test whether or not two ratios are equivalent.)

EXERCISES 2

Find out if the following proportions are true or false. Circle the true proportions.

1. $\frac{4}{7} = \frac{20}{35}$
2. $\frac{8}{10} = \frac{12}{15}$
3. $\frac{4}{6} = \frac{6}{8}$
4. $\frac{10}{4} = \frac{30}{10}$
5. $\frac{9}{12} = \frac{15}{20}$
6. $\frac{8}{5} = \frac{40}{26}$
7. $\frac{10}{15} = \frac{35}{52}$
8. $\frac{14}{20} = \frac{35}{50}$
9. $\frac{24}{27} = \frac{16}{18}$
10. $\frac{36}{21} = \frac{16}{9}$
11. $\frac{8}{3} = \frac{32}{20}$
12. $\frac{15}{27} = \frac{35}{63}$
13. $\frac{12}{21} = \frac{7}{12}$
14. $\frac{14}{9} = \frac{48}{32}$
15. $\frac{6}{15} = \frac{15}{37.5}$
16. $\frac{14}{8} = \frac{35}{22}$

✓ **POINT**

How is the cross-product used to show whether or not a proportion is true?

ANSWERS TO EXERCISES 2

1. $\frac{4}{7} = \frac{20}{35}$
2. $\frac{8}{10} = \frac{12}{15}$
3. $\frac{4}{6} = \frac{6}{8}$
4. $\frac{10}{4} = \frac{30}{10}$
5. $\frac{9}{12} = \frac{15}{20}$
6. $\frac{8}{5} = \frac{40}{26}$
7. $\frac{10}{15} = \frac{35}{52}$
8. $\frac{14}{20} = \frac{35}{50}$
9. $\frac{24}{27} = \frac{16}{18}$
10. $\frac{36}{21} = \frac{16}{9}$
11. $\frac{8}{3} = \frac{32}{20}$
12. $\frac{15}{27} = \frac{35}{63}$
13. $\frac{12}{21} = \frac{7}{12}$
14. $\frac{14}{9} = \frac{48}{32}$
15. $\frac{6}{15} = \frac{15}{37.5}$
16. $\frac{14}{8} = \frac{35}{22}$

ANSWERS TO ✓POINT

If the cross-products are equal, the proportion is true. If they are not equal, it is a false proportion.

SOLVING PROPORTIONS USING THE CROSS-PRODUCT

Some proportions that come from experiments and other problems contain numbers quite difficult to work with if you do not use the cross-product method for solution.

Example 1:

If a certain rubber ball bounces 6 inches when dropped from a height of 10 inches, how far will it bounce when dropped from 18 inches?

$$\frac{10}{6} = \frac{18}{x}$$

There is no convenient number to multiply by 10 to get 18.

Using the cross-product method:

$$10\,x \qquad\qquad 108$$
$$\frac{10}{6} = \frac{18}{x}$$

$$10 \cdot x = 6 \cdot 18$$
$$10 \cdot x = 108$$
$$x = \frac{108}{10}$$
$$x = 10.8 \text{ inches}$$

The ball bounces 10.8 inches when dropped from 18 inches.

SOLVING PROPORTIONS USING THE CROSS-PRODUCT

The point of the cross-product method is to:

1. Give students facility in testing whether a proportion is true or false (and therefore if two ratios are equivalent or not.)

2. Enable students to solve the many proportionality problems that involve "inconvenient" values.

Example: $\frac{4}{5} = \frac{N}{15}$

If the situation involves proportionality, then the ratios must be equivalent.

Then the products 4×15 and $5 \times N$ must be equal.

If $4 \times 15 = 5 \times N$, then $60 = 5 \times N$ and $N = 12$.

If $60 = 5 \times N$, then $N = 60 \div 5$

An example or two like the one above should provide students the understanding necessary to solve proportions.

Example 2:

Solve for N. $\frac{3}{5} = \frac{N}{32}$

$$5 \cdot N = 3 \cdot 32$$

$$5N = 96$$

$$N = \frac{96}{5}$$

$$N = 19\frac{1}{5} \text{ or } 19.2$$

EXERCISES 3

Solve these proportions for N:

1. $\frac{2}{6} = \frac{N}{15}$
2. $\frac{5}{7} = \frac{N}{20}$
3. $\frac{8}{3} = \frac{N}{20}$
4. $\frac{7}{10} = \frac{16}{N}$
5. $\frac{18}{8} = \frac{N}{3}$
6. $\frac{N}{12} = \frac{10}{18}$
7. $\frac{5}{6} = \frac{N}{20}$
8. $\frac{10}{18} = \frac{45}{N}$
9. $\frac{25}{75} = \frac{N}{300}$
10. $\frac{4}{8} = \frac{N}{10}$
11. $\frac{N}{35} = \frac{40}{100}$
12. $\frac{6}{8} = \frac{N}{75}$
13. $\frac{18}{27} = \frac{54}{N}$
14. $\frac{7}{22} = \frac{24.5}{N}$
15. $\frac{12}{25} = \frac{N}{20.5}$
16. $\frac{6.5}{20} = \frac{N}{110}$

EXERCISES 4

Go back to page 55 and solve the problems completely by solving the proportions you wrote there.

ANSWERS TO EXERCISES 3

1. N = 5	2. N = 14.28	3. N = 53.33	4. N = 22.85
5. N = 6.75	6. N = 6.66	7. N = 16.66	8. N = 81
9. N = 100	10. N = 5	11. N = 14	12. N = 56.25
13. N = 81	14. N = 77	15. N = 9.84	16. N = 35.75

ANSWERS TO EXERCISES 4

(Solutions to proportions written in Exercises 4, page 55.)

1. 384 miles	2. 110 minutes	3. 8.36
4. 24 minutes	5. 70.86 minutes	6. 150 pounds
7. 73.33 ft./sec.	8. 75 mph	9. 34.5 mph
10. 63.25 mph		

HUMAN MEASURES

CLASS ACTIVITY

For this activity you will be working in small groups (3 or 4 students per group).

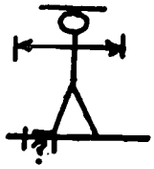
Read over the activity before beginning to do any measuring and check your equipment against the inventory.

Use proportions when making your predictions. This will let you know how well you can use proportions.

Equipment Inventory: 7 feet of brown wrapping paper, 1 meter stick, you, and masking tape.

Activity: The purpose of this activity is to measure humans and find ratios among the measures. Then the ratios are to be used to predict other human measures.

1. Use masking tape to attach the wrapping paper to the wall and floor. Attach it so you can measure your height, arm span, and foot.



HUMAN MEASURES

OBJECTIVES

1. Given an experimental situation, the student shall use ratios, equivalent ratios, proportions, and the solution of proportions to make predictions of additional trials of the experiment.

2. Given the components of a proportion which are based on a problem situation, the student shall form a valid proportion.

EQUIPMENT AND TEACHING AIDS

STUDENT

#1. 1 meter stick

#2. 7 feet of brown wrapping paper (8" wide).

#3. 10 inches masking tape

CONTENT AND APPROACH

Students are to be grouped in threes (4 is a maximum). Each group should go to a station located in convenient places in the room to perform the activity as a group. After the groups complete exercise 4, the activity is to be summarized and completed as a class discussion.

2. Measure to the nearest centimeter your:

   a. height _____
   b. arm span _____
   c. length of foot _____

3. Write the ratios:

   a. comparing length of foot to height. _____

   b. comparing arm span to height. _____

4. Write these ratios in decimal form.

   Example: $\frac{5}{4}$ can be written in decimal form by dividing

$$\begin{array}{r} 1.25 \\ 4\overline{)5.00} \\ \underline{4} \\ 10 \\ \underline{8} \\ 20 \end{array}$$

   a. Length of foot to height. _____

   b. Arm span to height. _____

CLASS DISCUSSION

1. Write on the chalkboard the ratios obtained by the boys in parts 4a and 4b above. How do the ratios compare? (Are they close to the same value?)

2. Write the girls' ratios on the chalkboard. How do they compare?

3. Find the average ratio for the boys' and girls' measurements.

	Ratio	
	Length of foot compared to height	Length of arm span compared to height
Boys		
Girls		

EXERCISES

1. A boy has an arm span of 178 cm. How tall do you think he is?

2. How tall do you think a girl is if her foot measures 23 cm?

3. Measure your teacher's arm span, then predict his or her height. Check your prediction by measuring your teacher's height directly.

CLASS DISCUSSION

After the ratios are listed on the chalkboard, look for variances and discuss why wide variance, if any, did occur. (All tall people in a group, or all short, etc.)

Then find the average ratios for boys and for girls.

Students are to use the averages to do the exercises at the bottom of page 73.

Students could be chosen and measured to see if the ratio applies well to them. In fact, each student could check the ratio against his own measurements to see how well the ratios apply.

✓ POINT

1. Use the cross-product method to test whether or not these proportions are true:

a. $\frac{9}{12} = \frac{15}{20}$ b. $\frac{7}{18} = \frac{21}{52}$ c. $\frac{12}{18} = \frac{16}{24}$

d. $\frac{20}{15} = \frac{16}{12}$ e. $\frac{24}{36} = \frac{16}{20}$ f. $\frac{21}{15} = \frac{27}{17}$

2. Solve the following proportions for N:

a. $\frac{5}{9} = \frac{N}{72}$ b. $\frac{6}{14} = \frac{N}{50}$ c. $\frac{22}{18} = \frac{64}{N}$

d. $\frac{N}{12} = \frac{8}{32}$ e. $\frac{4}{N} = \frac{26}{46}$ f. $\frac{7}{12} = \frac{N}{84}$

3. Write the proportions that would be used to solve the following:

a. A 28-inch bicycle wheel has a circumference of about 88 inches. What would be the circumference of a 20-inch bicycle wheel?

b. In a certain pulley system, a mechanic gives a 24-pound pull to lift a weight of 120 pounds. How much pull would be needed to lift an engine weighing 480 pounds?

c. For a pair of mother and daughter dresses, the girl's pocket measured $3\frac{1}{2}$ inches wide and $4\frac{1}{2}$ inches long. The width of the mother's pocket was $4\frac{1}{2}$ inches. How long was it?

ANSWERS TO ✓POINT

1. a. True b. False c. True
   d. True e. False f. False

2. a. N = 40 b. N = 21.43 c. N = 52.36
   d. N = 3 e. N = 7.08 f. N = 49

3. a. $\frac{28}{88} = \frac{20}{x}$ $x = 62\frac{6}{7}$ inches or 62.9 inches

   b. $\frac{24}{120} = \frac{x}{480}$ x = 96 pounds

   c. $\frac{3\frac{1}{2}}{4\frac{1}{2}} = \frac{4\frac{1}{2}}{x}$ $x = 5\frac{11}{14}$ inches or 5.79 inches

   The pocket would probably be about $5\frac{3}{4}$ inches long. Certainly measurements this precise would not be made.

SPRINGS AND BOLTS

CLASS ACTIVITY

There are some stations around the room where you will perform some experiments involving ratios.

After your teacher divides the class into groups,

1. Go to your station.

2. Follow the instructions for the activity. They begin on pages 76 and 78.

3. Use the equipment inventory to check your equipment.

4. Record your findings in the spaces provided.

When you finish your first activity, go to a station where the other activity is found.

SPRINGS AND BOLTS

OBJECTIVES

1. Given an experimental situation, the student shall use ratios, equivalent ratios, proportions, and the solution of proportions to make predictions of additional trials of the experiment.

2. Given the components of a proportion which are based on a problem situation, the student shall form a valid proportion.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

Stretching Springs -

#1. 1 spring

#2. 1 hook

#3. 1 meter stick

#4. 1 cardboard pointer

#5. 2 known weights (W_1, W_2)

#6. 2 unknown weights (W_3, W_4)

#7. 1 weight (x)

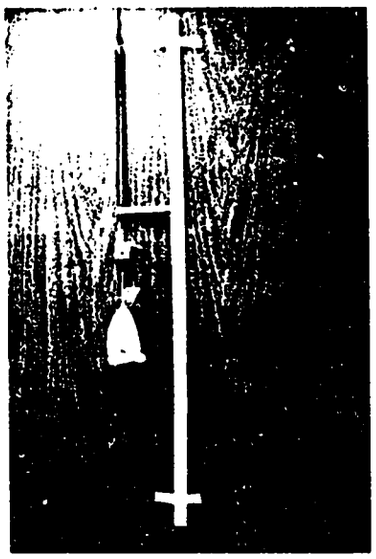
Nuts and Bolts -

#*1. 1 nut and bolt

#*2. 1 block of wood with holes A, B, C.

#*3. 1 strip of tagboard, one inch long

STRETCHING SPRINGS



Equipment Inventory:

1 spring (S), 1 hook,

1 meter stick, 1 cardboard pointer,

2 known weights (W_1, W_2),

2 unknown weights (W_3, W_4),

1 weight (x).

Activity: This activity centers on the question: How can a spring be used to predict the weight of an object?

1. Using the photograph as a guide,

   a. Place the spring on the hook,

   b. Put the meter stick in a position where measurements can be made,

   c. Attach weight x to the spring to provide some tension.

2. Attach W_1 to the spring or the bottom of weight x. How far does the pointer move when weight W_1 is placed on spring, S? _____

B. TEACHER

1. Overhead projector
2. Projection screen
*3. Transparency: ARP 11

ARP 11 is a summary form for discussing the activities.

CONTENT AND APPROACH

1. The students will be working two different experiments during the class period. Therefore some groundwork should be done prior to this time.

2. Each station should be clearly marked with the equipment at the station.

3. As a group finishes one experiment, they are to move to the other experiment when an opening occurs.

4. Your role: Refer to Lesson 4, pages T 37a - T 37h.

   While students are working on their activities, be available to help those having equipment trouble, and answer questions for those in difficulty. (Use a question to answer when possible.)

   Generally: Summarize the activities and capitalize upon the students' findings by applying it to the topics of simplifying ratios, equivalent ratios, and proportions.

   Specifically: Summarize the results by tabulating in a way that the entire class can see. (Overhead, chalkboard, etc.)

   "How do the results of the class compare?"

3. Remove W_1 and then attach W_2 to the spring. What distance does S move when W_2 is used?

4. Write these ratios:

   a. Compare weight W_1 to W_2. _____

   b. Compare the distances moved by W_1 and W_2. _____

5. How do the ratios in part 4 compare?

6. Attach W_3 to the spring and use a proportion to predict the weight of W_3. _____

7. Use W_4 and S to predict the weight of W_4. _____

8. Use a scale to check the weights of W_3 ____ and W_4 _____.

5. After the results of the class have been examined, the discussion should be turned to the topics of simplifying ratios, equivalent ratios, and proportions. (There are more experiments which enable the students to work with these ideas.)

NOTES ON THE ACTIVITIES

STRETCHING SPRINGS

1. Weight x is to be placed on the spring to provide some tension. Then the activity will produce proportional results.

2. The meter stick can be taped to the chalkboard so that the distances stretched can easily be determined.

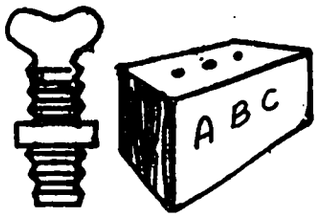
3. The question the student must answer is "How can I use a known weight to predict an unknown weight?"

4. Mathematically, the stretch of a spring is proportional to the weight attached to the spring.

5. The student might be able to recognize that as the weight is doubled, the distance the pointer moves is doubled. (Some students may be able to apply equivalent ratios at this point, but it should not be expected of the majority.)

6. If the student recognizes the principle in item 3, he may be able to verbalize the proportionality concept and apply it to other weights.

NUTS AND BOLTS



Equipment Inventory:

1 bolt and nut, 1 block of wood containing three holes, 1 strip of tagboard one inch long.

Activity: This activity centers around the question:

How can a bolt and nut be used to find the depth of a hole in a block of wood?

1. Use the tagboard strip which is one inch long and count the number of threads in one inch of the bolt. _____

2. Turn the bolt in the nut and count the turns as one inch of bolt passes through the nut. _____

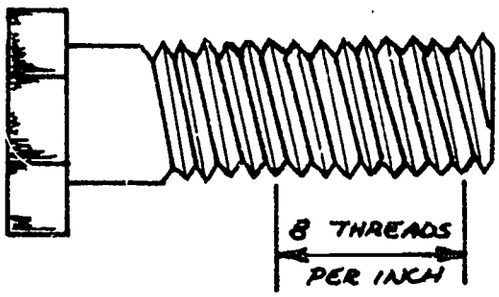
3. How far does the bolt move through the nut in one turn? _____

4. If the bolt is turned 6 times, how far does it move? _____

NUTS AND BOLTS

1. The nut and bolt is a crude version of a micrometer. Its only limitations are that the number of threads per inch is less than a micrometer, the nut does not fit on the bolt as closely as in the micrometer, and it is not graduated (though this could be done).

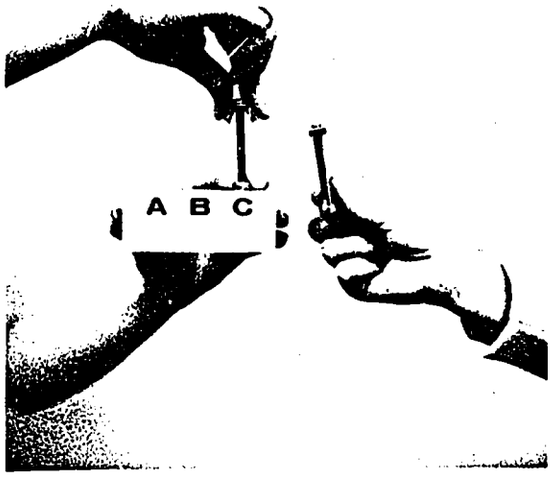
2. The number of threads per inch is calculated this way:



3. For each revolution of the nut on a bolt having N threads per inch, the nut advances $\frac{1}{N}$ of an inch. Therefore it takes N turns of the nut to advance one inch.

4. If the bolt end is started flush with the face of the nut, the bolt will advance through the nut $\frac{1}{N}$ of an inch for each turn of the bolt. This will allow the student to measure the depth of the holes in the block of wood. The student should recognize that the bolt is at the bottom of the hole when the nut begins to lift off the wooden block.

5. Find the depth of holes A, B, C in the wooden block.



a. Start with the end of the bolt even with the nut.

b. Set the nut and bolt over the hole and hold with your fingers.

c. Turn the bolt and count the turns until the bottom of the hole is reached.

Hole	A	B	C
Turns			

d. Calculate how deep the hole is by using the number of turns per inch and writing a proportion.

Hole	A	B	C
depth			

6. A certain screw-type truck jack has 4 threads per inch. How many turns would it take to lift the truck 7 inches off the ground?

7. Look for some information about a micrometer in a mathematics book or an encyclopedia. How are a micrometer and the nut and bolt related in operation?

MAKING COMPARISONS

RATIOS AND COMPARISONS

Proportions are useful when making comparisons of measurements, prices, and performances. One method is to use 100 as a basis for comparison.

Example 1:

During the 1968 baseball season Willie Horton got 146 hits out of 512 at bats while Al Kaline got 94 hits for 327 at bats. A convenient way to compare their performances is to find an equivalent ratio whose denominator is 100 for each player.

$$\frac{146}{512} = \frac{H}{100} \qquad \frac{94}{327} = \frac{K}{100}$$

$$512H = 14600 \qquad 327K = 9400$$

$$H = 28.5 \qquad K = 28.7$$

Their performances were extremely close. Batting at this rate, we could say that Kaline gets 28.7 hits for each 100 times at bat and Horton gets 28.5 hits.

Example 2:

Chef cooking oil comes in two containers like these.

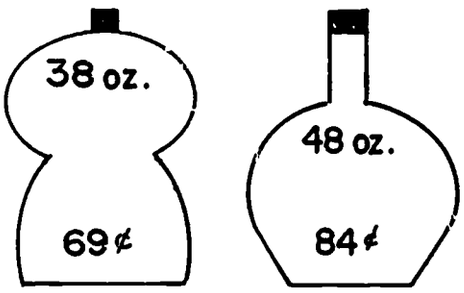
If the oils are of equal quality, which is the better buy?

$$\frac{38}{69} = \frac{X}{100} \qquad \frac{48}{84} = \frac{X}{100}$$

$$69X = 3800 \qquad 84X = 4800$$

$$X = \frac{3800}{69} \qquad X = \frac{4800}{84}$$

$$X = 55.1 \text{ oz. for } 100¢ \qquad X = 57.1 \text{ oz. for } 100¢$$



So the 48 oz. jar is the better buy.

MAKING COMPARISONS

OBJECTIVES

1. The student shall be able to use proportions to solve problem situations such as:

   a. Comparing the costs of two items using 100 as a basis for comparison.

   b. Determining the amount of medicine required to mix 40 oz. of a 35% solution.

   c. Comparing the performances of two athletes, or two automobiles.

2. The student shall be able to interpret rational numbers whose denominators are 100 as per cents. (Examples: $\frac{5}{100} = 5\%$; $\frac{15}{100}$ is 15 per cent)

3. The student shall give examples to show why 25% does not necessarily indicate a base of 100.

EQUIPMENT AND TEACHING AIDS

TEACHER

1. Overhead projector
2. Projection screen

*3. Transparencies: ARP 12A, ARP 12B

ARP 12A illustrates the meaning of a 40% solution as well as solving situations involving mixtures.

ARP 12B illustrates the meaning of 10% off which is often used in advertising.

CONTENT AND APPROACH

RATIOS AND COMPARISONS

It is intended to have students use ratios with 100 as their denominators by involving them in problems naturally using 100. Medicine dosages (given in per cents), athletic performances, and food costs lend themselves to the use of 100 as a basis for comparison.

Suggest to students that they bring in clippings from the newspaper or situations around their home that involve per cent. They should bring items such as a family need or desired purchase, per cent related to the father's (or mother's) occupation, or items of interest to them. Sharing such items with class members may provide opportunity to discuss comparisons.

It must be recognized that numbers other than 100 can be used as a basis for comparison. Example 2 determines the number of ounces of cooking oil that can be purchased for 100 cents. Many people calculate cost per ounce as a basis for comparison. One hundred is used in this lesson to orient the student's thinking in the direction of per cent, and this basis is as valid as any other.

EXERCISES I

The following exercises ask you to compare some information. Use proportions to make the comparisons using $\frac{X}{100}$ as the second ratio for each proportion.

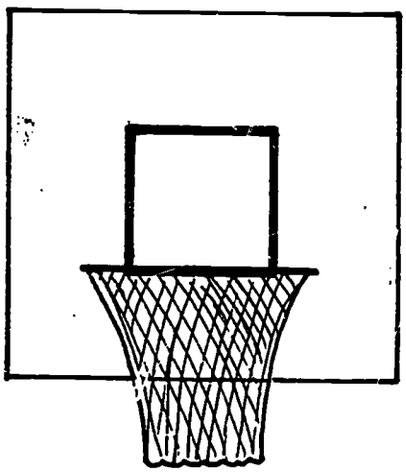
1. An instant coffee called Delight comes in several sizes. As long as spoiling is not a problem, which jar would be the best one to buy?

   A. 14 oz. for $1.79 B. 6 oz. for $.88 (use 179¢ and 88¢)

2. Compare the batting records of the 1968 Detroit Tigers (1292 hits, 5490 at bats) and the 1968 St. Louis Cardinals (1384 hits, 5561 at bats).

3. Among the free throw leaders in professional basketball during 1968 were Cazzie Russell of New York and Jerry West of Los Angeles. Russell made 282 free throws in 349 attempts while West made 391 throws out of 482 tries. Compare their performances.

4. Which would you buy? Delight Coffee: 14 oz. for $1.79, Delight Coffee: 6 oz. for $.88, or Swell Coffee: 10 oz. for $1.19?



5. In January the New York Knicks had won 18 of 47 basketball games. At the same time the Atlanta Hawks had won 17 of 44 games. Use proportions to compare these two records.

THINGS TO DISCUSS

A person does not always buy the item that cost the least. A large quantity may result in spoilage; the cheapest in price may be inferior in quality; an individual may prefer the taste of a particular brand of coffee and may be willing to pay the added expense. All things being equal, the cheapest cost may be the best buy. The point is that the student should be helped to recognize differences in packaging, and that it is the wise buyer who shops on a comparative basis.

ANSWERS TO EXERCISES I

1. Jar A. $\frac{14}{179} = \frac{N}{100}$, N = 7.82 oz. $\frac{6}{88} = \frac{N}{100}$, N = 6.82 oz.

2. Tigers: $\frac{1292}{5490} = \frac{N}{100}$, N = 23.53 hits

   Cardinals: $\frac{1384}{5561} = \frac{N}{100}$, N = 24.89 hits

3. Russell: $\frac{282}{349} = \frac{N}{100}$, N = 80.80 free throws

   West: $\frac{391}{482} = \frac{N}{100}$, N = 81.12 free throws

4. Swell: $\frac{10}{119} = \frac{N}{100}$, N = 8.40 oz.

   Delight: 14 oz. N = 7.82 oz.

   6 oz. N = 6.82 oz.

5. Knicks: $\frac{18}{47} = \frac{N}{100}$, N = 38.30 wins

   Hawks: $\frac{17}{44} = \frac{N}{100}$, N = 38.64 wins

RATIOS AND MIXTURES

Nurses and doctors use proportions when preparing medicines. The chemical or drug is not generally used full-strength. Usually a certain amount of drug is mixed with a given amount of liquid to make a solution.

If an alcohol solution is made up of 70 parts alcohol and 30 parts water, the entire solution contains 100 parts. $\frac{70}{100}$ represents the fractional part of the solution that is alcohol.

Example:

Patients may need different amounts of alcohol solutions. These different amounts still have the same strength, $\frac{70}{100}$. A patient needs 30 oz. of alcohol solution. How many ounces of alcohol should the nurse use?

$$\frac{70}{100} = \frac{X}{30}$$

$$100X = 2100$$

$X = 21$ ounces of alcohol is to be used. Then 9 oz. of water will be added. $(30 - 21 = 9)$

RATIOS AND MIXTURES

Although solutions are generally referred to in per cent language (70% solution), the ratio is used to emphasize 100 as the basis for comparison. On page 84, the ratios are interpreted as per cents. This is continued henceforth.

Paragraph two on page 82, "If an alcohol..." should be discussed carefully. A $\frac{70}{100}$ solution is one containing 70 parts solute and 30 parts solvent. A $\frac{70}{100}$ alcohol solution contains 70 parts alcohol for each 100 parts of solution (which means that the solvent is 30 parts of the solution.)

Discuss the example on page 82 fully with the students. Other examples may be necessary to insure that the ideas on this page are understood and that the students will be able to do the exercises on page 83.

Students could be asked to bring in problems oriented toward occupations. Their parents may be able to suggest problems or problems might be found in the newspaper.

EXERCISES 2

1. Twelve ounces of a $\frac{25}{100}$ solution of glycerin is to be prepared for a patient. The ratio $\frac{25}{100}$ means that 25 ounces of glycerin is to be used out of every 100 ounces of the solution. (Then 75 oz. of water will be used.) Since only 12 oz. of solution is needed, how much glycerin is required ?

$\frac{25}{100} = \frac{X}{12}$ Finish the problem.

2. How many ounces of boric acid are in 40 oz. of solution if the strength of the solution is $\frac{1}{500}$?

3. If you have 100 oz. of a $\frac{25}{100}$ solution of glycerin,
   a) how many ounces would be glycerin?
   b) how many ounces would be water?

4. If a solution contains 2 parts of a drug and 3 parts of water,

   a) what ratio represents the strength of the solution?
   b) how many ounces of drug would be contained in 100 oz. of solution?

5. Another boric acid solution is stronger, $\frac{55}{100}$.
   a) Out of every 100 oz. of solution, how many oz. of boric acid would be needed?
   b) How many ounces of boric acid would be needed to make 80 oz. of solution?

ANSWERS TO EXERCISES 2

1. X = 3 oz.

2. X = .08 oz.

3. a. 25 oz.

   b. 75 oz.

4. a. $\frac{2}{5}$ (refers to phrase: "For every ___ there are ___.")

   b. 40 oz.

5. a. 55 oz.

   b. 44 oz.

USING PER CENT

If a nurse knows the strength that is needed for a certain solution, she can use proportions to figure out the number of ounces of drug needed for any given amount of the solution. Using ratios such as $\frac{25}{100}$ or $\frac{55}{100}$ is common when comparing the amount of drug used to the amount of solution being prepared.

The ratio $\frac{55}{100}$ can be read "55 out of 100" or "55 of each hundred". It means that the drug makes up 55 out of 100 parts of the solution. You will recall the special name "per cent" that was used for this kind of comparison.

PER CENT IS A CONVENIENT METHOD OF MAKING COMPARISONS, AS WELL AS GIVING DIRECTIONS FOR MIXING INGREDIENTS (SUCH AS MEDICINES).

A boric acid solution which has a strength of $\frac{55}{100}$ is said to be a 55% solution. The symbol % represents $\frac{\ }{100}$. Remember: Per cent (%) means "for each hundred". A 55% boric acid solution means 55 oz. of boric acid for each 100 oz. of solution.

USING PER CENT

At this point the transition is made from the ratio $\frac{N}{100}$ to N%. This is not the first time the student has done this, but it is unwise to assume too much.

The example problem on page 85 should be discussed in detail so that the relationship between the two numerators (and two denominators) is understood.

The ratios used in per cent are similar to those in previous lessons which involved proportions. Proportions are still being used and the correspondences pointed out in Lesson 7 are still important.

The statement 55% of 375 emphasizes the phrase:

"___ out of ___ are ___".

55% of 375 is: 55 out of 100
55 out of 100
55 out of 100
☐ out of 75

adding: 165 + ☐ out of 375

Therefore the answer is more than 165.

If a coat is on sale for 10% off the regular price, it would sell for 10¢ off out of every 100¢ the coat cost ($\frac{10}{100}$). A coat with a regular price of $25.00 (2500¢) would sell for $22.50.

10% off

actual number of cents off

$$\frac{10}{100} = \frac{X}{2500}$$

out of each 100¢

of the 2500¢ regular price.

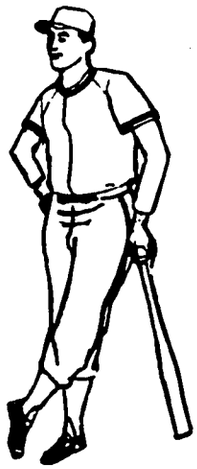
$100X = 25000$

$X = 250$ cents off regular price

2500¢ - 250¢ = 2250¢ or $22.50 (the sale price).

The symbol, %, is used mainly when writing about per cent and in advertising signs. When you use per cent in solving problems the fraction form is used. In calculating, 25% would be written as $\frac{25}{100}$.

AN IMPORTANT IDEA FOR YOU TO UNDERSTAND IS THAT 30% OR $\frac{30}{100}$ DOES NOT MEAN 100 WAS EXACTLY THE NUMBER OF TIMES AN EVENT TOOK PLACE OR THAN AN ITEM WAS ACTUALLY DIVIDED INTO 100 PARTS..



For instance, if a batter hits at a 30% pace ($\frac{30}{100}$) it does not necessarily mean that he was at bat 100 times. 100 is used because it is a <u>convenient</u> number to compare to and EVERYBODY does it! If a batter gets 6 hits for 20 tries at bat, he also hits 30%.

$$\frac{6}{20} = \frac{X}{100}$$

$$20X = 600$$

$$X = 30 \qquad \frac{30}{100} = 30\%$$

The idea can be illustrated another way. A batter hits safely 5 out of 12 times. What per cent did he hit? This question is asking: "How many hits would he get out of 100 at bats?

$$\frac{5}{12} = \frac{X}{100}$$

$$12X = 500$$

$$X = 41.7$$

41.7 hits out of 100 times at bat is 41.7%. Well! You know that you can't get part of a hit, so he must not be able to be at bat 100 times and <u>still</u> have a batting percentage of 41.7%. Yet, this is still a good way to compare one batter to another. How is a batter's performance reported in the newspaper?

It is important to discuss the idea that when per cents are given, the actual values of the relationship (sports data, clothing sales, etc.) are lost and per cent just tells <u>how</u> the items are related.

Sometimes it is obvious that 100 is not the base. For example, a batting "average" of .417 (41.7%).

(A batting average is not an average, but is a per cent if it is considered in the form 41.7% $\left(\frac{41.7}{100}\right)$. Newspaper reports give the performance as .417. Many people consider it in ratio form, $\frac{417}{1000}$, or in the per millage form of 417 ‰.)

A poll might report that 80% of a group of people reacted in a certain way. It should be recognized that 4 out of 5 people represent 80%.

The discussion on page 86 and the exercises on page 87 attempt to point out the limitations of reports based on per cents alone.

OTHER SUGGESTIONS

1. After reading the heavy black print at the top of page 80, tests can be used as an example.

   > Given 25 questions and 80% was the score, this would not mean that a person got 80 correct out of 100, rather 80 <u>if</u> 100 were the number of questions.

2. It might help the validity of the discussion on page 86 (as far as the students are concerned) if you discussed how batting averages are reported in the newspaper.

EXERCISES 3

Write the following ratios as per cents using the symbol, %.

1. $\frac{38}{100}$ 2. $\frac{25}{100}$ 3. $\frac{87}{100}$ 4. $\frac{65}{100}$ 5. $\frac{22\frac{1}{2}}{100}$ 6. $\frac{3}{100}$

Write these per cents as ratios.

7. 23% 8. 45% 9. 50% 10. 76% 11. 99% 12. 7%

13. How many ounces of glycerin would be added to 80 oz. of water to get a $\frac{20}{100}$ or 20% glycerin solution?

14. If a medicine is a 25% solution, how many oz. of drug is used to make 24 oz. of solution?

15. A solution is a 25% solution. Does this mean that you have 100 oz. of the solution? Explain.

DISCUSSION QUESTIONS

1. The basketball team made 65% of its free throw attempts.
   a. Write the per cent as a ratio.
   b. Discuss the question: How many free throws did the team attempt?
2. The basketball team had a field goal shooting record of 42.5%. Discuss the number of field goals made by the basketball team.
3. Why is 100 often used as a basis for comparing two situations, such as prices or athletic performances?
4. When 100 is used as a basis for comparisons, sometimes the result is half a person or half a time at bat. If this happens, how can 100 be used for comparison?

ANSWERS TO EXERCISES 3

1. 38% 2. 25% 3. 87% 4. 65% 5. $22\frac{1}{2}\%$ 6. 3%

7. $\frac{23}{100}$ 8. $\frac{45}{100}$ 9. $\frac{50}{100}$ 10. $\frac{76}{100}$ 11. $\frac{99}{100}$ 12. $\frac{7}{100}$

13. 20 oz.

14. $\frac{25}{100} = \frac{N}{24}$, N = 6 oz.

15. No. The ratio of number of ounces of drug to the ounces of solution is $\frac{25}{100}$. Any equivalent ratio could be used to describe the solution, such as $\frac{5}{20}$.

ANSWERS TO DISCUSSION QUESTIONS

1. a. $\frac{65}{100}$

   b. It cannot be determined how many free throws were attempted. It could have been $20\left(\frac{13}{20} = \frac{65}{100}\right)$ or $40\left(\frac{26}{40} = \frac{65}{100}\right)$.

2. $42.5\% = \frac{42.5}{100}$. It is certain that the team did not make 42.5 field goals. The number of field goals could have been $85\left(\frac{85}{200} = \frac{42.5}{100}\right)$ or $17\left(\frac{17}{40} = \frac{42.5}{100}\right)$.

3. 100 is a convenient number to multiply and divide by. 100 has numerous factors. When one number is used, there is a <u>basis</u> for comparison and its universal acceptance makes communication easier.

4. The point is that a comparison is being made using one number as the basis. 100 will still indicate <u>which</u> performance is best, and it is only a theoretical value.

STRETCHING RUBBER BANDS

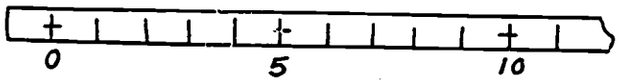
It is helpful to be able to estimate per cents such as the following: 75% of a line segment, 30% of a group of students, 25% of a length of cloth, etc. To help you see per cent, you are going to make a per cent calculator and work with line segments.

CLASS ACTIVITY

A rubber band can be made into a per cent calculator. With this device you can find per cents of line segments and be able to see what they are.

1. To make the calculator you will need a piece of rubber band.

2. Choose a partner. Stretch the rubber band the length of the ruler which is printed along the edge of this page. Then have your partner put a pencil mark on the rubber band every half inch.



3. When you let the rubber band relax, there is a set of evenly spaced marks. They will stay evenly spaced for any length the rubber band is stretched.

   Let's try it. Mark off 30% of the segment drawn below.

Here's how.

a. Stretch the rubber band next to the segment until it is divided into 10 units of length. (Each unit represents 10%, and there will be some marks on the rubber band that will not be used.)

STRETCHING RUBBER BANDS

OBJECTIVES

The student shall be able to

1. Estimate a whole number per cent of a given line segment.

2. Write a per cent in its various forms. (example: 50%, $\frac{50}{100}$, .50, $\frac{1}{2}$)

EQUIPMENT AND TEACHING AIDS

STUDENT

Rubber band, $\frac{1}{2}$ per student. Size 31: $\frac{1}{8}'' \times 2\frac{1}{2}''$

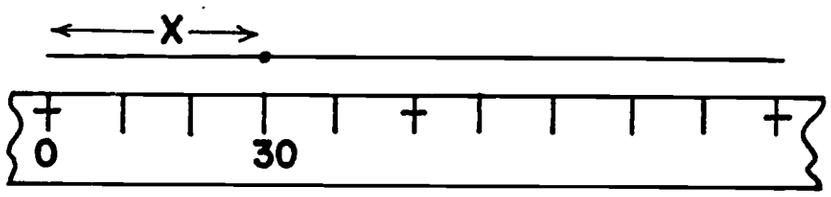
CONTENT AND APPROACH

PER CENT CALCULATOR

The per cent calculator is a device that enables the student to determine various percentages without computation. The calculator will be applied to line segments which will give the student a picture of various percentages. As a result, they can <u>see</u> how the per cents are related and possibly gain in their "feeling" of this concept.

The per cent calculator is made by stretching a rubber band and placing evenly spaced marks on it. These marks will remain evenly spaced over any stretching, thus segments of various lengths can be divided into percentages limited only by the graduations on the rubber band.

b. Place a dot on the segment after the third unit which will be the fourth mark on the rubber band.



c. The x on the segment indicates that the left hand part of the segment is 30% of the entire length.

4. Imagine each space having 10 smaller divisions. Then each small space represents 1 out of 100 or 1%. The fourth mark (at the end of the third unit) would then represent 30 out of 100 parts or 30%.

EXERCISES I

1. Place a mark on each segment so that the left hand part represents the given per cent.

a. 50% _____

b. 40% _____

c. 60% _____

d. 25% _____

e. 75% _____

f. 85% _____

g. 90% _____

h. 100% _____

i. 125% _____

Students should work in pairs. One person cannot mark the rubber band or indicate percentages on segments because stretching of the rubber band is required.

Give each student one-half of a rubber band. Each member of the pair will have a marked rubber band.

ANSWERS TO EXERCISES I

1. The segments will be marked using the per cent calculator. The answers given below should not be treated as the "exact" answers; rather a "reasonable" amount of error is to be allowed.

   a. 50%

   b. 40%

   c. 60%

   d. 25%

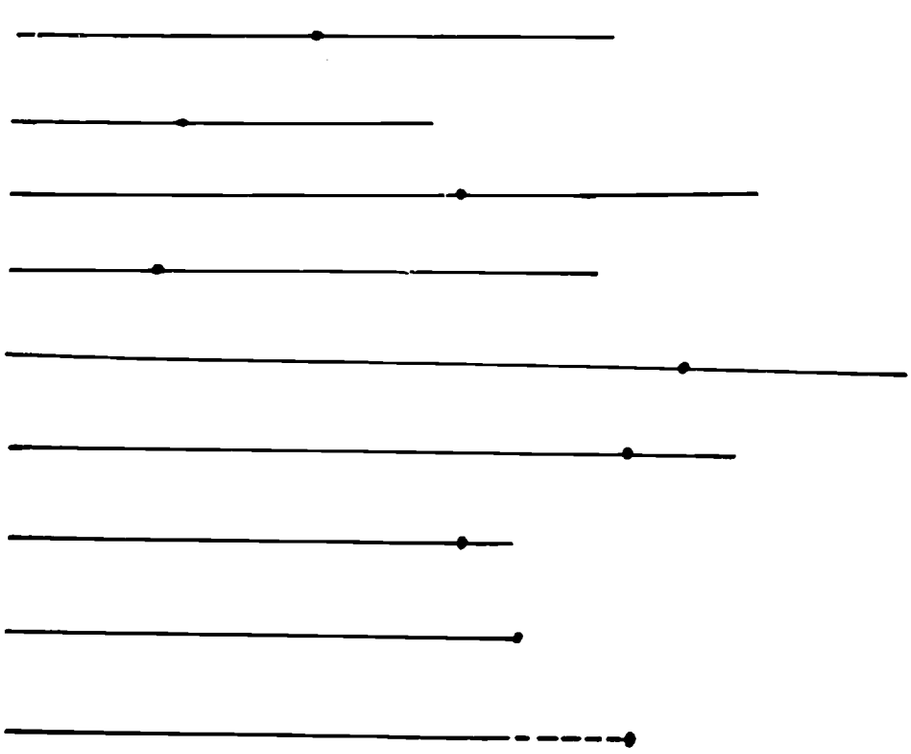
   e. 75%

   f. 85%

   g. 90%

   h. 100%

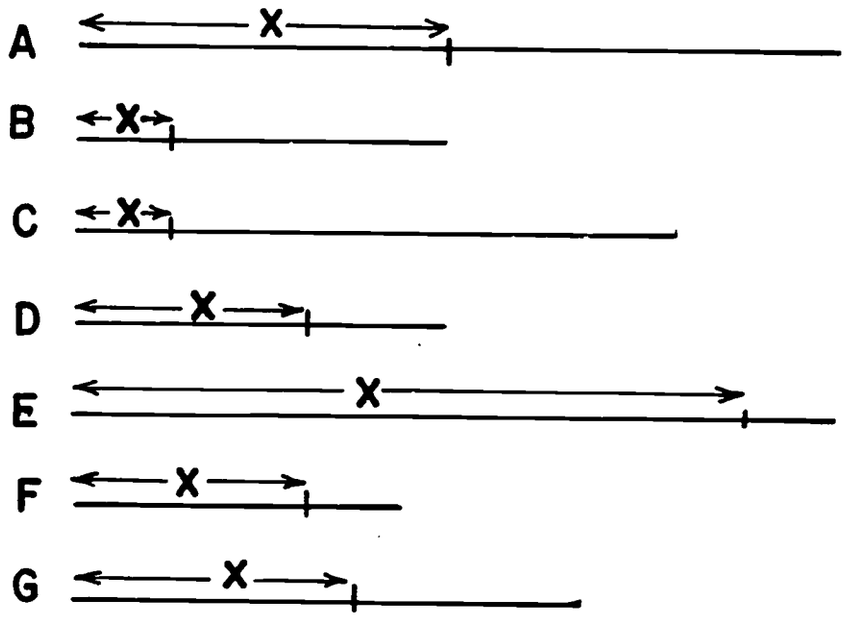
   i. 125%



2. (I) Estimate the per cent represented by the part of the segment indicated by |←—— x ——→|.

(II) Check your estimate using the rubber band.

	A	B	C	D	E	F	G
I ESTIMATE							
II CHECK							



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ANSWERS (CONT'D.)

2. (I) All estimates are acceptable. Students whose estimates are not very close should be given help in improving their estimation ability.

(II)

	A	B	C	D	E	F	G
I ESTIMATE*							
II CHECK	50%	25%	15%	60%	90%	70%	55%

* Answers will vary.

There are several ways in which a particular per cent can be written. Sometimes one particular form is easier to work with than others.

You know that 25% can be written as $\frac{25}{100}$.

1. If the ratio is simplified you get $\frac{25}{100} = \frac{1}{4}$. So $\frac{1}{4}$ of \$20 is the same as 25% of \$20 which is \$5.

2. To change a fraction into per cent form, use a proportion. For instance $\frac{2}{5} = \frac{x}{100}$, $x = 40$ so $\frac{2}{5} = \frac{40}{100} = 40\%$.

3. Per cent can be represented in decimal form.
$$\frac{25}{100} = 25 \div 100 = .25$$
A simpler way is to read $\frac{25}{100}$ as "twenty-five hundredths" and then write it in decimal form as .25.

4. The forms shown here are equivalent:
$$\frac{40}{100},\ 40\%,\ .40,\ \frac{2}{5}$$

EXERCISES 2

Complete the table by writing the various forms which represent the given per cents.

Per Cent form	50%	60%			20%		
Ratio form	$\frac{50}{100}$		$\frac{80}{100}$				
Decimal form	.50					.05	
Simplified fraction form	$\frac{1}{2}$			$\frac{3}{4}$			$\frac{1}{8}$

More examples than those provided on page 91 should be presented so the students have the opportunity to work with equivalent forms of per cent prior to Exercises 2.

ANSWERS TO EXERCISES 2

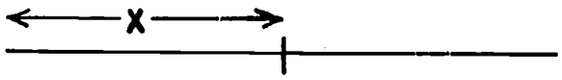
Per Cent form	50%	60%	80%	75%	20%	5%	12.5%
Ratio form	$\frac{50}{100}$	$\frac{60}{100}$	$\frac{80}{100}$	$\frac{75}{100}$	$\frac{20}{100}$	$\frac{5}{100}$	$\frac{12.5}{100}$
Decimal form	.50	.60	.80	.75	.20	.05	.125
Simplified fraction form	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{20}$	$\frac{1}{8}$

Some people, when estimating sizes or lengths, think of simplified fractions rather than forms such as decimal or per cent.

When estimating 25% of the length of a segment, a person would most likely think of $\frac{1}{4}$ of the segment (if only for the reason that they have worked more with fractions).

EXERCISES 3

Estimate the per cent or fractional part of the segment by placing a mark that distance from the left end. Indicate the part with an ←x→. Check your estimate using the rubber band.

Example: 50% 

75% _____

25% _____

40% _____

90% _____

3/4 _____

1/3 _____

30% _____

1/4 _____

80% _____

The students are supposed to do exercises 3 without aid of the rubber band. An estimation is asked for so answers can hardly be incorrect.

Students can best check their ability to estimate by comparing their estimates with others in the class.

ANSWERS TO EXERCISES 3

All estimates are acceptable. This exercise should be used to help students improve their ability to estimate.

You may wish to make a transparency of rather precise estimates so that the students can see how close they are. As you circulate around the room, you could lay the transparency on top of their work. An alternative to the transparency is the use of tracing paper. It is thin enough to see the students work through it.

ESTIMATION AND PER CENT

SALES, ESTIMATES, AND SHOES

A sale on shoes! 15% off. Stores often advertise sales in this way because it catches your eye. Some people will buy items even if they do not need them because the idea of "money off" impresses them. Estimation can help a person decide if the amount off is <u>really</u> enough to make purchase worthwhile.

Estimates of per cents can be made more easily if you have some guide for making the estimation. Per cents written in fraction form can be very helpful in estimating because you are quite familiar with fractional parts of numbers. For instance:

$50\% = \frac{50}{100} = \frac{1}{2}$. So 50% of \$25 is $\frac{1}{2} \cdot \$25$ or \$12.50.

ESTIMATION AND PER CENT

OBJECTIVES

The student shall be able to

1. Write a per cent as a simplified ratio.
2. Estimate a whole number per cent of a given number.

CONTENT AND APPROACH

The ability to estimate percentages easily is an asset because:

1. some practical situations (e.g. purchases) can be readily apprized.
2. it provides an opportunity for checking the reasonableness of answers.

Where it is convenient, a per cent is rewritten as a simple fraction and the estimation is then made. An extension of this application is to calculate a per cent as a multiple of another per cent. For example, 15% is 3 times 5% so $15\% = \frac{3}{20}$ because $3(5\%) = 3(\frac{1}{20})$.

A second method for estimating is to average two per cents. 15% is the average of 10% and 20%. These two per cents are easily obtained: $10\% = \frac{1}{10}$ and $20\% = \frac{1}{5}$.

After obtaining the fractional parts of the given number, their average is found.

Example: 15% of 30 is $4\frac{1}{2}$.

$10\% \ (\frac{1}{10})$ of $30 = 3$, $20\% \ (\frac{1}{5})$ of $30 = 6$, $\frac{3+6}{2} = 4\frac{1}{2}$

EXERCISES I

For each per cent, write a simplified fractional number.

Per Cent	Ratio: $\frac{N}{100}$	Simplified Ratio
50%	$\frac{50}{100}$	$\frac{1}{2}$
75%		
25%		
20%		
40%		
60%		
10%		
30%		

PER CENTS AS SIMPLIFIED RATIOS

As a simplified ratio, 20% = $\frac{1}{5}$ because $\frac{20}{100} = \frac{1}{5}$.

Using 20% as $\frac{1}{5}$, 20% of \$25.00 is \$5.00 because $\frac{1}{5} \times 25 = 5$.

It is not necessary to memorize all of the per cents in a table to use them. Compare 20%, 40%, 60%, and their corresponding ratios $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{3}{5}$. Note that the per cents are multiples of 20 and the ratios are multiples of $\frac{1}{5}$. If you know that 20% = $\frac{1}{5}$, then 60% = $\frac{3}{5}$ because $20 \times 3 = 60$ and $\frac{1}{5} \times 3 = \frac{3}{5}$.

A third method for obtaining 15% is to apply the distributive property. 15% = 10% + 5%, so 15% of $12.00 is 12(.10 + .05) = $1.20 + $.60 or $1.80. Actually a person could find 10% of $12.00 and take half of $1.20 or $.60 for then the sum of these two values is 15% of $12.00. This is an easy way to calculate a tip at a restaurant.

Estimation is helpful in correctly placing decimal points in percentage problems. Estimates are easy because they are usually whole numbers, and after calculations are made the estimate will determine the decimal point position. For example: 23% of 78

Estimate: 20% of 80 or $\frac{1}{5}$ of 80 = 16.

Calculation: 1794 are the digits obtained. The estimate locates the decimal point (17.94) in the proper place.

ANSWERS TO EXERCISES I

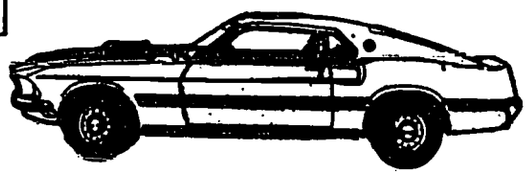
Per Cent	Ratio: $\frac{N}{100}$	Simplified Ratio
50%	$\frac{50}{100}$	$\frac{1}{2}$
75%	$\frac{75}{100}$	$\frac{3}{4}$
25%	$\frac{25}{100}$	$\frac{1}{4}$
20%	$\frac{20}{100}$	$\frac{1}{5}$
40%	$\frac{40}{100}$	$\frac{2}{5}$
60%	$\frac{60}{100}$	$\frac{3}{5}$
10%	$\frac{10}{100}$	$\frac{1}{10}$
30%	$\frac{30}{100}$	$\frac{3}{10}$

EXERCISES 2

Complete the table by applying the previous discussion.

Per Cent	Simplified Ratio
5%	
10%	
20%	
15%	
25%	
30%	
70%	
40%	
80%	

AUTO LOAN
8% INTEREST



ANSWERS TO EXERCISES 2

Per Cent	Simplified Ratio
5%	$\frac{1}{20}$
10%	$\frac{1}{10}$
20%	$\frac{1}{5}$
15%	$\frac{3}{20}$
25%	$\frac{1}{4}$
30%	$\frac{3}{10}$
70%	$\frac{7}{10}$
40%	$\frac{2}{5}$
80%	$\frac{4}{5}$

ESTIMATION...

It is frequently enough to estimate a per cent of a given number. When a sale is advertised, an estimation of the amount of savings is all that is necessary to decide if you want to buy the item or not.

In estimating, the symbol $\approx$ is used and means "approximately the same as".

Example: Estimate 15% of \$25.00.

Solution 1: $5\% = \frac{1}{20}$, so $15\% = \frac{3}{20}$

$\frac{1}{20}$ of 25 is $\frac{1}{20} \times 25 \approx 1$,

so $\frac{3}{20}$ of $25 \approx 3 \times 1 = \$3.00$

Solution 2: 15% is midway between 10% and 20% .

$10\% = \frac{1}{10}$ so $\frac{1}{10} \times 25 = \$2.50$

$20\% = \frac{1}{5}$ so $\frac{1}{5} \times 25 = \$5.00$

This means that 15% of \$25.00 is midway between \$2.50 and \$5.00 which is about \$3.50 . (\$3.75 to be exact.)

Solution 3: Find 10% of \$25.00 and then add half of the amount (5%) to it.
10% of \$25.00 = \$2.50, $\frac{1}{2}$ of \$2.50 = \$1.25.
\$2.50 + \$1.25 = \$3.75.

ESTIMATION . . .

The spirit in which estimates are handled should be this. An estimate is not incorrect, it just happens that some are better than others. A student would be able to see how he ranks with his classmates in estimating ability if all the estimates for a particular number were listed on the board in order of size so that those estimating close to one another will cluster. Students who make extremely high or low estimates will be able to see that they are "away from the pack". This way no student has to be embarassed by having his lack of estimating ability stated in front of the class. You can help these individuals improve their estimating ability.

A way to help students avoid embarassment is to have them write their estimates on a piece of paper and pass to another person. If, in turn, the estimates are passed again, a particular student's estimate will not be apparent.

EXERCISES 3

Estimate the following per cent of the given number.

Given Number	Per Cent	Estimated Amount
18	50%	
24	75%	
35	20%	
20	40%	
70	10%	
90	30%	
120	60%	
120	25%	
80	45%	

Another good use of estimation and per cent is to check whether or not an answer is reasonable.

ANSWERS TO EXERCISES 3

The answers given are not estimates. They are exact results. Answers given by students should be viewed in the spirit of estimation as indicated previously.

Given Number	Per Cent	Estimated Amount
18	50%	9
24	75%	18
35	20%	7
20	40%	8
70	10%	7
90	30%	27
120	60%	72
120	25%	30
80	45%	36

At this point the student's understanding of per cent can be checked. Some students might report the Estimated Amount (percentage) as $\frac{9}{100}$ or state that the 9 represents 9%. Careful observation of student results may do a great deal toward helping a student.

Example: A \$120.00 chair is on sale for 30% off the regular price. What is the amount of savings when buying the chair on sale?

Estimating the amount of savings:

30% is close to 25%

$25\% = \frac{1}{4}$

$\frac{1}{4}$ of \$120.00 is \$30.00

The savings is a little more than \$30.00

Calculate the actual amount of savings:

$$\frac{30}{100} = \frac{X}{120}$$

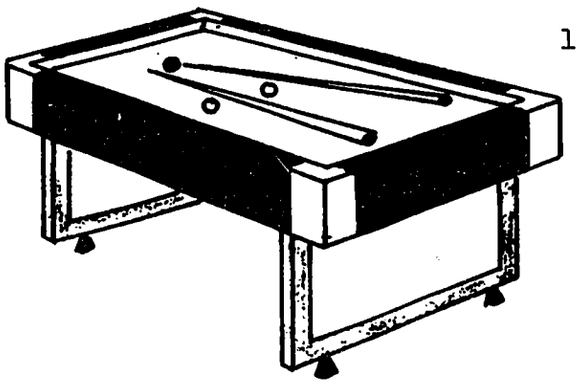
$$100X = 3600$$

$$X = \$36.00 \text{ in savings}$$

If you get an answer of \$3.60 or \$360.00 your estimate would tell you that these are incorrect.

EXERCISES 4

For the following problems, <u>estimate only</u> the answers. Do not actually calculate the correct answer.



1. A pool table regularly selling for \$79.95 is advertised on sale for 44% off. What is the amount of savings on this table?

This example demonstrates how an estimate can be used to check the reasonableness of an answer.

In addition, the estimate can give guidelines for decimal point placement. This method could be used rather than the traditional rules for positioning decimal points.

ANSWERS TO EXERCISES 4

1. $\$79.95 \approx \$80.00$

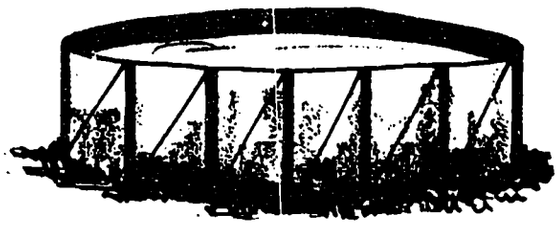
$44\% \approx 40\% = \frac{2}{5}$ $\quad \frac{2}{5}$ of $\$80.00 = \$32.00$

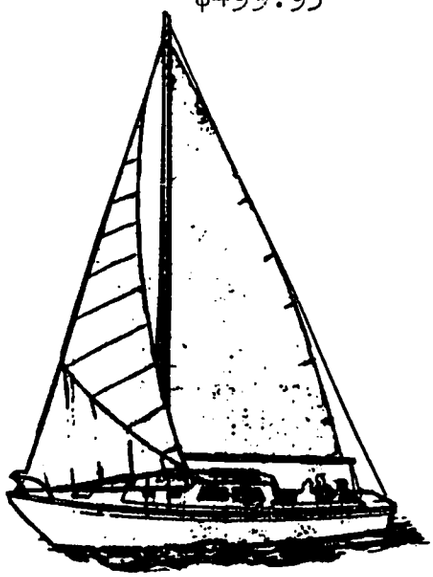
or

$44\% \approx 50\% = \frac{1}{2}$ $\quad \frac{1}{2}$ of $\$80.00 = \$40.00$

2. How much less does a pool table sell for if it regularly costs $695.00 and is reduced in price by 36% ?

3. These swimming pools are selling for 26% off. How much is saved?
   a) 20 ft. pool, regularly $399.95
   b) 24 ft. pool, regularly $499.95





The speeds of cars and boats cannot be compared directly. Boat speeds are reported in knots while car speeds are in miles per hour.

The way boat owners convert their boat speeds to mph is to use this formula: (Number of knots) + (15% of the number of knots) = mph.

So a boat traveling 20 knots is going 23 mph. (20 knots) + (15% of 20) = 20 + 3 = 23 mph.

15% of 20: 10% of 20 = 2, 5% of 20 = 1,
then 15% of 20 = 3

ANSWERS (CONT'D.)

2. $36\% \approx 40\% = \frac{2}{5}$ $\qquad$ $\frac{2}{5}$ of \$700.00 = \$280.00

   \$695.00 ≈ \$700.00

3. a. $26\% \approx 25\% = \frac{1}{4}$ $\qquad$ $\frac{1}{4}$ of \$400.00 = \$100.00

   \$399.95 ≈ \$400.00

   b. \$499.95 ≈ \$500.00 $\qquad$ $\frac{1}{4}$ of \$500.00 = \$125.00

4. a. Find in mph the speed of a boat traveling 30 knots.

   b. How many mph does a speed of 50 knots represent?

5. 

Coats! 20% off. How much would you save?

a) Fur-trim coats: regularly $68.00 and $109.00

b) All-weather coats: regularly $14.99 and $24.99

6. 

How much is saved on a one-day sale of ladies' diamond ring sets if they regularly sell for $310.00 and are advertised for 40% off?

7. A rayon instant wardrobe! A coat plus two coordinating shift dresses. They are selling for 30% off the regular price of $23.95. How much would be saved?

ANSWERS (CONT'D.)

4. a. $30 + (15\% \text{ of } 30) = 30 + (\frac{3}{20} \times 30) \approx 34$ or 35 mph.

   b. $50 + (\frac{3}{20} \times 50) \approx 57$ or 58 mph.

5. a. $20\% = \frac{1}{5}$

   $\$68.00 \approx \$70.00$ $\qquad \frac{1}{5} \times \$70.00 = \$14.00$

   $\$109.00 \approx \$110.00$ $\qquad \frac{1}{5} \times \$110.00 = \$22.00$

   b. $\$14.99 \approx \$15.00$ $\qquad \frac{1}{5} \times \$15.00 = \$3.00$

   $\$24.99 \approx \$25.00$ $\qquad \frac{1}{5} \times \$25.00 = \$5.00$

6. $40\% = \frac{2}{5}$

   $\$310.00 \approx \$300.00$ $\qquad \frac{2}{5} \times \$300.00 = \$120.00$

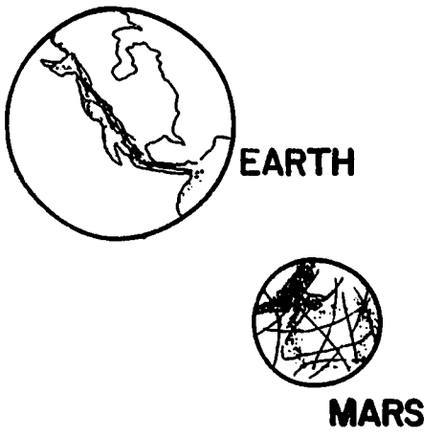
7. $30\% \approx \frac{1}{3}$

   $\$23.95 \approx \$24.00$ $\qquad \frac{1}{3} \times \$24.00 = \$8.00$

or

   $30\% = \frac{3}{10}$ $\qquad \frac{1}{10} \times \$24.00 = \$2.40$

   $\$23.95 \approx \$24.00$ $\qquad 3 \times \$2.40 = \$7.20$

8. 

The pull of gravity on Mars is 39% of the pull of gravity on Earth.

If you weigh 150 pounds on Earth, what would you weigh on Mars?

9. A ladies' pleated tennis outfit and racket usually sells for $29.99. How much less would it cost when on sale for 20% off?

DISCUSSION QUESTION

When we are estimating costs of items, why doesn't it matter if we do not agree exactly on the estimate?

ANSWERS (CONT'D.)

8. $39\% \approx 40\% = \frac{2}{5}$ $\qquad$ $\frac{2}{5} \times 150 = 60$ pounds

9. $20\% = \frac{1}{5}$ $\qquad$ $\frac{1}{5} \times \$30.00 = \$6.00$

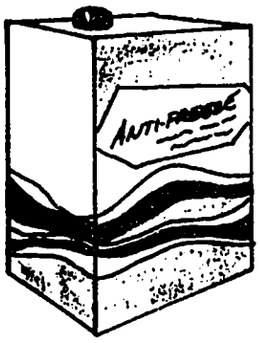
$\$29.90 \approx \$30.00$

ANSWER TO DISCUSSION QUESTION

The purpose of estimating costs of items is to determine if the purchase will really be a good purchase. Certainly exact answers are desired when the purchase is made, but an estimate is good enough for the initial step.

A CHALLENGE FOR EXPERTS

1.



Most car radiators are protected from freezing by permanent-type antifreeze. The amount of protection is calculated according to the per cent of antifreeze in the cooling system. For instance, if 40% of the coolant is antifreeze, then the car is protected to a temperature of -12° F.

This table shows the protection provided by different per cents of antifreeze.

ADAMS ANTIFREEZE

% of Antifreeze	Your car is Protected to, F
20	+16
25	+10
33	0
40	-12
45	-22
50	-34
55	-48
60	-62

Fill in the table below:

Size of Cooling System, Quarts	Protection Desired, °F	Amount of Antifreeze Required, Quarts
14	-12	
17	-34	
21	0	
16	-48	

A CHALLENGE FOR EXPERTS

1. If a temperature protection of -12° F is desired, the temperature is found in the right hand column of the antifreeze chart. Directly to the left of -12° F is the per cent of antifreeze required to have protection to that temperature. The amount of antifreeze required is 40%.

   If an automobile has a cooling capacity of 14 quarts, then 40% of 14 quarts must be antifreeze.

$$40\% = \frac{2}{5} \qquad \frac{2}{5} \times 14 = \frac{28}{5}$$

$$\frac{28}{5} = 5.6 \text{ quarts of antifreeze.}$$

To insure adequate protection, the next quart of antifreeze is given as the answer.

(The answer is calculated to two decimal places and is printed within parentheses.)

ANSWERS TO EXERCISES

Size of Cooling System, Quarts	Protection Desired, °F	Amount of Antifreeze Required, Quarts
14	-12	6 (5.60)
17	-34	9 (8.50)
21	0	7 (6.93)
16	-48	9 (8.80)

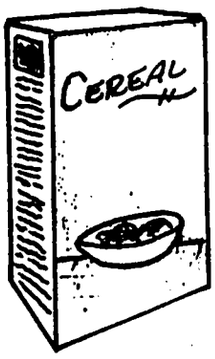
2. Adults require 10 mg. (milligrams) of iron per day in their food.

   1000 mg. = 1 gram, 1 gram $\approx$ .04 ounces, so 10 mg. $\approx$ .0004 oz.

   The amount of iron per ounce of cereal varies from brand to brand.

Example: Rice Krispies gives 5% of the adult daily requirement per ounce of cereal. How many mg. of iron is in each ounce of cereal?

Solution using estimation:



10% of the requirement (10 mg.) is found first. This is the same as $\frac{1}{10}$ of that number.

$\frac{1}{10}$ of 10 mg. is 1 mg.

Since 5% is one-half of 10%, Rice Krispies gives an adult .5 mg. of iron per ounce.

Fill in the table:

In One Ounce of Cereal	Per Cent of Adult Daily Iron Requirement	Milligrams of Iron Per Ounce of Cereal
Coco-Wheats	8.1%	
Cream of Wheat	120%	
All-Bran	30%	

2. It should be made clear that adults require 10 mg. of iron per day, and that the per cent in the table is the per cent of the adult requirement.

If a cereal provides 40% of the daily requirement, it provides 40% of 10 mg.

ANSWERS TO EXERCISES

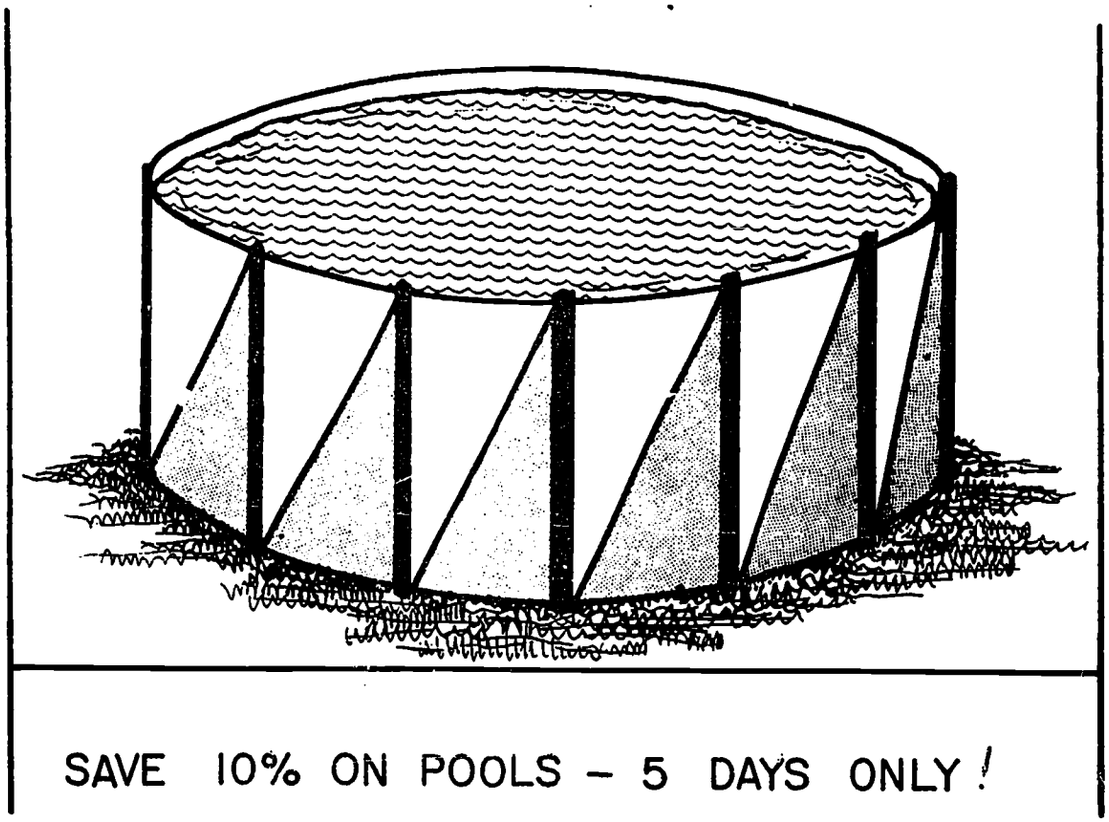
In One Ounce of Cereal	Per Cent of Adult Daily Iron Requirement	Milligrams of Iron Per Ounce of Cereal
Coco-Wheats	8.1%	1 (.81)
Cream of Wheat	120%	12
All-Bran	30%	3

$8.1\% \approx 10\% = \frac{1}{10}$ $\qquad \frac{1}{10} \times 10 = 1$

$120\% = \frac{6}{5}$ $\qquad \frac{6}{5} \times 10 = 12$

$30\% = \frac{3}{10}$ $\qquad \frac{3}{10} \times 10 = 3$

PUTTERS AND POOLS



PUTTERS AND POOLS

OBJECTIVES

The student shall be able to use proportions to calculate a whole number per cent of a given number.

CONTENT AND APPROACH

CALCULATING PER CENTS

Per cent calculations are made using proportions rather than the "formula method". It should be easier for the student to know how the problem is set up when proportions are used, because of the work dealing with correspondences between the terms of a proportion.

Example: How much is saved on an item advertised for 15% off if the regular price is \$38?

$15\% = \frac{15}{100}$, 15 cents off of every 100 cents of regular price.

We want to determine N cents off of the \$38 (3800¢) regular price, $\frac{N}{3800}$.

$\frac{15}{100} = \frac{N}{3800}$ ← refers to number of cents off
← refers to regular price

CALCULATING PER CENTS

Proportions can be used to find per cents of numbers.

Example: Find 56% of 283.

Estimation: Finding a per cent of a number is the same as finding a fractional part of the number. 56% is close to 50% or $\frac{1}{2}$.
$\frac{1}{2}$ of 283 is approximately 140.

Solution: $\frac{56}{100} = \frac{N}{283}$

$100N = 15848$

$N = 158.48$ This is close to the estimate.

EXERCISES I

<u>Estimate the answer</u> and then calculate the following percentages.

1. 25% of 88
2. 35% of 120
3. 48% of 130
4. 76% of 90
5. 88% of 64
6. 23% of 44
7. 65% of 150
8. 93% of 165
9. 18% of 58
10. 57% of 234

11. The generally accepted rate of tipping at a restaurant is 15% of the cost of the meal. If you and your date had meals totaling $4.80 at Rick's Restaurant, what tip would you leave for the waitress?

ANSWERS TO EXERCISES I

Accept <u>all</u> estimates.

1. 22	2. 42	3. 62.4
4. 68.4	5. 56.32	6. 10.12
7. 97.5	8. 153.45	9. 10.44
10. 133.38	11. $.72	

EXERCISES 2

Go back to Exercises 4 on pages 98-101 and find out the actual answers to the questions. Compare your answers to the estimates you made as a check on the reasonableness of your answers.

PER CENT IN ADVERTISING

GOLFER'S PARADISE SALE

MEN'S MATCHED SETS

MODEL	IRON-WOODS	REGULAR PRICE	SALE PRICE
'67 Spalding Silverlines	8-4	$210	$100
'67 Allied Tournaments	8-3	$150	$ 75
'68 Ram Aluminum	9-4	$325	$195
'67 Voit 300 Magnums	9-4	$313	$185
'68 MacGregor Tourney's	9-4	$284	$195
'67 Spalding Elites	8-4	$340	$225
'68 Ram Invitationals	8-3	$180	$ 95
'67 Wilson Signatures	8-4	$270	$150
'68 Wilson Signatures	8-4	$270	$185
'68 Wilson Dave Marr	8-3	$130	$ 70
'68 Spalding Dan Sikes	8-3	$150	$ 88
'68 Burke Bombers	8-3	$200	$115
'67 Slipstreams	9-4	$275	$100
'68 Fernquist-Johnson	9-4	$270	$180
'68 Paul Anthony's	8-3	$100	$ 45
'68 Wilson Shotmakers	8-3	$161	$115

SAVE TO 50% OFF BAGS, CARTS, BALLS, SHOES, etc.

Sale advertisements, especially those involving per cents, should be read carefully.

The amount an item is reduced is called the <u>amount of discount</u>.

For example, the '67 Spalding Silverlines are on sale for $100.00 while they sold regularly for $210.00. The discount is $110.00. When the discount is expressed as a per cent, it is called <u>discount rate</u> or <u>rate of discount</u>.

ANSWERS TO EXERCISES 2 (See top of student page 106)

1. $35.18
2. $250.20
3. a. $103.99
   b. $129.99
4. a. 34.5 mph.
   b. 57.5 mph.

5. a. $13.60, $21.80
   b. $ 3.00, $ 5.00
6. $124.00
7. $7.18

8. 58.5 pounds
9. $6.00

PER CENT IN ADVERTISING

The two advertisements used in this section are intended to help the student look at advertising more carefully.

The large print tends to focus attention of people in such a way that gross assumptions are made about the entire ad.

The exercises can be used as a class discussion and students should be encouraged to bring in advertisements that they think are questionable.

Question 4 is so time consuming in the calculations that it would be best to do it in class with a calculator or divide the problems among the class and tabulate as a group.

EXERCISES 3

Read the previous advertisement and answer the questions below.

1. How many sets of golf clubs advertised are on sale at a discount greater than 50%?

2. How many sets are on sale at a discount less than 50%?

3. How many sets are on sale for exactly 50%?

4. On which of the sets advertised will you receive the largest rate of discount? The smallest rate of discount? Be able to justify your answer.

5. On which of the sets advertised will you receive the largest amount of discount? The smallest amount of discount?

6. Compare your answers to exercises 4 and 5. Were your answers the same? Why or why not?

7. What is the meaning of the phrase "save to 50% off"?

8. Do you feel this advertisement is misleading? Why or why not?

ANSWERS TO EXERCISES 3

MODEL	REGULAR PRICE	SALE PRICE	DISCOUNT	DISCOUNT RATE, %
'67 Spalding Silverlines	$210	$100	$110	52.4
'67 Allied Tournaments	$150	$ 75	$ 75	50.
'68 Ram "Aluminum"	$325	$195	$130	40
'67 Voit 300 Magnums	$313	$185	$128	40.9
'68 MacGregor Tourney's	$284	$195	$ 89	31.3
'67 Spalding Elites	$340	$225	$115	33.8
'68 Ram Invitationals	$180	$ 95	$ 85	47.2
'67 Wilson Signatures	$270	$150	$120	44.4
'68 Wilson Signatures	$270	$185	$ 85	31.5
'68 Wilson Dave Marr	$130	$ 70	$ 60	46.2
'68 Spalding Dan Sikes	$150	$ 88	$ 62	41.3
'68 Burke Bombers	$200	$115	$ 85	42.5
'67 Slipstreams	$275	$100	$175	63.6
'68 Fernquist-Johnson	$270	$180	$ 90	33.3
'68 Paul Antony's	$100	$ 45	$ 55	55.
'68 Wilson Shotmakers	$161	$115	$ 46	28.6

1. [illegible]

4. Largest discount rate: [illegible] Slipstreams, 63.6%.
   Smallest discount rate: [illegible] Wilson Shotmakers, 28.6%.

5. Largest discount: '67 Slipstreams.
   Smallest discount: '68 Wilson Shotmakers.

6. They are the same because discount is dependent upon discount rates, and that the prices are quite close to each other. If the range of prices were great the largest discount would not necessarily result from the largest discount rate.

7. The discount rates could be any per cent from 0% up to and including [illegible]%.

8. Yes -- at face value. Variety of answers-discussion question.

SUMMER POOL CLOSEOUTS

ALL POOLS ARE GREATLY REDUCED!!

OPEN 9-9! COME OUT TODAY

DELUXE -- "Our Finest Pool"-- 10-Year Guarantee

SIZE	QUANTITY	SAVE	WAS	NOW
★24'x4'-6"	2	28%	$540	$390
18'x4'-5½"	19	27%	$400	$292
15'x4'-5½"	3	24%	$310	$232

CAPRI --"Lowest Priced Winterized Pool Ever"

SIZE	QUANTITY	SAVE	WAS	NOW
★24'x4'-5½"	41	34%	$420	$277
18'x4'-5½"	3	26%	$310	$229
15'x4'-5½"	13	27%	$250	$182

DELUXE OVAL -- 10-Year Guarantee

SIZE	QUANTITY	SAVE	WAS	NOW
32'x18'x4'	6	29%	$940	$671
25'x15'x4'	2	28%	$730	$522

ARISTOCRAT OVAL -- Includes 2 Redwood Decks

SIZE	QUANTITY	SAVE	WAS	NOW
★31'x16'x4'	3	25%	$700	$526
★24'x12'x4'	11	25%	$540	$405
18'x12'x4'	9	25%	$350	$262
15'x10'x3½'	7	27%	$140	$102

Save an "extra" 10% discount off these prices on any display model pool!

EXERCISES 4

The following exercises refer to the advertisements on the previous page.

1. For the swimming pools in the chart below (they are starred in the ad) calculate the sale price of each pool.

MODEL	SIZE	RATE OF DISCOUNT	REGULAR PRICE	SALE PRICE	ACTUAL CASH SAVINGS
Capri	24' X 4'5½"				
Deluxe	24' X 4'6"				
Aristocrat Oval	31' X 16' X 4'				
Aristocrat Oval	24' X 12' X 4'				

2. Compare the sale prices you calculated to those in the advertisement. Are the prices in the ad exactly what the discount rate indicated?

3. Since the sale price you calculated and the sale they quoted are not equal, the rates of discount in the advertisement are not exact. Why does the ad use discount rates that are not exact?

4. What would be the sale price of a Capri Model 24' × 4' 5½" if the pool were a display model?

ANSWERS TO EXERCISES 4

1.

MODEL	SIZE	RATE OF DISCOUNT	REGULAR PRICE	SALE PRICE	ACTUAL CASH SAVINGS
Capri	24' X 4'5½"	34%	$420.00	$277.20	$142.80
Deluxe	24' X 4'6"	28%	$540.00	$388.80	$151.20
Aristocrat Oval	31' X 16' X 4'	25%	$700.00	$525.00	$175.00
Aristocrat Oval	24' X 12' X 4'	25%	$540.00	$405.00	$135.00

2. No.

3. Reading is easier with whole numbers. The values are close.

4. $249.30 This is 10% off <u>their</u> sale price.

✓POINT

1. Using 100 as a basis, compare the following costs for coffee: 2 oz. for 29¢ and 6 oz. for 88¢.

2. Compare the 1968 passing records of two leading football quarterbacks:

   Len Dawson 224 attempts and 131 completions.
   Earl Morrall 317 attempts and 182 completions.

3. A patient needs 40 oz. of medication. The strength of the medicine is 30%. How many ounces of the drug are needed?

4. Write:

   a. $\frac{35}{100}$ as a per cent using the symbol, %.

   b. $\frac{40}{100}$ as a simplified ratio; in decimal form.

   c. .75 as a per cent using the symbol, %.

5. Use a mark to show your estimate of 40% of the line.

6. Estimate: a. 15% of 60

   b. 40% of 250

7. Calculate: a. 30% of 145

   b. 45% of 72

8. A store advertised that their $14.95 skirts were on sale for 12% off. How much would be saved on the skirt?

ANSWERS TO ✓POINT

1. $\frac{2}{29} = \frac{N}{100}$, $N \approx 6.90$ oz. $\frac{6}{88} = \frac{N}{100}$, $N \approx 6.82$ oz.

2. Dawson: $\frac{131}{224} = \frac{N}{100}$, $N \approx 58.5$ Morrall: $\frac{182}{317} = \frac{N}{100}$, $N \approx 57.4$

3. $\frac{30}{100} = \frac{x}{40}$, $x = 12$ oz.

4. a. 35%
   b. $\frac{2}{5}$; .40
   c. 75%

5. 

6. a. 10% of 60 is 6,
   20% of 60 is 12, then 15% of 60 is 9

   b. $40\% = \frac{2}{5}$, $\frac{2}{5} \times 250 = 100$

7. a. 43.5 b. 32.4

8. $1.79

(NOTE: Student page 111 and Teacher page T 111 are blank.)

VARIATION

Most of the work you have done with ratio and proportion has involved direct proportion.

DIRECT PROPORTION

Direct proportion behaves like this: As one quantity increases, a related quantity increases.

Example 1: The price of apples is 2 for 5¢. The more apples you buy, the higher the total price.

2 apples cost 5¢, 6 apples cost 15¢, 20 apples cost 50¢, etc.

Example 2: The activity, Stretching Springs, involves direct proportion. The more weight you hang on a spring, the longer the spring stretches.

For a particular spring, a weight of 8 ounces stretches the spring 3 inches. How far would a weight of 14 ounces stretch the spring?

VARIATION

OBJECTIVES

The student shall be able to:

1. Recognize whether or not a problem situation requires an inverse proportion.

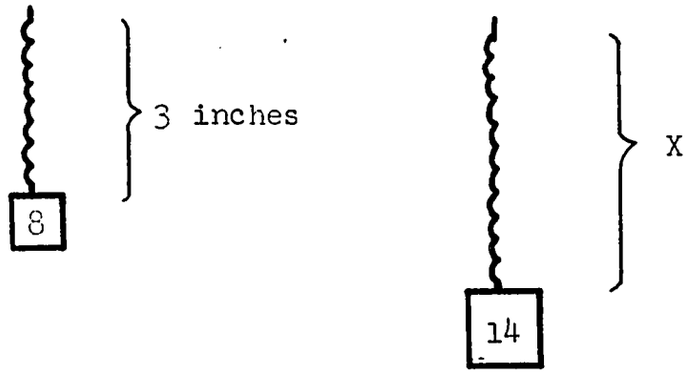
2. Solve problems requiring inverse proportions.

CONTENT AND APPROACH

Thus far, Activities With Ratio and Proportion has dealt with direct proportion. Inverse proportions often come up and are sufficiently different so that a clear distinction must be made.

The student booklet attempts to make the distinction clear. However the student should not be left to figure out the distinctions by himself. This lesson should be teacher-led to ensure success.

The main trouble spot regarding direct and inverse proportions is that a direct variation problem can result in two different proportions while an inverse variation problem has only one proportion. This is pointed out on student pages 115-118.



One proportion is $\frac{8}{3} = \frac{14}{X}$. From the problem situation you can see that the spring stretches as more weight is added.

increase in weight

$$\frac{8}{3} = \frac{14}{X}$$

increase in stretch

EXERCISES I

1. 25 pounds of a certain fertilizer is required for 5000 sq. ft. of lawn. How many pounds of fertilizer would be needed for 9500 sq. ft. of lawn?

For inverse variation proportions to produce correct results, the terms of each ratio must refer to like kinds of quantities.

For example: $\frac{MPH}{mph} = \frac{time}{TIME}$

This lesson should be carried out carefully and methodically.

ANSWERS TO EXERCISES I

1. 47.5 pounds

2. A set of pulleys requires 5 pounds of pull to lift a weight of 35 pounds. How much pull is required to lift 100 pounds?

INVERSE PROPORTION

Some situations work in a manner opposite to that of direct proportion, and they are called inverse proportions.

An inverse proportion works like this: as one quantity increases a related quantity decreases, or as the first quantity decreases the second quantity increases.

Example: As the age of a car increases, the value of the car decreases.

ANSWERS (CONT'D.)

2. 14.29 pounds

The key to solving proportion problems is to recognize whether the situation is direct or inverse.

The problem in solving proportion problems is that:

1. Direct proportion problems can be set up in two ways.

2. Inverse proportion problems can be set up in only one way.

Example: Direct Proportion

An alloy of copper and zinc makes brass. The ratio of the weight of copper to weight of zinc is 17 to 13. If 85 pounds of copper is used, how many pounds of zinc will be needed?

One proportion is $\frac{17}{13} = \frac{85}{N}$. $\frac{\text{copper}}{\text{zinc}} = \frac{\text{copper}}{\text{zinc}}$

Another proportion is

$$\frac{17}{85} = \frac{13}{N} \qquad \frac{\text{copper}}{\text{copper}} = \frac{\text{zinc}}{\text{zinc}}$$

N = 65 in both cases.

Direct proportions allow two methods to set up the proportion, each resulting in the correct answer.

Examples: Inverse Proportion

1. If 24 men can do a job in 30 days, how many days will it take 36 men to do the job?

   Since more men are working on the job, it will take fewer than 30 days to complete.

   The problem involves an inverse proportion since the number of days decreases as the number of men increases.

An inverse proportion can be set up in only one way. Each ratio must be a comparison of like quantities. In this case men to men and days to days.

The proportion is $\frac{24}{36} = \frac{N}{30}$.

N represents the number of days the 36 men will take to do the job.

Notice that the 24 and 30 (and the 36 and N) are not both in the numerator (in the denominator) although they correspond. This is what makes the proportion inverse. The second ratio appears upside down to what you might expect.

The reason the second ratio is written upside down is explained below.

In $\frac{24}{36} = \frac{N}{30}$, the numerators are smaller than the denominator ($24 < 36$ so $N < 30$).

We know that the number of days the 36 men take will be less than 30, so N must be in the numerator to obtain this result.

One way to set up the proportion is to write the terms so they correspond $\frac{24}{36}$ and $\frac{30}{N}$, then turn the second ratio upside down and write the proportion: $\frac{24}{36} = \frac{N}{30}$.

The number of days needed by the 36 men is $N = 20$.

If a proportion is made where the terms of each ratio are not like quantities, an incorrect answer will result. (You might try it and see.)

2. At 50 mph a car can travel from Huntsville to Carson City in 5 hours. How long will it take to make the trip at 65 mph?

This is an inverse proportion because as the speed increases the time will decrease.

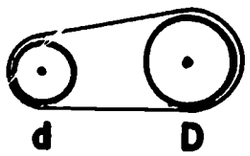
The proportion is $\frac{50}{65} = \frac{N}{5}$.

$$65N = 250$$

$$N \approx 3.8 \text{ hours}$$

EXERCISES 2

1. If the tension of a banjo string is kept constant, the frequency (vibrations per second) is inversely proportional to the length of the string. If a string 30 inches long vibrates 250 vps, how fast will it vibrate when 25 inches long?

2. 

A small drive wheel (d) is connected by a belt to a larger wheel and drives it (D is the driven wheel).

ANSWERS TO EXERCISES 2

1. $\frac{30}{25} = \frac{V}{250}$, $V = 300$ vps

2. $\frac{10}{4} = \frac{s}{100}$, $s = 250$ rpm

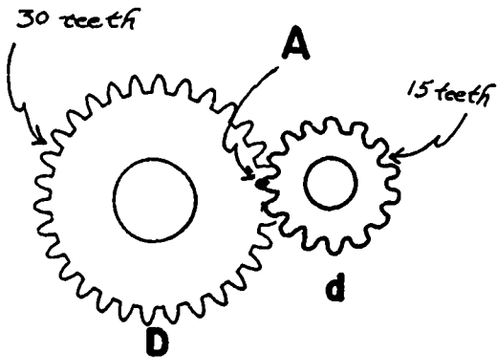
The drive wheel, d, turns faster than the driven wheel, D. The diameter of d is less than the diameter of D.

When two wheels (called pulleys) are connected in this way the number of revolutions per minute (rpm) of a wheel is inversely proportional to the diameter of the wheel.

What is the speed of pulley d if its diameter is 4 inches? D has a diameter of 10 inches and is turning 100 rpm.

3. Find the diameter of a pulley, D, having a speed of 80 rpm if it is driven by a 5 inch pulley having a speed of 200 rpm.

4. Gears behave somewhat like pulleys. Instead of being connected by a belt, they mesh using teeth.



The smaller gear turns faster than the larger gear in order for all of the teeth to mesh, yet the number of teeth in the smaller gear is less than in the larger gear.

a. The teeth meet at point A. If all 30 teeth of D pass point A, how many teeth from d must pass point A?

ANSWERS (CONT'D.)

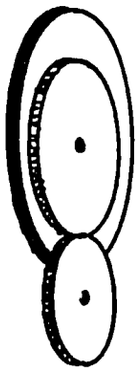
3. $\frac{D}{5} = \frac{200}{80}$, D = 12.5 inches

4. a. 30 teeth b. 2 revolutions c. 200 rpm

b. For every revolution of D, how many revolutions does d make?

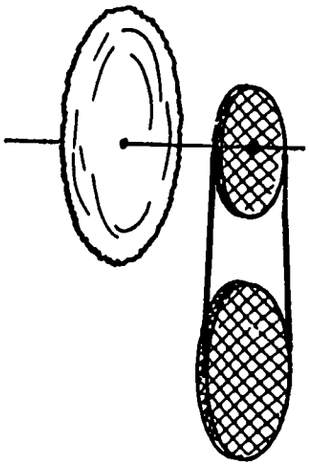
The number of revolutions per minute of two meshed gears is inversely proportional to the number of teeth in the gears.

c. If D turns 100 rpm, how many rpm will d make?

5. 

A grinding wheel turns at 850 rpm. It is attached to a 32 tooth gear and is driven by an 8 tooth gear. How fast is the drive gear turning?

6. A circular saw is connected to a 5 inch pulley and turns 1000 rpm. How large is the drive pulley if it turns 240 rpm?

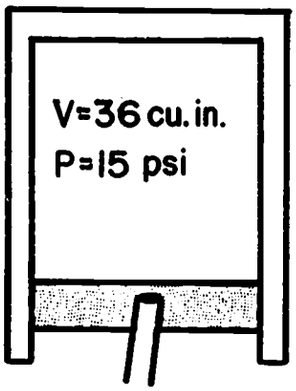


ANSWERS (CONT'D.)

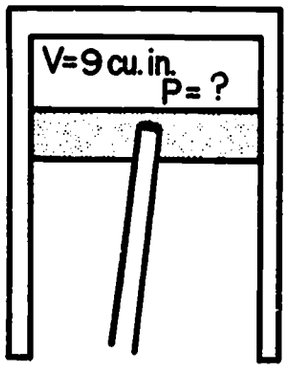
5. $\frac{32}{8} = \frac{r}{850}$, $r = 3400$ rpm

6. $\frac{5}{D} = \frac{240}{1000}$, $D = 20.83$ inches

7. The pressure, P, inside an automobile cylinder is inversely proportional to the volume, V.



piston at bottom of stroke



piston at top of stroke

What is the pressure in the compressed cylinder?
(psi means pounds per square inch.)

8. The 1968 Mustang Mach I has a compression ratio of 10.50 to 1. This means that if the volume of the cylinder is 1 when the piston is up, the volume is 10.50 when the piston is at the bottom of the stroke. What is the pressure when the piston is up, if the pressure is 15 psi when the piston is at the bottom of the stroke?

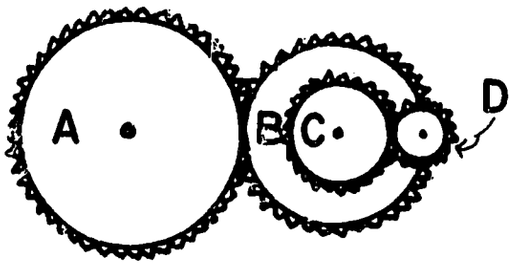
ANSWERS (CONT'D.)

7. $\frac{36}{9} = \frac{P}{15}$, $P = 60$ psi

8. $\frac{10.50}{1} = \frac{p}{15}$, $p = 157.5$ psi

A CHALLENGE FOR EXPERTS

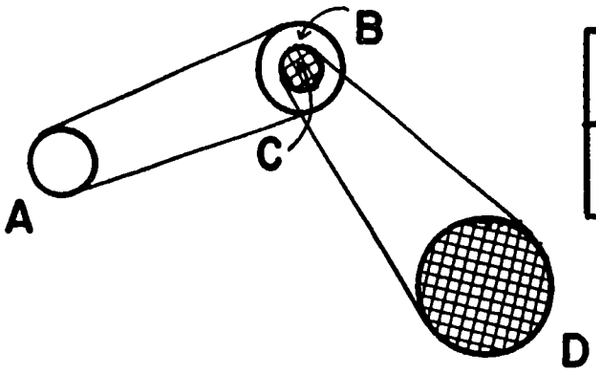
1. The diagram at the right shows a gear train (a train is a series of connected gears.) Gear A is turning at 70 rpm. How many rpm is Gear D turning?



Gear	A	B	C	D
Teeth	90	40	32	20

2. What is the effective gear ratio, $\frac{\text{gear A}}{\text{gear D}}$, in the gear train of exercise 1?

3. What diameter must pulley A have in order for it to turn 2400 rpm? Pulley D is turning 180 rpm.



Pulley	A	B	C	D
Diameter	?	5"	3"	16"

ANSWERS TO CHALLENGE PROBLEMS

1. $\frac{90}{40} = \frac{B}{70}$, $B = 157.5$ rpm

Since Gear C is attached to Gear B, C turns 157.5 rpm

$\frac{32}{20} = \frac{D}{157.5}$, $D = 252$ rpm

2. $\frac{70}{252} = \frac{x}{90}$, $x = 25$

The effective gear ratio is $\frac{90}{25}$ or $\frac{3.6}{1}$

3. $\frac{16}{3} = \frac{C}{180}$, $C = 960$ rpm

Since pulleys B and C are attached, pulley B turns 960 rpm.

$\frac{5}{a} = \frac{2400}{960}$, $a = 2$ inches

BEATERS AND BEAMS

CLASS ACTIVITY

There are some stations around the room where you will perform some experiments involving ratios.

After your teacher divides the class into groups,

1. Go to your station.
2. Follow the instructions for the activities which begin on the next page.
3. Use the equipment inventory to check your equipment.
4. Record your findings in the spaces provided.

When you finish your first activity, go to a station where the other activity is found.

BEATERS AND BEAMS

OBJECTIVES

1. Given an experimental situation, the student shall be able to set up and solve proportions related to the experiment.

2. The student shall be able to use proportions to make predictions about future trials of the experiment.

3. Given the components of an inverse proportion which are based on a problem situation, the student shall form a valid proportion.

EQUIPMENT AND TEACHING AIDS

A. STUDENT

The Egg Beater -

#1. 1 hand beater or hand speed drill

The Balance Beam -

#1. 1 meter stick.
#2. 2 malted milk cartons.
#3. 1 metal paper clamp (approximately $2\frac{1}{2}$").
#4. 1 wire rod, $\frac{1}{8}$" by 6".

THE EGG BEATER

Equipment Inventory: 1 Hand egg beater or
1 Hand speed drill

Activity:

This activity centers on the question: How many times does the beater revolve for each complete turn of the handle?

1. Turn the handle one complete turn and have your partner count the number of turns of the beater. _____

2. Turn the handle 4 times. How many beater revolutions were there? ____ For 6 handle turns, how many beater revolutions are there? _____

3. Using the information from parts 1 and 2, how many beater revolutions are there for one handle turn? _____

4. Predict the number of beater turns if the handle makes 8 turns. _____ Did you have to count the turns of the beater? Do you think there is a better way to make this prediction?

5. Write a ratio B to H, where B is the number of beater turns and H is the number of handle turns.

#5. 3 known weights: W_1, W_2, W_3

#6. 2 unknown weights: W_4, W_5

#7. 2 metal hooks

B. TEACHER

1. Overhead projector
2. Projection screen

*3. Transparency: ARP 17

ARP 17 is a summary form for discussing the activities.

CONTENT AND APPROACH

Students should work in groups of 3 or 4.

The stations should be clearly marked so the students can move from one to another with ease.

Refer to Teacher's Guide pages T 37a - T 37j for discussion of activity sessons.

THE EGG BEATER

1. The basic question for the student to answer is "How many revolutions does the blade make when the handle makes one revolution?

6. Count the teeth in each gear, and write a ratio comparing them. _____

7. How do the ratios you wrote in parts 5 and 6 compare?

8. What is the relation between the handle and beater revolutions and the number of gear teeth on each?

9. What is the precise number of turns the beater will make when the handle is turned one revolution? _____

10. If the beater turns 45 times, how many times will the handle turn? _____

11. Write a proportion which will show how handle and beater turns and number of gear teeth are inversely proportional.

12. An egg beater has gears with 64 and 14 teeth each. If the handle is turned 28 times, how many turns will the beater make?

2. The mathematical idea is that the revolutions of the handle and beater are inversely proportional to the number of gear teeth.

3. After predicting the number of blade revolutions, the students can verify this by counting.

4. The ratio obtained by comparing the number of teeth in each gear is equivalent to the ratio involving the revolution of the handle and blade.

5. Precise answers can be given to the question "How many revolutions does the blade make if the handle makes n revolutions?"

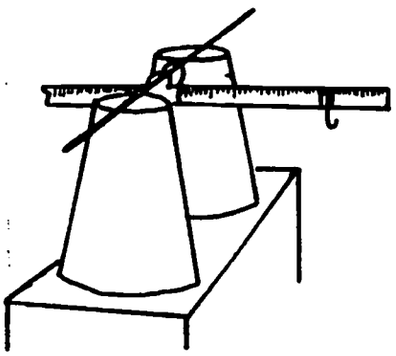
CHALLENGE PROBLEM:

A housewife making a lemon meringue pie must beat egg whites until they are stiff. This may take 4 minutes of rapid beating. If you were doing the beating, how many times would the blade turn during the 4 minute beating time?

THE BALANCE BEAM

Equipment Inventory: 1 meter stick, 2 dixie cups, 1 metal paper-clamp, 1 wire rod, 3 known weights (W_1, W_2, W_3), 2 unknown weights (W_4, W_5), 2 metal hooks.

Activity: This activity centers on the question: How can a beam balance be used to predict the weight of an object?



1. Assemble the beam balance including the hooks and move the clamp position until the meter stick balances.

2. Place weights W_1 and W_2 on hooks and slide the hooks until the meter stick balances.

3. Write as ratios:

   a. The distance W_2 is from the center of the stick to the distance W_1 is from the center. $\frac{D_2}{D_1}$:

   b. The weight of W_1 compared to the weight of W_2. $\frac{W_1}{W_2}$:

4. Use weights W_1 and W_3 and repeat parts 2 and 3.

$\frac{D_3}{D_1}$: $\frac{W_1}{W_3}$:

THE BALANCE BEAM

1. An immediate problem arises in the setting up of the experiment. (Yet, this is an aspect of problem solving.) The fulcrum should coincide with some convenient mark on the meter stick (for ease in reading distances), and then be made to balance.

2. This experiment deals with inverse variation because the weights and their distances from the center are inversely proportional.

3. Students may predict weights of the unknowns using direct proportions but should soon note that the calculated weights are incorrect by feel.

4. There are some physical situations you can point out as being inverse proportion situations.

    1. Teeter-totter (heaviest person sits nearest the fulcrum).

    2. Circular motion (swinging an object tied to a string in a circular motion. For a given effort on the part of the person holding the string, the shorter the string, the faster the object travels).

5. Are the ratios obtained in exercises 3 and 4 equivalent?

6. If a proportion is written $\frac{D_2}{D_1} = \frac{W_1}{W_2}$, what kind of a proportion is it? _____ _____

7. Place weights W_1 and W_4 on the beam and adjust until it balances. Use a proportion to predict the weight of W_4. _____

8. Check the weight of W_4 by weighing on a scale. _____

9. Predict the weight of W_5. _____

10. Check the weight of W_5. _____

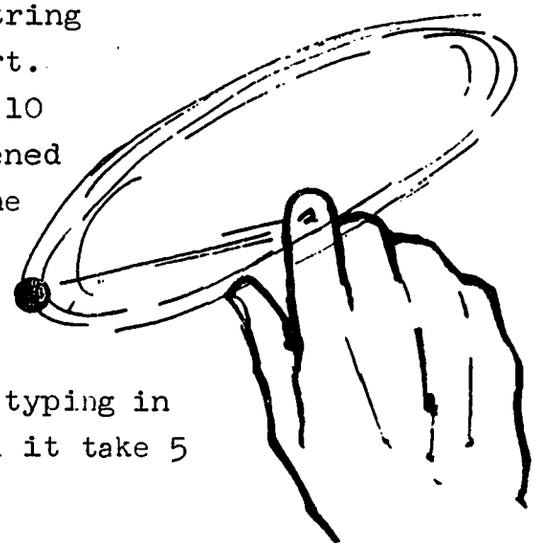
5. You may have to lend some guidance in this activity, but <u>let</u> them do it. They may be able to make discoveries on their own. Since the balance beam is quite sensitive, the weights (W_1, W_2, W_3 - known; W_4, W_5 - unknown) could be considerably lighter than those used for stretching springs. Beans or lead shot in cloth bags would work out nicely for this.

✓ **POINT**

1. Which of the following situations involve inverse proportions.

   a. Hanging various weights on a spring stretches the spring various lengths. This device can be used to predict the unknown weight of an object.

   b. A ball tied to the end of a string is twirled with constant effort. First the length of string is 10 inches, and then it is lengthened to 15 inches. The speed of the ball changes as the length of string changes.



   c. Three secretaries can do some typing in 16 hours. How many hours will it take 5 secretaries to do the job?

   d. A machine can grind 35 bolts in 15 minutes. How many bolts can it grind in 100 minutes?

2. How can you tell whether a situation involves direct or inverse proportions?

ANSWERS TO ✓POINT

1. a. Direct proportion

   b. Inverse proportion

   c. Inverse proportion

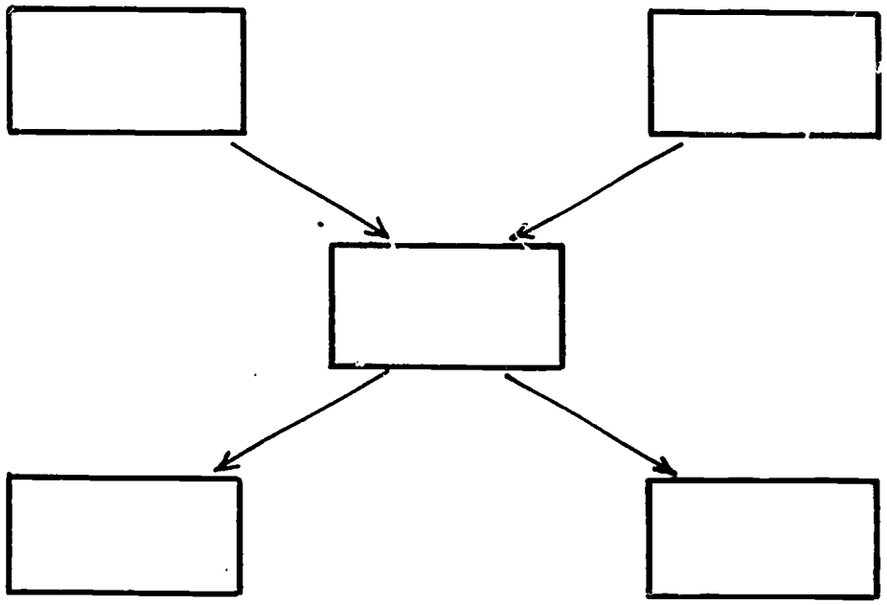
   d. Direct proportion

2. Direct proportion: An <u>increase</u> in one quantity causes a corresponding <u>increase</u> in a second quantity.

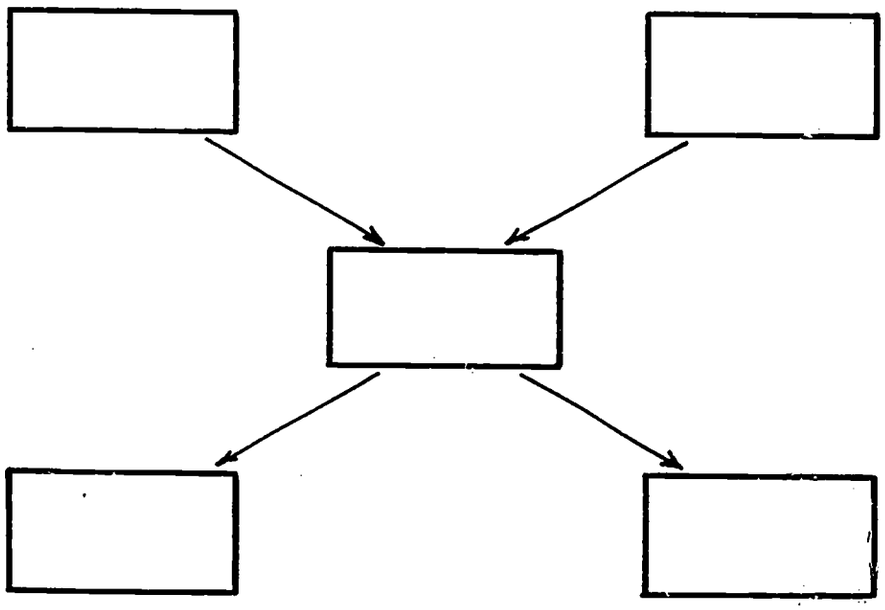
   Inverse proportion: An <u>increase</u> in one quantity causes a corresponding <u>decrease</u> in a second quantity.

Diagram outlines for Exercises 1, page 65.

1.

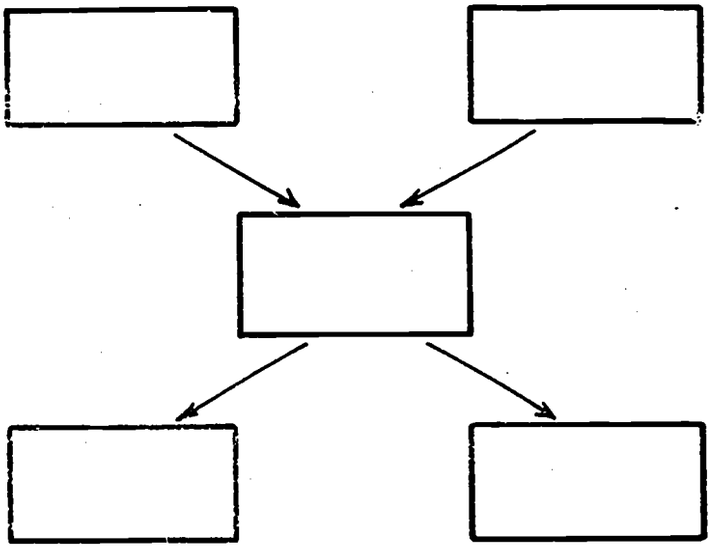


2.



ANSWERS TO EXERCISES 1 are on pages T 65 and T 65a.

3.



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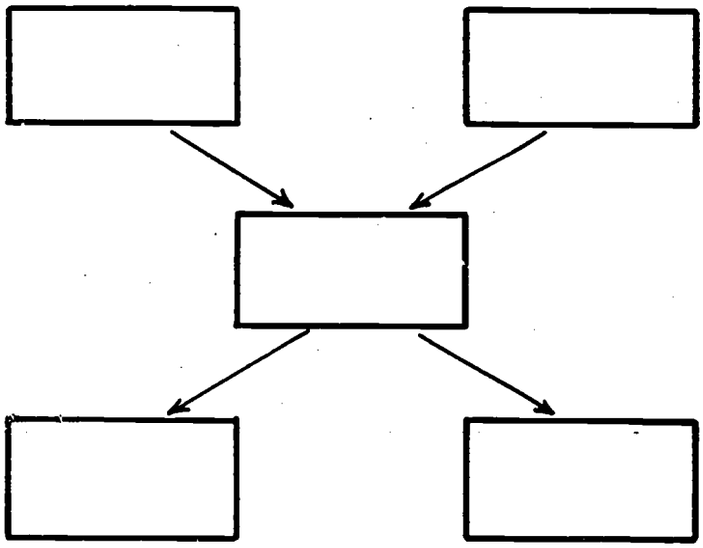

5.