

DOCUMENT RESUME

ED 067 395

TM 001 790

TITLE Relative Grading Methods: A Self-Help Text.
INSTITUTION Air Univ., Maxwell AFB, Ala.
PUB DATE Oct 67
NOTE 113p.; Supplementary Text for ECI Course 7521A

EDRS PRICE MF-\$0.65 HC-\$6.58
DESCRIPTORS *Autoinstructional Aids; *Evaluation Methods; *Grades (Scholastic); *Grading; *Instructional Materials; Self Evaluation; Workbooks

ABSTRACT

This supplementary text to a lecture course "Methods of Grading" is a self-help text. Some relative grading methods covered are: Centile Rank; Rank Order; Mean, Median and Mode; Range; Deviation; Standard Deviation; Standard Score; and T-Score. Section I of the book written as a "scrambled text," and Section II is a workbook. (DB)

ED 067395

774
T-100

ACADEMIC INSTRUCTOR COURSE

U. S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
OFFICE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRO-
DUCED EXACTLY AS RECEIVED FROM
THE PERSON OR ORGANIZATION ORIG-
INATING IT. POINTS OF VIEW OR OPIN-
IONS STATED DO NOT NECESSARILY
REPRESENT OFFICIAL OFFICE OF EDU-
CATION POSITION OR POLICY

Relative Grading Methods

A SELF-HELP TEXT

TM 001 790



AIR UNIVERSITY

MAXWELL AIR FORCE BASE, ALABAMA

ED 067395

7521A 01 S01 0367

Supplementary Text for ECI Course 7521A

Relative Grading Methods

A SELF-HELP TEXT



AIR UNIVERSITY

MAXWELL AIR FORCE BASE, ALABAMA

April 1966

Revised October 1967

This publication has been reviewed and approved by competent personnel of the preparing command in accordance with current directives on doctrine, policy, essentiality, propriety, and quality.

This self-help text, *Relative Grading Methods*, was written as a scrambled text and was specifically designed to support the lecture hours on *Methods of Grading* in the Academic Instructor Course.

It should be noted that it was not intended to replace the curriculum hours, but to bridge the gap between a large mass of technical material and the short time to cover it in the lecture. Some students in the past have had difficulty with the subject matter in these hours. The text is intended to be used as a remedial device, to clarify important points and to provide the opportunity for application of the various procedures. In addition, we have found it a help to those more serious students who feel that their retention will be improved with more practice and an additional opportunity to work with the grading methods. To this end, and at the request of many students, this book to be used with the text has been prepared. For these reasons, the introductory portion of the text has been materially shortened.

Those who examine or use this book outside of this context are asked to bear this explanation in mind.

Preface

Methods of Grading are varied and require a thorough understanding in order to apply them effectively. They are usually divided into two categories, absolute and relative. An absolute grading system is based on a fixed scale such as a raw score or percentage while a relative grading system reveals the individual's standing within the group. We are going to confine our activities here to some relative grading methods and in so doing cover the following areas:

Centile Rank
Rank Order
Mean, Median and Mode
Range
Deviation
Standard Deviation
Standard Score
T-Score

The index is at the back of the book.

This book is designed to assist you to understand better the materials covered in the Evaluation hours on Methods of Grading. We think you will find it both challenging and helpful if you follow the instructions and work conscientiously.

Section I is written as a "scrambled text" and works this way. After each problem or question you will be given several alternative answers each identified with a page number. After choosing the answer you believe to be correct, you should turn to the page number given for your choice. If your choice was correct you will find the next unit of information and/or the next problem, or, if the answer you chose was incorrect, you will find the correctional material appropriate to the answer you chose. You will usually be referred to the original choice page to try again. The page numbers in this section are assigned essentially at random, and you therefore cannot progress from one page to the next except by making an active choice of an answer.

Section II of the self-help text is a workbook and is intended to give you additional experience in the area of Methods of Grading and designed to be used in conjunction with Section I. Should you have difficulty with any of the problems, you may return to the specific part of Section I concerned with them by referring to the index on the last page of the text.

Take off to page 2.

SECTION I

Relative grading methods are those in which the grade reveals only the individual's relative standing in the group. A relative method you could use to report final examination grades would be to tell each student:

	Page
A. His raw score on the test.	8
B. How many of the class were better than he.	7

Your answer: The number of people worse than he *and* those tied with him.

The best choice you could have made. It would have involved an additional step if you had selected the total of the group. You have the three elements; those above, those tied and those below. We now have the basis for determining where any individual stands in the group.

You are now ready to move to page 9.

You came from page 7.

Your answer: The number of people tied with him.

Not enough to know only this.

This would be true in only one case, when this person has the lowest score in the group.

Turn back to page 7 and try again.

Your answer: All the raw scores in the group.

Yes! If you now arrange them in order from the best to the poorest, you will be able to tell each student how many of the group were better than he.

Go on to page 7.

Your answer: The number of people worse than he.

Now, now! Might there not be some people as good as he was?

Turn back to page 7 and try again.

Your answer: Tell each student how many of the class were better than he.

We know that in order to tell each student how many in the class were better than he, we would need to have all the scores arranged from the best to the worst.

This is a good start, but we still haven't enough information to make this a completely relative grading method. Which of the following must also be known?

- A. The number of people worse than he.
- B. The number of people tied with him.
- C. The total number in the group.

	Page
A and B	3
A and C	10
B and C	15
A only	6
B only	4
C only	14

Your answer: His raw score on the test.

It might be possible that you are confused about raw scores. Here's the straight poop. When you have corrected a paper and count the number of correct answers or responses, you have obtained the raw score for that paper. This is an absolute grade. In order to obtain a relative grade from a student's raw score you would need to know:

	Page
A. All the raw scores in the group.	5
B. The raw scores of the best and poorest students.	12

Well! We made it. We now have all the scores arranged in order and we are able to determine for any given score, the number of scores above, those tied and those below. We are ready to determine Centile rank for this group. The following formula is used:

$$\frac{\text{All those below} + \frac{1}{2} \text{ of those tied}}{\text{Total number in the group}} \times 100$$

To use it, count all the students below any given score; add to them half of the number tied at that score and divide by the total of the group. Then multiply this result by 100. This figure tells us (for any individual in the group) that he is equal to or better than a certain percent of the group.

Just remember one thing! If only one person has the particular score, the "1/2 of those tied" would equal .5. In other words the "1/2 of those tied" could be more precisely stated as "1/2 the frequency" or "1/2 the number" of students receiving that same particular score. Hope we didn't confuse you; we're trying to keep it simple.

Now try this one. Find Joe's centile rank.

Joe and 3 others got a score of 65.

12 had lower scores.

There were 56 people in the group.

Joe's centile rank is:

Page

A. 25	20
B. 24.1	11
C. 21.5	22
D. 17.8	13

Your answer: The number of people worse than he *and* the total number in the group.

OK! But you would have saved yourself one step if you had taken those tied with him rather than the total of the group. Now, you must subtract the total of those above and below him from the total number to get the number tied with him.

You are ready to move to page 9.

Your answer: 24.1

You forgot about Joe. Since there were three others tied with Joe that made a total of four at that score. Go back to page 9 and work it again using four tied.

Your answer: The raw scores of the best and poorest students.

This isn't enough. It would give you only the range of scores (the extremes) and not how all the scores were distributed between them. Go back to page 8 and take the other tack.

Your answer: 17.8

You got mixed up a bit. You should use $\frac{1}{2}$ of the people tied at Joe's score, *not* $\frac{1}{2}$ of the people below him.

If you remember to do this when you go back to page 9, you'll do it right.

Your answer: The total number in the group.

Well, now we know how many people were *better* than our student and the total number of people in the group. You see that if we subtracted those *better* than he from the *number* in the group we would have another sub group which contained *both* those *tied* with him and those *worse* than he. We still don't have enough information to tell exactly where our student stands.

Best you go back to page 7 and give it another go.

Your answer: The number of people tied with him *and* the total number in the group.

OK! But you would have saved yourself one step if you had taken those who were worse than he rather than the total of the group. Now you will have to subtract the total of those above and those tied from the total in the group to find out how many were worse than he.

You are now ready to go to page 9.

You came from page 7

Your answer: 52

Look! There were five people tied at the score of 65. Joe and four others. You are close, but not quite.

Try page 20 again.

Your answer: 80.8

Is this a new twist for you or have you done it again? In any event, you goofed when you divided by 65. Remember, we never use the raw scores in this computation, only the number of people in the group. This was 100.

Try page 20 again.

Your answer: 6/9

Good! You are on to the mechanics of the method and it isn't difficult to count down. However, all cases aren't as easy as this one.

When you have more than one person at any given score it would be unfair to assign different ranks to each of them. Let's assume we have three people with the same raw score. We must arrive at just one rank order for these three people as no one of them ranked higher or lower than the other two. We do this by determining which positions these scores would hold in the sequence of scores and average them. For example; if the score 84 was held by three people and there were eight other scores higher, these three would actually occupy positions 9, 10, and 11 in the sequence of scores (the eight higher scores would have used up positions

1 through 8). We would then average 9, 10, 11 $\frac{(9+10+11)}{3}$ and get 10.

This is the one rank order we would assign to *each* of the scores, 84. If there were 40 people in the group we would report this rank order, 10/40.

Try this:

In the following distribution of raw scores, 73, 68, 69, 74, 74, 67, 74, 70, 71, 75 what is the rank order for a score of 74?

	Page
A. 8/10	19
B. 4/10	23
C. 3/10	21
D. 2/10	30

You came from page 26.

Your answer: 8/10

You averaged positions, but you went in the wrong direction.
It is always from the best to the worst.

Now, reverse your direction when you count on page 18.

Your answer: 25

You used the formula correctly.

Let's firm this up now by working a more difficult problem.

What is Joe's centile rank now?

45 students better than Joe
Joe and 4 others had scores of 65
50 students had lower scores

	Page
A. 47.5	25
B. 52	16
C. 52.5	26
D. 80.8	17

You came from page 9

Your answer: 3/10

Yup! That's it.

The interesting thing about rank order is that some fractions are not used at all when there are ties in the scores. In this case, since we had averaged positions 2, 3, and 4 to get the rank order 3/10 we have no need for either 2/10 or 4/10.

Also, the numerator (rank indicator) is not always a whole number as we have had so far. For example: let us assume that two scores of 76 occupy positions 3 and 4 (there were two scores higher) in a sequence of 20 scores. Applying our rule of averaging the positions we would get $\frac{3+4}{2} = \frac{7}{2} = 3.5$. The rank order for each of the 76's would be written 3.5/20. This type of rank order will *always* result if an even number of people are tied at any given score. This final problem should square away this aspect of rank order.

Given the following distribution of raw scores, what rank order would you assign to a score of 31?

29,25,37,26,31,31,31,31,32,34,36,33

	Page
A. 6.5/12	34
B. 7.5/12	31

You came from page 18.

Your answer: 21.5

Whoops! Why did you use 65 for the total in the group? That was Joe's score. Scores are never used, only the number of students.

Now, go back to page 9 and remember to use the total number of people in the group.

Your answer: 4/10

You didn't arrange the scores from the best to the worst. Besides, you didn't average the positions occupied by the 74's, but took the first one you came to.

Now, arrange them properly and average the positions the three 74's occupy to get the right answer.

Back to page 18.

Your answer: 4/9

You must have started counting from the bottom instead of from the highest score down.

Page 26 is your destination.

Your answer: 47.5

Yes, there were 100 people in the group and you did take half of the people tied at Joe's score, but *WHY* did you take those above rather than those below Joe's score to get your answer? Do this properly and you are in.

Try page 20 again.

Your answer: 52.5

Good. Now you know how to compute centile rank. Memorize the formula:

$$\text{Centile rank} = \frac{\text{Below} + \frac{1}{2} \text{ tied}}{\text{Total}} \times 100$$

Another way of expressing a relative score is by means of Rank Order. We are now ready to compute it. Rank order is just another way of showing the relative position occupied by any individual in the group. It is expressed as a fraction in which the denominator indicates the number in the group; the numerator is the individual's position from the top of the group. For example, 3/75 tells us that this person is third highest in a group of 75 people.

In order to compute rank order we must first arrange the raw scores, as we did for centile rank, from the highest to the lowest, including all duplicate scores.

The following are the raw scores for a group of nine students:

38, 36, 35, 34, 32, 29, 27, 26, 22

What is the rank order for the person whose raw score is 29?

	Page
A. 4/9	24
B. 6/9	18
C. 2/3	28

You came from page 20

Your answer: 7/12 and 8/12

Remember, you used the positions 6, 7, 8, and 9 to get the rank order 7.5/12.

37,36,34,33,32,31,31,31,31,29,26,25

This means that all these positions were used so that each of the 31's could be assigned the rank order, 7.5/12. These positions are out of the running for any use here.

Try again, page 31.

Your answer: $2/3$

Your mathematical instinct fouled you up. This is one computation where you never reduce fractions.

You probably know the basic methods but go back to page 26 and be sure.

Your answer: $4.5/12$

Look at the distribution once more.

37,36,34,33,32,31,31,31,29,26,25

Your answer could only be arrived at if the fourth and fifth positions were tied. They aren't. Count down from the top. What position would the 32 occupy?

Right, the fifth position. So it would have to be rank order $5/12$.

Now go on to page 38.

You came from page 32.

Your answer: 2/10

You arranged the scores properly, but didn't average the positions occupied by the 74's. Your mistake was to take the first 74 you came to. It wouldn't be fair to assign different rank orders for the other 74's would it? Or, you didn't include the 75 which is the top score in the distribution.

You will get it right if you try page 18 again.

Your answer: 7.5/12

Yes! Arranging the scores properly, you got:

37,36,34,33,32,31,31,31,31,29,26,25

We found that the 31's occupied positions 6, 7, 8 and 9. Count them and see that those scores higher used up positions 1 through 5 in the sequence. The average of the position for the 31's is:

$$\frac{(6+7+8+9)}{4} = \frac{30}{4} = 7.5$$

A shorter, but just as accurate, method is to average only the extreme positions 6 and 9.

$$\frac{(6+9)}{2} = \frac{15}{2} = 7.5$$

Naturally, you get the same rank order.

In this distribution what would be the rank order preceding and following 7.5/12?

	Page
A. 7/12 and 8/12	27
B. 5/12 and 8.5/12	36
C. 5/12 and 10/12	38
D. 6.5/12 and 10/12	32

You came from page 21.

Your answer: $6.5/12$ and $10/12$

You are half right—positions 6, 7, 8, and 9 were used to get $7.5/12$ for all the 31's in the distribution.

37,36,34,33,32,31,31,31,31,29,26,25

Thus, you were correct in assigning $10/12$ to the position below the 31's. How about the one above? Would it be.

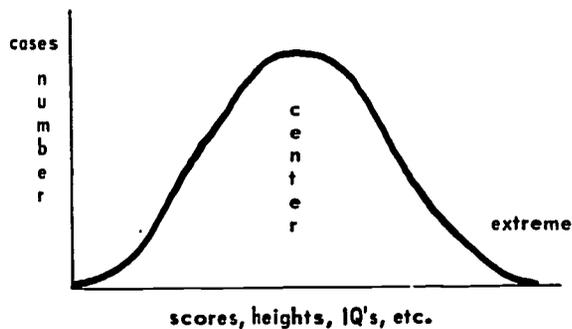
	Page
A. $4.5/12$	29
B. $5/12$	38

You came from page 31.

Whenever members of any large, unselected group are measured for a certain ability or trait (such as test scores or height) a distribution is obtained which tends to reflect the following features:

1. A massing of cases near the center.
2. Comparatively few cases are found at the extremes.
3. There is a tendency for the number of cases to decrease gradually as the extremes are reached.

This may be further illustrated by the typical normal probability curve.



Central tendency (massing of cases) at the center is represented by three measures, the mean, median and mode. The most frequently used measure is the MEAN. The mean, or arithmetical average, is derived by adding the scores of all the cases and dividing by the number of cases. What is the mean for these scores?

88, 80, 80, 80, 75, 73, 62, 61, 60, 51

	Page
A. 80	39
B. 74	35
C. 71	41
D. None of the above	44

Your answer: 6.5/12

Whoa! Are you still forgetting to arrange the scores first?
You averaged the positions just fine, but they weren't in the
right place.

Be sure you arrange the scores first when you go back to
page 21.

Your answer: 74

You didn't use the correct methods, but averaged the two mid scores, 73 and 75. You actually identified the **MEDIAN**, but we are getting ahead of ourselves—that will come later. The mean is what we are after.

Go back to page 33, read the instructions again and pick a better answer.

Your answer: $5/12$ and $8.5/12$

37,36,34,33,32,31,31,31,31,29,26,25

You are half right.—Since the positions 6, 7, 8, and 9 were used to get $7.5/12$ for the 31's, you were correct in assigning $5/12$ to the position above the 31's. How about the one below? Would it be.

	Page
A. $10/12$	38
B. $9.5/12$	42

You came from page 31.

Your answer: 70

You took the easy way out and got the wrong answer. You just can't average the scores as you did, but you must consider how many times that score appeared in the distribution. Here is the problem again:

Scores	Cases (frequency)
80	1
75	2
70	4
65	6
60	2

In order to do this properly we need to multiply each score by its frequency (80x1, 75x2, etc.) before we can add. Also we must add the cases column to find out how many are in the group.

Go back to page 41 and pick a better answer.

Your answer: 5/12 and 10/12

37,36,34,33,32,31,31,31,31,29,26,25

Good! That's it. Check yourself on the distribution above.

Raw score 32 has rank order 5/12. All 31's, averaging positions 6, 7, 8, and 9, have rank order 7.5/12. Raw score 29 has rank order 10/12.

Keep these rules straight:

1. Arrange the raw scores in descending order.
2. Count down and assign rank order to each score in turn.
3. For tied scores average the positions occupied by those scores and assign that rank order to each score.
4. Some positions, when used to compute ties, are not used as rank order.

What rank order would the raw score 36 have? 2/12 is correct, of course.

If you are interested in clearing up standard score and T scores, go on to page 33. **BUT**, before you go put a marker in your book and take a break—you've earned it.

You came from page 31, or 36; perhaps 29 or even 42. Anyway, you're here!

Your answer: 80

Did you use the correct method or just pick the score that appeared most frequently? You may not realize it, but you picked the MODE. We are getting ahead of ourselves—that will come later.

The mean is what we wanted.

Go back to page 33, read the instructions again and pick a better mean.

Your answer: 204

Whew! Why did you do this?

Your mean is way out beyond the range of scores where you would expect to find the mean. You did get the correct total of all the scores by adding after you had multiplied each score by its proper frequency. Then you divided by 5. That wasn't the number of cases. You must add all the frequency column to get the total of cases.

Try this correct step on page 41 and you will get the right mean.

Your answer: 71

That was easy. Just totaling all the scores and dividing by the number of cases gives the correct mean.

$$\frac{710}{10} = 71$$

In distributions where the same scores appear frequently, it is simpler to report them in this manner:

Scores	Cases (frequency)
80	1
75	2
70	4
65	6
60	2

What is the mean for the above distribution?

	Page
A. 3	43
B. 23.3	46
C. 68	45
D. 70	37
E. 204	40

You came from page 33.

Your answer: $9.5/12$

Look at the distribution again.

37,36,34,33,32,31,31,31,31,29,26,25

The $9.5/12$ could only be arrived at if the ninth and tenth positions had been tied—they weren't. Count down from the top. What position would the 29 occupy?

Right! The tenth position, so the rank order must be $10/12$.

Now go to page 38.

Your answer: 3

My, my, your system is much too easy. Besides, your mean isn't even within the range of raw scores. You divided the number of cases (15) by the number of different scores (5).

'Tis wrong!

Best you go back to page 41.

Your answer: None of the others.

What did you do? It is very likely that you used the correct method but goofed with your arithmetic.

Here is the distribution again:

88,80,80,80,75,73,62,61,60,51

Which of these answers do you get?

	Page
A. 80	39
B. 74	35
C. 71	41

You came from page 33.

Your answer: 68

Yes! The correct procedure is to multiply frequency times that score before adding; total the frequency column to obtain the number of cases before dividing. You divided 1020 by 15.

Another measure of *central tendency* is the **MEDIAN**. This is the mid-score of the distribution. It is easily understood. When a series contains either a few extremely high or a few extremely low scores relative to the majority of scores in the series, the median is perhaps the most representative measure of central tendency. To obtain the median we first arrange the scores in order (as we did for centile rank and rank order). The mid-score is that score which has as many cases above as below it. If we divide the total number of cases in the distribution into two halves we can determine the median score.

For instance we would determine the median score in a distribution of 65 cases by finding the score which is at the 33rd position in the sequence. There would be 32 above and 32 below, since this is an odd number of cases. If we have an even number of cases, 80, the median would be the score midway between those scores held by the 40th and 41st cases.

What is the median for the following distribution of scores?
27, 27, 25, 25, 25, 24, 23

	Page
A. 25	51
B. 25.5	48

You came from page 41.

Your answer: 23.3

Your mean isn't even within the range of raw scores for this problem. You just added the five different scores, got 350 and divided by the number of cases, 15. Some of these scores had more than one person making the score. One of them had 6. Therefore, in this event you must multiply the score by the number of cases before adding.

Look at this problem.

65	}	This is the same as	Scores × Cases
65			
65			{ 65 × 3 = 195
60			{ 60 × 2 = 120
60			{ 55 × 4 = 220
55			535
55			
55			
55			
535			

Best you go back to page 41.

Your answer: 30

This looked something like the problem you had just completed, but did you forget to arrange the scores in descending order. Bet you did.

Do that first on page 51

Your answer: 25.5

It's the middle score we want. Since there are seven cases it would have to be the one which has three cases above and three cases below. You probably averaged the 24 and 27 to get 25.5.

Don't let the fact that the score 25 appeared more than once throw you.

Back to page 45 to count again.

Your answer: 31

Yes! Found by arranging the scores in the correct order and finding the midway score between 30 and 32. Lucky you weren't led astray by the number of 30's we had in the distribution. This is the basis for another measure of *central tendency*, the Mode (most popular score).

The score which has the highest frequency is called the mode. It is a simple and easy method of determining, by inspection, a crude approximation of a representative score around which the rest of the scores tend to cluster. It is possible that more than one score will have the same highest frequency. Each of those scores is identified as a mode. This may often occur when the data considered is not homogeneous. For example, a mathematics background test given to a group which contained both students who had majored in mathematics as college undergraduates and those majoring in other fields would probably show two definite modes (bi-modal). When more than two modes exist in a distribution it is said to be multi-modal.

What is the mode for the following distribution?

Score	Frequency
65	2
63	9
62	8
61	7
59	5
58	9
55	3

	Page
A. 61	57
B. 63	55
C. 61, 63	52
D. 58, 63	60

You came from page 51

Your answer: 30.5

You probably arranged the scores correctly, but when you found the mid-way score you may have used the scores 30 and 31. Note that this distribution had no score of 31.

Check on page 51 what the next score above 30 is. This will give you the right answer.

Your answer: 25

Yes, the middle score is 25, three cases above and three below. It makes no difference how many times the 25 appears if the middle score is 25, that's it.

Now, this one. What is the median for this set of raw scores?

30, 36, 34, 30, 30, 30, 35, 32, 30, 33

	Page
A. 30	47
B. 30.5	50
C. 31	49

You came from page 45

Your answer: 61 and 63

63 is one of the modes because it has the highest frequency, 9, but why did you include 61? The score 61 is another measure of central tendency because it is both the median (midscore) and the mean. Did you go to the trouble of finding either of these? If so it was good practice, but be sure of what you did and how you got it.

Now, go back to page 49 and see if there isn't another mode?

Your answer: 62 to 46

Correct. Even though the highest score in this distribution was not as high as the others, the lowest score did extend the range. Hence the range was 16 raw score units as compared with 15 and 14 for the others.

We have already learned that the mean may be computed for any set of scores.

The distance that any one raw score is away from the mean is spoken of as its *deviation*. The greater the distance, the greater the deviation. Any score above the mean has positive deviation, scores below have negative deviation.

For example: the mean of this distribution is 58	66
(check it if you don't believe it). The deviation	64
for the raw score 63 is +5, $(63-58=+5)$. For the	63
raw score 50 the deviation is -8. $(50 - 58 = -8)$	59
	57
	56
	50
	49

Remember, deviations above the mean have plus signs, those below have minus signs.

In this set of scores:
84, 83, 80, 75, 71, 64, 53, 50, the mean is 70. Which score has the largest deviation?

	Page
A. 84	61
B. 53	64
C. 50	54

You came from page 60

Your answer: 50

Yes!! The score has a deviation of -20 ($50 - 70 = -20$) because it is below the mean. Positive or negative deviation was not the real issue here, only the distance from the mean.

Deviations need not always be expressed as whole numbers, but may be expressed as decimals as in the following example:

The mean is 59.5	68
	65
	63
The score 58 has a deviation	60
of -1.5 , the score 65 has a	58
deviation of $+5.5$.	57
	55
	50

What is the deviation of the score 60?

	Page
A. $+1.5$	59
B. $-.5$	63
C. $+.5$	66

You came from page 53

Your answer: 63

Yes, 63 is one mode, but you stopped too soon. Remember, a distribution may have more than one mode. This one did. Take a good look at it again and find the other.

It's on page 49

Your answer: +3

Yup! $86 - 68 = +18$ (the deviation above the mean). +18 divided by 6 gives you the number of standard deviations above the mean.

Rather than use the expression, "the number of standard deviations above or below the mean" an easier and shorter way is to simply call it the *Standard Score*.

For instance, the standard score +2.5 tells us that an individual has a raw score which is 2½ standard deviations above the mean.

Here is a formula which will help:

$$\text{Standard Score} = \frac{\text{Any raw score} - \text{Mean}}{\text{Standard Deviation}}$$

Standard scores normally have a range no greater than from -3 to +3.

Applying this procedure what is the standard score for a raw score of 58 when the:

Mean = 40

Standard deviation = 8

	Page
A. -5.0	92
B. +2.25	71
C. +12.25	74
D. -7.25	69
E. +1.45	72
F. +5.0	75

You came from page 67

Your answer: 61

The *mode* is the score with the highest frequency. It looks like you found the median (mid-score) or you might even have computed the mean. If so, you did too much work because the mode may be found by simply examining the distribution.

Go back to page 49 and be careful.

Your answer: 65 to 50

You let the fact that this set of scores contained the highest score influence you. We were interested in the greatest range. Check the sets on page 60 again and select the one with the greatest spread.

Your answer: +1.5

You are correct in assuming that the deviation is plus because 60 is greater than the mean, 59.5, but is it that much greater?

Look at the problem again, Page 54.

Your answer: 58 and 63

Yes. Both of these scores had the highest frequency (9) in the distribution. When there are two modes in a distribution it is said to be bi-modal.

So far we have discussed the three measures of central tendency, median, mode and mean. The mean is the most valuable and the most frequently used.

Although measures of central tendency are descriptive, they alone are inadequate to portray a distribution. Two sets of scores may have the same mean, but differ greatly in their spread. This variability may be indicated by how far the scores extend above and below the mean. The total distance from the lowest to the highest scores is expressed as the Range.

Which of the following sets of scores has the greatest range?

	Page
A. 62,62,59,62,51,46,54,62	53
B. 56,54,60,65,50,52,58,61	58
C. 63,49,59,49,49,54,49,50	62

You came from page 49

Your answer: 84

This score has a positive deviation, +14, since it is 14 units above the mean ($84 - 70 = +14$). We want the score with the greatest deviation. In this case if the negative deviations are farther away from the mean than the positive 84, then their deviation is greater. Don't let the signs fool you.

Take a look at the problem again—page 53.

Your answer: 63 to 49

Were you influenced by the number of 49's in this set?
The frequency of any one score has no real bearing on the
range of scores. We were concerned here only with the great-
est spread.

Check the sets on page 60 again.

Your answer: $-.5$

You are correct in determining that the 60 is just a half a unit away from the mean of 59.5, but you went in the wrong direction. Isn't 60 greater than the 59.5? That would make it a positive deviation, $+.5$. Clear?

If so go on to page 66.

You came from page 54.

Your answer: 53

At least you didn't get fooled by the difference in signs. This negative deviation, -17 , ($53 - 70 = -17$) is greater than the positive deviation of the 84, but it isn't the greatest of the three deviations.

Take another look at page 53.

Your answer: -1.6

Good! Check yourself.

$$\frac{50 - 63}{8.3} = \frac{-13}{8.3} = -1.56 = -1.6 \text{ (rounded)}$$

There is one case in which the standard score could be zero (0) and that is when the raw score we are concerned with is exactly at the mean. For example, if the raw score and the mean were both 65 and the standard deviation, 13, it would be worked this way;

$$\frac{65 - 65}{13} = \frac{0}{13} = 0$$

Here is an optional problem with a reverse twist to see if you understand what this is all about. If you prefer to move along more rapidly omit this one and go on to the regular problem.

Find the *raw score* when the:

Mean = 60
 Standard Score = -2.5
 Standard Deviation = 8

	Page
A. 80	84
B. 40	70

Regular problem:

Find the standard score when:

Mean = 62
 Standard Deviation = 4.5
 Raw Score = 55

	Page
A. +1.6	81
B. -1.56	78
C. -.16	76

You came from page 71

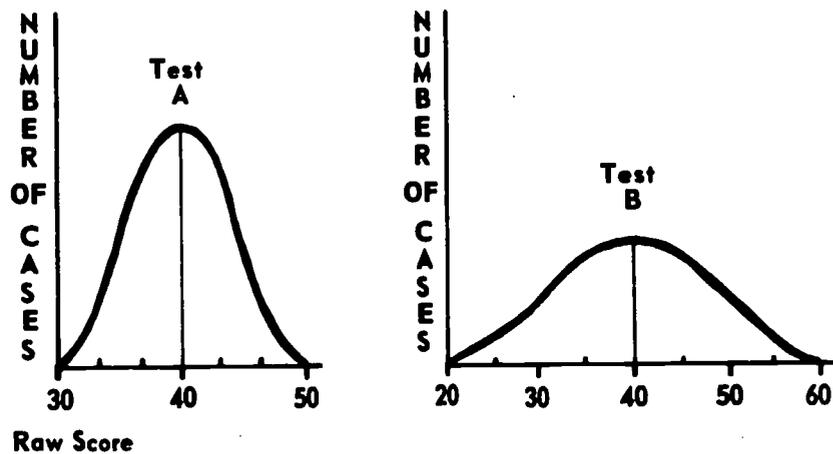
Your answer: $+ .5$

Yes. The score 60 is just a half unit above the mean of 59.5 so you are correct ($60 - 59.5 = +.5$) Let us go on! Just as the mean of a distribution of test scores can give us an idea of its difficulty (the higher the mean the easier the test, the lower the more difficult), so the deviation of the individual scores can give us some idea of the variability of the scores about the mean. If we look at the total deviations within a distribution and use them in a not too difficult mathematical computation we can determine the *Standard Deviation*.

This is a measure of the spread of scores about the mean and is used to identify the position of any one score in relation to the mean.

The standard deviation is expressed in raw score units and will be different in each set of scores. Normally, the standard deviation will be larger as the range of scores is greater and is usually equal to about one-fourth to one-sixth the total range of scores.

In test A below, since the range of raw scores is small ($30 - 50$), the standard deviation is much smaller than in Test B where the range of scores is greater, ($20 - 60$). Roughly speaking, the standard deviation in Test B would be about twice as large as in Test A.



Any raw score in a distribution may be expressed as so many standard deviations from the mean. For example, the mean of a set of scores is 55 and the standard deviation is 5. A score of 65 would be two standard deviations above the mean. The 65 has a deviation of +10 so we would divide that by the 5 (standard deviation) to get the +2. Similarly, a score of 40 would be three standard deviations below the mean. Dividing -15 by the 5 gives the -3 . Remember, the plus and minus signs indicate the direction from the mean.

Now, let's see if you can work one.

A set of raw scores has a mean of 68. The standard deviation is 6.

How would you indicate the number of standard deviations for a raw score of 86?

- A. -3
- B. $+3$

Page

85
56

Your answer: 43.3

This what you did? You found the deviation to be -18 , divided -18 into 12 to get $.666$ which you rounded off to $-.67$; then you multiplied by 10 to get -6.7 ; then added to 50 to get 43.3 .

Every step was OK except when you divided 18 into 12 . It is the deviation divided by the standard deviation to get standard score.

Do that step properly when you work the problem again and you will find the correct answer on page 80.

Your answer: -7.25

In all probability no distribution would have a standard score so large. You ignored the mean altogether in your computation and simply divided the raw score by the standard deviation. You must have guessed at the minus.

Remember, any raw score above the mean cannot have a minus standard score.

Try page 56 again.

Your answer: 40

You've got it. We know by inspecting the data that if the standard score was minus, the raw score would have to be below the mean. Properly substituting in the formula we would get:

$$\frac{X - 60}{8} = -2.5$$

$$X - 60 = -20$$

$$X = 60 - 20$$

$X = 40$, the raw score.

Now go back to page 65 and work the regular problem.

You came from page 65

Your answer: +2.25

Right! The raw score is above the mean so we know that the standard score will be positive. This was the proper application of the formula:

$$\frac{58 - 40}{8} = \frac{+18}{8} = +2.25$$

One important point to remember here is that when the mean is smaller than the raw score, the standard score will always be positive.

Now let us try another problem:

$$\text{Mean} = 63$$

$$\text{Standard Deviation} = 8.3$$

Find the standard score for the raw score, 50.

	Page
A. -.15	77
B. +.15	73
C. -1.6	65

You came from page 56

Your answer: +1.45

You didn't even consider the standard deviation in your computation, just divided the raw score by the mean. Yes, the standard score should be positive, but not your answer.

Check the formula again and try it again on page 56.

Your answer: $+.15$

You goofed! Wasn't the raw score below the mean? That *requires a minus sign*. Besides, if you did that arithmetic you have decimal problems.

Now, go back to page 71 and do it again.

Your answer: +12.25

No distribution could have a standard score so large. Check your answer next time and you won't make this mistake again. Anyway you added 58 and 40 before dividing. It should be 58 minus 40.

Go back to page 56 and do that first.

Your answer: +5

Too large. We don't have standard scores so big. Anyway, you can't get the right answer by dividing the mean alone by the standard deviation.

Look at the formula on page 56 and try again.

Your answer: $-.16$

Pretty close, but grading ain't horseshoes. Yes, the standard score had to be negative because the raw score is less than the mean. Your answer tells us that it is just barely below, only .16 standard deviation below. Is this logical? The mean is 62, and the raw score is 55 which gives us a deviation of 7 below. If the standard deviation is 4.5 how could you get only a fraction of one standard deviation?

Bet it's your decimals that give you trouble. Put the decimal in the right place when you divide.

Check page 65 again.

Your answer: $-.15$

The minus sign indicates that you realized that the score was below the mean, but your arithmetic went a bit sour. Watch the decimal points.

Find a better answer on page 71.

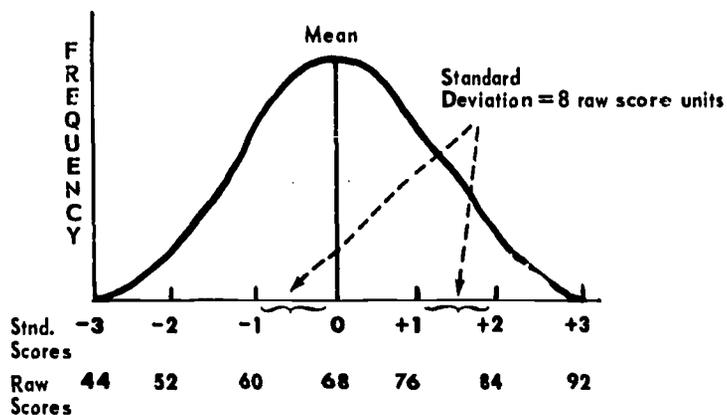
Your answer: -1.56

You have the method now and you weren't fooled by our rounding off at two decimal places. Did you do it this way?

$$\text{Standard Score} = \frac{X - M}{s} = \frac{55 - 62}{4.5} = \frac{-7}{4.5} = -1.555$$

Here we go with the next problem!

By way of review, the Standard Score is a relative method of grading where the position of any raw score is expressed in terms of deviation from the mean. A standard deviation, computed for any set of scores, gives us an idea of how those scores distribute themselves about the mean. A small standard deviation indicates that the distribution is tight and the range small. Standard scores above the mean are positive, those below, negative. The mean is expressed as zero. This graph will help you understand this:



STUDY IT CAREFULLY

You came from page 65

Go on to the next page

Standard scores need not always be whole numbers, but may be decimal fractions such as -1.5 . This tells us that the raw score is represented as one and five tenths standard deviations below the mean. If you care to work this out in the example above you will find the raw score to be 56.

Standard scores can be awkward to work with since we are dealing with positive and negative numbers. Students are often confused at having their grades reported in this way. Also instructors when recording grades might omit the minus. There is a difference between $+2$ and -2 , right?

In order to solve this problem it is easy to convert standard scores to a scale where we have no negative numbers. One such conversion is the T Score.

Here are key points to remember!

1. The mean on the T Score scale is always 50.
2. There are 10 T Score units in each standard score unit.

From this we can see that a standard score of zero (0), the mean, will equal a T score of 50. The standard score, $+1$, will equal a T score of 60. The standard, -1 , will equal a T score of 40.

Hence the rule, $T \text{ score} = 10 (\text{standard score}) + 50$

What is the T score equivalent to the standard score, $+1.5$?

	Page
A. 35	86
B. 48.5	89
C. 51.5	90
D. 65	87

Your answer: 27

Yes! -2.3 times 10 is -23
 -23 plus $50 = 27$

Adding numbers with different signs is actually finding the difference between them and assigning the sign of the larger.

Fortunately, in this process we always get positive T score values.

Just one more problem to clinch all this.

Find the T score for a raw score of 37 when:

Mean = 55
Standard Deviation = 12

	Page
A. 35	83
B. 48.5	91
C. 43.3	68
D. 65	82

Your answer: +1.6

Still having difficulty! You will get much help by remembering that if the raw score is below the mean the standard score will always be minus. Your computation was OK, but watch the signs. This is the proper substitution in the formula.

$$\text{Standard Score} = \frac{55 - 62}{4.5}$$

Your answer: 65

You found the deviation to be 18 all right and divided properly by the standard deviation (12). But wasn't the raw score below the mean? That should tell you that the standard score must be negative. This should make quite a difference in your answer.

Once more, page 80.

Your answer: 35

Good! Remember these steps.

$$37 - 55 = -18 \text{ (deviation)}$$

$$\frac{-18 \text{ (deviation)}}{12 \text{ (standard deviation)}} = -1.5 \text{ (standard score)}$$

$$-1.5 \times 10 + 50 = -15 + 50 = 35 \text{ (T score)}$$

This wasn't hard, was it?

THAT'S THE END OF THE LINE.

We have tried to clear up the various elements of some relative Methods of Grading which included rank order, centile rank, standard score and T score. Hope this has been a helpful learning experience for you.

You came from page 80



Your answer: 80

Didn't you notice that the standard score was negative?
Looks like you trapped yourself. In the formula

$$\text{Standard Score} = \frac{X - M}{s} \text{ you must}$$

have made the mistake of making it $M - X$. In this *incorrect*
approach you would have said:

$$\frac{60 - X}{8} = -2.5$$

$$60 - X = -20$$

$$-X = -20 - 60$$

$$\text{(raw score) } X = 80$$

Try it again and really understand why the raw score must
be 40.

Go on to page 78 then.

You came from page 65

Your answer: -3

Yes but! Subtracting the mean 68 from the 86 gives a deviation of $+18$ because it was above the mean.

$+18$ divided by 6 (the standard deviation) must make it $+3$ *not* -3 .

Your next problem is on page 56.

Your answer: 35

Multiplying 1.5 by 10 gave you 15. OK, *but* it was +1.5. We know that this score was well above the mean. If the T score mean is always 50 wouldn't it be 50 plus 15?

What do you get now?
Find the right answer on page 79.



Your answer: 65

Good!

+1.5 times 10 equals 15
15 plus 50 = 65

A good check —

If the standard score is negative, the T score will be below 50; if the standard score is zero the T score will be 50; and if the standard score is positive the T score will be above 50.

Now try another and remember all the rules.

Standard score is -2.3

What is the T score?

	Page
A. 73	88
B. 27	80

You came from page 79

Your answer: 73

Yes! -2.3 times 10 is -23 .

But, adding -23 and 50 can't give you 73 . What would it give? Work this out before you turn back to page 87 where you should find the correct answer.

Your answer: 48.5

You didn't remember to multiply the standard score by 10 did you? Besides, your answer tells us that this person is below the mean. The T score mean is always 50. His Standard score +1.5 is well above the mean so his T score must be well above 50.

Try page 79 again and pick a T score above the mean.

Your answer: 51.5

You realized that a standard score of +1.5 must be above the mean and that the T score therefore must be above 50 (the T score mean), but your error was in not multiplying +1.5 by 10.

Do that correctly and you will find the right answer on page 79.

Your answer: 48.5

It looks like you found the standard score properly, but did you multiply by 10 before you added it to 50?
Do that!

Try it once more, page 80.

Your answer: -5.0

You must have divided 40 by 8. This isn't the proper application of the formula since you didn't even use the raw score. First subtract the mean from the raw score.

Back to page 56.

SECTION II**CENTILE RANK**

1. Three people were tied at raw score 63; there were 60 people with lower raw scores. What was the centile rank for raw score 63?
A. 60
B. 61.5
C. 63
D. Not enough information
2. Fifty people took a test in which Jim tied, along with one other man, at a raw score of 82. What was Jim's centile rank?
A. 50.5
B. 52
C. 83
D. Not enough information
3. Out of a class of 90 students, 81 answered all questions correctly and 9 students missed 5 questions. If Jim was among the 9 students, what was his centile rank?
A. .5
B. 5
C. 10
D. Not enough information
4. The results of a test showed 20 people better than Joe and Harry. If none of the 50 people in the class had the same score as these two, what was Joe's centile rank?
A. 42
B. 51
C. 58
D. Not enough information
5. What is the centile rank of the top tied ten people in a class of 100?
A. 5
B. 10
C. 90
D. 95
6. What is the centile rank of the top two students tied in a class of 50?
A. 48
B. 49
C. 96
D. 98

13. Sgt. Smith and Sgt. Brown had the same raw score on a recent test. Twenty students in the class of 50 had scores lower than theirs. What was Sgt. Brown's rank order?
- A. 21.5/50
 - B. 29.5/50
 - C. 30/50
 - D. Not enough information

Use the following data on a given test for the next three problems (14, 15, and 16):

261 students in the class
The median score was 85
One student had a score of 85
The range was 62 - 135
Four students had scores of 62

14. What rank order would you assign the score 85?
- A. 85/261
 - B. 130.5/261
 - C. 131/261
 - D. Not enough information
15. What rank order would a 62 score have?
- A. 258/261
 - B. 259.5/261
 - C. 261/261
 - D. 2.5/261
16. What is the rank order for a raw score of 135?
- A. 135/261
 - B. 1/261
 - C. 261/261
 - D. Not enough information
17. How many students were better than Joe if his rank order was 25/75?
- A. 24
 - B. 25
 - C. I don't know

18. How many students might have tied if their rank order was 5.5/30?
- A. 7 C. 5
B. 3 D. 6

19. What rank order would you assign to the four people tied in a class of 75 if there were 22 better than they?
- A. 49/75 C. 24.5/75
B. 51.5/75 D. 26/75

Use the following distribution for the next six questions (20-25): 85, 85, 84, 83, 83, 83, 80, 79, 79, 75, 75, 75, 74, 74, 65

20. What is the rank order for the score 85?
- A. 1/15 C. 2/15
B. 1.5/15
21. What is the rank order for the score 65?
- A. 1/15 C. 15/15
B. 14/15
22. What rank order would you assign to 83?
- A. 4/15 C. 5/15
B. 4.5/15
23. What rank order would the score 80 have?
- A. 8/15 C. 7.5/15
B. 7/15
24. What is the rank order for the score 74?
- A. 13/15 C. 13.5/15
B. 14.5/15
25. What rank order would you assign to the median score?
- A. 8.5/15 C. 7.5/15
B. 8/15

CENTRAL TENDENCY AND DEVIATION

Use the data in the following distribution for problems 26-29:

x	f
50	1
45	4
40	8
35	9
30	6
25	5
20	4
15	2
10	1
	<hr/>
	40

26. The range of this distribution is:
- A. 9
 - B. 40
 - C. 30
27. The mode of this distribution is:
- A. 30
 - B. 35
 - C. 40
28. The median of this distribution is:
- A. the same as the mode.
 - B. larger than the mode.
 - C. the same as the range.
29. The mean of the distribution is:
- A. the same as the mode.
 - B. 35.25
 - C. 33.5
 - D. 32.25

3. B. Since the nine students had nobody below them it is:

$$\frac{4.5}{90} \times 100 = .05 \times 100 = 5$$

4. C. There were 28 below, hence

$$\frac{28 + 1}{50} \times 100 = \frac{29}{50} \times 100 = 58$$

5. D. $\frac{90 + 5}{100} \times 100 = \frac{95}{100} \times 100 = 95$

6. D. $\frac{48 + 1}{50} \times 100 = \frac{49}{50} \times 100 = 98$

7. B. $\frac{99 + .5}{100} \times 100 = \frac{99.5}{100} \times 100 = 99.5$

We take half of those tied even if one man is tied with himself.

8. C. 90 students below and 90 above;

$$\frac{90 + 1.5}{183} \times 100 = \frac{91.5}{183} \times 100 = 50$$

9. A. $\frac{50 + .5}{101} \times 100 = \frac{50.5}{101} \times 100 = 50$

10. B. You must first determine that there are 126 people in the class, then:

$$\frac{75 + .5}{126} \times 100 = \frac{75.5}{126} \times 100 = 59.9$$

11. D. The average of the top two positions: $1.5/65$
12. C. Not enough information — we don't know the number of students in the class.
13. B. If twenty people were below them, they had positions 29 and 30. You must average these positions:

$$\frac{29 + 30}{2} = 29.5$$

Sgt. Brown's rank order, then, (the same as Smith's) would be $29.5/50$

14. C. If 85 is the median score there were 130 above as well as below. That would make it $131/261$.
15. B. Average the four bottom positions:

$$\frac{261 + 260 + 259 + 258}{4} = 259.5$$

Rank order, then, is $259.5/261$

16. D. If you knew how many students had that score you could assign a rank order to it.
17. C. Don't know. You are correct here, but do you know why? If we had been told that nobody was tied with him, we could get the answer.
18. D. Only when an even number of students are tied does the averaging of positions yield a decimal; e.g., averaging positions 3, 4, 5, 6, 7, and 8 would yield 5.5.
19. C. They would be the average of positions 23, 24, 25, 26. The average of these positions is 24.5. Rank order, then, is $24.5/75$.
20. B. The average of positions 1 and 2 = 1.5. Rank order, then, is $1.5/15$.

21. C. The bottom man, without a tie, gets the largest number.
22. C. The 83's occupy positions 4, 5, 6. Average is 5. Rank order, then, is 5/15.
23. B. There were six better than the 80 so the rank order for 80 would be 7/15.
24. C. The 74's occupy positions 13, 14. The average would be 13.5. Rank order, then, is 13.5/15.
25. A. The median score is 79. The two scores here occupy positions 8 and 9. The average would be 8.5. Rank order, then, is 8.5/15.
26. B. The lowest score is 10, the highest, 50. $50 - 10 = 40$
27. B. The score with the highest frequency in this distribution is 35. In other words, 35 is the most popular score so 35 is the mode.
28. A. The mode is 35 and the median also falls at 35. There should be as many students below as above the median.
29. D. Multiply each frequency by its raw score (fx). The mean is the total of all fx 's divided by the number in the group (N).
- $$M = \frac{\sum fx}{N} = \frac{1290}{40} = 32.25$$
30. B. Don't be fooled by the -6 . This only means that the deviation is below the mean, but it is actually as far away from the mean as the $+6$ is.
31. A. If the -6 was six raw score units below the mean, the mean must be $55 + 6 = 61$.

32. B. If the mean is 61, the +3 deviation is $61 + 3 = 64$.

33. A. The 67 is higher than the mean so:
 $X - M = 67 - 43.5 = +23.5$

34. A. We know that this is below the mean so:
 $43.5 - 16.5$ yields a raw score of 27.

35. A. $63 - 7 = 56$

36. B. $\frac{X - M}{s} = \frac{42 - 63}{7} = \frac{-21}{7} = -3$

37. D. +1.0 is one standard deviation. Hence:
 $63 + 7 = 70$

38. B. Each student deviated 24, one above (+24) and the other below (-24). Their raw scores were 48 units apart:

$$97 - 49 = 48 \quad \frac{48}{16} = 3$$

Another way of looking at this is:

$$\frac{+24}{16} = +1.5 \text{ and } \frac{-24}{16} = -1.5$$

If one is +1.5 (above) and the other -1.5 (below) they must be 3 apart.

39. D. -2.5 standard deviations

$$\begin{aligned} -2.5 \times 16 &= -40 \\ 73 - 40 &= 33 \end{aligned}$$

40. B. One student (0) was at the mean, 73; the other (-1.5×16) was -24 below the mean or 49.

Now,

$$73 - 49 = 24 \text{ raw score units apart}$$

41. D. $10 (+1.0) + 50 = 60$

42. A. $10 (-1.3) + 50 = 37$

43. C. $\frac{42 - 48}{12} = \frac{-6}{12} = -.5$

$$10 (-.5) + 50 = 45$$

44. C. $\frac{66 - 48}{12} = \frac{18}{12} = 1.5$

$$10(1.5) + 50 = 65$$

45. D. A T score of 35 is equivalent to a standard score of -1.5 . A standard deviation is equal to 12 raw score units.

$$-1.5 \times 12 = -18 \quad \begin{array}{l} \text{Raw score units} \\ \text{below the mean} \end{array}$$

$$48 - 18 = 30$$

46. A. A T score of 25 is equivalent to a standard score of -2.5 .

$$-2.5 \times 12 = -30 \quad \begin{array}{l} \text{Raw score units} \\ \text{below the mean} \end{array}$$

$$-30 + 48 = 18$$

47. B. T score of 50 is the mean.

48. B. $65 - 53 = 12$ raw score units *below* the mean

$$\text{standard score} = \frac{X-M}{s}, \quad -1.5 = \frac{53-65}{s}$$

$$-1.5s = -12$$

$$s = 8$$

49. C. $2.3 \times 8 = 18.4$ raw score units above the mean
 $87 - 18.4 = 68.6$ (mean)

50. A. A T score of 45 is equivalent to a standard score of -5 . $8.6 \times -5 = -4.3$ raw score units. If his raw score was 80 and he was 4.3 raw score units below the mean, then the mean is:

$$80 + 4.3 = 84.3$$

51. D. A T score of 76 is equivalent to a standard score of 2.6.

$$76 - 50 = \frac{26}{10} = 2.6$$

$$2.6 \times 15 = 39$$

$$65 + 39 = 104$$

52. A. A T score of 34 is equivalent to a standard score of -1.6 .

$$34 - 50 = \frac{-16}{10} = -1.6$$

The student's deviation would = $60 - 73.6 = -13.6$ raw score units. His deviation, then, divided by his standard score would equal the standard deviation

$$\frac{-13.6}{-1.6} = 8.5, \text{ the standard deviation}$$

INDEX

Should you want to review any of the areas covered in this booklet, the page numbers indicate where you should begin.

Page

- 9 - Centile Rank
- 26 - Rank Order
- 33 - Mean
- 45 - Median
- 49 - Mode
- 60 - Range
- 53 - Deviation
- 66 - Standard Deviation
- 56 - Standard Score
- 79 - T Score