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AUTHOR Weaver, J. F.
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ABSTRACT

This investigation sought to ascertain whether significant differences in pupils' achievement when solving open-sentence types existed in relation to the following factors: school grade, sentence form as determined by the symmetric property of the equality relation, the operation specified in a sentence, the position of the placeholder in a sentence, and the existence of an open-sentence solution within the set of whole numbers. Tables present the results of performance of 3,268 pupils in grades 1, 2, and 3. (Author/DB)

SOME FACTORS ASSOCIATED WITH PUPILS' ACHIEVEMENT WHEN SOLVING SELECTED TYPES OF SIMPLE OPEN SENTENCES

J. F. Weaver
The University of Wisconsin-Madison

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Summary

Objectives. For open sentences derived from a well defined domain of basic number facts of the form a o b = c, this investigation sought to ascertain whether significant differences in pupils' achievement when solving such sentences existed in relation to the following factors: (A) school grade, (B) sentence form as determined by the symmetric property of the equality relation, (C) the operation specified in a sentence, (D) the position of the placeholder in a sentence, and (E) the existence of an open-sentence solution within the set of whole numbers. Table 1 identifies for each of these factors the particular levels considered in the present investigation.

Methods. Based upon factors B, C, and D (Table 1) there may be generated the 12 generic open-sentence forms identified in Table 2. Depending upon the whole numbers selected as constants, which takes into account factor E, these 12 generic forms may spawn the 20 particular open-sentence types also identified in Table 2.

A 32-item Inventory, Part 1* was developed which included two exemplars for each of the 12 types W1-W12 (Table 2) and one exemplar for each of the 8 types X3-X10. Each exemplar was derived by modifying appropriately a basic addition or subtraction fact selected from the set of such facts having sums between 10 and 18. Inventory Part 1 then was partitioned in a structured way into four 8-item Tests which were balanced with respect to levels of factors B, C, D, E (Table 1). The placeholder in each open sentence was shown as a square region (▩) rather than as a square (□) so that each Test could be cast in an appealing context of "numbers hiding under boxes."

Four additional open sentences--each using numbers less than 10--were developed as a set of sample items common to the four distinct Tests. In essence the following instructions were given, with simplicity of expression taking precedence over mathematical preciseness or pedantry:

3 + 2 = ▩ ______ "What whole number is hiding under the box?
"Write the number on the line.
"If no whole number is hiding under the box, mark a big X on the line."

Data source. Data were based upon performance of pupils from two classes at each of three grade levels--1,2,3--in each of 23 of 37 elementary schools using the same city-adopted basal mathematics textbook series. Each of the 3,268 pupils took one of the four group-administered Tests which had been distributed randomly among children within each intact mathematics class.

Design for data analysis. The levels of factor A (grade) may be viewed as defining three treatments designated as X1, X2, X3 in Figure 1. Factors B,C, D,E and their respective levels (Table 1) are inherent in observation O of Figure 1.

The factorial design used to analyze the data is represented by Figure 2. Since it is impossible to completely cross the levels of factors D and E, it was not feasible to generate a single ANOVA that embraced all four mathematical factors--B,C,D,E--in terms of the Figure 2 model. Consequently, separate ANOVAs were generated for particular factor/level combinations that could be

* This was in reality a two-part Inventory in which Part 2 also consisted of 32 items, but of a somewhat different nature: pairs of open sentences to be judged equivalent or nonequivalent. The present Summary relates only to Part 1 of the full Inventory.

completely crossed in keeping with the Figure 2 paradigm. The delimiting conditions for each of these ANOVAs is made explicit in connection with Table 4 (to be considered in the next section of this Summary).

The Figure 2 model makes it possible to test the statistical significance of the main effect and sundry interactions associated with factor S (school) in the case of any ANOVA having more than one replicate per cell. Although information regarding this factor may be of interest to the particular school district involved, factor S was of no interest per se in relation to the purpose(s) of the investigation. Hence no data pertaining explicitly to factor S have been included in this report.

Error terms for the ANOVA model (Figure 2) are specified in Figure 3.

Results. Table 3 gives an indication of mean correct responses for:
(1) factor A (grade)--across levels and by levels; and
(2) levels of mathematical factors B,C,D,E--across and by levels of factor A.

Table 4 characterizes five ANOVAs that were generated in accord with the Figure 2 model. For each of these ANOVAs the main effects and first-order interaction effects are identified, along with a probability value (p) to indicate the α-level at which H_0 for each effect could be rejected on the basis of the computed F. (It has not been feasible in this Summary to include the complete ANOVA table for each of the five ANOVAs generated.) Note that in Table 4 "$.10 < p$" is used as a broad catch-all category to embrace any instance for which the risk of making a Type I error would exceed an α-level of .10.

More details of the results, their interpretation, and a consideration of implications of the investigation will be incorporated in the oral presentation. Attention will be given to the fact that although there are highly significant main effects in connection with the first three ANOVAs (Table 4), there also are some highly significant interactions. Furthermore, the relative occurrence of highly significant effects (main and interaction) is not as marked for the fourth and fifth ANOVAs as for the first three.

Findings will be interpreted in relation to pupils' opportunity to learn.

J. F. Weaver
The University of Wisconsin-Madison
1 October 1971

TABLE 1

Factors and Levels for ANOVAs: NUMBER PUZZLES Inventory, Part 1

Factor	Level
A. Grade	1. First grade 2. Second grade 3. Third grade
B. Symmetric property of the equality relation	1. Sentence of the form $\underline{a} \circ \underline{b} = \underline{c}$ 2. Sentence of the form $\underline{c} = \underline{a} \circ \underline{b}$
C. Operation specified in the open sentence	1. Addition (+ for o) 2. Subtraction (− for o)
D. Position of the placeholder in the sentence	1. □ in the $\underline{a}$ position 2. □ in the $\underline{b}$ position 3. □ in the $\underline{c}$ position
E. Existence of a solution within set W	1. A solution exists in W 2. No solution exists in W
S. School	1-23. Schools 1,2,3,...,23

Note.--Set W = {0, 1, 2, 3, 4, 5, 6, 7, . . .}

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The University of Wisconsin-Madison
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TABLE 2

Generic Forms and Particular Types of Simple Open Addition and Subtraction Sentences

a o b = c			c = a o b		
Generic form of open sentence	Particular type		Generic form of open sentence	Particular type	
	A solution exists in W	No solution exists in W		A solution exists in W	No solution exists in W
1 a + b = □	W 1	#	2 □ = a + b	W 2	#
3 a + □ = c	W 3	X 3	4 c = a + □	W 4	X 4
5 □ + b = c	W 5	X 5	6 c = □ + b	W 6	X 6
7 a - b = □	W 7	X 7	8 □ = a - b	W 8	X 8
9 a - □ = c	W 9	X 9	10 c = a - □	W10	X10
11 □ - b = c	W11	#	12 c = □ - b	W12	#

Notes.--

Using whole numbers [W = {0, 1, 2, 3, 4, 5, 6, 7, . . .}] as constants, it is impossible to have an open sentence of this particular type.

Generic forms 1,2,7 and 8 may be referred to as canonical open-sentence forms.

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The University of Wisconsin-Madison
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TABLE 2.1

NUMBER PUZZLES Inventory, Part 1

Distribution of Items by Mathematical Factors and Levels and by Open-sentence Types

Mathematical factor and level				Item type	Number of items
B1: a o b = c	C1: a + b = c	D1: □ + b = c	E1: E2:	W 5 X 5	2 1
		D2: a + □ = c	E1: E2:	W 3 X 3	2 1
		D3: a + b = □	E1: E2:	W 1 —	2 0
	C2: a - b = c	D1: □ - b = c	E1: E2:	W 11 —	2 0
		D2: a - □ = c	E1: E2:	W 9 X 9	2 1
		D3: a - b = □	E1: E2:	W 7 X 7	2 1
B2: c = a o b	C1: c = a + b	D1: c = □ + b	E1: E2:	W 6 X 6	2 1
		D2: c = a + □	E1: E2:	W 4 X 4	2 1
		D3: □ = a + b	E1: E2:	W 2 —	2 0
	C2: c = a - b	D1: c = □ - b	E1: E2	W 12 —	2 0
		D2: c = a - □	E1: E2:	W 10 X 10	2 1
		D3: □ = a - b	E1: E2	W 8 X 8	2 1

The above distribution is embedded within each of the 3 levels of factor A and within each of the 23 levels of factor S.

J. F. Weaver
The University of Wisconsin-Madison
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My name is ______________________________

Grade ______ School ______________________________

0 1 2 ■ 4 5 6 7 8 9

10 11 12 13 14 15 ■ 17 18 19

■ . . . . . .

NUMBER PUZZLES

a. 7 - 2 = ■ ______

b. 9 + ■ = 10 ______

c. ■ = 3 + 5 ______

d. 0 - 4 = ■ ______

STOP

13

jfw

(2)

a. 13 = ■ + 4 ______

b. 8 + 9 = ■ ______

c. ■ = 11 - 7 ______

d. 9 - ■ = 14 ______

e. 6 + ■ = 11 ______

f. 5 = 12 + ■ ______

g. ■ - 9 = 7 ______

h. 8 = 12 - ■ ______

13

STOP

The 8 items in Part 1 of each of the 4 Tests were distributed so that ....

1. The same generic open-sentence form (Table 2) was represented by no more than one item.

2. Factors B and C (Table 1) were completely crossed and balanced, with 2 items for each of the 4 factor/level combinations.

3. There were 6 W-type sentences (factor E, level 1):
   3 items for each of the 2 levels of factor B;
   3 items for each of the 2 levels of factor C;
   2 items for each of the 3 levels of factor D.

   [Across the 4 Tests factors B, C, and D were completely crossed and balanced, with 2 items for each of the 12 factor/level combinations.]

4. There were 2 X-type sentences (factor E, level 2):
   1 item for each of the 2 levels of factor B;
   1 item for each of the 2 levels of factor C.

   [Across the 4 tests each of the 8 possible B,C,D factor/level combinations was represented once.]

Separate consideration was given to a balanced distribution (within and across Tests) of items with respect to the relative magnitude of sums, first addends, and second addends.

J. F. Weaver
The University of Wisconsin-Madison
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(3)

MORE NUMBER PUZZLES

a. 6 + 3 = ■
6 − 3 = ■ YES NO

b. 2 + 5 = ■
■ = 5 + 2 YES NO

c. ■ = 8 − 4
■ = 4 − 8 YES NO

d. 9 = 9 + ■
9 − ■ = 9 YES NO

STOP

13

(4)

a. 13 = ■ + 8

13 − 8 = ■ YES NO

b. 2 = 11 − ■

■ = 11 − 2 YES NO

c. 14 − ■ = 8

14 + 8 = ■ YES NO

d. 9 = ■ − 3

■ = 9 − 3 YES NO

e. ■ + 9 = 15

■ = 9 + 15 YES NO

f. 7 + ■ = 13

■ = 13 − 7 YES NO

g. 11 = 8 + ■

8 − 11 = ■ YES NO

h. ■ − 8 = 7

7 + 8 = ■ YES NO

STOP

01 11

	Y1	Y2	Y3	
X1	----------	----------	(G1)	0
X2	----------	(G1 ∪	G2)	0
X3	(G1 ∪	G2 ∪	G3)	0

Fig. 1. Research paradigm pertaining to factor A

Notes.--

Y1, Y2, Y3 refer to the 1967-68, 1968-69, 1969-70 school years respectively.

G1, G2, G3 refer to the instructional programs based upon the city-adopted mathematics textbook series for grades 1, 2, 3 respectively.

The same observation, O (NUMBER PUZZLES Inventory) was made for treatments X1, X2, X3 in the spring of the 1969-70 school year (Y3).

J. F. Weaver
The University of Wisconsin-Madison
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TABLE 3

Mean Correct Responses Across Schools: NUMBER PUZZLES Inventory, Part 1

Factor and level	Number of items	Across grades	Grade 1	Grade 2	Grade 3
A. Grade	32	18.12	12.79	19.07	22.51
B. Symmetric property of =					
1. <u>a</u> o <u>b</u> = <u>c</u>	16	10.01	7.52	10.70	11.80
2. <u>c</u> = <u>a</u> o <u>b</u>	16	8.11	5.27	8.36	10.71
C. Operation specified					
1. Addition (+)	16	10.48	7.32	11.04	13.06
2. Subtraction (-)	16	7.65	5.47	8.02	9.44
D. Placeholder position					
1. □ in <u>a</u> position	10	4.42	2.67	4.73	5.86
2. □ in <u>b</u> position	12	7.30	5.21	7.65	9.04
3. □ in <u>c</u> position	10	6.40	4.90	6.69	7.61
E. Solution in W					
1. A solution exists	24	14.37	9.47	15.28	18.37
2. No solution exists	8	3.75	3.32	3.79	4.13

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The University of Wisconsin-Madison
1 October 1971

TABLE 3.1

Mean Correct Responses Across Schools: NUMBER PUZZLES Inventory, Part 1

Generic form of open sentence	Sentences having a solution within set W [2 items/type]			Sentences having no solution within set W [1 item/type]		
	Grade 1	Grade 2	Grade 3	Grade 1	Grade 2	Grade 3
1 a + b = □	1.28	1.74	1.86	---	---	---
2 □ = a + b	.99	1.42	1.75	---	---	---
3 a + □ = c	1.06	1.57	1.84	.46	.53	.61
4 c = a + □	.80	1.34	1.71	.38	.53	.63
5 □ + b = c	.90	1.51	1.76	.43	.60	.59
6 c = □ + b	.63	1.33	1.69	.37	.48	.63
7 a - b = □	1.13	1.59	1.75	.49	.45	.46
8 □ = a - b	.70	1.21	1.49	.32	.27	.29
9 a - □ = c	1.00	1.59	1.79	.54	.57	.49
10 c = a - □	.64	1.16	1.53	.33	.36	.44
11 □ - b = c	.23	.55	.65	---	---	---
12 c = □ - b	.10	.26	.55	---	---	---

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1 October 1971

	A_1						A_2						A_3					
	J_1			J_p			J_1			J_p			J_1			J_p		
	K_1	··	K_q	K_1	··	K_q	K_1	··	K_q	K_1	··	K_q	K_1	··	K_q	K_1	··	K_q
S_1																		
S_2																		
S_3																		
S_4																		
··																		
S_{23}																		

Fig. 2. ANOVA model: 3 x p x q x ... x 23
(Repeated measures design)

Notes.--

This is a mixed model involving n factors, completely crossed, where:
A (grade) is a fixed factor (3 levels);
J (some mathematical characteristic) is a fixed factor (p levels);
K (some mathematical characteristic) is a fixed factor (q levels);
.......(any other mathematical characteristics as fixed factors); and
S (school) is considered to be a random factor (23 levels).

The number of replicates is the same for each cell of the matrix but may differ from one ANOVA to another. In any case, each replicate is a school mean: the mean correct responses for all pupils from a particular school and grade (pooled across classes) on a particular set of Inventory items.

ANOVAs computed on the basis of the above design were run at the Stanford University Computing Center using the BMDO8V Analysis of Variance program revised July 17, 1969 by the UCLA Health Sciences Computing Facility.

J. F. Weaver
The University of Wisconsin-Madison
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Source of variation	df	Error term	
		r = 1	r > 1
A	a - 1 = 3 - 1 = 2	AS	AS
J	j - 1	JS	JS
K	k - 1	KS	KS
S	s - 1 = 23 - 1 = 22	--	R(AJKS)
AJ	(a - 1)(j - 1)	AJS	AJS
AK	(a - 1)(k - 1)	AKS	AKS
AS	(a - 1)(s - 1)	---	R(AJKS)
JK	(j - 1)(k - 1)	JKS	JKS
JS	(j - 1)(s - 1)	---	R(AJKS)
KS	(k - 1)(s - 1)	---	R(AJKS)
AJK	(a - 1)(j - 1)(k - 1)	AJKS	AJKS
AJS	(a - 1)(j - 1)(s - 1)	----	R(AJKS)
AKS	(a - 1)(k - 1)(s - 1)	----	R(AJKS)
JKS	(j - 1)(k - 1)(s - 1)	----	R(AJKS)
AJKS	(a - 1)(j - 1)(k - 1)(s - 1)	----	R(AJKS)
R(AJKS)	(r - 1)ajks	----	-------
Total	rajks - 1 = N - 1		

*Fig. 3. Error terms for ANOVA model, Fig. 2.

Notes.--

The preceding patterns may be extended, of course, to include additional fixed factors.

We may view R ". . . 'replication' within the smallest cell of a design [as] a nested factor that is always random and is nested within all the other factors of the design." (Glass & Stanley, 1970; p. 474)

The "within cells" source of variation, R(AJKS), is nonexistent when r = 1.

*Oops! This is a table rather than a figure. Sorry about that. The mistake will be corrected in any future document.

J. F. Weaver
The University of Wisconsin-Madison
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TABLE 4

Significance Levels for Main Effects and for Two-factor Interactions:

NUMBER PUZZLES Inventory, Part 1 ANOVAs

ANOVA and Source of variation		Significance level of computed F
1. ANOVA based on all 32 sentences		
Main effect:	A	$p \leq .001$
	B	$p \leq .001$
	C	$p \leq .001$
	E	$p \leq .001$
Interaction:	AB	$p \leq .001$
	AC	$p \leq .001$
	AE	$p \leq .001$
	BC	$p \leq .001$
	BE	$.10 \geq p > .05$
	CE	$p \leq .001$
2. ANOVA based on 24 sentences having solutions in W [Factor E, Level 1]		
Main effect:	A	$p \leq .001$
	B	$p \leq .001$
	C	$p \leq .001$
	D	$p \leq .001$
Interaction:	AB	$p \leq .001$
	AC	$p \leq .001$
	AD	$.01 \geq p > .001$
	BC	$.01 \geq p > .001$
	BD	$p \leq .001$
	CD	$p \leq .001$

(Continued)

TABLE 4 (Continued)

3. ANOVA based on 8 sentences having no solution in W [Factor E, Level 2]		
Main effect:	A	$.01 \geq p > .001$
	B	$p \leq .001$
	C	$p \leq .001$
Interaction:	AB	$p \leq .001$
	AC	$p \leq .001$
	BC	$p \leq .001$
4. ANOVA based on 4 addition sentences [Factor C, Level 1] having no solution in W [Factor E, Level 2]		
Main effect:	A	$p \leq .001$
	B	$.10 < p$
	D	$.10 < p$
Interaction:	AB	$.10 \geq p > .05$
	AD	$.10 < p$
	BD	$.10 < p$
5. ANOVA based on 4 subtraction sentences [Factor C, Level 2] having no solution in W [Factor E, Level 2]		
Main effect:	A	$.10 < p$
	B	$p \leq .001$
	D	$.01 \geq p > .001$
Interaction:	AB	$.10 < p$
	AD	$.10 < p$
	BD	$.10 < p$

J. F. Weaver
The University of Wisconsin-Madison
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NUMBER PUZZLES Inventory, Part 1.
Comparison of F's with Factor A Included and Excluded:
ANOVA based on all 32 sentences
[ANOVAs 13 and 13.1; Table 17 (4.1)]

Source of variation	df for F	F			
		Factor A included	Factor A excluded		
			Grade 1	Grade 2	Grade 3
A	2,44	89.99***			
B	1,22	372.86***	160.84***	173.63***	53.17***
C	1,22	354.20***	36.05***	159.43***	282.55***
E	1,22	73.54***	.45	63.85***	219.38***
BC	1,22	25.86***	3.74#	12.37**	12.75**
BE	1,22	3.56#	.50	1.07	7.27*
CE	1,22	36.91***	32.84***	19.36***	4.45*
BCE	1,22	8.83**	1.95	1.90	3.74#
AB	2,44	22.13***			
AC	2,44	32.85***			
AE	2,44	44.99***			
ABC	2,44	.28			
ABE	2,44	.49			
ACE	2,44	7.63**			
ABCE	2,44	.068			

$.10 \geq p > .05$ * $.05 \geq p > .01$ ** $.01 \geq p > .001$ *** $p \leq .001$

If no coding is associated with a reported F, $p > .10$

J. F. Weaver
The University of Wisconsin-Madison
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NUMBER PUZZLES Inventory, Part 1. 5.2
Comparison of F's with Factor A Included and Excluded:
ANOVA based on 24 sentences having solutions in W (Factor E, Level 1)
[ANOVAs 8 and 8.1; Table 19 (4.2)]

Source of variation	df for F	F			
		Factor A included	Factor A excluded		
			Grade 1	Grade 2	Grade 3
A	2,44	246.86***			
B	1,22	399.82***	159.80***	169.88***	84.33***
C	1,22	669.83***	110.02***	213.97***	906.48***
D	2,22	698.72***	117.61***	353.11***	312.79***
BC	1,22	9.75**	.41	7.73*	9.30**
BD	2,44	10.17***	6.16**	4.22*	4.03*
CD	2,44	384.57***	28.66***	180.44***	439.46***
BCD	2,44	2.26	2.13	.68	.84
AB	2,44	15.17***			
AC	2,44	11.50***			
AD	2,88	4.38**			
ABC	2,44	1.13			
ABD	4,88	.14			
ACD	4,88	19.00***			
ABCD	4,88	1.02			

$.10 \geq p > .05$ * $.05 \geq p > .01$ ** $.01 \geq p > .001$ *** $p \leq .001$

If no coding is associated with a reported F, $p > .10$

J. F. Weaver
The University of Wisconsin-Madison
15 October 1971

NUMBER PUZZLES Inventory, Part 1. 5.3

Comparison of F's with Factor A Included and Excluded:
ANOVA based on 8 sentences having no solution in W (Factor E, Level 2)
[ANOVAs 6 and 6.1; Table 21 (4.3)]

Source of variation	df for F	F			
		Factor A included	Factor A excluded		
			Grade 1	Grade 2	Grade 3
A	2,44	6.86**			
B	1,22	79.79***	48.99***	49.78***	9.22**
C	1,22	53.19***	.020	41.89***	75.51***
BC	1,22	20.40***	3.10#	7.09*	8.99**
AB	2,44	11.57***			
AC	2,44	27.09***			
ABC	2,44	.024			

$.10 \geq p > .05$ * $.05 \geq p > .01$ ** $.01 \geq p > .001$ *** $p \leq .001$

If no coding is associated with a reported F, $p > .10$

J. F. Weaver
The University of Wisconsin-Madison
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For a preliminary, informal report of the investigation, see:

Weaver, J. Fred. Some factors associated with pupils' performance levels on simple open addition and subtraction sentences. *Arithmetic Teacher* 18: [redacted] 513-519; November 1971.

For subsequent reports which emphasize instructional implications of particular facets of the investigation, see:

Weaver, J. Fred. The ability of first-, second-, and third-grade pupils to identify open addition and subtraction sentences for which no solution exists within the set of whole numbers. (Accepted for publication in *School Science and Mathematics*.)

Weaver, J. Fred. The symmetric property of the equality relation and young children's ability to solve open addition and subtraction sentences. (Accepted for publication in *Journal for Research in Mathematics Education*.)