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ABSTRACT

This report consists of three main sections: (1) Introductory comments on the field of statistics and its study at the graduate level; (2) A recommended undergraduate program for prospective graduate students in statistics; and (3) Implications of the recommendations for departments of mathematics and their students. The recommendations are addressed to departments of mathematics of four year colleges and smaller universities which have no specialized departmental programs in statistics. (Author/MM)

PREPARATION for GRADUATE WORK in STATISTICS

A Report
of the
Panel on Statistics

COMMITTEE ON THE UNDERGRADUATE PROGRAM
IN MATHEMATICS

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The Committee on the Undergraduate Program in Mathematics is a committee of the Mathematical Association of America charged with making recommendations for the improvement of college and university mathematics curricula at all levels and in all educational areas. Financial support for CUPM has been provided by the National Science Foundation.

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> ...Both Computer Science and Statistics have dual sources of identity and intellectual force, only one of which is mathematical; hence they are more accurately described as partly mathematical sciences. ...
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> Modern statistics could not operate without mathematics, especially without the theory of probability. Equally, it could not exist without the challenge of inference in the face of uncertainty and the stimulus of the quantitative aspects of the scientific method ... statistics is both a mathematical science and something else.
>
> ...It is true that undergraduate preparation for majors in mathematics, with its traditional emphasis on core mathematics, provides an excellent foundation of knowledge for potential graduate students in statistics. It does not, however, provide nearly enough students with either motivation to study statistics or an understanding of the extra-mathematical aspects of statistics.[1]

This report consists of three main sections: (1) Introductory comments on the field of statistics and its study at the graduate level; (2) A recommended undergraduate program for prospective graduate students in statistics; and (3) Implications of the recommendations for departments of mathematics and their students. Our recommendations are addressed to departments of mathematics of four-year colleges and smaller universities which have no specialized departmental programs in statistics. At these institutions, the department is unlikely to have an experienced or trained statistician although it is often called upon to offer statistics courses as a service for students majoring in other fields.

I. STATISTICS AND GRADUATE STUDY

In our modern technological society, there is a continually increasing demand and necessity for quantitative information. This requires planning and skill in the collection, analysis, and interpretation of data. Statisticians deal with inherent variation in nature and measurement and are concerned with the planning and design of experiments and surveys, with methods of data reduction, and with inductive decision processes.

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[1] The Mathematical Sciences: A Report, by the Committee on Support of Research in the Mathematical Sciences of the National Research Council for the Committee on Science and Public Policy of the National Academy of Sciences, Washington, D.C., Publication 1681 (1968), pp. 84 and 157.

Statistics has made significant contributions to many fields, most notably to the experimental sciences, agriculture, medicine, and engineering. It has also had an important role in the development of other fields such as economics, demography, and sociology. Statistics and quantitative methods are assuming major roles in business and in the behavioral sciences, roles destined to receive more and more emphasis. The widespread use of computers in these fields increases the need for statisticians at all educational levels.

Although a demand exists in government and business for persons with only undergraduate training in statistics (especially in conjunction with training in computer technology and a subject-matter field), the attainment of competence in statistics at a professional level necessarily requires graduate study. Our recommendations deal with minimum undergraduate preparation for this study. It is generally agreed that broad knowledge of mathematics is required to proceed with graduate work; currently about two-thirds of all graduate students in statistics were undergraduate mathematics majors.* Advanced study in some field (physical, biological, or social science) in which data play an important role is also very helpful.

A significant part of graduate study in statistics is the attainment of a sound understanding of advanced mathematics and the theory of statistics and probability, since these are necessary for research in statistics and also for competent consultation on applications of statistics. The consultant seldom encounters textbook applications and is regularly required to modify and adapt procedures to practical problems.

Graduate study in statistics, at both the M.A. and Ph.D. degree levels, is available in a substantial number of universities. According to a survey published in The American Statistician (October, 1968), there are in the U.S. and Canada approximately 85 departments which offer undergraduate degrees in statistics. There are approximately 160 departments at 110 universities which offer programs leading to graduate degrees in statistics or subject-matter fields having a statistics option. These graduate programs of study lead to frontiers in both theory and application. The M.A. degree provides suitable qualifications for many positions in industry and government, often as a consultant or team member in research and development, as well as for positions in teaching at junior colleges. It is also useful for persons who will pursue advanced degrees in fields such as psychology and education in which statistical methodology plays a significant role. The Ph.D. degree provides additional preparation for research and teaching careers in universities as well as in government and industrial organizations.

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*Aspects of Graduate Training in the Mathematical Sciences, Vol. II, page 62, 1969. A report of the Survey Committee of the Conference Board of the Mathematical Sciences, 2100 Pennsylvania Avenue, N.W., Suite 834, Washington, D. C. 20037

II. THE RECOMMENDED PROGRAM OF STUDY

Our recommendations for undergraduate courses are subdivided into four areas: (A) probability and statistics, (B) mathematics, (C) computing, and (D) other requirements. In this section, we describe recommended courses in each of these areas.

A. Probability and Statistics Requirements

We recommend that students take at least a one-year course in probability and statistics and gain experience with real applications of statistical analysis.

1. Probability and Statistics Course (6 semester hours)

This key course is a one-year combination of probability and statistics. On the semester system, a complete course in probability should be followed by a course in statistics. If the course is given on a quarter system, it may be possible to have a quarter of probability, followed by two quarters of statistics or by a second quarter of statistics and a third quarter of topics in probability and/or statistics. In any case, these courses should be taught as one sequence.

Prerequisites for this one-year course are Mathematics 1, 2, and 4 (Calculus) from the CUPM report, A General Curriculum in Mathematics for Colleges (GCMC), 1965. Students should also be encouraged to have taken GCMC Mathematics 3 (Linear Algebra). For detailed course descriptions, see the GCMC report or the forthcoming Commentary on GCMC. All students in this key course, whether they be prospective graduate students of statistics, other mathematics majors, or students from other disciplines, should be encouraged to take the full year rather than only the first-semester probability course. Almost all students will have studied the calculus sequence and perhaps linear algebra (GCMC Mathematics 1-4) without interruption during their first two years in college. Although our recommended probability course and GCMC Mathematics 2P differ only little in content, our course assumes the additional maturity and ability of students who have successfully completed the three or four semesters of the core GCMC curriculum. Our statistica course differs in the same way from GCMC Mathematics 7 (version emphasizing statistics) which has only Mathematics 2P as its prerequisite course in probability.

The probability course should include the following topics:

Sample spaces, axioms and elementary theorems of probability, combinatorics, independence, conditional probability, Bayes' Theorem.

Random variables, probability distributions, expectation, mean, variance, moment generating functions.

Special distributions, multivariate distributions, transformations of random variables, conditional and marginal distributions.

Chebychev's inequality, limit theorems (Law of Large Numbers, Central Limit Theorem).

Examples of stochastic processes such as random walks and Markov chains.

The key course in probability should provide a wide variety of examples of problems which arise in the study of random phenomena. With this aim in mind, we recommend this course be taught so as to maintain a proper balance between theory and its applications.

The time allotted to the probability course will not permit detailed treatment of all topics listed above. We recommend that such topics as the Central Limit Theorem and the use of Jacobians in transformations of random variables be presented without proof. Also, discussion of multivariate distributions should include only a brief description of the multivariate normal distribution. Random walks and Markov chains may serve as useful topics for two or three lectures to illustrate interesting applications of probability theory. Even though the topics of this paragraph are not treated in depth mathematically, we recommend their inclusion to enrich the student's comprehension of the scope of probability theory.

The statistics course can be implemented in a variety of ways, giving different emphases to topics and, indeed, including different topics. Widely divergent approaches are acceptable as preparation for graduate work and are illustrated in the statistics books listed below, selected from many appropriate texts for this course:

Brunk, H. D. Introduction to Mathematical Statistics, 2nd ed. Blaisdell, 1965.

Freeman, H. A. Introduction to Statistical Inference. Addison-Wesley, 1963.

Freund, John E. Mathematical Statistics. Prentice-Hall, 1962.

Hadley, G. Introduction to Probability and Statistical Decision Theory. Holden Day, 1967.

Hoel, Paul G., Port, Sidney C., and Stone, Charles J. Introduction to Statistical Theory. Houghton Mifflin, 1971.

Hogg, Robert V. and Craig, A. T. Introduction to Mathematical Statistics, 3rd ed. Macmillan, 1970.

Lindgren, B. W. Statistical Theory, 2nd ed. Macmillan, 1968.

Mood, Alexander M. and Graybill, F. A.
Introduction to the Theory of Statistics,
2nd ed. McGraw-Hill, 1963.

Despite the diversity of possible approaches, most will include the following topics:

Estimation: consistency, unbiasedness, maximum likelihood, confidence intervals.
Testing hypotheses: power functions, Type I and II errors, Neyman-Pearson lemma, likelihood ratio tests, tests for means and variances.
Regression and correlation.
Chi-square tests.

Other topics to be included in the statistics course will depend on the available time and method of approach. Possible topics include:

Estimation: efficiency, sufficiency, Cramer-Rao Theorem, Rao-Blackwell Theorem.
Linear models.
Nonparametric statistics.
Sequential analysis.
Design of experiments.
Decision theory, utility theory, Bayesian analysis.
Robustness.

The above list of additional topics for the key course in statistics is much too large to be adequately covered in its entirety. The fact that many topics will have to be omitted or treated superficially gives the statistics course much more flexibility in approach and coverage than is possible in the probability course. The instructor's choice of topics may be influenced by the following factors. Decision theory, Bayesian analysis, and sequential analysis dealing with foundations of inference will appeal to the philosophically inclined students. The Cramer-Rao Theorem and the Rao-Blackwell Theorem appeal to mathematically oriented students and illustrate statistical theory. In design of experiments and estimation, one has an opportunity to apply techniques of optimization. Nonparametric techniques utilize combinatorial probability and illustrate the high efficiency that can be attained from simple methods. Analysis of variance provides an application of linear algebra and matrix methods and should interest students who have taken GCMC Mathematics 3.

Detailed outlines for the probability and statistics courses have not been presented on the assumption that the choice of texts, which is difficult to anticipate, will tend to determine the order of presentation and the emphasis in a satisfactory fashion. It may be remarked that most statistics texts at this level begin with a portion which can be used for the probability course.

To avoid a formal, dull statistics course and to provide sufficient insight into practice, we recommend that meaningful crossreferences between theoretical models and real-world problems be made throughout the course. Use of the computer would help to accomplish this goal. Three reports that are valuable in appraising the potential role of computers in statistics courses are:

Proceedings of a Conference on Computers in the Undergraduate Curricula, The University of Iowa, Iowa City, Iowa, 1970,

Development of Materials and Techniques for the Instructional Use of Computers in Statistics Courses, University of North Carolina, Chapel Hill, North Carolina, 1971, and

Conference on Computers in the Undergraduate Curricula, Dartmouth College, Hanover, New Hampshire, 1971.

2. Experience with Data

We believe that students should have experience with realistic examples in the use of the statistical concepts and theory of the key course. They should work with real data, consider the objectives of the scientific investigation that gave rise to these data, study statistical methods for answering relevant questions, and consider the interpretation of the results of statistical analyses. These goals are not easy to achieve, but various approaches are discussed below.

An appropriate course is not likely to be already available in a department of mathematics. The typical one-semester precalculus elementary statistics course, usually serving students from departments other than mathematics, does not fulfill the goals that we recommend.

Courses in research methodology or applied statistics in other subject matter areas may meet, at least in part, some of the objectives of exposing students to realistic statistical problems. For example, courses in biological statistics, research methods in behavioral science, economic statistics or econometrics, survey methods, sociological statistics, etc., can meet our intended objective if they are offered by practicing scientists who are familiar with the way data are generated, the complexities they usually exhibit, and the methods that help in their analysis. Although such a course will provide coverage of some topics in elementary statistical methods, it is important that the course be more than a catalog of methods. The student should see how some one field of science generates experimental data and copes with uncertainty and variability. Insight gained from relatively few examples or ideas can be considerably more valuable for the student than information obtained by covering a large number of separate topics. Modern computers could be useful in such a course.

If a faculty member with training and experience in applied statistics is available to the mathematics department, he can devise a data analysis course. This course could be based on a number of data sets of interest to the student, and it could use books on statistical methods as reference material for the appropriate statistical techniques. Basic texts on statistical methods which contain a variety of examples of applications of statistical methods include:

Dixon, Wilfrid J. and Massey, F. J., Jr. Introduction to Statistical Analysis, 3rd ed. McGraw-Hill, 1969.

Fisher, R. A. Statistical Methods for Research Workers, 13th ed. Hafner, 1958.

Guttman, Irwin and Wilks, Samuel S. Introductory Engineering Statistics. Wiley, 1965.

Li, C. C. Introduction to Experimental Statistics. McGraw-Hill, 1964.

Natrella, Mary G. Experimental Statistics, Handbook 91. U.S. Department of Commerce, National Bureau of Standards, 1966.

Snedecor, George W. and Cochran, W. G. Statistical Methods. Iowa State University Press, 1967.

Walker, Helen M. and Lev, Joseph. Statistical Inference. Holt, Rinehart and Winston, 1953.

Wallis, Wilson A. and Roberts, Harry. Statistics: A New Approach. Free Press, 1956.

Wine, Russell L. Statistics for Scientists and Engineers. Prentice Hall, 1964.

Winer, B. J. Statistical Principles in Experimental Design. McGraw-Hill, 1962.

Examples of data sources and/or statistical critiques of major scientific investigations are:

Tufte, Edward R. The Quantitative Analysis of Social Problems. Addison Wesley, 1970.

Kinsey, A. C., Pomeroy, W. B., and Martin C. E. Sexual Behavior in the Human Male. W. B. Saunders, 1948.

Report on Lung Cancer, Smoking, and Health. Public Health Bulletin 1103. Superintendent of Documents, U.S. Government Printing Office.

Cochran, William G., Mosteller, Frederick, and Tuckey, John W. Statistical Problems of the Kinsey Report. American Statistical Association, 1954.

"The Cochran - Mosteller - Tukey Report on the Kinsey Study: A Symposium." Journal of the American Statistical Association, September, 1955, p. 811.

Cutler, S. J. "A Review of the Statistical Evidence on the Association Between Smoking and Lung Cancer." Journal of the American Statistical Association, June 1955, pp. 267-83.

Brownlee, K. A. "Statistics of the 1954 Polio Vaccine Trials." Journal of the American Statistical Association, December 1955, pp. 1005-1014. (An invited address on the article "Evaluation of 1954 Field Trial of Poliomyelitis Vaccine: Summary Report." Poliomyelitis Vaccine Evaluation Center, University of Michigan, April 12, 1955.)

The following books use experimental and survey data to illustrate statistical concepts and techniques:

Bliss, Chester. Statistics in Biology. Vol. I(II). McGraw-Hill, 1967.

Cox, David R. Planning of Experiments. Wiley, 1958.

Davies, Owen L. Design and Analysis of Industrial Experiments, 2nd rev. ed. Hafner, 1956.

Ferber, Robert and Verdoorn, P. J. Research Methods in Economics and Business. Macmillan, 1962.

Stephan, Frederick F. and McCarthy, Phillip J. Sampling Opinion - An Analysis of Survey Procedures. Wiley, 1958.

Youden, William J. Statistical Methods for Chemists. Wiley, 1951.

When an experienced applied statistician is not available, the mathematics department may be able to develop a seminar with the assistance of faculty members in other disciplines. Typical experimental areas in a discipline may be discussed and illustrated with data from on-going faculty research or from student term projects.

Another opportunity to provide students with exposure to problems in data analysis is through laboratories associated with the key course in statistics. In such a laboratory, students may be asked to analyze data sets, followed by class discussion. Alternatively, term project assignments incorporating study, review, and critique of statistical studies with major data sets could be utilized. Still another opportunity would be through projects developed for independent study following the key course.

The important element in this recommendation is that the student obtain understanding of the role played by statistical concepts in scientific investigations and be motivated to continue the study of statistics.

3. Additional Courses

We recommend that additional courses in probability and statistics be offered to follow the key course whenever possible. Such courses could explore in detail a few topics which were omitted or treated lightly in the key course, e.g., analysis of variance, experimental design, regression, nonparametric methods, sampling, sequential analysis, multivariate methods, or factor analysis.

Other subjects which could serve as useful enrichment are stochastic processes, game theory, linear programming, and operations research. Courses in these subjects would be useful to students with specialized interests and also would help widen the knowledge and capabilities of the prospective graduate student in statistics.

B. Mathematics Requirements

We recommend that students take at least a complete 9-12 semester hour sequence in calculus, a course in linear algebra, and a course in selected and advanced topics in analysis.

1. Beginning Analysis (9-12 semester hours)

This sequence includes differential and integral calculus of one and several variables and some differential equations. It is desirable that prerequisites for calculus, including a study of the elementary functions and analytic geometry, be completed in secondary school.

For detailed course descriptions, we refer to the 1965 GCMC report of CUPM. This beginning analysis sequence is adequately described by GCMC's courses numbered 1, 2, and 4. Two versions of Mathematics 1 and two versions of Mathematics 2, 4 are offered in the GCMC report (pp. 31-34, 37-41). Either version is acceptable as long as a student has the complete 9-12 hour introductory analysis program including the basic concepts and techniques of differentiation, multiple integrals, and differential equations. We think it important to note that elementary probability theory is a rich source of illustrative problem material for students in this analysis sequence.

2. Linear Algebra (3 semester hours)

This course, which may be taken by students before they complete the beginning analysis sequence, includes the following topics: solution of systems of linear equations (including computational techniques), linear transformations, matrix algebra, vector spaces, quadratic forms, and characteristic roots. Two versions of such a linear algebra course (Mathematics 3) are presented in the GCMC report (pp.

42-47). Version 1, with "an algebraic and computational beginning exploiting the linear spaces on n-tuples, matrices, and elementary row operations" is preferred, although linear inequalities and convex sets with applications to linear programming, as in the alternate version syllabus, would be a desirable additional topic.

3. Selected Topics in Analysis (3 semester hours)

The GCMC course Mathematics 5 is not particularly appropriate for statistics students, and it is recommended that a course including the special topics listed below be offered for these students in place of Mathematics 5.

Such a course should give the student additional analytic skills more advanced than those acquired in the beginning analysis sequence. Topics to be included are multiple integration in n dimensions, Jacobians and change of variables in multiple integrals, improper integrals, special functions (Beta, Gamma), Stirling's formula, Lagrange multipliers, generating functions and Laplace transforms, difference equations, additional work on ordinary differential equations, and an introduction to partial differential equations.

It is possible that the suggested topics can be studied in a unified course devoted to optimization problems. Such a course, at a level which presupposes only the beginning analysis and linear algebra courses and which may be taken concurrently with a course in probability theory, would be a valuable addition to the undergraduate curriculum, not only for students preparing for graduate work in statistics but also for students in economics, business administration, operations research, engineering, etc. Experimentation by teachers in the preparation of written materials and textbooks for such a course would be useful and is worthy of encouragement.

4. Additional Courses

The following courses, listed in numerical order as described in GCMC, are not required but are desirable as choices for students who wish to have more than minimal preparation. A strong course in real variables is especially recommended for students interested in working for the Ph.D. in statistics.

Algebraic Structures	(GCMC Mathematics 6)
Numerical Analysis	(GCMC Mathematics 8)
Applied Mathematics	(GCMC Mathematics 10); see also CUPM's report - "A Curriculum in Applied Mathematics" (1966), especially the description of the course entitled "Introduction to Applied Mathematics (Optimization Option)," pp. 19-23.

Introductory Real Variable Theory (GCMC Mathematics 11-12)
Complex Analysis (GCMC Mathematics 13)

In general, the stronger a student's background in undergraduate mathematics, the better prepared he is for graduate work in statistics. If faculty members with special interests and competence are available, additional courses or seminars in the areas mentioned on page 9 would be valuable additions to the curriculum for a student interested in advanced work in statistics.

C. Computing Requirements

We recommend that students be familiar with a modern high-speed computer and what it can do, and that they have some actual experience analyzing data which are sufficiently obscure to require the use of a computer.

It is quite clear that the computer is becoming increasingly important to almost every academic discipline; in addition, it is an integral part of business, government, and even everyday affairs. These are strong enough reasons for every student who receives a baccalaureate degree to be acquainted with the computer and its potential. However, for someone who intends to be a professional statistician it is even more important that he know and understand the modern computer. Anyone who trains in statistics, who will handle data, or work with and advise people who handle data, must have a certain minimum competence in the use of a computer. While it is true that increased competence will be developed as the need arises and that some of the more sophisticated applications can be learned while doing graduate work, it is recommended that a student who comes into a graduate program in statistics begin his training in this area at the undergraduate level.

The most desirable way to be sure that sufficient competence is acquired would be through taking a regular course in computing such as Introduction to Computing, the course C1 described in the CUPM report, Recommendations for an Undergraduate Program in Computational Mathematics. It is, of course, possible to obtain an acceptable level of competence by attending the lectures or informal courses given in many computing centers, supplemented by sufficient additional computing experience. A student should understand and be able to use one of the major programming languages. He should learn enough of the nomenclature and characteristics of a computer to be able to stay abreast of the developments that will surely occur during his working lifetime. He should be made aware of and have some experience with library programs that are available. And, perhaps most important for our special purposes, students should have experience with actual data, with the numerical analysis and statistical problems they generate, and with the use of the computer for simulation. Some of this experience could be obtained by the techniques recommended in Section A2 above (page 6). The course CM1 of the above-mentioned report on computational mathematics is also based on simulation techniques, and would therefore be appropriate for students of statistics.

Finally, we suggest that in the next decade the availability of computers will change many subject-matter areas. If the statistician is to be an effective consultant in these areas, he must be aware of the way the computer is shaping the disciplines with which he will be associated.

D. Other Requirements

Statistics deals with the drawing of inferences from data and, in its applications, involves the statistician in working jointly with subject-matter specialists in the framing of relevant questions, the development of appropriate methodology for drawing inferences, and assisting in the analysis of final results. Whether a person will principally do research in statistical methodology, teach statistical applications or consult on statistical applications, knowledge of one or more areas of application and an understanding of the nature of statistical problems in them is highly desirable.

Undergraduate preparation for work in statistics should therefore include study of a variety of areas of application, with one studied in some depth. This will ensure that the student, upon graduation, will have an acquaintance with fundamental concepts in a variety of areas and technical competence to a moderate extent in at least one of the physical, life,or social sciences. Courses selected for study of a field in depth may include a statistical or research methodology course offered by that field in which the student will develop an understanding of data collection and data handling problems. The student's adviser may be particularly helpful in identifying such a course. Students who wish to undertake graduate study in specialized areas of statistics, such as econometrics or biostatistics, will find it desirable to take at least some advanced work in these areas as undergraduates.

III. IMPLICATIONS OF THE RECOMMENDATIONS

We are aware that at the present time most mathematics departments have few advanced courses in statistics available and few, if any, trained statisticians on their staffs. Because of the late date when a student may discover the field of statistics, he may not have time to elect many of our recommended courses even if these were available. Finally, some of the courses he takes will be designed not only for prospective graduate students, but for students with other majors and interests as well.

Were these and other limitations not present, we would expect our recommended program and the mathematics department to serve the needs of undergraduate students by not only imparting to them knowledge of the field of statistics, but also by enabling them to discover their abilities and interests and, if appropriate, by arousing their interest in graduate study in statistics or a related field.

But limitations do exist, and it is not to be expected that all of our recommendations can be implemented quickly, or that all the needs of students will be met by the programs in mathematics departments. It is difficult to meet all student needs under the best of circumstances. Realistically it is to be expected that severe limitations in time and facilities will prevent the student from obtaining a well-rounded understanding of the subject at the undergraduate level, but at least he can be exposed to some of the basic ideas in statistics. Perhaps most important, a program designed to achieve limited goals with relatively few courses, even if it falls short of the full program we recommend, can arouse the interest of students in statistics and related fields.

In the light of the recommendations in this and other CUPM reports (especially GCMC) dealing with courses in probability, statistics, and related areas, it is highly desirable, and we recommend, that each department of mathematics review its course offerings so as to establish appropriate courses in probability and statistics and arrange that these courses be staffed by a person competent in these fields. Ph.D.s in statistics are increasingly available to four-year colleges and smaller universities. With the establishment of courses in probability and statistics taught by a person competent in these fields, the mathematics department can serve the needs of prospective graduate students in statistics by:

i. arousing interest in and demonstrating the nature of the field of statistics;

ii. giving students an acquaintance with statistics, its theory, its applications, its traditions, even some of its open problems, and its relation to other fields such as probability, pure and applied mathematics, computer science, and operations research;

iii. counselling students as to courses, curricular choices, and graduate and career opportunities in probability, statistics, pure and applied mathematics, computer science, and operations research.

In creating this report, the Panel on Statistics confined its attention to recommending undergraduate programs for students who intend to do graduate work in statistics. In the development of appropriate recommendations, however, it became apparent that a broad program of study in the mathematical sciences was emerging, a program suitable, we believe, not only for graduate study of statistics but also for graduate study in the quantitative aspects of the social sciences and business and in newer areas such as operations research and computer science. The recommendations developed, therefore, can form the basis for an innovative degree program in the mathematical sciences different from the traditional programs in pure or applied mathematics.

The recommended program accomplishes important secondary objectives. These include: (i) a decision by students on the nature of their future graduate study can be made at a later point in the undergraduate program; (ii) mathematically gifted students are exposed to a wider range of potential careers than is presently the case; (iii) the possibility is created for a substantial emphasis in the mathematical sciences to be used as part of an undergraduate major, not only in mathematics and statistics but in other departments and in interdisciplinary programs.

The needs and opportunities for an innovative undergraduate program in the mathematical sciences are great. It can provide options in computer science, applied mathematics, econometrics, operations research, statistics, probability, and pure mathematics - all built around a solid core of training in mathematics. Early exposure to the concepts and possibilities of a variety of these options can lead to better choices of areas of concentration later on. A curriculum within this framework, combining mathematics, statistics, computing, and at least one field of application, has great potential for continued study in various graduate programs as well as value as a terminal degree program. The student may proceed from such an undergraduate program to advanced study in statistics, operations research, econometrics, psychometrics, demography, or computer science. He also will have excellent qualifications for advanced work in sociology, political science, business, urban planning, or education. If the undergraduate program in mathematical sciences is a terminal one, the student will have employment opportunities in computing, business, industry and government, with qualifications to meet many social needs.

The minimum preparatory program outlined in Section II can be supplemented in a variety of ways with additional work leading to undergraduate majors in

i. Mathematics,

ii. Statistics,

iii. Computational mathematics [See the CUPM document, Recommendations for an Undergraduate Program in Computational Mathematics (May, 1971)],

iv. Other fields (e.g., psychology, political science, economics, sociology, engineering, linguistics, business administration--especially management sciences--biological and physical sciences).