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ABSTRACT

In this report four models are formulated which forecast the enrollment and financial needs of students in higher education. The four models are: the undergraduate enrollment model, postbaccalaureate enrollment model, undergraduate student aid model, and postbaccalaureate student aid model. In addition to computing total financial needs, these models can be used to estimate the costs of alternative Federal aid programs. The amounts of Federal loans and grants given to undergraduates are reported by sex, income, and academic year. In estimating the parameters for the undergraduate and postbaccalaureate models, data from several national surveys of high school and college students were used. The estimation procedures are explained in the appendices. (JG)

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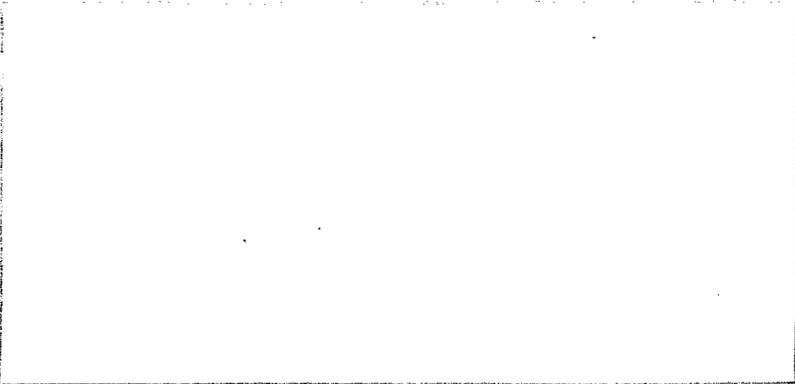
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ENROLLMENT AND
FINANCIAL AID MODELS
FOR HIGHER EDUCATION

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FINAL REPORT - PHASE II

by

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1. Introduction

In this report several models are formulated which forecast the enrollment and financial needs of students in higher education. There are four models altogether: the undergraduate enrollment model, postbaccalaureate enrollment model, undergraduate student aid model, and postbaccalaureate student aid model. Postbaccalaureate students include both graduate and first-professional students. Models were developed separately for undergraduate and postbaccalaureate students, because the student characteristics and available data are different for these two groups. These models are described briefly in this section.

In forecasting financial aid requirements for undergraduate students, it is necessary to know more than just aggregate enrollment projections. It is necessary to have enrollment projections by family income, because the parental contribution towards college expenses is the principal income source for most undergraduate students; and it is necessary to have forecasts by institution type and control, because the student college expenses will be dependent upon these variables. In this report, the control refers to either public or private, and the type refers to two-year colleges, universities, and all other four-year institutions. Figure 1.1 illustrates the undergraduate enrollment and student

STUDENT ENROLLMENT
MODEL

1. Estimate the number of high school graduates by sex and family income

2. Estimate the probability distribution for the number of years a student waits after high school graduation before first time college enrollment

3. Estimate the probability that a student is enrolled in various types of institutions in years following first enrollment

4. Estimate the total enrollment by sex and family income in various types of institutions

5. Estimate the expected contribution from parents and students by family income

7. Estimate the financial aid requirements for students in higher education

6. Estimate the expenses incurred in various types of institutions

STUDENT AID
MODEL

Figure 1.1:
Undergraduate Student Enrollment and Aid Models

aid models. The first step is to estimate the number of high school graduates by sex and by family income. The second step is to estimate the number of years a student in a particular sex and income group waits before enrolling for the first time in college. The first time freshmen enrollment can be estimated with the data from these first two steps. The third step is to estimate the probability that a student is still enrolled as an undergraduate during years following first enrollment. Since attrition rates differ for male and female students and for students in different income groups, these possibilities should be estimated separately by sex and by income. The distribution of students by type and control of institution is also determined in step three. Step four combines the data from the first three steps to estimate the total expected enrollment by sex, family income, and by type and control of institution. The first four steps represent the undergraduate enrollment model. The enrollment forecasts serve as an input to the financial aid model. The fifth step requires an estimate of the parental and student contribution to college expenses. The parental contribution depends on the total number of dependent children in the family, the total number of dependent children attending college from the family, and the family income. The student contribution is from summer earnings and student assets. The sixth step is to estimate the student expenses incurred, by type and control of institution. These expenses include tuition, fees, room, board, books, etc. Cost

estimates are made separately for resident and commuting students. Step seven combines the data from the previous steps to estimate the financial aid requirements of undergraduate students. Steps five through seven represent the student financial aid model.

The undergraduate enrollment model is formulated in Section 2, and the undergraduate financial aid model is formulated in Section 4. Enrollment and student need projections through 1975-76 are given in these sections.

In addition to computing total student financial needs, these models can also be used to estimate the costs of alternative Federal aid programs. Two examples are given in Section 4 to illustrate this use. The first example is for the following hypothetical aid package:

- (1) The first \$400 of the aid needed by a student would be met with Federal subsidized loans, similar to the National Defense Education Act (NDEA) loans;

- (2) Federal grants, either Educational Opportunity Grants (EOG), or College Work-Study Program (CWSP), not exceeding the difference between \$1,000 and the family contribution, would then be used to meet additional need; in other words, the maximum grant would be \$1,000 less the parents' contribution.

- (3) The remaining need would be met by State, institution, and private sources.

The amounts of Federal loans and grants given to undergraduates under this program are estimated in Section 4 by sex, income, and academic year. As a second example, we also examined this program:

(1) A student would be entitled to the maximum EOG payment consistent with the following conditions: the grant must not exceed \$1,200 less the family contribution; it must not exceed the student's need; it must not exceed 50% of the college expenses; and any positive payment must not be less than \$200.

(2) The remaining need would be met by State, institution, and private sources.

The conditions in this example may be more complex than would be considered in practice by the Federal government. Nevertheless, this example does illustrate the versatility of the model. The amounts of Federal grants under this second program are also estimated in Section 4 by sex, income, and year.

Currently, both the undergraduate enrollment and financial aid models are programmed on a time-sharing interactive system. This allows the user to determine:

(1) The impact of changes in enrollment model parameters (such as enrollment or attrition rates) on the financial need of students;

(2) The impact of changes in financial aid model parameters (such as student contribution from summer employment) on the financial needs of students;

(3) The costs of alternative Federal aid programs.

The postbaccalaureate enrollment and student aid models are illustrated in Figure 1.2. These models are different from the undergraduate models for several reasons:

(1) None of the available national student surveys included sufficient years to allow the attrition rates and other parameters to be determined in the way that was done for undergraduate students.

(2) There is evidence that postbaccalaureate study is best characterized as being capacity limited. The type of model used for projecting undergraduate enrollment works best when the enrollment is input or demand limited.

(3) The principal advantage of the undergraduate enrollment model is that it is able to forecast enrollment by parental income, while the postbaccalaureate model does not. The undergraduate student aid model converts the enrollment projections into estimates of financial aid requirements on the basis of the family income distribution. However, there is no generally accepted need analysis model at the postbaccalaureate level, as there is for undergraduates. Consequently, even if

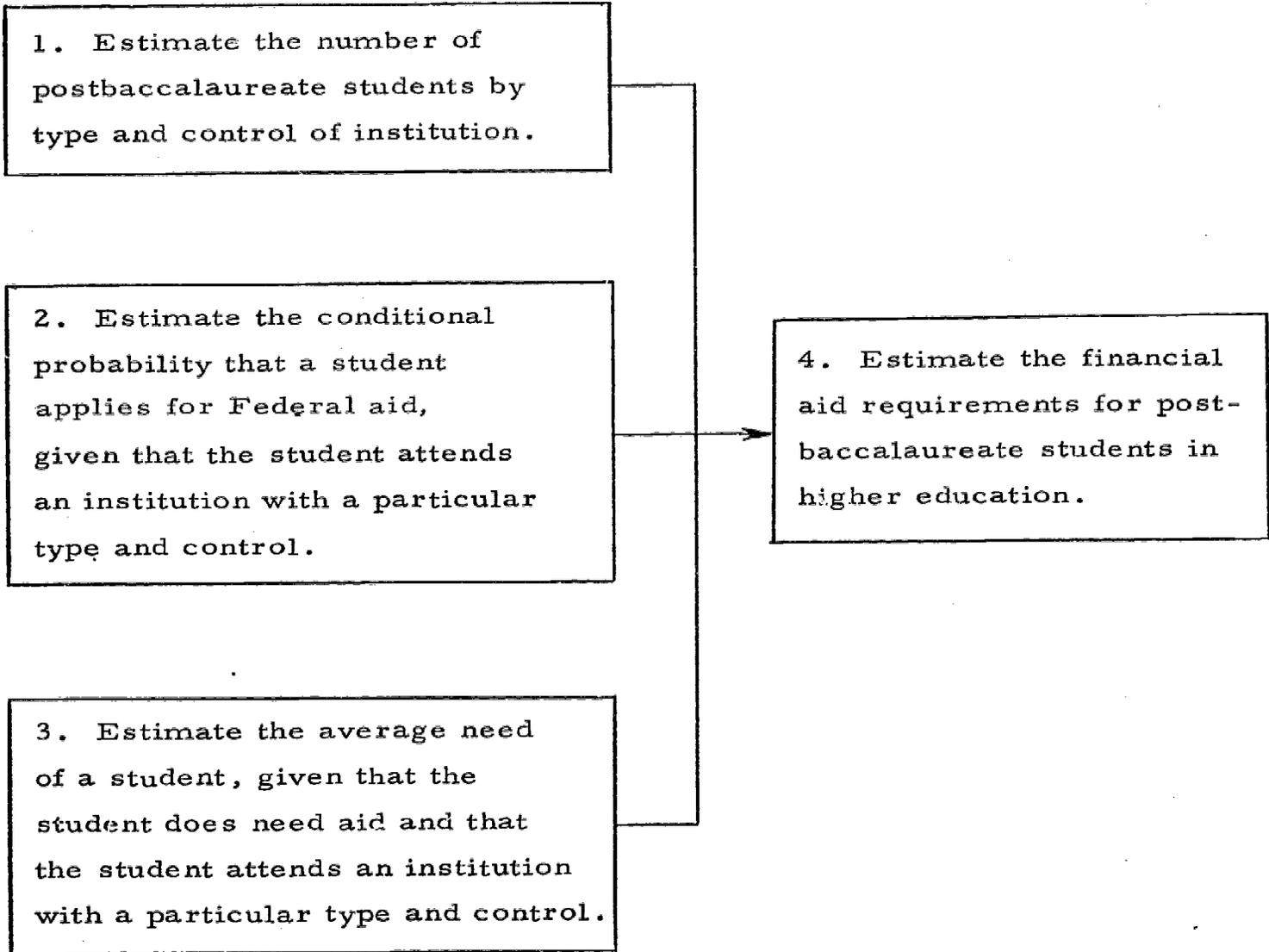


Figure 1.2:
Postbaccalaureate Student Enrollment and Aid Models

postbaccalaureate enrollment projections by family income were available, there would be no straightforward way of converting these estimates into projections of financial aid requirements.

Because of these differences, we have formulated different types of models than those used for undergraduate students. The postbaccalaureate enrollment model is represented as step one in Figure 1.2. This model simply projects trends in historical enrollment data to forecast enrollments by type and control of institution. The student aid model is represented by steps two, three, and four. Step two estimates the conditional probability that a student will apply for Federal aid, either from the National Defense Student Loan Program or from the College Work-Study Program. These probabilities are estimated by type and control of institution. The third step estimates the average need of a student, given that the student does apply for Federal aid. These average needs are also estimated by type and control of institution. And step four combines the data from the previous steps to estimate the financial needs of postbaccalaureate students who apply for Federal aid programs.

The postbaccalaureate enrollment model is formulated in Section 3, and enrollment projections through 1975-76 are given. The postbaccalaureate financial aid model is formulated in Section 5, and estimates of the need for Federal aid programs, either National Defense Student Loans or College Work-Study, are made through 1975-76.

In estimating the parameters for the undergraduate and postbaccalaureate models, we used data from several national surveys of high school and college students. Because these surveys were not entirely adequate for developing these models, some of our parameter estimates are only approximate. The characteristics of the longitudinal surveys that were used are described in Appendix A.1. In Section 6, we made recommendations on how future enrollment data should be collected so that more accurate parameter estimates can be made in the future.

2. Undergraduate Enrollment Model

The undergraduate enrollment model forecasts total undergraduate full-time enrollment in higher education by sex, family income, and institution type and control. In this section the model is formulated, and enrollment projections are given through 1975-76. These enrollment projections serve as an input to the undergraduate financial aid model discussed in Section 4. Refer to Appendix A for a detailed description of the data sources and estimation procedures used in calibrating this model. We used data from several national surveys of students, including the Project TALENT surveys, the American Council on Education (ACE) surveys, the ACE-Carnegie Commission on Higher Education (ACE-CCHE) follow-up surveys, the Bureau of Census-Columbia University (BC-CU) surveys, and the National Center for Educational Statistics (NCES) surveys. Unfortunately, all of the available longitudinal surveys had limitations of one kind or another: the Project TALENT, ACE, and ACE-CCHE follow-up surveys suffer from poor response rate and income information, while the BC-CU follow-up surveys had small sample sizes. The characteristics of these surveys are discussed in Appendix A.1. This enrollment model is similar to the one formulated by Pfeferman⁽¹⁾; the differences between these models are due to the fact that more recent data sources were available to us. Some of these differences are:

(1) Pfeferman's model forecasts enrollment by aptitude groups as well as by sex and income. The aptitude measure used was the score on Project TALENT's standardized test. We investigated using the student's self-reported high school grade average as an achievement measure, since this measure was used by the ACE, ACE-CCHE, and BC-CU surveys. However, this measure appeared to be unreliable for two reasons: many students did not know what their average was; and the grading standards of high schools vary considerably. As a result, this measure was not used.

(2) Our model forecasts enrollment by type and control of institution, while Pfeferman's does not. Also, our model forecasts undergraduate full-time enrollment directly, while the model in Ref. 1 first estimates total enrollment (full-time and part-time, undergraduate and postbaccalaureate) and then estimates undergraduate full-time enrollment using historical ratios.

The enrollment model is represented by the first four steps in Figure 1.1. The first step is to estimate the total number of high school graduates in each year by sex and family income. Let

$G_{s\tau}$ = the number of students with sex s who graduate from high school during the academic year beginning with year τ .

For example, if a student graduates from high school during the 1959-60 academic year, then $\tau = 1959$. Estimates and projections of $G_{s\tau}$ were

made by NCES^(2,3) and are given in Table A.1 in Appendix A.2.

Define

d_s^i = the conditional probability that a high school graduate is in the i th family income group, given that the student has sex s .

In our notation for conditional probabilities, we always use the convention that the superscripts define the random event and the subscripts define the conditioning event. The income classification, used in defining d_s^i and other parameters, refers to the family income distribution for graduating high school seniors. Our method for projecting this income distribution is described in Appendix B.2. Data were used from the first BC-CU⁽⁴⁾ follow-up survey and from several Bureau of Census publications⁽⁵⁻¹⁰⁾. Because of inflation, the income levels in one year are not comparable to the income levels in another year. Thus, the income index i will refer to quartiles, rather than specific income intervals. We use the notation: $i = 1$ corresponds to the first or low income quartile; $i = 2$ corresponds to the second income quartile; $i = 3$ corresponds to the third; and $i = 4$ corresponds to the fourth or high income quartile. According to BC-CU⁽⁴⁾ data, the values of d_s^i are approximately the same for male and female students; thus we will use $d_s^i = .25$ in this report.

The second step in the enrollment model is to estimate the probability distribution for the number of years that a student waits after high school graduation before first time college enrollment. The enrollment model will be designed to forecast opening fall enrollment (rather than

winter or spring), so that the model will be compatible with the opening fall enrollment surveys conducted by ACE, NCES, and the Bureau of Census. Since colleges have differing numbers of terms in an academic year (semester, trimester, or quarter systems), it would not be feasible to forecast enrollment by term on a national basis. Define

$h_{si\tau}^n$ = the conditional probability that the first fall in which a student is enrolled is during the n th year following high school graduation, given that the student graduated from high school during the academic year beginning in year τ , has sex s , and has family income i .

The following convention is used in the above definition: if a student graduates in June and enrolls during the following fall, then $n = 1$; if he graduates in June, first enrolls in the following spring, and continues to be enrolled during the second fall following graduation, then $n = 2$; etc. Since some students will never enroll in college,

$$\sum_n h_{si\tau}^n < 1 .$$

In Appendix A.3 data were used from Project TALENT⁽¹¹⁾, BC-CU⁽⁴⁾, Bureau of Census^(12,13), ACE⁽¹⁴⁾, and NCES^(3,15) to estimate this parameter.

The third step in the enrollment model is to estimate the probability that a student is enrolled in various types of institutions in years following first enrollment. Define

R_{si}^k = the conditional probability that a student is enrolled as a full-time undergraduate during the k^{th} fall after first fall enrollment, given that the student first enrolled full-time, has sex s , and has family income i .

Again, this definition refers to enrollment during the fall term. We allow the student to drop out and return to college, and thus this definition does not require the student to be enrolled for k consecutive years. The following convention is used: if a student first enrolls in fall 1970, then $k = 0$ refers to fall 1970; $k = 1$ refers to fall 1971; $k = 2$ refers to fall 1972; etc. A student may be enrolled as an undergraduate during the fourth or fifth falls following first fall enrollment, if he drops out for one or more terms, changes majors, enrolls in a five year bachelor's program, etc. Thus we will estimate R_{si}^k for $k = 0, 1, 2, \dots, 5$. Because estimates for R_{si}^k are affected by a poor follow-up response rate to a longitudinal survey, in Appendix A.4 we compared estimates that were derived from several surveys, including ACE-CCHE⁽¹⁶⁻¹⁸⁾, ACE⁽¹⁹⁾, and BC-CU^(4,20,21). There were some variability in these estimates. The values used in the enrollment projections in this report were chosen to be consistent with both the range of estimates from the longitudinal surveys and with the 1970 NCES⁽¹⁵⁾ enrollment data.

It is also desirable to estimate the enrollment by level and by type and control of institution. Define

π_{sik}^{ℓ} = the conditional probability that a student has attained to level ℓ (either lower division or upper division), given that the student has sex s , family income i , and is enrolled full-time during the k^{th} fall following first fall enrollment,

and

$\alpha_{sil}^{c\theta}$ = the conditional probability that an undergraduate student is enrolled in an institution with control c (either public or private) and type θ (either two-year, four-year college, or university), given that the student has sex s , family income i , and is enrolled at level ℓ .

In these definitions, the index ℓ refers to either lower division (freshman, sophomore) or upper division (junior, senior). The control c refers to public or private. The Office of Education classifies four-year institutions into two groups: universities and "all other four-year colleges." We refer to the second group as four-year colleges. According to Ref. 22, the four-year institutions which are classified as universities are those which give considerable stress to graduate instruction, which confer advanced degrees as well as bachelor's degrees in a variety of liberal arts fields, and which have at least two professional schools that are not exclusively technological. Consequently, the index θ in the definition for $\alpha_{sil}^{c\theta}$ refers to either two-year colleges, four-year colleges, or universities, and these are mutually exclusive categories. In Appendix A.5, π_{sik}^{ℓ} is estimated from ACE-CCHE^(16,17). And in Appendix A.6, $\alpha_{sil}^{c\theta}$ is estimated from ACE-CCHE⁽¹⁶⁾, ACE⁽¹⁴⁾, and NCES⁽¹⁵⁾.

After the foregoing parameters have been estimated, the total undergraduate enrollment can be projected by sex, family income, and institution. Define

$S_{sit}^{kc\theta}$ = the expected number of undergraduates with sex s and income i who are enrolled full-time in an institution with control c and type θ during the fall of year t and the k^{th} fall following first fall enrollment.

It follows from the previous definitions that

$$(2.1) \quad S_{sit}^{kc\theta} = \sum_n \sum_l h_{si,t-k-n}^n G_{s,t-k-n} d_s^i \pi_{sik}^l \alpha_{sil}^{c\theta} .$$

Projections of $S_{sit}^{kc\theta}$ are used as the input to the undergraduate financial aid model described in Section 4. And finally, let

E_{it} = the expected number of undergraduates with income i who are enrolled full-time in the fall of year t .

By summing $S_{sit}^{kc\theta}$ with respect to the indices s , k , c , and θ , E_{it} is computed as follows:

$$(2.2) \quad E_{it} = \sum_s \sum_k \sum_c \sum_\theta S_{sit}^{kc\theta} \\ = \sum_s \sum_k \sum_c \sum_\theta \sum_n \sum_l h_{si,t-k-n}^n G_{s,t-k-n} d_s^i \pi_{sik}^l \alpha_{sil}^{c\theta} .$$

Estimates of E_{it} , computed from Eqn. (2.2), are given in Table 2.1 for years 1967 through 1975. Also given in Table 2.1 are historical enrollments

Table 2. 1:

Undergraduate Full-Time Fall Enrollment,
By Year and Income Quartile

(In Thousands)

Year	Estimated					Actual	Percent Error
	Low Quartile	2nd Quartile	3rd Quartile	High Quartile	Total		
1967	611	923	1,290	1,708	4,532	4,377	3.5%
1968	673	966	1,358	1,804	4,801	4,779	.5%
1969	735	1,003	1,416	1,886	5,039	4,993	.9%
1970	798	1,038	1,473	1,970	5,280	5,280	0.0%
1971	859	1,081	1,539	2,064	5,544	-	-
1972	916	1,127	1,608	2,162	5,814	-	-
1973	965	1,173	1,675	2,255	6,068	-	-
1974	1,005	1,216	1,738	2,340	6,301	-	-
1975	1,041	1,257	1,797	2,418	6,513	-	-

Source: Actual values are from Refs. 15 and 22-24; estimated values were computed from Eqn. (2.2).

from NCES^(15,22-24) for years 1967 through 1970. Prior to 1967, the NCES fall enrollment surveys did not classify students as undergraduates or postbaccalaureates.

In this report, the enrollment model is used only to provide input data for the student aid model formulated in Section 4. Because this model attempts to simulate the actual flow of students through education, it could also be employed as a useful tool for studying the impact of changes in student behavior on college enrollment. For example, Froomkin⁽²⁵⁾ used the previous version of this model to estimate enrollment if all students entered college at the same rate as students from the highest income quartile.

3. Postbaccalaureate Enrollment Model

In this Section we describe the postbaccalaureate enrollment model. A different type of model will be formulated than that used for undergraduate students for several reasons:

(1) None of the available national longitudinal student surveys included sufficient years to allow the attrition rates and other parameters to be determined for postbaccalaureate students in the way that was done for undergraduate students.

(2) According to the Chronicle of Higher Education⁽²⁶⁾, several graduate schools are limiting or cutting back on graduate enrollment. In addition, applications for law or medical study far exceeds available space. In other words, the demand for postbaccalaureate study in these cases exceeds the capacity. The type of model used for projecting undergraduate enrollment works best when the enrollment is input or demand limited, rather than capacity limited.

(3) The principal advantage of the undergraduate model is that it is able to forecast enrollment by parental income, while the model discussed in this Section does not. The undergraduate student aid model discussed in Section 4 converts the enrollment projections into estimates of financial aid requirements on the basis of the family income distribution. However, there is no generally accepted need analysis model at the postbaccalaureate level, as there is for undergraduates. Consequently,

even if postbaccalaureate enrollment projections by family income were available, there would be no straightforward way of converting these estimates into projections of financial need requirements. In Section 5 a financial aid model is formulated which does estimate the aid requirements for postbaccalaureate students, and this model is able to use enrollment projections that are not classified by parental income.

For the first time in 1967, the Office of Education's annual fall enrollment survey classified students into undergraduate and postbaccalaureate categories. The postbaccalaureate estimates in Table 3.1 were taken directly from these surveys for 1967 through 1970. Since 1966, the Office of Education's annual survey of students enrolled for advanced degrees included students who were in programs leading to master's, doctor's and first-professional degrees. First-professional degrees include the first degrees given in law, medicine, theology, etc. However, there are individuals who are not enrolled for advanced degrees, and yet are classified as postbaccalaureate students. The 1966 postbaccalaureate estimates in Table 3.1 were obtained from the 1966 advanced degree enrollments, by using the assumption that the relationship between the 1967 postbaccalaureate and advanced degree enrollments would also be valid for 1966. This procedure could not be used to obtain postbaccalaureate estimates prior to 1966, because during those years the advanced degrees surveys did not include students who were in first-

Table 3.1:

Full-Time Fall Postbaccalaureate Enrollment,
By Year and By Institution Type and Control

Year	Public University	Public Other 4-Year	Private University	Private Other 4-Year
1966	190,781*	29,837*	117,012*	48,123*
1967	224,058	37,840	130,847	57,691
1968	239,935	41,362	132,886	57,510
1969	259,704	50,001	134,091	64,256
1970	262,651	66,739	139,281	69,379
P R O J E C T E D				
1971	289,242	70,945	145,158	74,115
1972	307,180	79,542	149,936	79,023
1973	325,119	88,138	154,715	83,930
1974	343,057	96,735	159,493	88,838
1975	360,996	105,331	164,271	93,746

*Estimated

Source: Refs. 15, 22-24 for 1967-1970; estimated from Refs. 22, 27, and 28 for 1966; estimated from Eqn. (3.1) for 1971-1975.

professional degree programs. Consequently, our enrollment projections will be made on the basis of the 1966 through 1970 data. In making these projections, the enrollment will not be classified by sex for the following reason: many postbaccalaureate institutions responded to the increased draft of men in 1968 by admitting more women; thus the fluctuations due to the draft in the combined enrollment series should be less than that for the male and female series considered separately.

Define

$K_t^{c\theta}$ = the total number of postbaccalaureate students who are enrolled full-time during the fall of year t in institutions with control c and type θ .

The control c refers to public or private, and the type θ refers to universities or other four-year colleges. Two different models were investigated for projecting $K_t^{c\theta}$:

$$(3.1) \quad K_t^{c\theta} = \alpha_1^{c\theta} + \alpha_2^{c\theta} \cdot (t - 1965)$$

and

$$(3.2) \quad \frac{K_t^{c\theta}}{N_t} = \alpha_3^{c\theta} + \alpha_4^{c\theta} \cdot (t - 1965) ,$$

where the $\alpha_i^{c\theta}$ are the calibration constants and N_t is the total population of the United States with ages between twenty-two and twenty-four on July 1

of year t . Estimates and projections of N_t are available from the Bureau of Census (29-31). We found that Eqn. (3.1) provided a substantially better fit than Eqn. (3.2) did. This may be due to the fact that postbaccalaureate enrollment is best characterized as being capacity limited, rather than as input or demand limited. Thus the population variable N_t should not be included in the model.

The constants $\alpha_1^{c\theta}$ and $\alpha_2^{c\theta}$ were estimated using linear regression analysis and are listed in Table 3.2, together with the corresponding values of the multiple correlation coefficient R^2 . The projections of postbaccalaureate enrollment made with Eqn. (3.1) are given in Table 3.1 for years 1971 through 1975. These enrollment projections are the input to the postbaccalaureate financial aid model discussed in Section 5.

Table 3.2:

Calibration Constants

Constant	Public University	Public Other 4-Year	Private University	Private Other 4-Year
$\alpha_1^{c\theta}$	181,610	19,366	116,489	44,669
$\alpha_2^{c\theta}$	17,939	8,597	4,778	4,907
R^2	.963	.966	.908	.968

4. Undergraduate Student Aid Model

The undergraduate enrollment model constitutes Steps 1-4 in Figure 1.1 and forecasts enrollment by sex, family income quartile, and institution type and control. The student aid model represents Steps 5 through 7 in Figure 1.1 and converts the undergraduate enrollment projections into estimates of financial aid requirements. This latter model is essentially a refinement of models formulated in previous studies of student financial need; see for example Refs. 32, 33, and 34. The financial need of a student is a function of several parameters including family income, college expenses, the number of dependent children in the family, and the number of dependent children attending college from the family. None of these parameters are fixed, but will vary from student to student. For the purposes of computing financial need, previous studies have assumed that these parameters were constant for students and colleges within certain groups or categories. The students were classified by family income intervals, and the institutions were classified by type and control. However, these student need parameters are best characterized as being random variables which can assume a range of values. Since the financial need of a student is a highly nonlinear function of these variables, it would be mathematically incorrect to evaluate the expected need of a student using only the mean values for these

variables. Consequently, our approach is to explicitly treat these parameters as random variables and to compute the expected financial need of a student using the distribution functions for these random variables.

In this section the undergraduate student aid model is formulated. Our discussion on how the parameters in this model are estimated will be postponed to Appendix B. Because of the unique requirements of this model, it was necessary to collect data from several sources, and much of these data have not been published previously.

The student aid model is based upon the need analysis method developed by the College Scholarship Service⁽³⁵⁾ (CSS). In their system, the expected parental contribution includes amounts from both parents' assets and incomes. Their approach is to convert the value of the parents' assets into a supplementary income flow on the basis of the age of the principal wage earner and the number of retirement plans that the family has. The supplementary income flow from the parents' assets is then added to the current income in order to compute the adjusted family income. Using data describing consumption patterns of families in the United States, the CSS then converts the adjusted family income into the expected contribution towards the college expenses of a student, as a function of the total number of dependent children in the family, and the number of dependent children attending college. Define

I = the family income of a student,

Δ = the total number of dependent children in the student's family,

λ = the total number of dependent children attending college,

$R_t(I, \Delta, \lambda)$ = the expected parental contribution during the academic year beginning in year t towards the expenses of a student attending college whose family has income I , total number of dependent children Δ , and number of dependent children λ attending college.

In Appendix B.1, the CSS need analysis method is described in more detail and the formula for $R_t(I, \Delta, \lambda)$ is derived. Since we used data from CSS⁽³⁶⁾ to estimate the average contribution from parents' assets as a function of family income, $R_t(I, \Delta, \lambda)$ does include the contribution from assets as well as from income. Because of inflation, we allow $R_t(I, \Delta, \lambda)$ to vary with respect to the year t .

The undergraduate enrollment model forecasts enrollments by family income quartiles, and not by specific income levels. However, the expected parental contribution computed with $R_t(I, \Delta, \lambda)$ will be different for different family income levels within a given quartile. Thus to estimate financial aid requirements, it is necessary to estimate the distribution of family income within each income quartile. Let

$F_{it}(I)$ = the conditional cumulative distribution function (c.d.f.) of family income in year t for students who graduate from high school during the academic year beginning in year t , given that the income falls within the i th income quartile.

Here, $i = 1, 2, 3,$ or $4,$ with $i = 1$ representing the low income quartile and $i = 4$ representing the high quartile. In Appendix B.2, our technique for estimating $F_{it}(I)$ is described. Data were used from the first BC-CU follow-up survey⁽⁴⁾ and from several Bureau of Census publications⁽⁵⁻¹⁰⁾. We will use $F_{it}(I)$ as the family income distribution for both students who are high school seniors in year t and students who graduated in previous years and are enrolled in college. It is important to understand that this procedure involves two approximations:

(1) The real family income of a student may be higher during years after high school graduation than before, because of promotions and raises received by his parents during the interim. Thus, it would be desirable to condition the c.d.f. with respect to the number of years since high school graduation. However, the available national longitudinal student surveys only asked for the family income during the first year in the survey, and thus it is not possible to estimate from these surveys the increase in real income of the parents as the student progresses through college.

(2) The c.d.f. should also be conditioned with respect to the type and control of institution which is attended, since low income students tend to enroll at public institutions rather than private, and at two-year institutions rather than four-year. The error incurred for not doing this will be small, because $F_{it}(I)$ is conditioned with respect to the income quartile $i,$ and because separate enrollment forecasts are made by income quartile and by type and control of institution.

The variables Δ (the number of dependent children in a family) and λ (the number of dependent children attending college) will vary from family to family and will be integer valued. Since these variables are not independent, their variation must be characterized by a joint probability mass function. Also, the distribution for Δ and λ should depend upon the number of years since first enrollment, because as time progresses, more of the children in a family will become self-supporting or enrolled in college. Let

$g_k^{\Delta\lambda}$ = the conditional joint probability mass function of the total number of dependent children Δ and the number of dependent children attending college λ in a family, given that the student is enrolled during the k^{th} fall following first fall enrollment.

The distributions $g_k^{\Delta\lambda}$ were estimated from data supplied by the American College Testing Program ⁽³⁷⁾ (ACT) and are given in Tables B.9 through B.12 in Appendix B.3.

In addition to the parents' contribution, we also assume that the student will contribute a portion of his summer's earnings and a portion of his savings. We allow this contribution to depend upon the sex and family income of the student and the number of years since first enrollment.

Let

$H_t(I, s, k)$ = the expected self-help contribution of a student during the academic year beginning in year t , given that the student is enrolled during the k^{th} fall following first fall enrollment and that the student has sex s and family income I .

The formula for $H_t(I, s, k)$ is derived in Appendix B.4 using data from CSS^(35, 38). According to the CSS system, the contribution from assets for a given year is computed by dividing the total assets by the number of years remaining plus one. Therefore, a prefreshman applicant's assets would be divided by five, a presophomore's assets by four, etc. The "plus one" factor provides the student with funds to begin graduate study or until receiving income from employment. Because of inflation, we allow $H_t(I, s, k)$ to vary with respect to the year t . Note that $H_t(I, s, k)$ should not include funds obtained from loans or school employment, as these are usually considered to be forms of student aid.

Since college expenses will differ for resident and commuting students, it is necessary to estimate the proportion of students in each category. Resident refers to any student living in dormitories, fraternities, sororities, or apartments. Define

$P_{c\theta si}^z$ = the probability that a student attends college with living status z , given that the student is enrolled full-time at an institution with control c and type θ , has sex s and that the student's family income lies in the i^{th} quartile.

The living status z refers to resident or commuter. The control c refers to public or private, and the type θ refers to university, other four-year colleges, or two-year colleges. These probabilities were estimated with data from the ACE⁽¹⁹⁾ one-year follow-up of 1966 entering freshmen, and they are listed in Tables B.15 and B.16 in Appendix B.5.

The college expenses incurred by a student will differ from college to college. The expenses at four year institutions tend to be higher than at two-year institutions, and the expenses at private institutions tend to be higher than at public institutions. Also, the expenses will depend upon the sex and living status of the student. Consequently, the distribution function for college expenses should be conditioned by the type and control of institution, by sex, and by living status. Let

C = the college expenses (tuition, room, board, books, etc.) for a student,

and

$B_{c\theta zst}(C)$ = the conditional c.d.f. of college costs C during the academic year beginning in year t , given that the student is enrolled full-time in an institution with control c and type θ and that the student has sex s and living status z .

In Appendix B.6, the c.d.f. $B_{c\theta zst}(C)$ was estimated using data from NCES (2,39) and from CSS (35).

If the college expenses exceed the resources of the student and his family, then the aid required by the student is positive; but, if the student's resources exceed his expenses, then the aid needed is zero. Thus the total aid required by a student is

$$(4.1) \quad [C - R_t(I, \Delta, \lambda) - H_t(I, s, k)]^+ ,$$

where the symbol $[w]^+$ is defined as follows:

$$[w]^+ = \begin{cases} w & \text{if } w > 0 \\ 0 & \text{otherwise.} \end{cases}$$

In our model, the financial aid requirements in (4.1) is a random variable, since it is a function of five random variables: $C, I, \Delta, \lambda,$ and z . The expected financial aid required may be computed from the distributions of these random variables. Define

$A_{sit}^{kc\theta}$ = the expected financial aid required during the academic year beginning in year t by a student with sex s and family income i who is enrolled full-time in an institution with control c and type θ and who is enrolled in the k th fall following first fall enrollment.

It follows from (4.1) and the foregoing definitions that $A_{sit}^{kc\theta}$ is given by the expression*

$$(4.2) \quad A_{sit}^{kc\theta} = \int_I \int_C \sum_z \sum_{\Delta, \lambda} [C - R_t(I, \Delta, \lambda) - H_t(I, s, k)]^+ g_k^{\Delta\lambda} P_{c\theta si}^z dF_{it}(I) dB_{c\theta zst}(C).$$

* Our notation refers to Riemann-Stieltjes integration, rather than the usual Riemann integration. In the case in which the c.d.f.'s $F_{it}(I)$ and $B_{c\theta zst}(C)$ both have continuous derivatives, then it is possible to replace the Riemann-Stieltjes integrals with Riemann integrals. However, it will be convenient for us to assume that $F_{it}(I)$ and $B_{c\theta zst}(C)$ are piecewise linear, and thus these distributions will not have continuous derivatives. Refer to Bartle⁽⁴⁰⁾ for a description of the Riemann-Stieltjes integral and its relation to the Riemann integral.

It is assumed in this expression that the random variables I and C are independent. This should not be a bad approximation, since enrollment and student aid calculations are made separately by income quartile and by type and control of institution. Numerical integration is used to compute $A_{sit}^{kc\theta}$ from the distributions of the random variables.

It follows from the definition of $A_{sit}^{kc\theta}$ that

$$(4.3) \quad \sum_i \sum_k \sum_c \sum_\theta S_{sit}^{kc\theta} A_{sit}^{kc\theta}$$

is the expected aid needed by full-time undergraduates with sex s during the academic year beginning in year t , where $S_{sit}^{kc\theta}$ is defined to be the total full-time undergraduate enrollment by sex, income quartile, year, and type and control of institution. The values of $S_{sit}^{kc\theta}$ are the output of the undergraduate enrollment model. In Table 4.1 are estimates of the expected need computed from (4.3) for academic years 1970-71 through 1975-76 using the enrollment projections developed in Section 2.

The need estimates in Table 4.1 are probably lower than the true values for the following reason: these estimates assume that the student will receive the parental support entitled to him according to the CSS need analysis formula. To the extent that some students are self-supporting or receive from their parents less than the assumed amount, these figures will underestimate the actual needs of undergraduates.

Table 4.1:

Estimated Financial Needs of Full-Time Undergraduate Students,
By Sex, Income Quartile, and Year

(In Thousands)

Academic Year	Sex	Low Quartile	2nd Quartile	3rd Quartile	High Quartile	Total
1970-71	Male	\$511,572	\$318,249	\$201,289	\$78,749	\$1,109,856
1970-71	Female	295,119	243,060	161,716	75,045	774,940
1971-72	Male	580,003	337,297	212,942	80,060	1,210,301
1971-72	Female	338,370	257,618	170,385	75,976	842,349
1972-73	Male	644,062	357,026	224,860	84,839	1,310,788
1972-73	Female	378,919	272,684	178,943	80,097	910,643
1973-74	Male	707,343	362,646	226,188	74,176	1,370,353
1973-74	Female	416,757	276,394	178,240	69,360	940,752
1974-75	Male	773,585	381,390	231,067	77,972	1,464,013
1974-75	Female	452,619	288,707	179,907	72,180	993,413
1975-76	Male	838,143	399,025	241,255	76,137	1,554,560
1975-76	Female	486,893	300,087	185,713	69,924	1,042,617

Source: Computed with (4.3).

The financial needs of students could be met by a combination of aid packages: part-time work during the academic year; loans from private, State, and Federal sources; and scholarships and grants from private, State, and Federal sources. One of the principal uses of the undergraduate enrollment and financial aid models will be to estimate the costs of alternative Federal aid programs. Two examples are given to illustrate this use. For the first example, consider the following aid package:

(1) The first \$400 of the aid needed by a student would be met by subsidized loans, similar to the National Defense Education Act (NDEA) loans;

(2) Federal grants, either Educational Opportunity Grants (EOG) or College Work-Study Program (CWSP), not exceeding the difference between \$1,000 and the family contribution, would then be used to meet additional need; in other words, the maximum grant would be \$1,000 less the parents' contribution.

(3) The remaining need would be met by State, institution, and private sources.

For this program, the amount of Federal loans given to a student is determined by the function

$$L(R_t, H_t, C) = \begin{cases} 400 & \text{if } C - R_t - H_t \geq 400 \\ 0 & C - R_t - H_t \leq 0 \\ C - R_t - H_t & \text{otherwise.} \end{cases}$$

And the amount of grants is determined by the function

$$G(R_t, H_t, C) = \begin{cases} 1000 - R_t & \text{if } 1000 \geq R_t \text{ and} \\ & C - H_t \geq 1400 \\ 0 & C - R_t - H_t \leq 400 \\ & \text{or } R_t \geq 1000 \\ C - R_t - H_t - 400 & \text{otherwise.} \end{cases}$$

Define

$AL_{sit}^{kc\theta}$ = the expected amount of NDEA-type loans given in year t to a student with sex s and family income i who is enrolled full-time in an institution with control c and type θ and who is enrolled during the k^{th} fall following first fall enrollment

and

$AG_{sit}^{kc\theta}$ = the expected amount of grants (EOG or CWSP) given in year t to a student with sex s and family income i who is enrolled full-time in an institution with control c and type θ and who is enrolled during the k^{th} fall following first fall enrollment.

Thus

$$(4.4) \quad AL_{sit}^{kc\theta} =$$

$$\int_I \int_C \sum_z \sum_{\Delta, \lambda} L[R_t(I, \Delta, \lambda), H_t(I, s, k), C] \cdot g_k^{\Delta\lambda} P_{c\theta si}^z dF_{it}(I) dB_{c\theta zst}(C),$$

and

$$(4.5) \quad AG_{sit}^{kc\theta} =$$

$$\int_I \int_C \sum_z \sum_{\Delta, \lambda} G[R_t(I, \Delta, \lambda), H_t(I, s, k), C] \cdot g_k^{\Delta\lambda} P_{c\theta si}^z dF_{it}(I) dB_{c\theta zst}(C).$$

It is necessary to evaluate these integrals numerically. Under this program the expected amounts of Federal loans and grants given to full-time undergraduates with sex s during the academic year beginning in year t are

$$(4.6) \quad \sum_i \sum_k \sum_c \sum_\theta S_{sit}^{kc\theta} AL_{sit}^{kc\theta}$$

and

$$(4.7) \quad \sum_i \sum_k \sum_c \sum_\theta S_{sit}^{kc\theta} AG_{sit}^{kc\theta} .$$

respectively. These sums were computed for academic years 1970-71, 1971-72, and 1972-73 and are given in Tables 4.2 and 4.3.

As another example of the use of the model in determining the cost of an aid program, consider the following:

(1) A student would be entitled to the maximum EOG payment consistent with the following conditions: the grant must not exceed \$1,200 less the family contribution; it must not exceed the student's need; it must not exceed 50% of the college expenses; and any positive payment must not be less than \$200.

(2) The remaining need would be met by State, institution, and private sources.

The conditions in this example may be more complex than that which would be used in practice by the Federal Government. Nevertheless, this example does illustrate the versatility of the model. The EOG payment is determined by the function

$$E(R_t, H_t, C) = \begin{cases} 1200 - R_t & \text{if } 1000 \geq R_t \text{ and} \\ & C - H_t \geq 1200 \text{ and} \\ & C/2 \geq 1200 - R_t \\ 0 & \text{if } C - R_t - H_t \leq 200 \text{ or} \\ & C/2 \leq 200 \text{ or} \\ & R_t \geq 1000 \\ C/2 & \text{if } 1000 \geq R_t \text{ and} \\ & C - R_t - H_t \geq C/2 \text{ and} \\ & C/2 \geq 200 \\ C - R_t - H_t & \text{otherwise.} \end{cases}$$

Table 4. 2:

Estimated Amount of Loans Given to Full-Time Undergraduate Students
From the First Hypothetical Federal Aid Program,
By Sex, Income Quartile, and Year

(In Thousands)

Academic Year	Sex	Low Quartile	2nd Quartile	3rd Quartile	High Quartile	Total
1970-71	Male	\$156,564	\$125,083	\$84,563	\$32,553	\$398,764
1970-71	Female	88,004	92,781	68,867	30,778	280,430
1971-72	Male	168,786	128,030	85,291	31,908	414,015
1971-72	Female	96,400	95,110	69,339	29,956	290,805
1972-73	Male	180,473	131,175	85,774	32,426	429,848
1972-73	Female	104,214	97,589	69,474	30,240	301,517

Source: Computed with (1.6).

Table 4. 3:

Estimated Amount of Grants Given to Full-Time Undergraduate Students
From the First Hypothetical Federal Aid Program,
By Sex, Income Quartile, and Year

(In Thousands)

Academic Year	Sex	Low Quartile	2nd Quartile	3rd Quartile	High Quartile	Total
1970-71	Male	\$250,000	\$73,468	\$8,436	0	\$331,904
1970-71	Female	136,516	54,261	7,142	0	197,919
1971-72	Male	274,247	65,947	4,143	0	344,337
1971-72	Female	149,984	48,153	3,495	0	201,632
1972-73	Male	287,546	55,649	995	0	344,190
1972-73	Female	157,122	40,103	842	0	198,067

Source: Computed with (4.7).

Define

$AE_{sit}^{kc\theta}$ = the expected amount of grants given in year t to a student with sex s and family income i who is enrolled full-time in an institution with control c and type θ and who is enrolled during the k^{th} fall following first fall enrollment.

Thus

$$(4.8) \quad AE_{sit}^{kc\theta} = \int \int \sum_C \sum_z \sum_{\Delta, \lambda} E[R_t(I, \Delta, \lambda), H_t(I, s, k), C] \cdot g_k^{\Delta \lambda} P_{c\theta si}^z dF_{it}(I) dB_{c\theta zst}(C).$$

The expected amount of Federal grants given to full-time undergraduates with sex s during the academic year beginning in year t are

$$(4.9) \quad \sum_i \sum_k \sum_c \sum_{\theta} S_{sit}^{kc\theta} AE_{sit}^{kc\theta}.$$

This sum was computed for academic years 1970-71, 1971-72, and 1972-73 and is given in Table 4.4 for these years.

Currently both the undergraduate enrollment and financial aid models are programmed on a time-sharing interactive system. This allows the user to determine:

- (1) The impact of changes in enrollment model parameters (such as enrollment or attrition rates) on the financial needs of students;

Table 4.4:

Estimated Amount of Grants to Full-Time Undergraduate Students
 From the Second Hypothetical Federal Aid Program,
 By Sex, Income Quartile, and Year

(In Thousands)

Academic Year	Sex	Low Quartile	2nd Quartile	3rd Quartile	High Quartile	Total
1970-71	Male	\$323,044	\$136,231	\$29,126	0	\$488,401
1970-71	Female	172,038	97,831	24,178	0	294,047
1971-72	Male	351,964	124,380	17,971	0	494,315
1971-72	Female	190,319	89,236	14,887	0	294,442
1972-73	Male	374,834	111,089	8,402	0	494,325
1972-73	Female	204,923	79,883	6,950	0	291,755

Source: Computed with (4.9).

(2) The impact of changes in financial aid model parameters (such as student contribution from summer employment) on the financial needs of students;

(3) And the costs of alternative Federal aid programs.

As discussed at the beginning of this section, the foregoing financial aid model contains features not included in previous models. Several previous studies have forecasted enrollment by income groups, which is what we have done. However, these studies then estimated the mean income in each income group, and assumed that all families within the income group have the mean income for the purposes of computing the parental contribution. This was done, for example, in Refs. 32, 33, and 34. Since expression (4.1) is a nonlinear function of I , this procedure would be in error, although the error would be small if there were enough income groups. Our use of the c.d.f. $F_{it}(I)$ allows the incomes to be distributed over the entire interval corresponding to an income group. Similarly, several studies estimated the mean college costs by institution type and control, and then assumed that the costs at all similar institutions would be equal to this mean value. This was done in Refs. 33 and 34. Since expression (4.1) is also a nonlinear function of the costs C , this procedure would also be in error. Again, our use of the c.d.f. $B_{c\theta zst}(C)$ avoids this problem. Also previous studies assumed a fixed number of dependent children in a family

and a fixed number of dependent children attending college from a family.

For example, Refs. 32 and 33 estimated the parental contribution by assuming that there were two dependent children altogether and only one attending college, while Ref. 34 assumed that there were two and a half dependent children in a family and only one attending college. We, however, allow these variables to vary and allow their distribution to depend upon the year of enrollment.

5. Postbaccalaureate Student Aid Model

For the purpose of developing a student aid model, there are several important differences between postbaccalaureate and undergraduate students:

(1) The principal source of income for undergraduates is from their parents. However, many postbaccalaureate students consider themselves to be independent and would not accept support from their parents.

(2) There is no generally accepted need analysis model at the postbaccalaureate level. This means that there is no straight-forward way of estimating parental support, as there was in the undergraduate case.

(3) Most undergraduate scholarships are awarded on the basis of need, while most postbaccalaureate scholarships are awarded on the basis of ability. Thus at the postbaccalaureate level it would not be permissible, in estimating unmet needs, to subtract the dollar amount of scholarships from the dollar amount of need, because many of the scholarships go to students who have little or no need.

Because of these changes, we will formulate a different type of model than that used for undergraduate students. We have used data from the Bureau of Higher Education which describe the characteristics of the student applicants for Federal aid programs: either National Defense Student Loans (NDSL) or the College Work-Study Program (CWSP). Postbaccalaureate students are not eligible for Educational Opportunity Grants.

Using these data, we will make estimates of the amount of Federal assistance required by postbaccalaureate students. Define

$\phi_{c\theta}$ = the conditional probability that a full-time postbaccalaureate student will apply for Federal assistance (either NDSL or CWSP), given that the student attends an institution with control c and type θ .

There are students who obtain aid from private, institution, and other sources, without having to apply for Federal assistance. Thus $\phi_{c\theta}$ is the probability that a student will apply for Federal assistance, which is less than the probability that the student needs assistance in addition to parental and self-help contributions. Define

$\Phi_{c\theta t}$ = the average difference between the college expenses and the parental and self-help contributions during the academic year beginning in year t , given that the student does need aid and that he attends full-time at an institution with control c and type θ .

For postbaccalaureate students, the institution type refers to university or other four-year colleges. As before, the control refers to public or private. An estimate of the total assistance (from institution, State, private and other sources) required by postbaccalaureate students who apply for Federal aid is

$$(5.1) \quad \sum_c \sum_{\theta} \phi_{c\theta} \Phi_{c\theta t} K_t^{c\theta},$$

where $K_t^{c\theta}$ is the number of full-time postbaccalaureate students by year and by institution type and control. The values $K_t^{c\theta}$ are the output of the postbaccalaureate enrollment model formulated in Section 3. Next

we estimate the need for Federal assistance. Define

$\Gamma_{c\theta t}$ = the average student need for NDSL and CWSP funds during the academic year beginning in year t , given that the student does apply for assistance and that the student attends full-time at an institution with control c and type θ .

Thus an estimate of the Federal aid needed by postbaccalaureate students is

$$(5.2) \quad \sum_c \sum_{\theta} \phi_{c\theta} \Gamma_{c\theta t} K_t^{c\theta} .$$

During the 1970-71 academic year, the institutions participating in Federal student aid programs submitted application forms to the Bureau of Higher Education which estimated the needs and resources of their students in 1971-72. This was the first year for which these particular forms were completed. Since these data had not yet been processed and were available only from the original application forms, we estimated the foregoing parameters by examining the forms from a sample of schools having postbaccalaureate programs, rather than using data from all schools. In Appendix C is the list of schools for which we obtained data. This list is based upon the sample of institutions used by ACE for their student surveys. However, the list that we used is not identical to the ACE sample, because not all of the institutions in the ACE sample have postbaccalaureate programs or submitted completed forms to the Bureau of Higher Education. We were able to obtain postbaccalaureate financial aid data from thirty-nine public

universities, twenty-nine public four-year colleges, twenty-four private universities, and thirty-five private four-year colleges.

In Table 5.1 are estimates of the probabilities that a full-time post-baccalaureate student will apply for Federal assistance, by type and control of institution. These probabilities are the averages of the corresponding probabilities for the individual institutions in the sample, weighted by the number of postbaccalaureate students attending each institution.

Table 5.1:

Probability That a Full-time Postbaccalaureate Student
Applies For Federal Assistance in 1971-72
By Institution Type and Control

Institution	Probability Student Requires Assistance
Public University	.34
Public Four-Year College	.30
Private University	.44
Private Four-Year College	.27

Source: Estimated from Ref. 42.

In Table 5.2 are estimates for 1971-72 of the average costs and contributions for students applying for Federal assistance, by type and control of institution. These estimates are the averages of the corresponding estimates from the individual institutions in the sample, weighted by the number of postbaccalaureate students who

Table 5.2:

Estimated Costs and Contributions in 1971-72 for Full-Time
Postbaccalaureate Students Who Require Financial
Assistance, By Institution Type and Control

	Public University	Public Other 4-Year	Private University	Private Other 4-Year
Student Cost of Attending Institution	\$3,015	\$2,123	\$4,675	\$4,561
Average Parental Con- tribution Per Student	192	111	455	181
Average Student Contribution	727	597	1,229	900
Support Per Student From All Sources Con- trolled by the Institution	1,292	479	984	1,615
Support Per Student From All Other Sources.	148	54	620	735
Support Per Student From Guaranteed Loans	285	367	570	272
Total Contribution Per Student From All Sources	2,645	1,607	3,857	3,703
Average Need Per Student	382	513	820	857

Source: Estimated from Reference 42.

require assistance at each college. The student cost of attendance in Table 5.2 represents the average cost for students who need aid. This estimate is the weighted average of the corresponding costs for residents and commuters, and it includes tuition and fees, room and board for residents, commuter travel and lunches, student personal expenses, books and supplies, round trip transportation for resident students, and any other necessary expenses. The average parental contribution figure reflects the institutions' estimates of the contribution from this source. As in the undergraduate model, the student's contribution includes savings and summer income. The support from sources controlled by the institution is the estimate of the average aid per student from loan, work, grant, and scholarship funds provided and administered by the institution. This aid does not include National Defense Student Loans (NDSL), College Work-Study Program (CWSP), or Guaranteed Student Loans (GSL), except for guaranteed loans made by the institution. Graduate students are not eligible for Educational Opportunity Grants made by the institution. The support per student from all other sources includes State and local scholarships, but does not include CWSP, NDSL, or GSL. The support per student from guaranteed loans includes all guaranteed loans except those made by the institution. The total contribution is the sum of the previous contribution estimates. Note that the average need per student given in Table 5.2 can not be evaluated simply by subtracting the average total contribution from the average student

cost, because the total contribution exceeded the student cost for some institutions. The average need estimates in Table 5.2 are the weighted averages of the corresponding estimates made by the institutions. These latter estimates were used by the institutions in applying for Federal aid programs (either NDSL or CWSP).

The data in Table 5.2 provide the desired estimates for $\Phi_{c\theta t}$ and $\Gamma_{c\theta t}$ for $t = 1971$. We have defined $\Phi_{c\theta t}$ to be the difference between the student cost and the sum of the parental and student contributions. And we have defined $\Gamma_{c\theta t}$ to be the difference between the college expenses and all sources, excluding CWSP or NDSL. Since data are available for only one year, we make the assumption that the same average need in constant dollars will be required in future years.

Define

$$\rho_t = \text{the Consumer Price Index for year } t.$$

Thus

$$\Phi_{c\theta t} = \frac{\Phi_{c\theta, 1971} \rho_t}{\rho_{1971}}$$

and

$$\Gamma_{c\theta t} = \frac{\Gamma_{c\theta, 1971} \rho_t}{\rho_{1971}}.$$

Values of ρ_t are given in Table B.4 in Appendix B.1.

In Table 5.3 are estimates, for years 1970-71 through 1975-76, of the total aid required by full-time postbaccalaureate students who apply for Federal assistance at the colleges attended. These estimates were computed from (5.1), where the enrollment estimates $K_t^{c\theta}$ are given in Table 3.1. In Table 5.4 are estimates of the needs of full-time postbaccalaureate students after they received aid from all sources except the CWSP and NDSL aid programs. These estimates were computed from (5.2). Of course these last estimates will be affected by a number of factors: changes in the GSL program; changes in the amount of available aid per student from each institution; changes in the levels and requirements of Federal fellowship programs that are not based upon need; etc.

Since the parameters in this model were estimated from application forms submitted by individual colleges, the accuracy of the need projections is based in part upon the accuracy of the data supplied by these institutions. Hopefully, these data were carefully derived by the institutions on the basis of their previous experience with postbaccalaureate financial aid applicants. Because 1971-72 is the first year for which these forms were completed, there is no convenient way of checking the accuracy of the institution's data. Note that expressions (5.1) and (5.2) do include estimates of self-help and parental contributions. Because there is no generally accepted need analysis model at the postbaccalaureate level, it would be difficult to estimate the parental contribution using other

Table 5.3:

Estimated Total Aid Required by Full-Time Postbaccalaureate
Students Who Apply For Federal Assistance,
By Institution Type and Control
(In Thousands)

Academic Year	Public University	Public Other 4 Year	Private University	Private Other 4 Year	Total
1970-71	\$179,800	\$27,214	\$176,077	\$ 62,620	\$445,711
1971-72	200,125	30,116	191,034	69,638	496,913
1972-73	227,535	35,096	205,098	77,175	544,904
1973-74	249,952	40,363	219,657	85,076	595,048
1974-75	273,376	45,918	234,711	93,339	647,344
1975-76	297,808	51,761	250,261	101,967	701,797

Source: Estimated from Eqn. (5.1)

Table 5. 4:

Estimated Requirements for Federal Aid (CWSP or NDSL) By
Full-Time Postbaccalaureate Students,
By Institution Type and Control

(In Thousands)

Academic Year	Public University	Public Other 4 Year	Private University	Private Other 4 Year	Total
1970-71	\$32,769	\$ 9,866	\$48,272	\$15,421	\$106,328
1971-72	37,567	10,918	52,373	17,149	118,007
1972-73	41,469	12,734	56,229	19,006	129,428
1973-74	45,554	14,633	60,220	20,951	141,358
1974-75	49,823	16,647	64,347	22,986	153,803
1975-76	54,276	18,765	68,611	25,111	166,763

Source: Estimated from Eqn. (5.2)

sources of data, such as the distribution of parental income for post-baccalaureate students. According to Ref. 35, a few theological seminaries have developed standards for self-help and parental support for postbaccalaureate students. A brief description of their system follows:

(1) The student is expected to provide \$2,000 a year from term or summer employment and to use portions of savings and other assets.

(2) In the case of a married couple without children, the spouse is expected to be employed and to contribute to the total family income.

(3) The family of a student, whether the student is unmarried or married, is expected to assist the student with his educational expenses; specifically, parents are to contribute one-third of the amount that would be expected at the undergraduate level.

However, these standards are not widespread and have been adopted at only a few colleges.

6. Updating the Parameters in the Models

We have formulated several models in the previous sections: the undergraduate enrollment model in Section 2; the postbaccalaureate enrollment model in Section 3; the undergraduate student aid model in Section 4; and the postbaccalaureate student aid model in Section 5. Due to the lack of reliable data, some of the parameter estimates in these models are only approximate. In addition, important variables, such as the rate of college expansion, draft law status, unemployment rate, size of financial aid programs, are not included explicitly in these models. As these variables change, the parameters in the models will also change. Thus it is very important that these parameter estimates be recomputed as new data become available. As we were unable to make accurate estimates for some of these parameters, we have made recommendations to the Office of Education as to how more reliable information could be obtained in the future. These recommendations are discussed in this section.

The Bureau of the Census has published enrollment data from their Current Population Survey for a number of years. In October of each year, approximately 50,000 occupied housing units are interviewed as part of the Current Population Survey, and data on the enrollment characteristics of the occupants are obtained. The results from these

interviews are then weighted to reflect the population of the United States as a whole. In the past the data from these surveys have not been very useful for developing enrollment and financial aid models, and in fact none of these data were used in this report. However, it would be possible to obtain a great deal of useful information from these surveys if the appropriate questions were asked. Since these surveys are given to the household, accurate family income information could be obtained. In fact, it would be possible to obtain accurate estimates of the following parameters: h_{sit}^1 and $\alpha_{sit}^{c\theta}$ in the undergraduate enrollment model; and $F_{it}(I)$, $P_{c\theta si}^z$, and $g_k^{\Delta\lambda}$ in the undergraduate student aid model. Currently, very little reliable data are available on h_{sit}^1 or $F_{it}(I)$. The estimates we made for $\alpha_{sit}^{c\theta}$ and $P_{c\theta si}^z$ were biased due to poor income information and poor follow-up response. And our estimates for $g_k^{\Delta\lambda}$ were biased because our data were limited to financial aid applicants only. Consequently, we recommend that questions be included so that estimates of the foregoing parameters could be made from the Current Population Survey. If estimates of these parameters were obtained on an annual basis, then up-to-date information would always be available, and it would be possible to determine trends in these parameters over time.

Although approximately 50,000 households are interviewed in the Current Population Survey, this is still a relatively small sample, because only a small portion of these households would have dependents

attending college. Thus, it would not be possible to estimate all of the model parameters from this source alone. In order to estimate such enrollment model parameters as R_{si}^k , π_{sik}^l , and h_{sit}^n (for $n \geq 2$) by sex and by income, it would still be necessary to rely on longitudinal follow-ups of high school and college students. The Office of Education is currently designing a new longitudinal study of high school seniors. The initial questionnaire in this study will be given in the Spring of 1972 to a sample of the high school seniors of that year. The follow-ups will be administered in successive years to determine the enrollment behavior of these students as they progress through higher education. As was discussed in Appendix A.1, the previous longitudinal studies were not adequate for the purposes of calibrating the undergraduate enrollment model. Consequently, we make the following recommendations:

(1) The sample size should be large enough to estimate the model parameters by sex and by income; this was not possible in the Bureau of Census - Columbia University surveys.

(2) An intensive effort should be made to follow-up the respondents in a way that would yield adequate response rates. The response rates for both the American Council on Education - Carnegie Commission on Higher Education and the Project TALENT surveys were very poor.

(3) Since the purpose of these models is to estimate financial aid requirements, it is important to obtain accurate family income

information for the survey respondents. If possible, the income information should come from the parents, as in the Bureau of Census surveys.

(4) The follow-up questionnaires should be designed so that the enrollment history of each respondent could be determined during the entire course of the study. Specifically, it should be possible to determine for each year in the survey: whether the respondent was enrolled full-time, part-time, or not enrolled; the student's attainment (freshman, sophomore, junior, senior, or graduate); and the type and control of the institution at which the respondent was enrolled. It was not possible to determine all of these items in any one of the previous longitudinal studies.

If these recommendations are carried out, then more reliable data should be available in the future to calibrate comprehensive enrollment and financial aid models of higher education.

APPENDIX A

Estimation of Parameters for the Undergraduate Enrollment Model

The undergraduate enrollment model was formulated in Section 2. The techniques used for estimating the parameters in this model are discussed in this Appendix. Data were used from several national surveys of high school and college students. While data from these surveys have been tabulated in a number of reports, it was necessary in some cases to perform special analyses of the original data banks because of the requirements of this model. The characteristics of the longitudinal surveys are discussed in Section A.1, and our estimation methods are described in Sections A.2 through A.6.

A.1 Characteristics of National Student Surveys

Most of our data is from the American Council on Education (ACE). Since 1966, ACE has administered comprehensive annual surveys of entering full-time freshmen attending more than three hundred institutions. In Refs. 42-47 are described the characteristics of these annual surveys, the questionnaires used, the institutions included in the samples, and the national norms computed from these surveys. ACE has also administered a one-year follow-up of a sample of students included in the 1966 freshmen survey. The response rate for this follow-up was approximately 58%. In

Ref. 48 is a description of the sampling procedure and questionnaire used in this follow-up. During the 1969-70 academic year, the Carnegie Commission on Higher Education (CCHE) and ACE jointly sponsored follow-up surveys that were given to a sample of full-time entering freshmen of years 1966 through 1969. These follow-up surveys provide one, two, three, and four-year longitudinal data. The follow-up response rates were 38%, 38%, 41% and 44% for the 1966, 1967, 1968, and 1969 freshmen respectively. Some limitations of the ACE surveys are:

(1) The only income information for a student is the student's own estimate of family income. Since the purpose of the model is to obtain enrollment projections by family income, it is desirable to have accurate income estimates for the survey respondents. Unfortunately, there is evidence which indicates that many students do not know what their family income is. In most of the ACE surveys, the student was not given the option of answering "have no idea" to the income question, but instead was required to make an estimate or leave the question blank. But in the 1967 freshmen survey⁽⁴⁴⁾, the student could answer "have no idea," and approximately 13% of the men and 28% of the women answered in this way. On a similar question in the Project TALENT study⁽⁴⁹⁾, 23% of the men and 42% of the women students indicated that they could not estimate their family income. Thus, parameters estimated by income from ACE data will be in error to some extent, because of the unreliability of the student estimates of family income.

(2) The sample of institutions used by ACE varies from year to year and may not provide representative data for all freshmen students during any given year.

(3) Because the follow-up response rates of the ACE and ACE-CCHE longitudinal surveys were quite low, parameters estimated from these surveys will be biased. In some cases, we corrected for non-response bias by using data from other surveys.

Project TALENT was a cooperative effort of the U. S. Office of Education, the American Institutes for Research, and the University of Pittsburgh. The purpose of Project TALENT was to gather continuing information about a great number of high school students throughout the United States. In 1960, the first year for which data was gathered, some 440,000 high school students in the ninth through twelfth grades were tested. The following year, a one-year follow-up study was administered to the graduating class of 1960. In 1962, a one-year (since graduation) study was done for the graduating class of 1961. Two more years resulted in similar studies for the classes of 1962 and 1963. In 1965, a five-year follow-up (after graduation) study was done for the class of 1960. During each of the next three years, a five-year follow-up study was done on classes of 1961, 1962, and 1963. Unfortunately, the response rates were quite low.

The one-year follow-up surveys had response rates between 37% and 69% for the four high school classes. The five-year follow-ups had response rates between 34% and 41% for the 1960 senior through sophomore classes. Because of the rapid decay in response rates, we have used data only from the follow-up surveys of the 1960 senior class. Refer to Refs. 49, 50, and 51 for a description of the Project TALENT program, the questionnaires used, and some of the results. Some limitations of the Project TALENT surveys are:

(1) As in the ACE surveys, the only income information is the student's own estimate of family income, which may be in error.

(2) Since the follow-up response rate of the Project TALENT longitudinal surveys were quite low, parameters estimated from these surveys will be in error to some extent.

(3) It is not possible to reproduce the enrollment history of the respondent from the five-year follow-up data. It is possible to determine the year of first enrollment in college, the year of graduation, and the attainment (freshmen, sophomore, junior, senior, or graduate) during the fifth year. However, it is not possible to determine the enrollment status (full-time, part-time, not enrolled) or the attainment during each of the interim years between the one and five-year follow-ups.

In October 1965, the Bureau of the Census (BC) conducted its monthly survey of a cross-section of U. S. households. Approximately

1,600 of these households included dependents who were enrolled in the senior year of high school at that time. The Bureau of Applied Social Research at Columbia University (CU) in conjunction with the Bureau of the Census administered follow-up surveys to these students in the spring of 1967, fall of 1968, and fall of 1969. These BC-CU follow-up surveys had relatively high response rates: 92% for the 1967 follow-up, 88% for the 1968 follow-up, and 81% for the 1969 follow-up. Refer to Refs. 12 and 52 for a description of the sampling procedures, the questionnaires used, and some of the results of the study. Because the family income estimates were made by parents, these income estimates should be quite accurate. Thus these surveys have two important advantages over the ACE, ACE-CCHE, and Project TALENT surveys: the relatively high follow-up response rates and the accurate family income information. Unfortunately, these surveys also had a significant limitation: because of the small sample size, it was not possible to estimate several parameters in the model by sex and by income quartile.

In general, none of the available longitudinal student surveys were adequate for the purpose of calibrating the undergraduate enrollment model. The ACE, ACE-CCHE, and Project TALENT surveys suffer from poor response rates and income information, while the BC-CU surveys had a small sample size. While we were able to estimate all of the enrollment model parameters, some of these estimates are only approximate, because of the poor quality of data. However, our survey of

education data sources did enable us to recommend to the Office of Education on how future enrollment data should be collected. These recommendations are given in Section 6.

A.2 Estimation of G_{st}

The parameter G_{st} was defined in Section 2 to be the number of students with sex s (male or female) who graduated from high school during the academic year beginning in year τ . For example, if a student graduated in 1965-66, then $\tau = 1965$. We have used the estimates and projections made by NCES, and these values are listed in Table A.1 for academic years 1959-60 through 1974-75. Except for 1969-70, all of these estimates were taken from the publication Projections of Educational Statistics to 1979-80, 1970 Edition⁽²⁾. For 1969-70, we used the most recent estimates made by NCES⁽³⁾.

A.3 Estimation of h_{sit}^n

We have defined h_{sit}^n to be the conditional probability that the first fall in which a student is enrolled full-time in college is during the n^{th} year following high school graduation, given that the student graduated from high school during the academic year beginning in year τ , has sex s , and the family income lies in the i^{th} quartile. Unfortunately, the available data is meager, and it is necessary to make additional assumptions in order to estimate h_{sit}^n . It is convenient to assume that

Table A. 1:

High School Graduates by Sex

(In Thousands)

Year	Total High School Graduates	Male Graduates	Female Graduates
1959-60	1,864	898	966
1960-61	1,971	958	1,013
1961-62	1,925	941	984
1962-63	1,950	959	991
1963-64	2,290	1,123	1,107
1964-65	2,665	1,314	1,351
1965-66	2,672	1,326	1,346
1966-67	2,680	1,332	1,348
1967-68	2,702	1,341	1,360
1968-69	2,839	1,408	1,431
1969-70	2,953	1,444	1,509
PROJECTED			
1970-71	3,102	1,541	1,560
1971-72	3,212	1,601	1,612
1972-73	3,312	1,654	1,658
1973-74	3,414	1,706	1,708
1974-75	3,507	1,756	1,751

Source: Refs. 2 and 3.

$$(A.1) \quad h_{si\tau}^n = a_{si}^n \cdot h_{si\tau}^1,$$

where a_{si}^n is independent of the year τ . This formula will be used to estimate $h_{si\tau}^n$, for $n \geq 2$, from projections of $h_{si\tau}^1$. We used Project TALENT⁽¹¹⁾ to estimate a_{si}^n , because this longitudinal survey extended over six years and the sample size was large enough to estimate a_{si}^n by sex and by income quartile. The values for a_{si}^n are given in Table A.2. The main deficiency of the Project TALENT survey is the poor follow-up response rate. According to two studies^(51,52) that have been made of non-respondents to longitudinal surveys of high school and college students, the students who do not enroll in college are the ones who tend not to respond to the follow-up questionnaire. Thus the response rate of students who enrolled in college should be significantly higher than the overall response rate. Since a_{si}^n is defined so that it may be estimated using data only from students who eventually enrolled in college, the estimates for a_{si}^n may be accurate in spite of the poor overall follow-up response rate. While the data in Table A.2 are for students who graduated from high school during the 1959-60 academic year, Eqn. (A.1) makes the approximation that a_{si}^n is constant over time. This approximation should be satisfactory for two reasons:

- (1) According to Project TALENT data⁽¹¹⁾, about 85% of all 1959-60 high school graduates who enrolled full-time in college did so during the

Table A.2:

Ratio of the Probability That a 1959-60 High School Graduate Enrolls Full-Time for the First Time in a Given Year to the Probability That the Graduate Enrolls Full-Time for the First Time in 1960, By Sex and Income Quartile

Sex	Income Quartile	Year First Enrolled					
		1960	1961	1962	1963	1964	1965
Male	1	1.000	.11	.05	.03	.02	.02
Male	2	1.000	.11	.05	.03	.01	.01
Male	3	1.000	.11	.05	.03	.01	.01
Male	4	1.000	.11	.02	.02	.02	.00
Female	1	1.000	.05	.04	.02	.00	.00
Female	2	1.000	.06	.05	.02	.00	.00
Female	3	1.000	.06	.03	.02	.00	.02
Female	4	1.000	.07	.01	.02	.00	.00

Source: Estimated from Ref. 11.

fall of 1960. Thus any error made in estimating $h_{si\tau}^n$, for $n \geq 2$, should be small compared to $h_{si\tau}^1$.

(2) The values for a_{si}^n in Table A.2 are roughly independent of the income quartile i . Thus any change in a_{si}^n due to expansion of financial aid programs should be small.

The next step is to estimate $h_{si\tau}^1$, the probability of enrolling in the year immediately following graduation. In Table A.3 are estimates derived from published Bureau of Census reports^(12,13) giving these probabilities for 1959 and 1965, but classified only by income quartile and not by sex.

Table A.3:

Probability of Enrolling in College During the Year
Immediately Following High School Graduation,
By Income Quartile and Year

Income Quartile	Probability of Enrollment for Graduates of Year	
	1959-60	1965-66
1	.19	.28
2	.38	.39
3	.47	.50
4	.62	.68

Source: Estimated from Refs. 12 and 13.

Estimates of $h_{si\tau}^1$, by sex and by income, were made for $\tau = 1965$ from the first BC-CU follow-up⁽⁴⁾ and are listed in Table A.4.

Table A.4:

Probability That A 1965-66 High School Graduate Enrolls Full-Time in College in Fall 1966 or Spring 1967, By Income Quartile and By Sex

Sex	Income Quartile	Probability of Enrollment
Male	1	.32
Male	2	.42
Male	3	.56
Male	4	.71
Female	1	.19
Female	2	.32
Female	3	.45
Female	4	.66

Source: Estimated from Ref. 4.

Other than the data in Tables A.3 and A.4, there is little additional reliable information. While $h_{si\tau}^1$ could be estimated from Project TALENT, these estimates would be biased by the poor response rate and income information. The 1959-60 data in Table A.3 includes students who enrolled full or part-time in either spring or fall 1960. The 1965-66 data

includes students who enrolled full or part-time in either fall 1965 or spring 1966. The data in Table A.4 are for students who enrolled full-time in either fall 1965 or spring 1966. We assume that $h_{si\tau}^1$ can be computed with the formula

$$(A.2) \quad h_{si\tau}^1 = h_{si,1965}^1 + \delta_i \cdot (\tau - 1965)$$

for $1959 \leq \tau \leq 1965$, where $h_{si,1965}^1$ is given in Table A.4, and δ_i is estimated from Table A.3 to be the following: $\delta_1 = .015$, $\delta_2 = .002$, $\delta_3 = .005$, and $\delta_4 = .010$.

For the first time in 1970, the ACE annual freshmen survey included a question which asked whether the student was enrolled during the year immediately following high school graduation. Using this data from ACE⁽¹⁴⁾ along with estimates of the number of high school graduates and first-time full-time college students from NCES^(3,15), the probability that a student who graduated in 1969-70 enrolled full-time in the fall of 1970 was estimated to be .56 for male students and .43 for female students. We assume that for $1965 \leq \tau \leq 1969$,

$$(A.3) \quad h_{si\tau}^1 = h_{si,1965}^1 + \frac{\delta_i}{\epsilon_s} \cdot (\tau - 1965) ,$$

where ϵ_s is .96 and 1.00 for male and female students respectively. The coefficient ϵ_s was chosen so that $h_{si,1969}^1$ would be consistent with the average estimates for each sex made for 1969. Projecting $h_{si\tau}^1$ for $\tau \geq 1970$ is a difficult problem for two reasons: the lack of reliable historical data; and the fact that $h_{si\tau}^1$ will be influenced by many factors such as the draft law, state of the economy, college expansion, and magnitude of student aid programs. It is expected that the current economic recession will reduce the rate of increase in $h_{si\tau}^1$. Since we are only making short-run projections in this report, we assume in these projections that $h_{si\tau}^1$ will remain constant at the 1969 level for $\tau \geq 1970$.

A.4 Estimation of R_{si}^k

The parameter R_{si}^k was defined to be the conditional probability that a student is enrolled as a full-time undergraduate during the k^{th} fall after first fall enrollment, given that the student first enrolled full-time, has sex s , and has family income i . By definition, $R_{si}^0 = 1$. We will use data from several longitudinal surveys to estimate R_{si}^k for $k = 1, 2, \dots, 5$. In Table A.5 are the probabilities that a student is enrolled full-time during the k^{th} fall following first enrollment, as estimated from the ACE-CCHE⁽¹⁶⁻¹⁸⁾, ACE⁽¹⁹⁾, and BC-CU^(4,20,21) longitudinal studies. These estimates do allow a student to drop out and then return to college.

Table A.5:

Probability That A Student is Enrolled
 During Years Following First Enrollment,
 As Estimated From Several Data Sources

Longitudinal Student Survey	Probability of Full-time Enrollment During the k^{th} Fall Following the First Full-time Fall Enrollment		
	$k = 1$	$k = 2$	$k = 3$
ACE-CCHE	.87	.77	.68
ACE	.85	---	---
BC-CU	.85	.66	.60

Source: Estimated from Refs. 4 and 16-21.

We did not include any estimates in Table A.5 from Project TALENT, as it is not possible to identify the enrollment characteristics of respondents during intermediate years in the Project TALENT study. It has been the experience in longitudinal surveys of college students that the non-respondents tend to be the ones who dropped out of college. Thus, because of the low follow-up response rates, the ACE-CCHE estimates are probably higher than the true values. This hypothesis is consistent with the data in Table A.5. It is not possible to estimate R_{si}^k by sex and by income quartile directly from the BC-CU data because of the small sample size.

In Table A. 6 are the values of R_{si}^k estimated from the ACE-CCHE⁽¹⁶⁻¹⁸⁾ surveys. The values for $k = 1, 2,$ and 3 were estimated directly from the 1969 follow-up of 1968, 1967, and 1966 entering freshmen respectively. It is also possible to estimate the probability that a student will be enrolled as an undergraduate during the fourth and fifth falls following first fall enrollment, because the follow-up questionnaire included a question asking for the expected date of graduation. A student would be an undergraduate during the k^{th} fall following first fall enrollment, for $k \geq 4$, if the student dropped out for a term, failed courses, changed majors, enrolled in a five year bachelor's degree program, etc. Define

Y_{si}^r = the conditional probability that a student expects to graduate during the r^{th} academic year following first fall enrollment, given that the student has sex s , family income i , and is enrolled full-time during the 3rd fall following first fall enrollment.

The probabilities Y_{si}^r were estimated from the 1969 ACE-CCHE follow-up⁽¹⁶⁾ of entering 1966 freshmen and are listed in Table A. 7 for $r = 4, 5,$ and 6 . Thus an estimate of the probability that a student is enrolled as an undergraduate during the k^{th} fall following first fall enrollment is

$$(A.4) \quad R_{si}^k = R_{si}^3 \sum_{r \geq k+1} Y_{si}^r$$

for $k \geq 4$. This formula assumes that the student will continue to enroll full-time until graduation. The values of R_{si}^4 and R_{si}^5 listed in Table A.6 were computed from Eqn. (A.4).

Table A.6:

High Estimates of the Probability That a Student is Enrolled as an
Undergraduate During Years Following First Enrollment,
By Sex and Income Quartile

Probability of Full-Time Enrollment During the kth Fall
Following the First Full-Time Fall Enrollment:

Sex	Income Quartile	k=0	k=1	k=2	k=3	k=4	k=5
Male	1	1.00	.835	.750	.639	.292	.046
Male	2	1.00	.876	.801	.662	.313	.053
Male	3	1.00	.891	.811	.705	.325	.057
Male	4	1.00	.905	.815	.722	.290	.049
Female	1	1.00	.796	.725	.653	.189	.011
Female	2	1.00	.847	.704	.577	.151	.016
Female	3	1.00	.865	.720	.669	.198	.030
Female	4	1.00	.887	.743	.686	.204	.020

Source: Estimated from Refs. 16-18.

Table A.7:

Probability That a Student Who is Enrolled Full-Time in Fall 1969
and First Enrolled Full-Time in Fall 1966 Expects to
Graduate During the Current or Future Academic Years,
By Sex and Income Quartile

Sex	Income Quartile	Probability* Student Expects to Graduate During:	
		1969-70	1970-71
Male	1	.450	.386
Male	2	.443	.393
Male	3	.494	.380
Male	4	.517	.333
Female	1	.648	.272
Female	2	.685	.235
Female	3	.656	.253
Female	4	.653	.268

*Probabilities may not sum to 1.00 because a student may not expect to graduate or he has already graduated.

Source: Estimated from Ref. 16.

Because of the poor response rates for the ACE-CCHE surveys, the estimates of R_{si}^k listed in Table A.6 are expected to be too high. We used the coefficients in Table A.8 to correct for this non-response bias. These coefficients are consistent with the range of average probabilities given in Table A.5 and with the 1970 NCES⁽¹⁵⁾ data in Table 2.1. Thus in this report, the values of R_{si}^k used for the enrollment projections are computed with

$$R_{si}^k = \delta_{sk} \cdot [\text{estimate in Table A.7}] .$$

Table A.8:

Coefficients Used to Correct For Non-Response Bias

Sex	k = 0	k = 1	k = 2	k = 3	k = 4	k = 5
Male	1.0	1.0	1.0	.89	.89	.89
Female	1.0	1.0	.89	.88	.88	.88

A.5 Estimation of π_{sik}^l

We have defined π_{sik}^l to be the conditional probability that a student has attained to level l (either under division or upper division), given that the student has sex s , family income i , and is enrolled full-time during

the k^{th} fall following first fall enrollment. Note that it would not be satisfactory to simply assume that a student would be an upperclassman during the second fall after his first fall enrollment, because the student may have dropped out for one or more terms, switched majors, enrolled part-time, etc. Unfortunately, none of the longitudinal studies--Project TALENT, BC-CU, or ACE-CCHE--asked for the student's attainment during each year in the survey. Project TALENT asked for the student's attainment during the fall of the sixth year following high school graduation, but not for the intermediate years. The ACE-CCHE follow-up surveys, however, included a question which asked when the respondent expected to graduate. Let the index $\ell = 1$ refer to under division, and $\ell = 2$ refer to upper division. We assume that $\pi_{sik}^1 = 1$ for $k = 0$ and 1, and that $\pi_{sik}^2 = 1$ for $k = 4$ and 5. Estimates of π_{sik}^{ℓ} , for $k = 2$ and 3, are given in Tables A-9 and A-10 respectively, and these estimates were obtained from ACE-CCHE^(16,17) using the following assumptions: if a full-time student expected to graduate within a year, he would be a senior; if he expected to graduate within two years, he would be a junior; etc.

A.6 Estimation of $\alpha_{sil}^{c\theta}$

The parameter $\alpha_{sil}^{c\theta}$ was defined to be the conditional probability that an undergraduate student is enrolled in an institution with control c (either public or private) and type θ (either two-year, four-year college,

Table A.9:

Probability of Upper or Under Division Enrollment for a
 Student Who is Enrolled Full-Time in Fall 1969 and
 First Enrolled Full-Time in Fall 1967,
 By Sex and Income Quartile

Sex	Quartile	Probability of Being Enrolled In:	
		Lower Division	Upper Division
Male	1	.41	.59
Male	2	.41	.59
Male	3	.39	.61
Male	4	.36	.64
Female	1	.38	.62
Female	2	.26	.74
Female	3	.22	.78
Female	4	.21	.79

Source: Estimated from Ref. 17.

Table A.10:

Probability of Upper or Under Division Enrollment for a Student
Who is Enrolled Full-Time in Fall 1969 and First Enrolled
Full-Time in Fall 1966, By Sex and Income Quartile

Sex	Income Quartile	Probability of Being Enrolled In:	
		Lower Division	Upper Division
Male	1	.11	.89
Male	2	.15	.85
Male	3	.12	.88
Male	4	.14	.86
Female	1	.06	.94
Female	2	.06	.94
Female	3	.07	.93
Female	4	.06	.94

Source: Estimated from Ref. 16.

or university), given that the student has sex s , family income i , and is enrolled full-time at level ℓ . Note that θ will always refer to four-year colleges or universities for upper division students; only for under division students is there a choice between two-year and four-year institutions.

For $\ell = 1$ (lower division), $\alpha_{si1}^{c\theta}$ was first estimated using data from the 1970 ACE⁽¹⁴⁾ freshmen survey. This would only be an approximation, because the survey did not include sophomores. We then normalized these estimates so that they would be consistent with the 1970 NCES⁽¹⁵⁾ data for lower division students. Our final values for $\alpha_{si1}^{c\theta}$ are listed in Table A.11.

For $\ell = 2$ (upper division), estimates of $\alpha_{si2}^{c\theta}$ were made from the 1969 ACE-CCHE⁽¹⁶⁾ follow-up of 1966 freshmen and are listed in Table A.12. These values are consistent with the 1970 NCES⁽¹⁵⁾ data for upper division students.

Table A.11:

Probability That A 1970 Full-Time Lower-Division Student
Attends A Particular Type and Control of Institution,
By Sex and Income Quartile

Sex	Income Quartile	Public		Private		Private	
		University	Other 4-Year	2-Year	University	Other 4-Year	2-Year
Male	1	.14	.21	.48	.03	.12	.03
Male	2	.20	.23	.38	.04	.13	.03
Male	3	.24	.22	.33	.05	.14	.02
Male	4	.32	.16	.22	.09	.19	.02
Female	1	.11	.26	.44	.03	.13	.03
Female	2	.19	.29	.32	.04	.14	.03
Female	3	.24	.28	.27	.04	.15	.03
Female	4	.31	.21	.16	.07	.21	.04

Source: Estimated from Refs. 14 and 15.

Table A.12:

Probability That a Student Who is Enrolled Full-Time in the Upper Division in Fall 1969 and First Enrolled Full-Time in Fall 1966 Attends a Particular Type and Control of Institution, by Sex and Income Quartile

Sex	Income Quartile	Probability Student Attends:			
		Public University	Public 4 Year	Private University	Private 4 Year
Male	1	.35	.33	.08	.24
Male	2	.36	.37	.08	.19
Male	3	.36	.35	.09	.20
Male	4	.38	.24	.14	.24
Female	1	.20	.47	.06	.27
Female	2	.30	.40	.07	.23
Female	3	.35	.40	.07	.19
Female	4	.38	.26	.10	.26

Source: Estimated from Ref. 16.

APPENDIX B

Estimation of Parameters for the Undergraduate Student Aid Model

The techniques employed for estimating the parameters in the undergraduate student aid model are discussed in this Appendix. Financial aid data from the American College Testing program (ACT) and the College Scholarship Service (CSS) were used, and income data from the Bureau of the Census and the Bureau of Labor Statistics were also used.

B.1 Estimation of $R_t(I, \Delta, \lambda)$

We have defined $R_t(I, \Delta, \lambda)$ to be the expected parental contribution during the academic year beginning in year t towards the expenses of a student whose family has income I , total number of dependent children Δ , and number of dependent children λ attending college. This parental contribution includes contributions from both current income and from assets. We have used the need analysis model developed by CSS⁽³⁵⁾ to derive the formula for $R_t(I, \Delta, \lambda)$. Their approach is to convert the value of the parents' assets into a supplementary income flow on the basis of the age of the principal wage earner and number of retirement plans the family has. Table B.1 gives the average parents' assets, by family income, for 1969 prefreshmen students. This data came from students who submitted scholarship applications through CSS. While CSS does have parental assets data for 1970 aid applicants, this

Table B.1:

Mean Parental Assets for 1969 Pre-Freshman Applicants,
As a Function of Family Income

Estimated Net Income (1970)	Mean Assets (1969)	Estimated Mean Assets (1970)
\$ 1 - 2,999	\$ 6,268	\$ 6,641
3,000 - 4,999	7,910	8,381
5,000 - 7,499	9,910	10,500
7,500 - 9,999	11,901	12,609
10,000 - 12,499	14,024	14,859
12,500 - 14,999	16,162	17,123
15,000 - 17,499	19,133	20,272
17,500 - 19,999	22,774	24,129
20,000 - 22,499	27,323	28,949
22,500 - 24,999	32,964	34,926
25,000 - 27,499	38,229	40,504
27,500 - 29,999	44,893	47,565
30,000 - Over	68,365	72,433

Source: Ref. 36 for Estimated Net Income (1970) and Mean Assets (1969).

information was not classified by income, and so it would not be useful for the financial aid model. The 1969 assets' values were converted into 1970 dollars using the Consumer Price Index (see Table B.4). These 1970 values were then converted into the supplementary income flows given in Table B.2 by using Table F in Ref. 35 and the following approximations: the age of the principal wage earner in the household is between forty-five and forty-nine, and the family has only one retirement plan. Define

$S(I)$ = the supplementary income in 1970 from parents' assets as a function of the 1970 family income I ,

and

Y = the 1970 adjusted income, which includes the current income plus the supplementary income.

Consequently,

$$(B.1) \quad Y = S(I) + I,$$

where $S(I)$ is given in Table B.2 as a function of the (estimated) income in 1970.

Using data describing consumption patterns of families in the United States, CSS developed tables which convert the adjusted family income Y into the expected contribution towards the college expenses of a student.

Table B. 2:

Supplementary Income Flow

Estimated Net Income (1970)	Estimated Supplementary Income (1970)
\$ 1 - 2,999	\$ 0
3,000 - 4,999	80
5,000 - 7,499	300
7,500 - 9,999	520
10,000 - 12,499	740
12,500 - 14,999	960
15,000 - 17,499	1,400
17,500 - 19,999	1,840
20,000 - 22,499	2,280
22,500 - 24,999	2,940
25,000 - 27,499	3,600
27,500 - 29,999	4,260
30,000 - Over	7,170

Source: Table B.1 and Table F in Ref. 35.

Define

$P(Y, \Delta)$ = the expected parental contribution towards a resident student's budget in 1970-71 from a family with adjusted family income Y , Δ dependent children, and only one dependent in college.

A resident student is a student who does not live with his parents, but instead lives in a dormitory, apartment, etc. Note that the definition of $P(Y, \Delta)$ applies specifically to the 1970-71 academic year and to families with only one dependent child attending college. Values for $P(Y, \Delta)$ are given in Table B. 3, and they were taken directly from Table A in Ref. 35. For each fixed value of Δ , we assume that $P(Y, \Delta)$ can be represented as a piecewise linear function of Y connecting the values in Table B. 3. It follows from the foregoing definitions that

$$(B.2) \quad R_{1970}(I, \Delta, 1) = P(S(I) + I, \Delta) .$$

If there are λ students attending college from a family, then CSS suggests the following as the parental contribution towards a resident student's budget:

$$(B.3) \quad R_{1970}(I, \Delta, \lambda) = \begin{cases} R_{1970}(I, \Delta, 1) & \text{if } R_{1970}(I, \Delta, 1) \leq 900 \\ 900 + \frac{R_{1970}(I, \Delta, 1) - 900}{\lambda} & \text{otherwise} . \end{cases}$$

Several assumptions are used in deriving this formula. A major assumption made by CSS is that parents are expected to continue to provide, as well as

Table B. 3:

Total Parents' Contribution (1970)
From Adjusted Income by Size of Family

Adjusted Income (1970)	Number of Dependent Children					
	1	2	3	4	5	6
\$ 5,000	\$ 260					
6,000	550	\$ 210				
7,000	820	430	\$ 210			
8,000	1,120	650	400	\$ 240		
9,000	1,420	870	580	410	\$ 330	\$ 270
10,000	1,720	1,110	770	570	490	420
11,000	2,090	1,340	960	740	650	570
12,000	2,490	1,580	1,160	900	800	710
13,000	2,870	1,810	1,350	1,080	960	860
14,000	3,260	2,120	1,540	1,250	1,130	1,010
15,000	3,640	2,420	1,730	1,420	1,290	1,170
16,000	4,020	2,730	1,970	1,590	1,450	1,320
17,000	4,390	3,030	2,230	1,760	1,610	1,470
18,000	4,760	3,330	2,480	1,970	1,770	1,620
19,000	5,130	3,620	2,730	2,190	1,960	1,770
20,000	5,490	3,920	2,980	2,420	2,170	1,950
21,000	5,850	4,200	3,230	2,640	2,380	2,150
22,000	6,200	4,490	3,470	2,860	2,590	2,350
23,000	6,560	4,770	3,720	3,080	2,800	2,550
24,000	6,900	5,050	3,960	3,290	3,000	2,740
25,000	7,240	5,330	4,200	3,510	3,200	2,930
26,000	7,580	5,590	4,430	3,710	3,410	3,130
27,000	7,910	5,860	4,650	3,920	3,600	3,320
28,000	8,240	6,130	4,880	4,120	3,790	3,500
29,000	8,550	6,380	5,110	4,330	3,980	3,680

Source: Table A in Ref. 35.

they are able, the basic essentials of life, whether the student lives at home or on the college campus. The first \$900 of R_{1970} (I, Δ , 1) represents the amount the parents save by having their child live away from home. This amount is assumed to be available for each dependent attending college. However, the portion of R_{1970} (I, Δ , 1) in excess of \$900 comes from money available for discretionary use, and it is evenly divided among all dependents attending college.

So far, Eqn. (B. 3) only provides the parental contribution for a resident student. The first \$900 of the total contribution in Eqn. (B. 3) represents the amount that the family is assumed to have used to provide the basic necessities for maintaining the student at home. If the student decides to live at home and commute to college, much of this amount cannot be considered to be available for payment of direct college expenses. Thus if the student lives at home, a reduction in the parental contribution should be made. However, certain Federal aid programs, such as Educational Opportunity Grants (EOG), use the parental contribution to determine eligibility. Because of possible confusion from using different contributions for residents and commuters, CSS⁽³⁵⁾ suggests using the same figure for both, and instead include the maintenance expenses (room and board at home) as part of the college expense budget for commuters. Thus we will use Eqn. (B. 3) for both residents and commuters. In Section B. 6, the first \$900 of R_{1970} (I, Δ , λ) will be included in the commuter expense budget to represent room, board, clothing, and other expenses at home.

Using Eqns. (B.1) through (B.3), the values for $R_{1970}(I, \Delta, \lambda)$ can be computed for any combination of I , Δ , and λ . By converting the income level in year t into 1970 dollars, future values of $R_t(I, \Delta, \lambda)$ can be estimated from the 1970 function. Define

ρ_t = the Consumer Price Index in year t .

Historical values of the Consumer Price Index are available from the Bureau of Labor Statistics⁽⁵³⁾ and are listed in Table B.4 for years 1959 through 1970. These values were projected to 1975 using the four-year linear trend between 1966 and 1970. This corresponds to an assumed inflation rate of 4.1% during 1971, which falls to a rate of 3.4% in 1975. Thus we assume that $R_t(I, \Delta, \lambda)$ can be computed with

$$(B.4) \quad R_t(I, \Delta, \lambda) = R_{1970}\left(I \frac{\rho_{1970}}{\rho_t}, \Delta, \lambda\right) \cdot \frac{\rho_t}{\rho_{1970}}$$

B.2 Estimation of $F_{it}(I)$

We have defined $F_{it}(I)$ to be the conditional c.d.f. of family income for students who graduate from high school during the academic year beginning in year t , given that the income falls within the i^{th} quartile. It is convenient to define the unconditional distribution as well. Let

$F_t(I)$ = the c.d.f. of family income I in year t for students who graduate from high school in the academic year beginning with year t ,

L_{it} = the lower bound for the i^{th} quartile of the family income distribution for students who graduated from high school in the academic year beginning with year t ,

Table B. 4:

Consumer Price Index
For Urban Wage Earners and Clerical Workers

Year	Consumer Price Index (1957-59 = 100)
1959	101.5
1960	103.1
1961	104.2
1962	105.4
1963	106.7
1964	108.1
1965	109.9
1966	113.1
1967	116.3
1968	121.2
1969	127.7
1970	135.3
	PROJECTED
1971	140.85
1972	146.40
1973	151.95
1974	157.50
1975	163.05

Source: Ref. 53 for years 1959 through 1970.

and

U_{it} = the upper bound for the i^{th} quartile of the family income distribution for students who graduated from high school in the academic year beginning with year t .

The income limits L_{it} and U_{it} can be computed from $F_t(I)$. If i refers to the low income quartile, then $L_{it} = 0$, and if i refers to the high income quartile, then $U_{it} = \infty$. The conditional c.d.f. $F_{it}(I)$ is defined in terms of $F_t(I)$, U_{it} , and L_{it} as follows:

$$(B.5) \quad F_{it}(I) = \begin{cases} 0 & I < L_{it} \\ \frac{1}{4} (F_t(I) - \frac{i-1}{4}) & L_{it} \leq I \leq U_{it} \\ 1 & U_{it} < I \end{cases} .$$

We describe next how to estimate $F_t(I)$.

In 1960, the Bureau of Census surveyed a sample of students who were high school seniors in October 1959 in order to determine their graduation status. Using published data ⁽¹³⁾ from this study, it is possible to estimate the c.d.f. $F_{1959}(I)$ at five points: $I = \$0, \$400, \$6,000, \$7,500,$ and $\$10,000$. These estimates are given in Table B.5.

Table B. 5:
Family Income Distribution of 1959-60 High School Graduates

Family Income I	F ₁₉₅₉ ^(I)
\$ 0	0.
4,000	.305
6,000	.550
7,500	.701
10,000	.860

Source: Estimated from Ref. 13.

Table B. 6:
Family Income Distribution of 1965-66 High School Graduates

Family Income I	F ₁₉₆₅ ^(I)
0	0.
1,000	.0148
2,000	.0541
3,000	.1059
4,000	.1739
5,000	.2591
6,000	.3753
7,500	.5254
10,000	.7336
15,000	.9333
25,000	.9826

Source: Estimated from Ref. 4.

In 1967, the Bureau of Census also surveyed a sample of students who were high school seniors in October 1965. We have used unpublished data from this survey to estimate the c.d.f. $F_{1965}(I)$ at eleven points, and these estimates are given in Table B.6.

In order to estimate $F_t(I)$ for years other than 1959 and 1965, it is convenient to make the following definitions:

M_t = the average in year t of the median income for families whose head is between thirty-five and forty-five years of age and the median income for families whose head is between forty-five and fifty-four,

and

I_t^r = the income level corresponding to the r^{th} percentile of the c.d.f. of family income for students who graduate from high school during the academic year beginning in year t .

The average incomes M_t can be estimated for several years from Bureau of Census data, including 1959 through 1969. By the definition of I_t^r ,

$$F_t(I_t^r) = 100 \cdot r$$

for any year t . We assume that the incomes I_t^r can be estimated with the formula

$$(B.6) \quad I_t^r = \frac{M_t}{M_{1965}} I_{1965}^r .$$

Note that this formula need not be accurate for very high and very low incomes, because no aid will be required for very high income families and no parental contribution will be expected from very low income families. Thus the expected financial need is independent of the shape of $F_t(I)$ for incomes near the edges of this distribution. This formula was tested using the data in Tables B.5 and B.6, and the results are summarized in Table B.7. In this latter table, the values of I_{1965}^r were estimated from Table B.6 using linear interpolation, and the values of I_{1959}^r were taken directly from Table B.5. Eqn. (B.6) was evaluated using estimates of the median incomes from the Bureau of Census^(5,6). In this case, the estimation formula was fairly accurate: within 2.8% over a six year interval.

In order to calibrate the enrollment and student aid models, it is necessary to estimate $F_t(I)$ for years subsequent to 1965. In Table B.8 the values of M_t were taken directly from Bureau of Census data for $t = 1965, \dots, 1969$. For $t \geq 1970$, M_t was estimated with the formula

$$(B.7) \quad \frac{M_t}{\rho_t} = \frac{M_{1969}}{\rho_{1969}} (1 + \gamma)^{t - 1969} ,$$

where γ is the real annual rate of increase for M_t . The value $\gamma = .0386$ was used, which was the mean value of γ between 1959 and 1969.

Table B.7:

Test of Estimation Formula

r	I_{1965}^r	I_{1959}^r	$(M_{1959}/M_{1965}) I_{1965}^r$	Percent Error
0.	\$ 0.	\$ 0.	\$ 0.	-
30.5	5,395.	4,000.	4,113.	2.8%
55.0	7,797.	6,000.	5,944.	.9%
70.1	9,619.	7,500.	7,333.	2.2%
86.0	13,165.	10,000.	10,037.	.4%

Source: See text.

Table B.8:

The Average of the Median Income for Families Whose Head is Between 35 and 44 and the Median Income for Families Whose Head is Between 45 and 54.

Year t	M_t
1965	\$ 8,053
1966	8,725
1967	9,458
1968	10,237
1969	11,279
PROJECTED	
1970	12,411
1971	13,419
1972	14,486
1973	15,616
1974	16,811
1975	18,075

Source: Refs. 6-10 for years 1965-1969;
Eqn. (B.7) for years 1970-1975.

Eqn. (B.6) only estimates the c. d. f. $F_t(I)$ at the values of I_t^r corresponding to the 1965 data given in Table B.6. We assume that the c. d. f. can be approximated with a piecewise linear function connecting these estimated values.

B.3 Estimation of $g_k^{\Delta\lambda}$

We have defined $g_k^{\Delta\lambda}$ to be the conditional joint probability mass function of the total number of dependent children Δ and the number of dependent children attending college λ in a family, given that the student is enrolled during the k^{th} fall following first fall enrollment. The distributions $g_k^{\Delta\lambda}$ were estimated using unpublished data ⁽³⁷⁾ from 1970 ACT financial aid applicants. If a student from a family requires financial assistance, then the number of dependents in that family tends to be larger than the number of dependents in the family of a student not requiring assistance. In other words, because the ACT data are from financial aid applicants, these data will be biased. Unfortunately, $g_k^{\Delta\lambda}$ cannot be estimated from any of the national student surveys. Nevertheless, using the ACT distributions will be a significant refinement over previous financial aid studies, such as in Refs. 32, 33, and 34, which assumed a fixed number of dependent children in a family and a fixed number of dependent children attending college.

In Tables B.9 through B.13 are the distributions of Δ and λ for freshman, sophomore, junior, and senior aid applicants respectively. These tables refer only to dependent children; married independent students and single independent students are not included. We will use the freshmen distribution for $g_0^{\Delta\lambda}$, the sophomore distribution for $g_1^{\Delta\lambda}$, the junior distribution for $g_2^{\Delta\lambda}$, and the senior distribution for $g_3^{\Delta\lambda}$, $g_4^{\Delta\lambda}$, and $g_5^{\Delta\lambda}$. Note that this procedure involves several approximations. The senior distribution was used for $g_3^{\Delta\lambda}$, $g_4^{\Delta\lambda}$, and $g_5^{\Delta\lambda}$, because most dependent undergraduate students who are enrolled in their third, fourth, or fifth fall following first fall enrollment are seniors. The junior distribution was used for $g_2^{\Delta\lambda}$, because most dependent undergraduate students who are enrolled in their second fall following first fall enrollment are juniors. The approximations for $g_0^{\Delta\lambda}$ and $g_1^{\Delta\lambda}$ are justified in a similar way.

B.4 Estimation of $H_t(I, s, k)$

We have defined $H_t(I, s, k)$ to be the expected self-help contribution of a student in year t towards his college expenses, given that the student is enrolled during the k^{th} fall after first fall enrollment, has sex s , and comes from a family with income I . One portion of a student's contribution

Table B. 9:

Distribution of the Number of Children in College by the
Number of Dependent Children in the Family for
Freshmen Aid Applicants

Number of Dependent Children in the Family	Number of Children in College			
	1	2	3	4 or more
1	.246			
2	.158	.066		
3	.119	.064	.014	
4	.083	.041	.016	.003
5	.040	.025	.010	.003
6 or more	.053	.036	.020	.005

Source: Ref. 37.

Table B. 10:

Distribution of the Number of Children in College by the
Number of Dependent Children in the Family for
Sophomore Aid Applicants

Number of Dependent Children in the Family	Number of Children in College			
	1	2	3	4 or more
1	.250			
2	.155	.071		
3	.121	.065	.012	
4	.063	.046	.017	.004
5	.040	.033	.012	.002
6 or more	.043	.040	.019	.007

Source: Ref. 37.

Table B. 11:

Distribution of the Number of Children in College by the
Number of Dependent Children in the Family for
Junior Aid Applicants

Number of Dependent Children in the Family	Number of Children in College			
	1	2	3	4 or more
1	.267			
2	.141	.093		
3	.096	.078	.021	
4	.048	.057	.019	.005
5	.020	.036	.017	.005
6 or more	.021	.038	.021	.017

Source: Ref. 37.

Table B. 12:

Distribution of the Number of Children in College by the
Number of Dependent Children in the Family for
Senior Aid Applicants

Number of Dependent Children in the Family	Number of Children in College			
	1	2	3	4 or more
1	.262			
2	.136	.121		
3	.064	.107	.017	
4	.047	.063	.019	.003
5	.016	.038	.015	.010
6 or more	.017	.028	.025	.012

Source: Ref. 37.

is from summer earnings. The CSS⁽³⁵⁾ estimates that the amounts in Table B.13 are available from this source in 1970-71.

Table B.13:
Contribution From Summer Earnings (1970)

Summer	Men	Women
Prefreshman	\$400	\$300
Presophomore	500	400
Prejunior	600	500
Presenior	600	500

Source: Ref. 35

In addition to summer earnings, there will also be a contribution from student savings. Define

$SS(I)$ = the mean prefreshman student savings in 1970, as a function of the 1970 family income I .

In Table B.14 are unpublished data from CSS⁽³⁸⁾ which give the average student assets for 1969 prefreshmen students, as a function of the estimated 1970 family income. These assets were converted into 1970 dollars using the Consumer Price Index (see Table B.4). The CSS⁽³⁵⁾ estimates that

$$\frac{SS(I)}{5}$$

will be used during each of the undergraduate years. Dividing the student's assets by five allows a small reserve to begin graduate study or to provide funds until receiving income from employment. Thus the formula that will be used for $H_{1970}(I, s, k)$ is:

$$H_{1970}(I, s, k) = \left\{ \begin{array}{ll} 400 + \frac{SS(I)}{5} & \text{for } k = 0 \text{ and } s = \text{male} \\ 300 + \frac{SS(I)}{5} & \text{for } k = 0 \text{ and } s = \text{female} \\ 500 + \frac{SS(I)}{5} & \text{for } k = 1 \text{ and } s = \text{male} \\ 400 + \frac{SS(I)}{5} & \text{for } k = 1 \text{ and } s = \text{female} \\ 600 + \frac{SS(I)}{5} & \text{for } k \geq 2 \text{ and } s = \text{male} \\ 500 + \frac{SS(I)}{5} & \text{for } k \geq 2 \text{ and } s = \text{female} \end{array} \right. ,$$

where $SS(I)$ is given in Table B.16. Future values of $H_t(I, s, k)$ will be estimated with the formula

$$(B.8) \quad H_t(I, s, k) = H_{1970}(I \frac{p_{1970}}{p_t}, s, k) \cdot \frac{p_t}{p_{1970}} .$$

B.5 Estimation of $P_{c\theta si}^z$

We have defined $P_{c\theta si}^z$ to be the probability that a student attends college with living status z , given that the student is enrolled full-time at

Table B. 14:

Mean Student Assets for 1969 Pre-Freshmen Applicants
As a Function of Family Income

Estimated Net Income (1970)	Mean Assets (1969)	Estimated Mean Assets (1970)
\$ 1 - 2,999	\$ 245	\$ 260
3,000 - 4,999	279	296
5,000 - 7,499	344	364
7,500 - 9,999	399	423
10,000 - 12,499	469	497
12,500 - 14,999	538	570
15,000 - 17,499	627	664
17,500 - 19,999	756	801
20,000 - 22,499	899	952
22,500 - 24,999	1,069	1,133
25,000 - 27,499	1,169	1,239
27,500 - 29,999	1,490	1,579
30,000 - Over	1,903	2,016

Source: Ref. 38 for Estimated Net Income (1970) and Mean Assets (1969).

an institution with control c and type θ , and has sex s , and that the student's family income lies in i^{th} quartile. The probability that a student is a resident or a commuter is given in Tables B.15 and B.16, by sex, income quartile, and institution type and control. These probabilities were estimated with unpublished data from the ACE⁽¹⁹⁾ one-year follow-up of 1966 full-time entering freshmen. However, these estimates may be biased due to the poor income information and follow-up response for this survey.

B.6 Estimation of $B_{c\theta zst}(C)$

We have defined $B_{c\theta zst}(C)$ to be the conditional c.d.f. of college costs C in year t , given that the student has sex s and is enrolled full-time with living status z in an institution with control c and type θ . We will use the tuition, room, and board data published by NCES and use the estimates for the other expenses (laundry, books, recreation, etc.) that were made by CSS. The expenses for a resident student ($z = 1$) includes tuition and required fees, room, board, books, clothing, laundry, recreation, incidentals, and travel. Resident students include students living in dormitories, fraternities, sororities, or in apartments. The expenses for a commuting student ($z = 2$) include tuition and required fees, books, on-campus meals, and miscellaneous personal expenses. As we discussed in Section B.1, the expenses for a commuter should also include the maintenance expenses at home, such as room, board, laundry, medical, etc.

Table B.15:

Probability That a Full-Time Male Student is Either a Resident
or Commuter, by Income Quartile and by
Institution Type and Control

Quartile	Institution	Probability That Student Is:	
		Resident	Commuter
1	Public University	.781	.219
1	Public 4 Year	.709	.291
1	Public 2 Year	.536	.464
1	Private University	.583	.417
1	Private 4 Year	.798	.202
1	Private 2 Year	.518	.482
2	Public University	.741	.259
2	Public 4 Year	.651	.349
2	Public 2 Year	.334	.663
2	Private University	.630	.370
2	Private 4 Year	.793	.207
2	Private 2 Year	.457	.542
3	Public University	.739	.261
3	Public 4 Year	.662	.338
3	Public 2 Year	.369	.631
3	Private University	.700	.300
3	Private 4 Year	.831	.167
3	Private 2 Year	.436	.564
4	Public University	.770	.230
4	Public 4 Year	.668	.332
4	Public 2 Year	.335	.665
4	Private University	.807	.193
4	Private 4 Year	.865	.135
4	Private 2 Year	.414	.586

Source: Estimated from Ref. 19.

Table B.16

Probability That a Full-Time Female Student is Either a
Resident or Commuter, by Income Quartile and by
Institution Type and Control

Quartile	Institution	Probability That Student Is:	
		Resident	Commuter
1	Public University	.696	.304
1	Public 4 Year	.714	.286
1	Public 2 Year	.346	.654
1	Private University	.610	.390
1	Private 4 Year	.810	.190
1	Private 2 Year	.628	.372
2	Public University	.747	.253
2	Public 4 Year	.660	.340
2	Public 2 Year	.254	.746
2	Private University	.723	.277
2	Private 4 Year	.782	.218
2	Private 2 Year	.574	.426
3	Public University	.757	.243
3	Public 4 Year	.658	.342
3	Public 2 Year	.298	.702
3	Private University	.762	.238
3	Private 4 Year	.864	.136
3	Private 2 Year	.740	.260
4	Public University	.842	.158
4	Public 4 Year	.692	.308
4	Public 2 Year	.178	.822
4	Private University	.828	.172
4	Private 4 Year	.890	.110
4	Private 2 Year	.802	.198

Source: Estimated from Ref. 19.

The distributions of tuition, room, and board costs, by sex, institution type, and institution control, were published by NCES⁽³⁹⁾ for the 1968-69 academic year. Unfortunately, college cost data for years subsequent to 1968-69 are not currently available from NCES. Define

$BSC_{sc\theta}^r$ = the cost corresponding to the r th percentile of the distribution of basic student charges (tuition and required fees, room, and board) for a student with sex s attending full-time at an institution with control c and type θ , during the 1968-69 academic year,

and

$TC_{c\theta}^r$ = the cost corresponding to the r th percentile of the distribution of tuition and required fees for a student attending full-time at an institution with control c and type θ during 1968-69.

Values of $BSC_{sc\theta}^r$ and $TC_{c\theta}^r$ are given in Tables B.17 and B.18 for $r = 10, 25, 50, 75, \text{ and } 90$. The tuition cost refers to the charge to undergraduate full-time students attending college within the college's district. Out of state students may pay higher costs at public institutions.

The mean values of the tuition, room, and board distributions are also available from NCES⁽²⁾ for several years. Define

$MBSC_{c\theta t}$ = the mean charges (tuition and required fees, room, and board) for a full-time student at an institution with control c and type θ during the academic year beginning in year t ,

and

$MTC_{c\theta t}$ = the mean tuition and required fees for a full-time student at an institution with control c and type θ during the academic year beginning in year t .

Distribution of Total Tuition, Room, and Board Costs (In-District) for 1968-69

	Public University		Public Other 4 year		Public 2 year		Private University		Private Other 4 year		Private 2 year	
	Students	Cost	Students	Cost	Students	Cost	Students	Cost	Students	Cost	Students	Cost
<u>Male Students:</u>												
10th Percentile	1,072	\$ 864	944	\$ 548	783	\$ 1,920	2,860	\$ 1,295	1,600	\$ 999	1,230	\$ 999
25th Percentile	1,140	944	783	783	783	2,350	2,860	1,600	1,600	1,230	1,230	1,230
Median	1,217	1,058	941	941	941	2,860	2,860	2,000	2,000	1,500	1,500	1,500
75th Percentile	1,377	1,186	1,145	1,145	1,145	3,120	2,450	2,450	2,450	2,000	2,000	2,000
90th Percentile	1,490	1,333	1,425	1,425	1,425	3,300	2,875	2,875	2,875	2,350	2,350	2,350
<u>Female Students:</u>												
10th Percentile	1,079	860	860	525	525	1,646	1,646	1,364	1,364	1,050	1,050	1,050
25th Percentile	1,140	948	948	776	776	2,340	2,340	1,675	1,675	1,320	1,320	1,320
Median	1,217	1,058	915	915	915	2,880	2,880	2,050	2,050	1,630	1,630	1,630
75th Percentile	1,393	1,186	1,115	1,115	1,115	3,174	2,500	2,500	2,500	2,200	2,200	2,200
90th Percentile	1,490	1,333	1,338	1,338	1,338	3,300	2,950	2,950	2,950	2,750	2,750	2,750

Source: Ref. 39.

Table B, 18:

Distribution of Tuition and Required Fees (In-District) for 1968-69

	Public University	Public Other 4 year	Public 2 year	Private University	Private Other 4 year	Private 2 year
10th Percentile	\$202	\$128	\$ 20	\$ 945	\$ 615	\$ 455
25th Percentile	318	214	108	1,400	850	600
Median	371	303	180	1,750	1,125	840
75th Percentile	433	375	270	2,000	1,500	1,095
90th Percentile	540	465	350	2,150	1,850	1,459

Source: Ref. 39.



Estimates of $MBSC_{c\theta t}$ and $MTC_{c\theta t}$ for 1961-62, 1962-63, 1963-64, 1966-67, and 1968-69 are given in Tables B.19 and B.20.

For the purposes of estimating future values of $MBSC_{c\theta t}$ and $MTC_{c\theta t}$, we assume that these parameters are related as follows:

$$(B.9) \quad \frac{MBSC_{c\theta t}}{\rho_t} = \alpha_{c\theta}^1 (t - 1960) + \alpha_{c\theta}^2$$

and

$$(B.10) \quad \frac{MTC_{c\theta t}}{\rho_t} = \beta_{c\theta}^1 (t - 1960) + \beta_{c\theta}^2,$$

where ρ_t is the Consumer Price Index in year t , and $\alpha_{c\theta}^1$, $\alpha_{c\theta}^2$, $\beta_{c\theta}^1$, and $\beta_{c\theta}^2$ are calibration constants. The foregoing equations may be interpreted as estimating the real (constant dollars) increase in college costs over time. The calibration constants were estimated with regression analysis from the data in Tables B.4, B.19, and B.20 and are given in Table B.21 with the corresponding multiple correlation coefficients. The projected values of $MBSC_{c\theta t}$ and $MTC_{c\theta t}$, computed with these equations, are given in Tables B.19 and B.20.

In addition to tuition, room, and board, there are also costs for books, laundry, recreation, etc. These costs depend upon whether the student commutes to college. Define

Table B. 19:

Mean Total Tuition, Room, and Board

Year	Public University		Public Other 4 year		Public 2 year		Private University		Private Other 4 year		Private 2 year	
	\$		\$			\$599		\$1,082		\$1,570		\$1,158
1961-62	947	788	814	615	630	638	2,022	2,105	1,810	1,700	1,608	1,271
1962-63	986	846	867	710	883	2,456	2,202	2,067	2,237	1,810	1,700	1,313
1963-64	1,026	846	867	710	883	2,456	2,202	2,067	2,237	1,810	1,700	1,313
1964-65	1,051	867	947	710	883	2,456	2,202	2,067	2,237	1,810	1,700	1,313
1966-67	1,171	947	947	710	883	2,456	2,202	2,067	2,237	1,810	1,700	1,313
1968-69	1,245	1,063	1,063	883	883	2,673	2,673	2,673	2,673	2,237	2,237	1,876
PROJECTED												
1970-71	1,465	1,229	1,229	998	998	3,194	3,194	3,194	3,194	2,659	2,659	2,290
1971-72	1,551	1,303	1,303	1,067	1,067	3,405	3,405	3,405	3,405	2,841	2,841	2,470
1972-73	1,639	1,379	1,379	1,139	1,139	3,622	3,622	3,622	3,622	3,029	3,029	2,555
1973-74	1,729	1,457	1,457	1,212	1,212	3,846	3,846	3,846	3,846	3,223	3,223	2,843
1974-75	1,821	1,537	1,537	1,289	1,289	4,076	4,076	4,076	4,076	3,423	3,423	3,049
1975-76	1,915	1,619	1,619	1,367	1,367	4,313	4,313	4,313	4,313	3,628	3,628	3,255

Source: Ref. 2 for years 1961-62, 1962-63, 1964-65, 1966-67, and 1968-69;

Eqn. (B. 9) for years 1970-71 through 1975-76.

Mean Tuition and Required Fees

Year	Public University		Public		Private University		Private	
	University	Other 4 year	2 year	Other 4 year	University	Other 4 year	2 year	
1961-62	\$205	\$182	\$ 88	\$1,059	\$ 538	\$ 537		
1962-63	258	192	97	1,149	869	609		
1963-64	281	215	97	1,216	935	642		
1964-65	298	224	99	1,297	1,023	702		
1966-67	360	259	121	1,456	1,162	845		
1968-69	377	281	170	1,638	1,335	956		
PROJECTED								
1970-71	461	347	196	1,983	1,622	1,199		
1971-72	494	374	214	2,131	1,752	1,304		
1972-73	528	401	233	2,284	1,886	1,414		
1973-74	564	430	253	2,443	2,026	1,529		
1974-75	601	459	273	2,606	2,170	1,648		
1975-76	638	489	295	2,776	2,320	1,771		

Source: Ref. 2. for years 1961-62, 1962-63, 1964-65, 1966-67, and 1968-69;
Eqn. (B.10) for years 1970-71 through 1975-76.

Table B. 21:

Calibration Constants

Constant	Public		Private		Other	
	University	Other 4 year	2 year	University	Other 4 year	2 year
$\alpha_{c\theta}^1$.18254	.16961	.20227	.56904	.51979	.60833
$\alpha_{c\theta}^2$	9.00441	7.38457	5.34973	17.90700	14.45450	10.84000
R^2	.953	.997	.908	.976	.991	.987
$\beta_{c\theta}^1$.10176	.08693	.07256	.47406	.44790	.40071
$\beta_{c\theta}^2$	2.38792	1.69625	.72014	9.91167	7.50879	4.84992
R^2	.933	.996	.918	.991	.994	.992

Source: See text.

AC_2 = the costs in addition to tuition, room, and board for a student attending college with living status α in year 1970-71.

For the resident student, CSS⁽³⁵⁾ suggests using \$500 for AC_1 ; this amount includes estimates for books, laundry, clothing, recreation, incidentals, and travel. For the commuting student, CSS⁽³⁵⁾ suggests using \$300 for AC_2 ; this amount includes \$150 for books and \$150 for on-campus meals.

We plan to approximate $B_{c\theta zst}(C)$ with a piece-wise linear function connecting the values that can be estimated with the foregoing data. Define

$C_{c\theta zst}^r$ = the cost corresponding to the r^{th} percentile of the distribution $B_{c\theta zst}(C)$.

For resident students, we assume that

$$(B.11) \quad C_{c\theta 1st}^r = MBSC_{c\theta t} + AC_1 \cdot \frac{\rho_t}{\rho_{1970}} + \frac{\rho_t}{\rho_{1968}} [BSC_{sc\theta}^r - MBSC_{c\theta, 1968}] ;$$

and for commuting students, we assume that

$$(B.12) \quad C_{c\theta 2st}^r = MTC_{c\theta t} + AC_2 \cdot \frac{\rho_t}{\rho_{1970}} + \frac{\rho_t}{\rho_{1968}} [TC_{c\theta}^r - MTC_{c\theta, 1968}] + \min \left[R_t(I, \Delta, \lambda), 900 \cdot \frac{\rho_t}{\rho_{1970}} \right] .$$

As we discussed in Section B.1, the first \$900 of $R_{1970}(G, A, \lambda)$ represents the amount that the family is assumed to save if the student lives away from home. But if the student commutes, this amount must either be subtracted from the parental contribution or, equivalently, added to the expenses. The latter option was used in Eqn. (B.12). With this approach there will be no confusion in using the parental contribution to determine eligibility for Federal aid programs.

APPENDIX C

Higher Education Institutions Used in Sample

In this Appendix we list the institutions for which graduate financial aid data were collected from the Bureau of Higher Education. This list is based upon the sample of institutions used by the American Council on Education (ACE) in their student surveys. Our list is not identical to the ACE sample because not all of the ACE schools have graduate programs or submitted completed forms to the Bureau of Higher Education. The Office of Education classifies four-year institutions into two groups: universities and "all other four-year colleges." We will refer to the second group as four-year colleges. According to Ref. 22, the institutions which are classified as universities are those which give considerable stress to graduate instruction, which confer advanced degrees as well as bachelor's degrees in a variety of liberal arts fields, and which have at least two professional schools that are not exclusively technological. The schools included in these two groups change over time. We have used the classification adopted by the 1970 NCES opening fall enrollment survey reported in Ref. 54. Our sample includes thirty-nine public universities, twenty-nine public four-year colleges, twenty-four private universities, and thirty-five private four-year colleges.

C.1 Public Universities

Ball State University, Indiana

Louisiana State University - New Orleans, Louisiana

Miami University - Oxford Campus, Ohio

Montana State University, Montana

New Mexico State University - University Park, New Mexico

Pennsylvania State University, Pennsylvania

Purdue University - Main Campus, Indiana

Southern Illinois University - Carbondale, Illinois

Southern Illinois University - Edwardsville, Illinois

St. Louis University - Main Campus, Missouri

University of Akron, Ohio

University of Alabama - Huntsville, Alabama

University of Alaska, Alaska

University of California - Los Angeles, California

University of California - Riverside, California

University of California - Santa Cruz, California

University of Colorado, Colorado

University of Delaware, Delaware

University of Illinois - Medical Center, Illinois

University of Illinois - Urbana, Illinois

University of Iowa, Iowa

University of Kansas, Kansas
University of Kentucky - Main Campus, Kentucky
University of Louisville, Kentucky
University of Massachusetts - All Campuses, Massachusetts
University of Missouri - Kansas City, Missouri
University of Missouri - St. Louis, Missouri
University of Nebraska - Lincoln, Nebraska
University of New Mexico, New Mexico
University of North Carolina - Chapel Hill, North Carolina
University of North Dakota - Main Campus, North Dakota
University of South Carolina - Main Campus, South Carolina
University of Tennessee - Knoxville, Tennessee
University of Vermont and State Agricultural College, Vermont
University of Virginia - Charlottesville, Virginia
University of Virginia - George Mason College, Virginia
Virginia Commonwealth University, Virginia
Virginia Polytechnic Institute - Main Campus, Virginia
Washington State University, Washington

C.2 Public Four-Year Colleges

Alabama A & M College, Alabama
Alabama State University, Alabama
California State College - Fullerton, California

California State College, Pennsylvania
Chicago State College, Illinois
Clarion State College, Pennsylvania
Coppin State College, Maryland
East Central State College, Oklahoma
Florida Technological University, Florida
Framingham State College, Massachusetts
Ft. Hays Kansas State College, Kansas
Georgia Institute of Technology, Georgia
Classboro State College, New Jersey
Idaho State University, Idaho
Louisiana Technological University, Louisiana
Montclair State College, New Jersey
Newark College of Engineering, New Jersey
North Carolina A & T State College, North Carolina
Northwest Missouri State College, Missouri
Oakland University, Michigan
Rhode Island College, Rhode Island
Stanislaus State College, California
SUNY College - Brockport, New York
SUNY College - Geneseo, New York
University of Northern Colorado, Colorado

Virginia State College - Main Campus, Virginia
Western Carolina University, North Carolina
Western Illinois University, Illinois
Wisconsin State University - All Campuses, Wisconsin

C.3 Private Universities

Adelphi University - Main Campus, New York
American University, District of Columbia
Baylor University - Main Campus, Texas
Boston College, Massachusetts
Bradley University, Illinois
Carnegie-Mellon University, Pennsylvania
Case Western Reserve University, Ohio
Columbia University, New York
Drake University, Iowa
Emory University, Georgia
John Hopkins University - Main Campus, Maryland
Loyola University, Illinois
Loyola University, Louisiana
New York University, New York
Northeastern University, Massachusetts
Northwestern University, Illinois

Pratt Institute, New York
Texas Christian University, Texas
Tufts University, Massachusetts
Tulane University of Louisiana, Louisiana
University of Denver, Colorado
University of Rochester, New York
Vanderbilt University, Tennessee
Washington University, Missouri

C.4 Private Four-Year Colleges

Abilene Christian College, Texas
Alfred University - Main Campus, New York
Augustana College, Texas
California Institute of Technology, California
Dartmouth College, New Hampshire
De Pauw University, Indiana
Dominican College of San Rafael, California
Eastern Mennonite College, Virginia
Fisk University, Tennessee
George Williams College, Illinois
Harding College, Arkansas
Hollins College, Virginia
Huntington College, Indiana

Illinois Institute of Technology, Illinois
Lewis and Clark College, Oregon
Lincoln Christian College, Illinois
Los Angeles Baptist College and Theo. Sem., California
Marywood College, Pennsylvania
Nazareth College of Rochester, New York
Ohio Northern University, Ohio
Our Lady of the Lake College, Virginia
Philadelphia College of Pharmacy and Science, Pennsylvania
Rider College, New Jersey
Saint Mary's College of California, California
Saint Xavier College, Illinois
Springfield College, Massachusetts
St. John's University, Minnesota
Transylvania College, Massachusetts
University of Hartford, Connecticut
University of Redlands, California
Valparaiso University, Indiana
Washington and Lee University, Virginia
Wesleyan University, Connecticut
Wittenberg University, Ohio

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