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AUTHOR	Levin, Henry M.
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ABSTRACT

Aimed at federal, State, and local educational administrators; educational researchers familiar with cost-benefit analysis and econometrics; and economists interested in problems of educational finance, this study explores the applicability of an econometric model of evaluation to the efficiency of schools. The emphasis is on assessing school input-output relations that appear to be maximizing educational outcomes. The standard constrained-maximum model is formulated for the schools where output is reflected by a verbal achievement measure; inputs are composed of student characteristics, personnel attributes, facilities, and organizational variables. The model is applied to a sample of white 6th graders attending schools in a large, eastern city during 1965-66. However the model also carries strong implications for those deal[illegible] [illegible]e problems of inner-city and low-income area schools. One o[illegible] [illegible] major implications of the findings is that evaluation results for any group of schools may not be generalizable to any particular school in the sample. The possibility of constructing efficiency rankings for schools to find out which ones are obtaining the largest outputs for their resource is also explored. (Author)

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FRONTIER FUNCTIONS: AN ECONOMETRIC APPROACH TO THE EVALUATION OF EDUCATIONAL EFFECTIVENESS

Henry M. Levin

School of Education
Stanford University
Stanford, California

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Introductory Statement

The Center is concerned with the shortcomings of teaching in American schools : the ineffectiveness of many American teachers in promoting achievement of higher cognitive objectives, in engaging their students in the tasks of school learning, and, especially, in serving the needs of students from low-income areas. Of equal concern is the inadequacy of American schools as environments fostering the teachers' own motivations, skills, and professionalism.

The Center employs the resources of the behavioral sciences--theoretical and methodological--in seeking and applying knowledge basic to the achievement of its objectives. Analysis of the Center's problem area has resulted in three programs: Heuristic Teaching, Teaching Students from Low-Income Areas, and the Environment for Teaching. Drawing primarily upon psychology and sociology, and also upon economics, political science, and anthropology, the Center has formulated integrated programs of research, development, demonstration, and dissemination in these three areas. In the Heuristic Teaching program, the strategy is to develop a model teacher training system integrating components that dependably enhance teaching skill. In the program on Teaching Students from Low-Income Areas, the strategy is to develop materials and procedures for engaging and motivating such students and their teachers. In the program on Environment for Teaching, the strategy is to develop patterns of school organization and teacher evaluation that will help teachers function more professionally, at higher levels of morale and commitment.

The aim of one component of the program on Teaching Students from Low-Income Areas is to build an econometric model of school effectiveness. This paper attempts to show how a particular econometric approach can be applied to evaluating the effectiveness of schools in achieving educational outcomes. The findings are particularly relevant for examining these relationships for the urban disadvantaged child, since the data are drawn from schools in a large city.

Contents

Abstract

The purpose of this paper is to explore the applicability of an econometric model of evaluation to the efficiency of schools. Emphasis is placed on assessing the input-output relations for schools that appear to be maximizing educational outcomes. The standard constrained-maximum model is formulated for the schools where output is reflected by a verbal achievement measure and inputs are composed of student characteristics, personnel attributes, facilities, and organizational variables.

The model is applied to a sample of white sixth graders who were attending schools in a large Eastern city in 1965-66. Separate estimates are made for schools that are on the efficiency frontier--those that are maximizing output--and all schools in that sample. Differences in estimates of the structural relationships between inputs and outputs are noted and their policy implications are assessed. One of the major implications of these findings is that evaluation results for any _group_ of schools may not be generalizable to any _particular_ school in the sample.

The possibility of constructing efficiency rankings for schools to find out which ones are obtaining the largest outputs for their resource inputs is also explored. It is noted that findings in this area are preliminary and subject to further refinement and replication. Accordingly, the attempt of this study is to further hone the tools of educational evaluation.

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Stanford University

Introduction

The conceptual model for evaluating a productive activity is well established in economics. The maximization of outcome for a given cost or, conversely, the minimization of costs for a given outcome leads to a specific criterion for determining which combination of inputs (or treatments) to employ. More formally, we can define the following production function (1).

(1) $$Y = f\,(X_1, X_2, \ldots, X_n)$$

(1) represents the most general function that describes a productive activity where Y represents the output and $(X_1, \ldots, X_n)$ represents the n inputs. Since the production function describes the maximum output that can be produced with each and every combination of inputs, we are concerned only with the surface of this polyhedron.[1] Of course, this is a static concept based on the state of technology available at the time.

This paper was prepared for the 1971 Meetings of the Psychometric Society, April 8, 1971, at St. Louis. The author wishes to express appreciation to Richard Carlson, Guilbert Hentschke, and Helen Loceff for their assistance.

[1] For an introduction to production and costs, see William J. Baumol, Economic Theory and Operations Analysis, 2d ed. (Englewood Cliffs, N. J.: Prentice-Hall, 1963), ch. 11. More advanced expositions are found in W. E. G. Salter, Productivity and Technical Change (Cambridge, Eng.: Cambridge University Press, 1960); Sune Carlson, A Study on the Pure Theory of Production (New York: Kelley and Millman, 1956); and Marc Nerlove, Estimation and Identification of Cobb-Douglas Production Functions (Chicago: Rand McNally, 1965).

If there is a maximum value for each set of inputs, the function is single-valued; and we assume that it is smooth and continuous, with partial derivatives at every point. Further, we assume that the partial derivatives or marginal physical products of the inputs are non-negative,

(2) $\partial Y/\partial X_i = f_i \geqslant 0$

since if they were negative, output would not be maximal. Thus, output can be increased by not using inputs with negative marginal products.

A second general assumption is that the second partial derivatives are negative. That is:

(3) $f_{ij} = \partial^2 Y/ \partial X_{ij} < 0$

(3) represents the convexity properties of the production function and corresponds to the well-known law of diminishing returns. The contribution of any input to output will be a diminishing one, the higher the level of utilization of that input.

Yet an infinite number of combinations of inputs will obtain a given output. Out of these possibilities, how does one obtain a unique set? It is only when a budget or cost constraint is imposed that a determinate solution to that problem can be obtained. Assume that the productive enterprise is faced with a budget, B, a fixed amount of dollars; and that this budget must be allocated among the n inputs into the production function. Clearly, the amount of any input that can be purchased depends on its price; and the obtainable combinations of all inputs depend on their prices. This can be expressed more formally by (4).

(4) $B = P_1 X_1 + P_2 X_2 + \ldots + P_n X_n + \sum_{i=1}^{n} P_i X_i$

Each input X_1 is associated with a price P_1, and we assume that the inputs are obtained in competitive markets so that the prices are invariant with regard to the amount of each input selected. (4) represents the budget constraint within which the producer must operate.

In order to maximize output (Y), subject to budget constraint (B), we define a new function (5).[2]

$$\text{(5)} \qquad W = (X_1, X_2, \ldots, X_n) + \lambda\,(B - P_1 X_1 - P_2 X_2 - \ldots - P_n X_n)$$

Since this is a constrained maximum, we use the method of the Lagrangean multiplier (λ). $\lambda \neq 0$. The necessary conditions for a maximum require our setting the first derivatives equal to zero, as in (6).

$$\text{(6)} \qquad \begin{aligned} \partial W/\partial X_1 &= 0 = f_1 - \lambda P_1 \\ \partial W/\partial X_2 &= 0 = f_2 - \lambda P_2 \\ \partial W/\partial X_n &= 0 = f_n - \lambda P_n \\ \partial W/\partial \lambda &= 0 = B - P_1X_1 - P_2X_2 - \ldots - P_nX_n \end{aligned}$$

Solving the first set of equations for λ we obtain

$$\text{(7)} \qquad \lambda = f_1/P_1 = f_2/P_2 = \ldots = f_n/P_n$$

(7) suggests that in order to maximize output for any given budget, <u>the marginal product of the last dollar spent must be the same for all inputs</u>. Stated another way, the ratios of marginal products to prices must be equal for all inputs. Second-order conditions for a maximum require that the bordered Hessian determinant in (8) be positive:

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[2] Readers not familiar with the formulation and solution of constrained maxima (minima) problems may wish to see Paul A. Samuelson, Foundations of Economic Analysis (Cambridge, Mass.: Harvard University Press, 1947), ch. 3 and 4.

$$(8)\quad \begin{vmatrix} f_{11} & f_{12} & \ldots & f_{1n} & -P_1 \\ f_{21} & f_{22} & \ldots & f_{2n} & -P_2 \\ f_{n1} & f_{22} & \ldots & f_{nn} & -P_n \\ -P_1 & -P_2 & \ldots & -P_n & 0 \end{vmatrix} > 0$$

If all our assumptions hold, then we need only satisfy the conditions in (7) to maximize the outcome that we seek for a given budget constraint. That is, we should allocate our budget among inputs or programs in such a way that the marginal contributions of all inputs or programs, relative to their prices, are equal.

How can this approach be applied to educational evaluation? The educational sector represents a set of productive activities that can be conceived of in the light of the production concepts introduced above. Inputs of students, personnel, facilities, and so on are used to produce an outcome called education.[3] Consider a general production function for education as it applies to each student.[4]

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[3]An excellent review of educational production functions is Samuel S. Bowles, "Towards an Educational Production Function," in W. Lee Hansen, ed., Education, Income, and Human Capital, Studies in Income and Wealth, vol. 35 (New York: National Bureau of Economic Research, 1970), pp. 11-61. A nontechnical summary of results is found in James W. Guthrie, George B. Kleindorfer, Henry M. Levin, and Robert T. Stout, Schools and Inequality (Cambridge, Mass.: MIT Press, 1971), ch. 4. Analyses of more recent technical developments are found in Alexander M. Mood et al., Do Teachers Make a Difference?, A Report on Recent Research on Pupil Achievement, OE-58042, U. S. Department of Health, Education, and Welfare (Washington, D. C.: Government Printing Office, 1970); see particularly the papers by Henry M. Levin, "A New Model of School Effectiveness" (ch. 3), and Stephan Michelson, "The Association of Teacher Resourcefulness with Children's Characteristics" (ch. 6).

[4]See Levin, "A New Model of School Effectiveness." The general approach and empirical work that follow are based on a model and a data set described in that paper. If the reader has questions on either of these, he may find it useful to refer to this earlier work.

(9) $A_{it} = g\ [F_i\ (t),\ S_i\ (t),\ P_i\ (t),\ O_i\ (t),\ I_{it}\ ]$

The i subscript refers to the ith student; the t subscript in parentheses (t) refers to an input that is cumulative to time t.

A_{it} = a vector of educational outcomes for the ith student at time t

$F_i\ (t)$ = a vector of individual and family background characteristics cumulative to time t

$S_i\ (t)$ = a vector of school inputs relevant to the ith student cumulative to t

$P_i\ (t)$ = a vector of peer or fellow student characteristics cumulative to t

$O_i\ (t)$ = a vector of other external influences (the community, for example) relevant to the ith student cumulative to t

I_{it} = a vector of initial or innate endowments of the ith student at t

Although only a few studies specify the general form of the educational production function as in (9), most explorations seem to be based on such a model.[5] In any event, there are many examples in the literature of such estimates, and there have also been several attempts to develop the methodology for estimating educational production functions.[6] Yet very little theoretical or empirical work has been done on a very im-

[5]See Eric Hanushek, "The Production of Education, Teacher Quality, and Efficiency," in Do Teachers Make a Difference?, ch. 4.

[6]See, for example, the works by Bowles, Levin, and Michelson cited in note 3.

portant aspect of educational production functions, that of seeking estimates of the maximum output that can be produced with a given set of inputs.

Average versus Frontier Estimates

The problem can be stated in the following way. Suppose we obtain survey data for a sample of children among a set of schools (e.g., sixth graders). For each child we observe measures of educational outcome at time t as well as proper measures for the elements of the family, school, peer, other influence, and innate endowment vectors. Assuming no errors in the variables (measurement errors) or in the equations (specification errors), we could estimate a structural relation between $A_{i\ t}$ and the input vectors, and the estimated coefficients would represent the marginal products of each input with regard to $A_{i\ t}$. But, in order for the relationship described by these points to depict a production function, all points must be based on each school being 100 percent efficient, producing as much A as can be attained from its inputs.

There is no reason to believe that schools are fully efficient in this sense. Indeed, schools are not likely to be educational output maximizers so long as there are few incentives for schools to maximize educational outcomes (such as verbal scores), managers exercise insufficient discretion over inputs, systematic information is not available to educational managers on inputs and outputs, and traditional ways of operating are revered. If this is so, then the statistical relationships desc

degr

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S_1

mat
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Soc

described above depict production relations among schools at varying degrees of efficiency rather than among schools at the frontier.[7]

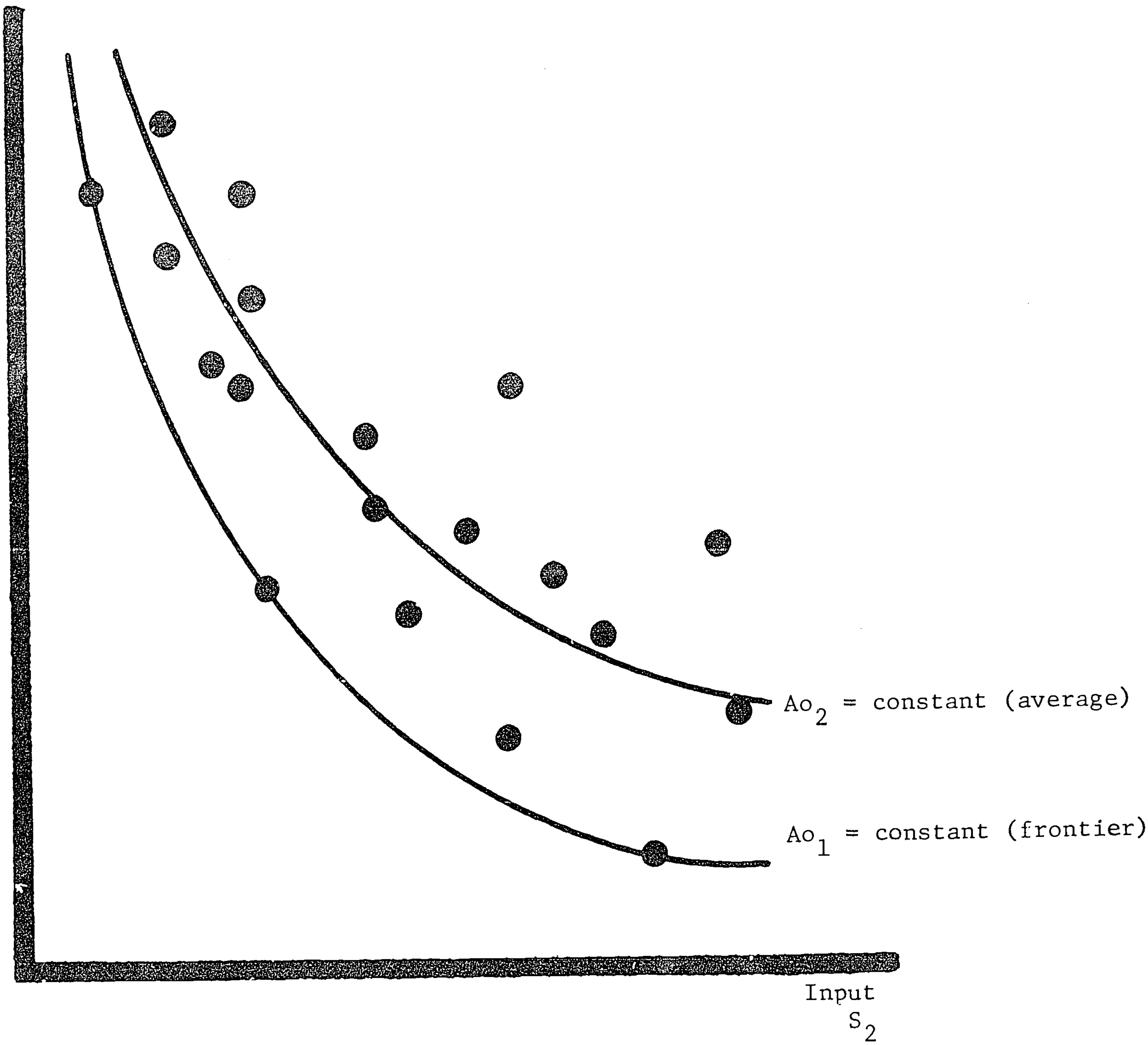

Fig. 1. Frontier and average production isoquants.

[7]This was not recognized explicitly even for private sector estimates of production functions until quite recently. See Michael Farrell, "The Measurement of Productive Efficiency," Journal of the Royal Statistical Society, Series A (General), vol. 120, part 3 (1957), pp. 253-81.

The situation is shown in Figure 1, which represents a hypothetical input-input space where S_1 and S_2 represent two different school inputs into the production of A. (It is assumed here that other inputs are being held constant; of course, this example can be generalized algebraically for n inputs.) Each observation represents the combination of S_1 and S_2 that a particular school is using to produce a given amount of output, Ao. That is, each school in the sample is using a different input mix even though output is the same.

Isoquant Ao_1 represents the production frontier defined as the locus of all observations that minimize the combinations of S_1 and S_2 required to produce constant product Ao. It represents a mapping of the most efficient points for producing Ao and is thus the production frontier. All observations to the northeast of Ao_1 are of inefficient schools that are using higher input levels to produce the same output. Now assume that we fit the observations statistically via normal regression procedures. We obtain the statistical equivalent of Ao_2 for all schools (both efficient and inefficient ones). Of course all points on Ao_2 are farther from the origin than those on Ao_1, showing that the average production relationship is a less efficient one than the frontier relationship.

Since virtually all estimates of educational productions have been based on the performance of both average and efficient schools rather than efficient ones only, the existing statistical studies of educational production are not production function studies in the frontier sense. Moreover, their results may suggest erroneous conclusions about which combination of inputs (program) maximizes outcomes for a given budget constraint. For example, assume the two-input production function:

(10) $A = h(s_1, s_2)$

In equilibrium we would wish to satisfy the conditions set out in (11), where P_1 and P_2 represent the prices of X_1 and X_2 respectively.

(11) $$\frac{\partial Y/\partial s_1}{P_1} = \frac{\partial Y/\partial s_2}{P_2} \quad \text{or} \quad \frac{h_1}{P_1} = \frac{h_2}{P_2}$$

Now consider two different values for h_1 and h_2. At the frontier, $h_1 = \hat{h}_1$, and for the average of all schools, $h_1 = \mathring{h}_1$. The symbols for h_2 can be defined in the same way. Rearranging the terms of (7) and applying it to the case at hand we obtain (12).

(12) $$\frac{\hat{h}_1}{\hat{h}_2} = \frac{\mathring{h}_1}{\mathring{h}_2} = \frac{P_1}{P_2}$$

(12) reiterates the necessary conditions for a maximum both for frontier estimates and for average estimates of the production function. In both cases we wish to select the combination of inputs that equates the ratios of marginal products (first derivatives) to the ratios of prices.

Implications for Evaluation

If we estimate only the average production function or only the frontier one, can the optimal ratio of inputs derived from one estimate also apply to the other? The answer to this question clearly depends on whether there are differences in the structural parameters associated with each input.

For example, it is possible that the inefficiencies of non-frontier schools are neutral among inputs so that at every level of input and for every combination of inputs the ratios of the marginal products are identical for both frontier and average functions. That is, (13) holds.

(13) $\hat{h}_i = \gamma \overset{\circ}{h}_i \qquad (i = 1,2)$

$o < \gamma \leqslant 1$

This can be represented by Figure 2, where Ao_1 signifies the production isoquant for Ao for all efficient schools and Ao_2 represents th[illegible]e

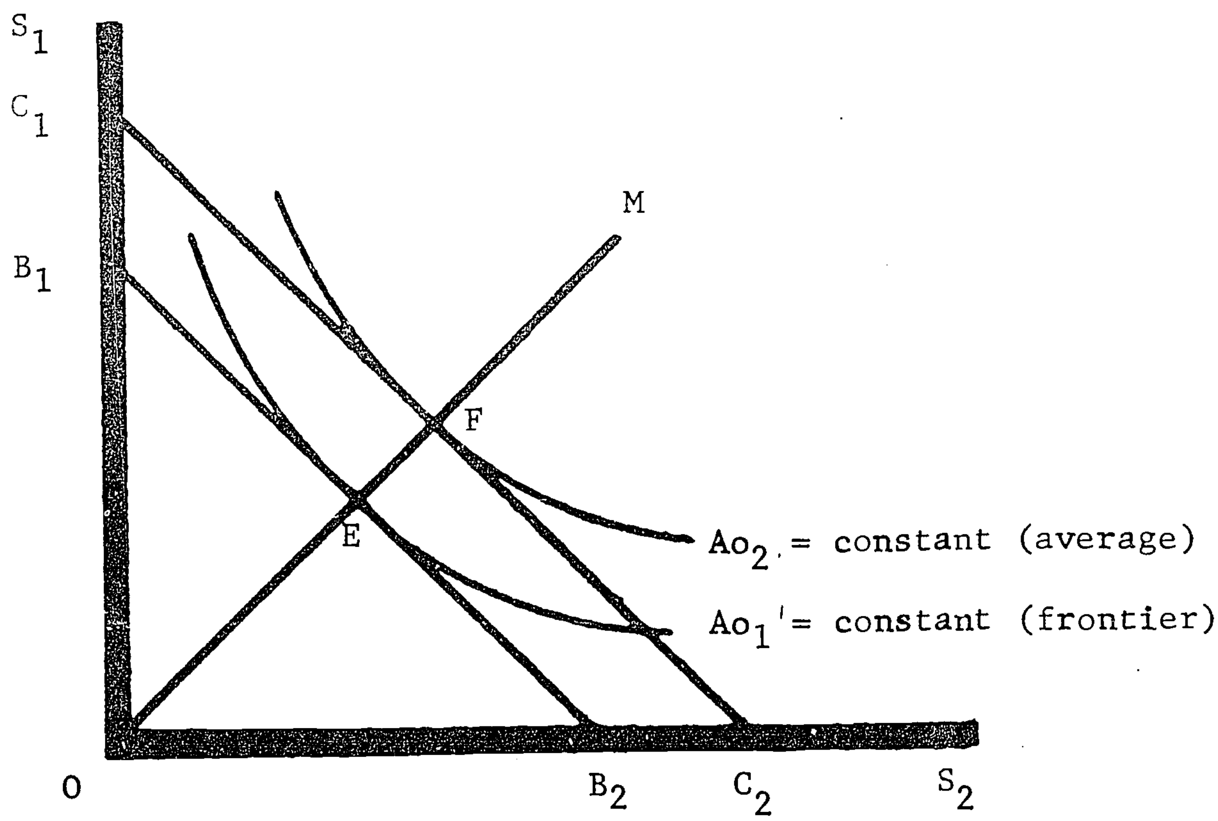

Fig. 2. Technical inefficiency that is neutral among inputs.

level of output for the entire set of schools, efficient and inefficient. $B_1 B_2$ and $C_1 C_2$ represent budget or isocost lines reflecting the various combinations of S_1 and S_2 obtainable for two given cost constraints, B and C, where $C > B$. The slope of the isocost lines is determined by the ratio of the prices, P_2/P_1 . Thus, E and F represent equilibrium points which reflect (12). That is, the combination of S_1 and S_2 that maximizes Ao for budget constraint B is determined by the tangency of A_{o1} to $B_1 B_2$

at point E for efficient or frontier schools and of A_{o2} to C_1 C_2 at point F for schools on the average.

It can be shown that the relative intensities of the two inputs will be identical for both groups of schools if a ray drawn from the origin intersects both points of tangency. O M satisfies that condition, so the same ratio of S_1/S_2 is optimal for both groups of schools. Whether we use the estimates of frontier schools or of all schools, the findings on the optimal combinations of S_1 and S_2 will be binding for both. In such a case it does not matter which group we use to estimate the production function, although the absolute product will be higher for the set of schools at the frontier for any input level.

On the other hand, there is a case where (13) does not hold. This can be shown in Figure 3, and it is also evident in Figure 1. Here the

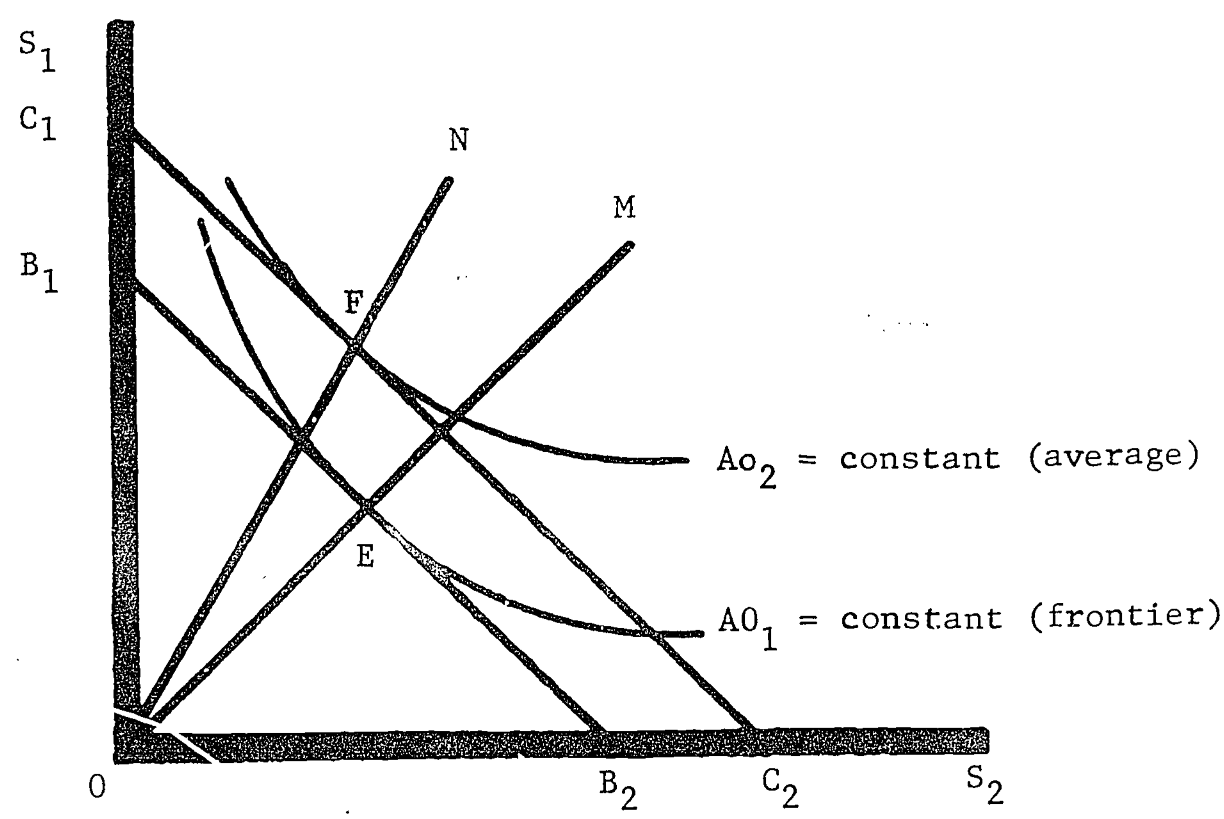

Fig. 3. Technical inefficiency that is biased between inputs.

relative inefficiency in the use of S_2 appears to be greater than that for S_1. For example, if S_1 represents physical school facilities and S_2 represents teachers, Figure 3 suggests that the organizational arrangements in inefficient schools are relatively more harmful to the productivity of the teachers than to that of the facilities. In this case a ray drawn through the origin representing a constant ratio of inputs will not intersect both points of tangency. That is, the optimal ratio of S_1/S_2 for frontier schools will be different from that for inefficient schools. In this case, the results obtained for one group of schools (e.g., frontier ones) cannot be applied to another group (e.g., non-frontier ones.). Rather, each set of schools will have their own optimal combination of S_1/S_2 depending on their relative efficiency.

An Empirical Application

One major difficulty in demonstrating some of the empirical implications of the foregoing analysis is that the necessary relationships are much easier to obtain mathematically than they are statistically. The particular problems in deriving educational production functions have been described elsewhere, so they will not be detailed here. It is useful to note, however, that the statistical work in this area is subject to both errors in the equations and errors in the variables. In the former case the proper specification of the model is still in the exploratory stage. The structure of the model, the specific variables to be included, and the relationship of the variables to one another have not been well established, and there are many gaps in our knowledge. Moreover, most of the operational variables used in the models are subject to varying degrees of measurement error.

Thus no strict application of the findings to public policy is warranted. Rather, the empirical aspects are meant to suggest new directions and to provoke new thought on the process of evaluation. The results derived must surely be subject to replication and further analysis before they can be considered acceptable for policy consideration.

The Sample

The data used in this analysis represent a subsample drawn from the Survey on Equal Educational Opportunity conducted by the U. S. Office of Education for the school year 1965-66. Specifically, the subsample is composed of some 597 white sixth graders who had attended only the schools in which they were enrolled at the time of the survey.[8] Since the data were reanalyzed and recoded extensively for the purpose of estimating present relationships, they differ in important ways from the data used in other studies that have drawn information from the same survey. Twenty-nine schools are represented in the sample, and the teacher characteristics identified are averages for each school for all teachers who were assigned to grades three through five. These averages were intended to reflect the teacher characteristics that had influenced student behavior up to the time of the survey. Moreover, it was assumed that the observed measures of family background and other educational influences were related systematically to the cumulative impact of each of these variables.

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[8] See Levin, "A New Model of School Effectiveness," for details.

The equation used to explore differences between frontier and average estimates is a linear equation based on (9). Linearity not only violates our assumptions about the second derivative, but it also runs counter to our intuition about the real world. Nevertheless, the difficulties of estimating particular nonlinear functions and the risk of creating even greater specification biases in the coefficients by imposing another arbitrary functional form suggest that the linear equation might yield reasonable first approximations to the estimates that we seek. Following this procedure, of course, limits our comparison of the frontier and average estimates to the linear marginal products and price ratios.

The variables in the equation are shown in Table 1. These variables are taken from the reduced-form equation for verbal achievement derived from a four equation system encompassing three simultaneous equations and one that represents a recursive relationship. Once that system is estimated, one can solve for the reduced-form equation for any of the three endogenous variables. Since the estimation of that system is discussed elsewhere[9], this paper is concerned only with the reduced form of the verbal equation. Since this equation was fitted to the entire sample of observations, it represents the average production relationship for the sample of schools. Results are shown for this estimate in the right-hand column of Table 2 (p. 19).

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[9] Ibid.

TABLE 1. List of Variables

Variable	Item Measured	Coding
Verbal Score	Student Performance	Raw Score
	Efficacy	Index compiled from questions 33-40 in the Sixth-Grade Student Questionnaire of the Equal Opportunity Survey "I can do many things well." (Well/No/Not Sure) "I sometimes feel I just can't learn." (Yes/No) The higher the value of the index, the greater the perceived efficacy of the student.
	Educational Expectations of Parents	Index based on three questions: (1) How good a student does your mother want you to be? (2) How good a student does your father want you to be? (3) Did anyone at home read to you when you were small, before you started school? (and how often?)
	Student Motivation	Grade level the student wishes to complete
Sex	Male-Female Differences	Male = 0 Female = 1
Age	Overage for Grade	Age 12 or over = 1 Less than 12 = 0
Possessions in Student's Home	Family Background (Socioeconomic Status)	Index of possessions: television, telephone, dictionary, encyclopedia, automobile, daily newspaper, record player, refrigerator, vacuum cleaner Yes = 1, No = 0 for each item. Index is sum.

(Table 1 cont'd. p.16)

Variable	Item Measured	Coding
Family Size	Family Background	Number of people in home
Identity of Person serving as Mother	Family Background	Real mother at home = 0 Real mother not living at home = 1 Surrogate mother = 2
Identity of Person Serving as Father	Family Background	Real father at home = 0 Real father not living at home = 1 Surrogate father = 2
Father's Education	Family Background	Number of years of school attained
Mother's Employment Status	Family Background	Has job = 1 No job = 0
Attended Kindergarten	Family Background	Yes = 1 No = 0
Teacher's Verbal Score	Teacher Quality	Raw score on vocabulary test
Teacher's Parents' Income	Teacher Socioeconomic Status	Father's occupation scaled according to income (1000's of dollars)
Teacher Experience	Teacher Quality	Number of years of full-time experience
Teacher's Undergraduate Institution	Teacher Quality	University or college = 3 Teacher institution = 1
Satisfaction with Present School	Teacher's Attitude	Satisfied = 3 Maybe prefers another school = 2 Prefers another school = 1
Percent of White Students	Student Body	Percentage estimated by teachers
Teacher Turnover	School	Proportion of teachers who left in previous years for reasons other than death or illness
Library Volumes Per Student	School Facilities	Number of volumes divided by school enrollment

Obtaining Frontier Estimates

The same set of data and variables can be used to obtain estimates of the equation for only the most efficient observations. There are several ways of doing this; the one used here is the programming approach in input-output space suggested by Aigner and Chu.[10] Since the individual observations are of students rather than schools, we wish to seek those students who show a particular outcome with the lowest application of resources. Using the general notation from (1), the problem is to minimize (14).

$$\sum_{i=0}^{n} \hat{\alpha}_i \bar{X}_i \tag{14}$$

where $\hat{\alpha}_i$ is the parameter for the ith input, $\bar{X}_i$ is the mean of X_i, and $\bar{X}_o = 1$ in order to obtain a constant term. More specifically, the problem is to minimize (14), which can be rewritten as (15) subject to the constraints (16).

$$\text{Min.} \quad \hat{\alpha}_o + \hat{\alpha}_1 \bar{X}_1 + \ldots + \hat{\alpha}_n \bar{X}_n \tag{15}$$

Subject to:

$$\begin{array}{llll} \hat{\alpha}_o + \hat{\alpha}_1 X_{11} & + \ldots + \hat{\alpha}_n X_{n1} & \geqslant & Y_1 \\ \vdots \quad\quad \vdots & \quad\quad\quad\quad \vdots & & \vdots \\ \hat{\alpha}_o + \hat{\alpha}_1 X_{im} & + \ldots + \hat{\alpha}_n X_{nm} & \geqslant & Y_m \\ \hat{\alpha}_i & \geq 0 & & \end{array} \tag{16}$$

[10] D. J. Aigner and S. F. Chu, "On Estimating the Industry Production Function," American Economic Review, 58 (1968): 826-39.

Since this is essentially a linear programming problem, there will be as many "efficient" observations as there are inputs into the production function (assuming that no two observations are identical). Unfortunately, some of the observations will appear to be "efficient" when in fact they represent measurement errors. Thus, it is impossible to know *a priori* whether a particular observation is efficient, or whether it is spurious. Following Timmer extreme observations have been discarded in order to eliminate what might be spurious points.[11] This is particularly important for the frontier estimates, since very few observations determine the structural coefficients.

Table 2 contrasts the frontier estimates with the estimates for the average function. Figures in parentheses beneath each coefficient for the average function signify the t statistics of those coefficients. Each of the coefficients represents the first derivative or marginal product of the function.[12] Four linear programming runs were used to obtain frontier estimates. Run 1 eliminated no observations; Run 2 discarded the nine most "efficient" points; Run 3 eliminated 23 observations; and Run 4 discarded the 38 most extreme points (about 6 percent of the sample). I will compare the frontier function from Run 4 and the average

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[11] C. Peter Timmer, "On Measuring Technical Efficiency." Unpublished Ph.D. dissertation, Department of Economics, Harvard University, 1969.

[12] Since $\alpha_i \geqslant 0$, those variables that showed negative coefficients for the average function represented problems for the programming estimates. The array for each such variable was multiplied by (-1) for the programming estimates, and the signs were reversed in turn when reporting the results in Table 2. The author is indebted to Richard C. Carlson for computing the programming estimates. See his paper, "Educational Efficiency and Effectiveness." Unpublished paper prepared for Seminar in Economics of Education, Stanford University, May 1970.

TABLE 2. Frontier and Average Production Relations for White Sixth Graders, Eastmet City

	Frontier Function				Average Function
Variable	Run 1	Run 2(-9)	Run 3 (-23)	Run 4(-38)	
Personal Variables					
Sex	0.0	1.649	0.982	0.01956	0.817 (1.41)
Age	-7.714	-4.642	-4.769	-5.553	-6.010 (-4.49)
Family Size	-0.502	-0.500	-0.089	-0.770	-0.552 (-3.50)
Father's Identity	0.0	0.0	-0.283	-0.420	-0.327 (0.64)
Mother's Identity	-0.878	-1.342	-1.190	-1.202	-0.433 (-1.90)
Father's Education	0.509	0.179	0.0	0.103	0.273 (3.22)
Mother's Employment	-1.726	-2.293	-1.089	-0.951	-0.509 (-1.31)
Possessions	1.865	1.464	1.070	1.020	1.229 (5.201)
School Variables					
Kindergarten	0.0	2.866	1.920	2.106	2.372 (2.47)
Teacher's Verbal Ability	0.810	0.218	0.695	0.791	0.250 (1.70)
Teacher's Parents' Income	0.0	0.0	0.0	-0.00006	-0.118 (-0.64)
Undergraduate Institution	3.736	5.269	1.991	8.307	6.525 (2.09)
Teacher Experience	0.0	0.392	0.368	0.616	0.787 (4.93)
Teacher Satisfaction	3.630	7.078	4.666	3.608	1.960 (1.50)
% White Students	0.0	-0.500	-0.264	-0.178	-0.047 (-.03)
Library Vols. Per Student	0.0	0.571	0.509	0.156	0.565 (1.53)
Teacher Turnover	0.0	0.0	0.0	-0.035	-0.101 (-1.27)
Constant	0.0	0.0	-0.944	-4.051	-7.902 (.84)

function and at the same time will examine three properties of the estimates: (1) the relative magnitudes of the coefficients; (2) the implications for allocative or price efficiency; and (3) an overall technical efficiency index.

Recall that in order for findings of optimal input intensities to yield the same relative applications of inputs for both average and frontier schools, the marginal products for both functions must bear a constant relation to each other as reflected in (13). Table 3 shows the ratios of marginal products for the two sets of estimates for all of the

TABLE 3. Ratio of Marginal Products at the "Frontier" to Marginal Products for the Entire Sample

School Variables	MP (frontier) / MP (average)	School Variables	MP (frontier) / MP (average)
Kindergarten	.888	Teacher Satisfaction	1.841
Teacher's Verbal Ability	3.164	Percent of White Students	3.787
Teacher's Parents' Income	.001 (.0005)	Library Volumes per Student	.276
Undergraduate Institution	1.273	Teacher Turnover	.347
Teacher Experience	.783	Constant	.513

school variables. According to this table there is no systematic relationship between the two sets. At the frontier, such inputs as the teacher's verbal facility and the proportion of white students to others show marginal products that are more than three times their counterparts derived for the sample as a whole. On the other hand, such variables as teacher turnover, teacher experience, and library volumes per student show much smaller coefficients for the frontier function.

If these estimates are truly unbiased, the implication is that so-called frontier schools are more efficient in the use of some inputs and less efficient in the use of others. Standard errors of the coefficients tend to be high relative to the differences in coefficients. Even so, the coefficient for teacher's verbal ability is significantly different between the two estimates, and other differences are on the margin of significance. This in turn suggests that the production isoquants for schools of different "efficiencies" may be intersecting within the relevant ranges of factor input substitution. Thus any optimal combination of inputs for any set of schools or any individual school is likely to be less than optimal for any other set of schools of individual school. In other words, for any given array of prices (P_1, P_2, . . . P_n) the optimal set of input proportions may vary significantly from school to school.

For purposes of generalizing, this is the worst of all possible worlds. That is, whereas we might be able to derive the optimal input structure for frontier schools or for schools on the average as represented by equilibrium conditions stated in (12), it is likely that the desirable combination of input intensities may differ between the two sets of schools (and may even differ significantly from school to school).

An illustration of this is found in Table 4, which shows the estimated ratios of prices for two inputs (teacher verbal score and teacher experience) as well as the two sets of marginal products for those inputs. The "prices" reflect the increments to annual teacher salaries for each of the teacher attributes; they were derived from an equation re-

TABLE 4. Relative Prices and Marginal Products for Teacher Verbal Score and Teacher Experience

	(1) Teacher Verbal Score	(2) Teacher Experience	(3) Ratio of (1) to (2)
Price (salary increment)	$24.00	$79.00	0.303
Marginal Product at Frontier	0.791	0.616	1.284
Marginal Product on Average	0.250	0.787	0.317

lating teacher attributes to earnings in the Eastmet teacher market.[13] The marginal products associated with a unit change in the two inputs are taken from Table 2. In equilibrium the ratios of the marginal products of the inputs should be equal to the ratios of their respective prices. For the average production estimates these ratios are almost identical, so that allocative or price efficiency is implied even though the average estimates are assumed to be based on technically inefficient (non-frontier) schools.

On the other hand, the frontier estimates show a ratio of marginal products four times as great as the price ratio for the two inputs. This suggests that the utilization of more verbally able teachers yields four times as much output per dollar as the utilization of additional teacher

[13]These are taken from Henry M. Levin, "Recruiting Teachers for Large-City Schools," mimeographed, The Brookings Institution, 1968. For a similar application of these prices, see Henry M. Levin, "A Cost-Effectiveness Analysis of Teacher Selection," Journal of Human Resources, 5, No. 1 (1970): 24-33.

experience. If this is correct, the schools on the frontier could increase total output by reallocating their budgets in favor of teachers' verbal scores rather than their experience.[14]

The significant aspect of this analysis is that the combination of inputs which maximizes output differs between the two estimates. If these differences persist among schools of different efficiencies, the hope of obtaining general decision rules that can be applied across schools seems to be frustrated. That is, the lack of similarities among the production techniques used by different schools may mean that neither average nor frontier findings can be applied to any particular school. Indeed, in the extreme case, each individual school is on its own production function, and evaluation results for any group of schools will not be applicable to individual schools in the sample.

Overall Efficiency Ratings

The foregoing analysis is rather pessimistic with regard to the possibility of establishing resource allocation criteria from estimates of the educational production relationship, whether the estimates are obtained from the sample as a whole or at the frontier. But there is still one interesting evaluation exercise that might have some validity. With a set of production coefficients for efficient schools, it is possible to calculate for each school what its output would be if it used its resources efficiently. This estimated output could be compared with actual output for each school to obtain a rating of efficiency, as in (17).

[14]For a similar finding see Ibid.

$$E_k = \frac{\text{actual output}_k}{\text{maximally efficient output}_k} \tag{17}$$

(17) is a technical efficiency index where E_k is the technical efficiency of the kth school. The index for schools is defined as in (18).

$$E_k = \frac{\frac{1}{m}\sum_{j=1}^{m} A_{jk}}{\frac{1}{m}\sum_{j=1}^{m}\left(\sum_{i=1}^{n} \hat{\alpha}_i X_i\right)_{jk}} \tag{18}$$

where jk = individual student j in the kth school ($j = 1, \ldots, m$). Thus the school efficiency rating is an arbitrary index of actual student performance relative to the predicted maximal level (both averaged over students for each school).

Using this measure of technical efficiency, the 29 schools in the sample showed fairly substantial variance. The mean efficiency for the sample was .76, but the index on a school-by-school basis varied between 1.00 and 0.13. Since these extreme values were realized for schools with only one child in the sample[15], they are not likely to be reliable. Using five students as a minimum sample size for establishing a range, the efficiency index varies between .46 and .91.

It is probably in the area of establishing overall efficiency ratings among schools that the frontier approach has the most promise. An index of that sort would yield an overall statistic on how efficiently a

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[15]Bear in mind that the sample selected from each school was limited to sixth-grade white children who had attended only the school in which they were enrolled at the time of the survey. In the cases of some inner-city schools, only one or two students fulfilled those prerequisites.

school is using its resources relative to schools on the frontier. If such indicators of efficiency are valid, they might have important implications for public policy. Efficient schools could be compared with inefficient ones to see how they differ, and policy recommendations for improving the latter schools might be forthcoming. Although it is tempting to make this application, the grounds for expecting large errors in the estimates are substantial. Thus, major refinements in measurement and structural specification will be needed before such an efficiency rating can be considered reliable enough for policy use.