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ABSTRACT

Tested was a method of learning numeration, addition, and subtraction using measuring operations in place of the more usual counting operations. It is claimed that an approach through "units of measurement" to continuous variables is mathematically more powerful than counting, which leads only to nominal and ordinal variables. Twelve children selected from the lowest achievement group of a middle class kindergarten were given a pretest of general mathematical ability and randomly assigned by matched pairs to experimental and control groups. The experimental group received 17 days of individual instruction by the experimeter, in which the student used an electro-mechanical measuring device as a learning aid. A conservation test was given to all students as a pre- and posttest, and the mathematical ability test was readministered as a further posttest. Analysis by a correlated t-test showed that the experimental group made significant gains on both tests; the changes in the control group's scores were not significant. (MM)

THE CHILD'S INTRODUCTION TO MATHEMATICS: A TRANSFER MODEL BASED IN MEASUREMENT[1]

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Children are usually taught to count objects, add, subtract, etc. as a sequence by which they are introduced to mathematics. Two difficulties with the counting method have been pointed out by the Russian Educational Psychologists P. Ya. Gal'perin and L. S. Georgiev (1969): (a) counting of discrete objects is more instructive of the characteristics of the physical object counted than of purely mathematical concepts--the child often fails to grasp "unit of measure," or continuity, and (b) the method leads to wide separation of the tasks. First, one arithmetic operation is taught, then another; but there is no ready means by which to integrate the concepts that should emerge as basic mathematical abstractions (Reitman, 1962).

This, therefore, was a study of mathematics learning based in the operations of linear measurement whereby five-year-old children learned to assign numbers by measuring rather than by counting. Certain measuring apparatus was used to teach numeration, addition, and subtraction in such a way that new concepts were always framed out of the same context; i.e., a measuring operation with the apparatus. For that reason, there was always overlap of the measuring exercise in terms of tangible or visual elements shared by the exercises. The objectives were to develop in the child (a) the concept of "unit of measure" as a power--a quantity isolated in a continuous variable; (b) addition and subtraction capabilities within linear measurement as a generalization of the rule $n \pm 1$; and (c) the conservation of quantities as evidence that the child has attained quantitative abstractions that replace the tendency to make only concrete visual comparisons of quantities, something children apparently do when they fail to conserve.

Rationale for a Different Teaching Method

Two bases are presented here in support of a tactic different from counting for introducing mathematics. From the standpoint of both

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mathematics theory and learning theory, current teaching method is deficient. In learning to count things, the child misses certain mathematical content. His attention is drawn away to the individuality of the enumerated object. He frequently fails to understand "unit of measure" and often does not grasp the meaning of continuity. Note that the first experiences and learnings of children are almost totally qualitative. In view of the child's extensive qualitative experience, one can gradually introduce to him quantitative concepts as is current practice or the introduction can be more abrupt. The method outlined in this paper presents quantitative concepts to the child rapidly in a context favorable to the integration of such concepts. The point is that when teaching method is sensitive to mathematics theory, method will supply the means to introduce children to their first quantitative concepts so that such concepts are genuinely quantitative and not readily confounded with qualitative concepts the child already knows. Herein lies the difficulty when teaching children to assign numbers by counting objects. The child is made aware of the discrete nature of the object counted, an extension of his qualitative experience. Numbers become identified with objects in the nominal sense; that is, qualitative attributes of the object associate with the number as if the child is without access to the number's potential mathematical attributes. The counting experience is devoid of the most basic of mathematical concepts, those of unit of measure as an arbitrary abstraction and continuity of a variable. That these concepts are foundational to mathematics learning is supported by the consideration of what defines quantity and what is quantitative. Implicit references to quantities pervade everyday life. From Ernest Nagel's (1960) discussion of the theory of measurement are these references: "He had a good sleep," and "The cake is too sweet." Both statements imply continuous variables. Nagel has referred to measurement as being ". . . the correlation with numbers of entities which are not numbers." The general grasp of such correlational relations of numbers with continuous variables is what the child requires in order to attain an orderly body of mathematical knowledge. The supposition of this paper is that the child will gain efficiently an understanding of basic mathematical concepts if he is appropriately introduced to measurement.

Measurements are made in a variety of ways and scales of measurement can be classified to accord with a hierarchy of increasing quantitativeness. S. S. Stevens (1960) has referred to four such scales in the following ascending order; nominal, ordinal, interval, and ratio scales. Counting operations are appropriate at the nominal level but no other operations that signify quantity are appropriate. Numbers at the nominal level have no meaning aside from "naming" as "1" = male or "2" = female. A ratio scale in contrast, has a number of attributes which are recognized as giving that scale a quantitative character. True zero, asymetrical transformation of numbers toward higher values, and equal size intervals all apply to the ratio scale. Perhaps the best attainable tangible representations of abstract mathematical thought are available in measurement operations at the ratio level of measurement. Expository interpretations of scales of measurement have been provided by Kaplan (1964) and Stevens (1960).

In the present study didactic measurement exercises at the ratio level were used to support the learning of the concept "unit of measure" as an abstraction quite apart from the unit selected to measure with. If a blackboard eraser, then a pencil, and following that, several different objects are used to measure the length of a desk the learner should come to realize that "unit of measure" is not the eraser or pencil. The unit is, rather, an arbitrarily or rationally selected quantity. The continuity feature of the variable and the infinite divisibility of the variable should also in time be realized by the subject. By measuring continuous variables on a ratio scale the teacher and learner are afforded the advantage that the subject matter is all there at the outset and as learning proceeds it is an elaborative acquisition process. There is less need for replacing learned concepts that are incorrect because the subject matter was inappropriate or incomplete. Such mislearning, according to Gal'perin and Georgiev (1969), is the case when children learn first to count, then to add, and then to subtract, and so on.

A school entering transfer base for mathematics. The devising of a means by which to make a teaching method agreeable with its subject matter (i.e., mathematics) and also make it consonant with learning theory is a demanding task. Sometimes, however, ideas from separate realms do fit together well and this is the case where transfer of learning is the core concept supplied by learning theory. Theorists generally value those activities which are said to assist the development of a large transfer base. Consider what the child will require in order to form such a favorable introduction to mathematics.

Ellis (1965) has listed empirically derived factors which facilitate transfer. Through a series of related measurement exercises to be performed by the learner, we have attempted to take into account these four transfer facilitators:

1. Practice should be exceptionally thorough in the early stages of skill and concept development. Initial work, falling short of such thorough practice, may lead to no transfer or even negative transfer. Harlow (1959) has suggested a working principle to ensure maximal efficiency: continue practice on those tasks where learning is greatest; when the returns become small it is wasteful to continue.

2. A variety of examples are important to the comprehension of concepts and principles. Stimulus variety has been valued for its motivational properties (Fowler, 1965), and should likewise be recognized as necessary to learning. The pupil must associate a response to many stimulus exemplars of its conceptual class to be reliably able to apply the concept to a new situation (Duncan, 1958; Travers, 1972, p. 173).

3. General principles must be understood before a significant amount of transfer can be expected. This is a caution against changing the rules of the learning game too often. The child must master one task before undertaking another as a protection against interference as when stimulus elements of a first task are confused with those of a second task.

4. Mediation due to a network of associative linkages between tasks will lead to transfer but the necessary preconditions include development of a relevant language (mathematics). In addition, the learner must have practiced with that language under varying conditions (Russell & Storms, 1955).

Careful ordering of subject matter in response to the principle of contiguity and in response to certain rules described by Gagné (1962) are helpful prerequisites for the formation of such networks of association. Wittrock (1968) has called attention to a neglected meaning of transfer: as the teacher teaches, transfer is mediated by the learner's history. Attention is thus drawn to individual differences and the need to individualize instruction is implied. Perhaps when children first enter school, and before individual differences widen, a more urgent requirement of instruction arises: that the instruction be so designed that more ready integration of basic concepts will occur. As applied to initial mathematics learning, this is a claim on the teacher's energies for more finesse in the selection and ordering of subject matter.

Wohlwill (1964) has referred to the common perceptual set of the small child in making visual comparisons of quantity as "a kind of vestigial form of reasoning." To overcome that difficulty he has urged the transfer merits of practice in measuring. Wohlwill did not suggest that the counting activities of small children may contribute to the "vestigial form of reasoning." Such might be so, however, and a case of negative transfer from counting experience to the learning of quantitative concepts. The learner may then require a very different learning set to be able to handle continuous quantities. Potentially, when the school-entering child acquires a transfer base which is appropriate for a lengthy period of his learning and the integrity of that base is not destroyed through obsolescence, the total of the child's learning will be a better organized body of information.

The course of early learning. It is reasonable to expect that the first learnings of children are symmetrically related to each other. By a symmetrical relation we mean that the association of one learned element to another is not transitive and that the association BA is as readily made as AB. During some initial period, the neophyte learner acquires information that is organized in only rudimentary ways, and those associations that form first are the easier ones. Following that, the rate at which learning progresses in the child will depend on favorable and unfavorable conditions of transfer. That initially there should be less transitive learning than later follows from theory and from research. According to Hebb (1949), the reason why children are slower than are animals to develop elaborate forms of behavior is found in the association-sensory ratio. The rate at which mature forms of behavior develop is inversely related to cortical mass and represented in the association-sensory ratio, A/S. For each sensory input the child has more potential associative links to be formed than do infrahumans who exhibit mature behavior early. Children do not exhibit organized or well developed behavior and learning is not rapid, that is, learning is not transitive and integrative, until a base of associative links has been formed.

In the transfer context, Travers (1972) has called attention to evidence that the initial period of learning differs from later periods, at least with some forms of learning. The evidence is best considered in relation to the form of the classical learning curve which is generally pictured as negatively accelerated with its initial segment showing a steep gradient. Indications are that this is not the true form of the curve; rather, it has the form of an ogive. Little or no elevation appears in the curve for a period in which basic experience is acquired with the relevant stimulus elements of a task. If such experience is well ordered, that is, favorable to transfer, integration of elements so as to form concepts will occur. Rapid integration which follows a period of no apparent progress is represented in the positively accelerated segment of the S-curve. Figure 1 is such a curve.

To influence those early mathematical experiences of the child that form the flat and positively accelerated portion of the curve, one will need to provide favorable conditions of transfer that are not usually present. The orderly progression of learning in monkeys as demonstrated by Harlow (1949) represent for those creatures and for that group of problems such unusually favorable conditions. Harlow assigned to "learning set" the responsibility for such orderly learning. According to Gagné (1962, 1968), when subordinate learning sets have been established in the learner, a hierarchical progression of knowledge will develop which is totally dependent upon those subordinate sets. Evidence was provided by Gagné to show that this was so. We think it reasonable to believe that when transfer occurs of a kind that relates numerous subordinate sets, the

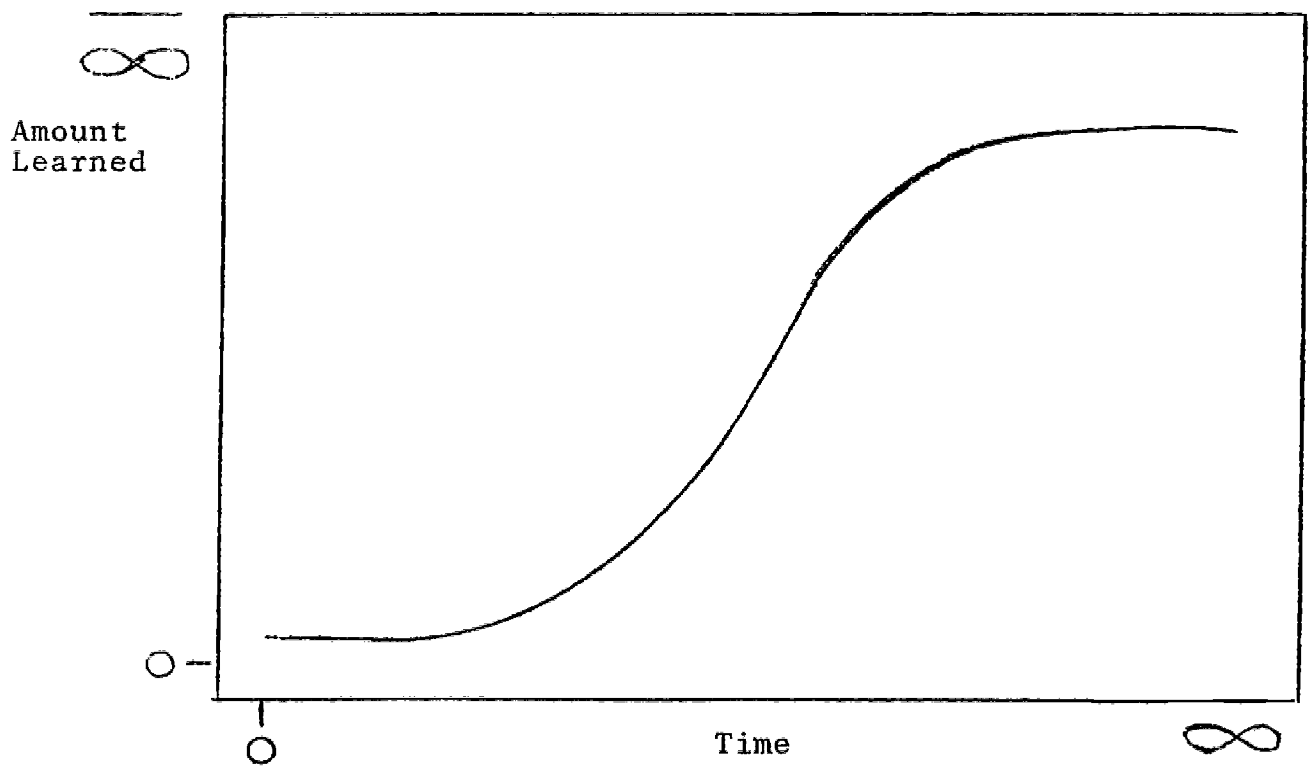

Fig. 1. A hypothetical learning curve in which the early learning is either not transitive or bilaterally transitive.

integration may be fitted to that temporal segment of the learning curve showing rapid acceleration. Under ordinary conditions such quick integration may be observed only rarely for the reason that obsolete learning sets are typically present and there is much negative transfer that the learner must overcome. We think that an organized sequence of measuring exercises will provide a basis for rapid integration of basic mathematical concepts.

Method

Subjects

Twelve children aged five years five months to six years one month were selected for study from a middle-class kindergarten. The kindergarten consisted of three achievement groups. Children selected for this study represented the least advanced group, providing the most mathematically naive subjects.

Apparatus

Measurement exercises were initiated by using common objects as units of measure. Subsequent measurement exercises were carried out on the device pictured in Figure 2. This electro-mechanical device was used to standardize and ease the teaching task. The function of the device is explained in Table 1.

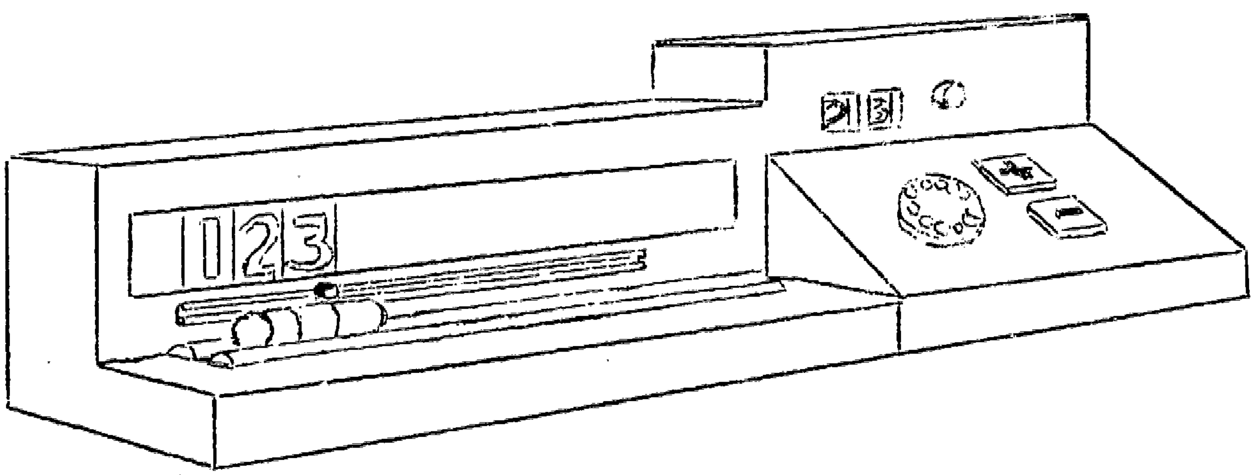

Fig. 2. An electro-mechanical measuring device the functions of which are given in Table 1.

TABLE 1

Function Outline for a Device to Teach Linear Measurement

I. Electro-mechanical measuring device

   A. Dowel segments--in horizontal track of Figure 1

      1. Cylindrical
      2. 1 1/4 inches in diameter
      3. Multiples of various lengths
      4. All segments of each length painted same color

   B. Measuring track

      1. Horizontal track for placement of dowel segments extending along the length of the device
      2. Thirty inches in length
      3. Contains concealed switches at 1" intervals--activated by weight of dowel segments

   C. Lighted number line

      1. Translucent plastic strip

         a. Mounted parallel to track on a plane perpendicular to it
         b. Numbers projected from behind at 1" intervals
         c. No distinct break between lighted number areas

      2. Lighted number areas advance in relation to the number (length) of dowel segments placed in the track

      3. Numbers can be deleted by substituting clear plastic sheet

      4. Can be set so that all the numbers along the line illuminate or final number only

      5. Light projections can be controlled either by dowel segments in the track or by a telephone dial at the far right of the device

      6. Zero position light

         a. Examiner controlled by separate switch
         b. Used only with numeral projections

TABLE 1 (cont'd)

D. Sliding marker above and parallel to track

1. Adjusted to mark the terminous of an object being measured
2. Can be used without dowel measuring units or other objects in the track

E. Number dial (telephone dial)

1. Used to add and subtract on the number line or a numeric counter

    a. Results indicated on lighted number line and/or numeric counter
    b. Number line connected to dial through stepping unit

        1) Dial pulses activate stepping unit
        2) Stepping unit has no zero reset (reset by subtraction)

2. Numbers subtracted from zero appear as light flashes

    a. Lamp located next to readout window
    b. Number line and/or counter remain at zero

F. Numeric readout (counter)

1. Located above and behind number dial
2. Two digit number viewed through window--reads 0-99
3. Numbers are added and subtracted by pulses from dial which rotate numbered wheels, changing number viewed in window

G. Add-subtract switches

1. Located next to number dial
2. Labeled + (plus) and - (minus)
3. Set by operator to select operation desired when using number dial
4. Face of selected plus or minus switch illuminates when pressed, indicating operation which will result

Experimental Design

A pretest, posttest control group design (Campbell & Stanley, 1963) was employed to study gain in mathematics and conservation abilities as a consequence of introductory mathematics instruction based on linear measurement:

$$\begin{array}{llll} R & M_1, C_1 & X & M_2, C_2 \\ R & M_3, C_3 & & M_4, C_4 \end{array}$$

R = randomization

M_1-M_4 = mathematics tests

C_1-C_4 = conservation tests

X = 17 day mathematics instruction period

The Pre- and Posttest Measures

Two dependent measures were employed in this study: a test of mathematics ability and a test of conservation ability. These measures were administered as pre- and posttest measures to the 12 children. The children were ranked and matched in pairs according to their mathematics pretest scores. The matched pairs were then randomly assigned to the control and experimental treatment groups. Matching was necessary because the sample was small and near equal mathematics ability between the two groups could be assumed only by matching. The pre- and posttests were administered on the seven school days prior to and the five school days following the 17-day instructional period. Repeated measure gains were tested by Sandler's A test, a t-test for correlated data (Sandler, 1955). These gains were also examined by the omega square statistic.

The tests were administered by a female graduate student who was experienced in standardized test administration. She did not know the treatment origin of the subjects. The mathematics test and the conservation test were administered in two separate sessions. The 61-item mathematics test, prepared by the authors as a means of assessing general mathematics ability, presented material different from that contained in the instruction. In this way, the Ss were not merely tested on the material from which they had received instruction but on application of more general mathematics concepts. The test covered recognizing and writing numerals, counting objects, determining numeral order, adding and subtracting (with objects and with paper and pencil), determining which is longer and shorter, and measuring linear distance. The test was longer than available standardized numbers tests.

The Concept Assessment Kit--Conservation, Forms A and C--24 items (Goldschmidt & Bentler, 1968) were administered as a measure of conservation abilities. Conservation scores were determined by (a) the objective correctness of S's response with respect to E's manipulations and (b) the appropriateness of his explanation for that response.

Procedure

The mathematics instruction was carried out on an individual basis to control and record the instructional interchange. Each individual session was started by having S perform again the one or two exercises learned in previous sessions, thereby the S was provided with additional, spaced practice and the E was given an opportunity to judge retention of these abilities from session to session. This evaluation afforded a means of providing practice where needed.

The initial operations, using common objects as units, were designed to teach "measurement" and "unit of measure." By using objects with which children are already familiar, and frequently changing such objects, the learner was intended to grasp the arbitrary and abstract meaning of "unit of measure." As the instruction developed, sequential tasks underwent slight changes to involve new behaviors. In order to further S's measurement abilities and develop in him the capacity to generalize his learnings a number of exercises were provided at each level incorporating a variety of objects and units. A description of the treatment levels follows.

Measurement exercises. The first exercise was that of measuring a table with a book, and presented a technique basic to subsequent forms of measurement. The E placed the book at the edge of the table, marked the table at the inside or forward edge of the book, and, lifting it, said: "That's one." The book was then advanced one "book-unit," placing the trailing edge of the book at the mark and again a mark was placed at its forward edge as the E said: "That's two." This was continued until the entire table length was traversed. Subjects were required to repeat this exercise. Additional demonstrations or other assistance was provided to those Ss unable to perform unassisted. The subsequent measurement exercises were conducted with other measured objects and new units of measure, eventually incorporating the subsequent treatment levels. The S's performance was typically initiated with E asking, for example: "How many erasers will it take to cross this desk?" This form of question was used throughout the various measurement tasks. The initial measurement exercises were conducted with the following objects used as unit measures: paper boxes, pencil, eraser, metal clips, chalk, strips of paper and a pen ranging in length from two inches to nine inches. Objects measured were: desk, table, blackboard, 8" box, 5" eraser, cigar box, a vertical shelf support, packages of paper, and a 30" box. At this level, as well as later when dowel segments were used as measuring units, all combinations of units and measured objects were preselected so that measurements would yield whole numbers. However, slight errors in measuring technique necessitated rounding off to the nearest whole unit on many exercises. Once S had correctly and independently performed three consecutive correct measurement exercises during the second session he was introduced to the use of multiple units and the use of the wooden dowel segments as measuring units.

Measurement using multiple arbitrary units and wooden dowel segments. The intent at this point was to exploit the learner's previously acquired skills to aid in instructing him on the use of the electro-mechanical device described earlier. To accomplish this, S was presented with a quantity of unit measures of the type used in previous exercises. For example, he was given a stack of erasers and asked to measure the desk again without using the marking pencil. By placing the erasers end to end this task was performed with a minimum of assistance from E. After S alternately demonstrated the ability to measure several objects using the repeated unit and multiple unit methods, the dowel segments were introduced.

The segments were first compared in length with the common units of book, shoe, etc., used previously. For example, S measured the desk with an eraser, then found a dowel segment equal in length to the eraser and measured the desk again, this time using the segment as a unit. Several objects were remeasured using the original arbitrary-object units and then with dowel segments of length equal to the arbitrary units. These measurements were performed with both the repeated and multiple unit methods. By alternating dowel segment measurement with the arbitrary object measurement, S became familiar with the use of the segments as multiple measuring units, developing a transferable skill necessary to measurement on the electro-mechanical device. Before proceeding further, S was required to demonstrate his competence when E asked: "How many of these red blocks (dowel segments) will it take to cross this book?" Generally S attempted a guess. When asked how he could determine the exact number he responded, "measure it," and proceeded to do so.

Measurement on the device using no projected numerals. At first the device projected a horizontal column of light on the number line which was parallel to and equal in length to the rods placed in the track. In this way the previous measurement tasks could be repeated with the addition of an illuminated line paralleling the series of rods which provided the child with a linear analogue in photic form. From the visual display he could interpret continuity and represent that continuity by various sized units of measure. The S became familiar with the operations of the device through performance of measuring tasks acquired previously but with new forms of information available to him from the responses of the device. Tangible objects could be measured on the device. These objects had to fit between the track and the vertical surface displaying the illuminated line. Pencils, pieces of rope, pens, stick and paper strips were all sufficiently narrow as to fit within the space.

Measurement on the device was demonstrated in the following manner: the object to be measured was placed behind and parallel to the track, its left end aligned with the start of the track. A sliding marker was then aligned with the right end of the object. The dowel segments were placed in the track one at a time, noting the number verbally as each was placed. The operation of the parallel light column was pointed out and explained as it responded. The segment placement was continued until the object was traversed and the ends of the rod column, the object, the slide, and the light column were linearly coterminous. S then repeated the exercise with any necessary help from E. Several additional exercises with a variety of objects and segment sizes were then performed with each S completing a minimum of three consecutively successful exercises before advancing to the next phase.

In preparation for a next learning step, the Ss were given practice at visual numeral recognition, something they could not already do. Numerals were presented in random order in two groups, 1-10 and 11-20, with each numeral printed on a separate card.

Measurement on the device using projected numerals. This phase of the instruction provided the addition of numeral projections on the number line. As before, S was familiar with the basic operations. One-inch dowel segments painted red were used when the numeral projections were added to the lighted horizontal column. Each of these dowel lengths advanced the number line one numeral.

The Ss repeated several measurements, noting the numerical response of the number line to each rod placement. This step was considered to aid transfer from the measurement concepts already developed to the use of the visual symbols. That the number line also responded by the disappearance of numerals as segments were removed was also noted by the learner. Soon the zero symbol was introduced to the subject by projecting a zero symbol when the last rod was removed. The concept of zero was explained in relation to the absence of rods and the fact that the symbol was projected to the left of the number line, the measured object, and the track. In this way, zero was established at the left terminous of measured linear distance.

Once S displayed an ability to perform prior exercises the task was again modified slightly. The slide was moved to the forward end of the measured object as before, the object was removed and S was required to measure the distance to the slide. The object was then returned to the measuring track to confirm that it was equal in length to the rod column. S performed exercises to accomplish the abstract generalization of measurement without the need for a physical object to be measured. It was pointed out to S that measuring the distance to the slide was like measuring an object; that the operation was the same. This ability was considered necessary to the accomplishment of later tasks.

Addition and subtraction of linear segments. The next task was to teach the subject addition and subtraction of linear segments. No attempt was made to teach the labeling terms themselves but to use these terms while illustrating the respective operations. To accomplish this an object was placed on the device and measured by S. Another, longer object was then added, the slide adjusted to it, and the S asked: "How many more blocks will we have to add to these (those already in the track used to measure the first object) to measure this?" The operation was reversed to illustrate subtraction. Several related exercises were provided to insure significant learning. Once the S was familiar with addition and subtraction in this form the exercises were altered by eliminating the second measured object. The exercise then consisted of adding and subtracting dowel segments while changing the location of the slide. Finally, S added and subtracted segments in response to changes in the location of the slide.

Measurement on the device with only the last number projected. Since in regular numerical usage the entire preceding sequence of numbers is not presented, the device was equipped to eliminate the column of numerals, thus approximating more closely this common usage. In this mode only the numeral corresponding to the last dowel segment placed remained projected, causing the antecedent numeral projection to disappear. The appropriate numeral projections advanced in relation to the rod placements, maintaining the established spacial orientations on the continuum but without the illumination of the entire column of numerals. As the rods were removed the device responded by continuing to project only the numeral corresponding to the last rod in the remaining series. As the last rod was removed the zero projection appeared.

To facilitate transfer of the skills acquired in the prior exercises the Ss were given practice performing measurement exercises alternating the old and new forms of presentation. The operations of addition and subtraction, as described above, were continued with the single numeral projections only. A few initial exercises utilized objects, but this practice was soon discontinued.

At this point in the instruction, the exercises required addition and subtraction operations in response to changes in the slide position and provided only single numeral projections. Establishment of this form of exercise provided a facile transition to a new source of change on the numeral projection: the number dial. Having accommodated the change from measuring physical objects to measuring only a corresponding linear distance the subject was next exposed to a still higher order abstraction; he dialed numerals representing linear distance, each numeral dialed provided a linear change equivalent to the addition or subtraction of one red dowel segment.

Exercise using the number dial. The function of the number dial was demonstrated to S. The one was dialed three times and the mechanical response of the dial and photic response of the number line was pointed out to S. A comparison to the corresponding rod manipulation was shown, i.e., adding three rods one at a time gave the same result as dialing three ones. The plus and minus buttons were presented in relation to their respective operations. The plus was demonstrated as causing the dialed values to affect the lighted number line the same way as when adding more rods to the track. The minus was identified with the opposite manipulation. To provide practice and facilitate transfer from the previously acquired abilities a series of addition and subtraction exercises were performed where E added or removed rods, one at a time, duplicating the prescribed operations that S performed using the number dial. As each one was dialed S noted verbally the value reached on the numberline and E made the appropriate adjustment to the rod column. To further this transfer, E then performed several operations using the dial and S was required to observe and duplicate these operations using the rods. Some of the earlier operations were interpolated to review them and to permit further generalization from the most concrete measurements to the more abstract ones.

Once this review was completed, numbers were dialed in multiples, rather than singly. This was demonstrated by singly adding three rods and dialing the one as each was placed. E then demonstrated that three rods could be added at one time, yielding the same result. Rod manipulation was accompanied by the appropriate dial operation; i.e., dialing the three. The dialing of multiple numbers was then practiced by S duplicating the multiple rod placements of E. These exercises utilized the operations of addition and subtraction. Two Ss were shown how the counter read-out could be operated but little expository use was made of the counter component due to lack of time.

Length of training sessions. Training sessions averaged 27 minutes per subject and had a range of 12 to 51 minutes. Table 2 contains information which summarizes the amount of time spent at the various levels of instruction. Instruction was given over 17 successive school days from 8:30 to 11:00 a.m. The mathematics and conservation tests were administered on the seven school days preceding and on the five school days following the instruction.

TABLE 2

Sequence of Measuring Exercises and Time Spent at Each

Exercise Sequence	Average Minutes
Initial exercises measuring various common objects	37.6
Exercises with multiple units of measure: common objects, dowels	25.3
Machine exercises with dowel segments and lighted column	41.7
Machine exercises with dowels and a series of projected numerals	60.5
Machine exercises with dowels and single numeral at terminous of lighted column	23.0
Machine exercises--addition and subtraction, first with dowel segments, then number dial	49.5
Total average time in minutes	237.6

Results

Mathematics test. Table 3 presents the means and standard deviations obtained from the 61-item mathematics test. The data evidence a 15 percent gain in performance in the treatment sample. That gain can be relied upon at the .02 level of confidence as demonstrated by Sandler's A test; i.e., a correlated t test (Sandler, 1955). As the sample size was small, the assumption of equal variances in the pre- and posttest measures was tested by a t-ratio for differences between correlated variances (Guilford, 1965). The resulting t-ratio was a common one ($t = .124$, $p > .2$), which supports the interpretation that the data meet the assumption of equal variances.

There was a 10.5 percent gain in the mathematics performance of the untreated group, but Sandler's A test demonstrated that gain to be non-significant ($p > .05$).

While the mathematics test contains various subject matter, the obtained Kuder-Richardson-20 values show it to give internally consistent results. The KR-20 values ranged between .87 and .93 (see Table 3).

TABLE 3

Mathematics Test and Conservation Test Performances of Treatment and Comparison Groups

Group	Pretest		Posttest	
	Mathematics	Conservation	Mathematics	Conservation
Treatment	$\bar{X}$ = 32.67 SD = 8.12 KR-20 = .93	$\bar{X}$ = 3.17 SD = 2.14 KR-20[a] Form A = .92 Form C = .92	$\bar{X}$ = 37.66 SD = 8.33 KR-20 = .93	$\bar{X}$ = 6.50 SD = 3.83
Comparison	$\bar{X}$ = 33.33 SD = 5.92 KR-20 = .87	$\bar{X}$ = 7.83 SD = 7.36	$\bar{X}$ = 36.50 SD = 7.09 KR-20 = .91	$\bar{X}$ = 6.17 SD = 6.21

[a]Publisher's Manual.

Conservation test. Table 3 presents means and standard deviations obtained from the 24-item conservation test. Significance of the 105 percent gain in conservation for the treatment group was demonstrated by Sandler's A test. These gains are reliable beyond the .05 level of confidence. As with the mathematics test, the variances of the pre- and posttest measures were tested for homogeneity by a t-ratio for differences between correlated variances. The resulting t-ratio ($t = 1.02$, $p > .2$), supported the assumption of equal variances.

Posttest conservation performance of the comparison group was lower than pretest performance. Sandler's A test demonstrated the change in the untreated sample to be non-significant ($p > .1$).

Importance of the treatment effects. Treatment conditions were imposed for only three weeks. As three weeks is a relatively short time in which to affect mathematics and conservation competence, the gains were examined by the omega square technique to ascertain what proportion of the total variance measured can be assigned to the treatment effects (Hays, 1963). The resulting omega square value for the mathematics test was .46, a relatively large value. A value of .37 was obtained for the conservation test.

Discussion

This research was not concerned with a critical test of the effects on behavior of one or two variables. Rather, it represented a field trial of an aggregate of pedagogical concepts having a complex rationale. The central difficulty in studying complex learning processes in the school is that the form of the behavior differs progressively and identification of functionally related independent variables is an uncertain matter. On the other hand, learning hierarchies studied in the laboratory (Gagné, 1962, 1968) ought to correspond in significant detail with those more complexly associated behaviors in real life. The present study does support that expectation.

We think there is cause for further development of a pedagogy of measurement for the early school years that will replace some of the content to which children are presently exposed. Concrete experiences that are partial or complete analogues of abstractions the child will be required to learn later is appropriate content. Mastery of a sizable amount of varied measurement experience should lead to integration of mathematical concepts. By integrated mathematical experience we mean that a transfer base has formed which will aid the child's subsequent learning; perhaps for as long as he continues to study mathematics.

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Abstract

Mathematics learning was introduced to kindergarten children by the operations of linear measurement. Arbitrary units of measure were used to teach the child "unit of measure" as a power, addition and subtraction by the rule n ± 1, and the application of these concepts to measurement of continuous variables. All instructional exercises were based in measurement and this common context for various concepts was intended to provide a transfer base for successive new concepts. Both an arithmetic concepts test of 61 items and the Concept Assessment Kit--Conservation, administered on entering and leaving, gave evidence of significant gains in the treated sample but not in a control sample.