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AUTHOR	Hirigoyen, Hector
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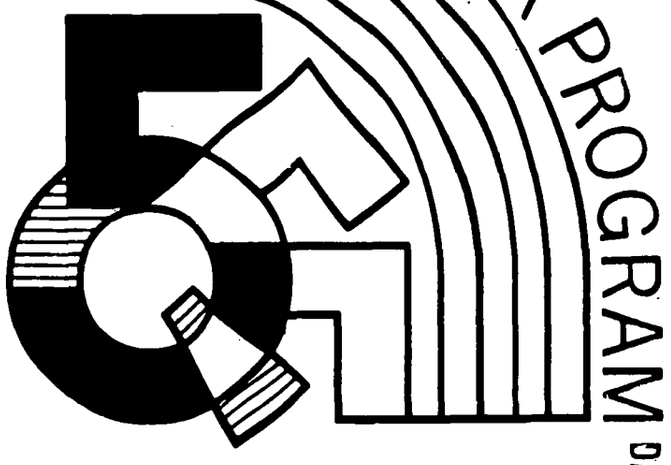
ABSTRACT

This is the second of the six guidebooks on minimum course content for first-year algebra; it includes the ordered field properties of the real number system, solution of linear equations and inequalities, verbal problems, exponents and operations with polynomials. Overall goals for the course are stated; performance objectives for each unit, a course outline, references to state-adopted texts, and teaching suggestions are given. A pretest and posttest are included, plus a list of eight references. For other booklets in the algebra sequence, see SE 014 897 and SE 014 875. (DT)

AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM



DADE COUNTY PUBLIC SCHOOLS

ALGEBRA 1Q

5215.12

MATHEMATICS

DIVISION OF INSTRUCTION • 1971

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

Algebra 1 q

5215.12

(EXPERIMENTAL)

Written by

Hector Hirigoyen

for the

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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

COURSE DESCRIPTION

A continuation of the development of the real number system and algebraic expressions and an introduction to problem solving. Includes the ordered field properties of the real number system, further work in the solution of linear equations and inequalities, an introduction to the solution of verbal problems, work with whole number exponents, and some work in the operations with polynomials.

Designed for the student who has mastered the skills and concepts in Algebra 1p.

TABLE OF CONTENTS

OVERALL GOALS

1. To understand and appreciate the basic structures of Algebra: The Real Number System and Polynomials.
2. To review the solution of linear open sentences and their relationships of equality and inequality.
3. To develop reasoning skills and reading ability through the use of verbal problems.
4. To introduce some of the basic manipulative skills necessary for the study of Algebra.
5. To recognize the manipulative skills of Algebra as reflections of its basic structure.
6. To develop the ability to communicate precisely through symbolic expressions.
7. To acquire facility in applying algebraic concepts and skills.

KEY TO REFERENCES

(* State Adopted)

* D - Dolciani, Mary; Wooten, William; Beckenbach, Edwin: Jurgensen, Ray; and Donnelly, Alfred. Modern School Mathematics, Algebra 1. New York: Houghton Mifflin, 1967.

N - Nichols, Eugene D. Modern Elementary Algebra. New York: Holt, Rinehart, and Winston, 1961.

* PL - Payne, Joseph; Zamboni, Floyd; and Lankford, Francis. Algebra One. New York: Harcourt, Brace, Jovanovich, 1969.

* PA - Pearson, Helen R. and Allen, Frank B. Modern Algebra: A Logical Approach, Book One. Boston: Ginn and Co., 1964.

[1] The number in the block preceding an objective indicates the number of the state assessment standard to which the objective is related.

PERFORMANCE OBJECTIVES

I. Properties

The student will

1. State or recognize the completeness property of real numbers.

2. Determine whether a given subset of the real numbers is dense or discrete.

[1] 3. Identify the properties of an ordered field that are also properties of a given subset of the real numbers.

[1] 4. Demonstrate an understanding of the structure of the real numbers. Identify the properties of the real number system and use them to verify given statements (ordered field properties).

5. Identify the properties of equality and use them to verify statements (substitution, symmetric, reflexive, transitive).

6. Identify the properties of inequality and use them to verify statements (transitive, trichotomy).

7. Write and read the properties and other mathematical expressions using set-builder notation and the quantifiers $\forall$ and $\exists$.

II. Open Sentences

The student will

[2] 1. Solve linear equations in one variable by using the addition and multiplication properties of equality.

   a. equations with similar terms

   b. equations with the variable on both sides of the equation

II. Open Sentences (continued)

2 2. Solve linear inequalities in one variable by using the addition and multiplication properties of inequality.

   a. inequalities with similar terms

   b. inequalities with the variable on both sides of the inequality

2 3. Graph the solution set of a linear open sentence in one variable given a replacement set that is a subset of the real numbers.

4. Solve the multi-variable linear equations for any one of the variables using the methods of Objective 1.

III. Problem Solving

The student will

12 1. Solve selected problems by writing a complete solution using linear equations and acceptable algebraic notation and form.

12 2. Solve selected problems by writing a complete solution using linear inequalities and acceptable algebraic notation and form.

IV. Polynomials

The student will

1. Classify a given expression as a polynomial or non-polynomial.

7 2. Evaluate a polynomial for a given number.

3. Classify a given polynomial according to:

   a. the number of terms

   b. the degree

4. Identify coefficients in the terms of a polynomial.

IV. Polynomials (continued)

5. Express a polynomial in ascending or descending order.

[5] 6. Add two or more polynomials.

[5] 7. Subtract one polynomial from another.

[8] 8. Apply the laws of exponents in multiplying two or more monomials with <u>positive</u> integral exponents.

9. Apply the laws of exponents in raising to a power a monomial with <u>positive</u> integral exponents.

[8] 10. Apply the laws of exponents in simplifying a rational expression whose numerator and denominator are monomials with <u>positive</u> integral exponents.

[5] 11. Find the product of a monomial and a polynomial.

[5] 12. Divide a polynomial by a monomial.

[9] 13. Factor a polynomial that has a common monomial factor.

[5] 14. Find the product of two polynomials.

COURSE OUTLINE

Related Objectives	(Roman numerals refer to Performance Objectives)
	I. Properties
I 1-4	A. Real Numbers
	1. Density vs. discreteness
	2. Ordered field properties
5	B. Equality
	1. Equivalence relation
	2. Addition, Multiplication, Cancellation
7	C. Mathematical Notation
	1. Quantifiers
	2. Set-builder
6	D. Order
	1. Definition $a < b \Leftrightarrow (\exists c)\ a + c = b$
	2. Transitive
	3. Trichotomy
	4. Addition and Multiplication
	II. First degree open sentences
II 1,2,4	A. Solving
	1. Equalities
	2. Inequalities
	3. Multi-variable equations
3	B. Graphing (number line)
III 1,2	III. Verbal Problems

COURSE OUTLINE

(continued)

Related Objectives	
	IV. Polynomials
IV 1-5	A. Vocabulary
	1. terms
	2. coefficients
	3. degree
6-14	B. Operations
	1. Addition and subtraction
	2. Laws of exponents
	3. Multiplication
	4. Division by monomials
	5. Common factors

REFERENCES

Course Outline		D	N	PL	PA
I	A	47-52 55-58 66-70	34-52	31-38 63-66 73-79	142-181
	B	71-76	188-196	34-35	139-140
	C	34-39 44-45	10,182 ∃ not covered	31-34	64,430,465 (quantifiers not covered)
	D	145-151	239-241	121-122	256-261 (I-6 not covered)
II	A	116-124 157-165	174-185	101-117	150-156 256-262
	B	157-164	188-196 239-244	101-117	39-40 66-68
III		124-134 169-178	211-238 245-258	128-150	156-162 251-256 176-178
IV	A	52-55	119-124	307-310	362-363 240-241
	B	102-106 267-283 307-314	199-201 384-388	257-273 310-317 324-328 341-343	340-350 360-370

SUGGESTED STRATEGIES

I. Properties

1. It may be useful to compare the properties of equality to those of inequality.

2. Students are apt to confuse the symmetric property of equality with the commutative properties; point out that the commutative properties involve "moving around" an operation sign, symmetry is just around the =.

3. Give frequent quizzes on the properties.

4. Quantifiers are only slightly covered in Dolciani and not at all in Pearson.

5. Set-builder notation is only slightly covered in the texts, except for Dolciani.

II. First-degree open sentences

1. Review properties of equality while this topic is being covered.

2. "Cover-up" technique suggested by Nichols (p. 178) is quite effective.

3. Compare the solution of equations to the solution of inequalities, paying special attention to multiplication and division by negative numbers. Point out, using the number line, that $3 < 4$ but $-3 > -4$.

III. Verbal Problems

1. Make sure enough time is spent on verbal problems. It may be wise to review translation of basic phrases into mathematical symbols, such as "3 more than a certain number" etc.

2. Pearson and Payne are particularly good in their approach to verbal problems.

3. You may want to try the reverse technique, that is, given an open sentence have the student make up a story to fit it.

4. Make sure you include problems which are solvable by inequalities.

IV. Polynomials

1. Make sure students can distinguish factored forms from polynomials and terms from factors. This will save some headaches when fractions are studied (particularly "cancelling through + and - signs").

2. Make sure enough drill is given on the properties of exponents, but intersperse it with other material so interest will be maintained.

3. Dodes, pp. 128-130, has excellent exercises on simplifying expressions using the laws of exponents.

PRE-TEST

(Sample items should show readiness for this quin)

1. List all the subsets of $A = \{a, b, c\}$

2. Graph the following on the number line:

   a. -1

   b. $\frac{5}{3}$

   c. $\sqrt{2}$

   d. 2.75

3. Write the symbol $<$ or $>$ that should be inserted between each pair of numbers to make a true statement.

   a. $-4 \quad 3$

   b. $-5 \quad -2$

4. Translate into a mathematical expression 6 more than $\frac{1}{3}$ of a certain number.

5. Solve:

   a. $x - 2 < 5$

   b. $3x = 15$

   c. $x + 6 = 24$

6. Simplify:

   a. $3(x - 4) + 6(3x + 5)$

   b. $3ab - 7ab + 9$

Pre-Test
(cont.)

7. Match the properties to the number statements:

___ a. $3 \times 4 = 4 \times 3$

___ b. $3 + (2 + 5) = (3 + 2) + 5$

___ c. $5(7 + 3) = 35 + 15$

___ d. $3 = 3$

A. Commutative for Addition

B. Associative for Addition

C. Commutative for Multiplication

D. Distributive

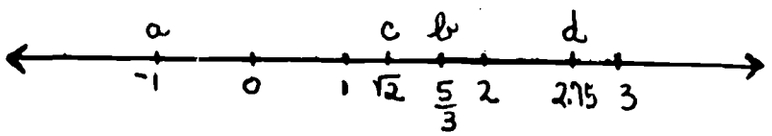
E. Symmetric

F. Transitive

G. Reflexive

KEY TO-PRE-TEST

1. $\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{b,c\}$, $\{a,c\}$, $\{a,b,c\}$.

2. 

3. a < b <

4. $\frac{1}{3}x + 6$

5. a) $x < 7$ b) $x = 5$ c) $x = 18$

6. a) $21x + 18$ b) $-4ab + 9$

7. a. C
   b. B
   c. D
   d. G

POST-TEST

(Keyed to objectives)

I.

1. State the completeness property of real numbers.

2. Classify as dense or discrete

   a. Rational numbers

   b. Whole numbers

   c. Real numbers

   d. Irrational numbers

   e. Integers

3. From the list on the right, choose the properties that apply to the given subset of the real numbers.

a. Integers	1. Closure Multiplication
b. Rationals	2. Additive inverses
c. Whole numbers	3. Additive identity
d. Irrationals	4. Multiplication inverses
	5. Trichotomy
	6. Commutative

4.-6. Identify the property that is being illustrated.

   a. If $3<4$ and $4<5$, then $3<5$

   b. If $3 = 2 + 1$, then $6 = 2(2 + 1)$

   c. $3 + 2 = 2 + 3$

   d. $3(3 + 1) = 9 + 3$

   e. $3(2 \cdot 5) = (3 \cdot 2)5$

Post-Test
(cont.)

7. Write the following using mathematical symbols:

   a. For all real numbers x, y, $x + y = y + x$.

   b. There exists a number c such that $a + c = b$.

   c. The set of all x's such that $3x = 5$.

II. Solve and graph on the number line:

1. $3x + 2 = 2x - 1$

2. $6x + 3 - 18\ x \leq 39$

3. $3x < 2x + 2$

4. Solve for x, $3\ a\ x + b = 7$

III. Solve the following problems:

1. A certain whole number is doubled and added to 4. The result is greater than 56 and less than 60. Find the number.

2. The sum of 2 numbers is 57. One number is 3 more than the other. What are the numbers?

IV. 1., 3., 4. Tell which of the following are polynomials; for the polynomials, identify each term, its coefficient and its degree, also give the degree of the polynomial.

   a. $3x^6 + 2x^2 + 4x - 8$

   b. $\frac{3}{x} - 5x^6 + 25$

   c. 39

   d. $3x^2 - 4x + 9$

2. Evaluate for $x = -2$; $3x^2 - 2x + 6$

5. Write in ascending order:

$$3x^5 + 2x + 3x^4 + 5x^6 + x^2 + 3x^3 + 6$$

Post-Test
(cont.)

Perform the indicated operations:

6. $(3x^2 + 2x + 5) + (2x^2 + 6 + 2x)$

7. $(2x^3 - 2x + 3) - (-4x^4 + 3x^3 - x^2 + x)$

Simplify each of the following using the laws of exponents:

8. $(15\ a^4b^3) \cdot (3\ a^2b)$

9. $(3x^2y^3)^4$

10. $\dfrac{(-2x^3)^4}{(-3xz^2)^3}$

Perform the indicated operations:

11. $3a(2x + 4a^2 + 4a^3)$

12. $(6x^4 + 12x^3 + 2x^2) \div 2x^2$

13. Factor: $3x^2 + 12x^4 + 27x^5 + 36x^7$

14. Find the product:

    a. $(5x + 3y)(3x^2 - 4y^3)$

    b. $(a^2 + ab + b^2)(a - b)$

    c. $(x + y)(x - y)$

KEY TO POST-TEST

I. 1.

2. a. dense c. dense e. discrete
   b. discrete d. dense

3. a. 1, 2, 3, 5, 6 c. 1, 3, 5, 6
   b. 1, 2, 3, 4, 5, 6 d. 2, 4, 5, 6

4.-6.

a. Transitive of inequality

b. Multiplication of equality

c. Commutative of addition

d. Distributive

e. Associative of multiplication

7. a. $\{\forall x, y \in R, x + y = y + x\}$

b. $\{\exists c, a + c = b\}$

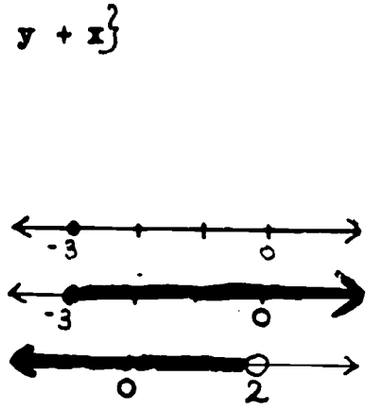
c. $\{x \mid 3x = 5\}$

II. 1. $x = -3$

2. $x \geq -3$

3. $x < 2$

4. $x = \frac{2-b}{3a}$



III. 1. 27

2. 27

Key To Post-Test
(cont.)

IV. 1, 3, 4.

a. polynomial; terms (degree in parentheses)

$3x^6$ (6th), $2x^2$ (2nd), $4x$ (1st), 8 (0)

coefficients 3, 2, 4
degree of polynomial: 6

b. non-polynomial
c. polynomial constant
d. polynomial; terms (degree in parentheses)

$3x^2$ (2nd), $-4x$ (1st), 9 (0)

coefficients 3, -4
degree of polynomial: 2

2. 22

5. $6 + 2x + x^2 + 3x^3 + 3x^4 + 3x^5 + 5x^6$

6. $5x^2 + 4x + 11$

7. $4x^4 - x^3 + x^2 - 3x + 3$

8. $45\, a^6\, b^4$

9. $81\, x^8\, y^{12}$

10. $\frac{-16x^9}{27z^6}$

11. $6ax + 12a^3 + 12a^4$

12. $3x^2 + 6x + 1$

13. $3x^2\,(1 + 4x^2 + 9x^3 + 12x^5)$

14. a. $15x^3 - 20xy^3 + 9x^2y - 12y^4$

b. $a^3 - b^3$

c. $x^2 - y^2$

BIBLIOGRAPHY

STATE-ADOPTED

Dolciani, Wooton, Beckenbach, Jurgensen, Donnelly, Modern School Mathematics, Algebra I. New York: Houghton-Mifflin, 1967.

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Payne, Zamboni, Lankford, Algebra One. New York: Harcourt, Brace, Hovanovich, 1969.

Pearson and Allen, Modern Algebra: A Logical Approach, Book I. Boston: Ginn & Co., 1964.

OTHER USEFUL REFERENCES

Barron's Educational Series: Regents' Exams and Answers: Elementary Algebra. New York: 1965. (excellent explanations and problems, as well as sample evaluations)

Dodes, Greitzer, Algebra I: Its Structure, Logic and Methods. New York: Hayden, 1967. (good strategies and calendar)

Dressler, Ninth Year Mathematics. New York: Amsco, 1966. (excellent explanations and problems, numerous test items)

SMSG, A First Course in Algebra. New Haven: Yale, 1961. (good for background and properties of Reals)