DOCUMENT RESUME

ED 063 161 SE 013 806

AUTHOR	Higgins, Jon L., Ed.
TITLE	Investigations in Mathematics Education, Volume 5 Number 2.
INSTITUTION	Ohio State Univ., Columbus. Center for Science and Mathematics Education.
PUB DATE	72
NOTE	74p.
AVAILABLE FROM	Ohio State University, Center for Science and Mathematics Education, 248 Arps Hall, Columbus, Ohio 43210 (Subscription, $6.00 year, $1.75 single copy)
EDRS PRICE	MF-$0.65 HC-$3.29
DESCRIPTORS	*Abstracts; Curriculum; Instruction; *Mathematics Education; Research; Research Reviews (Publications); Student Characteristics; Teacher Characteristics; Teaching Methods

ABSTRACT

This issue contains 17 expanded abstracts of recent research in mathematics education. Coverage is extended to papers and articles announced in "Research in Education", as well as journal articles. Each abstract reports the purpose, rationale, design, procedure, findings, and interpretation of the experiment, followed by an evaluation of the research. The research abstracted in this issue is largely concerned with teaching methods and with student and teacher characteristics. (MM)

Vol. 5, No. 2, Spring, 1972

INVESTIGATIONS IN MATHEMATICS EDUCATION

Expanded Abstracts and Critical Analyses of Recent Research

Center for Science and Mathematics Education
The Ohio State University
in cooperation with
the ERIC Science, Mathematics and
Environmental Education Clearinghouse

ERRATA

Investigations in Mathematics Education Volume 5, Number 1 contains two errors. A line is missing from the top of page 25. Conclusion (b) (beginning on page 24) should read:

"the two groups studying nondecimal numeration (G3 and G5) performed better than the group studying only decimal numeration."

The article on page 34 carries the wrong ERIC citation. The correct citation for the abstract, and a correction for the table of contents are listed below. For your convenience, you may wish to cut off these corrections and paste them in your copy of Volume 5, Number 1.

We apologize for the inconvenience to all concerned.

Jon L. Higgins
Editor

Correction for the table of contents:

Moody, William B.; Abell, Roberta, & Bausell, R. Barker. The effect of activity-oriented instruction upon original learning, transfer, and retention. Journal for Research in Mathematics Education 2: 207-212; May 1971.
Abstracted by ROLAND F. GRAY

Correction for the abstract on page 34:

EJ 038 323 340 SE 503 385
THE EFFECT OF ACTIVITY-ORIENTED INSTRUCTION UPON ORIGINAL LEARNING, TRANSFER, AND RETENTION Moody, William B.; and others, Journal for Research in Mathematics Education, v2 n3, pp 207-212, May '71
Descriptors--*Activity Learning. *Grade 3, *Multiplication, *Research, Achievement Gains, Mathematics Instruction, Retention Studies, Transfer or Training

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Published quarterly by

The Center for Science and Mathematics Education
The Ohio State University
1945 North High Street
Columbus, Ohio 43210

With the cooperation of the ERIC Science, Mathematics, and Environmental Education Information Analysis Center

Subscription Price: $6.00 per year. Single Copy Price: $1.75
Add 25¢ for Canadian mailings and 50¢ for foreign mailings.

Spring, 1972

NOTES . . .

from the Editor

This issue of INVESTIGATIONS IN MATHEMATICS EDUCATION expands our coverage to include not only journal articles, but papers and articles announced in the ERIC publication RESEARCH IN EDUCATION. ERIC is a national information system designed and supported by the U.S. Office of Education for providing ready access to results of exemplary programs, research and development efforts, and related information that can be used in developing more effective educational programs. Through a network of specialized centers or clearinghouses, each of which is responsible for a particular educational area, current significant information relevant to education is monitored, acquired, evaluated, abstracted, indexed, and listed in ERIC reference products such as RESEARCH IN EDUCATION.

Coverage of ERIC documents for I.M.E. brings at least two advantages. The ERIC system is designed for early identification and acquisition of reports, which will enable us to keep our coverage current. Secondly, ERIC reports usually contain more data and detail than condensed journal reports. This enables our abstractors to critique the research in greater depth.

From its inception, INVESTIGATIONS IN MATHEMATICS EDUCATION has invited abstracts of published research reports which were readily available to anyone who wished to read them. Expanding our coverage to ERIC documents does not change this policy. ERIC documents are available from ERIC Document Reproduction Service (EDRS) in two forms:

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For the convenience of our readers a pre-printed order blank is inserted as the last page in this issue. Copies of ERIC documents reviewed in this issue can be ordered by simply crossing our the numbers of those documents not wanted and completing the order blank.

Many institutions and libraries have standing orders for microfiche copies of all ERIC documents. Readers should check to see if microfiche copies are available locally. These copies may be located by using the ERIC accession number (ED 000 000) printed at the top of the abstract.

Journal articles abstracted here are located through the ERIC publication CURRENT INDEX TO JOURNALS IN EDUCATION. The use of CIJE provides us with access to 530 periodicals in the field of education. The access information carried at the top of the abstract is the same as that in CIJE. At the present time reprints of journal articles are <u>not</u> available through the ERIC Document Reproduction Service. Please <u>do not</u> attempt to order any documents whose accession number is of the form EJ 000 000. Inquiries about reprints of journal articles should be addressed to the journal involved.

The use of RIE and CIJE to locate research documents provides us with the widest possible range of coverage. The scope of INVESTIGATIONS IN MATHEMATICS EDUCATION restricts us to only a selective coverage of this material, however. For extensive coverage of mathematics education research, readers are encouraged to refer directly to RIE and CIJE. Readers who have questions about use of the ERIC system are invited to write the editor for a free copy of the publication <u>ERIC: How to Use It for Mathematics Education</u>.

Jon L. Higgins
Editor

EJ 033 810 310 SE 502 715
THE RELATIONSHIP BETWEEN THE ABILITY TO CONSERVE LENGTH AND CONCEPTUAL TEMPO Callahan, LeRoy G.; Passi, Sneh Lata, Journal for Research in Mathematics Education, v2 n1, pp36-43, Jan '71
Descriptors--*Conservation, *Concept Formation, *Elementary School Mathematics, *Geometric Concepts, *Mathematical Concepts, Grade One, Kindergarten, Learning

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Leslie P. Steffe, The University of Georgia

1. Purpose

Callahan and Passi investigated relationships between an impulsive-reflective response style and an ability to conserve length. The authors suggested an hypothesis that the reflective child would base his reasoning about the conservation task on the compensatory nature of the space occupied before and after transformation of a stick whereas the impulsive child would be easily seduced by perception. This hypothesis apparently served as a basis for the study, although it was not tested directly.

2. Rationale

Kagan et al, have examined the conceptual tempo of children and identified a response style they labeled impulsive-reflective. In the words of Callahan and Passi, "Their findings suggest that impulsive children...tend to report the first notion that occurs to them....; the quality of reflectivity has value especially in disciplines with deductive structures...." As noted, the authors suggested an hypothesis which, if true, would explain conservation and nonconservation of length in terms of response style.

3. Research Design and Procedure

A pool of 33 kindergarten and 39 first grade children were considered as a population. Two tests were administered, the Matching Familiar Figures (MFF) Test and a conservation of length test.

The MFF test included 12 items. For each item, a standard familiar figure unit was presented simultaneously with six variants. The child (S) was required to identify the variant which matched the standard unit. In case of a first incorrect response, S was told he was incorrect and was required to try again. The procedure was repeated until S made a correct response or six incorrect responses. The variables scored were

the response time to the first selection (response latency) and total errors.

The conservation of length test consisted of two items. In item 1, the Ss were presented with a stick and a string longer than the stick with respective endpoints coincident. S was required to make a judgment concerning the relative lengths. The string was then straightened and S was again asked to make a judgment concerning relative lengths. The string was then placed back in its original position and S was asked to make a final judgment concerning relative lengths. S was classified as a conserver only if he made all correct judgments or made an initial incorrect judgment and then two correct judgments. In the second item, S was presented with two sticks of equal length with respective endpoints adjacent. S was asked to make a judgment concerning relative lengths of the sticks both before and after one of the sticks was moved. S was classified as a conserver on this task only if both judgments were correct.

To be classified as "reflective," S had to score above the median on response latency and below the median on errors. To be classified as "impulsive," S had to score below the median on response latency and above the median on errors. The two remaining categories were termed "neutral."

Analysis of data involved use of 2x3 contingency tables. One dimension was conceptual tempo (impulsive-reflective) and the other conservation of length (conserve on no task, on exactly one, and on exactly two).

4. Findings

(1) Of the 33 kindergartners, nine were in the impulsive, nine were in the reflective, and 15 were in the neutral category. Of the 39 first-graders, 12 were in the impulsive, 11 were in the reflective, and 16 were in the neutral category.

(2) On the two Piagetian tasks, three kindergarten children were found to conserve on both tasks, 17 on exactly one task, and 13 on neither task. Of the first graders, 10 were found to conserve on both tasks, 13 on exactly one task, and 14 on neither task.

(3) $\chi^2 = 4.60$ (df = 2) was not significant in case of the 2x3 contingency table including all Ss. χ^2 (df = 2) also was not significant for the 2x3 contingency tables for only kindergarten Ss and for only first-grade Ss.

(4) A point biserial correlation coefficient ($r_{pb} = .18$) between response latency and conservation of length was not significant nor was a point biserial correlation coefficient ($r_{pb} = -.16$) between conservation of length and errors on the MFF test.

(5) Sixteen Ss scored above the median on response latency and above the median on errors, and 15 Ss scored below the median on response latency and below the median on errors. Little

deviation existed among frequencies of children scoring 0, 1, or 2 on conservation of length across these two categories.

5. Interpretation

The authors observed that although a close relationship would seem to exist between a child exhibiting a "reflective" conceptual tempo and an ability to conserve length, especially at the stage of development (5-7½ years), the evidence did not support the existence of such a relationship. The description of the sample in regard to conservation of length and the MFF test indicated it is quite representative of the general population at that age.

A replication of the study is desirable due to a rather strong but statistically nonsignificant trend in the data in the predicted direction. It may be the case that conceptual tempo is an important factor in conserving at the transitional stage, but is overwhelmed by maturational factors before and after this stage.

Abstractor's Notes

An attempt to relate an ability to conserve length with conceptual tempo would seem to be rather futile at the outset. An ability to conserve length, as advanced by Piaget, is a symptom of presence of an organizational structure called "groupings." As such, conservation of length is not reducible to the construct "conceptual tempo." Early in childhood, hardly any children are able to conserve length while in later childhood, almost all children are able to conserve length. At these two extremes there is little hope of observing a relationship between conceptual tempo and conservation of length. It is at the transitional stage, then, that any possibility of such a relationship exists. The Ss included in the study, because of the age range, were quite likely to have been going through a transitional stage. That no relationship was observed in the study between conceptual tempo and conservation of length supports the fact that the latter is not reducible to the former.

In crucial experiments where the experimenter seeks to relate two quite disparate theories, it would seem that the experimental methodology would be impeccable. There was no indication that the experimenters took methodological issues into consideration when measuring conservation of length. Many experimenters would consider it necessary to incorporate reasons given by the child in classifying him as a conserver, nonconserver, or transitional. Other experimenters would consider reasons as not being necessary but employ other standards, hopefully as "good" as the reasons. Such standards should be subjected to rigorous testing and validation before utilized in experimental or correlational studies.

Leslie P. Steffe
The University of Georgia

EJ 035 713 450 SE 501 406

PATTERNS OF CHARACTERISTICS OF SUCCESSFUL MATHEMATICS TEACHERS AND THOSE WHO LEAVE THE PROFESSION: A CASE STUDY. Chapin, June R., Mathematics Teacher, v63 n2, pp159-163, Feb 70

Descriptors--*Case Studies (Education), *Educational Research, *Mathematics Teachers, *Teacher Characteristics, Discriminant Analysis, Instructional Staff.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Thomas J. Cooney, University of Georgia.

1. Purpose

To determine, through the use of discriminant analysis, patterns of characteristics of mathematics student teachers who had attended a state college, and their subsequent occupational status.

2. Rationale

In view of the shortage of secondary mathematics teachers as reported by the NEA in 1966 and the concern over teachers leaving the profession, evidence is needed on the pattern of characteristics of individuals who are likely to enter, continue, and succeed in the teaching of mathematics and that of teachers who are likely to leave the profession.

3. Research Design and Procedure

The sample consisted of 179 (146M, 33F) secondary student teachers in mathematics (grades 7-12) from the period 1957-1966 who obtained either a major or a minor in mathematics. All students were enrolled in a public state college and successfully completed their student teaching program. Data on 40 characteristics such as marital status, grade point average, and supervisor's ratings were gathered from files maintained by the college on the student teachers. Of the original 179 student teachers, 136 were located (118M, 18F) and their employment status was ascertained. Ratings by principals were obtained for 120 of these subjects. This writer assumed that the other 16 subjects were no longer teaching at the secondary level. However,

because of incomplete data, only 95 subjects were included in the final analysis.

The criterion measure for teaching effectiveness was a confidential rating scale utilized by the principals under which the teachers were employed. Each teacher was rated on the following scale: 1-excellent, 2-very good, 3-satisfactory, 4-poor, 5-no basis for judgment.

The technique of discriminant analysis was used to attempt to identify patterns of characteristics of successful mathematics teachers (received ratings of 1 or 2 from principals), unsuccessful teachers (ratings of 3 or 4), those employed by the school district but not teaching mathematics (administrators, counselors, etc.), and those who had left teaching at the secondary level. Weights were obtained for the 40 characteristics and applied to compute a total score for the individual's characteristics. This score was used with the criterion of teacher effectiveness.

4. Findings

The proportion of females that could not be located as compared to the proportion of males not located was significant at the .01 level.

Of the 95 subjects involved in the final analysis, 48 were classified as successful, 16 as unsuccessful, 17 were employed by the school district but not as teachers of mathematics, and 14 were no longer teaching. Through discriminant analysis different patterns of characteristics of the 4 groups were distinguished at the .01 level of significance. Of particular interest was the fact that unsuccessful teachers tended to have extreme scores which did not overlap with any of the other 3 groups. The pattern of characteristics of teachers who left the profession was different from the other groups, but their scores were the closest to those of the successful teachers.

There were only a few variables which contributed significantly to the scores. In general, the student teaching supervisor's report on professional qualifications (understanding pupils, knowledge of subject, etc.) contributed to high positive weights while reports on personal characteristics (appearance, poise, dependability, etc.) contributed negative scores to the weights.

Of the 14 subjects who had left secondary school teaching, the majority of these subjects were males who were employed in industry with an emphasis on computer science.

5. Interpretation

It appears that the female mathematics teachers may be less likely than their male counterparts to pursue a teaching career in view of the results of the initial phases of this investigation.

The analysis seems to indicate that teachers who have left secondary school teaching are more similar to successful teachers than to unsuccessful teachers. Had these people stayed in teaching, it is probable that they would have been rated as successful by their principals and, hence, they represent a real loss to the teaching profession.

Abstractor's Notes

Research involving the mobility of mathematics teachers into and out of the profession is important if we are to understand the forces, both economic and sociological, which influence this mobility. Also of interest is the nature of teachers which are involved in this mobility. This study is an attempt to describe the characteristics of mathematics student teachers in relation to their current employment status.

The main finding of the study was: "it appears that the mathematics teachers who have left the profession would have been rated by their principals as successful and they are a real loss to the profession." This inference is predicated on the statement "that teachers who have left secondary school teaching are more similar to successful teachers than to unsuccessful teachers." The author's use of the terms "successful" and "unsuccessful" teachers is inappropriate in view of the fact that of the 16 teachers rated "unsuccessful" a maximum of 4 (or as few as 0 - no information was given) were rated as poor, the remaining subjects being rated as satisfactory. There must also be concern over the rating scales used since principals and supervisors may have different criteria for rating teachers.

For the given sample, the author did find a difference in characteristics between what he calls "unsuccessful teachers" and the other 3 groups. However, teachers who have left the profession can only be said to represent a real loss in that their characteristics seem to resemble "successful teachers" rather than the characteristics of the other groups. One can only conjecture as to whether the "dropouts" would be judged as effective by their

potential employers. Hence, the major conclusion given by the author extrapolates well beyond the existing data.

It is unfortunate that the author did not explicate in greater detail his tests of significance and other statistical procedures that were used. The author's statement that "the supervisor's ratings on professional qualifications contribute positive weights while ratings on personal qualifications contribute negative weights" needs further examination when one considers that the "unsuccessful" teachers had substantially higher scores than the other groups. How did these positive and negative factors contribute to the differential scores of the various groups?

Thomas J. Cooney
University of Georgia

ED 045 454 SE 010 532

A STUDY OF THE USE OF CERTAIN SOCIAL REINFORCERS IN COMPUTER-ASSISTED INSTRUCTION. Fejfar, James L., Pub. Date '70, Note--12p., EDRS Price MF-$0.65 HC-Not Available from EDRS.

Descriptors--*Computer Assisted Instruction, *Elementary School Mathematics, *Instruction, Negative Reinforcement, Positive Reinforcement, *Social Reinforcement, *Student Characteristics.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Max Jerman, Pennsylvania State University.

1. Purpose

The study sought to determine if the achievement of fourth-grade students at the Indiana State University Laboratory School with high or low dominance characteristics would be affected by differing social reinforcers during their study of multiplication facts through a computer-assisted instruction (CAI) system.

2. Rationale

It was assumed that the best model to follow in developing CAI programs is the set of behavior patterns used by live teachers. Since teachers believe differential social reinforcers are important in regular classroom instruction, social reinforcers must also be important in CAI. Therefore, in CAI programs the presentation of social reinforcers should correspond to both the accuracy of the response and the personality of the learner. Although some writers, such as Stolurow, suggest CAI systems use a student's ability, personality, and knowledge in making decisions while the student is receiving instruction, past research offers few guidelines when such factors as personality are taken into consideration.

3. Research Design and Procedure

Students in two fourth grade classes were given a timed test on the 100 multiplication facts. Those students whose scores indicated non-mastery at the 5 second response level were given a group personality test in order to identify the high (70th decile and above) and low (40th decile and

below) dominant children. The mean I.Q. of the 9 boys and 11 girls thus identified was 102, range 78 to 128. One of the two classes from which children were selected was for high ability children. Both teachers used the same text and remarkably similar methods of teaching. The students in each category, high or low dominance, were randomly assigned to two treatment groups. One group received reprimands alone for incorrect responses and the other received reprimands for incorrect responses and praise for correct responses.

The twenty students thus selected were given another paper-and-pencil test on the multiplication facts (the pretest). The treatment consisted of four 20 minute sessions via a CAI system, an IBM 1130 computer and an electric typewriter on which the multiplication facts were presented one at a time to each student. The student responded by typing their answer to which the computer would respond by typing the appropriate reinforcing statement. Examples of the social reinforcing statements typed back to students were of the following type.

Good	No, XX is the right answer
Fine	Error, XX is the right answer
You are doing very well	Wrong, XX is the right answer

If an item was answered correctly five consecutive times it was retired from the day's session. Items missed three times were also retired from the day's session. The multiplication facts were presented randomly within categories of difficulty.

A paper-and-pencil posttest on the 100 multiplication facts was given the fifth-week after the study began, one week after the last CAI session. The retention test was given 11 weeks after the last CAI session.

4. Findings

The Kuder-Richardson Formula 20 reliability coefficients for the pretest, posttest, and retention test were .89, .95, and .97 respectively. The means of the test scores for the students assigned to each cell are given below.

TABLE 1

Means of Test Scores

Dominance	Praise and Reproof			Reproof Alone		
	pre	post	ret	pre	post	ret
High	44.8	73.0	77.4	59.4	86.6	82.4
Low	47.2	70.0	68.6	42.0	67.3	64.0

An analysis of variance which assumed three fixed factors (dominance, reinforcement, achievement) with repeated measures on achievement, was performed on the data for 18 students, since the retention test scores for two students were not available. No significant differences ($p<.05$) between achievement test scores for each type of social reinforcement were found for either high or low dominance groups. No interactions at the .05 level were found. Posttest scores were found to be significantly higher ($p<.01$) than pretest scores but not significantly different ($p<.01$) than retention test scores.

5. Interpretations

The data show that all 18 students who completed the study learned and retained the knowledge taught after only four sessions of 20 minutes each on the CAI system. This finding adds to the growing body of literature relative to the efficacy of CAI.

The use of social reinforcers did not affect achievement, however; and, there were no interactions between dominance and type of social reinforcement. In particular, no statistical reason was found to reject the following hypotheses at the .05 level.

1. High-low dominant personality traits were not related to achievement in this situation.
2. The type of social reinforcement employed (praise and reprimand vs. reprimand alone) does not effect achievement.
3. There are no interaction effects.

7. Were there any differences in achievement for the students selected from the high achieving class as compared to the other class?
8. From the wording of the article, the author appears to have changed his view that the factors studied need not be considered in future research when it was discovered that some pupils were beginning to develop negative attitudes. The author seems to have modified his views as he wrote.
9. It seems somewhat unusual to find the hypotheses of the study stated near the end of the article. One cannot help but wonder if they were not an after thought.
10. Were the 20 fourth grade students who were the subjects for this article the same 20 fourth graders who were mentioned in the author's earlier article in The Arithmetic Teacher (March, 1969)? In the previous article the program was described as developmental-experimental with changes in the program being made as required in order to keep things running smoothly.

Max Jerman
Pennsylvania State University

EJ 033 165 140 SE 501 380
THE MATHEMATICAL UNDERSTANDINGS OF PRESERVICE AND INSERVICE TEACHERS Gibney, Thomas C.; And Others, Arithmetic Teacher, v17 n2, pp155-162, Feb '70
Descriptors--*Elementary School Mathematics, *Mathematics Education, *Teacher Education, Achievement, Evaluation, Inservice Teacher Education, Preservice Education, Teacher Experience

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Robert E. Reys, University of Missouri

1. Purpose

To compare preservice and inservice elementary teachers' understanding of basic mathematical concepts.

2. Rationale

It is hypothesized that preservice and inservice elementary teachers represent two different populations with respect to mathematical understandings. If this hypothesis is true, one implication for mathematics educators is that an appropriate mathematics education program for inservice elementary teachers might and perhaps should be quite different from the mathematics education program for preservice elementary teachers. Furthermore, there may also be a need for more specialized mathematics education programs that are tailored for grade levels.

3. Research Design and Procedure

Findings were based on a total of 887 preservice and 177 inservice elementary teachers. No sampling scheme was described. These subjects were all enrolled for college courses in mathematics education at Bowling Green State University, The University of Toledo or Eastern Michigan University during the 1967-1969 school years. The preservice elementary teachers had all completed at least one three semester hour course in mathematics. No mention was made of the mathematics background of the inservice teachers. The distribution of subjects by grade levels (either grade level presently teaching for inservice or grade level preferred by preservice) is summarized in Table 1.

The subjects' understanding of mathematics was assessed with an instrument developed by the investigators. Some precedures used in developing the test are discussed. It purports to measure "basic mathematical understandings,"

however the development and discussion of the instrument is too sketchy.

The investigators identified seven subtests. (Table 1) The total and subtest scores provide the criterion on which the analyses are based. A series of t-tests were made between preservice and inservice teachers for the seven sub scores and the total. t-tests were also used between level of teaching experience for inservice teachers and preferred level for preservice teachers. A one-way analysis of variance on total scores was also made among preservice teachers and inservice teachers with varying lengths of teaching experience.

4. Findings

The t-tests between preservice and inservice teachers produced significant ($p < .05$) mean differences that are summarized in Table 1.

TABLE 1

SUMMARY OF ANALYSES

	Grade[1]	K	1-2	3-4	5-6	7-8	TOTAL
	Preservice	137	302	267	154	27	887
Subtest	Inservice	11	30	36	85	15	177
Geometry			*				
Number Theory		*	*	*			
Numeration System							
Fractions				*			
Structural Properties			*	*	*		
Sets				*			
Operations							
TOTAL			*	*			

[1]Grade identifies level subjects were currently teaching or preferred.

In each case these significant differences were in favor of the preservice teachers. The analysis of variance of total test scores confirmed the significant difference ($p < .05$) between the preservice and inservice teachers, but did not reveal any significant differences among inservice teachers with varying lengths of teaching experience.

5. Interpretations

The authors claim these findings suggest that preservice and inservice teachers have significantly different levels of understanding in mathematics. Such an argument (if true) provides a strong rationale for using differently designed mathematics education courses for inservice and preservice teachers. A further examination of scores for inservice teachers revealed a trend between grade level taught and total mathematical understanding test scores. In other words, the higher the grade level taught, the higher the scores. This suggests, for example, that inservice elementary teachers in the primary grades and inservice elementary teachers in the intermediate grades represent different populations with respect to mathematical understandings. Consequently, inservice courses in mathematics education should be designed to reflect these populations.

Abstractor's Notes

This study pinpoints a crucial issue in teacher education. Should mathematics education courses for inservice and preservice elementary teachers be similar or designed differently? This research provides some evidence for offering different courses; however the evidence is far from conclusive. For example, scrutiny of Table 1 reveals that only 11 of the 40 cells reported significant differences.

Although this abstractor is in sympathy with the major implication, the article left a number of unanswered questions about the study itself and places some clouds on the credibility of the research. Here are several items that caused this abstractor some consternation.

1. The sample was restricted to preservice and inservice teachers enrolled in mathematics education courses. The practicality of this sampling scheme is understandable, but the comment that "An analysis of background information...of inservice teachers suggested that this group had about the same education and experience patterns as the entire population of elementary teachers in the areas..." provided little more than a testimony of the representativeness of the sample. Details of this analysis would help.

2. No information was provided on the mathematics background of the inservice teachers. Nor was this reflected in any discussion of the analyses.
3. When were the tests administered? Were they given at the same time each semester? Presumably the tests were given early in the course and approximately the same time each semester, but neither of these questions is answered in the article.
4. The testing instrument provides the only quantitative data used in the analysis. Therefore the quality of the research cannot exceed the quality of the instrument used to gather the data. No specific mention was made of test validity nor procedures used to insure test validity. Although several test areas are identified it is not clear that an item assesses only within a given area. The number of items in each subtest is not cited nor if these subtests were weighted in proportion to the emphasis placed on the related mathematical topics in elementary schools.
5. The test included 13 items with a discrimination index between .01 and .19 and two items between -1.00 and zero. Including these items without explanation or elaboration increases the instruments vulnerability.
6. Kuder-Richardson 21 formula estimated total test reliability to be .80, which is not surprising considering the number of test questions. No mention is made of the reliability of the various subtests. Such knowledge would be helpful.
7. Seven areas or subtests are identified and these results are used in the analysis. It is not clear that these subtests measure different mathematical understanding or a single common factor. A factor analysis would seem appropriate. If the factor analysis confirmed that the test does indeed measure seven different factors, the later analysis would take on added significance. If it does not, the value of some of the later analyses is severely limited.
8. The authors claim the subtests can be dichotomized into "modern" and "traditional" areas. The ambiguity of these terms among mathematics educators raises serious doubts about the value of this classification without additional elaboration.
9. The need for "different courses" for inservice and preservice teachers is mentioned throughout this report. It is realized that a detailed description of these "different courses" transcends the scope of the research; however some conjectures by the authors spelling out some course differences would be interesting. In fact, such conjectures would likely open a Pandora's box, thereby encouraging additional research into what this abstractor considers to be a very fertile area.

Robert E. Reys
University of Missouri

EJ 038 324 340 SE 503 386

TESTING STUDENTS' ABILITY TO DO GEOMETRIC PROOFS: A COMPARISON OF THREE OBJECTIVE ITEM TYPES Hanna, Gerald S., Journal for Research in Mathematics Education, v2 n3, pp213-217, May '71

Descriptors--*Achievement Tests, *Deductive Methods, *Geometry, *Research, *Tests, Mathematics Education, Secondary School Mathematics, Test Construction [Proof]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James M. Moser, The University of Wisconsin--Madison

1. Purpose

The stated purpose was "to compare the concurrent validity of three objective item types designed to measure students' ability to produce formal proofs. The extent to which three kinds of easily scored objective test items would correlate with a conventional test of student-produced formal proofs given at about the same time was investigated."

2. Rationale

High school geometry teachers must spend a great deal of time correcting formal proofs. For this reason and because of the desire for complete objectivity, teacher constructed final examinations and standardized geometry achievement tests usually fail to include items that test students' ability to do proofs. It was assumed that delineation of types of objective items with high predictive validity would be welcomed by most test writers.

3. Research Design and Procedure

A 45 minute criterion test consisting of four formal proofs drawn from material in a full year's geometry course was given to 570 geometry students in 24 classes from 5 schools. The test was administered near the end of the school year. Scores on this test were labeled as Variable C.

The following day, the 570 students were randomly assigned to two equal-size groups which took the following tests:

First Group

21 items for Variable I--statements based upon a diagram and given information to which the student responded that the statement could be proved, disproved, or neither proved or disproved.

20 items for Variable II--10 pairs of items consisting of a verbal statement and a diagram. For the first question of the pair, the student selects a multiple choice response which symbolically indicates the "Given"; the second question of the pair is similarly related to the "To Prove."

Second group

41 items for Variable III consisting of statements related to four proofs, each proof having a necessary diagram and statements in the "Statement" part of a standard two column proof. For each statement, the student selects the correct "Reason" from a given list.

For both groups, about 95% of the examinees completed the test in a 45 minute period. Variables III and C were matched for content. Variables I and II, when combined, were matched in content with Variable C; separately they were not.

The proofs on the criterion test were scored by two university seniors. Procedures were used to avoid contamination of scorers' judgments. Only 2 of the 4 proofs were scored by both scorers, inter-scorer reliability on these being .85 and .92.

Means and standard deviations for all tests were computed including performance on the criterion test by each of the subsamples. Correlations between each of the three objective tests and the criterion test were computed with appropriate corrections made for difference in lengths of test time and for differences in subsample variability.

4. Findings

The information for the variables is as follows:

	Correlation with C	Number of points in test	M	S.D.
Variable I	.41	21	12.21	2.83
Variable II	.54	20	13.70	4.77
Variable III	.62	41	34.68	6.76
Variable C	.78	100	40.21	27.83

The adjusted split-half reliability of the criterion test was reported as .78.

5. Interpretations

It was judged that Variables II and II were better assessors of students' ability to do proofs. The high mean performance on Variable III suggests the items may have been too easy.

Abstractor's Notes

I concur with the author's remark that it is "unfortunate that Variables I and II were not separately matched in content with the criterion variable." Because of this, the desirability of use of items of the type contained in these 2 tests cannot be fully assessed. It is also unfortunate that both scorers did not score all proofs on the criterion test. This might have been possible if the sample size had been reduced. Further, it is too bad that the author did not design his study so that combined predictive ability of two or three variables could have been computed. This would also have permitted computation of correlations between the three objective variables.

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ED 041 761 24 SE 009 294
A STUDY OF THE TEACHING OF NON-DECIMAL SYSTEMS OF NUMERATION IN THE ELEMENTARY SCHOOL, FINAL REPORT.
Kavett, Phyllis F., Rutgers, The State University, New Brunswick, N.J., Spons Agency--Office of Education (DHEW), Washington, D.C. Bureau of Research. Pub Date Apr 69, Note--225p. MF-$0.65 HC-$9.87
Descriptors--Arithmetic, Education, *Elementary School Mathematics, Evaluation, Grade 4, Grade 6, *Mathematical Concepts, *Mathematics, Number Concepts, *Research.
Identifiers--U.S. Department of Health, Education, and Welfare.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Richard Shumway, Ohio State University.

1. Purpose

This experiment was designed to investigate the effects of teaching non-decimal systems on the learning of decimal systems by fourth and sixth graders when computation and arithmetic achievement were used as evaluative criteria.

2. Rationale

It is now common practice to introduce numeration bases other than ten so that students may appreciate and understand base ten more fully (Rahmlow, 1965). Yet, the literature describing studies of effects of non-decimal instruction on decimal systems operations has been inconclusive. There is clearly a need for further investigation (Rosskopf, 1963; Shulman, 1967; Becker and McLeod, 1967). This study attempted to overcome the shortcomings of earlier research on the effects of teaching non-decimal systems on the learning of decimal systems.

3. Research Design and Procedure

The subjects were 430 students in 18 classes randomly selected from 45 classes whose teachers volunteered for the study; nine classes from grade four and nine from grade six. There were four treatments: Non-Decimal Systems; Non-Computation, Intuitive Geometry; Decimal

System: Visual-Manipulative Emphasis; and Decimal System: Regular Program; three, two, two, and two classes from each grade level were randomly assigned to the treatments respectively. The treatments were five-six weeks in duration. Pretest measures of mental maturity, vision, hearing, and coordination, and arithmetic achievement were used; posttests and retention tests on arithmetic achievement and non-decimal numeration were used. The retention tests were given seven weeks after the posttests. The data were analyzed seperately by grade using a one-way analysis of covariance procedure with student as the experimental unit of analysis. Three covariates were used, two pretests and one posttest. Experimental groups differed significantly in mean performance on the covariate measures. Not all tests of the assumptions underlying the analysis of covariance procedures were satisfied. Distributions of scores on the non-decimal tests were compared for the fourth and sixth grades with the Kolmogovov-Smirnov Two-Sample Test. Intercorrelations of scores were reported and compared among groups seperated on the following factors: treatment group, sex, race, degree of advantage.

4. Findings

(a) Equal arithmetic reasoning group mean scores were achieved by all groups on the posttest.
(b) Retention of knowledge of non-decimal systems of numeration was greater among sixth grade students than among fourth grade students.
(c) Intercorrelations of scores were slightly different for boys and girls, similar for different treatment and racial groups, and most dissimilar for groups separated according to teachers' judgments of degree of educational advantage.

5. Interpretations

(a) Teaching of non-decimal numeration systems in the upper elementary grades is recommended. Place-value and the relation to the decimal system should be stressed.
(b) Further research might explore grade placement of the topic, merits of different methods of teaching, and long-range effects of learning the topic.

Abstracter's Notes

(1) Shouldn't the experimental unit of analysis have been class instead of student?
(2) Shouldn't tests for the significance of the intercorrelations account for the fact that 12 independent variables were being examined? (For example, if n=72, for one independent variable correlations above .23 are significant (.05) but if the table has 4 variables, then only correlations above .35 are significant (.05).)
(3) What test was used to compare correlation coefficients for different groups? Was an adjustment made for the number of tests run?
(4) Shouldn't there be some doubt about the appropriateness of the analysis of covariance procedures? Can analysis of covariance be used as a replacement for random assignment? What about using posttests as covariates?
(5) Does the failure to find significant differences mean the means are equal?
(6) How does conclusion a) follow from the findings?
The problem of this investigation is an important one. The claims for transferability of non-decimal systems should be investigated. The author has identified several relevant variables which should be studied.

Richard Shumway
Ohio State University

The Stanford Achievement test, form X, was administered as a pre-test to all subjects. Form W was administered as a post-test. IPI Placement Tests were also used as a measure of achievement. The IPI pre-tests and post-tests were taken by the subjects in both the experimental and control groups at the Broad Street School, but not by the second control group.

The Otis-Lennon Mental Ability Test was administered to grades three and five, and the results of the test from the previous year were used as an IQ measure for grades four and six. The mathematics scores on the Stanford Achievement pre-test and these IQ scores were used to identify the control sections at the Gardner Road Elementary School. (The IQ data are not provided in the report so we must take the researchers word for the validity of the methods used for "matching".)

Student attitudes were measured by two tests developed by the school district. Test 1 was administered to all subjects in February. Test 2 was administered to the experimental group only.

Two attitude measures were given to the teachers of the experimental classes only. One was constructed by RBS and the other was developed locally. The locally constructed scale was administered in the fall and again the following spring.

Student achievement and student attitudes were to have been analyzed by a three way analysis of covariance. We are told, though, that the analysis was not completed and that "The results shown herein are presented in the best form possible under the circumstances." There is no further clue given to the statistical procedures employed.

4. Results

a. On the mathematics section of the Stanford Achievement Test, there was no difference in achievement between the control groups and the IPI group.
b. Using the IPI Placement Tests as a measure of achievement, it was found that the growth of IPI students was greater than that of the control group.
c. Low groups of pupils were as successful in IPI as were faster pupils.
d. IPI pupils liked mathematics better than non-IPI pupils.

e. All IPI teachers held very positive attitudes towards IPI.

5. Interpretations

It is implied, although not specifically stated, that the study demonstrates that the IPI method of instruction is a better way to teach mathematics than the conventional mode. The authors explain that the results on the SAT do not nullify this conclusion because it does not adequately measure IPI skills. They report that less than 30% of the IPI skills are measured by the SAT.

Abstracter's Notes

The reader is unable to form his own conclusions about the validity of this study because so little information is given. The following are among the more important questions that are not answered in the report.

1. What data were analyzed and how?

Not only are we left up in the air with respect to statistical procedures, but we're not even told if the variables of IQ and previous mathematics achievement were accounted for. The IPI tests on which the conclusions were based were administered only to Broad Street School students, and the control classes in this school were not matched with the experimental sections. Levels of significance at which the "no differences hypotheses" were rejected are also not given.

2. Was the content of the conventional program similar to that of the IPI program?

It was stated that the SAT was not a fair test of the IPI Program. Could it also be that the IPI tests are not a fair measure of the conventional program? Perhaps the results in favor of the experimental group on the IPI Placement Tests do not reflect a better method of teaching, only different objectives.

3. How were "low" students defined?

The abstracter concurs with the researchers that the performance of low ability students in an individualized program is of great concern. However, the positive results given in this report concerning the performance of "low" students should definitely not be generalized. In the Horsehead Elementary Schools, children are grouped according to ability for mathematics instruction. Since the term "low groups" was used in reporting the results above (rather than, say, "low ability students"), it is

suspected that these groups were the lower ability classes in the ability grouping structure. The abstracter averaged the pre-test Stanford Achievement scores for grades two through six over computation, concepts and applications and found that these "low ability" groups were about .1 of a year above grade level. On the New York State Mathematics Test given to grades 3 and 6, at most 11% of any one class (2 or 3 students) was below minimum achievement.

4. Has the data on student attitudes been interpreted correctly?

We are told that on the basis of interview data IPI students liked school better than non-IPI pupils. (Interviews were not included in the original design of the study.) We are also told that they liked math better. Was this based on the interviews or on the paper and pencil attitude measures? In the report, a copy of the Pupil Attitude Survey, Test 1, with results, is provided. The data are given in two columns originally labelled IPI and Control, in that order. These headings were crossed out and the titles "Pre-test" and "Post-test" are handwritten in. Since the information about the design of the study clearly indicates that Test 1 was to be administered only once, one certainly wonders where this information came from.

If the original headings were correct, the experimental group had considerably poorer attitudes towards mathematics than the control group. If the handwritten headings are correct, then there was an improvement in attitude towards mathematics - but by whom? and over what period of time?

5. How were teachers chosen for participation in the experimental classes?

No information is given about the selection of teachers. If all teachers were volunteers, the information that teacher attitude was positive has little value to the administrator or coordinator who is concerned with the staffing problems involved in implementing the IPI program.

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EJ 031 236 270 AA 508 346
MODERN AND TRADITIONAL MATHEMATICS TEACHING Rao, G. S.
Gopal; And Others, Educational Research, v13 n1, pp61-5, Nov '70.
Descriptors--*Mathematics Instruction, *Cognitive Processes, *Critical Thinking, *Tables (Data).

Abstract and Analysis Prepared for I.M.E. by James T. Fey, University of Maryland.

1. Purpose

To compare the effects of modern and traditional mathematics curricula on student critical and creative thinking abilities and attitudes toward mathematics.

2. Rationale

Despite popular attention to recent proposals for revision of the school mathematics curriculum, proponents of reform have increasingly been challenged to show that modern courses are more effective than those they replace. Achievement tests based on the topics common to modern and traditional syllabi do not fairly measure the important objectives of either course. But mathematics curriculum innovators have argued that a syllabus emphasizing the structural ideas and inductive/deductive methods of the subject is superior to the traditional program that focuses on training in specific skills. They claim that students who have followed the new curriculum will have acquired conceptual knowledge, broad cognitive skills, and positive attitudes toward mathematics that will be of great value in future learning and application of the subject.

3. Research Design and Procedure

Analysis of objectives for several modern mathematics curricula led the investigators to formulate three research hypotheses: Students who have studied a modern mathematics curriculum will have (1) greater critical thinking ability, (2) greater creative thinking ability, and (3) more positive attitudes toward mathematics than students following a traditional syllabus. The experiment involved comparison of third and fourth year students in British secondary schools.

The investigators selected 26 boys and 60 girls studying modern courses and 26 boys and 59 girls studying traditional couses in schools and classes judged to be of comparable background and ability. The girls were chosen from 4 intact classes; the boys were chosen from 2 intact classes. As far as could be determined, there were no differences of consequence in the teaching of subjects other than mathematics that could influence results of the tests of critical or creative thinking.

Each subject took a five part critical thinking test, a five part creative thinking test, and an attitude test. The critical thinking battery included measures of student ability in abstraction, analysis of data, classification and inference, correction of sign placement in algebraic expressions, and symbol manipulation. The creative thinking battery included the Thurstone "First and Last Letters" test of originality and word fluency, Guilford's test of "Unusual Uses", a "Hidden Word" test of Lovell, a card sorting concept formation test, and a "make up problems" test. The attitude instrument was original and of the Thurstone type.

On each of the eleven tests, experimental (modern) and control group means were compared using t-tests.

4. Findings

In four of the five critical thinking tests (abstraction, analysis of data, classification and inference, symbol manipulation) significant differences favored the experimental group (.01, .01, .01, .02). On the test of sign changes the difference--while favoring the experimental group--was slight.

In three of the creative thinking tests (Hidden Words, Unusual Uses, Concept Formation) significant differences also favored the experimental group (.01, .02, .05). The remaining tests favored the experimental group only slightly.

On the attitude instrument, scores were slightly in favor of the control (traditional syllabus) group. The means on a 9 point Thurstone scale were 4.970 (control) and 4.721 (experimental).

5. Interpretations

The investigators considered results from the critical and creative thinking tests to be impressive support for the cognitive process outcomes predicted by contemporary curriculum innovators. They suggest that the reverse

(though non-significant) trend in attitude data might be a result of working with intact groups under the influence of prior experience and attitude formation.

It is noted that, in experimental classes the style of instruction, as well as the syllabus, differed markedly from that in traditional classes. This suggested several questions for future research that appraises the interaction of curricular and instructional variables.

Abstracter's Notes

The report does not include careful description of the traditional and modern syllabi used in classes from which subjects were chosen. Furthermore, there is no data given to support the contention that subjects were from classes of comparable background and ability. In fact, possible differences in background are suggested as explanation for the attitude data patterns.

The most serious questions of the study center around the fact that subjects were chosen from only 6 intact classes. The investigators observed consistent differences of instructional style between experimental and control classes. Yet it is clear from the presentation of data that the experimental unit was the student, not the class. Differences in means that are highly significant for N=90 are much less impressive with N=3.

Nonetheless, the consistent collection of trends favoring experimental groups suggests a promising line for more extensive investigation. The reversal of this trend in attitude data can also be understood only after more thorough follow-up investigation in which the effect of teacher variables is better handled.

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EJ 011 987 140 SE 500 887

EVALUATING MATHEMATICS COURSES FOR PROSPECTIVE ELEMENTARY SCHOOL TEACHERS Moody, William B.; Wheatley, Grayson H., School Science and Mathematics, v69 n8, pp703-707, Nov '69

Descriptors--*College Mathematics, *Course Evaluation, *Evaluation, *Elementary School Teachers, *Teacher Education, Undergraduate Study, [University of Delaware]

Expanded Abstract and Analysis, Prepared Especially for I.M.E. by Thomas A. Romberg, The University of Wisconsin--Madison

1. Purpose

The purpose of this exploratory status study was to examine whether or not elementary education majors who had been exposed to especially designed mathematics courses would read mathematically oriented literature with a greater depth of comprehension than would students who had not had this exposure.

2. Rationale

One outcome of mathematics courses for prospective elementary school teachers, in addition to teaching certain concepts and skills, should be to enable the student to read with discrimination mathematically oriented literature.

3. Research Design, Procedure, Findings

Two hundred eight (208) subjects, 134 undergraduates and 74 inservice teachers at the University of Delaware, in the spring of 1967 were given an article to read and then asked to answer a 15-item comprehension test during a 50-minute (mathematics class) period. Results of the study were summarized by categorizing the subjects into four groups and calculating mean scores for each of the groups. The groups were:

a. Elementary education undergraduate major having completed the first course in the mathematics program. (EMY).
b. Elementary education major who had not been exposed to a mathematics course in college. (EMNO).
c. Liberal arts undergraduate major who had completed a basic college mathematics course. (LA).
d. Inservice elementary school teacher who had not taken a recent mathematics course. (IS).

A t-test for difference between mean scores of the EMNO, LA and IS groups with the mean score of EMY groups was used to

determine significance. In each case the differences were found to be significant in favor of the EMY group.

4. Interpretations

The authors are careful in warning the reader of the limitations of the study. They do feel the results suggest there is value in having students take the specially designed mathematics courses.

Abstracter's Notes

This is a nicely done and well reported study. The authors accurately describe what they did, why they did it, and were cautious in interpreting and generalizing the results.

As I read this article I was impressed with what needed to be done. The problem and rationale made sense to me. The data collected suggest results which need to be verified. Unfortunately, the authors have not indicated how they would design a study to verify these suggestive but very tentative results. Such a design would have to overcome three serious problems in this study: comparability of populations, limited and non-independent evidence, and inappropriate analysis of the data.

To infer that mathematics courses indeed had the hypothesized effect, subjects from a single population (such as juniors in an elementary education program) would need to be selected and randomly assigned to take or not to take the mathematics courses. The population categories used here could mean more than difference in exposure to mathematics (such as: age, recency of exposure, extent of content covered,...).

A single article and related items are not enough evidence to generalize about reading comprehension. In fact, technically the items of any reading passage are not independent items. The passage and items should be considered as a "superitem." (see Cureton, 1965). Non-independence of items results in spuriously high estimates of test reliability. Hence, a number of passages and items treated as superitems is called for.

Since the items used in the study are not independent, a different data analysis procedure is called for. A raw score difference of 1.5 points on 15 non-independent items is not much.(The means of the EMY and EMNO groups were 11.8 and 10.3 respectively.) Also, technically Dannett's test should have been used instead of the t-test.

These comments should not be interpreted that what the authors did was incorrect. In exploratory studies one often takes liberties with assumptions about populations, evidence or analytic procedures, for reasons of expedience. But, if suggestive results appear, a researcher is obligated to suggest how to verify the results.

Reference

Cureton, E. E. Reliability and validity...basic assumptions and experimental designs. Educational and Psychological Measurements, 1965, 25(2), 324-346.

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ED 040 849 24 SE 008 275

EFFECTS OF PRECISE VERBALIZATION OF DISCOVERED MATHEMATICAL GENERALIZATIONS ON TRANSFER, FINAL REPORT.

Retzer, Kenneth A., Illinois State University, Normal. Dept. of Mathematics, Spons Agency--Office of Education (DHEW), Washington, D.C. Bureau of Research, Pub. Date Oct. '69, Note - 61p, EDRS Price MF-$0.65 HC-$3.29.

Descriptors--Grade 8, *Instruction, *Learning, *Mathematical Logic, *Secondary School Mathematics, *Verbal Communication.

Expanded Abstract and Analysis Prepared especially for I.M.E. by Joseph M. Scandura, University of Pennsylvania.

1. Purpose

a) To test the effect of teaching certain concepts of logic on verbalization of discovered mathematical generalizations using a normally distributed research population of junior high school students.

b) To prepare a research population which has demonstrated the ability to verbalize newly discovered mathematical generalizations with precision.

c) To test the effect of an ability to verbalize discovered mathematical generalizations upon the ability to use that generalization.

2. Rationale

Hendrix (1961) feels that the transfer power of a generalization is available to a learner as soon as the non-verbal awareness of the generalization has occurred. She found that learners who completed correct verbalization of a discovery did no better on transfer tests than did learners for whom teaching was terminated at a level of non-verbal awareness, and that moreover, the verbalization of the discovery seemed for some persons to diminish their power to apply the discovery.

Ausubel (1961) agrees that subverbal awareness exists but feels that verbalizing a discovery is important to the thinking process. He says, "when an individual uses language to express an idea...he is engaged in a process of generating a higher level of insight that transcends by far...the previously achieved stage of subverbal awareness."

Retzer has previously developed a linearly programmed unit, Sentences of Logic, designed to teach junior high

school students the knowledge about language, hypothetically sufficient for the students to be able to write precise generalizations they had discovered, and found in an experiment with "college-capable" junior high school students as subjects, that the group which had studied Sentences of Logic did significantly better in verbalizing discoveries precisely than did a control group. This study attempted in part to replicate this finding with a wider cross section of ability among junior high school students. It also attempted to assess the degree of association between ability level (IQ) and ability to verbalize discovered mathematical generalizations. Once a population which could verbalize mathematical generalizations to some extent was identified, the study attempted to assess the association between this ability and the ability to use the generalizations.

The hypotheses to be tested in Phase I were:

H_1: Completion of the Sentences of Logic has no effect on the ability of junior high school students to verbalize discovered mathematical generalizations.
H_2: The ability level of junior high school students has no effect on their ability to verbalize discovered mathematical generalizations.
H_3: The effect of the completion of the Sentences of Logic unit on verbalization ability is independent of the ability level of junior high school students.

The hypotheses tested in phase II were:

H_1: Verbalization of discovered mathematical generalizations has no effect on the ability of the junior high school students to use the generalizations.
H_2: The ability to state discovered mathematical generalizations with precision has no effect on the ability of junior high school students to use the generalizations.
H_3: The effect of verbalizing discovered mathematical generalizations on ability of junior high school students is independent of the ability to state the generalizations with precision.

3. Research Design and Procedure

Subjects for Phase I were 120 eighth-grade students from a Normal, Illinois junior high school, 60 with IQ 116 or above, 60 with IQ 113 or below.

For Phase I, half the Ss completed the Sentences of Logic treatment; the other half did not. This factor was crossed with the factor of IQ. 4 groups of 30 Ss resulted, each S then completed a programmed unit A Short, Short, Story About Vectors. The dependent variable was the precision of verbalization of generalizations discovered during the vector story scored according to the guidelines in Retzer's original experiment. A 2 x 2 analysis of variance was used to test the hypotheses.

For Phase II, 60 of the Ss with non-zero verbalization scores were divided into two equal groups according to whether their verbalization scores were above or below the median score for the 60 Ss. Each S then completed Exponent program B which did not verbalize generalizations, C in which the text verbalized generalizations, or D in which the student was required to verbalize the generalizations. 6 groups of 10 Ss resulted and a 3 x 2 analysis of variance was used to test the hypotheses, using Ss' score on a transfer test given 1 month later as the dependent variable.

4. Findings

For Phase I, Hypothesis I was rejected ($F(1,116) = 8.923$, $p<.005$); H_2 was rejected ($F(1,116) = 36.426$, $p<.005$); and H_3 was rejected ($F(1,116) = 4.947$, $p<.05$).

TABLE 1

PHASE I

Mean Verbalization Scores

	IQ≥116	IQ≤113	Mean
Completed Sentences of Logic	29.8	3.6	16.7
Did not complete Sentences of Logic	13.3	1.2	7.3
Mean	21.6	2.4	12.0

For Phase II, Hypotheses H_1 and H_3 were not rejected ($F(1,54) = 18.353$, $p<.005$). Hypothesis H_2 was rejected ($F(1,54) = 18.353$, p .005).

TABLE 2

PHASE II

Mean Transfer Scores

	Phase I Verbalization Score above median	Phase I Verbalization Score below median	Mean
Completed exponent program B	11.2	5.3	8.3
Completed exponent program C	8.8	4.8	6.8
Completed exponent program D	9.3	4.0	6.7
Mean	9.8	4.7	7.25

5. Interpretations

The conclusions for Phase I were:

1. Those students who completed the programmed Sentences of Logic unit did significantly better in verbalizing newly discovered universal generalizations than those who did not.
2. Students with IQ 116 and above did precisely verbalize newly discovered generalizations better than did students with IQ 113 and below.
3. The verbalization of students with IQ 116 and above was aided more by the logic unit than was that of the other students.

On these bases then, if teachers could teach logical components of generalizations as an explicit part of the curriculum, then they could choose between delaying verbalization, or asking for immediate verbalization with the expectation that students would respond precisely. Also the ability to precisely state discovered mathematical generalizations seems to be a factor that can be manipulated for educational or experimental purposes.

The conclusions for Phase II were:

1. Students who have the ability to state generalizations precisely are better able to transfer their discoveries.
2. Superiority for one discovery strategy over another was not indicated.
3. The hypothesis (3), that as far as transfer power is concerned the ability to state a newly discovered mathematical generalization with precision is independent of the three discovery strategies used in Phase II, was not rejected.

Abstracter's Notes

Note that in Phase I, IQ was significantly associated with the ability to verbalize discovered mathematical generalizations precisely. In Phase II, precision of verbalization was an independent variable and it was significantly associated with ability to transfer. If in Phase II, IQ had been used as a covariable (to remove its effects on precision of verbalization), would there still have been a significant association with ability to transfer, or was the association due to IQ loading? Were there any effects due to history in the one-month period between treatments and transfer test?

Also Retzer's procedure for scoring precision of verbalization in Phase I in itself involved some instruction to the Ss. Scores were taken essentially during the procedure, but there may have been some effects due to Ss completion of the procedure. That is S's ability to verbalize precisely was probably better at the start of Phase II than it was when measured. The scores used as an independent variable in Phase II then, were probably not uncontaminated.

It should also be noted that relationships found in studies of this type generally are only symptomatic of underlying mechanisms of cognition, and as such are of peripheral interest if one is interested in clarifying

those mechanisms. (The results, of course, may be of practical interest at least until the underlying mechanisms are better understood.) In the present study, for example, the basic question is why verbalization has the effect on transfer it is purported to have, if indeed it does.

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The abstractor wishes to thank Mr. Wallace H. Wulfeck II for his assistance in the preparation of this report.

ED 055 852 SE 012 474
RESEARCH REPORT OF THE SPECIALIZED PROJECT 1969-1970.
San Diego County Dept. of Education, Calif., Spons Agency - California State Dept. of Education, Sacramento. Pub Date Jun 71, Note--65p., EDRS Price MF-$0.65 HC-$3.29
Descriptors--*Achievement, *Elementary School Teachers, *Inservice Programs, *Mathematics, Research, *Teacher Education.
Identifiers--Specialized Teacher Project.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Kenneth J. Travers, University of Illinois, Urbana.

1. Purpose

This project, one of four mathematics improvement programs established by the 1967 session of the California State Legislature, was devised to provide ways of strengthening mathematics instruction in the state's elementary schools. The aims of the workshop were very broad within the general framework of activity learning.

The project examined the effectiveness of five different in-service program formats, where effectiveness was defined primarily as improvement in pupil achievement in mathematics.

The various program formats involved a summer workshop (typically two weeks long) with varied instructor-participant ratios (from 2-20 to 4-150). One of the formats involved a one-week workshop followed by five Saturday sessions at two-week intervals during the fall.

2. Rationale

The training program emphasized individualized learning experiences in a laboratory setting in accordance with educational philosophies promoted by the Madison project, the Nuffield project and others utilizing an active approach to learning mathematics.

This project is built upon a study during the preceding year which reportedly demonstrated that the in-service training program for mathematics teachers significantly improved pupil achievement in mathematics.

3. Research design and procedure

Five separate in-service program formats were devised and their effectiveness (differences in pupil achievement measures) were studied.

Districts throughout the State of California were invited to submit applications for pairs of teachers (one teacher to become the participant, the other to become the control, that is, not attend the workshop).

Participants were reportedly assigned randomly to the various in-service formats. A total of 858 teachers at the second and fifth grades were selected for in-service workshops at three locations in the state. Prior planning and careful scheduling of activities were effected in an attempt to keep the workshop in the three locations as similar as possible. A follow-up session for the participants was held the next February.

The effectiveness measures, student achievement, were obtained from a battery of tests covering concept understanding, computational ability and attitude toward mathematics administered shortly after the beginning of the school year and a similar battery given near the end of the school year. The second-grade tests were from the SRA Primary Form D (mathematics). The fifth-grade tests were from the National Longitudinal Study of Mathematical Abilities battery.

Analysis of covariance using pretest scores as the covariates was employed.

4. Findings

An overall positive effect of the in-service program on the mathematics achievement of the pupils is reported. The data also are claimed to confirm the 1968-1969 finding that the in-service program is more effective in grade 2 than in grade 5. No overall significant differences in performance on any of the attitude scales were found. However, in the samples, in-service classes from the low socioeconomic group had the highest adjusted score on the fun versus dull attitude scale.

A cost-effectiveness analysis suggested that a large group team-taught format was most desirable (a teacher-participant ratio of 4 to 100 was recommended).

5. Interpretations

The investigators report that the data reveal that

pupils whose teachers received in-service training scored significantly higher on measures of comprehension and computation than did pupils in the contol group. The project was found to be particularly effective with pupils from low socioeconomic areas.

Abstractor's Notes

This is not a research project in a strict sense. The design, although commendable to contemplate, proved to be extremely difficult to effect. The statistical tests although technically correct (making the usual assumptions of random assignment of groups, homogeneity of regression, and so on) have yielded results which are puzzling when interpreted. For example, the investigator reports that "in grade 5 the two-week in-service programs produced significantly better pupil performance in several areas of mathematics achievement, although the effects were not large." Presumably, Table 10, page 17, of the report yields the results under discussion. Apparently, the tables were renumbered in the appendices, since the text refers the reader to Table 7, but those data are for second grade, and show no significant differences between the adjusted means for the five treatment groups (The control groups are not included.). In the analysis under discussion, in only two of the scales (Geometry-Informal and Graphs, Probability and Functions) did the control group have lower adjusted mean scores than at least one of the treatment groups. So although the differences between the six groups (five treatment and one control) are significant, the differences are not typically in the direction which supports the researcher's conclusions.

Later in the report, the researchers do recognize seeming anomolies in the scores, and point to an apparent organizational weakness in the plans for the treatment group which utilized the one week in the summer and the five Saturday follow-ups as the format. However, since according to Table 4, page 13, all those participants were in only one location, this reviewer is inclined to ask more about the nature of the school population, the teaching staff and other details concerning the educational setting of the district before making inferences about the format of the workshop itself.

Therefore, prospective readers of this report should not anticipate finding definitive research in in-service education. They will, however, find an accounting of an

interesting attempt to learn more about this important topic and some indications of the pitfalls of this kind of evaluation. Furthermore, they will find in the report records such as materials lists, reference books, films and filmstrips and other supplies of use to those actively involved in teacher education.

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ED 045 414 24 SE 010 179

AN EXPLORATORY STUDY OF THE INTERACTION OF THREE ELEMENTARY CONCEPTS OF PROBABILITY WITH STIMULI, SOCIOECONOMIC, GRADE, AND IQ DIFFERENCES. Shepler, Jack L., Wisconsin University, Madison. Research and Development Center for Cognitive Learning. Spons Agency--Office of Education (DHEW), Washington, D.C. Bureau of Research. Pub. Date, Mar. '70, Note--46p. EDRS Price MF-$0.65 HC-$3.29.

Descriptors--Educational Research, *Elementary School Mathematics, Grade 5, Grade 6, *Instruction, *Learning, *Mathematical Concepts, *Probability.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jack E. Forbes, Purdue University, Calumet Campus.

1. Purpose

To analyze (1) the status of three basic concepts of mathematical probability (points of a finite sample space, probability of an event for a finite sample space, most likely occurrence of an event in one trial where the event is a subset of each of two distinct sample spaces) with children of different IQs in Grades 5 and 6, (2) the effects of the presence or absence of visual aids on the child's ability to solve problems related to these three concepts, and (3) the differences of children's conception of these concepts when the school and socioeconomic environment are quite different.

2. Rationale

Most of those who recommend the teaching of probability concepts to elementary school children and those who write materials for such instruction assume the child has some intuitive notions about the three basic concepts considered in this study. It is important that both the general validity of this assumption and the possible dependency of this validity on socioeconomic, grade, and IQ differences be investigated.

3. Research Design and Procedure

This exploratory study was conducted in May, 1968 with

8 fifth and 8 sixth-grade students from an all white school in a middle socioeconomic section and 6 fifth and 8 sixth-grade students from a school in a low socioeconomic section of the same city. The second school (and the set of participants from it) included approximately 65% Black students. Subjects were selected by the teachers in approximately equal numbers from high, medium, and low IQ groups within each class.

The study employed an interview technique to determine the status of the three basic concepts of probability mentioned earlier. Socioeconomic, grade, and IQ differences were explored in relation to the sequence of steps used by the interviewer in an attempt to elicit a correct response and a statement of the reason for that response. Those steps were:

1) Verbal presentation of the problem (written and oral).
2) Verbal hints (oral).
3) Pictorial representation to aid solution.
4) Model to aid solution.

The interviewer continued until a correct response and an acceptable reason were obtained or until all steps above had been used. The data collected were the ratios of the number of correct responses (and reasons) to the number of responders. These data were reported in a 2 x 2 x 4 x 3 arrangement of cells - grade level x school x step at which correct response was obtained x IQ level. Informal conclusions were reached by comparison of these ratios.

4. Findings

"The study indicated that the status of these concepts is related to IQ, socioeconomic background, and to a lesser degree, grade differences. All differences were in the expected direction. The children had learned a significant amount concerning these three concepts without formal instruction. However, the children from a middle-class area were more successful at the verbal level than children from the low socioeconomic area."

5. Interpretations

"(From the performance of the children in the study)

one (sic) can thus be fairly confident that a unit on probability would be within the mental grasp of these children...The typical units on probability for the elementary school...would be appropriate for children from (the middle-class school). However, they would probably be very inadequate for the children from (the low socio-economic school)." The investigator stresses the limited generality of the study due to the small sample, the interdependence of the factors used to partition the set of subjects, and other aspects of the design.

Abstracter's Notes

The term "formative research" is often applied to a study that differs from "hard data" research only in the fact that the sample size is too small to hope for results that are statistically significant. This is not the case of the study reported here. The experimental design and procedures used in this study show that those involved clearly understand the purpose and limitations of such research. The results reported are not surprising. Of course, they are not conclusive. It is the design used rather than the results obtained which make this an interesting and important paper.

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EJ 038 325 340 SE 503 387
NEGATIVE INSTANCES AND MATHEMATICAL CONCEPT FORMATION: A PRELIMINARY STUDY Shumway, Richard J., Journal for Research in Mathematics Education v2 n3, pp218-227, May 71
Descriptors--*Concept Formation, *Grade 8, *Mathematical Concepts, *Research, Achievement, Learning, Mathematics Instruction, Teaching Procedures

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Ralph T. Heimer, The Pennsylvania State University

1. Purpose

The primary purpose of the investigation was to examine the effect of negative instances on the acquisition of mathematical concepts.

2. Rationale

Numerous studies have been conducted over the past half-century aimed at acquiring information about the role of negative instances in mathematical concept formation, but the issue remains unresolved in that the investigations have led to equivocal results. Interest in the problem has not subsided, however, and the present study constituted an attempt to reconcile former conflicting findings.

For purposes of the study, the investigator defined a concept as a partitioning of a class X into two disjoint classes X_1 and X_2 where the elements of X_1 are called positive instances of the concept, the elements of X_2 are called negative instances of the concept, and the class X is called the universal class over which the concept is defined. Furthermore, to say that a student knows the concept over the class X is to say that given any object from the class X the student is able to identify the object as a member of the class X_1 or the class X_2 associated with the concept over the class X.

3. Research Design and Procedure

Originally, 120 eighth grade students were randomly selected from a population consisting of 158 students, and the students comprising the sample were then randomly assigned to one of four classes taught by two instructors. Subsequent to the assignment of students to classes, but before the study was initiated, a number of student withdrawals occurred that caused

the sample size to drop to 84. The investigator reported that the withdrawals were evenly distributed over classes, the individual class enrollments finally being 22, 22, 18 and 22. There were two treatments, with each instructor teaching both an experimental and a control class. In the experimental classes, after each concept was defined, approximately the same number of positive as negative instances were studied by the students. In the control classes, after each concept was defined, only positive instances were studied by the students. The use of instances of the concept was the primary mode of instruction throughout the study; each instance was presented as a problem and the student was asked to classify the instance as positive or negative. The total number of instances studied for a given concept was the same for both the experimental and the control classes. The concepts taught during the experiment were drawn from the areas of geometry, exponents, and operations, and included the following topics: quadrilaterals, circles, integer exponents, modular arithmetic, general operations, closure, commutativity, associativity, identity elements, and distributivity. The study ran for a total of 65 class periods, 55 for instruction and 10 for testing.

The following pretests were used to determine the comparability of the four groups: Lorge Thorndike Verbal and Nonverbal Intelligence Tests, Form 4A; Sequential Tests of Educational Progress, Mathematics, Form 3A (STEP 3A); and a unit test on the content just prior to the beginning of the experiment.

The posttests were of two types, mathematics tests or general cognitive factor tests. The five mathematics tests used as posttests were the following: Sequential Tests of Educational Progress, Mathematics, Form 3B, (STEP 3B); (Ach-1), an achievement test on the geometry and the exponents units written by one of the instructors of the classes; Definitions: (Operations D-1), a test designed by the investigator to measure the student's ability to read definitions of random operations; Generalizations: Operations (G-1), a test designed by the investigator to measure the student's tendency to overgeneralize the properties of operations; and Operations: Properties (Pr-1) a test designed by the investigator to measure the student's tendency to overgeneralize the properties of operations to the basic operations of arithmetic.

Six general cognitive factor tests measuring the cognitive factors of induction, perceptual speed, and syllogistic reasoning were chosen from Kit of Reference Tests for Cognitive Factors (French, Ekstrom, & Price, 1962). The posttests for induction were Letter Sets Test (I-1) and Figure Classification Test (I-3). The posttests of perceptual speed were Number

Comparison Test (P-2) and Identical Pictures Test (P-3. The posttests of syllogistic reasoning were Nonsense Syllogisms Test (Rs-1) and Inference Test (Rs-3). Further descriptions of the tests can be found in Shumway (1970).

The null hypotheses were stated in the form: there is no significant difference in mean performance of students in eighth grade mathematics classes using both positive and negative instances of concepts (E groups) and eighth grade mathematics classes using only positive instances of concepts (C groups) on each of the eleven posttests.

Due to unusual discipline problems encountered by one instructor in both of his classes the investigator took over the two classes, one experimental and one control, after 15 of the 65 days of the experiment had elapsed. Because of this development the study was viewed as two separate experiments, Experiment A taught by the investigator and Experiment B taught by the remaining instructor. The data from each experiment were analyzed separately. On the basis of the pretest results, a one-way analysis of variance was chosen as the strategy for Experiment A and a two-way analysis of variance with blocking on the Lorge-Thorndike Nonverbal Intelligence Test and using the STEP 3A as the covariate was chosen as the strategy for Experiment B.

4. Findings

In Experiment A the mean score of the experimental class (E group) was significantly greater than the mean score of the control class (C group) on test G-1, a test of the student's tendency to overgeneralize the properties of operations ($p<.05$)--higher score means "less" tendency to overgeneralize. No other differences were significant.

In Experiment B the mean score of the experimental class (E group) was significantly greater than the mean score of the control class (C group) on test Pr-1, a test of the student's tendency to overgeneralize the properties of operations to the basic operation of arithmetic ($p<.05$). The mean score of the control class (C group) was significantly greater than the mean score of the experimental class (E group) on test P-3, a test of perceptual speed ($p<.05$). There were significant interactions between treatment and IQ level on tests D-1 and I-3 ($p<.05$). These interactions were not subjected to further analysis or discussion (the blocking was introduced only to control for pretest differences in Experiment B and random assignment was not made to individual cells).

5. Interpretation

The investigator reported that while there was a discrepancy in direction on test P-3 between Experiment A and Experiment B, the results of tests G-1 and Pr-1 suggest that the experimental groups tended to overgeneralize the properties of operations less frequently than did the control groups.

Abstractor's Notes

Numerous limitations of the study are cited all of which should be taken seriously. The investigator also reports that he is currently conducting a "more controlled" study--involving only the concepts of commutativity and associativity--in which an attempt is being made to correct the deficiencies of the present study.

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EJ 030 590 450 PS 500 713

ESTIMATION OF LINE LENGTH AND NUMBER: A DEVELOPMENTAL STUDY, Siegel, Alexander W.; McBurney, Donald H., Journal of Experimental Child Psychology, v10 n2, pp170-180, Oct 70

Descriptors--*Response Mode, *Mathematical Concepts, *Intellectual Development, *Research Methodology, Adults, Children, Situational Tests.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Marilyn N. Suydam, Pennsylvania State University.

1. Purpose

The purposes were to investigate (a) the possibility that handgrip force may be a subjective dimension that children can use reliably in estimating line length and number, and (b) that the functional relationships between these variables are dependent on the child's level of cognitive development and functioning.

2. Rationale

The use of number as the dependent variable may have obscured the possibility that direct techniques for scaling subjective magnitude might be applicable to children and others whose cognitive structure may not include the ability to deal with the ratio properties of number. If it could be demonstrated that children could reliably use some subjective dimension other than number, then direct scaling techniques could be employed to study a variety of sensory capacities and cognitive abilities.

3. Research Design and Procedure

A total of 96 children (ages 6-13) and 16 adults participated. An equal number of each sex were selected from (a) grades 4, 6, and 8 in a summer school for gifted students (Stanford Binet IQ, 119-165); (b) grades 1, 2, and 4 in a school in a black, lower socioeconomic area (Detroit IQ, 76-116); and (c) the staff and graduate students in a university psychology department.

Each subject was tested individually with a specially constructed handgrip apparatus, using his preferred hand. The procedure was introduced to younger children as a guessing game and to older subjects as a matching task.

The maximum and minimum grips of each were recorded. Stimuli for Part 1 were lines of 0.25 to 25.00 inches, each on a separate posterboard. Each subject was told that he was to squeeze the grip to show how long each line was. He saw each line in two different random orders. Stimuli for Part 2 were the numbers 1, 2, 5, 10, 20, 50, and 100, verbally presented in a different random order for each subject. He was asked to "pull as hard as you think a (5) should be".

4. Findings

For each type of stimuli, data were subjected to a mixed analysis of variance for Age x Sex x Subjects x Stimuli. As expected in each case, the main effect of Stimuli was highly significant, indicating that stronger pulls were given to the longer lines or the larger numbers. Also as expected in each case, the Age x Stimulus interaction was highly significant, indicating that the slopes of the curves differed as a function of age. No other main effect or interaction was significant at the .05 level.

The slopes for the data increased with age, mainly due to stronger pulls to the smaller stimuli by the younger children, who also gave somewhat weaker pulls to the larger stimuli. The individual slopes of all subjects were subjected to a simple Age x Subjects AOV that yielded a highly significant main effect for Age, indicating that the mean slopes for the various age groups were indeed different. Further analysis (using Duncan's Multiple Range Test and a t-test) indicated that although there was no significant difference between the slope of adults and eighth graders, the mean slopes of these groups were significantly greater than those of the other groups. Variances and intra-subject variability were also significantly greater in some cases for these two groups.

5. Interpretations

This study demonstrates that children are able to use handgrip force to match their perception of line length and number as reliably as adults.

A developmental trend might be reflected in the perception of subjective number, line length, and/or handgrip force. The most reasonable explanation for the regular increase in mean slope of the line length and number functions with age, however, was that younger children perceived subjective handgrip force to increase more

rapidly with the physical force they exerted than did the adults. They may have divided the total range of handgrip possible into the same subjective range despite differences in strength as a function of age. It is also possible that younger children may be more conservative in their assignment of handgrip forces to the stimuli. They might have been following category rather than magnitude estimation instructions.

Abstracter's Notes

Only the first purpose for the study is explored in the article; the second demands, obviously, much more study.

The procedures and data analysis are carefully presented and discussed. Selection of subjects seems to be the main difficulty--comprising less "what is needed" than "what is available". The explanation that "the second group of children was tested because the first group was no longer available" is interesting. That the children are black or non-black, bright or less-bright, is a function of happenstance, not planning. The confounding of race, SES, grade level, and IQ is something that can be avoided when more care is devoted to the selection of the sample. And care can be taken when stating the results: these are generalized by the authors to all children and all adults.

Interest in the study lies mainly in how it may be used by other researchers.

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EJ 033 808 310 SE 502 712
PERFORMANCE ON SOME DISCOVERY TASKS, GRADES 4-7 Sowder, Larry,
Journal for Research in Mathematics Education, v2 n1, pp5-11, Jan '71

Descriptors--*Discovery Learning, *Elementary School Mathematics, *Instruction, *Learning, Arithmetic, Mathematics Education

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James K. Bidwell, Central Michigan University

1. Purpose

The study investigated the amount of information students require in order to form generalizations on numerical discovery tasks. The study also provided data on how age and intelligence levels effect performance on these tasks. A linear model was used to determine predictability of performance from the usual independent variables.

2. Rationale

No previous research was cited and no discussion on discovery learning was given. The phrases "formed a generalization" and "made a discovery" were behaviorally defined to mean "the pupil gave two consecutive correct responses" on the test item.

3. Research Design and Procedure

An author-made discovery test was administered to students in grades 4, 5, 6, and 7. Grade, sex, and IQ level effects on the scores on the tests were analyzed by ANOVA. A linear regression analysis was done on the scores with age, IQ score, computation achievement quotient, concepts achievement quotient, creative interests, and routine interests.

The subjects were selected from the public school of a small Wisconsin community. Data was available for 272 subjects. Three subjects (and two reserves) were randomly selected to fill the 4 x 2 x 3 cells of grade-sex-intelligence levels. The middle intelligence level was defined by Kuhlmann-Anderson IQ scores (obtained from school records) from 104 to 114 inclusive. No upper (lower) limit was placed on the high (low) level.

Although the actual test was not reported, the general form of the test and one complete sample item was given. The eight

items were four "computational shortcuts" (i.e., $\sum_{k=1}^{n} 2k-1 = n^2$) and four "secret rules" (i.e., $n \rightarrow n(n+1)$), all presented as a sequence of numerical instances of the rule. The first instance (with answer) was presented for 7-10 seconds; the second instance (without answer) was presented for 15 seconds or until the subject responded, then the correct answer was written by the examiner followed by 15 seconds of study time. The next instance was exposed, etc. There were 10 incomplete instances. If subject gave second consecutive correct response on the kth instance he was scored k on the item. The total instance score (I) was the sum of all item scores. A performance measure (P) was defined as 10 G/I where G was the number of successful generalizations.

The test was given individually on consecutive days by three male graduate students. All subjects in sample at a given grade were tested on same day. Efforts were made to control interaction between examiners and IQ level and sex.

The computation achievement quotient and concepts achievement quotients were obtained from the Stanford Achievement Test (administered previous year) by dividing grade-equivalent score by grade level. The creative and routine interests scores were obtained from Mathematics Interests Questionnaire (SMSG) (administered one week before discovery test).

Investigation revealed that the subjects' practice at any discovery tasks had been minimal.

4. Findings

Mean I and P scores by grade, sex, and IQ levels were reported. The ANOVA for I scores and regression analysis for I scores were reported. When generalizations were made they required from three to six instances.

The mean I scores for 4th grade through 7th grade subjects were 67.4, 56.4, 48.6, 49.6, respectively. For performance scores, P, the same means were .38, .77, 1.06, 1.02 respectively. The G scores (not reported) were similar to P scores. The ANOVA showed significant differences ($p << .01$) for grade and IQ level for I and G scores. Under a one-way ANOVA grade effects for P scores was significant ($p < .001$). For I and G scores, age, IQ, and computational achievement quotient contributed most to the linear regression model.

A post ad hoc test was performed to determine if grade 5 and grade 6 levels were different. No significant differences were found.

5. Interpretation

There was an apparent plateau at grades 6 and 7. The differences between grades 5 and 6 were not significant. If a further study showed a plateau at grades 5 and 6, is it due

to computational proficiency?

"It is probably inappropriate to generalize the results beyond the content of the discovery items or the style of administering them."

Because the students need three to six instances to form generalizations and the fact that no generalizations were formed after six unsuccessful instances suggest that teachers should "not give up too soon, but avoid spending excessive time." The fact that low intellegence level students did generalize is encouraging.

Abstractor's Notes

The report did not give the complete form of the test items. This is crucial in order to know what the student generalized. Consider item three ($\sum_{k=1}^{n} 2k-1 = n^2$), if the sequence of instances was:

1+3=4
1+3+5= ___
1+3+5+7= ___
etc.

then it is obvious that the subject can give two correct responses by simply adding the next number to the previous answer. On the other hand if the instances were scrambled, as

1+3=4
1+3+5+7+9+11= ___
1+3+5+7= ___

then straight computation is more difficult but not inconceivable in 15 seconds. Thus without more detailed knowledge of the test, whether the students discovered the stated rule on the "shortcut" items is questionable to the reader.

A measure of the number of instances required per generalization by grade level would be helpful to the stated problem.

Even though the analysis of the data is carefully done, it would appear that the sample size (72 over four grade levels) combined with the test instrument and administration method restricts any extrapolation from the statistics. Another study with a modified test instrument might produce valuable information on the problem as stated: how much information do the students require to form generalization?

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ED 046 491 PS 003 921
UNDERSTANDING OF QUANTITATIVE CONCEPTS IN 3 1/2 - 4 1/2 YEAR-OLD CHILDREN. Tasaka, Masako N.; Chittenden, Edward A. Educational Testing Service, Princeton, New Jersey, Spons. Agency--Office of Child Development, Washington, D.C. Pub Date 3 March 70 Note--9 p.; Paper presented at the meetings of the American Educational Research Association, Minneapolis, Minnesota, March 3, 1970. EDRS Price MF - $0.65 HC - $3.29.
Descriptors--CONCEPT FORMATION, DEVELOPMENTAL PSYCHOLOGY, *EVALUATION, *MATHEMATICAL CONCEPTS, *MEASUREMENT INSTRUMENTS, *PRESCHOOL CHILDREN, PSYCHOMETRICS, RESEARCH NEEDS, *TASK PERFORMANCE
Identifiers--Counting Task, Enumeration Task, Spontaneous Correspondence Task

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Myron F. Rosskopf, Teachers College, Columbia University

1. Purpose

To analyze the nonverbal performance of preschool children on tasks involving discrete objects in order to:

(1) examine and describe some psychometric properties of measures appropriate for use with very young children;

(2) to relate the measures to theory in developmental psychology; and

(3) to consider some implications for educational research and practice.

2. Rationale

Piagetian investigations of the development of children's quantitative abilities show that the concept of number as adults understand it depends upon preconcepts that develop gradually over a number of years. Questions concerning the stage of development of pre-school children led to the construction of two tasks of spontaneous one-to-one correspondence and a third supplementary task of counting. The object being to find measures that would be simple to use and to interpret by teachers.

3. Research Design and Procedure

During 1969, from a larger group of subjects taking part in a longitudinal study involving disadvantaged children, a sample of 100 children (3 1/2 - 4 1/2) was tested. The

children of the sample were tested individually, presumably using a non-structured simple interview format.

The spontaneous correspondence task consisted of laying out an array of 1x1-inch ceramic tiles and asking the child to "put out the same number" or "just as many" tiles. The task was repeated four times: twice with 7 tiles, once with 8 tiles, and once with 10 tiles. For three presentations the tiles were in a line; for the remaining one, they were randomly placed. A child earned a score of 3 points if he put out the same number of tiles as the examiner; 2 points if the difference was 1 or 2 tiles; and 1 point if the difference was 3, 4, or 5 tiles. Thus, a possible score was 4x3, or 12, with a range of 0-12.

The enumeration task required no verbal response. The child was directed to point to each figure on a page of a test booklet once and once only. The figures consisted of colored circles arranged in 3 types of arrays: a single line, a row, or a random design. There were a total of 12 items in this test with 6, 7 or 9 figures for each item. An item was scored as correct if a child pointed to each figure of the item once and only once. Other responses were classified according to the type of error: omissions, repetitions, and combinations of these two. Possible score range was from 0-12.

The counting task consisted of one item. Seven figures were arranged in a row and the child was asked to count them aloud. Scoring took account of two aspects of the children's behavior: (1) ability to say the number names in ascending order, and (2) ability to give the correct number of number names regardless of order.

4. Findings

(a) Spontaneous correspondence. The mean score on the four-item task of spontaneous correspondence was 3.80 with a standard deviation of 2.90. Two subjects had perfect scores of 12. A K-R alpha coefficient of .64 indicated satisfactory internal consistency, considering the briefness of the test. Performance on the item of random arrangement of the tiles was as good as on the three items in which the tiles were in a line.

(b) Enumeration. The mean score for this 12-item test was 6.56 with a standard deviation of 3.57. A K-R 20 coefficient of .86 indicated good internal consistency. Over 90 percent of the children passed at least a few of the items.

(c) Counting. About 25 percent of the children were correct in both saying the number names in ascending order and in giving the correct number of number names; another 25 percent gave a correct sequence but not the correct number of number names; another 20 percent gave no response or said they did not know how to count. A few subjects gave seven number names but not in the correct order.

The correlation between the spontaneous correspondence and enumeration tasks was 0.28. A contingency table shows that subjects who scored high on spontaneous correspondence generally were quite successful on enumeration. However, the reverse did not appear to be true. An examination of the relationship of counting to enumeration seems to show a positive relationship, whereas there appeared to be almost no relationship between counting and spontaneous correspondence.

5. Interpretations

Performance on the spontaneous correspondence task suggests four different categories. (1) Some children put out tiles by emptying out all of them or by taking a few handfuls. (2) Many children take pains to match the configuration of the examiner but are not very accurate in matching his number of tiles. (3) Children match the configuration and are fairly accurate in matching the number. (4) There are those who are successful in matching the number of tiles but not the configuration.

In general the performance of the children in the sample support Piaget's similar investigations: The children respond on the basis of perception. The tasks are simple to administer and almost always secure a response from a child. Thus, they are useful to use as a means of ascertaining the level of development of the child.

The enumeration task, being a simple spatial enumeration requiring no verbal response, allows one to secure information on quantitative thinking that is distinct from the ability to recite numbers. The direction of development seems to be from spontaneous correspondence toward enumeration--at least as evidenced by these tasks--rather than the reverse.

Abstractor's Notes

From the point of view of one interested in Piaget's theory of cognitive development the methodology of the

investigation merits attention. Clearly, few directions for the subject were necessary. That is, the interviewing was about as meager as possible in individual testing. Thus, the tests proposed for pre-school children of ages 3 1/2 - 4 1/2 might be placed somewhere along the line from the Piagetian clinical interview to pencil-and-paper tests.

The random arrays in certain of the tasks deserves a comment. From the descriptions given in the paper it is not clear what the relative positions of subject and experimenter were. If the two were facing one another--say on opposite sides of a small table--then the random array task may have involved for the subject a geometric reflection. Such an extra dimension introduced at that age level would present a formidable problem for the children to solve.

The scoring of tasks presented to children is always difficult. How much and what sort of deviation from the model should be tolerated? The authors' examples, together with their discussion, seem to indicate that more deviation from the model was tolerated than can be gathered from their account of the scoring criteria. Careful descriptions of the tasks, even at the cost of extending the length of a paper, and enough detail on scoring procedures to allow replication are very important in connection with Piagetian investigations.

There is much promise in these preliminary results obtained by the authors, for it seems that minimizing the amount of interviewing in the tasks will promote wider use of such combinations of items to determine the developmental level of children on an individual basis. Also of interest is the authors' use of non-verbal response, for it allows a child to respond using only his own thinking. He does not have to take into account what an adult says. He has only to respond as he perceives a task.

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ED 048 002 SE 010 930

INDEPENDENT MATHEMATICS LEARNING AS A FUNCTION OF TEACHER BEHAVIORS. Ward, Beatrice A., Far West Lab. for Educational Research and Development, Berkeley, Calif. Spons Agency--Office of Education (DHEW), Washington, D.C. Pub Date 6 Feb 71. Note--22p.; Paper presented at the Annual Meeting of the American Educational Research Association (Feb. 4-7, 1971, New York City, N.Y.) EDRS Price MF-$0.65 HC-$3.29.

Descriptors--*Achievement, *Elementary School Mathematics, *Independent Study, *Instruction, Learning, Mathematics Education, Teacher Behavior, Teacher Characteristics.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Stephen S. Willoughby, New York University.

1. Purpose

To test two null hypotheses:

(a) There is no difference in the performance of pupils exposed to independent and group learning treatments.
(b) Teacher use of seven behaviors (see list below) will have no relation to prediction of pupil post-unit test scores.

2. Rationale

Individualization of instruction has received substantial attention during the 1960's. One of the variables introduced into the school setting by individualization of instruction, which has received particularly limited attention in research investigations, is the need for use of a unique set of teacher behaviors. The teacher behaviors investigated were:

(a) Specify what is to be learned.
(b) Specify how what is learned will be demonstrated.
(c) Identify resources to be used by the pupil.
(d) Specify the learning steps to be completed by the pupil.
(e) Establish Learning checkpoints.
(f) Establish deadlines.
(g) Describe the next activity the pupil can anticipate doing.

3. Research Design and Procedure

The study involved 34 teachers, 18 of whom received special training in the use of the seven teacher behaviors. Ten of these were randomly selected to teach the instructional unit using independent learning procedures, the other eight taught the same unit using group procedures. Of the 16 untrained teachers, nine were randomly selected to use independent procedures and the other seven used group procedures.

"The ten or so" children selected from each group instruction class were chosen randomly from all the pupils studying the unit, but for the independent learning treatment, "only the 10 or so pupils randomly selected for participation engaged in the unit activities". There is no indication of what the other pupils in those classes were doing at the time of the study. After elimination of the data for pupils who did not take all the tests, a sample population of 314 pupils was available.

The topic studied in the unit was measurement. The major learning objectives included were:

(a) To learn what characteristics of an object can be measured.
(b) To learn the need for likeness between what is to be measured and the unit of measure.
(c) To use measurement to determine equivalence of forms.
(d) To apply the transitive rule in describing relationships among measures of three or more objects.
(e) To relate self-developed measurement units to English standard units of measure and to metric units.

Teachers kept records of the number of minutes spent in unit activities and the percent of time spent in group or independent activities. According to these records, the number of minutes spent on the activities, and the percent of time involved in independent (as opposed to group) learning activities for each treatment were as follows:

	Group Trained	Group Untrained	Independent Trained	Independent Untrained
Mean number of minutes	608	527	523	395
Percent of time for independent study	40%	41%	83%	82%

The investigator expresses doubt regarding the records pertaining to percent of time devoted to independent learning because of apparent confusion regarding definitions.

Thirty minute videotapes were taken, and independent observers were used to score teachers on their use of the seven behaviors. Reliability of the scoring was .974 for the group treatment and .925 for the independent treatment. Pupil performance was measured by pre and post tests with reliability coefficients of .772 for the pretest and .896 for the post-test. The Iowa Tests of Basic Skills, Test A-1: Arithmetic Concepts, and Test A-2: Arithmetic Problem Solving were used to measure mathematics achievement. The California Short Form Test of Mental Maturity, 1963 revision was used to measure general learning ability. The post-test score on the measurement test was the dependent variable, and the other scores were independent variables.

4. Findings

Teacher mean total scores on the use of the seven teacher behaviors were as follows: trained, independent: 23.9; trained, group: 21.56; untrained, independent: 14.22; and, untrained, group: 9.88. The trained, independent teachers differed significantly ($p<.01$) from both untrained groups; and the trained group teachers differed significantly from the untrained group teachers.

Analysis of covariance was used to adjust for differences in pretest means when comparing post-test results. The adjusted combined means for the pupils of the trained and untrained teachers were essentially the same. However, the adjusted combined mean for the pupils exposed to the group treatment was significantly higher (using a 95% confidence interval) than that of the pupils exposed to the independent treatment.

Measures of the relationship between teacher use of the specified behaviors and pupil post-unit score were obtained through step-wise multiple regression analysis. Measurements of such behaviors did have specific relationships to pupil scores in this study, and these were different for the two treatments. For the group treatment, the largest positive coefficient was for establishing deadlines, and the largest negative coefficients (negative coefficients with greatest absolute value) were for describing an anticipated activity and for specifying what was to be learned. For the independent treatment, the largest

positive coefficient was for specifying how learning was to be demonstrated, and the largest negative coefficients were for organizing steps and describing an anticipated activity.

5. Interpretations

The two major contributions of this study are: (1) Under the conditions reported here, group learning was more effective than individual learning; and, (2) measurements of certain teacher behaviors may be used to predict pupil progress on test scores.

Because of the similarity of the materials used in this study to several individualized programs, the investigator belives the results comparing group and individualized study "may have some practical significance", but calls attention to the difference in time spent on the unit by the different treatment groups.

Although differing teachers, subject matter, pupils, etc., may be expected to result in emphasis on different teacher behaviors as good predictors of pupil progress, the results of this study suggest that measurement of precisely defined teacher behaviors may aid in definition of "effective teaching" for various instructional situations.

Abstracter's Notes

The most important immediate result of this study is to cast serious doubt on the effectiveness of the so-called individualized instruction programs that have been popular in recent years. Because of the kind of special teacher training and the specific pupil materials used, it would appear that the cards were originally stacked against group instruction. Yet, the only apparent advantage of the "individualized instruction" was more rapid coverage of the material by the pupils. The study does not indicate how many children learned the unit on measurement using each treatment, with a given amount of teacher time, but given the available data, a reasonable inference seems to be that fewer pupils learned less per teacher hour expended, using "individualized instruction".

The most important long-range result of this study may be the prediction of pupil achievement from precisely defined teacher behaviors. Follow-up studies comparing teacher behaviors and pupil achievement will undoubtedly

show that for different situations, different teacher behaviors are better predictors of pupil success. However, if enough situations and behaviors can be catalogued, the results could be a substantial aid in methods courses and in developing a theory of teaching.

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