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ABSTRACT

Presented is a general guide and syllabus for each of 14 secondary school mathematics courses offered by the Kent School District. Statements of the philosophy, organization, implementation, course sequences, requirements, guidance, and evaluation involved with the program are included. The courses for which a complete syllabus is offered include: Accelerated Math 7, Modern Math 7, Basic Math 7, Modern Math 8, Basic Math 8, Basic Math 9, Basic Math 1-2, Pre-Algebra 1-2, Algebra 1-2, Geometry 1-2, Algebra 3-4, Consumer Math, Trigonometry, and Mathematical Analysis. (JG)

THE PROFFESSIONAL'S GUIDE

FOR

INSTRUCTION

IN

SECONDARY MATHEMATICS

PREFACE

A Secondary Mathematics Curriculum Committee has been formed under the supervision and direction of the District Mathematics Coordinator to provide guidelines for the orderly evolution of an improved secondary mathematics program. This is a standing committee with representatives from each secondary school in the district.

The responsibilities of this Committee are to establish general principles of curriculum planning and revision; to develop and maintain the kind of curriculum that facilitates the implementation, coordination and administration of a good mathematics program; and to continue to evaluate the revised program.

The Committee has outlined, in this booklet, the courses of study offered in the secondary mathematics program of the Kent School District. This general introductory guide states the philosophy, organization, and general scope of our secondary mathematics program. The committee has and is developing, a detailed study guide for each course offered in the program. These guides are to serve as an aid to teachers in the presentation of the various secondary-level mathematics courses.

It is hoped that this series of publications will provide a basic framework within which each teacher may work and develop, through ingenuity and good teaching methods, a well-rounded satisfying program for the students he instructs.

THE SECONDARY MATHEMATICS CURRICULUM COMMITTEE

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PHILOSOPHY OF SECONDARY MATHEMATICS CURRICULUM

The Secondary Mathematics Curriculum is designed to extend to each student the opportunity:

to know as much of mathematics as he has the capability and interest to learn

to acquire the vocabulary and skills necessary to converse, read and write clearly and correctly about situations involving quantitative and spatial relations

to develop competency and accuracy in the analysis and solution of mathematical problems and to apply similar procedures to non-mathematical situations

to develop an awareness and appreciation of the pattern and structure of mathematics so that generalizations may be formulated as a foundation for further learning and practical application

to understand mathematics as a continuing creative endeavor with aesthetic values

to appreciate the interplay between the development of mathematics and the development of our cultural heritage

to appreciate the importance of mathematics to vocations, professions, and many future studies and to become aware of opportunities which are made available to individuals with various degrees of mathematics competence

ORGANIZATION

To insure the successful implementation of the Secondary Mathematics Program, the curriculum must be organized:

to provide a criteria for placement of the individual student at a level appropriate to his particular needs, interests, and abilities

to set up a basic minimum body of material to be covered and adequate standards of proficiency to be achieved in each course

to provide a continuing emphasis on desirable concepts with each successive course building on previously mastered material

to maintain adequate flexibility to allow for incorporation of recent findings of recognized authorities, new materials and teaching aids

to provide for evaluation and improvement of secondary mathematics curriculum by the Secondary Mathematics Curriculum Committee

IMPLEMENTATION

The most important single factor contributing to the effectiveness of the Secondary Mathematics Program is the classroom teacher. The teacher will aid in the implementation of this program most effectively:

by utilizing this general guide and a syllabus as developed for each course

by informing the students of grading standards and the level of performance required for successful completion of a course

by using interesting contemporary materials with challenging and appropriate mathematical content

by providing opportunities and encouragement for student imagination and creativity

by recognizing, fostering and utilizing student contributions

by making the mathematical processes meaningful rather than simply mechanical through continuing emphasis on the basic principles of mathematics and the processes of thinking which a student employs while solving a problem

by encouraging written and oral expression of thought processes in an orderly arrangement which is complete and accurate

by providing the student with adequate applications of the subject matter and with sufficient practice in problem solving to fix concepts and maintain skills

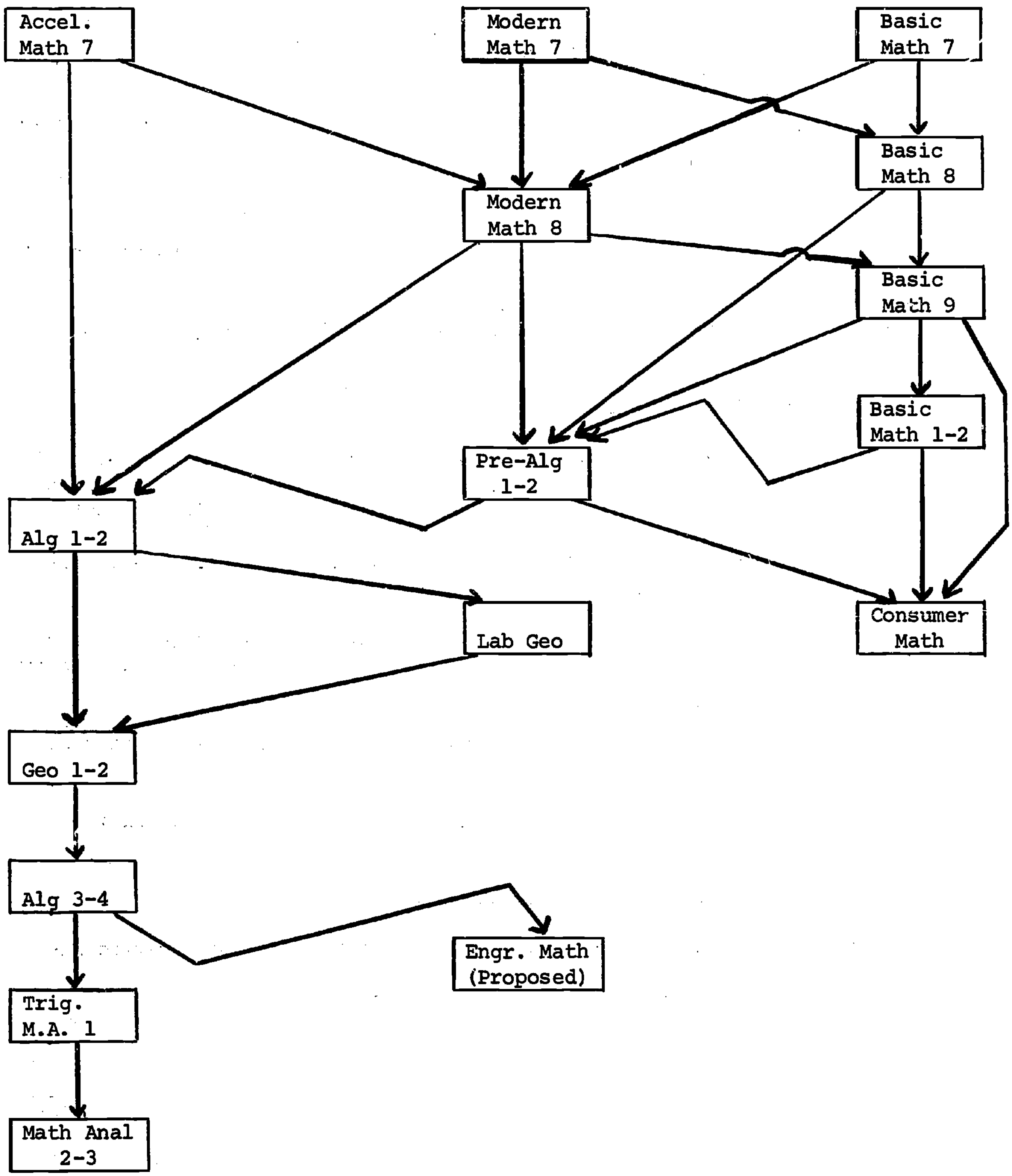
by employing a level of rigor appropriate to the mathematical maturity of the student

by utilizing tests which not only measure achievement but are of an instructional and diagnostic nature as well

by pursuing scholarship in relevant knowledge

by continuing to develop proficiency in the techniques of mathematics instruction

MATHEMATICS COURSE STRUCTURE



TYPICAL MATHEMATICS COURSE SEQUENCES

GRADE							
7	Accel. Math 7	Modern Math 7	Modern Math 7	Modern Math 7	Basic Math 7	Basic Math 7	Basic Math 7
8	Alg. 1-2	Modern Math 8	Modern Math 8	Modern Math 8	Basic Math 8	Basic Math 8	Basic Math 8
9	Geom. 1-2	Alg. 1-2	Alg. 1-2	Pre-Alg. 1-2	Basic Math 9	Basic Math 9	Basic Math 9
10	Alg. 3-4	Geom. 1-2	Lab Geom.	Alg. 1-2	Pre-Alg. 1-2	Basic Math 1-2	Basic Math 1-2 or No Math
11	Trig-Math Anal 1	Alg. 3-4	Geom. 1-2	Lab Geom	Alg. 1-2	Pre-Alg. 1-2	
12	Math Anal 2-3	Trig-Math Anal 1 or Engr. Math	Alg. 3-4	Geom. 1-2	Geom. 1-2 or Lab Geom.	Alg. 1-2	Consumer Math

These charts illustrate various course sequences available to mathematics students. It is not intended to be exhaustive as the curriculum has been structured to meet a variety of student interests and abilities.

The track which a student follows is not necessarily permanent. The student may move from one track to another or terminate his studies at any time after completing the district graduation requirement of three semesters of mathematics in grades 9 through 12.

SECONDARY MATHEMATICS COURSES

BASIC MATH 7: (2 semesters)

Purpose: To provide extensive review work for the student deficient in the basic skills of arithmetic.

Prerequisites: Seventh grade status and a grade placement two or more years below grade level as indicated by a standard test of mathematical achievement.

Topics to be included: Place value; the four fundamental operations of arithmetic as applied to whole numbers and gradually extending to fractions; measurement; principles of order; grouping, zero, one, and distributivity; elementary number theory; and an introduction to rational numbers.

MODERN MATH 7: (2 semesters)

Purpose: To provide an essential part of the mathematical background that all students will need as adults while at the same time laying a solid foundation for any future work in mathematics.

Prerequisites: Seventh grade status and mathematical grade placement above 5.0 as indicated by a standard test of mathematical achievement.

Topics to be included: A careful study of arithmetic with attention to both structure and skills; an intuitive study of the geometry of plane and space figures; and the properties of the set of whole numbers extended to the rational numbers.

ACCEL. MATH 7: (2 semesters)

Purpose: To provide an enriched and rigorous program preparing the superior student for the study of Algebra 1-2 in the 8th grade.

Prerequisites: Achievement of at least one year above grade level in mathematics and reading as indicated by a standard achievement test and preferably an I.Q. score of 115 or higher, or teacher recommendation.

Topics to be included: The development of the properties of whole, rational and real numbers; an intuitive study of the geometry of plane and space figures; and an introduction to open sentences, relations, functions and elementary probability.

BASIC MATH 8: (2 semesters)

Purpose: To continue the work of Basic Math 7.

Prerequisites: Eighth grade status and grade placement two or more years below grade level as indicated by a standard test of mathematical achievement.

Topics to be included: An extension of the Basic Math 7 topics; number bases; special products and quotients; estimation; fundamental operations with rational numbers; and decimal and percent.

MODERN MATH 8: (2 semesters)

Purpose: To continue the work of Math 7.

Prerequisites: Eighth grade status, mathematical grade placement above 6.0 as indicated by a standard test of mathematical achievement and *successful completion of a 7th grade math course.*

Topics to be included: The development of the number system; an intuitive study of the geometry of plane and space figures; an introduction to open number sentences, relations, functions and elementary probability.

BASIC MATH 9: (2 semesters)

Purpose: To provide extensive review for students deficient in mathematical skills. Although it is not intended that this course be directly followed by Algebra 1-2, hopefully, the course will serve the purpose of stimulating interest to the extent that many students will desire to continue into a pre-algebra program.

Prerequisites: Ninth grade status and grade placement two or more years below grade level as indicated by a standard test of mathematical achievement.

Topics to be included: Further study of topics developed in Basic Math 7 and 8; equations; selected topics in Euclidean Geometry; ratio; introduction to negative integers; and graphing number pairs.

PRE-ALGEBRA 1-2: (2 semesters - grades 8-12)

Purpose: To provide an adequate background for students who wish to continue in mathematics but who are inadequately prepared for Algebra 1-2.

Prerequisites: Completion of Math 8; completion of Basic Math 9 or Basic Math 1-2 with a grade of C or better; or teacher recommendation from Algebra 1-2.

Topics to be included: Basic operations and concepts of Algebra including set theory and notation; mathematical expressions and sentences; logic; factoring of integers; number line concepts; the real number system and exponential notation.

ALGEBRA 1-2: (2 semesters - grades 8-12)

Purpose: To provide an adequate knowledge of fundamental algebraic concepts both as a preparation for subsequent studies and as a basic mathematical tool.

Prerequisites: Completion of one of the following:

a. 7th grade accelerated math with at least a B grade and satisfactory performance on a standard algebra prognostic test.
b. 8th grade math with at least a C grade and satisfactory performance on a standard algebra prognostic test.
c. Pre-Algebra 1-2 with at least a C grade.

Topics to be included: Sets; logic; real numbers; equations and inequalities; factors and exponents; polynomials; the real number plane; radicals; and functions and realtions.

BASIC MATH 1-2 (2 semesters - grades 10 or 11)

Purpose: To provide remedial mathematical training for students deficient in mathematical skills.

Prerequisites: At least 10th grade status. This course is closed to students who have completed Algebra 1-2 or have completed Pre-Algebra 1-2 with a grade of C or better.

Topics to be included: Operations and concepts related to the basic arithmetic fundamentals and their application to fractions and decimals. Beyond this the student may pursue work in measurement and basic algebraic operations and concepts.

LAB GEOMETRY: (1 semester - grades 10-12)

Purpose: To provide an intermediate course of study between Algebra 1-2 and Geometry 1-2 for the student who requires more background in elementary space relationships and the basic concepts of abstract math. For those who have experienced difficulties with Mathematics but desire to take further math, Lab Geometry serves as a basic introduction to Geometry 1-2.

Prerequisites: Completion of Algebra 1-2 and teacher recommendation based upon achievement in Algebra 1-2 and a Geometry prognosis exam.

Note: After completion of Lab Geometry, Geometry 1-2 must be taken and satisfactorily completed to meet minimum college entrance requirements.

Topics to be included: Systems of linear and angular measurements; basic properties of polygons including areas and related algebraic computations; spatial equalities and inequalities; basic properties of circles including relationships of angles and circles; basic coordinate geometry.

GEOMETRY 1-2: (2 semesters - grades 9-12)

Purpose: To provide the students with an awareness of the structure and patterns of mathematics; to develop the students' capacity for analysis and orderly, consistent thinking; and to acquaint the student with the properties of plane and solid figures as a foundation for further studies.

Prerequisites: Satisfactory completion of Algebra 1-2. Students who elect to take Lab Geometry prior to Geometry 1-2 should achieve a grade of C or better in Lab Geometry.

Topics to be included: Point-line relationships; development of properties of plane and solid figures; distinction between inductive and deductive thought processes with extensive applications of both; areas of polygons and circles; similarigy; ratios and proportions; and coordinate geometry.

ALGEBRA 3-4: (2 semesters - grades 10-12)

Purpose: To provide a rigorous course primarily orientated toward the needs of those students planning to continue their education in a four year college program.

Prerequisites: Algebra 1-2 and Geometry 1-2 with a grade of "C" or better, or departmental approval.

Topics to be included: Further development of concepts from Algebra 1-2; real and complex number systems; polynomials and rational expressions; functions and relations; exponents and radicals; coordinate geometry; quadratic, exponential and logarithmic functions; systems of equations and inequalities; theory of equations; sequences, series and the binomial theorem; and permutations, and combinations.

TRIG-MATH ANAL 1: (2 semesters - grades 11-12)

Purpose: To provide a highly structured course of study primarily for students preparing for further academic work in mathematics, science or other technical areas.

Prerequisites: Completion of Algebra 3-4 with a grade of "C" or better, or departmental approval.

Topics to be included: Further development of concepts from Algebra 3-4; set theory; the real numbers; functions (exponential, logarithmic, and trigonometric); triangle solutions and vectors; inverse functions; complex numbers and polar coordinate forms; polynomial functions; sequences and mathematical induction; and matrices and determinants (systems of equations).

MATH ANALYSIS 2-3

Purpose: To provide the superior mathematics student with a preparation for calculus and a preview of advanced college-level mathematics.

Prerequisites: Completion of the mathematics course sequence through Trig-Math Analysis 1 with at least a "B" grade average and departmental recommendation.

Topics to be included: Extension and correlation of concepts developed in previous mathematics courses and an introduction of topics from modern abstract algebra, probability, analytic geometry, theory of limits, and integral and differential calculus.

CONSUMER MATH: (1 semester - grade 12 ONLY)

Purpose: To provide a course for seniors who need an additional mathematics credit to meet minimum high school graduation requirements or who desire some background in basic consumer mathematics.

Prerequisites: Twelfth grade status. This course is closed to students who have successfully completed Geometry 1-2.

Topics to be included: Topics will be selected from programmed materials on consumer buying, household budgeting, payroll and checking accounts, interest rates, taxes, savings and investments.

ENGINEERING MATHEMATICS 1-2 (Proposed)

REQUIREMENTS

The secondary mathematics program in the Kent School District is designed to offer each student a maximum amount of mathematics commensurate with his abilities and interests. A student must complete a mathematics course in grades 7, 8, and 9 with successful completion of three semesters of mathematics in grades 9-12 to satisfy a district graduation requirement.

College entrance requirements are greater than those for graduation from high school. Colleges and universities vary widely in their mathematics requirements for admission. A student planning to enter college should acquaint himself with the requirements for the college of his choice either by consultation with the counselor or by direct communication with the college. In general, three years of mathematics which should include Algebra 1-2, Geometry, and Algebra 3-4 are strongly recommended.

GUIDANCE

A function of the guidance program is to guide each student into a mathematics course which is consistent with his background, his maturity level, his vocational interests, and his level of ability. Guidance in mathematics is primarily the responsibility of the classroom teacher with assistance from the counselor, the department chairman, and the mathematics coordinator.

It should be recognized that various factors (parental pressures, status, and self-established goals) may cause a student to seek enrollment in a course for which he does not meet the prerequisites. If the student insists on taking the higher course after individual counseling, he should have the opportunity to do so. However, if unsuccessful during a trial period, he should be rescheduled into a course more consistent with his ability.

EVALUATION

In order to determine the effectiveness of the total secondary mathematics curriculum in the Kent School System, it is necessary that continuing evaluation procedures and practices be established. This evaluation will provide opportunity for indicated revisions to insure maintenance of a sound mathematics program.

Three basic areas to be covered are:

performance and attitude of the students including development of basic skills, understanding of fundamental concepts, applications to problem solving, and general enthusiasm for mathematics

administration of the program including consistency, flexibility, and provision for individual differences

teachability and suitability of materials and textbooks in classroom use

Among the techniques which might be employed in carrying out this evaluation are standardized tests, questionnaires to graduates of the Kent Schools, and professional judgment of teachers and administrators.

PREFACE

This resource guide for Accelerated Modern Math 7 is organized as follows:

1. A recommended sequence of units with a suggested time allotment for each unit.

2. Unit Outlines
   a. Specific objectives to be realized by the student on completion of the unit.
   b. A detailed breakdown of the unit into sub-units with suggestions for developing topics and hints for the teacher in presentation of topics.

The time allotment for each unit is included only as a guideline for the teacher. The specific objectives of each unit set a basic body of material to be covered in Accelerated Modern Math 7 and the standard of proficiency that should be expected from each student in the Kent School District.

The fact that this guide can be beneficially revised will become evident with use. We hope that teachers using it will critically analyze it and make comments and suggestions about its utility, content and structure.

COMMITTEE FOR ACCELERATED MODERN MATH 7 GUIDE

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INTRODUCTION

Statement of Purpose

The purpose of the accelerated course of Modern Mathematics 7 is to provide an essential part of the mathematical background that all students will need as adults, and, at the same time, build a solid foundation for any future work in mathematics. The contents of Accelerated Mathematics 7 provides for the pupil's steady growth in thinking mathematically, understanding the basic concepts, appreciating structure and pattern, and increasing proficiency in using mathematics. Specifically, students are prepared for a course in Algebra 1-2 in the eighth grade.

Objectives

The specific objectives of Accelerated Modern Mathematics 7 are to

- provide students with an understanding of the structure of the set of the real number system
- provide the students with an intuitive background in both metric and non-metric geometry in two and three dimensions
- develop the students' proficiency in using mathematical symbols
- develop the students' skill in computation and in simplification of expressions involving operations with real numbers
- enable students to apply their knowledge of the structure of the real number system, and the methods for the solution of equations in one variable, in problem-solving situations

Prerequisites and Selection of Students

Seventh grade status, achievement of at least one year above grade level in mathematics, and reading as indicated by a standard achievement test, as well as an IQ score of 115 or higher or teacher recommendation.

RECOMMENDED TEXTS AND REFERENCE MATERIALS

Basic Textbooks:

Dolciani, Wooton, Beckenbach, Chinn *Modern School Mathematics - Structure and Method*, Book 7; Houghton Mifflin Co. (Boston, 1967)

Dolciani, Wooton, Beckenbach, Makert *Modern School Mathematics - Structure and Method*, Book 8; Houghton Mifflin Co. (Boston, 1967)

Supplementary References:

Bergamini *Mathematics;* Time Inc. (New York, 1963)

Duncan, Capps, Dolciani, Quast, Zweng *Modern School Mathematics - Structure and Use;* Houghton Mifflin Co. (Boston, 1967)

Eicholz, O'Daffer, Brumfiel, Shanks, Fleenor *School Mathematics I;* Addison-Wesley Publishing Co. (Palo Alto, 1967)

Eicholz, O'Daffer, Brumfiel, Shanks, Fleenor *School Mathematics II;* Addison-Wesley Publishing Co. (Palo Alto, 1967)

Gundlach, Buffie, Denny, Kempf *Junior High School Mathematics;* Laidlaw Brothers (River Forest, Illinois, 1968)

Gardner *Scientific American Book of Mathematical Puzzles and Diversions;* Simon and Schuster (New York, 1959)

Gardner *Second Scientific American Book of Mathematical Puzzles and Diversions;* Simon and Schuster (New York, 1961)

Nichols, Flournoy, Kalin, Simon *Elementary Mathematics - Patterns and Structure;* Holt, Rinehart, Winston (New York, 1966)

Parker *The Structure of Number Systems;* Prentice-Hall (Englewood Cliffs, 1966)

Scheid *Elements of Finite Mathematics;* Addison-Wesley Publishing Co. (Reading, Mass., 1962)

School Mathematics Study Group *Mathematics for Junior High School, Volumes I and II;* Yale University Press (New York, 1961)

School Mathematics Study Group *Probability for Intermediate Grades;* Stanford University (Palo Alto, 1965 and 1966)

School Mathematics Study Group *Reprint Series;* Stanford University (Palo Alto, 1967)

Smith *Explorations in Elementary Mathematics;* Prentice-Hall (Englewood Cliffs, 1966)

Wirtz, Botel, Numley *Discovery in Elementary School Mathematics;* Encyclopedia Britannica Press (Chicago, 1963)

SUGGESTED TIME ALLOTMENTS

UNIT	First Semester	Suggested Time *
I	Sets and Numbers	4 days
II	Properties of Addition and Subtraction in the Set of Whole Numbers	4 days
III	Properties of Multiplication and Division in the Set of Whole Numbers	4 days
IV	Numbers and Numerals	4 days
V	Algorithms of Arithmetic	4 days
VI	Sets and Geometry	7 days
VII	Number Theory	8 days
VIII	Line and Angle Relationships	6 days
IX	Coordinate Systems on a Line	6 days
X	Fractions and Rational Numbers	6 days
XI	Decimals for Rational Numbers	6 days
XII	Measurement and Geometry	9 days
XIII	Percentage and Statistics	4 days

**This schedule gives 149 total days with the omission of the two optional chapters. Individual teachers may wish to vary the time allotments as necessary.*

SUGGESTED TIME ALLOTMENTS

UNIT	Second Semester	Suggested Time
XIV	The Set of Integers	5 days
XV	Rational Numbers	4 days
XVI	Addition and Subtraction of Rational Numbers	5 days
XVII	Multiplication and Division of Rational Numbers	4 days
XVIII	Geometric Figures in the Plane	3 days
XIX	Congruence and Measurement of Plane Figures	5 days
XX	Exponents and Scientific Notation	4 days
XXI	The Metric System	5 days
XXII	Precision and Accuracy	Optional
XXIII	Decimal Numerals and Real Numbers	5 days
XXIV	Open Number Sentences	5 days
XXV	Solving Open Number Sentences	5 days
XXVI	Using Equations	5 days
XXVII	Square Roots; Similar Figures	6 days
XXVIII	Pyramids and Prisms	6 days
XXIX	Cones, Cylinders, and Spheres	5 days
XXX	Relations, Functions, and Graphing	5 days
XXXI	Probability	Optional

UNIT I: SETS AND NUMBERS

Chapter 1 in Dolciani, Book 7
(4 days)

On completion of Unit I, the student should be able to

write relations between sets using the symbols of set notation ($\{\}$, $\cap$, $\cup$, $=$, $\neq$, $\in$, $\notin$, $\emptyset$)

form intersections and unions of given sets

form a one-to-one correspondence between two given sets

distinguish between equivalent sets and equal sets

use both the rule method and the roster method to identify finite and infinite sets

correctly classify sets as infinite or finite

correctly draw Venn diagrams showing given relations between sets

write all of the subsets of a given finite set

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Sets and Their Members	6	You will probably find that most of this unit will be a review for your students. You might determine how much they know by giving a pretest very similar to the chapter review before you plunge into the chapter. If test results are favorable, it is suggested that the first two sections be covered in one day and the remaining sections be covered the next day. Do not let the students become lazy in reading mathematical statements. For example, for the symbols $\{1,2\}$ read, "The set whose members are 1 and 2."
Sets and Subsets	10	The symbol for subset in this text is $\subset$; in the Pearson-Allen Algebra 1-2 textbook, it is $\subseteq$. In this text, if we wish to distinguish between proper and improper subsets, we will say so. Remember, $\emptyset$ and $\{\emptyset\}$ are not the same. You may wish to lead students to the discovery that a set of n elements has 2^n different subsets.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Comparing Sets	14	Students may be intrigued by those questions such as Exercise 16 which involve comparing an infinite set and one of its infinite subsets.
Intersection of Sets	19	Spend plenty of time with the students on diagrams of (BUA) ∩ C, etc.
Union of Sets	23	There is not enough stress on identification of ∩ and ∪, and differentiation between ∩ and ∪, especially in graphs.

You may wish to ask students to investigate, using Venn diagrams, the following questions:

1. Is finding the union or intersection of two sets a commutative operation? ($A \cup B = B \cup A$? $A \cap B = B \cap A$?)
2. Is taking the union or intersection of sets an associative operation? [$(A \cup B) \cup C = A \cup (B \cup C)$?]
3. Is there an identity element of set union? Of set intersection?
4. Is taking the union of sets distributive over taking the intersection of sets? [$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$?] Is taking the intersection of sets distributive over taking the union of sets? [$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$?]

You may also wish to discuss the idea of the universe $\underline{U}$ and of complements of sets. (The complement of a set A is denoted either by A' or cA). Here are relationships you might discuss with the students, using Venn diagrams.

$$A \cup A' = U$$
$$U \cup A = U$$
$$U \cap A = A$$
$$(A')' = A$$
$$(A \cup B)' = A' \cap B'$$
$$(A \cap B)' = A' \cup B'$$
$$\emptyset' = U$$
$$U' = \emptyset$$

UNIT II: PROPERTIES OF ADDITION AND SUBTRACTION
IN THE SET OF WHOLE NUMBERS
Chapter 2 in Dolciani, Book 7
(4 days)

On completion of Unit II, the student should be able to

insert >, <, =, +, or -, in a given set of incomplete whole-number statements, to make the statements true

identify, by listing, all the whole-number values that make true a given equation or inequality involving whole numbers

write an equation based on the whole-number facts stated in a given story problem and solve the equation, listing the properties (by name) illustrated in each step of its solution

write, using whole numbers, examples of the commutative and associative properties of addition

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Unit II, unlike Unit I, will be mostly review for the students. Again, it is suggested that you give the class a pretest to see just how much time you should spend on the unit. If results of your preassessment are favorable, allot four days for the unit. Pay special attention to the C exercises throughout.
Using Sets in Addition	34	Students are hard pressed to think of <u>any</u> arithmetical operation that is <u>not</u> binary. A <u>unary</u> operation worth mentioning is the correspondence of a set to its complement or a counting number to its reciprocal.
		"Simplify" and "get the answer" are <u>not</u> synonymous. The word "simplify" should evoke a response involving the replacement of an expression with a simpler name for the expression. By doing so, the implication drawn is that a correspondence exists between the expression and its replacement. "Get the answer" is an ambiguous command implying only that the operations be performed.
		As an application of addition, ask students to study the number pattern and try to find a rule to determine the sums without adding.

UNIT II (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS

Using Sets in Addition (cont.) — 34

$$\begin{aligned} 1 + 3 &= 4 \\ 1 + 3 + 5 &= 9 \\ 1 + 3 + 5 + 7 &= 16 \\ 1 + 3 + 5 + 7 + 9 &= 25 \end{aligned}$$

As another application of addition you might have students complete magic squares such as the one below. Explain that they are to insert numbers from 1 through 16 just once so that the sum of each row, column, diagonal, and the four corners is the same.

1	8		15
	13	3	
7			9
		5	4

The Commutative Property of Addition; Variables — 40

Emphasize that a variable represents all the elements of a particular set of elements making a statement true. Frequently, a variable is thought of as denoting a single number.

Students will consider classifying operations as commutative quite useless, unless they see that there are some operations which are not commutative. Ask them to make up operations that are not commutative.

Using Sets in Subtraction — 44

An additional type of problem you might use here:

$$\triangle + \square = 59$$
$$\triangle - \square = 15$$

What are $\triangle$ and $\square$?

The Associative Property of Addition. — 50

Symbols of association imply *an order of performance* of operations. The expression 6 - (4 + 1) requires, first, that "4 + 1" be replaced by its new corresponding sum.

Be sure students can show by example why subtraction is not associative. Ask them to "invent" other operations which are not associative.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Comparing Whole Numbers	56	See TM, page 13. The symbols < and > are also called "ordering" symbols. "Why isn't ≠ an ordering symbol?" Exercises 29-32 are examples of the <u>transitive</u> property of inequalities.

UNIT III: PROPERTIES OF MULTIPLICATION AND DIVISION IN THE SET OF WHOLE NUMBERS

Chapter 3 in Dolciani, Book 7
(4 days)

On completion of Unit III, the student should be able to

represent a given product of the form a x b = c, where a, b, and c are whole numbers, both as a repeated sum and as an array

identify correctly the product and factors, given a multiplication problem in the form a x b = c

identify correctly the dividend, divisor, numerator, denominator, and quotient when given an equation of the form a/b = c, where a and c are whole numbers and b is a natural number

match each of a given list of whole number equations with one or more of the following properties

- Commutative Property of Multiplication
- Associative Property of Multiplication
- Additive Property of 0
- Subtraction Property of 0
- Multiplicative Property of 1
- Multiplicative Property of 0
- Division Property of 1
- Division Property of 0
- Distributive Property

write an equation of the form a x (b + c) = a x b + a x c, where a, b, and c are whole numbers, as an array

write the division statements corresponding to a given multiplication statement, and vice versa

simplify numerical expressions involving addition, subtraction, multiplication, and/or division of whole numbers

solve word problems involving addition, subtraction, multiplication, and/or division of whole numbers

UNIT III (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		This unit may be chiefly a review for the pupils. Pretest as suggested in the discussion for Units I and II to see how much time you should spend. It is suggested that Section 3-5 be stressed. Even superior students have trouble with the distributive property.
Using Sets in Multiplication	70	Mention operations which are <u>not</u> commutative and associative such as the operation $*$, where $a * b = 2b$. Mention operations which are not binary such as the operation of squaring a number, which is unary, or the operation of finding the smallest of three numbers, which is ternary.
Using Sets in Division	75	Show that division can be thought of as repeated subtraction, just as multiplication can be thought of as repeated addition. For instance, $6 \div 2 = 3$, since $\underset{1}{\underline{6 - 2 = 4}}$, $\underset{2}{\underline{4 - 2 = 2}}$, $\underset{3}{\underline{2 - 2 = 0}}$. Show that division may be thought of as grouping an array. For instance, $6 \div 2 = 3$ since there are three groups of 2 in 6.
Simplifying Expressions	81	Tell students of the convention concerning order of operation when no grouping sumbols are present: Multiplications, divisions, additions, and then subtractions. Emphasize the clarity resulting if grouping symbols are used. Students may enjoy a "contest" in which they are given some digits, say 2, 3, 5, 6, 15, and the operations +, −, x, ÷, along with a pair of parentheses and are asked to find as many different expressions as they can using the given symbols.
The Properties of 0 and 1	84	Stress the difference between 0/a and a/0. Show that both division properties may be checked by multiplication. Mention similarities of the identity elements of addition, subtraction, multiplication, and division.

UNIT III (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Distributive Properties	89	This is a hard concept for many students. Colored chalk for blackboard examples might help: $\underline{4}$ (3 + 2) = $\underline{4}$ x 3 + $\underline{4}$ x 2 (underlining indicates same color). Stress that work can be checked by performing operation in parentheses first. That is, 4 (3 + 2) = 4 (5) = 20; 4 x 3 + 4 x 2 = 12 + 8 = 20. Warn students of a common mistake: 3 (5 + 2) = 3 x 5 + 3 x 2, <u>not</u> 3 x 5 + 2.

UNIT IV: NUMBERS AND NUMERALS
Chapter 4 in Dolciani, Book 7
(4 days)

On completion of Unit IV, the student should be able to

change Roman and Egyptian numerals to decimal numerals and decimal numerals to Roman and Egyptian numerals

write, if possible, any whole number in the form a^n, where a is a whole number and n is a counting number, and vice versa

write numerals in any base in expanded notation

write a numeral in a base other than ten as a base ten numeral

write a base ten numeral as a numeral in any given base

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Primitive Numbers (Optional)	104	Students should not be expected to memorize chart on page 105.
Roman Numerals (Optional)	107	Emphasize the principle of multiplication on page 109. The students probably have not seen it before.
Exponents and Powers	112	Be sure the students know that 2^3 = 2 x 2 x 2 and not 2 x 3.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Exponents and Powers (continued)	112	Students may be able to guess the simple numeral corresponding to 10^0. Do not discourage this guessing! Extending ideas is most important in mathematics. Some additional problems you may give students: 1. Find two different numbers x and y, so that $x^y = y^x$. (Answer: $x = 2$, $y = 4$.) 2. What number is 2^{2^3}? (Answer: $2^8 = 256$.) 3. Find n so that $4^3 = 2^n$. (Answer: $n = 6$.)
Base-Two Numeration	115	See TM, page 18, for some examples of "bicimal" fractions in case some students ask about them. Students who wish to read more about the use of base-two numerals in digital computers should be referred to the discussion on computing in the Life Science Library book, *Mathematics*. Mention that base-eight (octal) numbers are also used in computers.
Decimal Numerals	120	Point out to the students that a decimal numeral need not have a decimal point. An interesting point is described on page 19 in the TM concerning American and British terminology for numerals over one million.
Numerals in Other Place-Value Systems (Optional)	124	See TM, page 20, for a different method of converting base ten numerals to numerals in other bases.

UNIT V: ALGORITHMS OF ARITHMETIC
Chapter 5 in Dolciani, Book 7
(4 days)

On completion of Unit V, the student should be able to

simplify, using the appropriate algorithms, expressions involving decimal numerals or numerals in different bases

write the sum of two differences corresponding to a given difference of two sums involving whole numbers, or vice versa

show a complete check of the results of simplification of an expression involving multiplication or division.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Much of the material, with the exception of operating in different bases, in this unit will be review for your students. If you skipped the sections listed as optional in the previous unit, you will want to assign only the reading and problems which concern arithmetic algorithms for decimal numerals. Stress the <u>whys</u> of the algorithms throughout.
The Addition Algorithm	138	Pupils have difficulty with addition problems in other bases. Treat these problems with a light touch. Caution students that two numerals must be in the same base before they can be added together. Bright students may be fascinated, as well as frustrated, by this classic test of understanding of the addition algorithm. $\begin{array}{r} \text{SEND} \\ +\ \underline{\text{MORE}} \\ \text{MONEY} \end{array}$ Here the task is to replace each letter with a digit so that no two letters represent the same digit.
The Subtraction Algorithm	144	
The Multiplication Algorithm	149	Some automatic calculators work on the principle of Mr. Sand's computer.

<table>
<tr><th>TOPICS</th><th>PAGE</th><th>COMMENTS AND SUGGESTIONS</th></tr>
<tr><td>The Multiplication Algorithm (Continued)</td><td>149</td><td>Multiplying in bases other than 10 will be difficult for most students. Be sure they realize that two numerals must be in the same base before they can be multiplied together.

For variety you may want to ask students to fill in each blank with a digit so that the answers are correct.<pre>
                     - - -
     - -   x 9 9 9   9 9
   x 5 4   - - - -   x - -
   - - -   - - - -   - - -
 - - -   - - - -   - - -
 3 - 3 4   4 7 - - - 4   2 2 - -
</pre></td></tr>
<tr><td>The Division Algorithm</td><td>155</td><td>Provide some work with crytography if more drill or motivation is needed for any or all of the four algorithms.

Again, dividing in bases other than 10 will be hard for many students.

Since much planned practice is required to reinforce and maintain skills required, daily 5 to 10 minute computation drills may be extremely beneficial to your students. The advantage of providing this practice session at the beginning of class period is that it will encourage pupils to start work without delay. Each teacher will wish to tailor his own daily computation review to the requirements of his pupils. Types of exercises should vary from day to day.</td></tr>
</table>

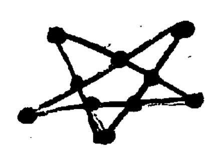
UNIT VI: SETS AND GEOMETRY
Chapter 6 in Dolciani, Book 7
(7 days)

On completion of Unit VI, the student should be able to

select names, numbers, and phrases from a list and insert them correctly in incomplete sentences relating to the terms and geometric properties discussed in this unit

classify given geometric sets as either infinite, finite but not empty, or empty

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Even your accelerated students will find many new concepts in this unit. Nevertheless, try to cover a section per day. Help students extend, generalize, and apply the ideas described wherever possible.
		Psychomotor activities are essential to the comprehension of geometric concepts. You will notice the profusion of illustrations in this unit, but total reliance on these planar illustrations of spatial ideas often frustrates the learning process. Individual student "geometry kits" consisting of several sheets of stiff paper or cardboard and straws or dowels will suffice. After an informal treatment of the Euclidean concepts presented in the unit, students should demonstrate with cardboard and dowels geometric situations--such as the possible intersections of parallel planes and a line. The "kits" can be retained for use during discussions of the geometric properties outlined in this unit.
Points and Space	168	If a figure is a subset of space, is the set containing all points in space a figure? And what of the set containing no points?
		An additional thought problem: Arrange ten dots in five rows with four in each row. Answer:



TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Lines	174	What does "determine" mean? Replacement with the more familiar and descriptive "locate" might be appropriate.

Mention that *at the very least* two points determine (or locate) a line in space.

Warn students that it is agreed to use the symbol ϵ, <u>not</u> the symbol $\subset$ to say that a point lies on a line.

Your students may discover (It's rather unlikely you say?) an interesting phenomenon concerning nonplanar figures such as those shown in Exercises 14, 17, 18, 20, 21, and 22 on pages 180–181.

The following table illustrates this phenomenon.

Points	Lines
4	6
5	10
6	15
7	21
8	28

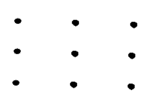
Have students look for a pattern. How many lines are determined by a ten-cornered nonplanar figure?

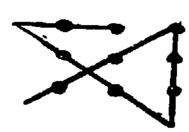
Also note the example on page 180. Students are directed to identify all lines determined by corner points. Emphasize that this means identifying even those lines not shown (for instance, $\overleftrightarrow{DB}$ in the example).

Extra problems you may wish to give your class:

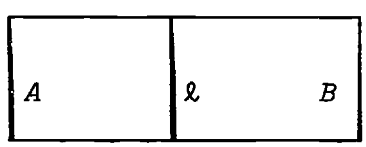
How many lines would you get from five points if no three of them were on the same line?

Draw four line segments, without lifting your pencil, so that each dot lies on one of the segments.

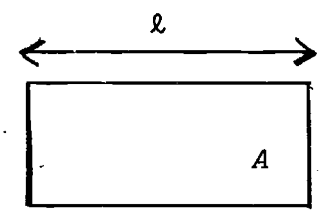


Answer: 

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Planes	182	Explain carefully the directions accompanying the written exercises on page 185. Common errors: R $\subset P$ instead of R $\in P$; $\ell \in P$ instead of $\ell \subset P$. Tell students that it has been agreed to use the symbol $\in$ when speaking of a point contained in a plane and to use the symbol $\subset$ when referring to a line lying in a plane. An additional question you may pose: How many planes are determined by five points? (Answer: 10)
Determining Planes	186	Again, the more familiar "locate" may be desired in place of "determine." Do Exercises 9-16 on page 190 before assigning them to students.
Intersections of Lines and Planes	192	Note the error in the illustration on page 194. Plane Z is denoted by the symbol "T". Do not assign the written exercises on pages 196 and 197 for homework. Invariably, students have difficulty drawing good three-dimensional sketches. Common errors to watch for:



Plane $A \cap$ Plane $B = \ell$



Plane $A \parallel \ell$
Plane $A \cap \ell = \emptyset$.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Extending Your Vocabulary	197	Remind students that Exercises 1-12 on pages 202 and 203 require identification of lines determined by points even though the lines are not drawn. Another reminder: Lower case italicized letters denote lines and script letters denote planes. Ask students to discuss the existence or non-existance of skew planes.

UNIT VII: NUMBER THEORY
Chapter 7 in Dolciani, Book 7
(8 days)

On completion of Unit VII, the student should be able to

identify the partial quotient and remainder, if there is one, in division problems of the form a/b, where a is a whole number and b is a counting number of less than four digits

classify a given number as prime or composite and odd or even

identify by roster the set of all divisors of a given whole number

state whether given whole numbers are divisible by 2, 3, 4, 5, 9, and/or 10, by using the corresponding divisibility tests

write the prime factorization of a given whole number

identify the GCF of a given pair of whole numbers by using the roster method

write the set of all factors of a given whole number using the prime factorization of the number

identify the LCM of a given pair of whole numbers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Extending the Division Algorithm	212	As an application, you may wish to lead students to the discovery that the square of an odd number when divided by 8 leaves 1 as a remainder. Also, you may want to lead students to the discovery that the square of a number is either divisible by 4 or leaves the remainder 1 when divided by 4. As an additional problem, you might ask students to find digits to replace the letters so that a correct division problem will result.

```
     b b
a ) b c c     (Answer:  a = 9, b = 1, c = 0).
    a
    ---
     b c
       a
     ---
       b
```

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Prime Numbers	216	Stress that there is no largest prime. Some students may be able to figure out why this is true. Emphasize that 1 is not considered a prime since it is divisible by itself only and since otherwise there would be no unique prime factorization of natural numbers. For example, if 1 were considered prime, we would have $6 = 3 \times 2 = 3 \times 2 \times 1 = 3 \times 2 \times 1^2$, etc. Students often confuse prime numbe[illegible]th odd numbers. Stress that all primes e[illegible] 2 are odd, but that the reverse is definitely not true. That is, all odd numbers are not prime.
Using Divisibility Tests	221	You may wish to lead students to the discovery that the last digits in the square of any number must be 0, 1, 4, 5, 6, or 9. The following problem will give students a chance to use reasoning involving their knowledge of tests of divisibility and prime numbers. Each letter names a prime number. $a \cdot b \cdot c = 1001$. Find a, b, and c. (Answer: 7, 11, 13)
Developing Divisibility Tests (Optional)	227	
Prime Factorization	231	Emphasize that a systematic approach to testing the primes in increasing order makes finding prime factors easier. Stress that each natural number has only one prime factorization.
Greatest Common Factor	235	You may wish to point out that finding the GCF of three numbers is an example of a <u>ternary</u> operation. Stress that two or more whole numbers may have many common factors, but only <u>one</u> greatest common factor.

UNIT VII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Greatest Common Factor (Continued)	235	Emphasize that two prime numbers are relatively prime, and that two numbers, *not necessarily prime*, may also be relatively prime. An example is 49 and 24: GCF (49,24) = 1. An additional problem: 12 was called an abundant number by Greek mathematicians, because the sum of its factors is greater than 12 + 12. 1 + 2 + 3 + 4 + 6 + 12 > 12. 12 is the smallest abundant number. What are the next two abundant numbers?
Euclidean Algorithm (Optional)	240	Emphasize the generality of this method. Tell students that this method may be used to check the results obtained when using the roster method. Outline the proof of this method, given on page 30 of the TM, for superior students.
Least Common Multiple	241	The generalization in Exercise 21, page 245, may be of interest even to the slower average student: GCF (a,b) x LCM (a,b) = a x b, where a and b are whole numbers. This property will help students check their work or to find the GCF of two numbers given the LCM (or vice versa).

UNIT VIII: LINE AND ANGLE RELATIONSHIPS
Chapter 8 in Dolciani, Book 7
(6 days)

On completion of Unit VIII, the student should be able to

identify from a diagram collinear, equal, opposite, parallel, skew, and coplanar rays

identify from a diagram an angle, the interior of an angle, the exterior of an angle, adjacent angles, and vertical angles

match, with its definition, each of the entries in the vocabulary and spelling list at the end of the chapter

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Separating a Line	252	The use of visual aids will be quite helpful in this chapter. Try to make them big enough for all to see. It may even help for all students to touch the geometric models. As a project, let some students make the models. Board work and student participation will be very useful in this chapter, also. Excellent models may be made easily and cheaply using transparent or colored lightweight plastic and masking tape.
Line Segments	257	Stress that $\overleftrightarrow{AB} = \overleftrightarrow{BA}$, $\overline{AB} = \overline{BA}$, but $\overrightarrow{AB} \neq \overrightarrow{BA}$.
Separations: Plane and Space	262	As an extension of the concept of a plane separating space into three sets, you might ask students to name two other types of space figures that separate space into three sets. (Some possible answers: spheres, rectangular solids.)
Congruent Segments and Angles	273	Stress the difference between "equal" and "congruent."

UNIT IX: COORDINATE SYSTEMS ON A LINE
Chapter 9 in Dolciani, Book 7
(6 days)

On completion of Unit IX, the student should be able to

graph a given number or a given set of numbers on a number line

state the coordinates of given points on a given number line using fractions if necessary

state the addition, subtraction, multiplication, or division fact pictured in a given number line diagram, and vice versa

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Numbers and Points on a Line	290	Much of this section will be a review. It should be emphasized that any pair of points may be used as endpoints for a unit segment, and that students should choose such points for their own convenience, depending upon what they want to show. Be certain to assign C exercises. Caution students that when they are asked to list or graph numbers <u>between</u> two given numbers, a and b, they should not include a and b.
Picturing Number Operations on a Line	294	Much of this section will be review. Proceed rapidly.
Using Coordinates in Problem Solving	301	Even superior students have difficulty understanding the importance of the methods introduced in this section. Emphasize that using coordinates to solve problems is not always the most efficient or straight-forward method, but give examples in which coordinates help to clarify the problem.
Midpoint Coordinates	305	Spend more time individually with students doing the procedure on page 306.
Other Division Point Coordinates	312	The lesson on page 312 will be a good change of pace. Note that the third entry in the Chapter Summary may confuse the students. The second sentence will not hold true when operations involving <u>integers</u> are discussed.

UNIT X: FRACTIONS AND RATIONAL NUMBERS
Chapter 10 in Dolciani, Book 7
(6 days)

On completion of Unit X, the student should be able to

identify from a list of expressions those that do not represent rational numbers

reduce given common fractions to lowest terms

state in lowest terms the improper fraction equal to a given mixed numeral and vice versa

name the LCD of a given set of fractions and use it to state the fractions equal to the given fractions

simplify given expressions by naming equivalent sums, differences, products, and quotients of fractions and/or mixed numerals

illustrate simple fractional equations involving either addition, subtraction, multiplication, or division by means of a number line diagram

identify the fraction that names the greater number in given pairs of fractions

write an equation based on the rational-number facts stated in a given story problem and solve the equation

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Most of your students will find this unit a review. Again, a short pretest is suggested to determine just how much work the students need. Throughout the chapter emphasize the whys and the structure of the positive rational number system, as well as the mechanics of operations with fractions.
Basic Laws of Rational Numbers	326	See the TM, pages 36-43, before proceeding with the unit. Note that "rationals" will be non-negative rationals.
		The need for numbers other than whole numbers is not explicitly stated. Once the student notes that the set of whole numbers is not closed under division, the establishment of rational numbers will follow easily. See S.M.S.G.'s *Introduction to Secondary School Mathematics*, Volume I, for further guidelines.

UNIT X (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Basic Laws of Rational Numbers (continued)	326	The text attempts to extend the properties of whole numbers to the fractions by examples such as the following: "Just as 3 + 5 = 5 + 3, so is 1/2 + 2/3 = 2/3 + 1/2." This treatment is unconvincing, if the student cannot simplify the fractional expressions. A satisfactory substitution might be: "Just as 3 + 5 = 5 + 3, so is 3/6 + 8/2 = 8/2 + 3/6."
Simplifying Fractions	330	Review the pitfall in the quotient a/b, where b = 0. (See page 87). On page 39 of the TM, see the remarks regarding "cancellation."
Adding and Subtracting Rational Numbers	336	Some students may know an alternate procedure for adding and subtracting two fractions with different denominators, namely, $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$ As a special case, $a \pm \frac{c}{d} = \frac{ad \pm c}{d}$, when b = 1. This equation may be developed later in the mixed numeral-improper fraction conversion discussion. Remind students of the <u>order</u> implied by parentheses.
Using Common Denominators	341	Review the procedure for identifying the LCM for sets of two and three numbers before assigning exercises. Two additional problems you might give: Fill in the missing squares so that each row, column, and the two diagonals add up to the same rational number for each of a and b.

(a) *Answers (b)

			Answers
$\frac{1}{2}$		$\frac{1}{6}$	← $\frac{7}{12}$
$\frac{1}{12}$	$\frac{5}{12}$		← $\frac{3}{4}$
$\frac{2}{3}$		$\frac{1}{3}$	← $\frac{1}{4}$

Answers			
5 →	$6\frac{1}{4}$		$11\frac{1}{4}$
$12\frac{1}{3}$, $2\frac{1}{2}$ →		$7\frac{1}{2}$	
$3\frac{3}{4}$, 10 →			$8\frac{3}{4}$

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Using Common Denominators (continued)	341	Find the first four sums. Use the resulting pattern to find the fifth sum. $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} = ?$ (Answer: $\frac{2}{3}$) $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} = ?$ (Answer: $\frac{3}{4}$) $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} = ?$ (Answer: $\frac{4}{5}$) $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \frac{1}{5\cdot 6} = ?$ (Answer: $\frac{5}{6}$) $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + --- + \frac{1}{99.100} = ?$ (Answer: $\frac{99}{100}$)
Using Mixed Numerals	345	Use the difference-sum property to simplify 24 - 15 before executing the more complex 11 2/5 - 4 9/10. (See Example 3, page 347.)
Multiplying Rational Numbers	349	No, Virginia, there is no <u>unique</u> reciprocal for 0. Remind students that products should be reduced to <u>lowest</u> <u>sums</u>. You may want to challenge your pupils with the following problems, which involve reciprocals. 1. Two numbers are reciprocals of each other. One of these numbers is four times as large as the other. Determine both numbers. (Answer: 1/2, 2) 2. If $a\cdot x = b$ and $b\cdot y = a$, and $a \neq 0$, $b \neq 0$, are x and y reciprocals? Why or why not?
Dividing Rational Numbers	354	Note that $\frac{a}{b} \div \frac{c}{d}$, $\frac{a}{b} \times \frac{d}{c}$, and $\frac{\frac{a}{b}}{\frac{c}{d}}$ are equivalent. Stress that division of positive rationals is neither commutative, nor associative.

UNIT X (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Dividing Rational Numbers (continued)	354	Two function rules for your students to guess:

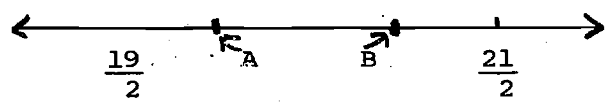
△	□
3	4/3
3 1/4	13/9
2/5	8/45
18/11	8/11
6/7	?

Answer: □ = △ ÷ 9/4 or △ · 4/9

△	□
5/6	1
4/7	24/35
10	12
2 2/3	?
?	27/40

Answer: □ = △ · 6/5 or △ ÷ 5/6

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Comparing Rational Numbers	354	As an application of the concepts in this chapter, you may wish to give the following problem:



Find the lowest-terms fraction for each point A and B.
(Answer: A = $\frac{59}{6}$, B = $\frac{61}{6}$.)

UNIT XI: DECIMALS FOR RATIONAL NUMBERS
Chapter 11 in Dolciani, Book 7
(5 days)

On completion of Unit XI, the student should be able to

- write numerals in the form $a\frac{b}{10^n}$, where a and b are whole numbers and n is a counting number, in decimal notation and vice versa

- write numerals written in expanded notation as decimals and vice versa

- arrange in ascending (or descending) order a given list of decimals

- simplify expressions involving addition, subtraction, multiplication, and/or division of positive decimals or of positive decimals and fractions

- round a given positive decimal to a specified decimal place

- write the common fraction in lowest terms that names a given positive terminating decimal

- write the terminating or the repeating decimal that names a fraction

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Most of your students will have had an introduction to decimals in elementary school, where place value concepts and operations with simple decimals are usually covered. Pretest your class to determine just where it stands.
Decimal Numerals	372	Stress that a number need not have a decimal point to be called a decimal. For instance, 53 is a decimal since it is in base 10. However, 53.0 is expressed in decimal notation.
		As motivation, help students to realize the many situations in real life in which decimals occur. Some examples are batting averages, mileages shown on an odometer, and amounts of money written in dollars and cents.
		You might ask your students this question:
		What is the greatest amount of money you can have (using pennies, nickels, dimes, quarters, and half dollars) and still not be able to give someone change for a dollar? (Answer: \$1.19; 1 half dollar, 1 quarter, 4 dimes, and 4 pennies, or 1 quarter, 9 dimes, 4 pennies.)

UNIT XI (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Decimals and Order	376	Expand on the statement in the text that says decimals may be compared by just looking at them. Tell students that to do this they must find the first corresponding place in the decimal numerals in which the digit differs and then make a decision. For instance, 54.712 < 54.723 since 1 < 2 in the hundredths place.
Algorithms for Addition and Subtraction	379	Students will need more practice than given in the text on problems involving the simplification of expressions involving both decimals and fractions, such as 1/2 + 0.45.
Approximations	383	Tell students that the symbol "≈" is also used to mean "approximately equal to." Stress that "rounding to one decimal place" is the same as rounding to the nearest tenth, "rounding to two decimal places" is the same as rounding to the nearest hundredth, and so on.

Point out to students the special problem arising when the number to be rounded is exactly halfway between two numbers. Tell them it is arbitrarily agreed in this book to "round up" when this is the case, that is, 12.5 to 13, instead of 12. Some books, however, agree to "round down" or to choose the decimal whose last digit is even.

Here is a rule you may wish to have students guess. (It involves a knowledge of rounding and of base-two numerals.)

X	Y
.75	1
2.1	10
$2.\overline{81}$	11
5.875	110
$15.\overline{3}$	1111
15.5	10000
13.4	1101

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Renaming Decimals and Fractions	386	Be sure students realize that in the process of converting common fractions to decimal numerals they are multiplying by convenient forms of 1. For instance, 1/2 x 5/5 = 5/10 = 0.5. The form of 1 here is 5/5.

UNIT XI (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Terminating and Repeating Decimals	389	Students may be tempted to write $0.68\overline{181}$ instead of $0.6\overline{81}$. Tell them that it is conventional to place a bar, often called a vinculum, over the smallest repeating block.
The Multiplication Algorithm	393	Urge students to check their answers for reasonableness by using rounding.
The Division Algorithm	397	Again, urge students to check their answers for reasonableness by using rounding.

UNIT XII: MEASUREMENT AND GEOMETRY
Chapter 12 in Dolciani, Book 7
(9 days)

On completion of Unit XII, the student should be able to

find the measure of a given line segment using a ruler

find the measure of a given angle using a protractor

classify a polygon, both from a given diagram and from a given description, as either a triangle, a certain type of quadrilateral, a pentagon, a hexagon, an octagon, or a decagon

classify a given triangle as scalene, isosceles, or equilateral and as acute, right, or obtuse

find the perimeter of a polygon, given the measure of each side

find the circumference of a circle, given either the diameter or radius of the circle

find the measure of each side of a regular polygon, given the perimeter

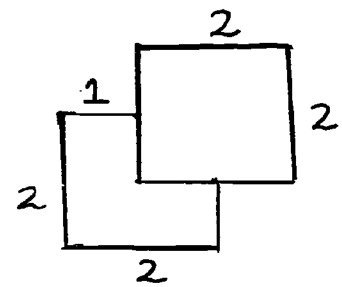
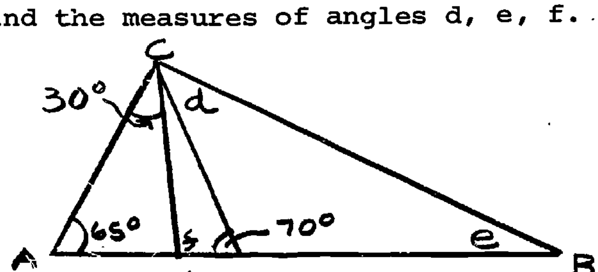
find the area of a rectangle, a parallelogram, a triangle, a trapezoid, and a circle, given appropriate measurements

find the volume and surface area of a rectangular prism, given appropriate measurements

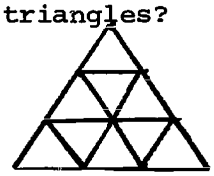
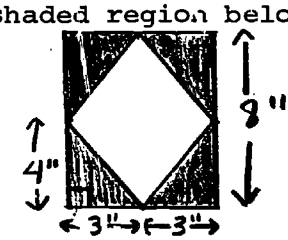
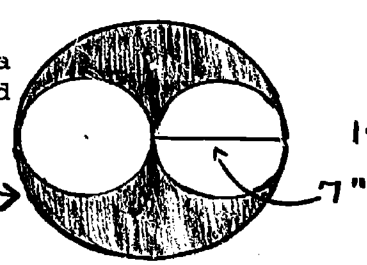
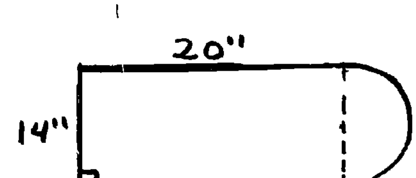
find the measure of the complement and supplement of an angle, given the measure of the angle

find the measure of the third angle of a triangle, given the measure of the other two angles

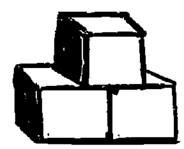
UNIT XII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		There is a lot of new material in this unit, but attempt to complete a section per day.
Length and Perimeter	410	See TM, page 47. Have students find perimeters of irregular shapes such as: 
Measuring Segments	417	If possible, have students measure large segments such as the length of a block, a building, or a parking lot. They will realize that measurement is an approximation when they compare their results.
Angle Measurement	423	Most students have spent very little time using a protractor. Therefore, you may want to prepare worksheets to give the students extra practice.
Triangles and Quadrilaterals	431	Point out the relationships between a square, a parallelogram, and a rhombus. Ask questions such as, "Is a square a rhombus?" Is a rhombus a square?" To apply the concepts in this section you might give students problems of the following type: Find the measures of angles d, e, f.  Answer: $m^\circ \angle d = 45^\circ$ $m^\circ \angle e = 25^\circ$ $m^\circ \angle f = 95^\circ$

UNIT XII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Triangles and Quadrilaterals (continued)	431	Here is another extra problem you may wish to give the class: How many triangles?  Answer: 13
Area	437	Have students draw geometric figures on graph paper and count the squares to find the area of the figure. This may lead students to discover the definitions for areas of rectangles, parallelograms, triangles, and trapezoids. You may also find geoboards of great value in this section. Have students find areas of unusual regions, such as the shaded region below. 
Circles: Circumference and Area	444	As a homework assignment you may wish to have each student measure the circumference C and diameter d of at least one circular object at home and then find the decimal corresponding to the ratio C/d. The next day have the students compare their results. You may wish to develop the concept of pi more carefully. Students will enjoy applying the ideas in the section to finding areas of regions such as those below. Find area of shaded region  27 

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Right Angles in Space	448	The ideas in this section will be more clear to students if geometric models are used. Students may have difficulty in drawing the diagrams required in the exercises.
Rectangular Prisms: Volumes and Surface Areas	454	You may find cuisenaire rods useful here. Attempt to give students unusual volume and surface area problems. Examples: Eight cubes of volume 1 cubic inch are to be joined face to face somehow. How can they be placed so that the least surface is exposed? Find the surface area if three cubes in the figure have sides 1 inch long. (Answer: 14 square inches)



UNIT XIII: PERCENTAGE AND STATISTICS
Chapter 13 in Dolciani, Book 7
(4 days)

On completion of Unit XIII, the student should be able to

write a ratio using a division sign ÷, a ratio sign :, a fraction, and a decimal

simplify a given expression involving percents

find the missing term in a given proportion

given either a decimal, a common fraction, or a percent, write equivalents to the given numeral in the two forms not given

set up a proportion to solve word problems involving rates

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		The last four sections of this chapter are optional. You may cover them if time permits. Students will have had a short introduction to percents in elementary school. However, they will find many challenging problems in this chapter.
Ratio and Proportion	464	Notice that there are no oral exercises in this chapter. You may want to make worksheets to give students extra practice. A discussion of the rule of three might create interest. See TM, page 52 (13-1). You might want to discuss the Golden Ratio which is 1:1.618. You will find students fascinated by the omnipresence of the ratio in art, in nature, and in mathematics. Three excellent references: *Mathematics*, part of the Life Science Library; *The Golden Measure*, part of S.M.S.G.'s Reprint Series; and *The Second Scientific American Book of Mathematical Puzzles and Diversions*, by Martin Gardner.
Ratio and Percent	468	The importance of estimating and making plausible guesses for a problem should be emphasized. Pick an incorrect answer to a simple sample problem to focus attention upon how absurd some answers may be. Pupils should avoid falling into the habit of being satisfied with <u>any</u> result they obtain.

UNIT XIII (CONTINUED)

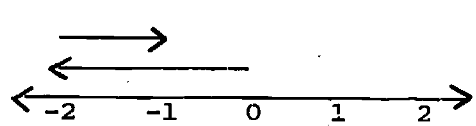
TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Percents and Percentages	473	Students should be told that percent frequently appears as two words, per cent, while percentage is always a single word. Students may be surprised at answers to problems of the following type. Have them attempt to explain the results. "On November 1 Charles weighed 125 pounds. During that month his weight decreased by 10%, but during December his weight increased by 10%. What did he weigh on January 1? What did he weigh on January 1 if his weight increased 10% during November and decreased 10% during December?
Percent in Banking and Buying	477	Optional Have students find articles in newspapers and magazines for oral presentation and discussion, if you discuss this section.
Percents and Graphs	481	Optional
Bar and Broken-Line Graphs	484	Optional
Frequency Distributions	490	Optional Spend considerable time discussing the vocabulary on page 491, if you cover this section.

UNIT XIV: THE SET OF INTEGERS
Chapter 14 in Dolciani, Book 7
(6 days)

On completion of Unit XIV, the student should be able to

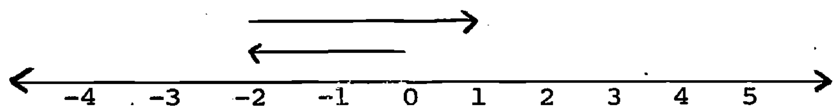
- locate any integer on the number line
- name the opposite of a given integer
- list a given set of integers in ascending or descending order
- express a given difference of integers as a sum of integers
- picture the sum or difference of any two integers in a number-line diagram
- simplify expressions involving addition, subtraction, multiplication, and/or division of integers
- solve word problems involving addition, subtraction, multiplication, and/or division of integers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Throughout this chapter emphasize the whys of operating with integers.
Negative Numbers As Coordinates	500	Stress that 0 is neither positive nor negative and that the opposite of 0 is 0.
		Explain to students that the raised sign is used in the notation for negative numbers to distinguish it from the sign for subtraction.
		You may want to give students additional work on vertical number lines.
		You might ask questions such as: "What is the opposite of the opposite of 3?" and "What is the opposite of the opposite of -7?"
Adding Integers	504	Students may find adding on the number line more clear if two arrows are used. That is, -2 +1 = -1 could be pictured:



TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Subtracting Integers	509	Emphasize the ever-present relationship between addition and subtraction.

Students may prefer to use two arrows to show subtraction on the number line. For instance, -2 - -3 = 1 could be pictured as:

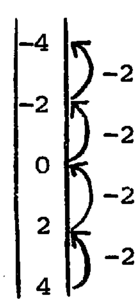


Stress that "·" and "x" are equivalent symbols for multiplication.

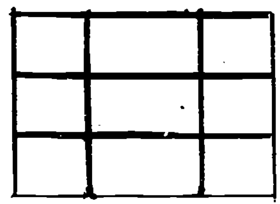
TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Multiplying Integers	513	You may wish to "justify" the statements on pages 515-516 by constructing a multiplication table and having the students fill in the table by using what they already know about arithmetic and by looking for patterns.

x	-2	-1	0	1	2
-2			0		
-1			0		
0	0	0	0	0	0
1			0	1	2
2			0	2	4

The blocked-off cells are readily filled by the student's knowledge of arithmetic. For instance, the 2 column may be filled in by noting that as one moves up, each number is two less than the number below:



Fill in the other cells with the students using similar reasoning, and also the commutative property. Then generalize.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Multiplying Integers (continued)	513	Two puzzles you may wish to give your students for extra practice:  Arrange fine ones (1) and four negative ones (-1) in the nine squares so that the product of the numbers in each row, each column, and the two diagonals is 1. Arrange four ones and five negative ones in the nine squares so that the product of the numbers in each row, each column, and the two diagonals is -1.
Dividing Integers	518	Help students to make generalizations for division of integers like the generalizations for multiplication of integers on pages 515-516. Emphasize the recurring relation between multiplication and division. Use counterexamples to emphasize that division of integers is neither associative nor commutative.

UNIT XV: RATIONAL NUMBERS
Chapter 1 in Dolciani, Book 8
(5 days)

On completion of Unit XV, the student should be able to

properly place a positive or negative rational number relative to two given points on a number line

show with a Venn diagram and by the roster method the union and intersection of the sets of positive, negative, nonpositive, and nonnegative rational numbers

name and define the opposite of any given rational number

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		The majority of your students will know most of the material in this unit, so you can probably plan to proceed rapidly.
Negative Numbers	2	The comments in the TM are quite explicit and should be of sufficient help in teaching this section.
Sets of Numbers	6	Except for set builder notation, much of this section will be review.
Opposites of Numbers	14	
Order	17	Many students will have trouble placing the correct sign (< or >) between two negative numbers.
Arrows on the Number Line	21	It is helpful if the students think of "absolute value" as being the number of units away from the origin on a number line. Even some of your bright students will have a difficult time understanding why -n might stand for a positive number. This idea is essential to understanding of the formal definition of absolute value which is given in algebra. Ask your students to explain why or why not for any integer n, $\|n\| \geq n$.

UNIT XV (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Arrows on the Number Line (continued)	21	You might want to have your pupils guess the following rule, which involves the absolute value.

x	y
1	2
-1	2
0	1
1/2	1 1/2
-1/2	1 1/2
-12	13
58	59

(Answer: $y = |x| + 1$)

UNIT XVI: ADDITION AND SUBTRACTION OF RATIONAL NUMBERS
Chapter 2 in Dolciani, Book 8
(6 days)

On completion of Unit XVI, the student should be able to

graph on a number line the sum or difference of two given rational numbers

write a mixed numeral in the form a/b, where a and b are integers, $b \neq 0$, and vice versa

write the absolute value of a given rational number

write another fractional name and decimal name for a given fractional number

write the sum or difference of any two rational numbers

write the generalizations for the closure property of addition, commutative property of addition, associative property of addition, additive identity, and the additive inverse property

match, from a list of numerical expressions, each of the above properties

write the generalization $a - b = a + (-b)$ and change given subtraction problems to equivalent addition problems

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Sums of Nonnegative Rational Numbers	32	This section is a review of material in Chapter 10, Book 7.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Sums of Negative Numbers	37	Students tend to have great difficulty with the concept of absolute value. The important thing is that the students learn to add signed numbers. The concept of absolute value is only one means for improving this skill. If the students can form the necessary generalizations on their own, this is sufficient.
Sums of Rational Numbers in General	41	You might ask students to show by counterexample why it is not true for any rational numbers a and b that $\lvert a + b\rvert = \lvert a\rvert + \lvert b\rvert$. A problem applying the ideas in this section: $1/x + 1/y + 1/z = 1$. x, y, and z are different whole numbers. Find them.
Properties of Addition	45	The closure property may give your students some trouble. Be certain to assign the exercises in part C.
Subtraction of Rational Numbers	51	Stress the fact that $a - b = a + (-b)$ and that each subtraction problem should first be changed to an equivalent addition problem before a solution is attempted. Even the brighter students may be surprised that subtracting a negative number is the same as adding the additive inverse of the number. Stress this fact.

UNIT XVII: MULTIPLICATION AND DIVISION OF RATIONAL NUMBERS
Chapter 3 in Dolciani, Book 8
(4 days)

On completion of Unit XVII the student should be able to

give the sign of the product of any two rational numbers by inspection

find the product of two or more rational numbers

match the properties of multiplication with statements illustrating these properties

give the sign of the quotient of any two rational numbers by inspection

find the quotient of any two rational numbers by inspection

find the quotient of two rational numbers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Product of a Rational Number and a Nonnegative Rational Number	62	Use the concept of multiplication as repeated addition to lead into the rules for the product of a rational number and a nonnegative rational. Be certain students realize the reasons why they may use "slant-bars" in the process of multiplying or dividing rational numbers.
The Product of Two Negative Rational Numbers	65	The table approach on page 66 is excellent. Might use "Friend of a Friend" as shown in Algebra 1-2 guide on page 20.
Properties of Multiplication	68	The book's discussion will have to be expanded. Even superior students will need plenty of practice with problems such as the following, which involve the distributive property: 2/3 (-1/2 + 5/6) = ?
Division of Rational Numbers	73	You may wish to discuss the properties of division of rational numbers with the students. Some other problems involving division of rational numbers: 12/10 is how many times as large as 12/100? 4/7 is how many times as large as 7/4?

UNIT XVIII: GEOMETRIC FIGURES IN THE PLANE
Chapter 4 in Dolciani, Book 8
(3 days)

On completion of Unit XVIII, the student should be able to

identify from a drawing: a) transversal, b) corresponding angles, c) exterior angles, d) interior angles, e) supplementary angles, f) vertical angles, g) alternate interior and exterior angles

draw all axes of symmetry for a given figure and label any point of symmetry the figure has

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Drawing Geometric Figures	84	If time permits you may wish to cover the constructions in this unit, but this knowledge is not essential to the algebra course for which you are preparing students. If you discuss these constructions, it is recommended that the teacher become somewhat skilled in the use of the blackboard compass. This may take some practice. For instructing the students in using a protractor, use an overhead projector and transparencies. In this section, stress the properties of corresponding and alternate interior angles formed by a transversal and parallel lines, as well as the properties of vertical angles and the sum of the measures, in degrees, of the angles of any triangle.
Basic Constructions	92	Optional
Symmetry	98	

UNIT XIX: CONGRUENCE AND MEASUREMENT OF PLANE FIGURES
Chapter 5 in Dolciani, Book 8
(5 days)

On completion of Unit XIX, the student should be able to

state from given information whether or not two given figures are congruent

construct a figure congruent to a given figure

state the ASA, SAS, and SSS properties of congruence

list the corresponding parts of two congruent figures

identify the following from an appropriate drawing: radius, chord, diameter

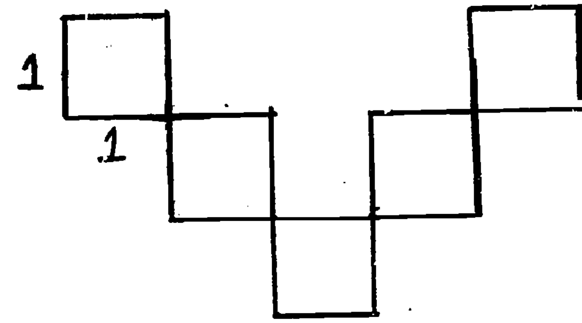
find the perimeters of given polygons or closed figures composed of segments and/or semi-circles

find the area of a given rectangular, circular, triangular, or trapezoidal region

define a circle and find the circumference of a circle given radius or diameter

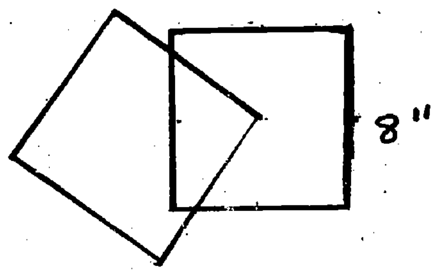
TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Congruent Figures	112	Stress the fact that although a correspondence may be set up between any two polygons of the same number of sides, this correspondence will be a congruence if sides and angles are congruent. Some time should be spent establishing the concept of congruence of segments and angles.
Congruent Triangles	118	
Constructing Congruent Triangles	124	Optional See the geometry overlay notebook for sample constructions that can be used on the overhead projector.
Perimeter	132	Here is an example of a different sort of perimeter problem you might present: Five squares of the same size arranged in this way have a perimeter of 20. What is the smallest perimeter you can get by rearranging the squares? (Diagram on next page)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Perimeter (continued)	132	(Answer: 10)

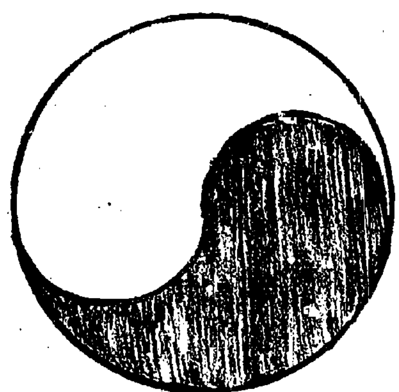


Area	137	Here are two area problems which your pupils may enjoy:

Two squares of the same size overlap so that the corner of one is at the center of the other. What is the area of the overlapping region?



Find the area of the shaded region if the two small curves are semicircles and the radius of the big circle is 14".



(Answer: A = 98 π sq. inches.)

UNIT XXII: PRECISION AND ACCURACY
Chapter 8 in Dolciani, Book 8
(Optional)

On completion of Unit XXII, the student should be able to

write the unit measure for any given measurement

determine the greatest possible error for a given measurement

select the more precise of two given measures

determine the number of significant digits in a given measure

round off a given measurement to a given number of significant digits

find the relative error of a given measurement

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		This entire unit is optional. If you skip it, be certain that students are aware that measure is only approximate.
Units of Measure and Greatest Possible Error	202	
Significant Digits	208	
Relative Error	212	

UNIT XXIII: DECIMAL NUMERALS AND REAL NUMBERS
Chapter 9 in Dolciani, Book 8
(5 days)

On completion of Unit XXIII, the student should be able to

name a rational number corresponding to a given repeating or terminating decimal

name a terminating or repeating decimal corresponding to a given fraction

compare the size of two given numbers in both decimal and fractional notation

name a rational and an irrational number between two given numbers

graph a given set of real numbers

use set-builder notation to name the set of real numbers on a given graph

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		The majority of this section is a review of material in Chapter 11, Book 7. Cover it rapidly.
Decimal Numerals for Rational Numbers	224	Stress that the "vinculum" is *only* over the repetend. Note that $2.\overline{45} = 2.\overline{4545}$ but that $2.\overline{45}$ is simpler.
Rational Numbers Named by Repeating Decimal Numerals	229	The process discussed will be difficult for some students.
Real Numbers	234	You might use Venn diagrams to stress the relationships between the real numbers, the rational numbers, the integers, the whole numbers, and the natural numbers.
		Students may not believe the statement on page 237 which says that $759.\overline{9} = 760.\overline{0}$. Have them justify this equation by using the process of the preceeding section. You might ask students to generalize from this example.
Completeness and Graphs	245	Point out that density and completeness are not equivalent.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Completeness and Graphs (continued)	245	Warn students that a common mistake in graphing the sets in the exercises is to forget that the variables represent real numbers and graph points, instead. You might ask students to graph the intersections of two sets given in set builder notation.

UNIT XXIV: OPEN NUMBER SENTENCES
Chapter 10 in Dolciani, Book 8
(5 days)

On completion of Unit XXIV, the student should be able to

write open number phrases expressing sums, differences, products, and quotients

evaluate open number phrases when the value of the variable is known

determine truth or falsity of open number sentences in one or more variables when the values of the variables are known

solve simple equations in one unknown by inspection

write an open number sentence to express a relationship in a simple word problem

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		A mastery of the concepts in this chapter will prove invaluable, not only in the next two units, but also in algebra.
Number Phrases	258	Students may wish to devise their own "mind reading games" similar to those on page 258. Stress also the difference between "phrases" and "sentences." Subject and verb give a sentence, as in English.
Values for Open Number Phrases	262	

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Statements and Open Number Sentences	266	The definitions on page 267 in the colored boxes are important. They should be studied carefully. Be certain to assign the C exercises. You might want to give the students more of this type, since they will probably find the rest of the set very easy.
Solving Equations By Inspection	271	Encourage students to express answers in set notation {} in these exercises. It is a good habit to acquire as it will be used extensively later.

UNIT XXV: SOLVING OPEN NUMBER SENTENCES
Chapter 11 in Dolciani, Book 8
(5 days)

On completion of Unit XXV, the student should be able to

solve simple linear equations in one variable using the transformations listed on page 283 of the text and acceptable set notation

write the addition and multiplication properties of equality

solve and graph simple inequalities using the transformations on page 286 in the text

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Mastery of the material in this section is essential for success in algebra. Although most of the students in the accelerated program will be able to solve the given equations by inspection, insist that they show their work and check it until they feel at ease with the transformations discussed.
Solving Equations By Transformations	282	Most students will require a great deal of practice in just the mechanics of solving equations. Supplementary problems can be taken from any algebra text.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
More About Variables and Equations	288	Warn students that the solution sets of the equations in the C exercises may have more than one member.
Solving Inequalities	294	
Compound Inequalities	300	You might want to have the class "help" to construct the following truth table to be used in deciding whether a given compound sentence is true or false. Let A and B stand for sentences.

A	B	A and B	A or B
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

You may wish to find the entries in the table by using as examples both simple math compound sentences as well as simple English compound sentences.

Emphasize the connection between "or" sentences and unions of sets as well as "and" sentences and intersections of sets.

UNIT XXVI: USING EQUATIONS
Chapter 12 in Dolciani, Book 8
(5 days)

On completion of Unit XXVI, the student should be able to

translate word sentences into number sentences

represent two unknowns in terms of one variable

solve problems by use of an equation

solve percentage problems by use of an equation

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		The material in this section will be difficult even for the superior student. If a pupil has demonstrated that he can go through the steps in the solution of an equation, do not insist that he show <u>every</u> step. However, have students state for what a variable stands when solving a word problem. Urge them to draw diagrams wherever possible as an aid to setting up the proper equations.
Translating Word Sentences into Number Sentences	312	Emphasize that the members of an equation represent numbers. Students will have considerable trouble with age problems and money problems. Do the Oral Exercises very carefully with the class.
Applying Number Sentences in One Variable	316	Go through the steps of problem solving carefully. Again, spend considerable time on the Oral Exercises with the class.
Percentage Problems	322	Use a proportion with a variable to solve percentage problems. It is unlikely that many students will correctly solve Exercise 21. It is, however, a good problem. Be sure to assign it.

UNIT XXVII: SQUARE ROOTS; SIMILAR FIGURES
Chapter 13 in Dolciani, Book 8
(6 days)

On completion of Unit XXVII, the student should be able to

find the square roots of numbers up to 1000 having rational roots

determine whether three numbers are Pythagorean triples

find the square root of any three-digit number to two decimal places

write the square root of any three-digit positive integer as the product of the greatest possible integer and the smallest possible radical

use the Pythagorean Theorem to find the third side of any right triangle of which two sides are known, and write the answer in simplified form

apply the Pythagorean Theorem to such problems as finding the diagonal of a rectangle

use the idea of similar polygons to solve simple problems involving proportion in measurement

identify corresponding parts of similar polygons and name the criteria for determining similar triangles

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Pythagorean Property	332	This property is the basis for most of the work done in this unit on radicals. It should be treated thoroughly. Students should receive considerable practice in verifying Pythagorean triples. You might ask students how they would construct a line segment with a length of $\sqrt{2}$ units, given a line segment of length 2 units.
Square Roots	336	Stress breaking down radicals into prime factors as an aid to simplification of irrational square roots. If this is done properly now, future troubles may be avoided. Be sure that students realize that an expression such as $3\sqrt{2}$ is the same as $\sqrt{18}$ and vice versa.

UNIT XXVII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Special Right Triangles	339	Students may be overwhelmed by the material in this section. The 30°-60° right triangle discussion may prove especially difficult. Go over the development of the formula $b = a\sqrt{3}$ with the class, asking for reasons for each step. Then give examples to show the significance of the results. Urge students to draw diagrams to help with solutions wherever possible.
Rational Approximations to Square Roots	343	In general, do not worry about finding square roots any more accurately than two decimal places. Again, stress the idea of breaking down the radicals into simpler form.
Similar Polygons	347	Stress use of corresponding parts in solving word problems.
Trigonometric Ratios	353	Optional as time permits.
Scale Drawings	359	Optional.

UNIT XXVIII: PYRAMIDS AND PRISMS
Chapter 14 in Dolciani, Book 8
(6 days)

On completion of Unit XXVIII, the student should be able to

define a line perpendicular to a plane

define the distance between two parallel planes

sketch three-dimensional prisms and pyramids

identify the base, altitude, and slant height of a given polyhedron

identify from a given set of drawings, a tetrahedron, triangular and rectangular prisms, parallelpiped, and oblique and regular prisms

find the surface area of a right rectangular prism, a right triangular prism, and a regular pyramid from a given drawing

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Even accelerated students may be overwhelmed by the new concepts in this unit. Do not require memorization of the formulas. It will be much more beneficial to students if they are taught to find the volumes and surface areas requested by reasoning of their own. Stress the whys of the formulas throughout.
Points, Lines, and Planes in Space	374	Students invariably have trouble drawing three-dimensional figures. For suggestions as to how to help them learn, see TM, pages 32-33.
Tetrahedrons and Other Pyramids	379	When discussing a polyhedron, do not depend upon the students' ability to picture it mentally. The teacher should have models close at hand throughout this section. (Plastic models may be made easily and inexpensively.)
Prisms	385	For additional help in visualization, see the geometry transparencies book.
Surface Areas of Prisms and Pyramids	390	In order to gain a better understanding of surface areas and how the formulas are derived, the teacher should have the students construct models such as those on pages 384-385 and 389.
Volumes of Prisms and Pyramids	395	The emphasis should be on application of the formulae. Students should not be expected to memorize them.
		Many of the students will enjoy playing three-dimensional tic-tac-toe which is described on pages 404 and 405. Challenge them to find ways of assuring wins. The pupils may even wish to organize tournaments.

UNIT XXIX: CONES, CYLINDERS, AND SPHERES
Chapter 15 in Dolciani, Book 8
(5 days)

On completion of Unit XXIX, the student should be able to

name the parts of a cone, cylinder, and sphere

find the surface area and volume of a given cone, cylinder, and sphere

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Students should not be expected to memorize all formulas in this chapter. The emphasis should be on the whys of the formulas.
Cones and Cylinders	408	Models are essential here.
Spheres: Great and Small Circles	412	The use of a world globe is good.
Surface Area of Cylinders, Cones, and Spheres	417	Have the students construct cylinders and cones to illustrate the principles of this section.
Volumes of Cylinders, Cones, and Spheres	422	Relationships between cylinder and prism, cone and pyramid should be stressed and generalized.

UNIT XXX: RELATIONS, FUNCTIONS, AND GRAPHING
Chapter 16 in Dolciani, Book 8
(5 days)

On completion of Unit XXX, the student should be able to

locate on the number plane any point given by an ordered pair (x,y)

determine by inspection in what quadrant a point named by an ordered pair will be found

determine whether a given ordered pair determines a point which is in the truth set of an equation in two unknowns

graph the truth set of any linear equation in two unknowns

solve two simultaneous linear equations by graphing (optional)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		The material in this unit will be discussed in more detail in the Pearson-Allen algebra text. Treat this unit as an introduction to coordinate geometry and a formulized discussion of the extremely important concept of function.
Solutions of Equations in Two Variables	432	Emphasize the idea of ordered pairs. The students have a tendency to ignore order, possibly as a result of continuous exposure to the commutative property.
Solution Sets of Open Number Sentences in Two Variables	434	
Rectangular Coordinate Systems	438	It might be interesting for the superior students to find the distance between two points on the number plane by use of the Pythagorean Theorem. Also, a little information on Descartes might be of interest here.
Relations and Functions	443	You might want to give other examples of functions from everyday life such as postage as a function of weight and height as a function of age. Students will need many examples to show the relevance of the definition of function on page 445. Stress that each element of the domain is paired with only one element in the range. You might ask them why the test at the top of page 466 works.
Graphing Linear Equations	447	The graphs of equations which are not linear such as $y = x^2$ and $y = 1/x$ might be shown here.
Graphing Linear Inequalities	452	Optional
Systems of Linear Equations	454	Optional

UNIT XXXI: PROBABILITY
Chapter 17 in Dolciani, Book 8
(Optional)

On completion of Unit XXXI, the student should be able to

list the possible outcomes and give the probability of each outcome of an experiment involving "equally likely" outcomes

list the set of favorable outcomes and give the probability of one of these occurring in an experiment

establish the odds in favor of an event knowing the probability of the event

find the probability of $E \cup F$ where E and F are mutually exclusive events

find the probability of $E \cup F$ where E and F are events which are not mutually exclusive

examine a random sample and establish empirical probability for an event

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		This unit is an enjoyable one to cover during the last days of school. Do so, if time permits! An excellent reference: S.M.S.G.'s *Probability for Intermediate Grades, Teacher's Commentary.*
The Probability of an Outcome	470	Emphasize that all possible distinct outcomes of an experiment must be considered.
The Probability of an Event	473	Distinguish between an outcome and an event, which may be a set of favorable outcomes.
The Probability of E or F for Disjoint Sets	478	Work on "mutually exclusive." Be sure students equate this idea with disjoint sets. Venn diagrams are a useful tool for illustrating this concept.
The Probability of E or F in General	481	Intersection and union of sets should be reviewed if necessary.
Empirical Probability: Samples	485	Example from TM page 38 is excellent.

PREFACE

This resource guide for Modern Mathematics 7 is organized as follows:

1. A recommended sequence of units with a suggested time allotment for each unit.

2. Unit Outlines
   a. Specific objectives to be realized by the student on completion of the unit.
   b. A detailed breakdown of the unit into sub-units with suggestions for developing topics and hints for the teacher in presentation of topics.

The time allotment for each unit is included only as a guideline for the teacher. The specific objectives of each unit set a basic body of material to be covered in Modern Mathematics 7 and the standard of proficiency that should be expected from each student in the Kent School District.

The fact that this guide can be beneficially revised will become evident with use. We hope that teachers using it will critically analyze it and make comments and suggestions about its utility, content and structure.

COMMITTEE FOR MODERN MATHEMATICS 7 GUIDE

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TABLE OF CONTENTS

INTRODUCTION

Statement of Purpose

The purpose of Modern Mathematics 7 is to provide an essential part of the mathematical background that all students will need as adults, and, at the same time, build a solid foundation for any future work in mathematics. The content of Mathematics 7 provides for the pupil's steady growth in thinking mathematically, understanding the basic concepts, appreciating structure and pattern, and increasing proficiency in using mathematics.

Objectives

The specific objectives of Modern Mathematics 7 are to:

- provide students with an understanding of the structure of the set of nonnegative rational numbers and the set of integers

- provide the students with an intuitive background in both metric and nonmetric geometry in two and three dimensions

- develop the students' proficiency in using mathematical symbols

- develop the students' skill in computation and in simplification of expressions involving operations with nonnegative rational numbers and/or integers

- enable students to apply their knowledge of the structure of the nonnegative rational numbers and integers, as well as metric and nonmetric geometry, in problem-solving situations

Prerequisites and Selection of Students

Seventh grade status and mathematical grade placement above 5.0 as indicated by a standard test of mathematical achievement.

Basic Textbook:

Dolciani, Wooton, Beckenbach, Chinn *Modern School Mathematics - Structure and Method;* Houghton Mifflin Co. (Boston, 1967)

Supplementary References:

Duncan, Capps, Dolciani, Quast, Zweng *Modern School Mathematics - Structure and Use;* Houghton Mifflin Co. (Boston, 1967)

Eicholz, O'Daffer, Brumfiel, Shanks, Fleenor *School Mathematics I;* Addison-Wesley Publishing Co. (Palo Alto, 1967)

Gundlach, Buffie, Denny, Kempf *Junior High School Mathematics;* Laidlaw Brothers (River Forest, Illinois, 1968)

Nichols, Flournoy, Kalin, Simon *Elementary Mathematics - Patterns and Structure;* Holt, Rinehart, Winston (New York, 1966)

Parker *The Structure of Number Systems;* Prentice-Hall (Englewood Cliffs, 1966)

Scheid *Elements of Finite Mathematics;* Addison-Wesley Publishing Co. (Reading, Mass., 1962)

School Mathematics Study Group *Mathematics for Junior High School;* Yale University Press (New York, 1961)

Smith *Explorations in Elementary Mathematics;* Prentice-Hall (Englewood Cliffs, 1966)

Wirtz, Botel, Numley *Discovery in Elementary School Mathematics;* Encyclopedia Britannica Press (Chicago, 1963)

SUGGESTED TIME ALLOTMENTS

UNIT	First Semester	Suggested Time *
I	Sets and Numbers	8 days
II	Properties of Addition and Subtraction in the Set of Whole Numbers	12 days
III	Properties of Multiplication and Division in the Set of Whole Numbers	11 days
IV	Numbers and Numerals	8 days
V	Algorithms of Arithmetic	8 days
VI	Number Theory	13 days
VII	Coordinate Systems on a Line	15 days

UNIT	Second Semester	Suggested Time *
VIII	Fractions and Rational Numbers	16 days
IX	Decimals for Rational Numbers	15 days
X	The Set of Integers	7 days
XI	Sets and Geometry	15 days
XII	Line and Angle Relationships	8 days
XIII	Measurement and Geometry	16 days
XIV	Percentage and Statistics	11 days

**This schedule gives 163 total days. Individual teachers may wish to vary the time allotments as necessary.*

UNIT I: SETS AND NUMBERS
Chapter 1 in Dolciani, Book 7
(8 days)

On completion of Unit I, the student should be able to

write relations between sets using the symbols of set notation ($\{\}$, $\cap$, $\cup$, $\subset$, $=$, $\neq$, $\in$, $\notin$, $\emptyset$)

form intersections and unions of given sets

form a one-to-one correspondence between two given sets

distinguish between equivalent sets and equal sets

use both the rule method and the roster method to identify finite and infinite sets

correctly classify sets as infinite or finite

correctly draw Venn diagrams showing given relations between sets

write all of the subsets of a given finite set

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Sets and Their Members	6	Do not let students become lazy in reading mathematical statements. For example, for the symbols $\{1,2\}$ read, "The sets whose members are 1 and 2."
Sets and Subsets	10	The symbol for subset in this text is $\subset$; in the Pearson-Allen Algebra 1-2 textbook, it is $\subseteq$. In this text, if we wish to distinguish between proper and improper subsets, we will say so. Remember, $\emptyset$ and $\{\emptyset\}$ are not the same.
Comparing Sets	14	Students may be intrigued by those questions such as Exercise 16 which involve comparing an infinite set and one of its infinite subsets.
Intersection of Sets	19	Spend plenty of time with the students on diagrams of $(B \cup A) \cap C$, etc.
Union of Sets	23	There is not enough stress on identification of $\cap$ and $\cup$ and differentiation between $\cap$ and $\cup$, especially in graphs.

UNIT II: PROPERTIES OF ADDITION AND SUBTRACTION IN THE SET OF WHOLE NUMBERS
Chapter 2 in Dolciani, Book 7
(12 days)

On completion of Unit II, the student should be able to

insert >, <, =, +, or -, in a given set of incomplete whole-number statements, to make the statements true

identify, by listing, all the whole-number values that make true a given equation or inequality involving whole numbers

write an equation based on the whole-number facts stated in a given story problem and solve the equation, listing the properties (by name) illustrated in each step of its solution

write, using whole numbers, examples of the commutative and associative properties of addition

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Using Sets in Addition	34	Students are hard pressed to think of any arithmetical operation that is not binary. A unary operation worth mentioning is the correspondence of a set to its complement or a counting number to its reciprocal. "Simplify" and "get the answer" are not synonymous. The word "simplify" should evoke a response involving the replacement of an expression with a simpler name for the expression. By doing so, the implication drawn is that a correspondence exists between the expression and its replacement. "Get the answer" is an ambiguous command implying only that the operations be performed.
The Commutative Property of Addition; Variables	40	Emphasize that a variable reprsents all the elements of a particular set of elements making a statement true. Frequently, a variable is thought of as denoting a single number.
Using Sets in Subtraction	44	Verbalization of the expression "take away" should be discouraged. Continued usage of "take away" will result in later confusion in the calculation of expressions of the form a - b where a < b. After all, "taking away" two kumquats from one is a very clever theft.

UNIT II (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Associative Property of Addition	50	Symbols of association imply *an order of performance* of operations. The expression 6 - (4 + 1) requires, first, that "4 + 1" be replaced by its new corresponding sum. Be sure students can show by example why subtraction is not associative.
Comparing Whole Numbers	56	See TM, page 13. The symbols < and > are also called "ordering" symbols. "Why isn't ≠ an ordering symbol?"

UNIT III: PROPERTIES OF MULTIPLICATION AND DIVISION IN THE SET OF WHOLE NUMBERS

Chapter 3 in Dolciani, Book 7
(11 days)

On completion of Unit III, the student should be able to

represent a given product of the form a x b = c, where a, b, and c are whole numbers, both as a repeated sum and as an array

identify correctly the product and factors, given a multiplication problem in the form a x b = c

identify correctly the dividend, divisor, numerator, denominator, and quotient when given an equation of the form a/b = c, where a and c are whole numbers and b is a natural number

match each of a given list of whole number equations with one or more of the following properties

- Commutative Property of Multiplication
- Associative Property of Multiplication
- Additive Property of 0
- Subtraction Property of 0
- Multiplicative Property of 1
- Multiplicative Property of 0
- Division Property of 1
- Division Property of 0
- Distributive Property

write an equation of the form a x (b + c) = a x b + a x c, where a, b, and c are whole numbers, as an array

write the division statements corresponding to a given multiplication statement, and vice versa

simplify numerical expressions involving addition, subtraction, multiplication, and/or division of whole numbers

solve word problems involving addition, subtraction, multiplication, and/or division of whole numbers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Using Sets in Multiplication	70	Mention operations which are <u>not</u> commutative and associative such as the operation *, where a * b = a + 2b.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Using Sets in Multiplication	70	Mention operations which are not binary such as the operation of squaring a number, which is unary, or the operation of finding the smallest of three numbers, which is ternary.
Using Sets in Division	75	Show that division can be thought of as repeated subtraction, just as multiplication can be thought of as repeated addition. For instance, $6 \div 2 = 3$, since $\underbrace{6 - 2 = 4}_{1}$, $\underbrace{4 - 2 = 2}_{2}$, $\underbrace{2 - 2 = 0}_{3}$. Show that division may be thought of as grouping an array. For instance, $6 \div 2 = 3$ since there are three groups of 2 in 6.
Simplifying Expressions	81	Tell students of the convention concerning order of operation when no grouping symbols are present: multiplication, division, addition, and then subtraction. Emphasize the clarity resulting if grouping symbols are used.
The Properties of 0 and 1	84	Stress the difference between 0/a and a/0. Show that both division properties may be checked by multiplication. Mention similarities of the identity elements of addition, subtraction, multiplication, and division.
The Distributive Properties	89	This is a hard concept for many students. Colored chalk for blackboard examples might help: $\underline{4}\,(3 + 2) = \underline{4} \times 3 + \underline{4} \times 2$ (underlining indicates same color). Stress that work can be checked by performing operation in parentheses first. That is, $4(3 + 2) = 4(5) = 20$; $4 \times 3 + 4 \times 2 = 12 + 8 = 20$. Warn students of a common mistake: $3(5 + 2) = 3 \times 5 + 3 \times 2$, not $3 \times 5 + 2$.

UNIT IV: NUMBERS AND NUMERALS
Chapter 4 in Dolciani, Book 7
(8 days)

On completion of Unit IV, the student should be able to

change Roman and Egyptian numerals to decimal numerals and decimal numerals to Roman and Egyptian numerals

write, if possible, any whole number in the form a^n, where a is a whole number and n is a counting number, and vice versa

write numerals in any base in expanded notation

write a numeral in a base other than ten as a base ten numeral

write a base ten numeral as a numeral in any given base

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Primitive Numerals	104	Students should not be expected to memorize chart on page 105.
Roman Numerals	107	Emphasize the principle of multiplication on page 109. The students probably have not seen it before.
Exponents and Powers	112	Be sure the students know that $2^3 = 2 \times 2 \times 2$ and not 2×3.
Base-Two Numeration	115	See TM, page 18, for some examples of "bicimal" fractions in case some students ask about them.
Decimal Numerals	120	Point out to the students that a decimal numeral need not have a decimal point. An interesting point is described on page 19 in the TM concerning American and British terminology for numerals over one million.
Numerals in Other Place-Value Systems	124	See TM, page 20, for a different method of converting base ten numerals to numerals in other bases.

UNIT V: ALGORITHMS OF ARITHMETIC
Chapter 5 in Dolciani, Book 7
(8 days)

On completion of Unit V, the student should be able to

simplify, using the appropriate algorithms, expressions involving decimal numerals or numerals in different bases

write the sum of two differences corresponding to a given difference of two sums involving whole numbers, or vice versa

show a complete check of the results of simplification of an expression involving multiplication or division

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Addition Algorithm	138	Students may have difficulty with addition problems in other bases. Treat these problems with a light touch. Caution students that two numerals must be in the same base before they can be added together.
The Subtraction Algorithm	144	Note that "borrowing" as used in the subtraction algorithm is similar to "carrying" in the addition algorithm. You may want to use "steal" instead of "borrow," since the word "borrow" implies an eventual return.
The Multiplication Algorithm	149	Some automatic calculators work on the principle of Mr. Sand's computer. Multiplying in bases other than 10 will be difficult for most students. Be sure students realize that two numerals must be in the same base before they can be multiplied together.
The Division Algorithm	155	Provide some work with crytography if more drill or motivation is needed for any or all of the four algorithms. Again, dividing in bases other than 10 will be hard for many students. Since much planned practice is required to reinforce and maintain skills acquired, daily 5 to 10 minute computation drills may be extremely beneficial to your students.

UNIT V (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Division Algorithm (continued)	155	The advantage of providing this daily practice session at the beginning of the class is that it will encourage pupils to start work without delay. Each teacher will wish to tailor his own daily computation review to the requirements of his pupils. Types of exercises should vary from day to day.

UNIT VI: NUMBER THEORY
Chapter 7 in Dolciani, Book 7
(13 days)

On completion of Unit VI, the student should be able to

identify the partial quotient and remainder, if there is one, in division problems of the form a/b, where a is a whole number and b is a counting number of less than four digits

classify a given number as prime or composite and odd or even

identify by roster the set of all divisors of a given whole number

state whether given whole numbers are divisible by 2, 3, 4, 5, 9, and/or 10, by using the corresponding divisibility tests

write the prime factorization of a given whole number

identify the GCF of a given pair of whole numbers by using the roster method

write the set of all factors of a given whole number using the prime factorization of the number

identify the LCM of a given pair of whole numbers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Extending the Division Algorithm	212	As an application, you may wish to lead students to the discovery that the square of an odd number when divided by 8 leaves 1 as a remainder. Also, you may want to lead students to the discovery that the square of a number is either divisible by 4 or leaves the remainder 1 when divided by 4.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Prime Numbers	216	Stress that there is no largest prime. Emphasize that 1 is not considered a prime since it is divisible by itself only and since otherwise there would be no unique prime factorization of natural numbers. For example, if 1 were considered prime, we would have $6 = 3 \times 2 = 3 \times 2 \times 1 = 3 \times 2 \times 1^2$, etc. Students often confuse prime numbers with odd numbers. Stress that all primes except 2 are odd, but that the reverse is definitely not true. That is, all odd numbers are not prime.
Using Divisibility Tests	221	You may wish to lead students to the discovery that the last digits in the square of any number must be 0, 1, 4, 5, 6, or 9.
Developing Divisibility Tests (optional)	227	
Prime Factorization	231	Emphasize that a systematic approach to testing the primes in increasing order makes finding prime factors easier. Stress that each natural number has only one prime factorization.
Greatest Common Factor	235	You may wish to point out that finding the GCF of three numbers is an example of a ternary operation. Stress that two or more whole numbers may have many common factors, but only one greatest common factor. Emphasize that two prime numbers are relatively prime, and that two numbers, *not necessarily prime*, may also be relatively prime. An example is 49 and 24: GCF (49,24) = 1.
Euclidean Algorithm (optional)	240	Emphasize the generality of this method. Tell students that this method may be used to check the results obtained when using the roster method. Outline the proof of this method given on page 30 of the TM for superior students.

UNIT VI (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Least Common Multiple	241	The generalization in Exercise 21, page 245, may be of interest even to the slower average student: GCF (a,b) x LCM (a,b) = a x b, where a and b are whole numbers. Tell students that this property will help them check their work or to find the GCF of two numbers given the LCM (or vice versa).

UNIT VII: COORDINATE SYSTEMS ON A LINE
Chapter 9 in Dolciani, Book 7
(15 days)

On completion of Unit VII, the student should be able to

graph a given number or given set of numbers on a number line

state the coordinates of given points on a given number line using fractions if necessary

state the addition, subtraction, multiplication, or division fact pictured in a given number line diagram, and vice versa

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Numbers and Points on a Line	290	Emphasize to students that any pair of points on the line may be used as endpoints for a unit segment, and that they should choose such points for their own convenience, depending upon what they want to show.
Picturing Number Operations on a Line	294	
Using Coordinates in Problem Solving	301	Students may have difficulty understanding the importance of the methods introduced in this section. Emphasize that using coordinates to solve problems is not always the most efficient or straightforward method, but give examples in which coordinates help to clarify the problem.
Midpoint Coordinates	305	Spend some time individually with students doing the procedure on page 306.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Other Division Point Coordinates	312	The lesson on page 312 will be a good change of pace. Note that the third entry in the Chapter Summary may confuse the students. The second sentence will not hold true when operations involving <u>integers</u> are discussed.

UNIT VIII: FRACTIONS AND RATIONAL NUMBERS
Chapter 10 in Dolciani, Book 7
(16 days)

On completion of Unit VIII, the student should be able to

identify from a list of expressions those that do not represent rational numbers

reduce given common fractions to lowest terms

state in lowest terms the improper fraction equal to a given mixed numeral and vice versa

name the LCD of a given set of fractions and use it to state the fractions equal to the given fractions

simplify given expressions by naming equivalent sums, differences, products and quotients of fractions and/or mixed numerals

illustrate simple fractional equations involving either addition, subtraction, multiplication, or division by means of a number line diagram

identify the fraction that names the greater number in given pairs of fractions

write an equation based on the rational-number facts stated in a given story problem and solve the equation

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Basic Laws of Rational Numbers	326	See the TM, pages 36-43, before proceeding with the unit. Note that "rationals" will be non-negative rationals.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Basic Laws of Rational Numbers (continued)	326	The need for numbers other than whole numbers is not explicitly stated. Once the student notes that the set of whole numbers is not closed under division, the establishment of rational numbers will follow easily. See S.M.S.G.'s *Introduction to Secondary School Mathematics*, Volume I, for further guidelines. The text attempts to extend the properties of whole numbers to the fractions by examples such as the following: "Just as 3 + 5 = 5 + 3, so is 1/2 + 2/3 = 2/3 + 1/2." This treatment is unconvincing, if the student cannot simplify the fractional expressions. A satisfactory substitution might be: "Just as 3 + 5 = 5 + 3, so is 6/3 + 8/2 = 8/2 + 6/3."
Simplifying Fractions	330	Review the pitfall in the quotient a/b, where b = 0. (See page 87.) On page 39 of the TM, see the remarks regarding "cancellation."
Adding and Subtracting Rational Numbers	336	Some students may know an alternate procedure for adding and subtracting two fractions with different denominators, namely, $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$ As a special case, $a \pm \frac{c}{d} = \frac{ad \pm c}{d}$, when b = 1. This equation may be developed later in the mixed numeral-improper fraction conversion discussion. Remind students of the order implied by parentheses.
Using Common Denominators	341	Review the procedure for identifying the LCM for sets of two and three numbers before assigning exercises.
Using Mixed Numerals	345	Use the difference-sum property to simplify 24 - 15 before executing the more complex 11 2/5 - 4 9/10. See example 3, page 347.

UNIT VIII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Multiplying Rational Numbers	349	No, Virginia, there is no unique reciprocal for 0. Remind students that products should be reduced to lowest sums.
Dividing Rational Numbers	354	Note that $\frac{a}{b} \div \frac{c}{d}$, $\frac{a}{b} \times \frac{d}{c}$, and $\frac{\frac{a}{b}}{\frac{c}{d}}$ are equivalent.
Comparing Rational Numbers	359	

UNIT IX: DECIMALS FOR RATIONAL NUMBERS
Chapter 11 in Dolciani, Book 7
(15 days)

On completion of Unit IX, the student should be able to

write numerals in the form $a\frac{b}{10^n}$, where a and b are whole numbers and n is a counting number, in decimal notation and vice versa

write numerals written in expanded notation as decimals and vice versa

arrange in ascending (or descending) order a given list of decimals

simplify expressions involving addition, subtraction, multiplication, and/or division of positive decimals or of positive decimals and fractions

round a given positive decimal to a specified decimal place

write the common fraction in lowest terms that names a given positive terminating decimal

write the terminating or the repeating decimal that names a fraction

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Decimal Numerals	372	Stress that a number need not have a decimal point to be called a decimal. For instance, 53 is a decimal since it is in base 10. However, 53.0 is expressed in decimal notation. As motivation, help students realize the situations in real life in which decimals come up. Some examples are batting averages and amounts of money written in dollars and cents.
Decimals and Order	376	Expand on the statement in the text that says decimals may be compared by just looking at them. Tell students that to do this they must find the first corresponding place in the decimal numerals in which the digit differs and then make a decision. For instance, $54.712 < 54.723$ since $1 < 2$ in the hundredths place.
Algorithms for Addition and Subtraction	379	Students will need more practice than given in the text on problems involving the simplification of expressions involving both decimals and fractions, such as $1/2 + 0.45$. If extra drill is needed, have students do addition and subtraction problems involving money.

UNIT IX (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Approximations	383	Tell students that the symbol "≈" is also used to mean "is approximately equal to." Stress that "rounding to one decimal place" is the same as rounding to the nearest tenth, "rounding to two decimal places" is the same as rounding to the nearest hundredth, and so on. Point out to students the special problem arising when the number to be rounded is exactly halfway between two numbers. Tell them it is arbitrarily agreed in this book to "round up" when this is the case, that is, 12.5 to 13, instead of 12. Some books, however, agree to "round down" or to choose the decimal whose last digit is even.
Renaming Decimals and Fractions	386	Be sure students realize that in the process of converting common fractions to decimal numerals they are multiplying by convenient forms of 1. For instance, $\frac{1}{2} \times \frac{5}{5} = \frac{5}{10} = 0.5$. The form of 1 here is $\frac{5}{5}$.
Terminating and Repeating Decimals	389	Students may be tempted to write $0.6\overline{8181}$ instead of $0.6\overline{81}$. Tell them that it is conventional to place a bar, often called a vinculum, over the <u>smallest</u> repeating block.
The Multiplication Algorithm	393	Urge students to check their answers for reasonableness by using rounding.
The Division Algorithm	397	Again, urge students to check their answers for reasonableness by using rounding.

UNIT X: THE SET OF INTEGERS
Chapter 14 in Dolciani, Book 7
(7 days)

On completion of Unit X, the student should be able to

locate any integer on the number line

name the opposite of a given integer

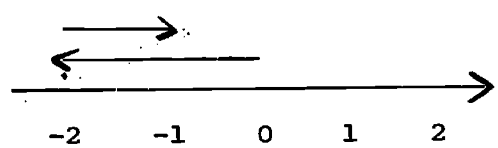
list a given set of integers in ascending or descending order

express a given difference of integers as a sum of integers

picture the sum or difference of any two integers on a number-line diagram

simplify expressions involving addition, subtraction, multiplication, and/or division of integers

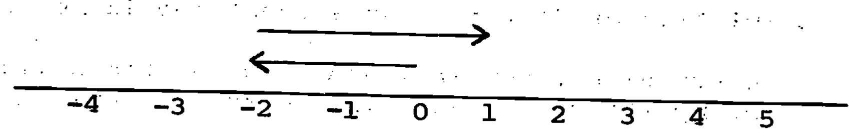
solve word problems involving addition, subtraction, multiplication, and/or division of integers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Negative Numbers as Coordinates	500	Stress that 0 is neither positive nor negative and that the opposite of 0 is 0. Explain to students that the raised sign is used in the notation for negative numbers to distinguish it from the sign for subtraction. You may want to give students additional work on vertical number lines.
Adding Integers	504	Students may find adding on the number line more clear if two arrows are used. That is, $^-2 + 1 = ^-1$ could be pictured: 
Subtracting Integers	509	Emphasize the ever-present relationship between addition and subtraction. Students may prefer to use two arrows to show subtraction on the number line. For instance, $^-2 - ^-3 = 1$.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS

Subtracting Integers (continued) — 509

This could be pictured as:



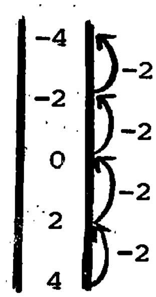
Stress that "·" and "x" are equivalent symbols for multiplication.

Multiplying Integers — 513

You may wish to "justify" the statements on pages 515-516 by constructing a multiplication table and having the students fill in the table by using what they already know about arithmetic and by looking for patterns.

x	-2	-1	0	1	2
-2			0		
-1			0		
0	0	0	0	0	0
1			0	1	2
2			0	2	4

The blocked-off cells are readily filled by the student's knowledge of arithmetic. For instance, the 2 column may be filled in by noting that as one moves up, each number is two less than the number below:



Fill in the other cells with the students using similar reasoning, and also the commutative property. Then generalize.

Dividing Integers — 518

Help students to make generalizations for division of integers like the generalizations for multiplication of integers on pages 515-516.

Emphasize the recurring relation between multiplication and division.

UNIT XI: SETS AND GEOMETRY
Chapter 6 in Dolciani, Book 7
(15 days)

On completion of Unit XI, the student should be able to

select names, numbers, and phrases from a list and insert them correctly in incomplete sentences relating to the terms and geometric properties discussed in this unit

classify given geometric sets as either infinite, finite but not empty, or empty

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Psychomotor activities are essential to the comprehension of geometric concepts. You will notice the profusion of illustrations in this unit, but total reliance on these planar illustrations of spatial ideas often frustrates the learning process. You will need classroom models. Individual student "geometry kits" consisting of several sheets of stiff paper or cardboard and straws or dowels will suffice. After an informal treatment of the Euclidean concepts presented in the unit, students should demonstrate with cardboard and dowels geometric situations - such as the possible intersections of parallel planes and a line. The "kits" can be retained for use during discussions of the geometric properties outlined in this unit.
Points and Space	168	If a figure is a subset of space, is the set containing all points in space a figure? And what of the set containing no points?
Lines	174	What does "determine" mean? Replacement with the more descriptive and familiar "locate" might be appropriate.
		Mention that at the very least two points determine (or locate) a line in space.
		Warn students that it is agreed to use the symbol $\in$, not the symbol $\subset$ to say that a point lies on a line.

UNIT XI (CONTINUED)

<table>
<tr><th>TOPICS</th><th>PAGE</th><th>COMMENTS AND SUGGESTIONS</th></tr>
<tr><td>Lines (continued)</td><td>174</td><td>Your students may discover (It's rather unlikely you say?) an interesting phenomenon concerning nonplanar figures such as those shown in Exercises 14, 17, 18, 20, 21, and 22 on pages 180 and 181.

The following table illustrates this phenomenon.

<table><tr><th>Points</th><th>Lines</th></tr><tr><td>4</td><td>6</td></tr><tr><td>5</td><td>10</td></tr><tr><td>6</td><td>15</td></tr><tr><td>7</td><td>21</td></tr><tr><td>8</td><td>28</td></tr></table>
Have students look for a pattern. How many lines are determined by a ten-cornered nonplanar figure?

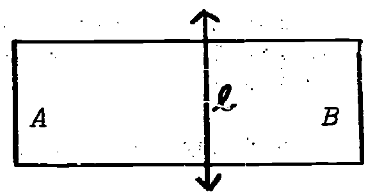
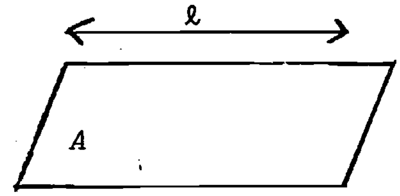
Also note the example on page 180. Students are directed to identify all lines determined by corner points. Emphasize that this means identifying even those lines not shown (for instance, $\overleftrightarrow{DB}$ in the example).</td></tr>
<tr><td>Planes</td><td>182</td><td>Explain carefully the directions accompanying the written exercises on page 185.

Common errors: $R \subset P$ instead of $R \in P$; $\ell \in P$ instead of $\ell \subset P$. Tell students that it has been agreed to use the symbol $\in$ when speaking of a point contained in a plane and to use the symbol $\subset$ when referring to a line lying in a plane.</td></tr>
<tr><td>Determining Planes</td><td>186</td><td>Again, the more familiar "locate" may be desired in place of "determine."

Do Exercises 9-16 on page 190 before assigning them to students.</td></tr>
<tr><td>Intersections of Lines and Planes</td><td>192</td><td>Note the error in the illustration on page 194. Plane Z is denoted by the symbol "T".

Do not assign the written exercises on pages 196 and 197 for homework. Invariably, students have difficulty drawing good three-dimensional sketches.</td></tr>
</table>

UNIT XI (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Intersections of Lines and Planes (continued)	192	Common errors to watch for:  Plane $A \cap$ Plane $B = \ell$  Plane $A \parallel \ell$ or Plane $A \cap \ell = \emptyset$.
Extending Your Vocabulary	197	Remind students that Exercises 1-12 on pages 202 and 203 require identification of lines determined by points even though the lines are not drawn. Another reminder: Lower case italicized letters denote <u>lines</u> and script letters denote <u>planes</u>.

UNIT XII: LINE AND ANGLE RELATIONSHIPS
Chapter 8 in Dolciani, Book 7
(9 days)

On completion of Unit XII, the student should be able to

identify from a diagram collinear, equal, opposite, parallel, skew, and coplanar rays

identify from a diagram an angle, the interior of an angle, the exterior of an angle, adjacent angles, and vertical angles

match, with its definition, each of the entries in the vocabulary and spelling list at the end of the chapter

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Separating a Line	252	The use of visual aids will be quite helpful in this chapter. Try to make them big enough for all to see. It may even help for all students to touch the geometric models. As a project, let some students make the models. Board work and student participation will be very useful in this chapter, also.
Line Segments	257	Stress that $\overleftrightarrow{AB} = \overleftrightarrow{BA}$, $\overline{AB} = \overline{BA}$, but $\overrightarrow{AB} \neq \overrightarrow{BA}$.
Separations: Plane and Space	262	
Angles	267	
Congruent Segments and Angles	273	Stress the difference between "equal" and "congruent."

UNIT XIII: MEASUREMENT AND GEOMETRY
Chapter 12 in Dolciani, Book 7
(16 days)

On completion of Unit XIII, the student should be able to

find the measure of a given line segment using a ruler

find the measure of a given angle using a protractor

classify a polygon, both from a given diagram and from a given description, as either a triangle, a certain type of quadrilateral, a pentagon, a hexagon, an octagon, or a decagon

classify a given triangle as scalene, isosceles, or equilateral and as acute, right, or obtuse

find the perimeter of a polygon, given the measure of each side

find the circumference of a circle, given either the diameter or radius of the circle

find the measure of each side of a regular polygon, given the perimeter

find the area of a rectangle, a parallelogram, a triangle, a trapezoid, and a circle, given appropriate measurements

find the volume and surface area of a rectangular prism, given appropriate measurements

find the measure of the complement and supplement of an angle, given the measure of the angle

find the measure of the third angle of a triangle, given the measure of the other two angles

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Length and Perimeter	410	See page 47, TM.
Measuring Segments	417	If possible, have students measure large segments such as the length of a block, a building, or a parking lot. Students will realize that measurement is an approximation when they compare their results.
Angle Measurement	423	Most students have spent very little time using a protractor. Therefore, you may want to prepare worksheets to give the students extra practice.

UNIT XIII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Triangles and Quadrilaterals	431	Point out the relationships between a square, a parallelogram, and a rhombus. Ask questions such as, "Is a square a rhombus?" and "Is a rhombus a square?"
Area	437	Have students draw geometric figures on graph paper and count the squares to find the area of the figure. This may lead students to discover the definitions for areas of rectangles, parallelograms, triangles, and trapezoids. You may also find geoboards of great value in this section.
Circles: Circumference and Area	444	As a homework assignment you may wish to have each student measure the circumference C and diameter d of at least one circular object at home and then find the decimal corresponding to the ratio C/d. The next day have the students compare their results.
Right Angles in Space	448	The ideas in this section will be more clear to students if geometric models are used. Students may have difficulty in drawing the diagrams required in the exercises.
Rectangular Prisms: Volumes and Surface Areas	454	You may find cuisenaire rods useful here.

UNIT XIV: PERCENTAGE AND STATISTICS
Chapter 13 in Dolciani, Book 7
(11 days)

On completion of Unit XIV, the student should be able to

write a ratio using a division sign ÷, a ratio sign :, a fraction, and a decimal

simplify a given expression involving percents

find the missing term in a given proportion

given either a decimal, a common fraction, or a percent, write equivalents to the given numeral in the two forms not given

set up a proportion to solve word problems involving rates

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Ratio and Proportion	464	Notice that there are no oral exercises in this chapter. You may want to make worksheets to give students extra practice. A discussion of the rule of three might create interest. See TM, page 52 (13-1).
Ratio and Percent	468	The importance of estimating and making plausible guesses for a problem should be emphasized. Pick an incorrect answer to a simple sample problem to focus attention upon how absurd some answers may be. Pupils should avoid falling into the habit of being satisfied with any result they obtain.
Percents and Percentages	473	Students should be told that percentage is always a single word.
Percent in Banking and Buying	477	Optional Have students find articles in newspapers and magazines for oral presentation and discussion, if you discuss this section.
Percents and Graphs	481	Optional
Bar and Broken-Line Graphs	484	Optional
Frequency Distributions	490	Optional. Spend considerable time discussing the vocabulary on page 491, if you cover this section.

PREFACE

This resource guide for Basic Mathematics 7 is organized as follows:

1. A recommended sequence of units with a suggested time allotment for each unit.

2. Unit Outlines
   a. Specific objectives to be realized by the student on completion of the unit.
   b. A detailed breakdown of the unit into sub-units with suggestions for developing topics and hints for the teacher in presentation of topics.

The time allotment for each unit is included only as a guideline for the teacher. The specific objectives of each unit set a basic body of material to be covered in Basic Mathematics 7 and the standard of proficiency that should be expected from each student in the Kent School District.

The fact that this guide can be beneficially revised will become evident with use. We hope that teachers using it will critically analyze it and make comments and suggestions about its utility, content and structure.

COMMITTEE FOR BASIC MATHEMATICS 7 GUIDE

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Kentridge Senior High School

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TABLE OF CONTENTS

INTRODUCTION

Statement of Purpose

The purpose of Basic Math 7 is to provide an extensive review of basic arithmetic fundamentals for students whose level of achievement is significantly below that expected of the entering seventh-grade student. As this is basically a review course, new material will not be introduced except as a tool for reinforcing previously covered topics. It is essential that a great deal of flexibility be maintained in conducting these classes as the students enrolled in the Basic Math program are likely to have widely differing backgrounds and abilities. The course should be limited exclusively to those students who need this extensive review and any students who do not fall into this category should be placed in other classes as soon as possible.

Many of the students in Basic Math 7 are likely to have had difficulty with mathematics throughout their schooling and it is anticipated that many of them will be suffering from a negative attitude toward mathematics in general. Perhaps the primary function of the teacher in the Basic Math program is to try to overcome this negative attitude and build in the student a feeling that mathematics is a subject worthy of consideration.

Objectives

The major objective of Basic Math 7 is to provide an extensive review for the student deficient in the basic skills of arithmetic. This will include the following topics:

Place Value
The Four Fundamental Operations of Arithmetic
Measurement
Principles of Order
Elementary Number Theory
An Introduction to the Rational Numbers

Prerequisites and Selection of Students

Seventh grade status and a grade placement two or more years below grade level on a standard test of mathematical achievement (or below tenth percentile).

RECOMMENDED TEXTS AND REFERENCE MATERIALS

Basic Textbook:

Eicholz, O'Daffer, Brumfiel, Shanks *Basic Modern Mathematics - First Course;* Addison-Wesley, Inc. (Palo Alto, Calif., 1965)

Supplementary References:

Eicholz, O'Daffer, Brumfiel, Shanks *Supplementary Experiences - First Course;* Addison-Wesley, Inc. (Palo Alto, Calif., 1965)

Longley, Cook *Fun With Brain Puzzles;* Fawcett Publications, Inc., (Greenwich, Conn., 1965)

Meyer *Fun With Mathematics;* Fawcett Publications, Inc., (Greenwich, Conn., 1961)

National Council of Teachers of Mathematics *Enrichment Mathematics for the Grades, Twenty-seventh Yearbook;* National Council of Teachers of Mathematics, Inc., (Washington, D. C., 1964)

National Council of Teachers of Mathematics *Topics in Mathematics for Elementary School Teachers, Twenty-ninth Yearbook;* National Council of Teachers of Mathematics, Inc., (Washington, D. C., 1964)

Spitzer *Enrichment of Arithmetic* Webster Division - McGraw-Hill Book Co., (San Francisco, 1964)

Spitzer *The Teaching of Arithmetic* Houghton Mifflin Co. (Boston, 1961)

SUGGESTED TIME ALLOTMENTS

UNIT	First Semester	Suggested Time *
I	Place Value	8 days
II	Addition and Subtraction	12 days
III	Adding and Subtracting	13 days
IV	Multiplication	13 days
V	Division	10 days
VI	Measurement	10 days
VII	Basic Principles	9 days
	Second Semester	
VIII	Multiplying	18 days
IX	Dividing	20 days
X	Number Theory	7 days
XI	Fractions	15 days
XII	Rational Numbers	15 days

**This suggested time schedule is based on a school year of 150 teaching days. Thirty days have been allowed for testing, review, and other activities.*

UNIT I: PLACE VALUE
Chapter 1 in Basic Modern Mathematics Textbook 7
(8 days)

On completion of Unit I, the student should be able to

graph a given array by tens and write the whole number corresponding to the array

identify the place value of a designated digit of a six-digit whole number

list a given set of whole numbers in ascending or descending order

write the numeral corresponding to a verbally stated number

read aloud a given list of numerals of less than seven digits

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Grouping by Tens	1	Emphasize the fact that the number for which a given digit stands depends upon its place in the numeral.
2-Digit Numerals	2	
Hundreds	3	
3-Digit Numerals	4	
Thousands	5	
Reading the Odometer	9	
5 and 6-Digit Numerals	10	
One Million	11	
7, 8, and 9-Digit Numerals	12	
Mountain Peaks in North America	14	Spend plenty of time reviewing during and at the end of this and every unit.
Planets	16	

UNIT II: ADDITION AND SUBTRACTION
Chapter 2 in Basic Modern Mathematics Textbook 7
(12 days)

On completion of Unit II, the student should be able to

write two addition and two subtraction equations associated with two sets

recite from memory the sum of any pair of one-digit numbers and state the subtraction facts corresponding to each combination

write the addition or subtraction equation for a given number line illustration, and conversely write the twelve addition combinations for three different whole numbers by ordering and grouping

state the operations (addition and/or subtraction) necessary to the solution of short sentence word problems

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Addition and Subtraction are Related	20	Have the students compare two sets by a one-to-one correspondence to show subtraction facts.
Comparing Sets	22	Demonstrate equations by using sets of objects.
Collecting Shells	23	Provide games and puzzles using addition and subtraction facts through 18.
Addition and Subtraction on the Number Line	24	Discuss the structure of the number line with the students.
Addends and Sums	26	Discuss the function machine. It is used throughout the text.
Differences and Missing Addends	28	
Treasure Map	31	Have an oral discussion about scale drawings.
The Order Principle for Addition	32	
The Grouping Principle for Addition	33	
Rearranging Addends	34	
Sums Between 10 & 19	36	Number riddles can be used to create interest and participation by the students.

UNIT II (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Finding Differences When You Know the Addends	39	
Another Way to Find Differences	40	Begin using variables in discussions and written work. Let students subtract problems such as "15 - 8 = n" by doing two easy problems such as 15 - 5 = 10 and 10 - 3 = 7.
Short Stories	42	
Guess-the-Rule Game	43	Have students make up their own rules. Then each student should have the opportunity to present his rule to the class with the teacher participating. However, the teacher should allow the students to guess the rule as much as possible. This activity can take several days, if the students participate and continue to show interest.
The Function Game	44	

UNIT III: ADDING AND SUBTRACTING
Chapter 3 in Basic Modern Mathematics Textbook 7
(13 days)

On completion of Unit III, the student should be able to

state the value (in cents and in dollars and cents) of two coin collections and write the decimal numerals that represent their sum and difference

find the sum or difference (requiring "carrying" or "borrowing") of any two whole numbers

solve sentence word problems which are statements of addition and multiplication facts

insert the symbols <, >, or = appropriately between two whole number sums or differences

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Pennies and Dimes	50	Stress grouping (dimes with dimes and pennies with pennies).
Value of Coin Collections	52	
Think, Think, Think	54	
Sums: 10, 20, 30, 40	56	
Addition: 2-Digit Numbers	58	Work through problems on page 58 in class. Progress slowly to column addition.
Subtraction	62	Several problems should be demonstrated before assignment is made.
Inequalities	66	
Sums: 2-Digit Numbers	68	Show "re-grouping" to explain "carrying." Good place to use abacus.
Reasoning in Subtraction	71	
Place Value	72	
Place Value and Subtraction	73	Again "re-grouping" is the key.

UNIT III (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Subtraction: 2-Digit Numbers	74	Formalize "borrowing" from "re-grouping."
Working With Dollars and Cents	76	Note: Students may not be sure of basic "combinations." Check this, use flash cards, etc., if necessary. Have students work at board, use contests and games to stimulate interest.

UNIT IV: MULTIPLICATION
Chapter 4 in Basic Modern Mathematics Textbook 7
(13 days)

On completion of Unit IV, the student should be able to

group a rectangular array by rows and by colums and write the two procucts that represent each grouping

replace a repeated addition expression with its equivalent product

recite from memory the product of any pair of one-digit numbers

solve short sentence word problems which are statements of multiplication facts

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Meaning of Multiplication	82	Give many examples to show relationship between addition and multiplication.
Products and Factors	84	As it will be used later, make sure students know what we mean by <u>product</u> and <u>factor</u>.
The Order Principle for Multiplication	85	It may help to take a multiplication table and show it is symmetric about the diagonal.
Collecting Pennies	86	
Collecting Dimes	87	
Multiplication and Repeated Addition	88	
Throwing Darts	89	

UNIT IV (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Multiplication and The Number Line	90	Large number line on the board may help.
Multiplication Review	92	
Short Stories	93	
One in Multiplication	94	
Zero in Multiplication	95	
Multiplication Facts	98	Have the students make up their own multiplication tables on graph paper.
Reasoning to Find Products	100	
Practice Finding Products	102	
The Forest Ranger	103	
Finding Missing Factors	104	Could be a good exercise to play competitive games with in order to increase the students' speed in multiplication.
The Post Office	105	
Short Short Stories	106	
The Function Game	107	Another game type oral exercise.

UNIT V: DIVISION
Chapter 5 in Basic Modern Mathematics Textbook 7
(10 days)

On completion of Unit V, the student should be able to

graph a rectangular array by rows and columns and write the two quotients associated with each grouping

replace a repeated subtraction expression with its equivalent quotient

find quotients by writing the missing one-digit factors

state the operations (addition, subtraction, multiplication, or division) necessary to the solution of short sentence word problems

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Division and Sets	114	Emphasis here is on grouping of various sets into subsets. Students should have practice in writing division equations for simple word problems.
At the Ball Game	116	These word problems should be done in class. Allow time for discussion and checking.
Division and Subtraction	117	Here we emphasize that division is merely repeated subtraction. Stress again the writing of division equations.
Division and the Number Line	118	This is another means of graphically representing division. A thorough treatment is not necessary.
The Function Game	119	Function machines are interesting tools to familiarize students with basic operations. A discussion of different kinds of function machines might be valuable.
Division and Multiplication	120	Emphasize that division equations can be written as multiplication equations. Time might be spent going from one form to the other.
Quotients and Missing Factors	121	This provides excellent practice in manipulation of division and multiplication equations.
Zero in Division	124	Division by zero guarantees instant disaster. Let students know this in no uncertain terms.
One in Division	125	Here we emphasize that $a/a = 1$ when $a \neq 0$. This is very important for later work.

UNIT V (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Division And Sets Again	127	
Short Stories	128	All these problems should be done to provide practice in application of division in word problems. Insist that equations must be written.
Quotients and Missing Factors	129	Emphasize exercise 3 to show relationship between division and multiplication.
Skills in Division	130	More practice.
At the Dairy	131	Here are more word problems. You may wish to add supplementary exercises from other sources. This is where more practice may be necessary.

UNIT VI: MEASUREMENT
Chapter 6 in Basic Modern Mathematics Textbook 7
(10 days)

On completion of Unit VI, the student should be able to

choose from a list a unit segment, square, unit, or cubic unit appropriate to a given line segment to the nearest half unit of measurement

draw a grid of one inch (or centimeter) squares over an irregularly shaped figure and estimate its area

find the volume of a solid by counting the cubes

state the larger of two liquid measures

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Ways of Measuring	134	Stress that units of measure are arbitrary and that measuring is a form of counting.
Units for Measuring	136	Show the need for 3 different types of units (for length, area and volume).
Measuring With Different Unit Segments	138	
Measurement is Not Exact	142	

UNIT VI (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Measuring to the Nearest Inch	143	Show interval within which length must fall.
Measuring to the Nearest Half-Inch	144	
Finding Area	146	Have students find areas of desks, the room, etc. by using a square foot or a square yard piece of paper.
Estimating Area	149	
Finding Volume	150	Three dimensional models are very helpful here.
Liquid Measure	152	Models helpful again.

UNIT VII: BASIC PRINCIPLES
Chapter 7 in Basic Modern Mathematics Textbook 7
(9 days)

On completion of Unit VII, the student should be able to

find the missing elements in equations that are examples of the order, grouping and multiplication-addition principles

solve short sentence word problems that involve the use of the order, grouping and multiplication-addition principles

find the sum or product of a given list of numbers by ordering or grouping into pairs whose sums are multiples of ten and then adding these multiples

write the equation assiciated with a given pair of rectangular arrays that is an example of the order, grouping or multiplication-addition principle

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Order Principle for Addition	156	Use of the number line may help. It may be mentioned that this is also called the commutative property.
The Order Principle for Multiplication	157	

UNIT VII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Grouping Principle for Addition	158	Also called associative property. Show how grouping can sometimes make the operations easier.
The Grouping Principle for Multiplication	159	
Rearranging Addends	160	
Rearranging Factors	161	
Review of Principles	162	
Working in the Book Store	163	
A New Principle	164	Also called distributive property of multiplication over addition.
The Multiplication-Addition Principle	166	More practice may be needed.
Multiplication-Addition Exercise	168	
The Multiplication-Addition Principle Again	169	Could be used for oral exercises.

UNIT VIII: MULTIPLYING
Chapter 8 in Basic Modern Mathematics Textbook 7
(18 days)

On completion of Unit VIII, the student should be able to

find the product of a single and a multiple digit number

solve short sentence word problems involving the use of multiplication

solve equations involving the use of the multiplication-addition principle for missing elements

write, without computation, the product of a whole number and 10, 100 or 1000

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Factors of 10 and 100	177	The aim here is to enable students to perform mentally, multiplications by 10 and 100. Extensive treatment may not be necessary.
Factors of 10, 20, 30	179	
Related Products	180	Just more practice.
The Multiplication Addition Principle	182	You may wish to refer to this as the distributive property.
Multiplication-Addition Problems	183	Students should do all these problems.
Using the Multiplication-Addition Principle	184	This simply involves using the distributive property to simplify computation. Students may need practice.
Multiplication-Addition Problems	185	
2-Digit Factors	186	This teaches the same thing in vertical form. No special treatment is needed.
A Short Cut	187	
Miles Per Gallon	189	It is felt that students can profit by as much practice as possible in solving word problems.
Time	191	

UNIT VIII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Finding Products	193	This section is just more drill. Use if needed. Possibly some students will not need it.
Moon Trip	194	
How Many...	195	This might provide a welcome change in that the approach is a little different.
3-Digit Factors	196	For students having trouble, it would be helpful to go through the explanation in the text carefully.
At the Airport	197	
Factors With 4 or More Digits	198	
Short Stories	199	
Estimating Products	200	This is a valuable section as it provides a means for checking the reasonableness of answers. It should be given thorough treatment.
Making the Best Guess	201	
Map Problems	202	Although the arithmetic required for this section is not rigorous, it does provide an introduction to using maps. An extensive class discussion may prove valuable.
Chapter Review	204	Students will probably need this.

UNIT IX: DIVIDING

Chapter 9 in Basic Modern Mathematics Textbook 7

(20 days)

On completion of Unit IX, the student should be able to

find the quotient with and without remainders of two numbers when the divisor is a single digit number

solve short sentence word problems involving the use of division

write a division problem associated with a given rectangular array

find a missing digit in the quotient of a given division problem

write the two division equations related to a given multiplication equation

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Quotients With Zero Endings	208	Also use divisions by 10 and 100.
About How Fast Can They Go	210	
At Camp and Short Stories	212	Assign all story problems (either as class discussion or assigned work).
The Function Game	214	You may follow this by asking each student to make up several rules and see if the class can find this rule.
Reasoning in Division	216	Strive for the correct multiple of 10.
Division and Addition	220	Good examples of the distributive property of division over addition.
Estimating the Quotient	222	
Finding the Tens Digit for Quotients	224	These are all leading to the division algorithm.
Finding the Ones Digit in Quotients	226	
Finding 2-Digit Quotients	229	Go over these steps carefully.
Delivering Newspapers and Short Stories	230	

UNIT IX (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Comparing Prices and Short Stories	232	Story problems are very important.
Money Problems	235	
Remainders in Division	235	Stress that the remainder must be less than the divisor.
Division and Sets	238	Another approach to remainders.
Checking Division	239	Stress this.

UNIT X: NUMBER THEORY
Chapter 10 in Basic Modern Mathematics Textbook 7
(7 days)

On completion of Unit X, the student should be able to

identify the even or odd numbers from a given list

list the multiples of given whole numbers that are less than 100

list the factors of a whole number less than 50 and find the greatest common factor of two whole numbers less than 50

identify from a given list of whole numbers that are less than 50, the prime and the composite numbers

completely factor a whole number less than 50

state whether a number is divisible by 2, 3 or 5 by using the divisibility tests

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Even and Odd Numbers	246	
Sums and Products-Even or Odd	248	Provide plenty of time for class discussion on even, odd, multiples, etc.
Multiples	249	Be sure students say "twelve is the product of four and three," rather than simply saying
Factors and Products	250	"twelve is the product."

UNIT X (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Finding Factors	251	An analogous situation would be to say that "the number equal is equal." Such a statement is meaningless. Similarly, when we say that the number four is a factor, the natural question arises "a factor of what?"
Common Factors	252	
Greatest Common Factors	253	
How many factors?	254	
Prime Numbers	255	
The Function Game	256	Plenty of oral drilling may be used in reviewing even, odd, prime and composite numbers.

UNIT XI: FRACTIONS

Chapter 11 in Basic Modern Mathematics Textbook 7
(15 days)

On completion of Unit XI, the student should be able to

state the fraction that names a designated part of an array

choose from a list those fractions that are improper

express a given fraction in lowest terms

tell whether or not two given fractions are equivalent

write a set of six fractions equivalent to a given fraction

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Number Pairs and Fractions	260	Sometimes reading the vinculum as "of" is helpful. Thus 3/4 is read 3 of 4 parts.
Parts of an Object	261	Now we must insert "equal" parts.
Fractions and Parts of an Object	262	Models for this work would be helpful.
Fractions and Sets	264	Do this work carefully as it leads to equivalent fractions.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Equivalent Fractions	266	The "discovery" approach works well here if several examples are shown.
Sets of Equivalent Fractions	268	
Building Sets of Equivalent Fractions	270	You may wish to go to 3/4 is equivalent to 3x/4x.
Numerator and Denominator	272	Just associating numerator with top and denominator with bottom is sufficient here.
Fractions (Numerator) > or = Denominator	276	"Improper" fractions are not "improper" at all. Also stress that we have 4/3 circles, not 4/3 of a circle.
More About Fractions	277	
Zero Numerators	279	0/4 can be read as zero fourths and thought of as no fourths.
A Check for Equivalent Fractions	280	You may or may not wish to use the formal definition for equivalent fractions i.e. a/b is equivalent to c/d if and only if ad = bc.
Equivalent Fractions Again	282	
Lower and Higher Terms	284	Stress that lower and higher terms are applied only to equivalent fractions.
More About Lower and Higher Terms	285	Notice there is no mention of "reduce."
Lowest Terms	286	
More About Lowest Terms	287	"Common factor" is the key idea.
Lowest Term Fractions	288	

UNIT XII: RATIONAL NUMBERS
Chapter 12 in Basic Modern Mathematics Textbook 7
(15 days)

On completion of Unit XII, the student should be able to

match a given class of equivalent fractions with a point on a given number line

insert the symbols >, <, or = appropriately between two rational numbers

give the lowest-terms fractions corresponding to evenly spaced points in the interval from 0 to 4 on the number line

find the lowest-terms fraction for the sum and difference of pairs of fractions having the same denominator and decimals having the same place value

find the lowest-terms fraction for the product of pairs of common fractions

convert common fractions and whole number percents to decimals, and conversely

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Fractions	294	Consider doing the activity described on page 332 of teacher's edition from the preparation section.
From Fractions to Numbers	296	
Rational Number Exercises	298	
Names for Rational Numbers	300	Once the students have completed the test for equivalence, you can have them write the equality sign between the equivalent fractions and the inequality sign ($\neq$) if they are not equivalent. Do not ask the students to indicate less than and greater than.
Inequality	304	Use physical objects such as measuring cups, to demonstrate inequality concepts.
Rational Numbers Greater Than 1	306	A rational number such as 8/5 is in lowest terms. There is no need of changing it to 1 3/5. However, 8/6 should at least be put in lowest terms of 4/3.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Addition and Subtraction	308	
Whole Numbers and Rational Numbers	310	Many games and puzzles are available for use in regard to rational numbers.
Mixed Numerals	312	
Measurement Using Mixed Numerals	313	
More About Addition and Subtraction	314	
Multiplication of Rational Numbers	316	Use material from supplementary experiences and other sources to give the student more work with operations of rational numbers. Methods for using these exercises are given on page 365 of teacher's manual.
Finding Products	317	
Decimals	318	
Addition and Subtraction Using Decimals	320	
Percents	321	

PREFACE

This resource guide for Modern Mathematics 8 is organized as follows:

1. A recommended sequence of units with a suggested time allotment for each unit.

2. Unit Outlines
   a. Specific objectives to be realized by the student on completion of the unit.
   b. A detailed breakdown of the unit into sub-units with suggestions for developing topics and hints for the teacher in presentation of topics.

The time allotment for each unit is included only as a guideline for the teacher. The specific objectives of each unit set a basic body of material to be covered in Modern Mathematics 8 and the standard of proficiency that should be expected from each student in the Kent School District.

The fact that this guide can be beneficially revised will become evident with use. We hope that teachers using it will critically analyze it and make comments and suggestions about its utility, content and structure.

COMMITTEE FOR MODERN MATH 8 GUIDE

Cedric G. Hannon, Chairman
Kentridge Senior High School

Charles A. Standen
Sequoia Junior High School

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Meridian Junior High School

TABLE OF CONTENTS

INTRODUCTION

Statement of Purpose

The purpose of Modern Math 8 is to provide a background for those eighth grade students who plan to take Algebra in the ninth grade. The course is a continuation and expansion of Modern Math 7 and offers students a sound development of the structure of the real number system.

This knowledge together with the basic mathematical skills form an integral part of the math background that such students will need as adults and at the same time provides the necessary background for future math courses.

Objectives

Modern Math 8 is essentially a course intended to prepare students for subsequent work in the mathematics program. To accomplish this objective, the following topics will be introduced and developed:

fundamental operations with rational numbers, both positive and negative

comparison and measurement of plane and solid geometric figures

radicals and the Pythagorean Theorem

exponents and scientific notation

writing and solving equations with one variable and using these equations to solve word problems

representing truth sets of equations and inequalities on the number line

Prerequisites and Selection of Students

Because from 70 to 80 per cent of all eighth grade students will be enrolled in Modern Math 8 and because of the wide range of abilities and aptitudes in this number of students, it is anticipated that some grouping will be attempted in this course.

To be placed in Modern Math 8 the student must have satisfied one of the following:

a. Completion of Modern Math 7 with a grade of C or better.
b. Completion of Basic Math 7 with a grade of B or better and teacher recommendation.
c. Completion of Modern Math 7 with a grade of D or X and teacher recommendation.

Students who do not achieve to a satisfactory level may be transferred to Basic Math 8.

RECOMMENDED TEXTS AND REFERENCE MATERIALS

Basic Textbook:

Dolciani, Wooton, Beckenbach, Chinn *Modern School Mathematics - Structure and Method;* Houghton Mifflin Co. (Boston, 1967)

Supplementary References:

Gundlach, Buffie, Denny, Kempf *Junior High School Mathematics;* Laidlaw Brothers (River Forest, Illinois, 1968)

Nichols *Pre-Algebra Mathematics;* Holt, Rinehart, Winston (New York, 1965)

Nichols, Flournoy, Kalin, Simon *Elementary Mathematics - Patterns and Structure;* Holt, Rinehart, Winston (New York, 1966)

Longley, Cook *Fun With Brain Puzzles;* Fawcett Publications, Inc. (Greenwich, Conn., 1961)

Parker *The Structure of Number Systems;* Prentice-Hall (Englewood Cliffs, 1966)

Scheid *Elements of Finite Mathematics;* Addison-Wesley Publishing Co. (Reading, Mass., 1962)

School Mathematics Study Group *Mathematics for Junior High School;* Yale University Press (New York, 1961)

Smith *Explorations in Elementary Mathematics;* Prentice-Hall (Englewood Cliffs, 1966)

Wirtz, Botel, Numley *Discovery in Elementary School Mathematics;* Encyclopedia Britannica Press (Chicago, 1963)

SUGGESTED TIME ALLOTMENTS

UNIT	First Semester	Suggested Time *
I	Rational Numbers	10 days
II	Addition and Subtraction of Rational Numbers	10 days
III	Multiplication and Division of Rational Numbers	10 days
IV	Geometric Figures in the Plane	8 days
V	Congruence and Measurement of Plane Figures	12 days
VI	Exponents and Scientific Notation	6 days
VII	The Metric System	11 days
VIII	Precision and Accuracy	Optional (10 days)
IX	Decimal Numerals and Real Numbers	10 days
	Second Semester	
X	Open Number Sentences	10 days
XI	Solving Open Number Sentences	11 days
XII	Using Equations	7 days
XIII	Square Roots; Similar Figures	15 days
XIV	Pyramids and Prisms	10 days
XV	Cones, Cylinders, and Spheres	7 days
XVI	Relations, Functions, and Graphing	14 days
XVII	Probability	Optional (11 days)

**This schedule gives 151 total days with the omission of the two optional chapters. Individual teachers may wish to vary the time allotments as necessary.*

UNIT I: RATIONAL NUMBERS
Chapter 1 in Dolciani, Book 8
(10 days)

On completion of Unit I, the student should be able to

properly place a positive or negative rational number relative to two given points on a number line

show with a Venn diagram and by the roster method the union and intersection of the sets of positive, negative, nonpositive, and nonnegative rational numbers

name and define the opposite of any given rational number

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Negative Numbers	2	The comments in the TM are quite explicit and should be of sufficient help in teaching this section.
Sets of Numbers	6	
Opposites of Numbers	14	
Order	17	Many students will have trouble placing the correct sign (< or >) between two negative numbers.
Arrows on the Number Line	21	It is helpful if the students think of "absolute value" as being the number of units away from the origin on a number line. A common mistake: -n may stand for a positive or a negative number, not just a negative number as some students will think.

UNIT II: ADDITION AND SUBTRACTION OF RATIONAL NUMBERS
Chapter 2 in Dolciani, Book 8
(10 days)

On completion of Unit II, the student should be able to

graph on a number line the sum or difference of two given rational numbers

write a mixed numeral in the form a/b, where a and b are integers, $b \neq 0$, and vice versa

write the absolute value of a given rational number

write another fractional name and decimal name for a given fractional number

write the sum or difference of any two rational numbers

write the generalizations for the closure property of addition, commutative property of addition, associative property of addition, additive identity, and the additive inverse property

match, from a list of numerical expressions, each of the above properties

write the generalization $a - b = a + (-b)$ and change given subtraction problems to equivalent addition problems

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Sums of Nonnegative Rational Numbers	32	A brief review of the addition and simplification of fractions will be helpful. Review also how to change a mixed number to a fraction and how to derive a decimal numeral and percent equivalent to a given fraction.
Sums of Negative Rational Numbers	37	Students tend to have great difficulty with the concept of absolute value. The important thing is that the students learn to add signed numbers. The concept of absolute value is only one means for improving this skill. If the students can form the necessary generalizations on their own, this is sufficient.
Sums of Rational Numbers in General	41	The teacher will find that particularly for a slower class extensive use of the number line in introducing the addition of positive and negative rationals will be extremely beneficial. The discovery method can be used very successfully here. A supplemental list of problems may also be required as a thorough mastery of this unit is necessary before the introduction of subtraction.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Properties of Addition	45	The closure property may give some students trouble.
Subtraction of Rational Numbers	51	Stress the fact that $a - b = a + (-b)$ and that each subtraction problem should first be changed to an equivalent addition problem before a solution is attempted. Emphasize that subtracting a negative number is the same as adding the additive inverse of the number.

UNIT III: MULTIPLICATION AND DIVISION OF RATIONAL NUMBERS
Chapter 3 in Dolciani, Book 8
(10 days)

On completion of Unit III, the student should be able to

give the sign of the product of any two rational numbers by inspection

find the product of two or more rational numbers

match the properties of multiplication with statements illustrating these properties

give the sign of the quotient of any two rational numbers by inspection

find the quotient of two rational numbers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Product of a Rational Number and a Nonnegative Rational Number	62	Use the concept of multiplication as repeated addition to lead into the rules for the product of a rational number and a nonnegative rational. Be certain students realize the reasons why they may use "slant bars" in the process of multiplying or dividing rational numbers.
The Product of Two Negative Rational Numbers	65	The table approach on page 66 is excellent. Might use "Friend of a Friend" as shown in Algebra 1-2 guide on page 20.
Properties of Multiplication	68	The book's discussion will have to be expanded.
Division of Rational Numbers	73	

UNIT IV: GEOMETRIC FIGURES IN THE PLANE

Chapter 4 in Dolciani, Book 8

(8 days)

On completion of Unit IV, the student should be able to

identify from a drawing: a) transversal, b) corresponding angles, c) exterior angles, d) interior angles, e) supplementary angles, f) vertical angles, g) alternate angles

use a protractor to measure and construct given angles with no more than one degree of error

bisect a segment and an angle using only compass and straightedge

duplicate a given triangle using only a straightedge and a compass

draw all axes of symmetry for a given figure, and label any point of symmetry the figure has

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Drawing Geometric Figures	84	It is recommended that the teacher become somewhat skilled in the use of the blackboard compass. This may take some practice. For instructing the students in using a protractor, the use of an overhead projector and transparencies are strongly advised.
Basic Constructions	92	See TM page 21.
Symmetry	98	

UNIT V: CONGRUENCE AND MEASUREMENT OF PLANE FIGURES
Chapter 5 in Dolciani, Book 8
(12 days)

On completion of Unit V, the student should be able to

state from given information whether or not two given figures are congruent

construct a figure congruent to a given figure

state the ASA, SAS, and SSS properties of congruence

list the corresponding parts of two congruent figures

identify the following from an appropriate drawing: radius, chord, diameter

find the perimeter of a given polygon or closed figures composed of segments and/or semi-circles

find the area of a given rectangular, circular, triangular, or trapezoidal region

define a circle and find the circumference of a circle of given radius or diameter

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Congruent Figures	112	Stress the fact that although a correspondence may be set up between any two polygons of the same number of sides, this correspondence will be a congruence if sides and angles are congruent. Some time should be spent establishing the concept of congruence of segments and angles.
Congruent Triangles	118	
Constructing Congruent Triangles	124	See the geometry overlay notebook for sample constructions that can be used on the overhead projector.
Perimeter	132	The teacher need not expect the student to memorize all of the formulae in this chapter but should expect the student to apply them.
Area	137	

UNIT VI: EXPONENTS AND SCIENTIFIC NOTATION
Chapter 6 in Dolciani, Book 8
(6 days)

On completion of Unit VI, the student should be able to

write a number which is an integal power of 10 by use of exponents

multiply and divide numbers which are powers of 10 by use of exponents

write numbers in scientific notation

multiply and divide numbers written in scientific notation

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Exponents	154	Emphasize note on top of page 155 in TM.
Zero and Negative Exponents	158	See TM.
Scientific Notation	161	Emphasize multiplying and dividing by powers of 10.
Scientific Notation in Products and Quotients	166	Note that answer must sometimes be converted before it is in scientific notation. In the exercises, remind students that their answers in "decimal numeral" form need not have a decimal. Stress that it is very easy to estimate products and quotients when expressions are written in scientific notation. You might ask students to estimate answers before they actually work problems.

UNIT VII: THE METRIC SYSTEM
Chapter 7 in Dolciani, Book 8
(11 days)

On completion of Unit VII, the student should be able to

- convert from one metric unit of length, volume, or area, to another
- multiply and divide by integral powers of 10 mentally
- use scientific notation to convert from one metric unit to another
- convert from one unit to another in the English system
- convert units of linear measure from English to metric and vice-versa

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Throughout this unit, do not require memorization of equivalent units.
Units of Length	174	Stress that the yard is <u>defined</u> as .9144 meters. This is <u>exact</u>. The same is true with 1 in = 2.54 cm. Other conversion factors are approximate.
Conversion of Linear Units	180	This section will give students much practice in mental multiplication by powers of 10.
Metric Units of Area	184	One approach that can be used to convert units of measure is to multiply by fractions equal to "one" in which the numerator and denominator are equivalent units of measure. For example, to change 3 cm. to feet. The problem would be set up as follows: $3 \text{ cm.} \times \frac{1 \text{ in.}}{2.54 \text{ cm.}} \times \frac{1 \text{ ft.}}{12 \text{ in.}}$ Notice that the sequence is set up in such a way that all the labels cancel except that one to which we wish to change.
Metric Units of Volume	188	
Metric Units of Mass and Capacity	192	

UNIT VIII: PRECISION AND ACCURACY
Chapter 8 in Dolciani, Book 8
(10 days)

On completion of Unit VIII, the student should be able to

write the unit of measure for any given measurement

determine the greatest possible error for a given measurement

select the more precise of two given measures

determine the number of significant digits in a given measure

round off a given measurement to a given number of significant digits

find the relative error of a given measurement

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Unit of Measure and Greatest Possible Error	202	The teacher should not expect complete mastery of this unit but rather should treat it as a first exposure stressing the idea that a measure is not exact.
Significant Digits	208	
Relative Error	212	

UNIT IX: DECIMAL NUMERALS AND REAL NUMBERS
Chapter 9 in Dolciani, Book 8
(10 days)

On completion of Unit IX, the student should be able to

name a rational number corresponding to a given repeating or terminating decimal

name a terminating or repeating decimal corresponding to a given fraction

compare the size of two given numbers in both decimal and fractional notation

name a rational and an irrational number between two given numbers

graph a given set of real numbers

use set-builder notation to name the set of real numbers on a given graph

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Decimal Numerals for Rational Numbers	224	Stress that the "vinculum" is *only* over the repetend. Note that $2.\overline{45} = 2.4545$ but that $2.\overline{45}$ is simpler.
Rational Numbers Named by Repeating Decimal Numerals	229	This process will be difficult for some students and mastery by all should probably not be expected.
Real Numbers	234	You might use Venn diagrams to stress the relationships between the real numbers, the rational numbers, the integers, the whole numbers, and the natural numbers. Students may not believe the statement on page 237 which says that $759.\overline{9} = 760.\overline{0}$. Have them help you justify this equation by using the process of the preceeding section.
Density	241	
Completeness and Graphs	245	Point out that density and completeness are not equivalent. Warn students that a common mistake in graphing the sets in the exercises is to forget that the variables represent real numbers and graph points, instead.

UNIT X: OPEN NUMBER SENTENCES
Chapter 10 in Dolciani, Book 8
(10 days)

On completion of Unit X, the student should be able to

write open number phrases expressing sums, differences, products, and quotients

evaluate open number phrases when the value of the variable is known

determine truth or falsity of open number sentences in one or more variables when the values of the variables are known

solve simple equations in one unknown by inspection

write an open number sentence to express a relationship in a simple word problem

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Number Phrases	258	Students may wish to devise their own "mind reading games" similar to those on page 258. Stress also the difference between "phrases" and "sentences." Subject and verb give a sentence as in English.
Values for Open Number Phrases	262	
Statements and Open Number Sentences	266	The definitions on page 267 in the colored boxes are invaluable here. They should be studied carefully.
Solving Equations by Inspection	271	Encourage students to express answers in set notation {} in these exercises. It is a good habit to acquire as it will be used extensively later.

UNIT XI: SOLVING OPEN NUMBER SENTENCES
Chapter 11 in Dolciani, Book 8
(11 days)

On completion of Unit XI, the student should be able to

solve simple linear equations in one variable using the transformations listed on page 283 of the text and acceptable set notation

write the addition and multiplication properties of equality

solve and graph simple inequalities using the transformations on page 296 in the text

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Many students will be able to solve the given equations by inspection. However, require them to show their work as a demonstration of their understanding of the transformations discussed.
Solving Equations by Transformations	282	Most students will require a great deal of practice in just the mechanics of solving equations. Supplementary problems can be taken from any Algebra text.
More About Variables and Equations	288	If you assign any of the exercises in part C, warn students that the solution sets of the equations may have more than one member.
Solving Inequalities	294	
Compound Inequalities	300	This section is optional. Include only if time and class ability allow.

UNIT XII: USING EQUATIONS
Chapter 12 in Dolciani, Book 8
(7 days)

On completion of Unit XII, the student should be able to

translate word sentences into number sentences

represent two unknowns in terms of one variable

solve problems by use of an equation

solve percentage problems by use of an equation

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		The material in this section will be difficult even for superior pupils. Urge students to state for what their variables stand and to show their work. Also encourage them to draw diagrams as an aid to setting up the proper equations.
Translating Word Sentences into Number Sentences	312	Emphasize that the members of an equation represent numbers.
Applying Number Sentences in One Variable	316	Go through the steps of problem solving carefully.
Percentage Problems	322	Use the proportion with a variable to solve percentage problems.

UNIT XIII: SQUARE ROOTS; SIMILAR FIGURES
Chapter 13 in Dolciani, Boook 8
(15 days)

On completion of Unit XIII, the student should be able to

find square roots of numbers up to 1000 having rational roots

determine whether 3 numbers are Pythagorean triples

find the square root of any 3-digit number to two decimal places

write the square root of any 3-digit positive integer as the product of the greatest possible integer and the smallest possible radical

use the Pythagorean Theorem to find the third side of any right triangle of which two sides are known, and write the answer in simplified form

apply the Pythagorean Theorem to such problems as finding the diagonal of a rectangle

use the idea of similar polygons to solve simple problems involving proportion in measurement

identify corresponding parts of similar polygons and name the criteria for determining similar triangles

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Pythagorean Property	332	This property is the basis for most of the work done in this unit on radicals. It should be treated thoroughly. Students should receive considerable practice in verifying Pythagorean triples.
Square Roots	336	Stress breaking down radicals into prime factors as an aid to simplification of irrational square roots. If this is done adequately now, future troubles may be avoided.
Special Right Triangles	339	Students may be overwhelmed by the material in this section. The 30°-60° right triangle discussion may prove especially difficult. You may need to provide reasons for each step in the development of the formula $b = a\sqrt{3}$. Urge students to draw diagrams to help with solutions wherever possible.

UNIT XIII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Rational Approximations to Square Roots	343	In general, do not worry about finding these any more accurately than two decimal places. Again, stress the idea of breaking down the radicals into simpler form.
Similar Polygons	347	Stress use of corresponding parts in solving word problems.
Trigonometric Ratios	353	Optional as time permits.
Scale Drawings	359	Optional. May provide welcome relief both for students and teacher.

UNIT XIV: PYRAMIDS AND PRISMS
Chapter 14 in Dolciani, Book 8
(10 days)

On completion of Unit XIV, the student should be able to

define a line perpendicular to a plane

define the distance between two parallel planes

sketch a 3-dimensional figure, prism, and pyramid

identify the base, altitude, and slant height of a given polyhedron

identify from a given set of drawings, a tetrahedron, a triangular and rectangular prism, a parallelpiped, and an oblique prism and a regular prism

find the surface area of a right rectangular prism, a right triangular prism, and a regular pyramid from a given drawing

find the volume of a prism and a pyramid from a given drawing

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Even faster pupils may feel swamped by the concepts in this unit. Do not require memorization of the formulas.
Points, Lines, and Planes in Space	374	Students invariably have trouble drawing 3-dimensional figures. For suggestions as to how to help them learn, see TM, pages 32-33.
Tetrahedrons and Other Pyramids	379	When discussing a polyhedron, do not depend upon the students' ability to picture it mentally. The teacher should always have a model close at hand throughout this section. (Plastic models may be made easily and inexpensively.)
Prisms	385	For additional help in visualization, see the geometry transparencies book.
Surface Areas of Prisms and Pyramids	390	In order to gain a better understanding of surface areas and how the formulas are derived, the teacher should have the students construct models such as those on pages 384-385 and 389.
Volumes of Prisms and Pyramids	395	The emphasis should be on application of the formulae. Students should not be expected to memorize them.

UNIT XV: CONES, CYLINDERS, AND SPHERES
Chapter 15 in Dolciani, Book 8
(7 days)

On completion of Unit XV, the student should be able to

name the parts of a cone, cylinder, and sphere

find the surface area and volume of a given cone, cylinder, and sphere

It may be desirable to teach this unit after Chapter 16 or 17 if the students' interest in area and volume seems to vanish at the end of Chapter 14.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Cones and Cylinders	408	Models are essential here.
Spheres: Great and Small Circles	412	The use of a world globe is good.
Surface Area of Cylinders, Cones, and Spheres	417	Have the students construct cylinders and cones to illustrate the principles of this section.
Volumes of Cylinders, Cones, and Spheres	422	Relationships between cylinder and prism, cone and pyramid should be stressed and generalized. Students should not be expected to memorize all formulae in this chapter. The emphasis should be on application.

UNIT XVI: RELATIONS, FUNCTIONS, AND GRAPHING
Chapter 16 in Dolciani, Book 8
(14 days)

On completion of Unit XVI, the student should be able to

locate on the number plane any point given by an ordered pair (x,y)

determine whether an ordered pair determines a point which is in the truth set of an equation in two unknowns

graph the truth set of any linear equation in two unknowns

solve two simultaneous linear equations by graphing

determine by inspection in what quadrant a point named by an ordered pair will be found

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Solutions of Equations in Two Variables	432	Emphasize the idea of ordered pairs. The students have a tendency to ignore order, possibly as a result of continuous exposure to the commutative property.
Solution Sets of Open Number Sentences in Two Variables	434	
Rectangular Coordinate Systems	438	It might be interesting for more able students to find the distance between two points on the number plane by use of the Pythagorean Theorem. Also, a little information on Descartes might be of interest here.
Relations and Functions	443	You might want to give other examples of functions from everyday life such as postage as a function of weight and height as a function of age. Students will need many examples to show the relevance of the definition of function on page 445. Stress that each element of the domain is paired with <u>only one</u> element in the range. This explains why the test at the top of page 446 works.
Graphing Linear Equations	447	Possibly the graphs of equations which are not linear could be shown here. Examples are: $y = x^2$, $x^2 + y^2 = 1$, $y = 1/x$, $y = x^3$, $y = 1/x^2$.

UNIT XVI (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Graphing Linear Inequalities	452	
Systems of Linear Equations	454	Possibly, another method of solving these systems could be introduced, but take care that the students do not try to do them all by addition. Remember that the purpose of the chapter is largely an introduction to graphing.

UNIT XVII: PROBABILITY
Chapter 17 in Dolciani, Book 8
(11 days)

On completion of Unit XVII, the student should be able to

list the possible outcomes and give the probability of each outcome of an experiment involving "equally likely" outcomes

list the set of favorable outcomes and give the probability of one of these occurring in an experiment

establish the odds in favor of an event knowing the probability of the event

find the probability of $E \cup F$ where E and F are mutually exclusive events

find the probability of $E \cup F$ where E and F are events which are not mutually exclusive

examine a random sample and establish empirical probability for an event

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		This unit is an enjoyable one to cover during the last days of school. Do so if time permits! An excellent reference: S.M.S.G.'s *Probability for Intermediate Grades, Teacher's Commentary.*
The Probability of an Outcome	470	Emphasize that all possible distinct outcomes of an experiment must be considered.
The Probability of an Event	473	Distinguish between an outcome and an event, which may be a set of favorable outcomes.
The Probability of E or F for Disjoint Sets	478	Work on "mutually exclusive." Be sure students equate this idea with disjoint sets. Venn diagrams are a useful tool for illustrating this concept.
The Probability of E or F in General	481	Intersection and union of sets should be reviewed if necessary.
Empirical Probability: Samples	485	Example from TM page 38 is excellent.

PREFACE

This resource guide for Basic Mathematics 8 is organized as follows:

1. A recommended sequence of units with a suggested time allotment for each unit.

2. Unit Outlines
   a. Specific objectives to be realized by the student on completion of the unit.
   b. A detailed breakdown of the unit into sub-units with suggestions for developing topics and hints for the teacher in presentation of topics.

The time allotment for each unit is included only as a guideline for the teacher. The specific objectives of each unit set a basic body of material to be covered in Basic Mathematics 8 and the standard of proficiency that should be expected from each student in the Kent School District.

The fact that this guide can be beneficially revised will become evident with use. We hope that teachers using it will critically analyze it and make comments and suggestions about its utility, content and structure.

COMMITTEE FOR BASIC MATHEMATICS 8 GUIDE

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INTRODUCTION

Statement of Purpose

The purpose of Basic Math 8 is to provide an extensive review of the fundamental operations of arithmetic for students who are significantly below the level of most entering 8th graders in arithmetic skills. Although the course does introduce some new material, particularly in the areas of geometry and number theory, the primary purpose of the course is to provide a review of previous work. For this reason, students who do not need this type of course should be transferred to the regular Math 8 course as soon as possible.

We anticipate that some of the students in Basic Math 8 will be taking Pre-Algebra in the 9th grade and, therefore, the course material is a little more abstract than that in Basic Math 7. As is often the case in review courses, motivation may be the single most important problem encountered by the teacher. For this reason, the teacher of Basic Math 8 may be called upon to exercise his utmost powers of imagination and creativity.

Objectives

The major objective of Basic Math 8 is to provide an extensive review for the student deficient in the basic skills of arithmetic. This review will include the following topics:

- Place Value
- The Four Fundamental Operations of Arithmetic
- Measurement
- Principles of Order
- Elementary Number Theory
- Rational Numbers
- Decimals and Percents

Prerequisites and Selection of Students

Eighth grade status and a grade placement two or more years below grade level on a standard test of mathematical achievement (or below tenth percentile).

RECOMMENDED TEXTS AND REFERENCE MATERIALS

Basic Textbook:

Eicholz, O'Daffer, Brumfiel, Shanks *Basic Modern Mathematics - Second Course;* Addison-Wesley, Inc. (Palo Alto, California, 1965)

Supplementary References:

Eicholz, O'Daffer, Brumfiel, Shanks *Supplementary Experiences - Second Course;* Addison-Wesley, Inc. (Palo Alto, California, 1965)

Longley, Cook *Fun With Brain Puzzles;* Fawcett Publications, Inc. (Greenwich, Conn., 1965)

Meyer *Fun With Mathematics;* Fawcett Publications, Inc. (Greenwich, Conn., 1961)

National Council of Teachers of Mathematics *Enrichment Mathematics for the Grades, Twenty-seventh Yearbook;* National Council of Teachers of Mathematics Inc. (Washington, D. C., 1963)

National Council of Teachers of Mathematics *Topics in Mathematics for Elementary School Teachers, Twenty-ninth Yearbook;* National Council of Teachers of Mathematics, Inc. (Washington, D. C., 1964)

Spitzer *Enrichment of Arithmetic;* Webster Division - McGraw-Hill Book Co. (San Francisco, 1964)

Spitzer *The Teaching of Arithmetic;* Houghton Mifflin Co. (Boston, 1961)

SUGGESTED TIME ALLOTMENTS

UNIT	First Semester	Suggested Time *
I	Place Value and Number Bases	11 days
II	Addition and Subtraction	14 days
III	Multiplication and Division	13 days
IV	Measurement	11 days
V	Special Products and Quotients	7 days
VI	Estimation	7 days
VII	Multiplying	11 days
	Second Semester	
VIII	Dividing	16 days
IX	Number Theory	9 days
X	Fractions	13 days
XI	Rational Numbers	9 days
XII	Addition and Subtraction of Rational Numbers	13 days
XIII	Multiplication and Division of Rational Numbers	7 days
XIV	Decimals and Percents	9 days
		150 days

**This suggested time schedule is based on a school year of 150 teaching days. Thirty days have been allowed for testing, review, and other activities.*

UNIT I: PLACE VALUE AND NUMBER BASES
Chapter 1 in Basic Modern Mathematics Textbook 8
(11 days)

On completion of Unit I, the student should be able to

write a numeral representing a number given in words

state from a given numeral how many ones, tens, hundreds, thousands, or millions it represents

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Grouping by Tens 2-Digit Numerals	2	Provide containers of beads or beans, bundles of sticks or straws, etc. for students to physically do some of the problems in the first three sections.
3-Digit Numerals	4	
4-Digit Numerals	6	
Comparison-Place Value	8	
Inventions	10	Spend time discussing and working the section on inventions to provide a stimulating experience in work and place value concepts.
Thousands	12	
Millions	14	Discuss thoroughly the excellent comparisons of a million on page 15. The best time for a discussion on pages 16 and 17 would be after the students try working the exercises. The chief value of this is the explanation which the students might give to tell how they arrive at the correct answer.
Population	18	Objects such as beads or sticks can be very helpful when working with base four.
Billions and Trillions	20	
Base Ten and Base Four	21	
More About Base Four	23	Provide plenty of opportunity for the students to discover the grouping concept of base ten and base four.

UNIT I (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Operations Using Base Four Numerals	24	Provide plenty of activities such as games and puzzles throughout this unit and the remainder of the book. Consult textbooks and reference list for recommended sources of mathematical side trips.

UNIT II: ADDITION AND SUBTRACTION
Chapter 2 in Basic Modern Mathematics Textbook 8
(14 days)

On completion of Unit II, the student should be able to

- write the union and intersection of two sets
- add and subtract numbers that involve carrying and borrowing
- write an addition fact as a related subtraction fact and vice versa
- simplify addition problems by changing the order and grouping of numbers
- add and subtract numbers with three or more digits

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Sets	28	For some topics such as carrying and borrowing in addition and subtraction, it may be helpful, especially for less able students, to use a few set demonstrations to show "breaking apart" sets of ten and regrouping.
Addition	29	
Subtraction	30	
At the Service Station	32	Mastery of addition of single digit numbers is of utmost importance since we base subtraction facts on knowledge of addition facts.
Short Stories	33	
The Function Game	34	
The Order Principle for Addition	36	Let students make up their own function rules and give each student an opportunity to lead the function game.
The Grouping Principle for Addition	37	

UNIT II (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Using the Order and Grouping Principles	38	
Reasoning-Addition	39	Give the students some exercises in which you read three numbers and they give the sum. Students can become quite good at mental addition with a little practice.
Column Addition	40	
Sums - 2-Digit and 3-Digit	42	
Column Addition - Larger Numbers	44	
Energy	45	Allow plenty of discussion as some students may have interesting comments to contribute because of their exposure to public concern for dieting and calorie counting.
Money - Finding The Total Amount	46	
Planning the Camping Trip	47	
Regrouping	48	
Subtraction - 2-Digit Numbers	49	
The Human Skeleton	50	
Subtraction - 3-Digit Numbers	51	
Science Short Stories	52	
History Short Stories	53	
Shopping Problems	54	
More Regrouping	55	
More Subtraction	56	
Animal Weights	57	
Waterfalls of North America	58	
Bridges	59	

UNIT III: MULTIPLICATION AND DIVISION
Chapter 3 in Basic Modern Mathematics Textbook 8
(13 days)

On completion of Unit III, the student should be able to

write simple multiplication facts as related division facts and vice versa

simplify multiplication by using the multiplication - addition principle

write multiplication facts through 9 x 9

solve word problems using multiplication and division

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Multiplication	64	Oral work in class may be all that is needed on these five topics but exercise 3 on page 68 should be covered thoroughly so the students know why they should multiply or divide and not just guess.
Division	65	
Multiplication and Addition		
Division and Subtraction	67	
Multiplication and Division are Related	68	
The Order Principle For Multiplication	70	
The Multiplication-Addition Principle	71	Show how this principle can be used to "break apart" factors to simplify multiplication.
0 and 1 in Multiplication	72	
Reviewing Multiplication Facts	73	As suggested in the book, have each student make and keep his own multiplication table.
Threes, Fours, & Fives	74	
Sixes, Sevens, Eights, and Nines	75	
Multiplication Exercises	76	
Number Line Exercises	77	Good exercises to play games with.

UNIT III (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
More Number Line Exercises	78	
Animal Short Stories	80	
Missing Factor Exercises	81	
Using Multiplication to Divide	82	Make sure the students can handle this division concept before moving on.
Products and Factors	83	
0 and 1 in Division	84	Remind students that zero is the Multiplicative Annihilator and its use in division will bring disaster and devastation. DO NOT DIVIDE BY ZERO.
Using Division to Solve Problems	86	
At the Grocery Store	87	
The Function Game	88	Good exercise to use as competitive class game.
Weights on the Moon	90	
Short Space Stories	91	

UNIT IV: MEASUREMENT
Chapter 4 in Basic Modern Mathematics Textbook 8
(11 days)

On completion of Unit IV, the student should be able to

write or state orally whether linear, square, or cubic measuring units are needed in a problem

measure the length of an item to the nearest half unit

find the perimeter and area of a rectangle with given length and width

find the surface area of a rectangular solid having given dimensions (whole numbers only)

write a ratio comparing the measure of two objects

solve a simple problem involving ratio

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Length, Area, and Volume	96	Models would be helpful here and throughout this unit.
Finding Length	98	The unit for measurement of length is arbitrary.
Inch and Centimeter Rulers	99	The students should either have or make centimeter rulers.
Estimating Length	100	Stress the interval within which a measurement lies.
A Buried Treasure Map	101	
Finding Area	102	We start with counting to find area.
Area: Rectangles	104	Due to past experience, classes will come up with the formula, or at least a rule, for area of a rectangle.
Finding Volume	106	We now use counting to find volume.
Estimating Perimeter	108	A few examples on the board with student participation will fix the idea of perimeter.
Measuring to Find Perimeter	110	
Surface Area	112	Models are necessary here.

UNIT IV (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Surface Area Exercises	114	Here again, we count to find surface area.
Liquid Measure	116	Models filled with sand or water, though messy, are very helpful.
Inches, Feet, Yards, and Miles	117	
Ratio and Scale Drawing	118	Exercise 7 on page 119 should make an interesting assignment.

UNIT V: SPECIAL PRODUCTS AND QUOTIENTS
Chapter 5 in Basic Modern Mathematics Textbook 8
(7 days)

On completion of Unit V, the student should be able to

find products and quotients involving factors which are multiples of 10

simplify multiplication problems using the order and grouping principles

simplify computation problems using the multiplication-addition principle

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Multiples of 10, 100, and 1000	122	The symbolism used in this section is introduced on page 77. If you choose to use this section, it might be wise to review first.
Using the Grouping Principle to Find Products	124	Here we use the associative property to simplify computation to enable us to multiply by 10 or 100. This is a worthwhile tool and should be given thorough treatment.
Special Products	126	These sections provide practice in using the rules for multiplying by 10 and 100. Even the poorer students should have some success here.
Special Quotients	127	
Reasoning	128	This is an introduction to a little number theory which should help students to determine whether or not an answer is reasonable. Additional practice may be indicated.

UNIT V: (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Buying and Selling Clothes	129	This is application of previously covered material in the unit. All exercises should be done.
Using the Order and Grouping Principles	130	Applying commutative and associative properties is extremely important in later work. It should be stressed.
The Multiplication Addition Principle and Its Use	132	The distributive property may be the most important topic in the understanding of algebra. Although many students in Basic Math 8 will not be taking algebra, the property's value in simplifying computation makes its inclusion worthwhile.

UNIT VI: ESTIMATION
Chapter 6 in Basic Modern Mathematics Textbook 8
(7 days)

On completion of Unit VI, the student should be able to

estimate sums, products, differences and quotients with a reasonable degree of accuracy

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Estimating	138	Notice * on page 166 of Teacher's Edition.
Estimating Sums And Differences	140	Good oral exercise.
Estimating Products	142	
Which Estimate is Best	144	Optional topic.
Estimating Quotients	145	
Short Stories - Estimation	146	
Estimation for Fun	147	

UNIT VII: MULTIPLYING
Chapter 7 in Basic Modern Mathematics Textbook 8
(11 days)

On completion of Unit VII, the student should be able to

multiply 3-digit numbers

solve word problems involving 3-digit multiplication

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
One 2-Digit Factor	152	Be sure the students understand the idea of "breaking apart" when they multiply.
Factors With 3 and 4 Digits	154	This provides good practice in the "breaking apart" method. Show carefully why the "short cut" works.
Area Exercises	156	Review the rule for areas of rectangles.
City Altitudes	157	
2-Digit Factors	158	One of these factors is a multiple of 10.
More About 2-Digit Factors	160	This is the development of the multiplication algorithm for 2-digit numbers. Be sure the students have mastered the preceeding topic before doing this.
Volume Exercises	161	Do not insist on use of the volume formula. Number of blocks in a layer times the number of layers is sufficient.
Records	162	
Short Stories	163	
Multiplication Exercises	164	
Two 3-Digit Factors	166	Review multiplication by multiples of 100. Show how the algorithm parallels the "breaking apart" method.
The Statue of Liberty	167	
Multiplying Practice	168	
Speed of Sound	169	

UNIT VIII: DIVIDING
Chapter 8 in Basic Modern Mathematics Textbook 8
(16 days)

On completion of Unit VIII, the student should be able to

divide problems with one-, two-, or three-digit quotients and one- or two-digit divisors

solve word problems using division

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Estimation	176	This gives a good introduction to non-exact division.
Quotients and Remainders	178	Note the use of repeated subtraction to develop the division algorithm.
Subtraction and Quotients	180	Help the students arrive at a good first guess.
Multiples of 10 in Estimation	182	These problems will help in first estimates.
Finding the Quotient	184	Here is the simple form of the division algorithm.
Finding and Checking Quotients	186	Insist that students do check.
Finding a Substitute	188	Just try guessing and checking the guess until the students (hopefully) come up with a rule.
Finding the Average	189	
Multiples of 100 in Estimation	190	
Finding 3-Digit Quotients	191	All of these problems should be assigned.
A short cut -- 1-Digit Divisors	192	Go over this development of short division carefully.
Finding Special Averages	194	
Temperature	195	

UNIT VIII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Alaska and Hawaii	197	
More Estimation	198	
Multiples of 10 As Divisors	199	
Larger Quotients	200	
Trains	201	
Estimation Exercises	202	
2-Digit Divisors	203	Again try for best estimates.
Short Stories	204	
Estimation - Larger Factors	205	These are necessary as background to division.
Division & Quotients Between 10 and 100	206	
Finding 2-Digit Quotients	207	Being able to solve these problems is the objective of this entire unit.
Multiplication and Money	208	Try to work intuitively into money notation with multiplication and division.
Division and Money	209	

UNIT IX: NUMBER THEORY
Chapter 9 in Basic Modern Mathematics Textbook 8
(9 days)

On completion of Unit IX, the student should be able to

- state whether or not one number is a factor of another
- construct a "factor tree"
- factor a composite number into a product of primes
- find the greatest common factor and the least common multiple of numbers less than 100

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Even and Odd Numbers	216	Students should understand that an even number is two times a whole number and that an odd number is one more than an even number.
Factors	218	
Finding the Factors of a Number	219	
Factor Trees	220	
Prime Numbers	222	As a reinforcement activity for prime numbers, you might find it helpful to list additional prime numbers, perhaps those between 50 and 100.
Union and Intersection of Sets	224	
Greatest Common Factor	226	If the students are unable to list all the factors of a given number, they will likely have little success with this lesson.
Common Multiples	228	
Least Common Multiple	229	Least common multiple means least common non-zero multiple.
Let's Review	230	
Find the Number	231	

UNIT X: FRACTIONS

Chapter 10 in Basic Modern Mathematics Textbook 8
(13 days)

On completion of Unit X, the student should be able to

name the fraction represented by a figure

identify the numerator and denominator of a given fraction

write a set of equivalent fractions from a given fraction

check whether two given fractions are equivalent by using the cross-product method

reduce a fraction to lowest terms

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Number Pairs and Fractions	236	Models would be helpful here and throughout this unit. Cutting up poster paper to represent fractions is good.
Fraction Exercises	238	
Numerator and Denominator	239	This should be review.
Numerator (> or =) Denominator	240	Explain that "improper" is a term held over from older math and does not have the usual connotation.
Zero Numerators	241	
More About Fractions	242	0/4 can be thought of as no-fourths.
Equivalent Fractions	244	Use an intuitive approach here. Save the formal definition for later.
Sets of Equivalent Fractions	246	
Building Sets of Equivalent Fractions	248	You should be able to generalize $a/b \sim ax/bx$.
A Check for Equivalent Fractions	250	This is the time for the formal definition for equivalence of fractions.
More About Equivalent Fractions	252	

UNIT X (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Lower and Higher Terms	254	These are all leading to "reducing" fractions.
Lowest Terms	255	
More About Lowest Terms	256	
Reducing to Lowest Terms	258	A fraction is said to be in lowest terms (or reduced), if the numerator and denominator have no common factor except "1".
Reducing Fractions	260	
Fractions Short Stories	261	

UNIT XI: RATIONAL NUMBERS
Chapter 11 in Basic Modern Mathematics Textbook 8
(9 days)

On completion of Unit XI, the student should be able to

name a class of equivalent fractions using one rational number and graph that class as a point on the number line

insert the proper comparison symbol between two given rational numbers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
From Fractions to Numbers	268	Exhibit on the chalkboard several lowest termed fractions and have the students build sets of equivalent fractions.
Names for Rational Numbers	270	Emphasize that we can select any fraction from the set of equivalent fractions and use it for the name of the rational number.
Equality	272	
Inequalities	274	It may benefit some of the less able students to experiment with various physical devices.
Rational Numbers Greater Than One	276	

UNIT XI (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Rational Number Exercises	278	The most important thing you can do to prepare students for this lesson is to give them the opportunity to use different fractions to represent a single rational number.
Short Stories Rational Numbers	279	
Let's Review	280	
The Panama Canal	281	
Tall Buildings	282	
Short Stories (Weights)	283	If time permits, provide some review of measurement concepts, especially those of surface area and volume.

UNIT XII: ADDITION AND SUBTRACTION OF RATIONAL NUMBERS
Chapter 12 in Basic Modern Mathematics Textbook 8
(13 days)

On completion of Unit XII, the student should be able to

add two rational numbers which are represented by fractions having different denominators

subtract rational numbers, using the same familiar inverse relationship that exists between addition and subtraction of whole numbers

add rational numbers using mixed numeral notation

simplify the answers to addition problems for which the fractional part of the answer represents a rational number that is greater than or equal to one

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Addition and Subtraction	288	
Whole Numbers and Rational Numbers	290	At this time, it is not necessary to have students put fractions in lowest terms.

UNIT XII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Mixed Numerals and Improper Fractions	292	
Rational Number Problems	293	
Sums of Rational Numbers	294	Some of the students may want to add rational numbers another way, i.e., $a/b - c/d = ad - bc/bd$. If they don't understand this fairly well, then don't insist on doing it the textbook way.
Sums and Differences	296	
Finding Sums	298	
Finding Sums and Differences	300	
Let's Review	302	
Temperature	303	
Least Common Denominator	304	Allow the students considerable freedom in the methods they use for adding and subtracting rational numbers of different denominators.
Methods for Adding and Subtracting	306	
Short Stories	307	
Basic Principles (Addition)	308	Emphasize that using the commutative and the associative principles together allows us to rearrange addends in any way we please.
More About Addition	310	Try some activities such as mathematical baseball, puzzles, etc. to create interest.
More About Subtraction	312	
Short Stories	313	
Let's Review	314	
The World	315	
Orbital Speeds of Planets	317	

UNIT XIII: MULTIPLICATION AND DIVISION OF RATIONAL NUMBERS
Chapter 13 in Basic Modern Mathematics Textbook 8
(7 days)

On completion of Unit XIII, the student should be able to

find the product of two rational numbers

find the quotient of two rational numbers using the "finding the missing factor" method

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Regions and Multiplication	318	Show many examples at the board or on the overhead projector. Students should associate the word "of" with multiplication.
The Number Line and Multiplication	320	Have students use the number line at the board to find various products. Notice the different examples of multiplying a whole number by a fraction and a fraction by a fraction.
Basic Principles	322	Students should be aware that the basic principles of whole numbers also hold true for rational numbers.
Some Special Products	323	
Finding Products	324	Some more explanation of the example on page 324 may be needed.
Multiplication and Division	326	Some review of whole number division may be needed to get this concept across, i.e., $2 \times 3 = 6$, $6 \div 3 = 2$.
The Function Machine	327	
Short Stories	328	

UNIT XIV: DECIMALS AND PERCENTS
Chapter 14 in Basic Modern Mathematics Textbook 8
(9 days)

On completion of Unit XIV, the student should be able to

read orally numbers written in decimal notation

add, subtract, and multiply numbers using decimal notation to thousandths

solve money problems and other word problems with decimals in them

interchange fractional and percent notation

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Fractions & Decimals	332	Be sure to expand a decimal, i.e., .736 = 7/10 + 3/100 + 6/1000 = 736/1000.
Decimal Exercises	334	
More About Decimals	335	
More Decimal Exercises	336	
Addition and Subtraction	337	Notice that .6 + .5 = 11/10 = 10/10 + 1/10 = 1.1.
Adding	338	Lining up the decimal points is simply a way of assuring us of having the same denominators.
Decimals & Money	340	48¢ = $.48 = 48/100 dollars.
Decimal Problems	341	
Multiplying With Decimals	342	Show the parallel between multiplying denominators of common fractions and the placement of the decimal point in multiplication of decimal fractions.
Multiplying - A Short Cut	343	
Percent Notation	344	Stress that this is simply another way to represent rational numbers.
Rational Number Problems	346	Insist that students use both decimals and fractions in the solution of these problems.
Short Stories	347	

PREFACE

This resource guide for Geometry 1-2 is organized as follows:

1. A recommended sequence of units with a suggested time allotment for each unit.

2. Unit Outlines
   a. Specific objectives to be realized by the student on completion of the unit.
   b. A detailed breakdown of the unit into sub-units with suggestions for developing topics and hints for the teacher in presentation of topics.

The time allotment for each unit is included only as a guideline for the teacher. The specific objectives of each unit set a basic body of material to be covered in Geometry 1-2 and the standard of proficiency that should be expected from each student in the Kent School District.

The teacher will note that this guide consists of 12 units whereas the textbook consists of 14 chapters and two appendices. Two of the chapters and one appendix have been incorporated with other chapters under single unit headings. The incorporated sections have not necessarily been de-emphasized. But since much of the material in these sections is optional, the concepts which are considered important have been grouped with topics to which they are logically related.

It is very important that the teacher make himself completely familiar with the comments in the teachers' text (green pages). The authors have many valuable suggestions for the presentation of ideas and give explanations of objectives for the study of certain units. This guide is intended to supplement the authors' guidelines and coordinate the teaching of Geometry in the Kent School District.

The fact that this guide can be beneficially revised will become evident with use. We hope that teachers using it will critically analyze it and make comments and suggestions about its utility, content and structure.

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INTRODUCTION

Statement of Purpose

The purpose of this course is to provide the students with an awareness of the structure of mathematics; to develop the students' capacity for analysis and orderly, consistent thinking and to acquaint the student with the properties of plane and solid figures as a foundation for further studies.

Although complete mastery of all phases of Geometry is not expected, certain minimal standards of proficiency must be met for successful completion. The rigcr of the course, however, should be sufficient to satisfy basic college entrance requirements.

Objectives

The general objectives of Geometry 1-2 are to have students:

understand the basic structure of geometry and through geometry further understand the nature of mathematics

develop the ability to analyze geometric figures and build a knowledge of the relationships among geometric elements

grow in the understanding of the inductive and deductive methods

gain an appreciation for the need of and develop a precise geometric vocabulary

use and strengthen algebraic skills

gain a knowledge of coordinate geometry and the relationship of algebra and geometry

use and appreciate creative thinking

Prerequisites and Selection of Students

Before enrolling in Geometry 1-2 the student should satisfy one of the following:

a. Satisfactory completion of Algebra 1-2.
b. Completion of Lab Geometry with a grade of C or better.

Geometry 1-2 is normally considered a 10th grade course, however, a limited number of carefully chosen students will be enrolled in the 9th grade as part of the accelerated program. At the 10th grade level students will be grouped according to the recommendations of their previous math instructors. Should 10th grade students not achieve a satisfactory level during the first semester they should, with teacher approval, be transferred to Lab Geometry.

RECOMMENDED TEXTS AND REFERENCE MATERIALS

Basic Text:

Jurgensen, Donnelly, Dolciani *Modern Geometry - Structure and Method;* Houghton Mifflin Co. (Boston, 1965)

Supplementary References:

Lewis *Geometry - A Contemporary Approach;* D. Van Nostrand Co. (Princeton, New Jersey, 1964)

Moise *Elementary Geometry from an Advanced Standpoint;* Addison-Wesley Publishing Co. (Reading, Mass., 1964)

Moise, Downs *Geometry;* Addison-Wesley Publishing Co. (Reading Mass., 1964)

School Mathematics Study Group *Geometry - Parts I & II;* Yale University Press (New Haven, Connecticut, 1960)

SUGGESTED TIME ALLOTMENTS

First Semester

UNIT	First Semester	Suggested Time *
I	Basic Elements	9 days
II	Induction	9 days
III	Deduction and Logic	13 days
IV	Angle Relationships and Perpendicular Lines	10 days
V	Parallel Lines and Planes	8 days
VI	Congruence	14 days
VII	Similarity	20 days

Second Semester

UNIT	Second Semester	Suggested Time *
VIII	Circles, Arcs, and Angles	15 days
IX	Constructions and Loci	10 days
X	Introduction to Coordinate Geometry	19 days
XI	Areas in the Plane	13 days
XII	Areas and Volumes of Solids	10 days
		150 days

**It may be noted that the time schedule is based upon 150 instructional days. Thirty days are allowed for testing, review, and other activities. The time schedule as presented suggests more instructional days for the first semester than the second. This is done so that a block of time could be devoted to a general review of Geometry at the end of the second semester.*

UNIT I: BASIC ELEMENTS
Chapter 1 in Jurgensen Textbook
(9 days)

On completion of Unit I, the student should be able to

define and illustrate, where possible, the vocabulary words in this unit

use Venn diagrams and the roster method to represent unions, intersections, and complements of given sets

find the distance between pairs of points on the real number line, when coordinates are given

find the measure of an angle, with the aid of a protractor, when the zero scale does or does not lie on one side of the angle

compute angular measures, including sums and differences, in degrees, minutes, and seconds

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Representing Sets; Relationship Between Sets; Venn Diagrams	1	It is suggested that sections 1-3, which should be largely review, be assigned the first day; second day, Test 1 of Progress Tests to be administered as a diagnostic test to determine class level of understanding.
Real Numbers; Number Line; Distance Between Points; Absolute Value	11	Test 2 (Progress Test) may be used for a grade score and basis for review.
Points, Lines and Planes; Basic Undefined Terms; Essential Definitions	22	Emphasize the difference in meaning and notation between a set and the measure. We should agree that in this assigned course, "=" (equality) means identity; i.e., either the same set of points when considering geometric sets or the same number.
Angles and Their Measurement, 1-9 Angle, 1-10, Measurement of Angles, 1-11, Special Angles and Angle Relationships	30	The text's definition of half-plane is inconsistent with previous instruction. Insist that 1 line divides the plane into 3 disjoint sets analogous to point on a line, figure in a plane, and plane in space. The edge is not to be considered a part of a half-plane. From this argument, we must conclude that the interior of a straight angle has no meaning.

UNIT I (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		In view of our definition of a half-plane APB in the illustration at top of page 35 does not lie in H but the interior of APB does.
		Definition of a dihedral angle on page 40 should read "the union of two half-planes with a common edge and their common edge."

UNIT II: INDUCTION
Chapter 2 in Jurgensen Textbook
(9 days)

On completion of Unit II, the student should be able to

describe the inductive process

define and illustrate, where possible, the vocabulary words in this unit

analyze geometric figures, make reasonably accurate measurements, and state conclusions based on his observations and measurements

state the limitations of the inductive argument

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Meaning of Induction	54	The teacher should use numerous examples to reinforce the definition of induction on page 54. It might be pointed out that a valid inductive generalization may be made from the investigation of a specific case. A generalization so derived is said to be intuitive.
Applications of the Inductive Process	55	Page 61 has a very good set of exercises which will reinforce the students' understanding of induction. In this set of exercises the student should draw a number of different test figures (at least 3 or 4) for each problem. After measurement and analysis, he should attempt to make a generalization. This set of exercises (page 61) and similar exercises would best check the students' capacity for analysis and ability to use induction.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		At the same time, this type of exercise can be used to point out the limitations of induction in attempting to form general truths.
Vocabulary and Spelling	77	All of the terms contained in these pages are defined in this unit. The student is expected to have a mastery of this basic vocabulary. The teacher should emphasize the limitations of induction as a process of arriving at general truths. The necessity and desirability of some other process which provides us with a greater degree of certainty should be pointed out--namely, the deductive process.

UNIT III: DEDUCTION AND LOGIC
Chapter 3 in Jurgensen Textbook
(13 days)

On completion of Unit III, the student should be able to

describe the deductive process

identify a conditional statement and rephrase declarative statements in "if-then" form

write a deductive argument supporting an algebraic hypothesis

describe sequentially the necessity for undefined terms, defined terms, and postulates as the basis of a mathematical system

state the definition, theorem, or postulate justifying an indicated deduction

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Deductive Reasoning	87	In discussing deductive reasoning, more extensive examples of the syllogistic order of a deductive argument should be presented than appear in the text. The teacher will have to provide students with additional exercises in this area. At this time Appendix 2 (pages 564-582) could be covered. At <u>least</u> the section of Appendix 2 on

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Conditionals and Rules of Inference (pages 569-572) should be discussed.
Hypothetical Statements ("if-then" statements)	91	The teacher may want to develop further exercises in which the students restate declarative sentences in hypothetical form and, conversely, in a more specifically geometric context.
Definitions and Postulates	94	In discussing the meaning and significance of definitions and postulates in a mathematical system, the necessity for basic undefined terms should be emphasized. At this time it would be best to discuss the meaning of "theorem" also. Be sure to point out that an idea may be accepted as a theorem *only after it has been proved.*
Properties of Real Numbers, Equality and Inequality	99	The students should be thoroughly familiar with these properties from Algebra 1-2. Thus, discussion of the properties should be only by way of review. The use of these properties in developing a formal (ledger type) algebraic proof cannot be overemphasized. Numerous exercises in addition to those presented in the text should be assigned applying these properties in the formal proof.
Initial Postulates and Theorems	104	Thorough familiarity with the postulates presented should be expected of each student. Because we are attempting to emphasize the straight-forward deductive argument in this unit, the development (proofs) of the theorems presented should be deferred until as many theorems as possible using such an argument have been established. It is suggested that theorems numbered 1-10 be discussed so that the proofs, as presented in the text, need not be dwelt upon as examples of straight-forward deductive arguments. Good Luck!

UNIT IV: ANGLE RELATIONSHIPS AND PERPENDICULAR LINES
Chapter 4 in Jurgensen Textbook
(10 days)

On completion of Unit IV, the student should be able to

draw a figure from the data indicated in a conditional statement and list the "given" and "to prove"

complete a ledger-type direct proof by supplying the reasons when the statements are given

solve problems involving complementary, supplementary, right, and vertical angles given a figure and appropriate data

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Initial Postulates and Theorems	123	In Postulate 9 on page 124 make certain the students understand that the given set includes both the half-plane and its edge. In Theorem 7 on page 125 it should be pointed out that the author has made the arbitrary assumption that $x > y > z$ and that the proof is equally valid when the order is reversed. We have stated that a line divides a plane into three disjoint sets, thus excluding the edge from a half-plane. When a theorem involving this definition is encountered, the teacher should take this into account.
Straight Angles, Right Angles, and Perpendicular Lines	130	Re-emphasize at this time the fact that a definition is reversible, while a theorem may, or may not, have a "true" converse.
Supplementary Angles, Complementary Angles, and Vertical Angles	137	The students could well be given more problems involving proofs than are available in this section. Fill-in proofs where the student supplies the "reasons" could be very helpful.
The Demonstration of a Theorem	143	Initially, the complete demonstrations of a theorem will be required. Upon mastery of this format the requirement may be relaxed.

UNIT V: PARALLEL LINES AND PLANES
Chapter 5 in Jurgensen Textbook
(8 days)

On completion of Unit V, the student should be able to

state concisely the basic properties of parallel lines and planes developed in this unit

compare and contrast parallel and skew lines

establish a statement using a short indirect proof

state the converse of a given theorem and determine its truth or falsity by deductive argument

name angles of equal and supplementary measure from a drawing of parallel lines and transversals

define and illustrate, wherever possible, the vocabulary words in this unit

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Basic Properties	153	Historically, parallel lines have been defined in the following ways: (1) Parallel lines have the same or like direction or directions. (2) Parallel lines have a constant distance between them (equidistant). (3) Parallel lines are coplanar and have no common point. The fallacy of definition (1) lies in the fact that "direction" is derived from knowledge of properties of parallels; it is a case of explaining a thing in terms of itself. To emphasize the need for consistency in defining terms, have students state their own definitions of skew lines, parallel lines, etc.
Transversals and Special Angles	157	Point out that exercises 17 and 18 on page 162 can be considered to be corollaries to previously proven theorems and as such are useful in the proof of later theorems.
*Indirect Proof	163	The method of indirect proof is sometimes a more difficult method to apply than other rules of inference. When establishing the statement "if P, then Q", assume the negation of "Q", which is the simplest case.

UNIT V (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Remind students that an indirect proof must lead to a contradiction (P is true and P is false), and by definition every contradiction is false; i.e. the negation of Q if false and Q is true.
Parallel Postulates	166	For further reading: SMSG Studies in Mathematics; *Euclidean Geometry Based on Ruler and Protractor Axioms;* Vol II; page 147-52. Heath, Sir Thomas; *Euclid's Elements;* New York, Dover Publications Inc. 1956, Vol I; Pages 202-220.
Converses of Earlier Statements About Parallels	171	Students should not, habitually, refer to theorems by number.
Applying Parallels To Triangles	176	

UNIT VI: CONGRUENCE
Chapter 6 in Jurgensen Textbook
(14 days)

On completion of Unit VI, the student should be able to

list the corresponding parts of two congruent triangles named in such a way that the order of the letters indicates corresponding vertices

prove two triangles congruent given data sufficient to form three equalities between their corresponding parts

prove corresponding parts of two triangles equal to given data sufficient to establish congruence of the triangles

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Corresponding Parts of Two Triangles	190	Emphasize that a 1:1 correspondence between the vertices of two triangles may always be established but correspondence doesn't imply congruence. Several trials may be necessary to insure that a congruence was not overlooked by initially mismatching the vertices.
Formal Treatment of Congruent Triangles	193	Demand the 1:1 correspondence used by the student agrees with the order he uses in writing the letters (See T.E. page 25).
More Ways of Proving Congruent Triangles	198	It may be pointed out that many of these "postulates" may be deduced from one postulate and we may return, if time permits, and reduce the number of "postulates".
Overlapping Triangles	201	The student must remain exact in his notation even though the teacher may relax in the amount of detail required in a proof. (See page 26 in T.M.) The exception to the comment on page 202 is when the conclusion to be reached involves position or description of a point set. For example, "the medians or angle bisectors of a triangle are concurrent."
Proving Corresponding Parts	204	This is somewhat like a 1¢ sale. The postulates and theorems on congruent triangles cut the requirements of the definition of congruence in half. But once congruence is established, the definition is reversible and all equations on correspondence immediately follow.

UNIT VI (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Isosceles Triangles	207	The problems on page 215 and 216 develop all the basic properties of parallelograms etc. Although these properties are not listed as theorems, many of the properties are equal in importance to the theorems developed. The student should be responsible for these properties in the discussion of subsequent ideas. In addition exercises involving numerical computations related to the properties of quadrilaterals should be given to the students to reinforce these ideas.
Applying Properties of Congruent Triangles to Quadrilaterals	213	Note the table on page 217, the preface to the problem on page 215 and the comments in T.M. page 28 (2). Practical applications on page 217-219 should be discussed, especially #11-14. This is a useful technique in developing insight into locus problems.

UNIT VII: SIMILARITY
Chapter 7 and 8 in Jurgensen Textbook
(20 days)

On completion of Unit VII, the student should be able to

write a formal proof establishing similarity of two given triangles

set up and solve proportions involving corresponding parts of similar polygons

find the measure of a side of a right triangle by the Pythagorean Theorem and, using its converse, establish whether or not a given triangle is a right triangle

solve problems involving a 30-60-90 or a 45-45-90 right triangle

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Ratio and Proportion	229	Use this section to review and reinforce previous learnings.
Special Properties of a Proportion	234	Stress properties 1 and 2 and give the student ample opportunities to apply them.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Properties 3, 4, and 5 may be introduced at this time but should be stressed at a later time when problems including their use are encountered.
Similar Polygons	238	Stress the point that a correspondence does not necessarily imply a similarity or a congruence. (See T.M. pages 30-31).
Similar Triangles	244	Note paragraphs II and III on page 245 and stress this point. (Also see T.M. page 31). To provide for a complete coverage of this topic include proofs of the converse of Theorem 34, the SAS Theorem and the SSS Theorem for similarity of triangles. The teacher may also include the AAA relationship as a theorem instead of introducing Postulate 16 as the author has done. See Lewis, chapter 11, for these proofs.
Properties of Special Segments in a Triangle	251	
Properties of the Altitude Drawn to the Hypotenuse in a Right Triangle	258	
The Pythagorean Theorem	265	See the accompanying book of transparencies for other proofs of the Pythagorean Theorem.
Right Triangles in Three-Dimensional Figures	274	A complete mastery of the vocabulary in this section should not be expected.
Projections Into a Plane	278	A complete mastery of this section should not be expected.
Trigonometry - Chapter 8	293	This section is optional. It should only be included if time and the rigor of the course permit.

UNIT VIII: CIRCLES, ARCS, AND ANGLES
Chapter 9 in Jurgensen Textbook
(15 days)

On completion of Unit VIII, the student should be able to

find the measure of central angles, inscribed angles, angles formed by intersecting chords and intersecting secants and angles formed by a secant and a chord, and the arcs related to these angles from a given figure

compute the measures of the segments of intersecting chords given the measures of three of the four segments

compute the measures of tangents and/or secants given the measures of two of the three segments

prove those theorems and corollaries in the chapter that have not already been proved and use all the theorems to prove statements involving circles or related parts

write an informal argument featuring the tangent to a circle from an external point as the limiting case for the secant from the point

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Review	321	A short oral treatment of this section during class will probably be sufficient.
Measures of Arcs and Angles	322	See T.M. page 35.
Inscribed Angles and a Limit Case	327	Make certain that all the students can readily distinguish between the arc in which an angle is inscribed and the arc which the angle intercepts. It may prove helpful to give the students a case in which the vertex of the angle is in the interior or exterior of the circle.
Other Angles Formed by Secants and Tangents	336	
Chords of the Same Circle or Equal Circles	340	Note the importance attached to problems 15, 16, 19, and 20 on page 344. These exercises should be proven as theorems.
Proportions Involving Chords, Secants, and Tangents	345	

UNIT IX: CONSTRUCTIONS AND LOCI
Chapter 10 in Jurgensen Textbook
(10 days)

On completion of Unit IX, the student should be able to

determine by construction the set of points which satisfies a given set of conditions

perform all the basic constructions using straight edge and compass

describe and draw a figure illustrating a given locus of points

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
What Construction Means: Permissable Instruments and Basic Angle Constructions	359	At the outset point out the direct relationship between accuracy of representation and neat, orderly techniques. Performance of all the basic constructions in this unit should be sufficient. Justification of <u>every</u> construction may prove to be tedious.
Constructing Parallel Lines and Perpendicular Lines	364	
Constructions Involving Circles	368	
Constructing Special Segments	372	Before leaving constructions a discussion of constructability (See *The Human Angle*, page 387) may be in order.
Meaning of Locus	375	Discuss loci in the plane and in space concurrently.
Intersection of Loci	379	Emphasize that the conjunction "and" in compound conditions implies the <u>intersection</u> of sets.
Construction by Means of Loci	382	This section is an excellent test of the student's comprehension of the interrelation of constructions and geometric properties.

UNIT X: INTRODUCTION TO COORDINATE GEOMETRY
Chapters 11 and 12 in Jurgensen Textbook
(19 days)

On completion of Unit X, the student should be able to

graph solution sets of simple equations and inequalities in one variable

plot points and determine from a graph the coordinates of points in a rectangular coordinate system

define and use the terminology associated with a rectangular coordinate system

apply the distance and midpoint formulae

write the equation of a circle given the radius and the coordinates of the center

determine the slope of a line

write the equation of a line given the slope and point on the line

state the relationship between the slopes of lines which are parallel or perpendicular to a given line

find the point of intersection (if possible) of two given lines by graphing and by algebraic means

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Graph on One Axis	393	Carefully assess the level of maturity and skill of your students before assigning exercises on page 395.
Plotting Points in Two Dimensions	396	
Symmetry	399	Delete oral exercise #1, page 400. Optional.
Graphs Meeting Certain Conditions	402	Note comment to teacher on top of page 404.
The Distance Formula	404	
The Circle	407	Students generally have difficulty correlating the relationship between a specific numerical example and the general <u>form</u> of a circle.

UNIT X (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Midpoint Formula	410	Use the "average" of the coordinates of the end-points.
The Slope of a Line	412	If trigonometry was covered, point out the tangent-slope relationship. The slope is not defined for vertical lines but the lines have a well-defined direction.
Parallel and Perpendicular Lines	415	The proof of Theorem 55 is not trivial for the average geometry student.
Writing the Equations of Lines	419	Proof of Theorem 56 is optional.
Additional Properties of Lines	424	
Placing Coordinate Axis	437	Chapter 12: Optional
Parallel and Perpendicular Lines	443	
Distances	445	
Triangles	448	
Parallelograms	450	
Trapezoids and General Quadrilaterals	452	

UNIT XI: AREAS IN THE PLANE
Chapter 13 in Jurgensen Textbook
(13 days)

On completion of Unit XI, the student should be able to

solve in terms of any variable particular cases of the area formulae (page 507) for triangles, special quadrilaterals, circles, and regular polygons

determine the ratio of the areas of two similar polygons given the ratio of two corresponding segments, and conversely

solve in terms of any variable particular cases of the circumference, arc length and sector area formulas for the circle

describe the conditions under which a circle may be circumscribed or inscribed in a polygon and indicate which type of polygon can always have an inscribed and circumscribed circle

state the limits of the measures of apothems, perimeters and areas of regular inscribed polygons

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Areas of Basic Polygons	471	Since the basic area computations will be review for most students, emphasis should be placed on the development of proofs. In order to appreciate and recognize the need for the postulates stated in this section, the distinction between units of linear measure and units of area measure must be made clear. Point out that there is no necessary correlation between linear and area units and that the conjunctional relationship between perimeter measure (linear) and area (square units) is accidental.
Areas of Regular Polygons	487	The students need not be required to develop formal (ledger) proof for these theorems. The students may be directed to write informal (paragraph) proofs. In any case they should be very familiar with the ideas in and applications of Theorems 62, 63, 64 and 65.
Circles, Limits and Area	494	In discussing the proof of Theorem 66 it is good to use an inductive analysis of the concept of limit.

UNIT XI (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		The 3 (bold print) assumptions stated on page 494 should be stated as postulates arrived at after the analysis of the idea of limits.
Arcs, Sectors and Segments	500	They should be completely familiar with the distinction between a circle segment and a circle sector.
Area Constructions	504	Optional.

UNIT XII: AREAS AND VOLUMES OF SOLIDS
Chapter 14 in Jurgensen Textbook
(10 days)

On completion of Unit XII, the student should be able to

solve for the total surface area and volumes of cubes, rectangular solids, regular prisms, cylinders and cones given sufficient dimensional measures

solve for the ratio of the areas and volumes of similar solids

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Prisms	517	Development of computational skills and analysis of cubes, rectangular solids, and regular prisms should be emphasized. The teacher should discuss the properties of pyramids as extensively as time allows. However, the student should not be held responsible for too much computation with respect to pyramids.
Area and Volume of Cylinders and Cones	524	Since the majority of students have the greatest difficulty with concepts related to the sphere, it should not be emphasized in this course, if time is of great concern at this point.
		If time permits cones and conic sections may be treated more extensively.
Areas and Volumes of Similar Solids	531	Should the class reach this unit with only two weeks or less remaining in the school year, the time might best be spent with limited discussions of volumes and surface area of the most common solids and the majority of time devoted to review exercises such as those on pages 542-549.

POSTULATES AND THEOREMS

For the use of the Jurgensen, Donnelly and Dolciani *Modern Geometry* textbook in the Kent School District, the postulates and theorems have been reprinted on separate ditto sheets.

It is hoped that every geometry teacher will look upon the author's list of postulates and theorems critically and suggest appropriate revisions.

GEOMETRY 1-2 POSTULATES

P_1 A line contains at least two points; a plane contains at least three points not all on one line; and space contains at least four points not all in one plane.

P_2 There is exactly one line through two different points.

P_3 There is exactly one plane through any three points which are not on one line.

P_4 If two points lie in a plane, then the line containing them lies in that plane.

P_5 If two different planes intersect, then their intersection is a line.

P_6 Between any two points there is a unique distance.

P_7 The set of points on a line can be put in one-to-one correspondence with the real numbers in such a way that:

1. Any point can be paired with zero.
2. The distance between any two points is equal to the absolute value of the distance between the numbers corresponding to those points. (Ruler Postulate)

P_8 To every angle there corresponds a unique real number greater than 0 and less than or equal to 180. (Angle-Measurement Postulate)

P_9 In a half-plane the set of rays with a common end point in the edge of the half-plane can be put in one-to-one correspondence with the real numbers from 0 to and including 180 in such a way that:

1. Either ray in the edge of the half-plane can be paired with 0.
2. Any angle whose sides are rays of the given set has a measure equal to the absolute value of the difference between the numbers corresponding to its sides. (Protractor Postulate)

P_{10} If two parallel lines are cut by a transversal, corresponding angles are equal.

P_{11} Through a point outside a line there is exactly one parallel to that line.

P_{12} If three sides of one triangle are equal to three sides of another triangle, the triangles are congruent. (SSS)

P_{13} If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, the triangles are congruent. (SAS)

P_{14} If the hypotenuse and a leg of one right triangle are equal to the hypotenuse and leg of another right triangle, the triangles are congruent. (HL)

P_{15} If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, the triangles are congruent. (ASA)

P_{16} If two angles of one triangle are equal to two angles of another triangle, the triangles are similar. (AA)

P_{17} If the intersection of arcs DE and EF of a circle is the single point E, then $\overset{\frown}{DE} + \overset{\frown}{EF} = \overset{\frown}{DEF}$ (the Arc-Addition Postulate).

P_{18} To every polygonal region there corresponds a unique positive number.

P_{19} If two triangles are congruent, they have the same area.

P_{20} If a plane figure which encloses a region can be separated into a number of non-overlapping polygons, its area is the sum of the areas of those polygons. (Area-Addition Postulate)

P_{21} The area of a rectangle is equal to the product of its base and its altitude. $(A = bh)$

P_{22} If two polygons are similar, they can be separated into the same number of triangles similar each to each and in corresponding positions.

GEOMETRY 1-2 THEOREMS

Th. 1 If two lines intersect, they intersect in exactly one point.

Th. 2 If a point lies outside a line, exactly one plane contains the line and the point.

Th. 3 If two lines intersect, exactly one plane contains both lines.

Th. 4 If three different points are on a line, at most one is between the other two.

Th. 5 On a ray there is exactly one point at a given distance from the end point of the ray.

Th. 6 A segment has exactly one midpoint.

Th. 7 If ray OE lies between rays OD and OF in a half-plane, then $\angle$ DOE + $\angle$ EOF = $\angle$ DOF. (Angle-Addition Theorem)

Th. 8 In a half-plane, through the end point of a ray lying in the edge of the half-plane, there is exactly one other ray such that the angle formed by the two rays has a given measure.

Th. 9 In a half-plane, an angle has exactly one bisector.

Th. 10 All straight angles are equal.

Th. 11 If the exterior sides of two adjacent angles are opposite rays, the angles are supplementary.

Th. 12 All right angles are equal.

Th. 13 If two lines are perpendicular, they meet so as to form right angles.

Th. 14 If two lines meet so as to form a right angle, the lines are perpendicular.

Th. 15 If two adjacent acute angles have their exterior sides in perpendicular lines, the angles are complementary.

Th. 16 In a plane, through a point in a line, there is exactly one perpendicular to the line.

Th. 17 If two angles are supplementary to the same angle or to equal angles, they are equal to each other.

Th. 18 If two angles are complementary to the same angle or to equal angles, they are equal.

Th. 19 If two lines intersect, the vertical angles formed are equal.

Th. 20 If two parallel planes are cut by a third plane, the lines of intersection are parallel.

Th. 21 If a transversal is perpendicular to one of two parallel lines, it is perpendicular to the other one also.

Th. 22 If two parallel lines are cut by a transversal, alternate interior angles are equal.

Th. 23 Through a point outside a line, exactly one perpendicular can be drawn to the line.

Th. 24 If two lines are cut by a transversal so that corresponding angles are equal, the lines are parallel.

Th. 25 In a plane, if two lines are perpendicular to a third line, they are parallel to each other.

Th. 26 If two lines are cut by a transversal so that alternate interior angles are equal, the lines are parallel.

Th. 27 The sum of the measures of the angles of a triangle is 180.
Cor. 1 - If two angles of one triangle are equal to two angles of another triangle, the third angles are equal also.
Cor. 2 - Each angle of a equiangular triangle has a measure of 60.
Cor. 3 - A triangle can have at most one nonacute angle.
Cor. 4 - The acute angles of a right triangle are complementary.
Cor. 5 - The sum of the measures of the angles of a quadrilateral is 360.

Th. 28 The measure of an exterior angle of a triangle is equal to the sum of the measure of the two remote interior angles.

Th. 29 If the legs of one right triangle are equal to the legs of another right triangle, the triangles are congruent. (LL)

Th. 30 If two angles and a not-included side of one triangle are equal to the corresponding parts of another triangle, the triangles are congruent. (AAS)
Cor. 1 - If the hypotenuse and an acute angle of one right triangle are equal to the hypotenuse and an acute angle of another right triangle, the triangles are congruent. (HA)
Cor. 2 - If a leg and an acute angle of one right triangle are equal to the corresponding parts of another right triangle, the triangles are congruent. (LA)

Th. 31 If two sides of a triangle are equal, then the angles opposite those sides are equal. (Base angles of an isosceles triangle are equal.)
Cor. 1 - An equilateral triangle is also equiangular.
Cor. 2 - Each angle of an equilateral triangle has a measure of 60.

Th. 32 If two angles of a triangle are equal, then the sides opposite those angles are equal.
Cor. - - An equiangular triangle is also equilateral.

Th. 33 If two polygons are similar, the ratio of their perimeters equals the ratio of any pair of corresponding sides.

Cor. 1 - If two right triangles have an acute angle of one triangle equal to an acute angle of the other, the triangles are similar.

Cor. 2 - If two isosceles triangles have their vertex angles equal, the triangles are similar.

Cor. 3 - If two triangles are similar to a third triangle, the two triangles are similar to each other.

Th. 34 If a line is parallel to one side of a triangle and intersects the other two sides, it divides them proportionally.

Cor. 1 - If three or more parallel lines intersect two transversals, they divide them into proportional segments.

Th. 35 If a ray bisects one angle of a triangle, it divides the opposite side into segments which are proportional to the other two sides.

Th. 36 If two triangles are similar, their corresponding altitudes have the same ratio as any pair of corresponding sides.

Th. 37 If the altitude is drawn to the hypotenuse of a right triangle, the two triangles formed are similar to the given triangle and to each other.

Cor. 1 - A leg of a right triangle is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse.

Cor. 2 - The altitude drawn to the hypotenuse of a right triangle is the mean proportional between the segments of the hypotenuse.

Cor. 3 - In a right triangle the product of the hypotenuse and the altitude to it is equal to the product of the legs.

Th. 38 In any right triangle the square of the hypotenuse is equal to the sum of the squares of the legs.

Th. 39 If the sum of the squares of two sides of a triangle is equal to the square of the third side, the triangle is a right triangle. (Converse of the Pythagorean Theorem)

Th. 40 If two central angles are equal, their arcs are equal.

Th. 41 If two minor arcs are equal their central angles are equal.

Th. 42 The measure of an inscribed angle is one-half the measure of its intercepted arc.

Cor. 1 - An angle inscribed in a semicircle is a right angle.

Cor. 2 - If a quadrilateral is inscribed in a circle, opposite angles are supplementary.

Cor. 3 - If two inscribed angles intercept the same arc or equal arcs, the angles are equal.

Th. 43 The measure of an angle formed by a secant ray and a tangent ray drawn from a point on a circle is one-half the measure of the intercepted arc.

Th. 44 The measure of an angle formed by two secants intersecting within a circle equals one-half the sum of the measures of the intercepted arcs.

Th. 45 The measure of an angle formed by two secants intersecting outside a circle equals one-half the difference of the measures of the intercepted arcs.

Th. 46 The measure of an angle formed by a tangent and a secant, or by two tangents, intersecting outside a circle equals one-half the difference of the measures of the intercepted arcs.

Th. 47 A diameter perpendicular to a chord bisects the chord and its two arcs.

Th. 48 If two chords intersect within a circle, the product of the segments of one chord equals the product of the segments of the other.

Th. 49 If two secants are drawn to a circle from an outside point, the product of one secant and its external segment is equal to the product of the other secant and its external segment.

Th. 50 If a tangent and a secant are drawn to a circle from an outside point, the tangent is the mean proportional between the secant and its external segment.

Th. 51 The distance between two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Th. 52 The circle with center (a,b) and radius r has the equation

$$(x - a)^2 + (y - b)^2 = r^2$$

Th. 53 The midpoint of the segment joining the points (x_1, y_1) and (x_2, y_2) is the point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Th. 54 Two nonvertical lines are parallel if and only if they have equal slopes.

Th. 55 Two nonvertical lines are perpendicular if and only if the slope of one line is the negative reciprocal of the slope of the other line.

$$m_1 = -\frac{1}{m_2}, \text{ or } m_1 \cdot m_2 = -1$$

Th. 56 The graph of any equation which can be written in the form $ax + by = c$, a and b not both zero, is a line.

Th. 57 The equation of the line passing through the point (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$.

Th. 58 The area of a parallelogram is the product of any base and the corresponding altitude. $(A = bh)$

Cor. Parallelograms having equal bases and equal altitudes are equal in area.

Th. 59 The area of a triangle is equal to one-half the product of its base and altitude. $(A = 1/2\ bh)$

Cor. 1 - Triangles with equal bases and equal altitudes have equal areas.

Cor. 2 - The area of an equilateral triangle with side s is one-fourth the product of the side squared and $\sqrt{3}$. $A = \frac{s^2}{4}\sqrt{3}$

Cor. 3 - The area of a rhombus is one-half the product of its diagonals.

$$\left(A = \frac{d_1 \cdot d_2}{2}\right)$$

Th. 60 The area of a trapezoid is equal to one-half the product of its altitude and the sum of its bases. $A = 1/2\ h\ (b_1 + b_2)$

Th. 61 The ratios of the areas of two similar triangles is the square of the ratio of any two corresponding sides.

Th. 62 A circle can be circumscribed about any regular polygon.

Th. 63 A circle can be inscribed in any regular polygon.

Th. 64 The area of a regular polygon is equal to one-half the product of the apothem and the perimeter. $(A = 1/2\ ap)$

Th. 65 The ratio of the areas of two similar polygons is equal to the square of the ratio of a pair of corresponding sides.

Th. 66 The ratio of the circumference to the diameter is the same for all circles.

Cor. - The circumferences of two circles have the same ratio as their radii and their diameters.

Th. 67 The area of a circle is equal to the product of π and the square of the radius of the circle. $(A = \pi r^2)$

PREFACE

This resource guide for Algebra 1-2 is organized as follows:

1. A recommended sequence of units with a suggested time allotment for each unit.

2. Unit Outlines
   a. Specific objectives to be realized by the student on completion of the unit.
   b. A detailed breakdown of the unit into sub-units with suggestions for developing topics and hints for the teacher in presentation of topics.

The time allotment for each unit is included only as a guideline for the teacher. The specific objectives of each unit set a basic body of material to be covered in Algebra 1-2 and the standard of proficiency that should be expected from each student in the Kent School District.

The fact that this guide can be beneficially revised will become evident with use. We hope that teachers using it will critically analyze it and make comments and suggestions about its utility, content and structure.

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TABLE OF CONTENTS

INTRODUCTION

Statement of Purpose

The purpose of Algebra 1-2 is to provide an adequate knowledge of fundamental algebraic concepts both as a preparation for subsequent studies and as a basic mathematical tool. The student's basic knowledge of fundamentals of arithmetic will be extended and integrated with the language and skills of Algebra.

Certain minimal standards of proficiency must be met for successful completion of the course, although it is not expected that each student will achieve complete mastery of all phases of Algebra. For most students, Algebra 1-2 should not be considered an end in itself and therefore must provide sufficient background for later courses in the mathematics sequence. Many of the students will be pursuing education beyond the high school level and hence the rigor of this course should be sufficient to satisfy basic college entrance requirements.

Objectives

The specific objectives of Algebra 1-2 are to have the student:

understand the development and structure of the real number system

recognize the value and use of the deductive process in Algebra

acquire the skills and vocabulary necessary for solving mathematical problems by algebraic techniques

develop proficiency in operations with polynomials and rational algebraic expressions

Prerequisites and Selection of Students

Because the most able students take Algebra 1-2 in the 8th grade as part of the accelerated program, it is felt that no further ability grouping is necessary. To be placed in Algebra 1-2, a student must have completed one of the following:

a. 7th grade accelerated math with at least a B grade and satisfactory performance on a standard algebra prognostic test.

b. 8th grade math with at least a C grade and satisfactory performance on a standard prognostic test.

c. Pre-Algebra 1-2 with at least a C grade.

Algebra 1-2 is normally considered a 9th grade course but a limited number of 8th graders will be enrolled as part of the accelerated program. Special care should be taken to identify students who are having difficulty in Algebra, particularly in the 8th grade, so that they can be transferred as soon as possible to a class more appropriate to their level of ability. In 8th grade, this would be Math 8. In 9th grade or beyond, they should be placed in Pre-Algebra 1-2.

RECOMMENDED TEXTS AND REFERENCE MATERIALS

Basic Textbook:

Pearson, Allen *Modern Algebra - A Logical Approach;* Ginn & Company (Boston, 1964)

Supplementary References:

Allendoerfer, Oakley *Principles of Mathematics;* McGraw-Hill (New York, 1963)

Brant, Keedy *Elementary Logic For Secondary Schools;* Holt, Rinehart & Winston (New York, 1962)

Christian *Introduction to Logic And Sets - Preliminary Edition;* Ginn & Company (Boston, 1958)

Dolciani, Wooton, Beckenbach, Juergensen, Donnelly *Modern School Mathematics - Algebra I;* Houghton Mifflin Company (Boston, 1967)

Dressler *Ninth Year Mathematics - Review Guide;* Amsco School Publications, Inc. (New York, 1966)

Rourke, Syer *Algebra I;* Ginn & Company (Boston, 1967)

School Mathematics Study Group *First Course in Algebra;* Yale University Press (New Haven, 1961)

Van Engen, Hartung, Trimble, Berger, Cleveland *Seeing Through Mathematics - Book Three;* Scott Foresman & Co. (Chicago, 1964)

Vannatta, Goodwin, Fawcett *Algebra One - A Modern Course;* Charles E. Merrill Books, Inc. (Columbus, Ohio, 1966)

See pages M-30 to M-36 of Pearson-Allen textbook for a detailed list of teaching aids that may be used in teaching this course.

SUGGESTED TIME ALLOTMENTS

UNIT	First Semester	Suggested Time *
I	Sets and the Number Line	8 days
II	Expressions and Sentences	5 days
III	Logic	5 days
IV	Operations With the Numbers of Arithmetic	20 days
V	Real Numbers	20 days
VI	Division of Real Numbers	15 days
	Second Semester	
VII	Factors and Exponents	17 days
VIII	Polynomials and Rational Expressions	30 days
IX	Real Number Plane	16 days
X	Radicals	14 days
XI	Functions and Other Relations	as time allows
		150 days

**This suggested time schedule is based on a school year of 150 teaching days. 30 days have been allowed for testing, review, and other activities.*

UNIT I: SETS AND THE NUMBER LINE
Chapter 1-2 in Pearson-Allen Textbook
(8 days)

On completion of Unit I, the student should be able to

describe a set in both words and set notation

classify given sets as finite or infinite

write all the possible subsets of a given set

draw a Venn diagram to show a given set relationship

write a set relationship from a given Venn diagram

classify a given set as closed or not closed under a particular operation

identify given numbers as members of the sets of whole, natural, or rational numbers of arithmetic

demonstrate a one-to-one correspondence between sets of numbers and certain points on the number line

order a given pair of numbers by proper use of comparison symbols

identify given statements as being either equivalent or not equivalent

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Meaning, Defining, Naming of Sets	1	This unit is a review of material previously covered in grades 7 and 8. The time spent on this unit should be limited to topics indicated by pre-test results. Tests 1-B and 2-B from the booklet of supplementary achievement tests should be used for this purpose. These tests will be provided with suggestions for interpretation of results. Individual students having trouble with Unit I may profit by the use of Pearson & Allen's programmed materials, part one.
Belonging to a Set	2	
Equal Sets	2	Stress that members of equal sets must be identical. Order irrelevant.
Number of Elements in a Set	4	Stress that $\emptyset$ and $\{\emptyset\}$ are <u>not</u> the same.

UNIT I (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Subsets	5	
Universal Sets	6	
Venn Diagrams	9	
Operations With Sets	10	See comment on page 11 in Teachers' Edition.
Closure	15	
Numerals	18	Note distinction between number and numeral. Example: 2 or 9. Which is larger?
Variables	18	Although the students will have had previous experiences with variables, a quick but intensive review may be helpful.
Numbers of Arithmetic	29	
The Whole Numbers and The Number Line	29	Emphasize the idea of one-to-one correspondence.
Natural Numbers and The Number Line	31	
Rational Numbers of Arithmetic and the Number Line	32	Stress that numbers are rational if they can be written in the form a/b where $b \neq 0$.
Betweenness on the Number Line	35	
Order on the Number Line	37	
Graphs of Sets	39	Stress uniform use of notation in the book. Students may have been exposed to graphing in a slightly different form.
Comparison Symbols	42	See comments in T.E. pages 42 and 43.

UNIT II: EXPRESSIONS AND SENTENCES
Chapter 3 in Pearson-Allen Textbook
(5 days)

On completion of Unit II, the student should be able to

evaluate numerical expressions involving more than one arithmetic operation

find and graph the solution set of simple and compound sentences

write, in set-builder notation, the set indicated by a given graph

use the proper connective - and or or - to join two simple sentences into a compound sentence whose graph is given

match a list of the italicized words in the chapter with a list of appropriate definitions

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Algebraic Expressions	52	Point out difference between numerical and open expressions and that both are algebraic expressions.
Order of Operation	53	See comments in T.E. pages 53-54. To help students remember, a little gimmick such as My Dear Aunt Sally may help. Additional exercises may be necessary. Touch on vinculum.
Indicating Multiplication With Parentheses	55	
Evaluation	55	Oral work in class should be sufficient.
Simple Numerical Sentences	57	Students should be aware of the difference between a sentence and an expression.
Open Sentences	60	
Solution Sets	61	Students should become accustomed to stating the domain of the variable involved. Show examples whose solution set includes the domain and the empty set. Formal methods of finding solution sets will be developed leter. Systematic guessing is o.k.
The Set Builder	64	Sometimes called by different name in other textbooks. Students should know this is standard notation and is used throughout the book.

UNIT II (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Graphs of Truth Sets	66	Emphasize domain in graphing.
Compound Sentences	68	Note comments on page 71 in T.E. and stress.
Graphs of Truth Sets of Compound Sentences	72	Emphasize domain. Unlabeled number lines may be duplicated for student use.

UNIT III: LOGIC
Chapter 4 in Pearson-Allen Textbook
(5 days)

On completion of Unit III, the student should be able to

form the disjunction, conjunction, or an implication from two simple statements

identify the hypothesis and conclusion of an implication although it may be expressed in a variety of forms

write an implication by applying the transitive property of implication to a sequence of two or more implications

complete a simple direct proof of no more than three steps

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		Time limits more than a very basic familiarity with logic. However, as a minimum, a cursory exploration of logic is essential to the understanding of later material. All topics except those listed are optional and should be considered only if time and rigor permit.
Statements	80	Only declarative sentences are used in logic.
Equivalent Statements	81	
Special Notation for Contradiction	87	
Conditional Sentences Implications	97	

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Variation in Stating An Implication	99	See page 25, Example #1, *Elementary Logic*, by B-K for other forms of implication.
Proof	103	See pages 79-81, *Elementary Logic*, by B-K for other forms of syllogistic inference. It should be noted that a syllogism is true regardless of the truth or falsity of the component propositions. A syllogism is a type of tautology.
Forms For Writing A Syllogism	105	
Transitive Property of Implication	107	
Forms of Proof	109	Emphasize that the ledger-type proof is a series of syllogisms.
Need for Assumption and Definitions	121	This topic can be best used as an introduction to the following unit.

UNIT IV: OPERATIONS WITH NUMBERS OF ARITHMETIC
Chapter 5 in Pearson-Allen Textbook
(20 days)

On completion of Unit IV, the student should be able to

identify the following properties as reasons in simple proofs and apply them in the writing of equivalent algebraic statements

a. Prop. for the No. of Arith:
- Commutative
- Associative
- Distributive
- Identity
- Closure
- Inverse

b. Prop. of Equality:
- Reflexive
- Symmetric
- Transitive
- Substitution
- Addition
- Multiplication

simplify fractional expressions by applying the above properties

translate verbal problems to algebraic statements and determine the truth sets of these statements

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Need for Assumptions and Definitions	121	This topic serves as an introduction. Have the students read this page on their own time.
The Numbers of Arithmetic and Closure	134	See comments in T.E., pages 134-34. Stress non-divisibility by zero.
Binary Operations	137	See comments in T.E., pages 137-38.
Properties of Equality	139	Students should understand these properties as they are the basis to solving equations and proofs.
Evaluating Open Expressions	145	Much drill is helpful in fixing this concept.
Inverse Operations	148	See comment in T.E., pages 148-49.
Solving Equations With One or More Operations	150	Since this is the first time a formal solution is presented, the examples should be covered thoroughly. Students should show each step in solution and check.
Using Equations to Solve Problems	157	Emphasis should be on the solving of the problems rather than the answer. Suggest setting up the open sentence for many of the problems orally.

UNIT IV (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Equivalent Equations and Equation Solving	160	
Identity Elements	162	
The Multiplicative Inverse	163	It should be noted that reciprocal is a special form of multiplicative inverse.
The Distributive Property	166	
Equivalent Expressions	169	
Using the Distributive Property to Write Equivalent Expressions and Solve Equations	170	See comment in T.E. page 171. Many of the problems starting on page 175 should be assigned.
Multiplication Property of Zero	178	Stress Theorem 3a, page 179, as it will be used later in solving quadratics.
Expressing Division by the Multiplicative Inverse	181	
Adding, Multiplying and Simplifying Fractions	182	Use of multiplication property of 1 should be stressed, i.e., 1 = c/c, etc.
Equations Involving Fractions	190	Re-emphasize no zero in denominator.
Boolean Algebra	202	May be used as enrichment with the superior students if time permits.

UNIT V: REAL NUMBERS
Chapter 6 in Pearson-Allen Textbook
(20 days)

On completion of Unit V, the student should be able to

graph any given real number on the number line

identify the eleven basic properties of a number field (P.A. page 210) both by listing and by listing them as reasons in simple proofs

solve simple equations and inequalities involving the concept of absolute value

multiply and factor monomials and polynomials over the integers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Real Numbers	204	Emphasize introduction of negative numbers as additive inverses of the numbers of arithmetic.
Integers and Rational Numbers	206	See comment page 206.
Real Numbers and The Number Line	207	
Order on the Real Number Line	208	See comment page 208.
Properties of the Real Number	209	Take special time to emphasize that these are the properties of a number field. See notes on pages 210 and 211 in T.E.
Uniqueness of the Additive Inverse	212	See note on page 212 in T.E. Stress that this uniqueness is one of number rather than of numeral.
Addition of Real Numbers	215	See note on page M-15 in T.E.
Absolute Value	217	Stress definition on page 218.
Rules for Addition of Real Numbers	219	See note on page 219 in T.E.
Addition on the Number Line	221	
Subtraction on the Number Line	222	

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Multiplication of Real Numbers	227	See comments on page M-15 in T.E. Cover the rules for multiplication in detail. Here is a possible tool: Let us call a positive number a "friend." Let us call a negative number an "enemy." Let us substitute the word "of" for "times." A friend of a friend is a friend. [(+)·(+) = (+)] A friend of an enemy is an enemy. [(+)·(−) = (−)] An enemy of a friend is an enemy. [(−)·(+) = (−)] An enemy of an enemy is a friend. [(−)·(−) = (+)] Zero is not considered here as his function in multiplication is unique. In his real identity as the Multiplicative Annihilator, he has the power to destroy both sides and hence, neither side would dare offend him.
Signs of a Real Number and its Multiplicative Inverse	230	
Theorems Concerning Multiplication of Real Numbers	231	
Equivalent Expressions Containing Parentheses	235	Provide plenty of practice on this. Students tend to have trouble with $a - (b - c) = a - b + c$.
Polynomials	240	Stress definitions. A few examples of polynomials over something other than the integers might be helpful here.
Products of Polynomials	242	Be sure of this material as it is basic to a knowledge of factoring and subsequent work.
Factoring a Polynomial	247	Practice is necessary here. Cover this material thoroughly. Stress use of <u>distributive property</u> in factoring.
Problem Solving	251	See comments on pages 251-52 in T.E. Be sure that students are writing equations for these problems. Much practice is needed.

UNIT V (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Addition Property of Inequality	256	
Multiplication Properties of Inequality	259	This concept is more difficult than the addition property for students to grasp. See comment on page M-16 in T.E.
Repeating Decimals (Optional or enrichment)	268	This section provides some interesting insights into the nature of repeating decimals. It may also serve as an introduction to Unit VI.

UNIT VI: DIVISION OF REAL NUMBERS
Chapter 7 in Pearson-Allen Textbook
(15 days)

On completion of Unit VI, the student should be able to

express a fraction as an equivalent fraction by properly changing the sign of the fraction, its numerator and/or its denominator

solve simple proportions and use correct terminology to describe proportions

find and graph the solution set of equations and inequalities involving fractions with at least integral or monomial denominators, which includes recognizing and stating the conditions which exclude extraneous roots

translate verbal problems involving fractions into algebraic statements

change the subject of a formula and evaluate formulas

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Definition of Division of Real Numbers	270	
Zero, Not a Divisor	271	Be sure students recognize the pitfall in example 2, page 271.
Rules for Division of Real Numbers	274	
The Signs of a Fraction	276	Place particular emphasis upon the fact that the distributive property must be applied when changing the signs of the numerator or denominator of a fraction involving polynomials.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Rational Numbers as Quotients of Integers	278	
Applications of Theorems on Fractions	279	
Adding Fractions in The Real Number System	282	Do not place undue emphasis upon Theorem 24. Point out its application to the addition of simple fractions but emphasize the method of adding fractions by changing to a common denominator and using Theorem 23 to add more complicated fractions.
Ratio and Proportions	285	Review the procedures for solving word problems as listed on page 157, Chapter 5 of P.A.
Complex Fractions	288	Instead of using Theorem 26 as a formula, you will probably find it best to simplify complex fractions by using either Theorem 7 or Theorem 4.
Equations Involving Fractions	289	Re-emphasize the importance of checking solutions.
Fractional Equations	293	Refer to page M-17 of T.E.
Problems Whose Equations Involve Fractions	296	This section will probably require more time than a typical section.
Formulas Involving Fractions	301	A greater emphasis should be placed upon changing the subject of a formula than on solving for particular values.
Inequalities Involving Fractions	306	Special attention should be given to the sign of the expression used to clear the fractions. Each problem will be handled by cases in order to apply Theorem 19 or 20 and each will introduce a compound statement.
Sentences Involving Absolute Value	308	Optional. (For Merit or Rhodes Scholars).

UNIT VII: FACTORS AND EXPONENTS
Chapter 8 in Pearson-Allen Textbook
(17 days)

On completion of Unit VII, the student should be able to

select proper factors of a given number from a list of factors of that number

find the complete factorization of a given number using divisibility tests for 2, 3, 4, 5, 9, 10.

find the greatest common factor and the least common multiple of a set of integers and a set of monomials containing variables

multiply and divide monomials using the laws of exponents

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Fundamental Operations with Polynomials Over the Integers	320	
Factoring Integers	321	See comment M-18 in T.E.
An Important Theorem	325	Proof of Theorem 27 is a good example of proof by definition.
Even and Odd Integers	326	See Comment (1) in T.E., page 326.
Tests for Divisibility	327	May be interesting for students to find tests for divisibility other than those given.
Prime Numbers	329	Stress that "one" is not a prime number. Interested students may read P.A. 360 and 426.
Complete Factorization	331	See comment M-18 in T.E.
Least Common Multiple	332	See comment in T.E., page 332.
Greatest Common Multiple	333	See comment in T.E., page 333.
Adding Fractions Having Integral Denominators	334	

UNIT VII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Some Theorems About Factors and Products of Integers	337	
Exponents	340	Stress $3a^2 \neq (3a)^2$. This is often confused by students and must be treated thoroughly.
Laws of Exponents - Positive	342	See comment in T.E., page 344 and M-19.
Laws of Exponents - Zero and Negative	347	See comment in T.E., page 348 and M-19.
Scientific Notation	350	
Monomials Involving Variables	353	Students should have a good working knowledge of LCM and GCF before proceeding to next topic.

UNIT VIII: POLYNOMIALS AND RATIONAL EXPRESSIONS
Chapter 9 in Pearson-Allen Textbook
(30 days)

On completion of Unit VIII, the student should be able to

identify the degree of a polynomial in one or more variables

add and multiply polynomials of first, second, and third degree with integral coefficients in one or more variables

find the solution set of a quadratic equation given in factored form

find <u>all</u> proper monomial factors of a polynomial over the integers, factor a binomial which is the difference of two squares and factor a quadratic trinomial which has binomial factors

solve a quadratic equation with rational roots by factoring

express a rational algebraic expression in simplest terms

add, subtract, multiply and divide two rational expressions where the degree of the numerator or denominator of each expression is two or less

find the solution set of a fractional equation where the LCM of the fractions involved is a polynomial of degree two or less

divide a polynomial in one variable by a binomial

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Degree of a Polynomial	362	To check understanding, have the students construct a polynomial of a given degree.
Adding Polynomials	363	
The Meaning of "Factorable"	365	Point out that a non-factorable polynomial is prime.
Multiplying and Factoring	366	Re-emphasize the fact that the multiplying of polynomials is merely the application of the distributive property
More About Multiplication of Polynomials	368	
More About Equations and Inequalities Involving Products	370	

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Grouping Before Factoring	373	This section is unusually difficult for most students. The teacher should present sufficient examples and point out such difficulties as encountered in Example #3 and Problems 9 and 15.
Difference of Two Squares	375	Exercise caution in assigning problems 5 and 6 on pages 377-78 to an average or below average class.
Perfect Squares	378	To emphasize the pattern of a perfect square, do exercises i through xi on page 379 orally.
Factoring After Completing the Square	381	Extend to cover both rational and irrational roots and the derivation and use of the quadratic formula. See Scott-Foresmar, Book 3, Page 362.
Quadratic Polynomials	382	
Products Which Can Be Expressed as Quadratic Polynomials	383	This section is to emphasize speed with accuracy.
Factoring Quadratic Trinomials	385	
Factoring $ax^2 + bx + c$; $a \neq 0$, $a \neq 1$	387	For all but the most able students, all of these exercises should be assigned and perhaps even supplemented.
Roots of an Equation	389	
Solving Quadratic Equations Using Factoring	389	Re-emphasize Theorem 3.
Summary of Factoring Methods	392	
More About Solving Equations Using Factoring	393	
Polynomials Over The Rational Numbers	395	Point out that the first step in factoring over the rationals is to rewrite each term of the polynomial as an equivalent expression having the same denominator.
Rational Expressions	397	

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Simplification of Rational Expressions	399	See T.E., page M-21. Re-emphasize the difference between a factor and a term.
Multiplication and Division of Rational Expressions	401	
Least Common Multiple of Polynomials	403	
Addition and Subtraction of Rational Expressions	403	Emphasize the comment in the solution of example 5.
More About Fractional Equations and Inequalities	407	The solution of the problems involving inequalities is optional.
Meaning of Division of One Polynomial By Another Polynomial	410	
How To Divide a Polynomial by a Polynomial	412	Choose for a first example a more elementary problem than that shown in example 1. When subtracting to obtain the new dividend, make sure that the student changes each sign and shows this change.

UNIT IX: THE REAL NUMBER PLANE
Chapter 10 in Pearson-Allen Textbook
(16 days)

On completion of Unit IX, the student should be able to

set up a coordinate system and graph an ordered pair of numbers

name the coordinates of any point on a graph

determine if an ordered pair is in the truth set of an open sentence in two unknowns

form the Cartesian product of two given sets of numbers

solve simple simultaneous equations by graphing, addition, and substitution

use a system of equations to solve word problems in two unknowns

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Ordered Pairs of Numbers	428	Emphasize the concept of order. For simplicity call the first member x and the second y.
Open Sentences In Two Variables	429	Practice is needed.
Cartesian Products	431	The number of products in M x N is equal to the number of elements in M times the number of elements in N.
Graphs of Sets of Ordered Pairs	432	All problems should be done to this concept.
The Graph of I x I, Q x Q, and R x R	437	See comment on page M-22 in T.E.
Distance Between Two Points	443	Optional: Note distance formula is given for d^2 because radicals have not been introduced.
Directed Distance	446	Optional. See note on page M-22 and M-23 in T.E.
Graphs of Open Sentences In Two Variables	448	
Straight Lines	452	See note on page M-23 in T.E.
Y-Intercepts	453	Optional. See note on page M-23 in T.E.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Slope of a Straight Line	454	Optional. See comment on page 455 in T.E. Stress Theorem 33.
Graphing Equations By the Slope - Intercept Method	457	Optional. See comment on page 457 in T.E.
Determining the Equations of a Line By the Slope-Intercept Method	458	Optional. See comment on page M-23 in T.E.
Parallel and Perpendicular Lines	460	Optional. This topic might be of interest to the better students but extensive consideration is not necessary.
Disjunctive Sentences Whose Clauses Are Equations	464	
Systems of Equations	465	This topic provides an introduction to simultaneous equations.
Solving Systems of Equations by the Addition Method	467	This section is extremely important and merits an extensive consideration. Most of the exercises should be assigned and discussed. See note on page M-23 in T.E.
Solving Systems of Equations by the Substitution Method	477	
Summary of Ways to Solve Systems of Equations	478	The students will tend to solve all of the examples by the addition method. They should be encouraged to use the other methods for some problems.
Types of Systems of Equations	480	See comment on page M-24 in T.E.
Problems Involving Systems of Equations	482	If possible, assign all problems as practice is needed in applying these concepts. Encourage utilization of equations containing both one and two variables.
Graphs of Open Sentences Involving Inequalities and Absolute Value	487	These can be regarded as optional topics to be covered as time and student interest dictate. They are not considered particularly vital to the development of the unit.

UNIT X: RADICALS
Chapter 11 in Pearson-Allen Textbook
(14 days)

On completion of Unit X, the student should be able to

simplify sums, products, and quotients involving square roots by use of the product and quotient properties of radicals

find the approximate square root of a number using the square root table and the "divide and average" method

use the proper symbols (radical and exponent) to denote "raising to a power" and "extracting a root"

write the principal square root of a positive number

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Roots of Real Numbers	501	
Square Roots of Real Numbers	502	To help emphasize why only the principal root is considered, use the following example: If $2 = \sqrt{4}$ and $\sqrt{4} = -2$ therefore, by the transitive property, $2 = -2$
Cube Roots and Other Roots	504	Emphasize the generalizations on page 505.
Irrational Numbers	506	At this point the student should be aware of the existence of irrational numbers such as $\sqrt{2}$. It should be pointed out that the square root of any positive integer that is not a perfect square is irrational. Demonstrate the proof that $\sqrt{2}$ is irrational but do not expect the student to master such proofs.
Products of Radicals	508	Be sure that the student grasps the full meaning of Theorem 39.
Quotients of Radicals	511	
Sums Involving Radicals	515	
Multiplication and Division of Binomials Containing Radicals	516	

UNIT X (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Square Root	519	Use only the square root table and the "divide and average" methods for finding square roots.
Rational Numbers as Exponents	526	Optional.
More About Rational Exponents	529	Omit.
Equations Containing Radicals	530	Omit.
More About the Pythagorean Theorem	532	Omit.

UNIT XI: FUNCTIONS AND OTHER RELATIONS
Chapter 12 in Pearson-Allen Textbook
(As Time Allows)

On completion of Unit XI, the student should be able to

define a relation

define a function

determine if a relation is a function by examining its graph

state the domain and range of a given function

graph both linear and quadratic functions

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Relations	542	Stress definition.
Domain and Range of Relations	545	Stress distinction between domain and range. An excellent operational definition is: Domain - first number Range - second number See italicized comments on page 549. Other function machines may be developed for further clarification.
Function Notation	554	See note on page M-27 in T.E.

UNIT XI (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Graphs of Functions	556	
Linear Functions	559	Notice horizontal and vertical lines are not treated as graphs of linear functions in this text. Do not cover this topic unless you have covered the optional material from Unit IX.
Quadratic Functions	588	Remind students that f(x) is used as y in equation $f(x) = ax^2 + bx + c$.
Graphing Quadratic Functions	588	A general discussion of symmetry will be helpful here.
		The rest of this chapter, if considered at all, should be used as enrichment or supplementary material.

THEOREMS FROM PEARSON-ALLEN TEXTBOOK

Th. 1 (page 211) If $a \in R$ and $a \neq 0$, then a has only one multiplicative inverse.

Th. 2 (page 211) If a is a real number, then $a \cdot 0 = 0$.

Th. 3 (page 211) If a and b are real numbers such that $ab = 0$, then $a = 0$ or $b = 0$.

Th. 4 (page 211) If a and b are real numbers such that $b \neq 0$, then $\frac{a}{b} = \frac{1}{b} \cdot a$

Th. 5 (page 211) If a and b are real numbers such that $a \neq 0$ and $b \neq 0$, then $\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab}$

Th. 6 (page 211) If a, b, c and d are real numbers such that $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

Th. 7 (page 211) If a, b and c are real numbers such that $b \neq 0$ and $c \neq 0$, then $\frac{a}{b} = \frac{ac}{bc}$

Th. 8 (page 212) If $a \in R$, then a has only one additive inverse.

Th. 9 (page 223) If a and b represent real numbers, then $a - b = a + (-b)$.

Th. 10 (page 231) If a is any real number, then $(-1)a = -a$.

Th. 11 (page 232) If a and b are real numbers, then $(-a)b = -(ab)$.

Th. 12 (page 232) If a and b represent real numbers, then $(-a)(-b) = ab$.

Th. 13 (page 235) If a, b and c represent real numbers, then $a - (b + c) = a - b - c$.

Th. 14 (page 236) If a, b and c represent real numbers, then $a(b - c) = ab - ac$.

Th. 15 (page 236) If a, b and c represent real numbers, then $(a - b)(c - d) = ac - bc - ad + bd$.

Th. 16 (page 237) If a and b represent real numbers, $-(a - b) = b - a$.

Th. 17 (page 257) For real numbers a, b and c, if $a + c > b + c$, then $a > b$.

Th. 18 (page 257) For real numbers a, b and c, if $a > b$ and $b > c$, then $a > c$.

Th. 19 (page 259) For real numbers a, b and c, if $a > b$ and $c > 0$, then $ac > bc$.

Th. 20 (page 260) For real numbers a, b and c, if $a > b$ and $c < 0$, then $ac < bc$.

Th. 21 (page 276) If a and b represent numbers $(b \neq 0)$, then $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$

Th. 22 (page 276) If a and b represent real numbers $(b \neq 0)$ then $\frac{-a}{-b} = \frac{a}{b}$

Th. 23 (page 282) For real numbers a, b and c when $b \neq 0$, then $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$

Th. 24 (page 283) If a, b, c and d represent real numbers $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

Th. 25 (page 286) For real numbers a, b, c and d, $\frac{a}{b} = \frac{c}{d} \rightleftarrows ad = bc$ provided $b \neq 0$ and $d \neq 0$.

Th. 26 (page 288) For real numbers a, b, c and d, if $b \neq 0$, $c \neq 0$ and $d \neq 0$, then

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

Th. 27 (page 325) If a is a factor of b and b is a factor of c, then a is a factor of c when $a, b, c, \in I$.

Th. 28 (page 326) If t is an odd integer, then $t = 2k + 1$ where k is an integer.

Th. 29 (page 337) If a is an odd integer, then a^2 is an odd integer.

Th. 30 (page 338) If a^2 is an even integer, then a is an even integer.

Th. 31 (page 338) If a is a factor of b and a is a factor of c, then a is a factor of $b + c$ when $a, b, c, \in I$.

Th. 32 (page 338) If a is a factor of $b + c$ and a is a factor of b, then a is a factor of c when $a, b, c, \in I$.

Th. 33 (page 455) The slope of the line which is the graph of the equation $y = mx + k$ is m.

Th. 34 (page 460) Two lines having different y-intercepts are parallel if and only if they have the same slope.

Th. 35 (page 462) Two lines, neither of which is parallel to the y-axis, are perpendicular to each other if and only if the product of their slopes is -1.

Th. 36 (page 471)

$$\text{If } S = \begin{cases} ax + by + c = 0 \\ dx + ey + f = 0 \end{cases}$$

$$\text{And } T = \begin{cases} ax + by + c = 0 \\ k_1(ax + by + c) + k_2(dx + ey + f) = 0 \end{cases}$$

where $k_2 \neq 0$, then systems S and T are equivalent.

Th. 37 (page 503) If a is a non-negative real number, then there is a unique non-negative real number denoted by $\sqrt{a}$ such that $(\sqrt{a})^2 = a$.

Th. 38 (page 508) If a and b are real numbers such that $a \geq 0$ and $b \geq 0$, then $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$.

Th. 39 (page 509) If a is any real number, $\sqrt{a^2} = |a|$.

Th. 40 (page 511) If a and b are real numbers such that $a \geq 0$ and $b > 0$ then

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

PREFACE

This resource guide for Pre-Algebra 1-2 is organized as follows:

1. A recommended sequence of units with a suggested time allotment for each unit.

2. Unit Outlines
   a. Specific objectives to be realized by the student on completion of the unit.
   b. A detailed breakdown of the unit into sub-units with suggestions for developing topics and hints for the teacher in presentation of topics.

The time allotment for each unit is included only as a guideline for the teacher. The specific objectives of each unit set a basic body of material to be covered in Pre-Algebra 1-2 and the standard of proficiency that should be expected from each student in the Kent School District.

The fact that this guide can be beneficially revised will become evident with use. We hope that teachers using it will critically analyze it and make comments and suggestions about its utility, content and structure.

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TABLE OF CONTENTS

INTRODUCTION

Statement of Purpose

Pre-Algebra 1-2 is organized specifically for the student who is inadequately prepared for Algebra 1-2, but is likely to take it at a subsequent time. The purpose of the course is to give the student a familiarity with the basic terminology and fundamental operations of Algebra. The course is intended to provide an introduction to most of the topics found in Algebra 1-2, but with a less rigorous development.

Objectives

The general objectives of Pre-Algebra 1-2 are to have the student:

develop an understanding of our number system including the field properties

develop competence in the fundamental operations of whole numbers, fractions, decimals, and percents

broaden and clarify the concept of ratio and proportions

gain a knowledge of the elementary concepts of Algebra, including simple linear equations and signed numbers

develop competence and skill in analyzing and solving word problems

Prerequisites and Selection of Students

Because of the nature of the track program, no ability grouping will be necessary in Pre-Algebra. To be placed in Pre-Algebra, the student must have satisfied one of the following:

a. Completion of Math 8 and teacher recommendation.
b. Completion of Basic Math 9 with a grade of C or better.
c. Completion of Basic Math 1-2 with a grade of C or better.

Students who are not achieving at a satisfactory level may be transferred from Algebra 1-2 upon teacher recommendation.

RECOMMENDED TEXTS AND REFERENCE MATERIALS

Basic Textbooks:

Pearson & Allen *Foundations for Modern Algebra, A Programmed Course;* Ginn and Company (Boston, 1965)

Wilcox *Mathematics - A Modern Approach - Second Course;* Addison-Wesley (Palo Alto, California, 1966)

Supplementary References:

Dressler *Reviewing Elementary Algebra;* Amsco School Publications Inc. (New York, 1963)

Drooyan, Wooton *Programmed Beginning Algebra;* John Wiley and Sons Inc. (New York, 1963)

Heimer, Kocher, Lottes *A Program in Contemporary Algebra;* Holt, Rinehart and Winston Inc. (New York, 1963)

Kingston House *Preparing for Algebra;* Encyclopedia Britannica Press Inc. (U.S.A., 1964)

Nichols *Pre-Algebra Mathematics;* Holt, Rinehart and Winston Inc. (New York, 1965)

Nichols, Kalin, Garland *Arithmetic of Directed Numbers;* Holt, Rinehart and Winston (New York, 1962)

Nichols, Kalin, Garland *Equations and Inequalities;* Holt, Rinehart and Winston (New York, 1962)

Nichols, Kalin, Garland *Introduction to Exponents;* Holt, Rinehart and Winston (New York, 1962)

Nichols, Kalin, Garland *Introduction to Sets;* Holt, Rinehart and Winston (New York, 1962)

School Mathematics Study Group *Introduction to Algebra Parts 1 & 2;* Yale University Press (New Haven, 1962)

SUGGESTED TIME ALLOTMENTS

UNIT		SUGGESTED TIME*
	First Semester	
I	Sets	10 days
II	Properties of Addition and Multiplication	10 days
III	Fractions, Decimals and Percents	12 days
IV	Simplifying Computations	10 days
V	Negative Numbers	15 days
VI	Equations	15 days
	Second Semester	
VII	Review and Extension of Sets	10 days
VIII	Inequalities	12 days
IX	Review of Number Line	8 days
X	Exponents	6 days
XI	Expressions and Sentences	9 days
XII	Polynomials	15 days
XIII	Ratio, Proportion, Variation	8 days
XIV	Percent	10 days
		150 days

** This suggested time schedule is based on a school year of 150 teaching days. Thirty days have been allowed for testing, review and other activities.*

UNIT I: SETS
Chapter 1 in Wilcox Textbook
(10 days)

On completion of Unit I, the student should be able to

describe a set in both words and set notation

state whether a given set is null, finite, or infinite

state whether two given sets are equal

state whether a given number is a member of the set of whole numbers, natural numbers, rational numbers, or irrational numbers

express using set notation the intersection and union of two given sets

write a rational number between two given numbers

graph sets of numbers on a number line

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Sets	1	A set may also be defined by stating a rule. Stress that $\emptyset$ and $\{\emptyset\}$ are <u>not</u> the same.
Kinds of Numbers	3	
Finite and Infinite Sets	5	
Subsets and Equal Sets	6	Symbol " $\subseteq$ " may be introduced.
Intersection of Sets	8	Venn diagrams may be used to reinforce the concept of intersection. Universal, disjoint, and complementary sets may be introduced at this point.
Least Common Multiple	10	L.C.M. is developed later in the text but is important here for the illustration of sets of numbers.
Union of Sets	11	Venn diagrams may be used to reinforce the concept of union. Provide numerous examples of compound statements, if exercises 19-24 are assigned.
Names for a Number	13	

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Rational Numbers	14	It may be useful to demonstrate that the expressions a/b and c/d where a, b, c, and d are integers and $b \neq 0$, $d \neq 0$, represent the same rational number if and only if ad = bc.
Rational and Irrational Numbers	15	Differences should be pointed out by example and not by proof. Rigorous treatment is not appropriate at this point in the course.
Betweenness	17	
Graphs of Numbers	19	Other exercises may be needed to augment the minimum list on page 20.
Variables	21	This topic may well be reserved as an introduction to Unit VI.

UNIT II: PROPERTIES OF ADDITION AND MULTIPLICATION
Chapter 2 in Wilcox Textbook
(10 days)

On completion of Unit II, the student should be able to

state whether a given set is closed under the operations of addition and multiplication

simplify numerical expressions involving the commutative and associative properties of addition and the distributive property for multiplication over addition

simplify expressions involving the special properties of one and zero

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Closure Under Addition	25	The closure property is often difficult for the slower student to fully understand. Additional exercises and examples will probably be necessary.
Closure Under Multiplication	26	
Commutative Property of Addition	27	Stress the idea of counter-example.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Commutative Property of Multiplication	29	Because of the emphasis placed on the structure of number systems a complete understanding of the field properties is essential. To be sure of a complete mastery of these properties it may be advisable to give the students several more complex examples of each.
Associative Property of Addition	30	
Associative Property of Multiplication	31	
Distributive Property of Multiplication Over Addition	32	
The Number Zero	34	
The Number One	35	

UNIT III: FRACTIONS, DECIMALS AND PERCENTS
Chapter 3 in Wilcox Textbook
(12 days)

On completion of Unit III, the student should be able to

write a given number as a fraction, decimal, and percent

add, multiply, and divide fractions and decimals

round quotients to a specified accuracy

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Notation	41	You may wish to show $4/5 = 4 \div 5 = .8$.
Percent Notation	43	
Percents Greater Than 100%	44	Examples may be necessary here to show meaning of percents greater than 100%.
Fractions and Percents Notation	45	Again division may be thought of as $4/5 = 4 \div 5 = .8 = 80\%$.

UNIT III (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Addition of Fractions With Unlike Denominators	48	
Rounding Numbers	50	
Multiplicative Inverses	51	
Multiplication - Fraction Notation	52	
Multiplication - Decimal Notation	54	
Division of Fractions	56	If students already "invert and multiply," this is all right--but the book's method gives more meaning. Students may need guidance in the "form of 1" needed.
Division - Decimal Notation	57	Note continued use of multiplication by "1".
Rounding Quotients	59	

UNIT IV: SIMPLIFYING COMPUTATIONS
Chapter 4 in Wilcox Textbook
(10 days)

On completion of Unit IV, the student should be able to

factor whole numbers into prime factors

find the greatest common divisor and least common multiple by using prime factors

simplify a numerical expression such as (8)(18) + (20)(9) using the GCD and the distributive property

find square roots by using the square root table

find the measure of the hypotenuse of a right triangle using the pythagorean relation given the lengths of the legs

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Factors of Numbers	65	
Prime Factors	66	Avoid the use of exponents unless the students are clamoring for them. We shall use the desire to shorten the writing of factors as motivation for exponents in a later section.
Least Common Multiple	67	
Fractions With Large Numbers as Denominators	68	Have students work sample problems on the board and go into the idea of least common multiple to help with large denominators.
Using the GCD	71	GCD is also called greatest common factor.
The Square Root Table	72	
Factors and Square Roots	73	Pick your example problems of square root carefully as far as large primes are concerned.
Pythagorean Relation	75	
Area of a Triangle	76	Optional

UNIT V: NEGATIVE NUMBERS

Chapter 5 in Wilcox Textbook
(15 days)

On completion of Unit V, the student should be able to

state the additive inverse of a given number

add, subtract, multiply, and divide rational numbers

state the greater or lesser of two given rational numbers

substitute positive and negative values for variables in open number expressions and evaluate the resulting expression

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Negative Numbers	83	The teacher will find it useful to make extensive use of the number line in introducing the addition and subtraction of positive and negative rationals.
Addition of Negative Numbers	84	The method presented in the book may cause problems for the students and another approach to adding a negative and positive number may be advisable.
Addition Including Negative Numbers	85	After an introduction to adding, subtracting, multiplying, and dividing rational numbers it may help to introduce gimmicks for the students to remember the rules. One such device for multiplication is: Let us call a positive number a friend. Let us call a negative number an enemy. Let us substitute the word "of" for "times". A friend of a friend is a friend. $[(+)\cdot(+) = (+)]$ A friend of an enemy is an enemy. $[(+)\cdot(-) = (-)]$ An enemy of a friend is an enemy. $[(-)\cdot(+) = (-)]$ An enemy of an enemy is a friend. $[(-)\cdot(-) = (+)]$ Zero is not considered here, as his function in multiplication is unique. In his real identity as the Multiplicative Annihilator he has the power to destroy both sides and hence, neither side would dare offend him. Another device that can be used would be one of patterns which is illustrated by the following table.
Addition Including Fractions	86	
Subtraction	88	
Subtraction With Fractions	89	
Subtraction of Negative Numbers	90	
Multiplication With Negative Numbers	91	
Multiplication of Two Negative Numbers	93	
Division	95	

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
		$(2)\cdot(2) = (4)$ $\quad (-2)\cdot(2) = (-4)$ $(2)\cdot(1) = (2)$ $\quad (-2)\cdot(1) = (-2)$ $(2)\cdot(0) = (0)$ $\quad (-2)\cdot(0) = (0)$ $(2)\cdot(-1) = (-2)$ $\quad (-2)\cdot(-1) = (2)$ $(2)\cdot(-2) = (-4)$ $\quad (-2)\cdot(-2) = (4)$
Order of Rational Numbers	96	
Letters Replaced by Numbers	97	

UNIT VI: EQUATIONS
Chapter 6 in Wilcox Textbook
(15 days)

On completion of Unit VI, the student should be able to

determine truth or falsity of mathematical sentences involving both equalities and inequalities

find truth sets of simple open sentences by the addition and multiplication methods

solve equations with fractions using the multiplication property of equality

simplify equations anc check solution sets using the distributive property

write an open mathematical sentence from an English sentence and use this technique to solve simple word problems

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Open Sentences	103	
Solution Sets	104	Re-emphasize the concept of domain.
Solving Equations	105	Be sure to assign some problems from each set as students will need practice to finalize concepts. Be sure to emphasize that X means 1X.

UNIT VI (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Equations With Fractions	111	The basic idea is that of multiplying by the LCD. Emphasize that this must be done to both sides of the equation.
Distributive Property and Equations	113	The distributive property is essential here in both simplifying by factoring and checking solution sets. It cannot be over stressed.
Review of Equations	116	
Problems About Numbers	117	This section is important as it introduces students to translation from English sentences to mathematical sentences. This is very difficult for most students and should be covered carefully.
Angles of a Triangle and Perimeters of Polygons	118	The purpose of this section is to provide more practice in writing equations from word problems. This practice will be needed!
More Equations	124	Use this section if class appears to need additional practice.

UNIT VII: REVIEW AND EXTENSION OF SETS
Chapter 1 in P.A. Foundations
(10 days)

On completion of Unit VII, the student should be able to

describe a set in both words and set notation

state whether a given set is null, finite, or infinite

state whether two given sets are equal

find the complement of a given set with a given universe

list all subsets of a given set

state which subsets are proper subsets of a given set

find the intersection and union of two given sets

state whether a given set is closed under addition, subtraction, multiplication, or division

draw a Venn diagram to show the union or intersection of two given sets

draw a Venn diagram to show two disjoint sets

COMMENTS AND SUGGESTIONS

Text Materials: Chapter 1 in Pearson & Allen *Foundations of Modern Algebra*.

This unit is considered to be a review and expansion of learnings from Unit I. Unit VII should be reserved for the start of the second semester since it is at this time that several students should be expected to "drop in" as a result of failure in Algebra 1-2.

*General Suggestions Related to the Use of Programmed Materials.** These comments will apply as well to use of Programmed Materials in Units IX and XI.

The student should be shown, possibly by overhead projector or chalkboard, that a definite written response should be made for each frame. Responses to frame items should be made on another sheet of paper, possibly in a spiral notebook. While response to a frame is being made the answer portion of the frame should be covered (3x5 cards are helpful for covers; after initial issuance, new ones should be supplied by the student). After the response for a frame is made, the student should immediately check his answer with the answer portion of the frame. If the student's answer is incorrect, he should try to find the answer from previous frames or request teacher assistance if he cannot find this resolution himself. Students should be told that just "reading" the program or copying the answers will not help them learn the materials.

UNIT VII (CONTINUED)

COMMENTS AND SUGGESTIONS

The time limit for completion of a unit should be clearly demonstrated to the students. If keeping up with class work becomes a problem, the teacher might require the students to show daily evidence of completed work (having students put their responses in a spiral notebook which can be collected and checked during the last 2-3 minutes of class is helpful; a minimum of 60 frames per day is necessary to complete this unit on time). Reserve 2-3 minutes at the end of class for turning in materials since most of the work on programs should be done in class.

The teacher should help individuals with particular frames or sets of frames upon request by students; as a general rule, however, unrequested help may interfere with the students' learning.

The students who finish the program section early should be given supplementary lessons from the Algebra 1-2 textbook which deal with the same topic. A brief discussion with the students would help determine weak areas in his programmed learning experience.

UNIT VIII: INEQUALITIES
Chapter 7 in Wilcox Textbook
(12 days)

On completion of Unit VIII, the student should be able to

write open number sentences using the inequality sumbols

list elements from a given domain which belong to the solution set of a given inequality

represent finite and infinite solution sets using set notation

graph finite and infinite solution sets

solve an inequality by comparing it with an equation

solve inequalities using the addition and multiplication properties of inequalities

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Inequalities	131	Review the meaning of domain, open sentences, etc. Emphasize the difference between equations and inequalities.

UNIT VIII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Open Sentences - Inequalities	131	
Solution Sets of Inequalities	133	May also be read "member of" or belongs to.
Finite Solution Sets	135	
Variables With Infinite Domains	136	Set builder notation may be brought in when the domains are infinite.
Graphs of Solution Sets	137	Be sure the stude re neat and orderly about their graphing.
Addition Property of Inequalities	141	
Multiplication Property of Inequalities	143	Make sure the students understand that if $a > b$, and $c < 0$, then $ac < bc$.
Solving Inequalities	145	

UNIT IX: REVIEW OF NUMBER LINE
Chapter 2 in P.A. Foundations
(8 days)

On completion of Unit IX, the student should be able to

write a given set of numbers in order from least to greatest or vice versa.

graph a given set of numbers on the number line when the set is described verbally

graph the set of numbers of arithmetic, whole numbers, counting numbers, and rational numbers and given subsets of these sets

find a rational number between two given rational numbers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS

Text Materials: Chapter 2 in Pearson & Allen *Foundations of Modern Algebra*.

See *General Suggestions For Use of Programmed Materials* which follows Unit VII.

UNIT X: EXPONENTS
Chapter 11 in Wilcox Textbook
(6 days)

On completion of Unit X, the student should be able to

write a fraction as an expression using negative exponents and vice versa

simplify expressions involving zero exponents

write a decimal numeral in expanded form

write numerals in scientific notation

simplify the multiplication and division of exponential expressions by applying the rules of exponents

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Exponents	219	It may be necessary to provide additional practice for some students.
Zero and Negative Exponents	220	
Base 10	222	
Scientific Notation	227	
Computation Using Scientific Notation	229	
All other topics in this chapter are optional		

UNIT XI: EXPRESSIONS AND SENTENCES
Chapter 3 in P.A. Foundations
or Chapter 3 in P.A. Text
(9 days)

On completion of Unit XI, the student should be able to

evaluate numerical expressions involving more than one operation

evaluate expressions by substituting and simplifying given values

graph truth sets of simple sentences

graph truth sets of compound algebraic sentences

define a set using set builder notation

differentiate which connective - and or or - would be used to join two simple sentences to form a compound sentence whose graph is given

COMMENTS AND SUGGESTIONS

Text Materials: Chapter 3 in Pearson & Allen *Foundations of Modern Algebra.*

See *General Suggestions For Use of Programmed Materials* which follows Unit VII.

As students tend to tire of the use of programmed material, the teacher may wish to delete the use of this material and teach the unit by worksheets of material from Chapter 3 in the Pearson & Allen Text.

UNIT XII: POLYNOMIALS
Chapter 12 in Wilcox Textbook
(15 days)

On completion of Unit XII, the student should be able to

state whether a given mathematical expression is a polynomial

simplify a polynomial expression by combining like terms

add and multiply polynomials

solve first degree equations involving the use of the distributive property

solve first degree equations involving the use of several properties

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Polynomials	241	Further exercises may be necessary to insure a complete understanding of the definition of a polynomial. Particular attention should be given to such expressions as x + 5 and 3/x + 7 as students do not readily identify these as not belonging to the set of polynomials.
Addition of Terms	243	
Addition of Polynomials	244	
Multiplication	246	This section affords an excellent opportunity to re-stress the associative and commutative properties.
Multiplication With Polynomials	247	Stress the fact that the multiplication of polynomials involves using the distributive property.
Equation and the Distributive Property	248	To cover this section adequately will probably require at least two days. Provide the student with several sample solutions and provide the opportunity for him to solve problems during class.
Equations Using Several Properties	250	Spend several days on this section. Stress the importance of using the correct order of operations.
Second Degree Equations	251	Optional - introduce as time allows.

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Equations With Rational Roots	254	Optional - introduce as time allows.
The Quadratic Formula	256	Optional - introduce as time allows.
Finding Rational Roots	259	Optional - introduce as time allows.
Irrational Roots	260	Optional - introduce as time allows.

UNIT XIII: RATIO, PROPORTION, VARIATION
Chapter 9 in Wilcox Textbook
(8 days)

On completion of Unit XIII, the student should be able to

write the measures of two quantities as a ratio

solve ratio problems using algebraic methods

solve equations that are in the form of proportions

solve word problems involving proportions

solve direct variation problems using the proportion form

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Ratio	173	Stress the importance of order. i.e. $1:2 \neq 2:1$
Ratio Problems	175	This section may need careful attention.
Proportions	176	Recall that $a/b = c/d$ if and only if $ad = bc$ and $d \neq 0$.
Direct Variation	180	It might be helpful to show the graph for direct variation.
Problems Involving Direct Variation	183	All of the unstarred problems should be assigned.
Inverse Variation	185	Optional
Problems With Inverse Variation	186	Optional

UNIT XIV: PERCENT
Chapter 10 in Wilcox Textbook
(10 days)

On completion of Unit XIV, the student should be able to

select the best approximation to a given percentage problem from a list of possible numerical answers where each answer after the first is ten times the preceeding one

translate verbal problems involving percentage into proportions, solve, and check for reasonableness

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Percent	193	Emphasize that 1% means 1/100, 2% means 2/100, etc.
Estimating Percents	194	
Estimating Answers in Verbal Problems	196	
Proportions and Percent	198	Have students get in the habit of checking answers for reasonableness. Insist upon it!!
Decimals and Percent	200	
Fractions and Percent	202	Tackling a problem involving a percent less than one might prove interesting at this point.
Similar Percent Problems	203	To get students started, establish some concrete patterns on formulas such as: $\%\text{ increase} = \frac{\text{amount of increase}}{\text{original amount}} = \frac{\text{increased amount} - \text{original amount}}{\text{original amount}}$
Interest and Tax	206	
Problems Involving Interest, Tax, Discounts, and Commissions	208	
Other Percent Problems	211	Optional

PREFACE

This resource guide for Basic Mathematics 9 is organized as follows:

1. A recommended sequence of units with a suggested time allotment for each unit.

2. Unit Outlines
   a. Specific objectives to be realized by the student on completion of the unit.
   b. A detailed breakdown of the unit into sub-units with suggestions for developing topics and hints for the teacher in presentation of topics.

The time allotment for each unit is included only as a guideline for the teacher. The specific objectives of each unit set a basic body of material to be covered in Basi Mathematics 9 and the standard of proficiency that should be expected from each student in the Kent School District.

The fact that this guide can be beneficially revised will become evident with use. We hope that teachers using it will critically analyze it and make comments and suggestions about its utility, content and structure.

COMMITTEE FOR BASIC MATHEMATICS 9 GUIDE

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TABLE OF CONTENTS

INTRODUCTION

Statement of Purpose

The purpose of Basic Mathematics 9 is to provide an extensive review of the fundamentals of arithmetic for those 9th grade students whose level of achievement is significantly below that expected of the entering 9th grader. In general, the course is a continuation of the work begun in the Basic Mathematics 7 and 8 but with a slightly different emphasis. By the time a student is a 9th grader, his patterns of academic achievement are rather well established. If, by this time, he is still well below grade level in mathematics achievement, he is likely to remain so unless the teacher can somehow stimulate his flagging interest.

As this is still basically a review course, many students will have severe problems in maintaining whatever enthusiasm for mathematics they may have. For this reason, the teacher of Basic Mathematics 9 should be prepared to provide a variety of experiences for the student and to be sure that they have something specific to do at all times while in the classroom. The teacher should strive to give the student some feeling of confidence in his own ability, as well as provide him with some of the basic tools necessary to further mathematical study, should he decide to undertake it. In addition, the teacher should be trying at all times to build in the student a feeling that there is something worthwhile to be gained from the study of mathematics.

Objectives

The major objective of Basic Mathematics 9 is to provide an extensive review for the student deficient in the basic skills of arithmetic. This will include the following topics:

- Place Value and Number Bases
- The Four Fundamental Operations of Arithmetic
- Measurement
- Principles of Order
- Elementary Number Theory
- Rational Numbers
- Decimals and Percents
- Ratio
- Equations

Prerequisites and Selection of Students

Ninth grade status and a grade placement two or more years below grade level on a standard test of mathematical achievement (or below tenth percentile).

RECOMMENDED TEXTS AND REFERENCE MATERIALS

Basic Textbook:

Eicholz, O'Daffer, Brumfiel, Shanks *Modern General Mathematics;* Addison-Wesley Publishing Co. (Palo Alto, Calif., 1965)

Supplementary References:

Eicholz, O'Daffer, Brumfiel, Shanks *Supplementary Experiences - Second Course;* Addison-Wesley Publishing Co. (Palo Alto, Calif., 1965)

Longley, Cook *Fun With Brain Puzzles;* Fawcett Publications, Inc. (Greenwich, 1965)

Meyer *Fun With Mathematics;* Fawcett Publications, Inc. (Greenwich, 1961)

National Council of Teachers of Mathematics *Enrichment Mathematics for the Grades, Twenty-Seventh Yearbook;* National Council of Teachers of Mathematics, Inc. (Washington, D. C., 1963)

National Council of Teachers of Mathematics *Topics in Mathematics for Elementary School Teachers, Twenty-Ninth Yearbook;* National Council of Teachers of Mathematics, Inc. (Washington, D. C., 1964)

Spitzer *Enrichment of Arithmetic;* Webster Division - McGraw-Hill Book Co. (San Francisco, 1964)

Spitzer *The Teaching of Arithmetic;* Houghton Mifflin Co. (Boston, 1961)

SUGGESTED TIME ALLOTMENTS

UNIT	First Semester	Suggested Time *
I	Place Value and Number Bases	11 days
II	Equations and Operations	8 days
III	Basic Principles for Whole Numbers	6 days
IV	Estimation	4 days
V	Computing	20 days
VI	Geometry	12 days
VII	Number Theory	8 days
VIII	Fractions and Rational Numbers	6 days
	Second Semester	
IX	Addition and Subtraction of Rational Numbers	11 days
X	Multiplication and Division of Rational Numbers	20 days
XI	Ratio	5 days
XII	Decimals	17 days
XIII	Percent	10 days
XIV	Integers	7 days
XV	Graphing	5 days
		150 days

**This suggested time schedule is based on a school year of 150 teaching days. 30 days have been allowed for testing, review and other activities.*

UNIT I: PLACE VALUE AND NUMBER SETS
Chapter 1 in Modern General Mathematics Textbook 9
(11 days)

On completion of Unit I, the student should be able to

- read aloud and write large numbers
- write any given 4-digit number in expanded notation
- round off whole numbers to given multiples of ten
- use the proper symbol (>, =, <) in comparing two whole numbers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Abacus	1	Provision of an abacus for student experiment would be helpful.
Using an Abacus	2	
Base Ten	4	Provide plenty of review on place value and bases.
Place Value	5	
Place-Value Exercises	6	
Inequality Exercises	8	
Millions and Billions	10	Have students read large numbers orally and be sure they read them excluding "and" as a break.
Larger Numbers	11	
Approximation - Rounding	12	
The First Thirteen States	13	
Exponents	14	Students will understand, once it is pointed out, that a googol has 101 digits because its symbol would be a one followed by one hundred zeroes.
More About Exponents	16	
The Universe	17	A very good class discussion may center on this section.

UNIT I (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Roman Numerals	18	
Other Numerals	19	The important objective of this page is to give the students an enjoyable experience in working with these different types of numerals without having to memorize the meaning of any of these symbols.
Different Number Bases	20	These last sections are optional depending on the level of the class.
Base Ten & Base Six	22	
Computing With Base Six Numerals	23	
Larger Numbers - Base Six	24	

UNIT II: EQUATIONS AND OPERATIONS
Chapter 2 in Modern General Mathematics Textbook 9
(8 days)

On completion of Unit II, the student should be able to

solve simple equations involving at most two of the fundamental arithmetic operations

write equations in one unknown from simple word problems

evaluate numerical expressions containing at least one set of parentheses

solve simple word problems involving two arithmetic operations

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Function Machine	30	This is not new to the students, having been used in past work. It is interesting and gives good practice.
Multiplication and Addition Equations	32	It may be necessary to explain exercise 4 rather carefully.
Using Parentheses in Equations	33	

UNIT II (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Subtraction and Addition	34	Show the relationship between subtraction and addition.
Difference and Missing Addends	35	
Division and Multiplication	36	Show that multiplication and division are inverse operations.
Quotients and Missing Factors	37	
Combining Operations	38	Do several exercises of this type orally before assignment.
Find the Number	40	Very good practice.
Finding the Number Pair	41	Very good practice.
Short Stories - Sets	42	Very good practice.
Add, Subtract, Multiply and Divide	43	
Number Line Exercises	44	Be sure students understand the notation of these exercises.
More Number Line Exercises	46	Students may need quite a lot of help for those jumps that run off the number line.

UNIT III: BASIC PRINCIPLES FOR WHOLE NUMBERS
Chapter 3 in Modern General Mathematics Textbook 9
(6 days)

On completion of Unit III, the student should be able to

simplify computation with whole numbers using the commutative, associative, and distributive properties

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Associative Principle for Addition and Multiplication	54	Application more important than names. The thing to emphasize is that this unit is intended to make computation easier for the student. Only by this approach can motivation be accomplished.
Distributive Principle	58	
Multiples of 10, 100 and 1000	60	
Special Quotients	62	
Using Exponents	64	
Using the Distributive Principle	65	This is a very important concept for any later work in mathematics. A thorough treatment is indicated.

UNIT IV: ESTIMATION
Chapter 4 in Modern General Mathematics Textbook 9
(4 days)

On completion of Unit IV, the student should be able to

estimate sums, products, differences and quotients with a reasonable degree of accuracy

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Estimates	70	Notice * on page 94 of teacher's edition.
Multiples of 10 in Estimation	71	Make sure students understand the rounding off procedure in problem 4.
Multiples of 100 in Estimation	72	

UNIT IV (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Estimating	73	All of the exercises on this page should be assigned and make sure students write an equation and not just answers.
Estimating Time, Weight and Distance	74	Could be used for class discussion and have students say why they picked the answer they did.
Sequoia Trees	75	
Estimation for Fun	76	Oral exercise.

UNIT V: COMPUTING
Chapter 5 in Modern General Mathematics Textbook 9
(20 days)

On completion of Unit V, the student should be able to

solve word problems involving more than one step, using addition, subtraction, multiplication and division of whole numbers

find the arithmetic mean of a series of numbers

solve division problems with up to two-digit divisors

solve multiplication problems with up to three-digit factors

solve word problems involving dollars and cents

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Finding Sums	82	
Vacation Travel	83	
Finding Differences	84	
Transportation	85	
Money	86	This section may merit careful consideration. The word problems are particularly good and should be done whenever they appear.
Finding Products	88	

UNIT V (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Two and Three-Digit Factors	90	Because some of the problems in this section are so long, care must be taken that the students do not become discouraged.
Birds	92	These are good story problems, some of which require more than one step. Some interest may be generated.
Exponents Again	94	
Washington, D. C.	95	These are all further practice on word problems. This material may be helpful.
The Sports Shop	96	
Height and Depth	97	
Temperature	99	
Dividing	100	Inasmuch as many students seem to have difficulty with division, a thorough treatment of this material seems indicated. The short cut should be handled only with students capable of understanding it.
Short Cuts - Divisors Less Than 10	101	
Divisors Less Than 100	102	
Divisors and Quotients Less Than 100	104	
Average - Arithmetic Mean	106	This is a concept which may be of interest in that the arithmetic involved is not difficult but the idea is often used. It may be profitable to introduce the idea of median and mode at the same time. A discussion of uses and misuses of averages may prove valuable.
Rockets	107	
The Oceans	108	
Short Cuts - Divisors Between 10 and 100	110	
More Short Cuts	111	

UNIT V (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Zeroes in the Quotient	112	Many students find this difficult. Treat it thoroughly.
Dividing Exercises	113	
Horses	114	
Money Problems - Multiplication and Division	116	
Time, Rate, Distance	118	This section is particularly valuable. If possible, give students practice in changing the distance formula (D = RT) from one form to another. You may wish to have them do the same thing with other simple formulas.
Find the Number	122	More practice.
North America	124	These word problems provide application of material covered in the chapter. They may be supplemented with problems from other sources.
Land Expansion of the United States	125	

UNIT VI: GEOMETRY
Chapter 6 in Modern General Mathematics Textbook 9
(12 days)

On completion of Unit VI, the student should be able to

name segments, rays and lines from a drawing using proper notation

state in words and writing the general concept of congruence for segments, angles and triangles

do simple geometric constructions including copying figures, bisecting segments and angles, perpendicular lines and parallel lines

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Sets of Points	130	Many parallels could be drawn between the study of geometry and the study of arithmetic.
Segments, Rays and Lines	132	The symbol for a ray always points to the right, i.e. $\overleftarrow{AB}$. The first letter is the starting point.
Congruent Segments	134	
Rays and Angles	136	You should not overemphasize space figures at this time since this will be taken up later in the chapter.
Comparing Angles	138	
Constructions	140	
Congruent Triangles	142	Stress the notation "≅" for congruence.
Copying Triangles	144	
Right Angles and Perpendicular Lines	145	
Parallel Lines	146	
More About Parallels	148	Provide an intuitive proof that the sum of the angles of a triangle make up a straight angle.
Geometric Illusions	150	Visual impressions are sometimes misleading as will be seen from this lesson.

UNIT VI (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Space Figures	152	The students might observe that a blown up inner tube reminds them of a torus or that a specially constructed box reminds them of a cube or a square prism.
The Cube	154	
Cross Sections	155	
Constructing a Space Figure	156	Have the students make up their own models as directed on page 156.
Space Figures Again	157	

UNIT VII: NUMBER THEORY
Chapter 7 in Modern General Mathematics Textbook 9
(8 days)

On completion of Unit VII, the student should be able to

state whether a given number is a factor of another number

state whether a number is a prime or a composite number

construct a factor tree

write a complete factorization of a number with as many as three digits

find the greatest common factor and the least common multiple of two numbers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Factors of Numbers	166	
2, 3 and 5 as Factors	168	If all the digits of a numeral add up to a multiple of 3 then the numeral itself is a multiple of 3.
Factor Trees	169	
Prime Numbers	170	
Prime Factorization of a Number	171	Stress that each number that appears on a factor tree is a factor of the given number. Note also that we do not always get every factor of a number on a given factor tree.

UNIT VII (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Greatest Common Factor	173	
Least Common Multiple	174	You probably should point out at this time that while zero is certainly smaller than 12, we do not consider zero the least common multiple of 3 and 4.
Clock Arithmetic	176	
Addition in Clock Arithmetic	177	
Subtraction in Clock Arithmetic	178	
Multiplication in Clock Arithmetic	179	
Just for Fun	180	Some students may for the first time really understand the importance of thinking about missing addends for subtraction and missing factors for division.

UNIT VIII: FRACTIONS AND RATIONAL NUMBERS
Chapter 8 in Modern General Mathematics Textbook 9
(6 days)

On completion of Unit VIII, the student should be able to

name a fraction to represent a part of a region

name a fraction to represent a part of a set

write a set of equivalent fractions from a given fraction

check whether 2 fractions are equivalent

reduce a fraction to lowest terms

place the proper order sign (>, =, <) between 2 given rational numbers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Fractions and Regions	188	Stress 3/8 of a region means 3 of 8 equal parts of the region.
Fractions and Sets	189	Here we are concerned only with the number of items in a set.
Equivalent Fractions	190	Try to get different forms of the correct answer from the class, i.e. 6/9 and 2/3 to example B.
Sets of Equivalent Fractions	191	Try building several more equivalent sets of fractions as in exercise 4.
Lowest Terms	192	A fraction is in lowest terms when the numerator and denominator are relatively prime, i.e., greatest common factor is one.
Reducing to Lowest Terms	193	"Dividing out" common factors is the usual method.
From Fractions to Rational Numbers	194	Stress than an infinite set of equivalent fractions is associated with just one rational number and one point on the number line.
Names for Rational Numbers	195	
Equality	196	If 2 fractions are equivalent, the rational number represented by them is the same. If 2 fractions are not equivalent, the rational numbers represented by them are not equal.
Inequalities	197	
More About Rational Numbers	198	

UNIT IX: ADDITION AND SUBTRACTION OF RATIONAL NUMBERS
Chapter 9 in Modern General Mathematics Textbook 9
(11 days)

On completion of Unit IX, the student should be able to

add and subtract rational numbers with different denominators

add and subtract rational numbers with like denominators

write an improper fraction for a whole number

reduce answers to lowest terms

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Short Stories	204	
The Number Line	205	Write equations and help the students gain a feeling for the relationship between the jumps on the number line and the rational number equations that such jumps suggest.
Sums and Differences	206	Spend plenty of time building sets of equivalent fractions.
Finding Sums and Differences	207	
Least Common Denominator	208	Allow the students to use any method they please for finding the least common multiple.
Methods for Adding and Subtracting	210	
Let's Review	212	
Cape Kennedy	213	
Mixed Numerals and Improper Fractions	214	
Basic Principles (Addition)	216	
Adding and Subtracting	217	
Adding - 3 or More Addends	218	

UNIT IX (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Short Stories	219	
The Function Machine	220	
Recipes	221	This sounds very good. If Betty Crocker is in class, have her make a yellow cake with cherry frosting.
Track Records	222	A good class discussion on these sections can stimulate interest.
Automobiles	223	
Mixed Numerals and Improper Fractions Again	224	
Pre-Historic Animals	227	

UNIT X: MULTIPLICATION AND DIVISION OF RATIONAL NUMBERS
Chapter 10 in Modern General Mathematics Textbook 9
(20 days)

On completion of Unit X, the student should be able to

find the product of two rational numbers

find the quotient of two rational numbers using the "finding the missing factor" method

state the reciprocal of a rational number

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Regions and Multiplication	232	Be sure students can relate the word "of" with multiplication.
The Number Line and Multiplication	234	
Basic Principles for Rational Numbers	236	Students should know the principles by name.
Using the Basic Principles	237	

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Special Products	238	More practice may be useful.
Finding Products	239	
Multiplication Exercises	240	
Short Stories	241	
Special Rational-Number Products	242	
Short Cuts in Multiplication	243	Due to the type of student in Basic Math 9 they will probably be highly motivated for this topic.
The Distributive Principle	244	Very important principle.
Function Exercises	246	Good oral exercise.
Number-Line Exercise	247	Optional exercise that could be used to stimulate a little abstract thinking.
Our Solar System	248	
Track Records in Space	250	
Missing-Factor Exercise	252	
Division of Rational Numbers	253	This is still the intuitive approach to division. Problem 6 on page 255 gives the formal rule.
More About Division	254	
Whole Numbers, Rational Numbers and Division	256	
Using Division to Solve Problems	258	
Short Stories	259	
Diamonds	260	
Average Temperatures	261	

UNIT X (CONTINUED)

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Animal Length	262	
Function Exercises	264	
Short Stories - Estimation	266	
Estimation for Fun	267	
Short Stories - Time	269	

UNIT XI: RATIO
Chapter 11 in Modern General Mathematics Textbook 9
(5 days)

On completion of Unit XI, the student should be able to

write a ratio to compare two measures

write different equivalent ratios for a given ratio

write ratios from simple word problems

solve word problems using ratios

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
The Language of Ratio	274	Relate work with ratios to previous work with fractions and rational numbers.
Equal Ratios	276	
Short Stories - Ratio	277	
Ratio in Geometry	278	
Ratio Fantasy	279	The students should be quite amused by the idea of a man being able to lift some of the tremendous weights that are indicated by the various ratios here.

UNIT XII: DECIMALS
Chapter 12 in Modern General Mathematics Textbook 9
(17 days)

On completion of Unit XII, the student should be able to

- interchange between fractional and decimal notation
- put the proper comparison symbol between two given decimals
- add, subtract, multiply and divide using decimal notation
- write a given number in scientific notation
- solve problems involving decimal numerals

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Other Names For Rational Numbers	284	Help students to think of the units place as the central digit. Then relate the various places in pairs. i.e. Tenths with tens, hundredths with hundreds, etc.
Fractions and Decimals	286	Show that 2.345 = 2 345/1000 by the definition and addition. i.e. 2.345 = 2 + 3/10 + 4/100 + 5/1000 = 2 + 300/1000 + 40/1000 + 5/1000 = 2 345/1000.
Decimals and the Number Line	288	Showing the divisions on a demonstration slide rule might be good.
Inequalities - Decimals	289	It may be necessary to change to fractional notation on some of these exercises.
Adding and Subtracting	290	Review whole number adding and subtracting with emphasis on "carrying" and "borrowing."
Decimals and Money	291	
Short Stories	292	
Density	293	
A String Problem	295	
More About Decimals	296	Relate pairs again to extend decimal notation.
Approximation - Rounding	297	

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Multiplying by 10, 100 and 1000	298	Develop this carefully so that students understand why they can "move" the decimal point.
Multiplying Decimals	300	Show several examples done by changing to fractional notation to show the denominators.
Multiplying - Short Cut	301	
Short Stories - Space	302	
Money Around the World	303	
Estimation Exercises	304	This is important for the next topic.
Dividing - Whole Number Division	305	Insist that students always check by estimation.
Dividing	306	Again show why we can "move" decimal points.
Short Stories	308	
Solving Decimal Problems	309	
Fractions and Decimals Again	310	Changing fractional notation to decimal notation by division.
Repeating Decimals	311	
More About Division	312	You may wish to omit example "B" on page 312.
Scientific Notation	313	
Gold	315	
Chapter Review	316	
Speed and Sound	317	

UNIT XIII: PERCENT
Chapter 13 in Modern General Mathematics Textbook 9
(10 days)

On completion of Unit XIII, the student should be able to

write a rational number as a fraction, decimal and percent

solve word problems involving percent

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Percent Notation	320	Could be used as an oral class exercise.
Using Percent Notation	322	
Short Stories	323	Try to get the students to understand what the problem is about, rather than just make a guess on how to get the answer.
More About Percent Notation	324	Some of the slower students may need individual help here.
Problem Solving - Percent Notation	326	Be sure to cover both solutions to the example problem.
Short Stories - Water	327	
Greenland	328	
Percent Discount	329	Maybe students could bring newspaper ads showing discounts.
Interest	330	
Percent Problems	331	
Percents in Equations	332	Optional
Solving Percent Problems	333	Optional
The Human Body	335	

UNIT XIV: INTEGERS
Chapter 14 in Modern General Mathematics Textbook 9
(7 days)

On completion of Unit XIV, the student should be able to

represent distances in opposite directions by means of integers

add, subtract and multiply integers

place the proper comparison symbol between 2 given integers

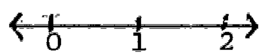
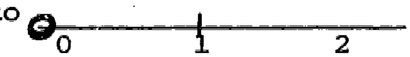
TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Exploration Exercises	342	This is a preparation lesson to provide an intuitive approach to the study of integers.
The Integers	344	Stress the word <u>opposite</u> and the idea of additive inverse.
Basic Principles for Addition of Integers	345	Mention that zero is called the additive identity and the commutative and associative principles together allow you to add in any order you choose.
Addition of Integers	346	Use both approaches as outlined in the text.
Addition and Subtraction of Integers	348	This will take careful development. Plan on quite a bit of time on this topic.
Multiplication of Integers	350	Stress the relationship between multiplication and addition and the use of the multiplicative identity to develop intuitively the rule for multiplication of integers.
Integer Inequalities	351	Stress that position on the number line determines order.
Chapter Review	351	

UNIT XV: GRAPHING (OPTIONAL)
Chapter 15 in Modern General Mathematics Textbook 9
(5 days)

On completion of Unit XV, the student should be able to

graph rational numbers on the number line

graph whole number pairs on a number plane

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
Graphs of Numbers	355	It may be clearer for the student if the number line is made with dashes instead of dots, i.e.  instead of 
Graphs of Rational Numbers	356	You may want to change the notation of an open interval that the text shows from x  to 
Number Pairs	358	
Graphs of Number Pairs	360	You may want to provide the student with graph paper.
Graphing Pairs of Rational Numbers	361	
Graphs of Functions	362	This is a little too abstract for some of the students. Do not expect complete mastery of this topic.
More About Graphing	364	

PREFACE

This resource guide for Algebra 3-4 is organized as follows:

1. A recommended set of units with specific objectives to be realized by the student on completion of the unit.

2. A suggested time allotment for each unit.

The sequence of the units may vary according to text used and teacher preference. The specific objectives of each unit set a basic body of material to be covered in Algebra 3-4 and the standard of proficiency that should be expected from each student in the Kent School District.

We hope that this preliminary guide will be analyzed and comments made about length of time required per unit, additional objectives, and suggestions for developing topics, etc. These comments will be used in subsequent revision and completion of this guide.

COMMITTEE FOR ALGEBRA 3-4 GUIDE

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INTRODUCTION

Statement of Purpose

Algebra 3-4 is organized to provide a rigorous course primarily orientated toward the needs of those students planning to continue their education in a four year college program.

Objectives

The general objectives of Algebra 3-4 are to have the student

develop an understanding of the real and complex number systems including the field properties

use the laws of exponents extended to real numbered exponents

develop computational skills including the use of logarithms and slide rule

extend his knowledge of functions and relations to include quadratic equations and logarithms and exponential functions

further develop the techniques of graphing linear and quadratic relations and systems of linear and quadratic relations

gain a knowledge of sequences, series, determinants, and elementary probability

Prerequisites and Selection of Students

To be placed in Algebra 3-4 the student must have completed Algebra 1-2 and Geometry 1-2 with grades of C or better, or have departmental approval.

RECOMMENDED TEXTS AND REFERENCE MATERIALS

Basic Textbooks:

Dolciani, Wooton, Beckenbach, Sharron *Modern School Mathematics - Algebra and Trigonometry II*; Houghton Mifflin Company (Boston, 1968)

Vannatta, Goodwin, Fawcett *Algebra Two: A Modern Course*; Charles E. Merrill Books, Inc. (Ohio, 1966)

Supplementary References:

Brumfiel, Eicholz, Shanks *Algebra II*; Addison-Wesley Publishing Co. (Palo Alto, 1962)

Halberg, Devlin *Fundamental Structures: Elementary Functions*; Scott, Foresman and Co. (Chicago, 1967)

Keedy, Griswold, Schacht, Mamary *Algebra and Trigonometry*; Holt, Rinehart and Winston Inc. (New York, 1967)

Nichols, Heimer, Garland *Modern Intermediate Algebra*; Holt, Rinehart and Winston Inc. (New York, 1965)

Pearson & Allen *Modern Algebra, A Logical Approach, Book Two*; Ginn and Company (Boston, 1965)

School Mathematics Study Group *Intermediate Mathematics, Parts I & II*; A. C. Vroman, Inc. (U.S.A., 1962)

Welchons, Krickenberger, Pearson *Algebra, Book Two, Modern Edition*; Ginn and Company (Boston, 1962)

SUGGESTED TIME ALLOTMENTS

V = Vannatta, Goodwin, Fawcett Textbook

D = Dolciani, Wooton, Beckenbach, Sharron Textbook

UNIT			SUGGESTED TIME *
I	Computational Aids and Shortcuts Supplementary Material		13 days
II	Properties of Sets and Numbers V, Chapters 1, 2	D, Chapter 1	10 days
III	Operations With Polynomials and Rational Expressions V, Chapter 3	D, Chapter 6	15 days
IV	Linear Relations and Functions V, Chapters 4, 5	D, Chapters 2, 4	20 days
V	Linear System of Open Sentences V, Chapter 7	D, Chapter 5	10 days
VI	Radicals and Exponents V, Chapter 8	D, Chapter 7	10 days
VII	Quadratic Relations and Functions V, Chapter 6	D, Chapters 8, 10	18 days
VIII	Quadratic Systems of Open Sentences V, Chapter 7	D, Chapter 10	5 days
IX	Exponential and Logarithm Functions V, Chapter 9	D, Chapter 9	10 days
X	Complex Numbers V, Chapter 11	D, Chapter 8	6 days
XI	Determinants V, Chapter 13	D, Chapter 14	8 days
XII	Sequences and Series (Progressions) V, Chapter 14	D, Chapter 3	12 days
XIII	Principles of Counting and Probability (Binomial Theorem) V, Chapter 15	D, Chapters 15, 16	15 days

**This suggested time schedule is based on a school year of 152 teaching days. Twenty-eight days have been allowed for testing, review, and other activities.*

UNIT I: COMPUTATIONAL AIDS AND SHORTCUTS
(13 days)

On Completion of Unit I, the student should be able to

convert a number from decimal to scientific notation, and, conversely, convert a number from scientific to decimal notation

compute products, quotients and/or rational powers of numbers which are all powers of 2 (or 3) by converting the problem to exponential form using a table of powers of 2 (or 3) and applying the laws of exponents

simplify products and quotients of monomials of the form x^p, where p is rational, using the laws of exponents

perform the necessary computation without pencil and paper in order to select the best approximation to a given product from a list of possible numerical answers where each answer, after the first one, is 10 times the preceeding answer

Change a monomial raised to a fractional exponent to radical form and conversely

use a four place table of mantissas to find logarithms of numbers with no more than three significant digits and to find antilogarithms not requiring interpolation

multiply, divide, raise to positive integral powers, and take positive integral roots of numbers using common logarithms in selected computational problems requiring no interpolation

perform the following operations on a 10" slide rule given an allowance of ± 0.2% error

set or read a number with 3 significant digits on the D scale

find the product or quotient of two numbers using the C and D scales

solve for the fourth term in a proportion using the C and D scales when given the other three terms

find the square (or cube) of a number using the D and A (or K) scales

set the radicand on the proper section of the A (or K) scale, and find the square (or cube) root of a number on the D scale

UNIT II: PROPERTIES OF SETS AND NUMBERS
(10 days)

On completion of Unit II, the student should be able to

name the eleven field properties of the real numbers and illustrate each with an example

name which of the above properties do not hold for given subsets of the reals

describe a set in set-builder and roster notation

find the union and intersection of given sets

form the complement of a given set with respect to a given universal set

name the properties of equality

evaluate numerical expressions involving several operations and including positive and negative numbers and zero

write the converse of a condition statement

write the negation of a simple, a conjunctive, or a disjunctive statement

select the hypothesis and the conclusion of a rational statement

UNIT III: OPERATIONS WITH POLYNOMIALS AND RATIONAL EXPRESSIONS (15 days)

On completion of Unit III, the student should be able to

fill in or match an incomplete definition with a word or phrase from the list of vocabulary terms (printed in red type) introduced in the unit

state the degree of a polynomial in one or more variables and state the degree of a polynomial with respect to a particular variable

classify a given polynomial (or monomial) as a polynomial (or monomial) over the integers, or a polynomial over the rationals or a polynomial over the real numbers

state whether or not a given polynomial has a given monomial as a term or a factor or merely a factor of only some of the terms

multiply and divide monomials over the rationals and raise such monomials to positive integral powers using the laws for operating with exponents

add, subtract, multiply, and divide two polynomials over the rationals

find the factors of a polynomial whose complete factorization consists of monomials and binomials over the integers

find the least common multiple and greatest common factor of a set of polynomials which have monomial or binomial factors over the integers

reduce a rational expression to lowest term by dividing the numerator and denominator of the expression by the highest common factor of the polynomials comprimising the numerator and denominator

change the signs of the factors in the numerator and denominator of a rational expression without changing the sign of the entire expression

add, subtract, multiply, and divide rational expressions whose numerators and denominators are polynomials over the integers

find the solution set of an equation involving polynomials or rational expressions

UNIT IV: LINEAR RELATIONS AND FUNCTIONS
(20 days)

On completion of Unit IV, the student should be able to

from a given set of relations, tell which are functions

state the domain and range of a function defined by a given set of ordered pairs

set up a coordinate system and graph a given ordered pair of numbers

name the coordinates of a given point on a graph

sketch the graph of a given linear function or relation

transform a given linear equation into slope-intercept form and graph it

solve linear equations in one variable

solve a verbal problem which can be translated into a linear equation

find the distance between two given points using the distance formulae

find the midpoint of a line segment given the coordinates of the two endpoints

solve conditional linear inequalities algebraically and graph the solution set

UNIT V: LINEAR SYSTEMS OF OPEN SENTENCES
(10 days)

On completion of Unit V, the student should be able to

state the relationship between the solution sets of equivalent open sentences

classify a system of linear equations using the terms dependent, independent, consistent and inconsistent

find the solution set of a system of two linear equations in two variables by the algebraic methods of addition and subtraction of the two equalities and also by substitution

find, by algebraic methods, the solution set of a system of three linear equations in three variables

draw the graph of the solution set of a system of open linear sentences in two variables

sketch a three dimensional coordinate system and plot points given by ordered triples

draw the graph of the solution set of a linear equation in three variables by drawing that part of the solution set in each of the three coordinate planes

UNIT VI: RADICALS AND EXPONENTS
(10 days)

On completion of Unit VI, the student should be able to

multiply and divide monomials using the laws of operation with exponents

evaluate expressions involving zero and negative exponents

simplify radicals; a radical is simplified if

the index is reduced when possible, the degree of each factor in the radicand is less than the index, and no fraction occurs in the radical

rationalize denominators of fractions when the denominator is a monomial containing a radical or binomial containing indicated square roots

add, subtract, multiply and divide radicals

solve radical equations with index 2

convert fractional exponents to radical notation and vice versa

UNIT VII: QUADRATIC RELATIONS AND FUNCTIONS
(18 days)

On completion of Unit VII, the student should be able to

solve a quadratic equation in one variable by factoring when the roots to the equation are rational

solve a quadratic equation in one variable by completing the square when the roots of the equation are real

derive the quadratic formula by completing the square when given the general form of a quadratic equation in one variable

solve a quadratic equation in one variable by using the quadratic formula

use the discriminant of a quadratic equation to determine the number of roots to the equation and whether the roots are rational, irrational or neither

state the relationship between the coefficients and the sum and the product of the roots of a quadratic equation in one variable

solve fractional and radical equations which can be transformed into quadratic form

solve a quadratic inequality in one variable algebraically and represent its solution set graphically on a number line

draw the graph of the solution set of a quadratic open sentence in two variables whose solution set either is or is bounded by a circle, parabola, ellipse or hyperbola

find the equation of a circle, parabola, ellipse or hyperbola given the coordinates of the center and the radius or coordinates of each focus and vertex and conversely

UNIT VIII: QUADRATIC SYSTEMS OF OPEN SENTENCES
(5 days)

On completion of Unit VIII, the student should be able to

solve both graphically and algebraically a system of equations involving one linear and one second degree equation in two variables

solve a system of two second degree equations in two unknowns, which can be solved algebraically by the addition method

find the solution set of a system of inequalities involving second degree relations graphically

solve verbal problems which can be translated into a system of linear open sentences

UNIT IX: EXPONENTIAL AND LOGARITHMIC FUNCTIONS
(10 days)

On completion of Unit IX, the student should be able to

evaluate an exponential function for integral and rational values of the exponent

write the equation of an exponential or logarithmic function in both exponential and logarithmic form

graph an exponential or logarithmic function with a positive integer greater than one as the base for the exponential or logarithmic function

use a four place table of mantissas to interpolate the logarithms of numbers with four significant digits and use the table to interpolate antilogarithms

multiply and divide and find powers and roots of numbers with four significant digits using logarithms and give the solution correct to four significant digits

solve exponential equations in one variable using logarithms

UNIT X: COMPLEX NUMBERS
(6 days)

On completion of Unit X, the student should be able to

express a root of a negative number in simplest form, involving i whenever applicable

reduce powers of i to ± 1 or $\pm i$

write the general form of a complex number

add, subtract, multiply, and divide complex numbers

state the conjugate of a given complex number

solve quadratic equations having imaginary roots and cubic equations having one integral and two imaginary roots

form an equation given the number of roots and the necessary real and complex roots

represent a complex number graphically

add and subtract complex numbers graphically

UNIT XI: DETERMINANTS

On completion of Unit XI, the student should be able to

evaluate the determinant of a 2 x 2 square matrix

write the minor of any element of a 3 x 3 determinant

evaluate the determinant of a 3 x 3 square matrix by expanding the determinant by minors

solve a system of two linear equations in two variables or a system of three linear equations in three variables using Cramer's Rule

UNIT XII: SEQUENCES AND SERIES (PROGRESSIONS)
(12 days)

On completion of Unit XII, the student should be able to

find the common difference or ratio and state the formula for the n^{th} term of a given arithmetic or geometric sequence

write an arithmetic or geometric sequence given a specified term and the common difference or ratio

insert a stated number of arithmetic or geometric means between two given numbers

find the sum of a stated number of terms of a given arithmetic or geometric series

write in expanded form a series given in Sigma notation

find the sum of an infinite convergent geometric series

solve verbal problems involving sequences and series

UNIT XIII: PRINCIPLES OF COUNTING (BINOMIAL THEOREM) AND PROBABILITY (15 days)

On completion of Unit XIII, the student should be able to

evaluate n!, nPr, and nCr for given nonnegative integral values of n and r

expand a binomial expression to a positive integral power using the binomial theorem

find a specified term of the expansion of a binomial to a positive integral power using the formula based on the binomial theorem for determining a specific term

calculate the mathematical probability of simple events using a given mathematical model

compute the probability of mutually exclusive and/or independent events given the probability of the related simple events

list the possible outcomes and give the probability of each outcome of an experiment involving "equally likely" outcomes

list the set of favorable outcomes and give the probability of one of these occurring in an experiment

establish the odds in favor of an event knowing the probability of the event

calculate the empirical probability for an event by examining the given data from a random sample

PREFACE

This resource unit for Consumer Mathematics is organized around the following pattern:

1. An introduction which gives specific guides to classroom procedure.

2. Definition of Core Units and Extra Core Units.

It is hoped that this resource unit will assist the classroom teacher in his daily planning. The objectives of the course set a basic body of material to be covered by continuous progress in Consumer Mathematics.

The fact that this guide can be beneficially revised will become evident with use. We hope that teachers using it will critically analyze it and make comments and suggestions about its utility, content and structure.

COMMITTEE FOR CONSUMER MATHEMATICS GUIDE

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Miss Shelly Parker, Typist

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INTRODUCTION

Statement of Purpose

The purpose of Consumer Mathematics is to provide a course for seniors who need an additional mathematics credit to meet minimum high school graduation requirements, or who desire some background in Consumer Mathematics.

Objectives

The specific objectives of Consumer Mathematics are:

1. To provide the student with a basic framework of skills, concepts and perceptions which will help him in vocational preparation and choice.

2. To provide the student with a chance to expand his knowledge into consumer related areas of purchasing, budgeting, investing, buying insurance, and tax paying.

Selection of Students

Twelfth grade students may take Consumer Mathematics. If a student has followed another track of study and has successfully completed Geometry 1-2 (C grade or higher), he may not take Consumer Mathematics.

Recommended Texts and Reference Materials

Knowles, David *Consumer Mathematics;* Behavioral Research Laboratories (Menlo Park, California, 1966)

*Book 1 - Vocational Opportunities and Lifetime Earnings
Book 2 - The Pay Check
Book 3 - The Household Budget
Book 4 - The Wise Buyer
Book 5 - Income Tax
Book 6 - Insurance
Book 7 - Investments

**This program is the required Core program for Consumer Mathematics, other programs are elective.*

Suggested Time Allotments
and
Continuous Progress Procedure

The total time alloted for Consumer Mathematics is normally one semester. A student may elect to take a second semester of Consumer Mathematics on approval of the Consumer Mathematics teacher and the Chairman of the Mathematics Department.

1. During the first quarter, the Core Program should be Book 1, *Vocational Opportunities and Lifetime Earnings*.

2. Upon completion of Unit I, the student must elect a second Core Unit from the remaining programs in the Consumer Mathematics series.

3. Upon completion of his second Core Unit for the semester, the student may select other units from the Consumer Mathematics series or other work approved for extra credit.

4. Students who take a second semester of Consumer Mathematics must select two new programs to serve as Core Units. Students may gain extra credit for the second semester as outlined in 3 (above).

5. Work during any semester must be formalized by a contract between the student and the Consumer Mathematics teacher.

Grading Procedures

A distinction is made between grading of Core Units and Extra Credit Units.

1. Core Units
   a. Core Units will be graded according to the following scale (average scores for Progress Tests in the Unit):

      90% - 100% = B
      75% - 89% = C
      50% - 74% = D (Student may choose to review and take a like form of the test and have the highest of the two scores serve for the grading).
      0% - 49% = X (Student must repeat the Unit and be retested).

2. Extra Credit Units
   a. Successful completion of an Extra Credit Unit from the Consumer Mathematics series is defined as an average score of 80% for all Progress Tests for that Unit.
   b. Extra Credit for work in material other than that of the Consumer Mathematics series will be made at the discretion of the instructor.
   c. Extra Credit will allow the raising of the average of the two quarter's work by a maximum of one letter grade.

Class Procedures

Two possibilities exist for handling a group of students taking Consumer Mathematics.

1. The class could meet as a large group with one teacher having the responsibility for the class as a unit.

2. The class could initially meet for orientation and then be allowed to complete work on their programs at different class periods of the day (possibly corresponding to their first study hall). To complete work in their programs they would work in the Mathematics Resource Center and be responsible to the teacher in control of the resource center at that time. This procedure would faciliate the student scheduling whatever other classes he needs and still gain Consumer Mathematics credit.

3. Responsibility for grading and feedback rests with the one teacher with Consumer Mathematics as his teaching load.

Related Experiences

A concerted effort should be made by the instructor assigned to Consumer Mathematics to bring in resource persons (such as car salesmen, tax consultants, vocational counselors, etc.), to use audio-visual experiences, and to provide outside experiences (field trips, etc.). Individual students should not be overlooked, since they may have experiences to share or know of some outside resource which will be helpful.

CORE UNIT I: CONSUMER MATHEMATICS
Vocational Opportunities and Lifetime Earnings

On completion of Unit I, the student should be able to

read and interpret graphically presented material

discriminate meanings of graphic and statistical terms (per capita, scale, bar graph, line graph, median, mode, average)

discriminate at an elementary level accurate presentations of statistics, particularly statements of central tendency

discriminate meaning of terms related to vocations and vocational choice (wages, salary, annual income, lifetime earnings, hourly, weekly, unemployment, regions)

perform the mathematical operations of addition, subtraction, multiplication, and division, and extend these operations to decimal numbers

TOPICS	PAGE	COMMENTS AND SUGGESTIONS
I. Gathering Information for Vocational Decisions	1	I. Two reviews are found in this chapter (1-2). Upon completion of a review, the student should be given the Progress Test which corresponds to that part of the program (after review one, give the students Progress Test One). Within a day after his completion of a test, the student should be informed about his test results individually or in a small group.
II. Averages Communicate Information	137	II. One review is found in this chapter (3). See note in I.
III. How to Get Sensible Answers About Annual Earnings	230	III. One review is found in this chapter (4). See note in I.
IV. Finding Lifetime Earnings and Factors Influencing Their Amounts	336	IV. One review is found in this chapter (5). See note in I. The final grade should be based upon the mean of all Progress Tests by the student.

CORE UNIT II

To provide uninterrupted continuous progress for the student, a second Core Unit must be selected by the student upon his completion of Core Unit I. Results of work on this second Core Unit will provide grading for the second quarter of the semester in which Consumer Mathematics is taken. If the student takes Consumer Mathematics for a second semester, he may select two new Core Units for that semester.

Programs which may be selected as second Core Units are:

Knowles *Consumer Mathematics;* Behavioral Research Laboratories (Menl Park, California, 1966)

- Book 2 - The Pay Check
- Book 3 - The Household Budget
- Book 4 - The Wise Buyer
- Book 5 - Income Tax
- Book 6 - Insurance
- Book 7 - Investments

EXTRA CREDIT UNITS

Extra Credit Units may be undertaken at the student's discretion. If he is satisfied with his semester grade as an average of grades for the two Core Units, he may choose to use his remaining time for study in other subject areas. If the student wishes and time permits, he may elect to complete another Consumer Mathematics Units for extra credit. (See list following Core Unit II). Extra Credit may be gained for other work with permission of the instructor. Completion of approved extra credit would raise the student's semester average grade by one letter grade.

PREFACE

This resource guide for Basic Mathematics 1-2 is organized as follows:

1. An introduction with recommendations which should facilitate establishing this course as a continuous progress learning experience.

2. Unit outlines which establish the sequence of units to be followed:
   a. Specific units to be completed by the student.
   b. Detailed breakdown of each of the units into sub-topics.
   c. Suggestions for selection of students for each unit.

The fact that this guide can be beneficially revised will become evident with use. We hope that teachers using it will critically analyze it and make comments and suggestions about its utility, content and structure.

COMMITTEE FOR BASIC MATHEMATICS 1-2 GUIDE

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INTRODUCTION

Statement of Purpose

The course purposes and objectives of Basic Mathematics 1-2 are to provide remedial training for students who are deficient in mathematical skills; to provide a mathematical background for those students who may be taking their last course in mathematics; to provide the student a background of addition, subtraction, multiplication and division, and these operations extended to fractions and decimal numbers.

Objectives

The student will be expected to complete as many units listed below as he is capable of doing while maintaining an average of two units per quarter.

The student will be expected to show mastery of a unit by learning the specific skills and concepts as implied by the list below of individual units. The student should be able to:

add, subtract, multiply and divide with whole numbers

use fraction concepts in problem solving

add, subtract, multiply and divide with fractional numbers

add, subtract, multiply and divide with decimal numbers

extend fractional and decimal concepts to percentage problems

The student will be expected to show mastery of a unit by scoring 80% or higher on the test from the Basic Math Achievement Series (BMAS) which applies to that unit. A score of 40% or lower should indicate the need to repeat the programmed unit (possibly with another published program). Scores between 40% and 80% should indicate the need to do the test problems again and show work.

Prerequisites and Selection of Students

In order to be accepted for Basic Mathematics 1-2, the student must be in grades 10-12, and have scored below the mean on more than four of the subsets of the Basic Math Diagnostic Series.

Classroom Procedure

During class, students will be working on different units depending on their position on the sequence of units. The teacher can at this time circulate among the students giving individual attention to situational learning problems. After testing, the student should be given as immediate knowledge of results as is possible either by the teacher or by a student assistant. During this feedback session, an individual lesson can be given to help consolidate understanding of the unit.

General Suggestions Related to the Use of Programmed Materials

The students should be shown, possibly by overhead projector or chalkboard, that a definite written response should be made for each frame.

1. Responses to frame items should be made on another sheet of paper, possibly in a spiral notebook.

2. While response to a frame is being made, the answer portion of the frame should be covered (3 x 5 cards are helpful for covers; after initial issuance, new ones should be supplied by the student).

3. After the response for a frame is made, the student should immediately check his answer with the answer portion of the frame.
   a. If the student's answer is correct, he can go on to the next frame.
   b. If the student's answer is incorrect, he should try to find the answer from previous frames or request teacher assistance if he cannot find the resolution himself.

4. Students should be told that just "reading" the program or copying the answers will not help them learn the materials.

The time limit for completion of a unit should be clearly stated to the students:

1. If keeping up with classwork becomes a problem, the teacher might require the students to show daily evidence of completed work (having students put their responses in a spiral notebook which can be collected and checked during the last 2-3 minutes of a class is helpful). A minimum of 50 frames per day is necessary to complete a unit on time.

2. Reserve 2-3 minutes at the end of class for turning in materials since most of the work on programs should be done in class.

The teacher should help individuals with particular frames or sets of frames upon request by students. As a general rule, however, unrequested help may interfere with the students' learning.

An effort should be made to contact all students working in the same unit (such as multiplication of whole numbers) as a group at least once weekly. A separate large table should be used for this purpose. During this section, visual and manipulative aids should be used to help the students develop an understanding and a concrete basis for mathematics. Also during these group sessions individual and group problems can be brought up and brought to closure.

The student who finished the full programs early should be given supplementary lessons from the Pre-Algebra programs or from some other program or undertaking of his choice. A brief discussion with the student would help determine weak areas in his learning experience.

Mean scores for scales on Basic Math Diagnostic Series for a group of 124 ninth grade students during the spring of 1968. Students included were those who had a choice of attending Basic Math 1-2 or Pre-Algebra 1-2 for their tenth grade mathematics.

Scale	Area Covered	Actual Mean	Cut Off Score Used*
BMDS #1	Addition	81.40	81
BMDS #2	Subtraction	86.12	86
BMDS #3	Multiplication	71.35	71
BMDS #4	Division	61.44	61
BMDS #5	Fraction Concepts	72.32	72
BMDS #6	Fraction Operations	40.37	40
BMDS #7	Decimals	29.29	29
BMDS #8	Percentage	24.43	24

**Those who scored the cut off score or lower were below the mean score for determination of placement in either Basic Math 1-2 or Pre-Algebra 1-2.*

Suggested Time Allotments

Since programmed materials are used and each student is expected to move at his own rate, no particular time requirement for each unit is necessary. A certain sequence of units, however, should be followed. Initial assignment to a unit should be on the basis of the results of Diagnostic Tests given during the first few days of class. Once assigned to a unit, the student should complete the sequence of units which follow.

RECOMMENDED TEXTS AND REFERENCE MATERIALS

Bobrow, Daniel *Basic Mathematics - Addition and Subtraction of Whole Numbers;* Encyclopedia Britannica Press (U.S.A., 1962) [Temac 1]

Bobrow, Daniel *Basic Mathematics - Multiplication and Division of Whole Numbers;* Encyclopedia Britannica Press (U.S.A., 1962) [Temac 2]

Bobrow, Daniel *Basic Mathematics - Fractions and Mixed Numbers;* Encyclopedia Britannica Press (U.S.A., 1962) [Temac 3]

Bobrow, Daniel *Basic Mathematics - Decimals and Percentage;* Encyclopedia Britannica Press (U.S.A., 1962) [Temac 4]

Bobrow, Daniel *Basic Mathematics - Measurement;* Encyclopedia Britannica Press (U.S.A., 1962) [Temac 5]

Hauck, Moore, Smith *Fractions Part I;* McGraw-Hill Inc. (U.S.A., 1966) [Frac 1]

Hauck, Moore, Smith *Fractions Part II;* McGraw-Hill Inc. (U.S.A., 1966) [Frac 2]

Hauck, Moore, Smith *Fractions Part III;* McGraw-Hill Inc. (U.S.A., 1966) [Frac 3]

Hauck, Moore, Smith *Percentage;* McGraw-Hill Inc. (U.S.A., 1966) [Percentage]

Hauck, Moore, Smith *Decimals;* McGraw-Hill Inc. (U.S.A., 1966) [Decimals]

Nichols, Kalin, Garland *Arithmetic of Directed Numbers;* Holt, Rinehart and Winston Inc. (U.S.A., 1962)

Nichols, Kalin, Garland *Equations and Inequalities;* Holt, Rinehart and Winston Inc. (U.S.A., 1963)

Nichols, Kalin, Garland *Introduction to Sets;* Holt, Rinehart and Winston Inc. (U.S.A., 1962)

Odom, Nichols *Introduction to Exponents;* Holt, Rinehart and Winston Inc. (U.S.A., 1964)

Pearson & Allen *Foundations for Modern Algebra - Books 1, 2, and 3;* Ginn and Company (U.S.A., 1965)

Sullivan *Programmed Math for Adults - Book 1;* McGraw-Hill Book Co. (U.S.A., 1965)

Sullivan *Programmed Math for Adults - Book 2;* McGraw-Hill Book Co. (U.S.A., 1965)

Sullivan *Programmed Math for Adults - Book 3;* McGraw-Hill Book Co. (U.S.A., 1965)

Sullivan *Programmed Math for Adults - Book 4;* McGraw-Hill Book Co. (U.S.A., 1965)

Sullivan *Programmed Math for Adults - Book 5;* McGraw-Hill Book Co. (U.S.A., 1965)

TOPICS	PROGRAMS	COMMENTS & SUGGESTIONS
I. Addition of Whole Numbers A. Simple non-carrying addition B. Carrying C. Addition when not all numbers have the same number of digits D. Long column addition E. Commutativity of addition	Sullivan 1-2; Temac 1	I. A student should not be assigned to this unit if he has scored above the cut off score on the Addidion Scale of the Basic Math Diagnostic Series. Temac units should be reserved for those students who show a high ability to deal with verbal materials. Test: BMAS #1.
II. Subtraction of Whole Numbers A. Simple non-borrowing subtraction B. Borrowing C. Borrowing when there are zeros in the minuend D. Subtracting when there are zeros in the subtrahend E. Non-commutativity	Sullivan 3; Temac 1	II. A student should not be assigned this unit if he has scored above the cut off score on the Subtraction Scale of the Basic Math Diagnostic Series. Same consideration verbal/non-verbal consideration as in I. Test: BMAS #2.
III. Multiplication of Whole Numbers A. Simple Multiplication facts up to 10 x 10 B. Multiplication by a single digit multiplier C. Multiplication when the multiplier has two or more digits D. Multiplication when one or more digits in the multiplicand are zero E. Commutativity of multiplication	Sullivan 4; Temac 2	III. A student should not be assigned to this unit if he has received a score above the cut off score on the Multiplication Scale of the Basic Math Diagnostic Series. The same verbal/non-verbal assignment. Test: BMAS #3.
IV. Division of Whole Numbers A. Division by a single digit divisor-- no remainder B. Division by divisor with more than 1 digit--no remainder C. Division with a remainder D. Division when one or more of the digits are zero	Sullivan 5; Temac 2	IV. A student should not be assigned to this unit if he scored above the cut off score on the Division Scale of the Basic Math Diagnostic Series. Same verbal/non-verbal consideration as in I. Test: BMAS #4.

TOPICS	PROGRAMS	COMMENTS & SUGGESTIONS
V. Basic Fraction Concepts A. Fraction Identification 1. like fractions 2. unlike fractions B. The fractions as an indicated division of the numerator by the denominator 1. improper fractions 2. mixed numbers C. The fraction as a number 1. equivalent fractions 2. finding a lowest common denominator	McGraw-Hill Frac. I; Temac 3	V. A student should be assigned to this unit if he scored 70% or lower on both scale 5 and 6 of the Basic Math Diagnostic Series. McGraw-Hill programs are slightly less verbal at this level than are the Temac programs (therefore are preferable to Temac). Test: BMAS #5.
VI. Operations With Fractions A. Multiplication of fractions 1. multiplying two proper fractions 2. multiplying a mixed number by a fraction 3. multiplying two mixed numbers 4. reducing fractions and/or forming a mixed number B. Addition and subtraction 1. need for common denominator (numbering system concept) 2. adding and subtracting numerators (commutativity or non-commutativity of the operation) C. Division of fractions 1. non-commutativity of division 2. inverting divisor for a multiplication problem 3. reducing or forming a mixed number D. Identity property of multiplication of fractions (reciprocal)	McGraw-Hill Frac. II & III; Temac 3	VI. See comment on V. The student should have a refresher on fraction concepts before working on operations with fractions. Test: BMAS #6.

TOPICS	PROGRAMS	COMMENTS & SUGGESTIONS
VII. Decimal Concepts and Operations A. Place value B. Fractions as decimals C. Extension of previous learnings about addition and subtraction to decimals D. Extension of previous learnings about multiplication 1. how to find number of decimal places in product E. Extension of previous learning about division 1. how to get a whole numbered divisor 2. continuing division to many decimal places	McGraw-Hill Decimals; Temac 4	VII. A student should be assigned to this unit if he has scored 70% or lower on scale 7 and 8 of the Basic Math Diagnostic Series. The unit on decimals should precede the unit on percentages, even though scale 7 score is over 70%. Much of the material in both programmed series on percentages depends on prior learning in decimals. Test: BMAS # 7.
VIII. Percentage Problems A. Changing decimal notation into percentage notation B. Changing percentage notation into decimal notation C. Changing fractional notation into either decimal notation or percentage notation D. Finding the fractional equivalent of decimals and percentages E. Applying percentage knowledge into practical problems	McGraw-Hill Percentage; Temac 4	VIII. This unit represents the terminal point for Basic Math 1-2. If the student completes this part of the course, he may be given enrichment material from Unit IX.
IX. Enrichment Materials A. General survey of algebraic operations B. Specific topics such as directed numbers	Pearson-Allen, Part 1-3, Encyc. Brit., *Preparing for Algebra;* Holt, Rinehart & Winston Programmed Series; Temac 5	