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ABSTRACT

This guidebook covers Boolean algebra, matrices, linear transformations of the plane, characteristic values, vectors, and algebraic structures. Overall course goals and performance objectives for each unit are specified; sequencing of units and various time schedules are suggested. A sample pretest and posttest are given, and an annotated list of 14 references is included. (DT)

AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

DADE COUNTY PUBLIC SCHOOLS

MODERN ALGEBRA

5293.36

MATHEMATICS

DIVISION OF INSTRUCTION•1971

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

Modern Algebra

5293.36

(EXPERIMENTAL)

Written by

Raymond J. Edwards

for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
1971-72

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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

A formal development of beginning matrix algebra and the abstract algebra of groups, rings, fields, integral domains, etc. Designed for those interested in higher mathematics.

Designed for the student who has mastered the skills and concepts of Analytic Geometry 2 or Mathematical Analysis 3.

TABLE OF CONTENTS

GOALS

I. To survey the basic concepts of modern algebra in order to gain insight into the selection of college-level mathematics courses in this area.

II. To examine matrices as they relate to algebraic structures and to various scientific, commercial, or business studies.

III. To examine examples of abstract algebraic structures, their interrelationships and applications to other branches of mathematics and science.

PERFORMANCE OBJECTIVES

The student will:

I. BOOLEAN ALGEBRA

1. Define and identify the operations and properties of a Boolean algebra.

2. Prove Boolean algebra theorems by the use of step-reason arguments and by the use of truth value tables.

3. Apply Boolean arguments to special cases such as logical statements, set relations, or switching circuits.

II. MATRIX OPERATIONS

1. Define, identify or determine each of the various items related to or making up matrices; such as order, transpose, ij element of, symmetric matrix.

2. Determine the sum and product of two given conformable matrices.

3. Determine the scalar product of a given matrix and a given real number.

4. Identify the properties of matrix operations and apply them to find the simplest form of a given matrix expression.

III. SYSTEMS OF EQUATIONS

1. Given a system of linear equations, write the system in matrix form and vice-versa.

2. Solve a system of linear equations by Gauss-Jordan elimination or by elementary row transformations.

3. Identify dependent and inconsistent systems.

IV. MATRIX INVERSION

1. Determine if a given matrix is singular or non-singular.

IV. MATRIX INVERSION (Continued)

2. Find the inverse of a given non-singular matrix by adjoint formula if it is second or <u>third</u> order and by Gauss-Jordan reduction if it is <u>third</u> order or higher.

3. Solve a system of linear equations by use of the inverse of the coefficient matrix.

V. LINEAR TRANSFORMATIONS OF THE PLANE

1. Given a simple linear homogeneous transformation in the xy-plane:

a. In equation or matrix form, describe its geometric effects.

b. In equation form, write a matrix for the transformation.

c. In descriptive form, write a set of equations and a matrix for the transformation.

d. Find its inverse transformation.

e. Express it as the product of elementary transformations and interpret geometrically.

2. Given several linear transformations in sequence, use matrix multiplication to obtain one equivalent transformation.

3. Given a transformation of the plane, indicate whether it is linear or non-linear.

VI. DETERMINANTS

Given an nth order matrix:

1. Define its determinant.

2. Determine the minor of any indicated element.

3. Determine the cofactor of any indicated element.

4. Determine its adjoint matrix and use the result to find its inverse.

VI. DETERMINANTS (Continued)

5. Evaluate its determinant by expansion or by reduction using elementary row and/or column transformations.

6. Use properties of determinants to evaluate the determinant of some related matrix, such as its transpose or its inverse.

VII. CHARACTERISTIC VALUES AND THEIR APPLICATIONS

1. Given an nth order matrix, define its characteristic polynomial, equation, and values.

2. Given second or certain third order matrices, determine the characteristic polynomials, equations, and values.

3. State the Cayley-Hamilton theorem; prove it for second order matrices; and use its results to determine the nth power of a given matrix and to solve problems in progressions, minimum functions, infinite series, Markov chains or others as selected by the teacher.

VIII. INTRODUCTION TO VECTORS

1. Define a geometric (free) vector in 2 dimensions as a row or column matrix, and relate the definition to the equivalence classes of directed segments.

2. Define and graph the sums and scalar products of 2-dimensional vectors.

3. Define and be able to find the components, magnitude and direction angle, (or direction cosines,) of a two-space vector.

4. Define and evaluate the inner product of two 2-space vectors and use the results to determine:

a. if they are collinear or not,

b. if they are orthogonal or not and

c. the angle between them.

IX. ABSTRACT ALGEBRA

1. Define and identify some of the following: Groupoid, semi-group, monoid, group, Abelian group, ring, ring with unity, integral domain, field, ordered field, vector space, algebra. (Teacher's choice.)

2. Define isomorphism and give examples of isomorphisms between some of the above-mentioned structures.

3. Give examples of the above structures and list some of their properties.

4. Prove simple theorems about the structures studied.

SEQUENCE, SOURCES AND STRATEGIES

A key to the text code will be found in the list of references on p-29. Obviously the material in the following sequence is too extensive for a 9-week quinmester. The teacher is expected to select a sequence from the listed material. Do not interpret the estimated time allowances as requirements. These estimates are included to help the teacher select a sequence of units that will fit a 9-week, (36-lesson, 9-test) quinmester.

OPTION	UNITS	TOTAL TIME (without tests)
A	I, II, IV, VI, VIII, IX	36 days
B	II, III, IV, VI, VII, IX	40 days
C	II, III, IV, V, IX	39 days

Perhaps the most thorough reference, and one with which the teacher should be thoroughly familiar, is Birkhoff and MacLane's A Survey of Modern Algebra. It should be used as a source of examples as well as background material.

SEQUENCE

I. BOOLEAN ALGEBRA

A. Introduction, Operations and Properties, Duality

B. A Simple Model of a Boolean Algebra, $(\{0,1\}, \vee, \cdot)$

C. The Algebra of Sets, $(2^{U}, \cup, \cap, ')$

D. The Algebra of Logical Propositions

E. Application to Switching Circuits (Optional)

SOURCES AND STRATEGIES

First choice as text: Nahikian: pp 53-97. (5 days)

Reference: Andree: pp 54-77.

Birkhoff: pp 313-319.

II. MATRIX OPERATIONS

A. Introduction, Nomenclature, Equality

B. Addition and Subtraction

C. Scalar Multiplication

D. Matrix Multiplication

E. Properties of Matrix Multiplication

F. Applications (Time permitting)

First choice as text: Davis: pp 1-71. (11 days.)

Second choice as text: SMSG: pp 1-51. (Not as deep as Davis, time may be reduced by using this text, or its exercises.)

References: Nahikian: pp 159-178. (Too brief.)

McCoy: pp 232-239.

Andree: pp 104-137, p-168 for transpose.

Birkhoff: pp 194-203.

III. SYSTEMS OF EQUATIONS

A. Matrix Form of a System of Linear Equations

B. Gauss-Jorden Elimination

C. Elementary Row Transformations

First choice as text: Davis: pp 57-83. (4 days)

Second choice as text: SMSG: pp 103-132.

References: McCoy: pp 221-245.

Andree: pp 145-158. (adjoint matrix p-173)

Birkhoff: pp 159-166.

IV. MATRIX INVERSION

A. Singular and Non-Singular

B. Algebra of Inverses

C. Gauss-Jordan Reduction

D. Identities (Optional)

E. Solution of Systems by Inversion

First choice as text: Davis: pp 83-116. (7 days)

Second choice as text: SMSG: pp 62-85, 113-123.

References: Andree: pp 114-118. (Read the problems for examples of proofs. pp 173-176)

Birkhoff: pp 207-222

V. LINEAR TRANSFORMATIONS OF THE PLANE

A. Relations, Correspondences, Transformations, and Mappings, (Reviewed)

B. Transformations of the Plane, (Simple Transformations)

C. Matrices as Transformations

1. Products of transformations
2. Inverse transformations

D. Linear Homogeneous Transformations

1. Define and described
2. Degenerate forms, projections

E. Non-homogeneous Linear Transformations

First choice of text: Davis: pp 125-169. (13 days)

Second choice of text: SMSG: pp 177-218, (Time can be reduced)

References: Nahikian: pp 199-211. (Brief)

McCoy: pp 268-299.

References: (Continued)

Andree: p-136.

Birkhoff: pp 166-175.

VI. DETERMINANTS

A. Introduction. Some Properties

B. Determinants. Cofactors and Inverses

C. The Properties of Determinants

D. Geometry of Determinants, (Optional)

E. Cramer's Rule

First choice of text: Davis: pp 171-206. (5 days)

References: McCoy: pp 246-267.

Andree: p-159.

Birkhoff: pp 280-312.

VII. CHARACTERISTIC VALUES AND THEIR APPLICATION

A. Characteristic Polynomial, Equation, (and Vector, Optional)

B. Cayley-Hamilton Theorem

C. Finding Characteristic Value

D. High Power of a Matrix

E. Difference Equation

First choice as text: Davis: pp 265-295. (9 days)

References: Nahikian: pp 214-222. (More Markov chains if the student has some knowledge of probability)

Andree: pp 133-134. (Very good example of a Markov chain, an application of high powers of a matrix)

VIII. INTRODUCTION TO VECTORS

A. Column Matrices as Vectors

B. Components, Magnitude and Direction

C. The Inner Product and Angles, Lines, Half-Planes, Convex sets (Be selective)

First choice as text: Davis: pp 207-235 (5 days)

Second choice as text: SMSG: pp 133-175.

References: Nahikian: pp 135-140, 156-158.

Birkhoff: pp 149-177.

IX. MATRICES AND ABSTRACT ALGEBRA

A. The Complex Number Field

B. An Example of a Group, Transformations or Non-singular Matrices. (2x2)

First choice as text: SMSG: pp 53-71, 85-102. (3 days)

References: Davis: pp 296-312. (Skimpy)

Nahikian: pp 127-135. (Best examples of structures 'smaller' than a group)

The state adopted texts are very brief on the above material. Following is a list of structures and references where the structures are defined and some examples given. From this list, section IX can be expanded if that is desired.

ALGEBRAIC STRUCTURES

A. Groupoid, Semi-group, Monoid...Nahikian p-127, Birkhoff Chapter 6.

B. Group.......... . . Davis (no def.) p-310, Nahikian p-127, McCoy p-167, Andree p-78, SMSG p-90, Crouch and Beckman p-25, Birkhoff Chapter 6.

ALGEBRA STRUCTURES (Continued)

C. Abelian Group.	McCoy p-169, Andree p-185, SMSG p-90, Crouch and Beckman p-31, Birkhoff Chapter 6.
D. Ring	Nahikian p-130, McCoy p-12, Andree p-183, SMSG p-58, Birkhoff 1-10, 25-30 Chapter 13.
E. Integral Domain.	McCoy p-36, Andree p-7, Birkhoff 1-11 (compare these.)
F. Field.	Davis p-299, Nahikian p-131, McCoy p-73, Andree p-180, SMSG p-55, Crouch and Beckman p-12, Birkhoff Chapter 2.
G. Vector Space	Nahikian p-141, McCoy p-197, SMSG p-166, Crouch and Beckman p-69, Birkhoff Chapter 7.
H. Algebra.	McCoy p-271. Birkhoff pp 222-228.

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SAMPLE PRETEST

1. Simplify: $\frac{1}{y} + \frac{2}{3 + y}$, (assume $y \neq -3$ and $y \neq 0$).

2. For what real numbers x does $3x^2 - x - 4 = 0$?

3. If $\frac{x}{3} + \frac{5}{6} = 2$, then $x =$ (?).

4. If $f(x) = 2x^3 - x + 2$, then $f(-1) =$ (?).

5. If $\frac{3}{3k + x} = \frac{1}{k + 1}$ and $k \neq -1$, then $x =$ (?).

6. If $m = \sqrt{3}$, then $m^{-4} =$ (?).

7. Simplify: $\sqrt[3]{\frac{27}{8}}$.

8. If $x^3 - 8x^2 = 0$, then $x =$ (?).

9. For what value of k is $x - 1$ a factor of $3x^5 - k$?

10. - 12. Given: $A = \{-4, -3, -2, -1, 0\}$, $B = \{0\}$,
$C = \{0, 1, 2, 3, 4\}$, and $D = \{1, 2, 3, 4\}$.

10. $A \cap C =$ (?). 11. $D \subset$ (?). 12. $B \cup D =$ (?).

13. Simplify: $\frac{2\sqrt{2}}{1 + 3\sqrt{5}}$ 14. $\frac{9}{n} + \frac{7}{n} = \frac{1}{6}$, find n.

15. Solve the equation: $x^2 - 9x + 18 = 0$, for x, over the reals.

16. Find the complex conjugate of $7 + 2i$.

17. Add: $3 + \sqrt{-2}$; $4 - \sqrt{-8}$.

18. Evaluate: $|5 - 12i|$.

19. Find all three complex cube roots of eight.

20. What is the slope of the line whose equation is $2y + 6x = 1$?

21. When 72 is divided by a certain positive integer, the result is twice this integer. Find it.

SAMPLE PRETEST (Continued)

22. $\text{Log}\ \frac{x}{y} = (?)$ 23. If log 2 $\approx$ .3010, what is log 8?

24. If $d^3 - 7d - 6$ is divided by $d + 1$, the remainder is (?).

25. Solve over R: $(x - 3)(x + 1) > 0$

26. If (2, -3) and (-6, 9) are ordered pairs of the solution set of $y = mx + b$, then $m = (?)$.

27. If $f(x) = 4x + 3$ and $g(x) = x^2 - 2$, then $f(g(x)) = (?)$.

28. Find the vertices of the graph of: $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

29. If the solution set of the equation $x^2 + 4x + c = 0$ contains two real elements, describe c.

30. Solve for x, over the reals: $|x - 2| < 4$.

31. - 34. Find the solution set of all (x, y), or (x, y, z):

31. $3x + 5y = -12$

$4x - 2y = 10$

32. $6x + 14y = 22$

$9x + 21y = 33$

33. $2x + 3y - 5z = 9$

$-4x + 9y + 2z = 3$

$5x - 3z = 1$

34. $7x + y = 1$

$14x + 2y = 1$

35. - 38. Write the name of the property indicated or exemplified in each of the following statements. Literal factors represent real numbers unless otherwise indicated.

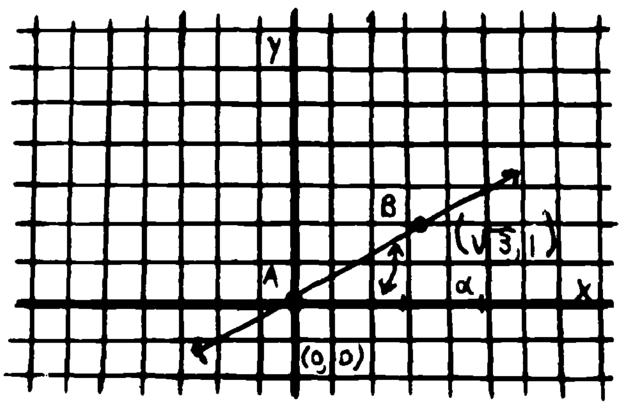
35. $ab = ba$. 36. $3x + 2x = (3 + 2)x$. 37. $(ab)c = a(bc)$.

38. If $a \in \{\text{All integers}\}$, and $b \in \{\text{All integers}\}$, then ab is integral.

39. The set $\{0, 2, 4, 6, 8, \ldots\}$, is closed for what operations?

SAMPLE PRETEST (Continued)

40. - 42. Given the figure below:



40. Find the distance AB. 41. Find sin α

42. Find α in degrees or radians.

43. Describe the graph of $x^2 + y^2 = 36$.

44. Sketch the graph of $4(y - 2) = (x - 5)^2$.

45. Sketch the graph of $|x + 1| + |y - 3| < 5$.

ANSWERS TO PRETEST

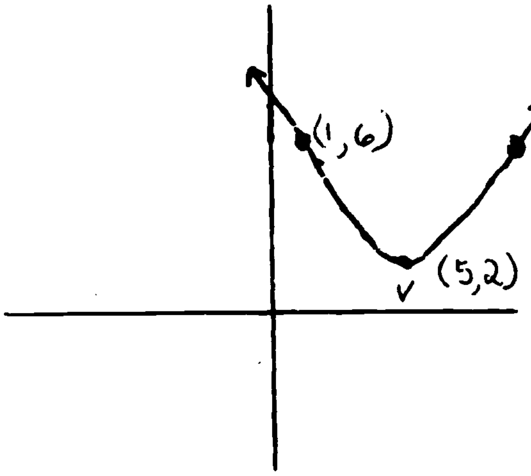
1. $\frac{3 + 3y}{3y + y^2}$
2. $\left\{\frac{4}{3}, -1\right\}$
3. $\frac{7}{2}$
4. -2
5. $x = 3$
6. $\frac{1}{9}$
7. $\frac{3}{2}$
8. $\{0, 8\}$
9. $k = 3$
10. B
11. C
12. C
13. $\frac{3\sqrt{5} - \sqrt{2}}{22}$
14. 72
15. $\{3, 6\}$
16. $7 - 2i$
17. $7 - i\sqrt{2}$
18. 13
19. $\{2, -1 \pm i\sqrt{3}\}$
20. -3
21. $n = 6$
22. $\log x - \log y$
23. 0.9030
24. -14
25. $\{x: x > 3 \vee x < -1\}$
26. $\frac{-3}{2}$
27. $4x^2 - 5$
28. $(-4, 0)$; $(4, 0)$
29. $c < 4$
30. $\{x: -2 < x < 6\}$
31. $\{(1, -3)\}$
32. $\{(x, y): 3x + 7y = 11\}$
33. $\left\{\left(-1, \frac{1}{3}, -2\right)\right\}$
34. $\emptyset$
35. Commutativity, multiplication
36. (Left-hand) distributivity
37. Associativity, multiplication
38. Closure over the integers, multiplication
39. Addition and multiplication (and raising to powers.)
40. 2
41. $\frac{1}{2}$

ANSWERS TO PRETEST (Continued)

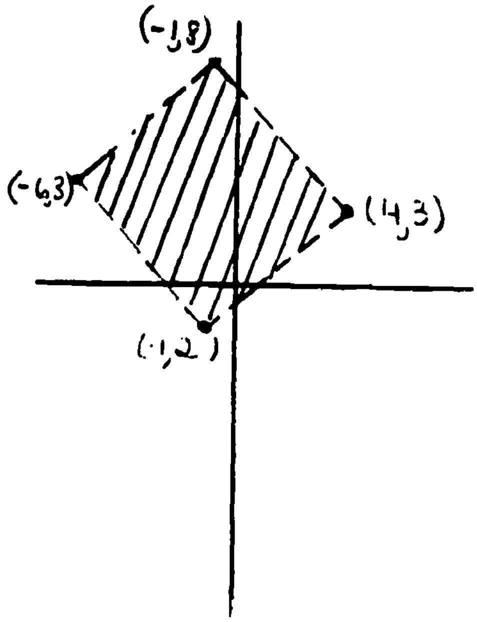
42. 30° or $\frac{\pi}{6}$ rad.

43. Circle with center at the origin and radius of 6.

44.



45.



SAMPLE POSTTEST

I. Boolean Algebra:

1. The _____ property says $a \vee a = a$.

2. The dual of $(x' \cdot y') = x \vee y$ is _____.

3. Complete the table of truth values:

a	b	a'	b ∨ a'
0	0		
0	1		
1	0		
1	1		

4. Construct a truth table to prove:

   a) $a \cdot (b \vee c) = (a \cdot b) \vee (a \cdot c)$.

   b) $(a \cdot b)' = a' \vee b'$.

   c) $(p \rightarrow q) \leftrightarrow (p' \vee q)$.

II. Matrix Operations:

5. Find the sum of each pair of matrices below.

   a) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$; $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

   b) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$; $\begin{pmatrix} -1, & 5, & 7 \end{pmatrix}$

   c) $\begin{pmatrix} 4 & 0 \\ 3 & 0 \end{pmatrix}$; $\begin{pmatrix} 0 & 0 \\ 1 & 2 \end{pmatrix}$

   d) $3\begin{pmatrix} 4 & 1 \\ -3 & 2 \end{pmatrix}$; $\frac{1}{2}\begin{pmatrix} -2 & 0 \\ 1 & 0 \end{pmatrix}$

   e) $\begin{pmatrix} 0 & 1 & 4 \\ 2 & 0 & 3 \\ 5 & 6 & 0 \end{pmatrix}$; $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 5 \\ 3 & 4 & 1 \end{pmatrix}$

6. Find the product of each pair of matrices above.

SAMPLE POSTTEST (Continued)

7. If $A = \begin{pmatrix} 3 & 1 \\ 2 & 8 \\ 4 & 1 \end{pmatrix}$, find A'.

8. If $B = \begin{pmatrix} 3 & 1 \\ x & 5 \end{pmatrix}$ and B is a symmetric matrix, find x.

9. If A = B and $A = \begin{pmatrix} x & y & z \\ 3 & 1 & 6 \end{pmatrix}$ while $B = \begin{pmatrix} 4 & 0 & 7 \\ m & n & p \end{pmatrix}$, find y.

10. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, then a_{21} = (?).

11. Find (a) $\left(r \begin{pmatrix} u & v \\ w & x \end{pmatrix} \right) \begin{pmatrix} u & v \\ w & x \end{pmatrix}$ (b) $\begin{pmatrix} u & v \\ w & x \end{pmatrix} \left(r \begin{pmatrix} u & v \\ w & x \end{pmatrix} \right)$

III. Systems:

12. Write a matrix equation for each system below:

$$x + y - z = 0$$

a. $$2x + 4y - z = 0$$

$$3x + 2y + 2z = 1$$

$$2x_1 - 3x_2 + 6x_3 + 2x_4 - 5x_5 = 3$$

b. $$x_2 - 4x_3 + x_4 = 1$$

$$x_4 - 3x_5 = 2$$

13. Solve by Gauss-Jordan reduction:

$$m + 2n - 2p = 10$$
$$-2m + 3n + 2p = 1$$
$$4m + 5n + 3p = 4$$

14. $2x + 3y = 1$

$5x + 7y = 3$. Write a matrix equation and solve by inversion.

15. Solve the following system by elementary row operations:

(x, y)

$ax + by = e$

$cx + dy = f$

IV. Inverses:

16. If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ find:
   (a) The minor of a_{31}.
   (b) The cofactor of a_{32}.
   (c) The adjoint of A.
   (Section VI.)

17. Find the inverse of each of the following:

(a) $\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -1 & -1 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 3 & 2 & 4 \\ 4 & 6 & 5 & 5 \\ 3 & 5 & 2 & 14 \\ 2 & 2 & -3 & 14 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -4 \\ 3 & 4 & 0 \end{bmatrix}$

V. Linear Transformations

18. The curve: $y = 2x^3$ is reflected in the line: $y = x$. Find the new equation.

19. The curve: $y = f(x)$ is rotated 90°. Find the new equation.

20. If $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is orthogonal, then $a^2 + b^2 = (?)$

21. Write the matrix form of the reflection:

$x' = y$

$y' = x$

22. Given the expansion: $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$.

Describe its effect on the graph of $x^2 + y^2 = 9$.

23. What happens to the line $3x + 2y = 5$ under the shear given by $x' = x$ and $y' = \frac{1}{2}x + y$?

24. Write the matrix of transformation corresponding to the combined effects of the rotation:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

followed by the shear:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

VI. Determinants

25. Find the determinant of:

(a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 3 \\ 4 & 3 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 1 \\ 5 & 1 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 1 & 0 \\ -3 & 0 & -5 \\ -4 & -2 & -6 \end{pmatrix}$

26. If the determinant of A is three, find the determinant of: (a) $A^T = A'$ (A transpose) (b) A^{-1} (c) $3A$

27. Use Cramer's Rule to solve:

(a) $\begin{aligned} 3x + 34y &= 7 \\ 9x - 17y &= 0 \end{aligned}$ (b) $\begin{aligned} x + 2z &= 1 \\ 3x + y + 3z &= 1 \\ y + 2z &= 0 \end{aligned}$

SAMPLE POSTTEST (Continued)

VII. Characteristic Values:

28. Find the characteristic roots of the matrix $\begin{pmatrix} 4 & 7 \\ 2 & -1 \end{pmatrix}$.

29. Find the characteristic roots and the corresponding vector for each root of $\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$.

30. Write the characteristic polynomial for $\begin{pmatrix} 2 & 7 \\ 0 & 4 \end{pmatrix}$.

31. Find the characteristic equation of $A = \begin{pmatrix} 3 & 0 & 6 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ and demonstrate the Cayley-Hamilton theorem for matrix A.

32. Given that the characteristic roots of $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ are $\lambda_1 = -1$ and $\lambda_2 = 5$. Use the Division Algorithm and the Cayley-Hamilton theorem to find A^3 and verify your answer by direct computation.

VIII. Introduction to Vectors

33. On a coordinate plane, draw the standard position directed segment representing each vector:

(a) A, if $A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (b) $-B$, if $B = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$

(c) 3C, if $C = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ (d) D, if $D = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \oplus \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

(e) Show D as the resultant of the addends given in problem d.

ANSWERS TO POSTTEST

I. 1. Idempotent

2. $x' \vee y' = x \cdot y$

3.

a	b	a'	b ∨ a'
0	0	1	1
0	1	1	1
1	0	0	0
1	1	0	1

4. a.

a	b	c	b ∨ c	a . (b ∨ c)	a . b	a . c	(a . b) ∨ (a . c)
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

b.

a	b	a . b	(a . b)'	a'	b'	a' ∨ b'
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

ANSWERS TO POSTTEST (Continued)

c.

p	q	$p \rightarrow q$	p'	$p' \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

II. 5. a. $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ b. impossible c. $\begin{pmatrix} 4 & 0 \\ 4 & 2 \end{pmatrix}$ d. $\begin{pmatrix} 11 & 3 \\ -8\frac{1}{2} & 6 \end{pmatrix}$

e. $\begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 8 \\ 8 & 10 & 1 \end{pmatrix}$

6. a. impossible b. $\begin{pmatrix} -1 & 5 & 7 \\ -2 & 10 & 14 \\ -3 & 15 & 21 \end{pmatrix}$ c. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

d. $\begin{pmatrix} -10\frac{1}{2} & 0 \\ 12 & 0 \end{pmatrix}$ e. $\begin{pmatrix} 12 & 17 & 9 \\ 11 & 16 & 3 \\ 5 & 16 & 30 \end{pmatrix}$

7. $\begin{pmatrix} 3 & 2 & 4 \\ 1 & 8 & 1 \end{pmatrix}$

8. $x = 1$

9. $y = 0$

10. $a_{21} = 4$

11. a. $\begin{pmatrix} ru^2 + rvw & ruv + rvx \\ ruw + rwx & rwv + rx^2 \end{pmatrix}$ b. $\begin{pmatrix} ru^2 + rvw & ruv + rvx \\ ruw + rwx & rwv + rx^2 \end{pmatrix}$

III. 12. a. $\begin{pmatrix} 1 & 1 & -1 \\ 2 & 4 & -1 \\ 3 & 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

ANSWERS TO POSTTEST (Continued)

12. b. $\begin{pmatrix} 2 & -3 & 6 & 2 & -5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 0 & 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

13. $m = 2$, $n = 1$, $p = -3$.

14. $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $x = 2$, $y = -1$

15. $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{ad - bc} \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} \begin{pmatrix} e \\ f \end{pmatrix} = \frac{1}{ad - bc} \begin{pmatrix} de - cf \\ af - be \end{pmatrix}$

IV. 16. a. $\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$ b. $a_{21}a_{13} - a_{11}a_{23}$

c. $\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$ where A_{ij} is the cofactor of a_{ij}

17. a. $\frac{1}{7}\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}$ b. $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{pmatrix}$ c. $\begin{pmatrix} -23 & 10 & 1 & 2 \\ 29 & -12 & -2 & -2 \\ -\frac{64}{5} & \frac{26}{5} & \frac{6}{5} & \frac{3}{5} \\ -\frac{18}{5} & \frac{7}{5} & \frac{2}{5} & \frac{1}{5} \end{pmatrix}$

d. There is none, the matrix is singular.

V. 18. $x = 2y^3$

19. $x = f(-y)$

20. $a^2 + b^2 = 1$

21. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

ANSWERS TO POSTTEST (Continued)

22. Contraction along the y-axis by a factor of $\frac{1}{3}$; so the circle becomes the ellipse $\frac{x^2}{9} + \frac{y^2}{1} = 1$

23. $4x + 2y = 5$

24. $\begin{pmatrix} \cos\theta + 3\sin\theta & 3\cos\theta - \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

VI. 25. a. 1 b. -6 c. 9 d. -18

26. a. 3 b. $\frac{1}{3}$ c. 9

27. a. $\left(\frac{1}{3}, \frac{3}{17}\right)$ b. $\left(\frac{1}{5}, \frac{-4}{5}, \frac{2}{5}\right)$

VII. 28. $(6, -3)$

29. $1 + 2i \rightarrow (-1, 1)k$
$1 - 2i \rightarrow (1, -1)k$

30. $\begin{vmatrix} x+2 & 7 \\ 0 & x+4 \end{vmatrix}$ or $x^2 + 6x + 8$

31. $x^3 - 9x^2 + 26x - 24 = 0$

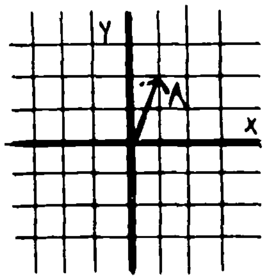
$A^3 - 9A^2 + 26A - 24I = 0_3$, substitute to verify.

32. See Davis p. 275;

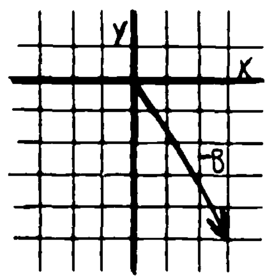
$$A^n = \frac{\lambda_2 \lambda_1^n - \lambda_1 \lambda_2^n}{\lambda_2 - \lambda_1} I + \frac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1} A, \quad \lambda_1 \neq \lambda_2$$

Substitution yields: $A^3 = 20I + 21A = \begin{pmatrix} 41 & 84 \\ 42 & 83 \end{pmatrix}$

VIII. 33. a.

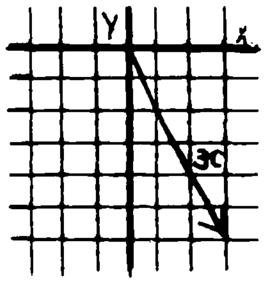


b.

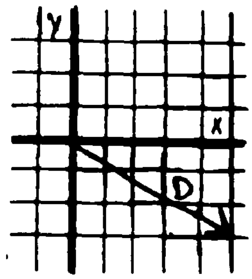


ANSWERS TO POSTTEST (Continued)

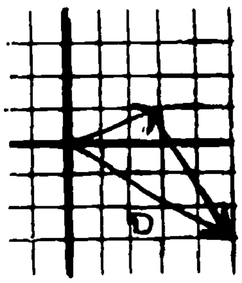
33. c.



d.



d.



BIBLIOGRAPHY

I. Books Mentioned in the Sources.

Code Name:	Book:
Andree	Andree, Richard V. Selections From Modern Abstract Algebra. New York: Holt, Rinehart and Winston, 1958. A very thorough reference. Too difficult for a text for high school.
Birkhoff	Birkhoff, Garret, and MacLane, S. A Survey of Modern Algebra, (3rd edition). New York: MacMillan, 1965. This is the classic reference; a must for the teacher. (Not to be confused with the more modern - 1967 - book by the same authors, Algebra, which is considerably more abstract).
Crouch and Beckman	Crouch, Ralph and Beckman, David. Fundamental Mathematical Structures-Linear Algebra. Chicago: Scott, Foresman and Co., 1965. Could be used as a text for an outstanding group of students. Very structured, rigorous but clear.
Davis	Davis, Philip J. The Mathematics of Matrices. 2nd Edition. New York: Ginn and Co., 1965. Primary text for this quin. Excellent, but contains more material than can be covered in 9 weeks.
McCoy	McCoy, Neal H. Introduction to Modern Algebra. College Mathematics Series. Boston: Allyn and Bacon Inc., 1960. Good reference. Advanced in level and in notation.

BIBLIOGRAPHY

I. Books Mentioned in the Sources.

Code Name:	Book:
Nahikian	Nahikian, Howard M. Topics in Modern Mathematics. London: The Macmillan Company, Collier-Macmillan Ltd., 1966. Good source of problems. Text material is too brief in some areas, so that the topic is not well developed for this level of student. Needs supplementing.
SMSG	School Mathematics Study Group. Introduction to Matrix Algebra. Stanford, California: A. C. Vroman, Inc., 367 Pasadena Avenue., Pasadena, California, 1960. Secondary text for parts of this quin. Must be supplemented.

II. Other Source-books:

Ayres, Frank Jr., Theory and Problems of Matrices. Schaum's Outline series. New York: McGraw-Hill Book Company, 1962. Lots of problems with solutions.

Glicksman, Abraham M. and Ruderman, Harry D. Fundamentals For Advanced Mathematics. New York: Holt, Rinehart and Winston, Inc., 1964. Very good reference. Problems are sometimes more difficult than they look on first reading.

Kemeny, John G.; Snell, J. Laurie and Thompson, Gerald L. Introduction to Finite Mathematics. 2nd Edition. Prentiss-Hall Mathematics Series. Englewood Cliffs, N. J. Prentiss-Hall Inc., 1956. Matrices begin as vectors. Some good problems. Reference for Markov Chains.

Lipschutz, Seymour. Linear Algebra. Schaum's Outline Series. New York: McGraw-Hill Book Co., 1968. Many problems with solutions. Goes far beyond this course.

II. Other Source-books.

Meserve, Bruce E., Pettofrezzo, Anthony J. and Meserve, Dorothy T. Principles of Advanced Mathematics. Syracuse, N. Y: The L. W. Singer Co., a division of Random House, Inc., 1964. Covers a wide range of topics in greater depth than Nahikian.

Stabler, Edward R. An Introduction to Mathematical Thought. Reading, Mass: Addison-Wesley Publishing Co., Inc., 1948. Excellent reference for mathematical structures and especially good on Boolean Algebra.

Western, Donald W. and Haag, Vincent H. An Introduction to Mathematics. New York: Henry Holt and Co., Inc., 1959. Brief. Some good problems.