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AUTHOR Travers, Kenneth J.; And Others  
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ABSTRACT

This paper reviews teaching approaches and general resource materials for low achievers in both elementary and secondary mathematics classes. A survey of reported characteristics of low achievers is divided into two classes: (1) social and emotional problems, and (2) learning difficulties. Characteristics related to class 1 problems include: high rate of absence, goals for the immediate future only, low motivation, antisocial behavior, short interest span, and inability to see the practical use of mathematics. Characteristics related to class 2 problems include: a record of failure in mathematics, a fear of the subject, achievement scores at least two years below grade level, reading difficulties, inability to follow directions, tendency to leap to conclusions, and inability to generalize. Teaching approaches which have been reported as being successful include the use of computational aids, manipulative devices, and laboratory techniques. Also reported was the development of individualized short-term curriculum units, emphasizing success and immediate reward. The two bibliographies included are: (1) a bibliography of general resource material, and (2) an annotated bibliography of articles which have appeared in "The Arithmetic Teacher" and "The Mathematics Teacher" which suggest lessons for low achievers. (RS)

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MATHEMATICS EDUCATION REPORTS

TEACHING RESOURCES FOR  
LOW-ACHIEVING MATHEMATICS CLASSES

by

Kenneth J. Travers

John W. LeDuc and Garth E. Runion

ERIC Information Analysis Center for  
Science and Mathematics Education  
1460 West Lane Avenue  
Columbus, Ohio  
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This paper reviews general resource materials and approaches for low achievers in mathematics in both elementary and secondary grade levels. It includes an extensive bibliography of general resource materials, and an annotated bibliography of mathematics topics and content approaches for low achievers treated in articles appearing in The Arithmetic Teacher and The Mathematics Teacher. The paper was commissioned by the ERIC Information Analysis Center for Science and Mathematics Education to provide resources for the classroom teacher of low achieving mathematics students. It is with pleasure that we make it available as a Mathematics Education Report.

F. Joe Crosswhite  
and  
Jon L. Higgins  
editors ..

## Mathematics Education Reports

Mathematics Education Reports are being developed to disseminate information concerning mathematics education documents analysed at the ERIC Information Analysis Center for Science and Mathematics Education. These reports fall into three broad categories. Research reviews summarize and analyze recent research in specific areas of mathematics education. Resource guides identify and analyze materials and references for use by mathematics teachers at all levels. Special bibliographies announce the availability of documents and review the literature in selected interest areas of mathematics education. Reports in each of these categories may also be targeted for specific sub-populations of the mathematics education community. Priorities for the development of future Mathematics Education Reports are established by the advisory board of the Center, in cooperation with the National Council of Teachers of Mathematics, the Special Interest Group for Research in Mathematics Education, the Conference Board of the Mathematical Sciences, and other professional groups in mathematics education. Individual comments on past Reports and suggestions for future Reports are always welcomed by the editors.

## Introduction

The dust which was generated by the mathematics curriculum reform movement of the 1960s has settled, and some definite statements concerning the current scene can now be offered. First, it is clear that the curriculum has been submitted to a thorough-going revision. Mathematical content has been radically upgraded, and grade levels at which it is presented have been lowered. More and better mathematics has been made available to students at an earlier age.

There is, however, another side of the story. By and large, the majority of students in our schools have not been a party to the proclaimed benefits attending the mathematics reform movement. While the more mathematically able students have gleaned much from newer curriculums, the less able students have found little which meets their needs, appeals to their interests, or is accessible within their conceptual frameworks. Carl B. Allendoerfer has remarked that the first revolution, the new mathematics movement of the last decade, was "guided by college people and was largely aimed at the college-capable student." (91, P. 690.) The emerging need now, he continues, is for "new types of structures" which are more properly suited to the remaining seven-eighths of the school population, and "particularly to the lower one-third" (P. 693.). This second revolution will be initiated, he claims, when serious attention is paid to the growing body of knowledge on how young children learn mathematics.

Few would contend that a "second revolution" in school mathematics has now been set in motion. But there are indications, at least, that groundwork for a genuine reformation is being laid. For example, there appears to be a growing point of view that while the first reform movement started with a body of knowledge to be taught and attempted to fit the students into it, a more successful and pedagogically sound approach is to begin with the student, assess his learning styles, interests and talents, and then search for subject matter and instructional strategies which appear to be appropriate to him at his particular stage of development. As Jean Piaget, who is having great impact on current thinking in curricular matters, has remarked:

"It is a great mistake to suppose that a child acquires the notion of number and other mathematical concepts just from teaching. On the contrary, to a remarkable degree he develops them himself, independently and spontaneously. When adults try to impose mathematical concepts on a child

prematurely, his learning is merely verbal; true understanding of them comes only with his mental growth." (30, P. 74.)

In a prophetic article of some five years ago which has proved to be remarkably accurate, Davis (100) outlined four changes in elementary school mathematics which he saw as "most urgent."

1. Greater use of physical materials in mathematics classes.
2. Greater diversity of types of experiences in the child's day.
3. Identification and early introduction of basic mathematical ideas.
4. More emphasis on student originality and creativity within the school mathematics program. (P. 355.)

These four needs have received increased attention over the intervening years since Davis' article was published, and they are now considered particularly important to the low-achieving child not only in the elementary school, but throughout his formal educative experience.

The present paper is an attempt to put together a collection of ideas, practices and techniques which have been used with low achievers in recent years. Although there has been, of course, an element of judgement used in selection as far as possible decisions as to the effectiveness of a particular teaching resource are left to the individual. Such a policy appears to be consonant with the complexity of the teaching task presented by the low achiever. So many factors come into play in a given situation, and so many decisions have to be made by the teacher during any one lesson, that little predictive power exists at present concerning the effectiveness of particular strategies and materials.

The challenge of teaching children having learning difficulties in mathematics is great. But so are the rewards. Another curricular "happening" during the 1960s, made possible by extensive funding by federal government and other agencies, was that of special projects devoted to low achievers and their problems. Over 40 such projects were identified by the Committee on Mathematics for the Noncollege Bound, appointed by The National Council of Teachers of Mathematics. And one message from these groups is loud and clear: Once a teacher has managed to generate a classroom situation highly conducive to learning mathematics and has witnessed the responses of "slower" students to these situations, remarkable things happen. Enthusiasm grows, spirits lift and low-achieving classes become an opportunity and challenge for both students and teachers.

Many of the innovations and points of view of the projects

for low achievers are reflected in the review of teaching resources which follows. Those of us who prepared this paper are hopeful that other teachers of low achievers can find help and encouragement here for bringing new opportunities and challenges to their own classes.

## I. Review of general resource material

### A. Characteristics of low achievers

The literature abounds with lists of characteristics of students who have difficulties in learning mathematics. (See, for example, Hoffman (19).) Most such lists can be divided into two sections: characteristics which arise from social and emotional problems, and characteristics arising from learning difficulties or achievement failures in mathematics. We will focus on social and emotional characteristics first, and return to learning and achievement problems later.

Social and emotional problems are often reflected in the following characteristics of low-achieving students:

High rate of absence.

Goals based on a view of the immediate future.

Low motivation.

Antisocial behavior.

Short interest span.

Inability to see practical uses of mathematics.

Sobel (41), in an article in The Mathematics Teacher, focuses particularly upon the slow learner in junior high school, and observes that:

"1. In the first place, these are the years of rapid, uneven growth and there is need for both teachers and students alike to understand this growth and to realize that it is both variable and individual in nature.

2. Youngsters of this age, continuing a drive which begins at birth, seek personal independence from both parents and teachers.

3. They seek peer acceptance and have a need to belong to some social group.

4. They are insecure, primarily due to the tremendous physical changes taking place at this age, and they crave security and success.

5. They want recognition, approval, and status.

6. Their interests change rapidly; they crave new experiences while at the same time longing for the security of the old." (P. 347.)

He then summarizes:

"...the slow learner in the junior high school has the same characteristics as other pupils of the same age; the same basic needs and interests. However, more than the average child, he needs to be given the chance to experience success and approval: more than the others he needs to feel that he is a member of a group with a contribution to make; he needs status; his confidence must be re-established, his interest stimulated, his attitude toward mathematics made favorable, his ego flattered." (P. 348.)

Henry (18), in an address to a 1963 conference on the low achiever in mathematics commented that homes of low achievers

"...are physically and personally disorganized, life does not run on a time schedule, and so on. Thus, emotionally and cognitively, they lack the structure on which a conventional educational system can build." (P. 8.)

Henry suggests that these students need emotional calming down before any attempt at work, and recommends that the student to teacher ratio should be between ten and fifteen to one, and that additional supernumerary personnel should be available.

Low achievers are often equated with disadvantaged students, but Sasse (123), writing in The Mathematics Teacher, points to distinctions which should be made between various categories of students who are behind grade level in mathematics. She notes, for example, that the "unwilling, psychological dropout has not lagged behind for the same reasons as has the slow learner," and further comments that "the culturally deprived responds to different experiences than does the emotionally starved."

Regardless of the classification or label assigned to a student or group of students, the critical factor lies in the point of view assumed by the teacher toward these students. Robert Davis (5), Director of the Madison Project, writing some 15 years ago on the role of emotions and attitudes in teaching, commented that

"Good teaching can not be built on the attitude that students are stupid...The good teacher builds on the correct part, whereas the poor teacher attacks the weaker part. Hence, the good teacher leads the student along the path to self-reliance and self-confidence, while the poor teacher induces feelings of inadequacy and distaste for the subject." (P. 140.)

He emphasizes further that

"As a sheer guess, we might estimate that the teacher

is well advised to devote one-tenth of his effort to the objective presentation of material (which should be an easy task for him, anyway, if he is well qualified), and to spend the remaining nine-tenths in establishing a suitable emotional background for the students' efforts. Obviously, not everyone would agree with this estimate, but I think there is much evidence to support it." (P. 133.)

Fremont (11) suggests that many "disadvantaged" children have built up a wall of indifference concerning the value of school to them. Suggestions he makes for breaking down this wall include the feeling on the part of each teacher that each student is important and is capable of learning. Fremont further proposes working with small groups of children, rather than with the class as a whole, using games and puzzles, manipulative devices, and other interesting activities. For each child, a personal folder containing all written work, test papers, ditto work and any other items of interest may be prepared and kept up to date.

A concerted effort to provide materials which take account of the social and emotional characteristics of low achievers in upper elementary and junior high school is the writing project of The National Council of Teachers of Mathematics, "Experiences in Mathematical Ideas" (EMI) (159). The introduction to these materials states:

"The EMI project materials are designed so that teachers may implement them in conventional self-contained classrooms as well as in other organizational patterns--for example, in team teaching, multi-unit programs, nongraded structures. Hence EMI materials are dependent only on the willingness of teachers to offer pupils opportunities and freedom to learn those basic mathematics concepts that have been provided. Teachers are a crucial and central element in implementing student-centered and activity oriented materials. They must be exceptionally kind, warm, and fair, displaying a sincere empathy for pupils with learning problems." (P. ix.)

Proctor (121) feels that "...we must help the slow learner develop faith in himself. He must have hope for the future and confidence in his ability if he is to experience any degree of success" (P. 118.).

Daniel Prescott (32) has said, "The first factor in a security-giving relationship is a sincere valuing of the child by the teacher." This means, "he points out, "having an inner conviction that the child has good in him, has potentialities that can be realized, and is worth the thought and effort involved in helping him."

Fremont (11) feels that

"Sit-down teaching, with the teacher hiding safely behind her desk, can be expected to achieve little in this situation (i.e., in teaching the low achiever). The psychological distance between teacher and pupil has to be reduced and the involvement of the teacher with each of his students increased. Many teachers are quick to point out that group work with these students is foolhardy. This is the attitude of the frightened ones. When you are working with these children, and not against them, any approach that enables you to work with individuals is helpful and necessary." (P. 321)

Incentives and rewards are being tried in more and more low achiever programs as a basis for increasing the student's confidence in his abilities. Preliminary feedback from performance contracting projects in mathematics funded by O.E.O. indicate that most of these programs use incentive rewards to obtain improved academic performance by low achievers. The initial contractor in this area, Dorsett Educational Systems, used

"...a reward system under which students receive green stamps, transistor radios, and chances on a color TV for academic improvement." (33, P. 8)

Six contractors funded by O.E.O. for the 1970-1971 school year use incentive rewards to varying degrees, and anticipate the potential for meaningful data on the relationship between performance in mathematics and incentive rewards.

A program operated by the Teacher Corps is training interns from St. Cloud College also uses incentive methods with nonachievers in Minneapolis junior high schools (33). The report states:

"The program is designed to improve educational opportunities for poor children and to attract good teachers to schools in poor neighborhoods. Techniques used by the interns to help motivate their pupils include: paying them for school attendance by giving them credit to purchase items at a local store; deducting for tardiness or rowdiness from these earnings; encouraging students to participate in planning and conducting classes, allowing each student to select ten books for his school library; providing personalized letterheads to students to encourage letter writing. Twenty-four college-graduate interns with non-education degrees work in teams of four to six under an experienced teacher." (P. 3)

In contrast to external motivation of low achievers by

by rewards and incentive methods, Allen (46) makes a case for self-motivation through the curriculum itself. He argues that the important factors are the

"...intricacies and the effects upon motivation that can be achieved by appropriately designed materials for arranging a learning situation in which participants are actively responding to challenging problems constructed by themselves in a way that is both enjoyable and self-paced. Self-motivated learning is the lofty goal of the efforts to construct such autotelic learning situations. If such techniques ultimately turn out to be applicable in some manner to the learning of other skills, can anyone reasonable doubt the usefulness of such efforts to the educational profession?" (P. 220.)

Fremont and Ehrenberg (12) has had success in motivating low achievers by focusing upon "the use of the language of algebra to describe relationships in the world in which we live: the study of functions, although the students never heard this word" (page 552). As a result of their work with ninth graders, they suggest that "skill in carrying out fundamental operations and ability in mathematics may not be one and the same" (page 554). Attempts to change the students' attitudes of incompetency and inadequacy in mathematics were made by showing them that they could do the same mathematics as the other students.

Jones (63) describes a pilot study on 38 low achievers in the Parish Schools in Lafayette, Louisiana. Partially programmed mathematics material and mathematical games were used in an attempt to improve mathematical achievement and attitudes toward mathematics of the students involved. Results of the pilot study indicated that

"the modified programmed lecture and mathematical games instructional approach was effective in teaching ninth-grade general mathematics. The effectiveness was reflected both in the achievement scores and in the evaluation of the attitude data." (page 607)

As the preceding accounts have indicated, the affective (attitudinal, emotional) and cognitive (knowledge, skill) aspects of learning are closely related. However, focusing for the moment on the latter categories, it is possible to identify some specific kinds of learning difficulties. Because much of school mathematics is a hierarchy of prerequisite skills, concepts, and principles, many lists of low achiever characteristics typically include such items as:

A record of failure in mathematics and a fear of the subject.

Achievement scores at least two years below grade level.

Reading and comprehension difficulties in many cases.

Inability to follow directions well.

Tendency to leap to conclusions.

Inability to generalize.

Ross (34) found the following results in a study conducted on sixth and seventh grade students who were two years below their grade level in arithmetic achievement:

1. Mean percents of accuracy in the basic computational processes were as follows: addition, 88 percent; subtraction, 79 percent; multiplication, 60 percent; division, 33 percent; addition of fractions, 18 percent; subtraction of fractions, 14 percent; multiplication of fractions, 8 percent; division of fractions, 2 percent.
2. Students evidenced satisfactory reasoning in word problems involving addition and subtraction of whole numbers, but made frequent reasoning errors in problems involving multiplication and division of whole numbers as well as in all the processes involving common fractions.
3. Sixteen (out of 20) of the students were one or more years below their mental grade level in functional reading ability as measured by the Durrell Analysis.
4. Teacher assessment and cumulative records indicated that students were underachieving generally in school subjects other than arithmetic.
5. Teacher assessment and personality tests indicated that the students characteristically were withdrawn and defeated in their attitudes toward school and society.
6. Sixty-three percent of the causes of underachievement identified by classroom teachers were of an emotional nature, involving lack of interest, home or school maladjustment, short attention span, or limited initiative.
7. Fifteen of the students had shown immaturity or slowness of general development, while thirteen of them manifested abnormal physical conditions, ranging from low vitality to rheumatic fever.

8. Parents tended to be from lower socio-economic classes. Three parents owned small business, two were salesmen, twelve were skilled or unskilled laborers, and three were unemployed.
9. Parents of twelve of the students tended to hold one or more teachers responsible for their child's inadequacies. (P. 241-2)

Homan (21) has discussed five steps for a diagnostic remedial process as outlined by Batesman (1) and then suggests methods and aids which may help the child with a learning disability in arithmetic. The five steps dealt with are:

1. Determine whether a learning problems exists.
2. Obtain a behavioral analysis and description of the disability.
3. Study the relevant correlates of the disability.
4. Formulate (done by diagnostician) a clear, concise, and accurate diagnostic hypothesis.
5. Make educational recommendations. (P. 199.)

Homan deals with the following difficulties associated with learning disabilities in arithmetic: Lack of perceptual skills; motor disinhibition; perseveration; language disabilities; low level of abstract thinking (P. 199ff). She then outlines specific remedial techniques for dealing with the various difficulties. For example, a child with motor disinhibition (inability to coordinate and integrate his visual, auditory and motor activity) may be helped by counting on a magnetic board or felt board, or by use of class activities involving identification and counting. Interested readers are urged to refer to this informative article for further details.

An article by Sarah Greenholz (16) is directed to the teaching of pupils who rank below the 30th percentile of the student population in mathematical achievement. Some interesting and helpful suggestions are presented in the areas of problems with underachievers (e.g., "How do you grade them?"), general techniques which have been used successfully in the classroom (e.g., "Give these pupils immediate satisfaction by checking their work as they do it."), techniques in mathematics which also have been used successfully in the classroom (e.g., "Fold a strip of newspaper to represent 50%, 25%, 12 1/2%, 33 1/3%, 16 2/3% of the whole.), and finally in increasing the teacher's competence at this level (e.g., Have some exchange sessions with other teachers on what has worked in your classroom.).

The following teaching suggestions selected from Encouraging Children to Learn, by Dinkmeyer and Dreikurs, (7) summarize well the philosophy of many of the more promising directions in working

with low achievers.

1. Accept children as they are. Like them as they are so they can like themselves.
  2. Show a faith in the child that enables the child to have faith in himself.
  3. Make him feel that it is all right to try. If he fails, failure is not crime.
  4. Be pleased with a reasonably good attempt.
  5. Show confidence in his ability to become competent.  
(p. 118-119)
- B. Activity learning for low achievers (including use of calculators and computers)

The difficulties encountered by low achievers in mathematics classrooms have been attacked on a massive scale by the use of computational aids and by involving the student in learning activities where he is an active participant. The U.S.O.E. Report on the Low Achiever in Mathematics edited by Woodby, found that common elements in programs for the low achiever throughout the country are:

1. A mathematics laboratory--whether it be center for the school, a formal laboratory for the use of a few selected classes, or a classroom laboratory.
2. The use of calculators to help the student find his pattern or error in computation and to enable him to get past simple computational blocks to basic mathematical understandings.
3. A regulated program with a pattern of activities for security, but with a change of activities to accommodate the short attention-span of the slow learners and the unit-a-day pattern for the satisfaction of a task completed and evaluated on the spot.
4. Provision for reinforcement of early basic concepts, which may be weak, utilizing the method and techniques of the more modern programs in mathematics--the exploration of the structure of the number system, the experimentation and discovery of patterns and their utilization.
5. The use of many manipulative devices, such as the abacus, cuisenaire rods, geoboards, etc.

6. The proper and controlled use of games, puzzles, and other motivational techniques.
7. The use, where possible, of remote terminals tied into computers for computer-aided instruction units. (p. 96)

The report also made four specific recommendations for beginning teachers of low achievers:

- a. Learn as much mathematics as you can. You must know mathematics unusually well to be able to split up the usual processes into simple elements.
- b. Study the recent developments in the curriculum and acquire experience in teaching the new courses. You should know the best mathematics in the best expositions available before you can adapt these ideas to the special needs of low achievers.
- c. Request from your administration an adequate supply of manipulative materials, and learn how to use them in teaching low achievers.
- d. Become acquainted with what these students are doing in their industrial arts, business education, and home economics classes, and use these courses as sources of applications of mathematics. (p. 93)

#### Computational aids

The School Mathematics Study Group explored the use of charts and tables in assisting junior high school students to overcome the "barrier of computation" as they approached the subject matter of the regular SMSG curriculum (103, 104).

Van Atta (88) very briefly describes four kinds of problems that he feels lend themselves quite well to calculator- (or computer-) aided class work. They are: (1) an intuitive approach to the laws of exponents (i.e., having students prepare tables of power of some number, say 5, and then answer a question such as " $5^3 \times 5^4 = ?$ "); (2) an intuitive approach to Pythagoras' theorem; (3) work involving the evaluation of roots of numbers; and (4) work related to logarithms.

Stenzel (42) found that motivation of low achievers was greatly improved by the introduction of desk calculators and grocery store type computing scales into her classroom.

A report in Today's Education (126) gives an account of how in the Los Angeles and Kansas City School systems computers are being used in mathematics instruction.

"...Eighth graders in Bingham Junior High School in

the latter system spend from 30 minutes to two-and-a-half hours a week participating in an instructional dialogue on supplementary and enrichment topics in grade 8 science and mathematics. They use the IBM 1500 Instructional System.

Computer and calculator firms in the Greater Los Angeles area have provided nearly \$200,000 worth of equipment for experimental purposes at no cost to the schools. Without exception, the desk-top calculators (which can be programmed) have been highly successful and are enthusiastically received by both the gifted students and the slow learners, many of whom were potential dropouts. "Nothing turns them on like these calculators," claims the spokesman. (p. 51)

Shoemaker (38) has developed materials employing the desk calculator in the high school classroom, claiming

The calculator is a very important segment of the mathematics laboratory. During the past few years, hundreds of instructors have found very favorable results in using the calculator as a means of reaching the low achievers. (p. 24)

Shoemaker outlines three stages of machine usage in the classroom:

At the outset, the students are overwhelmed with the complexities of the machine, followed in a few weeks by enthusiasm and desire to master the "mechanical genius." With proper guidance, learning soon moves into a problem-solving and discovery learning atmosphere, creating the desired result--using the calculator as a tool of mathematics. (p. 25)

Materials developed by teachers in Arkansas, called "Drop-In Mathematics," (106) involve exercises in which students use flow charting as well as printing calculators as aids in handling troublesome arithmetic in the solution of problems. The materials are written so that a unit per day may be taught. The teacher may cover sequential units on consecutive days or non-related units. Each unit has suggested procedures for presentation. The intended level for usage is ninth and tenth grade general mathematics.

Hoffman (19) has identified directions for special projects for the slow learner which involve

use of a mathematics laboratory, with all of its ramifications, including calculators, remote terminals for computer, and flow-charting for problem analysis. (P. 87)

At the elementary school level, less work with computers and calculators appears to have been done to date. However, some have urged (see, for example, Travers and Knaupp (86)) the early introduction of computational aids in the curriculum.

Teachers and administrators will find themselves faced with many decisions (budget, in-service training, sequencing of topics, for example) as they consider using calculators and computers in their schools. One technological development which is assuming growing importance is the emergence of programmable desk calculators. Such machines are able to execute rather elaborate series of instructions (programs) and in many cases may adequately serve the instructional needs of the students.

#### Laboratory technique

Perhaps the most ardent proponent of the laboratory movement has been the English educator, Miss Edith Biggs (53, 54). The philosophy of the laboratory approach to mathematics, together with an abundance of illustrations is its use at various grade levels, elementary through junior high school, can be found in Biggs and Maclean's Freedom to Learn (54). The book espouses learning mathematics in an "active, creative way," with the following aims:

1. to free students, however young or old, to think for themselves.
2. to provide opportunities for them to discover the order, pattern and relations which are the very essence of mathematics, not only in the man-made world, but in the natural world as well.
3. to train students in the necessary skills. (p. 3)

Sections of the book are devoted to such topics as: location and use of equipment and materials; identification of reference books; use of the audio-visual aids; role of the teacher in the classroom; typical programs for teacher-training workshops.

May (71) describes the junior high learning laboratory in Winnetka, Illinois, as a place where students can explore, question and investigate, and where there is no failure. The slow learner finds that he can come to the laboratory to get help and gain strength in some area so that he can continue his work in the regular classroom, according to this point of view.

There are some slow learners who have never succeeded in the classroom in the area of numbers, but find great satisfaction in learning how to construct tetrahedrons, octahedrons, bisect line segments, etc. The glow in their faces as they hold up their geometric models that they have constructed by following the simple instructions is worth

a great deal." (p. 11)

Sweet (83) says of the mathematics laboratory:

There are a number of reasons for operating a mathematics laboratory. One is that the laboratory method of teaching is a pedagogically sound technique. It utilizes an experimental approach that requires each student's participation, but allows him to work at his own rate. This leaves the instructor free to administer more individual help. Also, because the laboratory is conducted on a less formal level than the regular classroom, it provides an opportunity for more discussion among the students, and they learn from one another. Most of all, it adds variety as a change from the usual classroom routine.

The author then goes on to point out the wide appeal of the laboratory.

...it (the mathematics laboratory) can be the focal point of interest--equally appealing to the slow learner and the capable student. It can be a time for field trips or group projects. It can provide the time for several students to investigate elementary group theory, or to make a filmstrip of optical illusions. One student might wish to use some of the laboratory periods to study computer programming. Another could calculate a value for pi using the Monte Carlo method. The class might study a unit on surveying, or produce a mathematical movie. Any of these activities will help students to develop mathematical concepts. However, their conclusions will be based on their own observations, rather than upon facts which were spoon-fed to them in a classroom lecture. (p. 117)

Davidson and Fair (56) describe in some detail how an elementary school mathematics laboratory was started in the Oak Hill Elementary School in Newton, Massachusetts. Two interesting aspects of their program are the buddy system and the mathematics lab fair.

In the first program, sixth grade students volunteer and are then trained to help first-grade children who are either low achievers or are in need of some enrichment. The "buddies" work together for an entire school year about 90 minutes per week. Both sets of children seemed to profit from this arrangement.

The second project involved a large number of students working on a wide variety of mathematics projects and then inviting parents in for an evening of observation and participation.

Phillips (73) has reported teachers' reactions to working in a mathematics laboratory.

In the mathematics laboratory, the teacher can individualize instruction, and get children to work voluntarily on a higher level of accomplishment than they otherwise would. Children can prove to themselves that what they are learning is correct and consequently rely less on authoritarian proof from teachers and textbooks. (p. 2)

The author then states enthusiastically, concerning the evaluation of laboratory approaches to instruction.

Some research is now being done to determine whether the teacher using the laboratory gets better results than the teacher who doesn't. But even before the results are known, it seems safe to say that, where the laboratory technique is used by teachers who like it and believe in it, it works. (p. 2)

Myers (72) has broken down laboratory teaching into two levels. One level deals with the specific activities of pupils while the other deals with larger, goal-seeking projects which involve specific activities. He asserts, "Too often in traditional classes pupils are engaged in aimless, specific activities without unity, significance, and motivation provided by larger, goal-seeking projects" (p. 103).

He enumerates some of the specific activities as measurement (length, weight, volumes, angles, time, temperature, revolutions per unit of time, heat, electrical units and power), constructions (both plane and solid), drawing, making graphs and presenting data, coming up with conclusions from measurements, constructions or drawings.

Some of the larger goal-seeking projects which could be adapted for use with low achievers have to do with statistical studies in the school. Meyers suggests, for example, devising tables and graphs of attendance, breaking the data down by classes, by weeks or months, correlating absence with weather, with boys and girls, with distance from school; displaying enrollment figures by showing distribution of the student body geographically; producing scale drawings of floor plans and rooms, maps of the school grounds; graphing budget figures of school overhead, data on library usage, fees and expenses of students, average growth curves of students by grades involving weights and heights showing adolescent acceleration. Such a study could be summarized by committees of students, duplicated, bound in a report and distributed to the faculty, administration, and parents.

Hamilton (61) in attempting to help mathematics teachers develop discrimination in choosing and using manipulative devices suggests the following characteristics of good devices:

1. Outcome and organization must not be obscure.

2. Variety must be provided.
3. Devices must be simple to operate. (Remember that children often open a toy and then play with the box.)
4. They should be simple to assemble and easy to store.
5. Parts must not be easily lost.
6. The device should not be an end in itself.
7. The device should encourage communication of some sort. (p. 462)

He also suggests concepts that seem amenable to illustration by manipulative devices are those that have these characteristics:

1. Substitutions are made.
2. Juxtaposition is important.
3. Relations between ideas are paramount.

In an article by Haggerty (60), a number of suggestions are given for mathematics projects at the junior high level. Typical are a marble device used to illustrate the normal distribution and a chart to show the relation of the laws of probability to genetics. While the article is not directed specifically to the low achiever, some ideas suggested might be appropriate for such students.

The Eighteenth Yearbook of the National Council of Teachers of Mathematics, Multi-Sensory Aids in the Teaching of Mathematics, includes many suggestions for activities which may prove to be effective for low achievers. Typical articles are: "Curve-Stitching in Geometry," by C. V. McCamman (69); "Window Transparencies" by J. A. Tennyson (85); and "The Mathematical Collection" by George Wolff (89).

Smart (78) gives a partial listing of various inexpensive (under \$10) weighing and measuring devices appropriate for elementary school pupils. He also gives some sample exercises which would make use of these devices (e.g., "Place square inch tiles on the top of a shoe box until it is covered. How many square inches does it take to cover the box?").

Fox (58) has suggested many opportunities for activities which may "make arithmetic more meaningful in the intermediate grades." (p. 141-2) Equipment inviting experimentation in computing and measuring:

Scales (both types)  
Liquid measures  
Thermometers

Erector sets  
Balloons  
Varied lengths of pipes

Rulers  
Tuning fork  
Levers  
Pulleys  
Perpetual calendars  
Measuring cups and spoons  
Directional compass  
Compass for drawing circles  
Pedometer  
Map meter  
Stop watch  
Weights  
Clocks  
Flannel board & illustrations (fractional parts, magic square numbers, etc.)

Barometer  
Egg cartons  
Hour glass  
Sun dials  
Tape measures  
Metronome  
Toy piano  
Pitch pipe  
Catalogues  
Toy money  
Chess  
Dominoes  
Monopoly (game)  
Globe, maps

Experiences which involve number, counting, computing, or measuring:

Physical Education:

Marching by 2's, 3's, 4's.  
Rhythmic counts  
Square dancing  
Keeping score  
Setting dimensions for court or field  
Keeping time for relays  
Number Ball toss (Two teams with members numbered; when number is called, child must catch tossed ball before it touches ground to score a point for team.)

Music:

Arranging varied lengths of pipe so tune can be played  
Filling glasses with water enough so scale can be played  
Raising or lowering of pitch by tightening or loosening a stretched wire  
Locating pitch of sound made by tapping glass, block of wood, piece of metal, etc.

Art:

Enlarging by means of squares a small picture (vice versa)  
Design a movable color wheel so that as you spin parts, each section on top is smaller than one below; label fractional parts.

Arithmetic:

Design a multiplication wheel for the lower grades (or a division wheel)  
Make personal playing cards with additional facts on them; put answers on back. Work with a pal who will flash the cards  
What 2 objects here equal the weight of three others? Use the scales to check your guess.  
Cut the numbers one to nine from a large calendar. Arrange them in three even columns so that they equal fifteen horizontally, vertically and diagonally. Use colored spools, beads, and string to make an abacus.

Social Studies, etc.:

Record daily weather data  
Keep records of food fed  
hamsters during nutrition  
experiment; chart weight  
changes.

Clocks and maps. If every  
15 degree equals one hour  
of time, set the clock to  
the time it is now in London,  
San Francisco, Chicago.

C. Curriculum Consideration for Low Achievers

Both individual school systems and larger curriculum organiza-  
tions are now turning their attention to the production of curriculum  
guidelines and materials for low achievers.

In review of attempts of major U.S. cities to deal with "needs  
problems, and capabilities of big-city children," Today's Education  
reported on the work of five school systems. In this article, Rickey  
(126), a mathematics consultant to the Dade County Public Schools,  
Florida, reported effective results from the placement of junior high  
school students in four levels of mathematics instruction. Writing  
teams of teachers have produced a curriculum bulletin, Guide of Objec-  
tives and Activities for Learning, and Success in Mathematics (Goals),  
which make it possible to help individual students progress.

Many of their teachers of low achievers have developed self-  
instructional materials and techniques.

First they defined the content and sequence of topics in  
general mathematics; then they worked out ways for students  
to take different paths through the sequence. They also  
explore techniques that would combine individualized  
instruction and group interaction. As a basis for develop-  
ing multimedia packages for ninth grade general mathema-  
tics course, committees of teachers outlined units of  
instruction. Appropriate commercially developed materials  
are included in these packages. When necessary, committee  
members wrote short programmed sequences compatible with  
the reading level, attention span and emotional maturity  
of the students.

A typical package contains the teacher's commentary and  
the student's package of multimedia activities. A proce-  
dure flow charts directs the student through each unit.  
Students also receive information from films, audiotapes,  
text references, games, laboratory activities, and supple-  
mentary activities. (p. 51)

Longshore (126), secondary school mathematics consultant in the Kansas Public Schools, reported use of individualized instruction and flexible scheduling, to attempt to meet the needs of low achieving students.

To accommodate the varied backgrounds of their students, three Kansas City secondary schools have adopted flexible modular scheduling. Under this plan students meet in large groups for presentation of new material by the teacher, for summaries of student reports, for speakers from industry, for instruction through audiovisual materials, or for testing. Laboratory periods provide for study with teachers or other students and for learning by discovery through manipulative devices or through games or puzzles. Open laboratories and library resource centers allow for independent study or for review. The formulation of performance objectives enables the student to know exactly what he is expected to learn. (p. 50)

The National Council of Teachers of Mathematics has produced materials for low achievers in grades five through eight. This project, Experiences in Mathematical Ideas, emphasizes the importance of "experiences (which) grow out of the physical environment of the student and are developed through an assortment of teaching strategies which frequently culminate in laboratory-oriented activities" (167, p. 196). Key words used to "give further insight regarding its (the N.C.T.M. Project's) orientation and purpose, are "activity," "individuality," "success," "meaning" and "novelty."

An earlier N.C.T.M. Project, Experiences in Mathematical Discovery, was directed to the task of producing appropriate materials for ninth grade students in the 25th to 50th achievement percentile range. The writing committee agreed that such materials "should provide opportunities for genuine mathematical discovery at the maturity level of the students and should stimulate them to continue their study of mathematics in the regular sequence" (166, p. 63). Units have been published by N.C.T.M. on the following topics:

1. Formulas, Graphs, and Patterns
2. Properties of Operations with Numbers
3. Mathematical Sentences
4. Geometry
5. Arrangements and Selections

Another group, the Central Iowa Low Achievers Mathematics Project, has developed materials for students in grades seven through nine. Sixteen booklets of field-tested materials have been available from the project at a cost of approximately \$1.25 per booklet. A typical

publication is Gimmicks (163) which is divided into three major areas--drill exercises, games and puzzles, and motivation of new concepts. The booklet is essentially a collection of ideas which have been found to be successful by the teachers in the project. Attempts are made to have each activity in the booklet self-explanatory. Other references for further resource material are provided. Other booklets available include Measurements, ESP (Enrichment Student Projects), Probability, and Mathematics in Sports.

Stenzel (42), in a course which she has devised, attempts to capture the interest of her students by introducing topics in modular arithmetic, geometry, algebra, coordinate geometry, graphing and topology. She also encourages the use of games, projects, and gadgets as well as having the students move around the room, to work together and to use the chalk rather than scratchpaper.

Hoffman (19) in a description of the Jefferson County Public School's program for low achievers, outlines some procedures designed to meet the needs of the low achiever.

1. Multiple Activities--All work is completed and evaluated during the class period. The student is provided with at least two changes of activity during a period.
2. Electric Printing Calculators--One for each two students is provided.
3. Local Business Problems--Contributed by local businessmen and presented to the student on letterhead paper.
4. Flow-Charting--To aid students in analyzing an algorithm.
5. a. Mathematics Laboratory Experiments--Performed with inexpensive and easily assembled physical apparatus.  
b. Laboratory Multisensory Aids--e.g. Standard audio-visual equipment, calculators, scales, slide rules, mathematical equipment and models, and much equipment for individual student use.  
c. Laboratory Involvement Projects--e.g. puzzles and kits for individual use. (p. 89, 90)

Hoffman continues, "The Jefferson County School system has evaluated the work done under this special program and feels that it has aroused the interest and developed the skills of students who were formally disinterested and unmotivated" (p. 90).

In working with 66 high school dropouts in the Houston area, Kneitz and Creswell (65) were able to produce an average gain of seven months achievement in two months of instruction as well as noticeable changes in attitude, poise, and dress.

Techniques and materials used included beginning instruction with materials which would allow students to succeed, use of individualized instruction, programmed instruction booklets, SRA Computational Skills Kits, and crossnumbers puzzle kits, the World Book Cyclo-Teacher, frequent changes of pace, filmstrips, math games and real life problems.

Adler (90) suggests that "a new approach to teaching the low achiever in mathematics is beginning to take hold in the secondary schools of the United States."

Inherent in this new approach is the assumption that the low achiever is probably capable of doing better if only the causes of his poor performance can be identified and counteracted. On the basis of this new approach, the school tries to overcome the student's handicaps instead of perpetuating them. It also tries to prepare him and encourage him to enter the regular sequence of mathematics courses at a later time.

The new general mathematics curriculum has the same educational goals as the curricula for the average and above average students, i.e., it tries to develop the kind of literacy in math that is needed by a citizen in the present age of automation and atomic energy and it tries to cultivate the intellect and liberate the mind. (p. 29)

Adler lists several things which he feels the new low achiever programs do to achieve these goals: (1) presents significant new mathematical ideas to the low achievers; (2) helps him relearn arithmetic from a new, more meaningful and more mature point of view; (3) seeks to arouse and maintain his interest in mathematics; (4) provides him with opportunities to make genuine mathematical discoveries at his level of knowledge and performance; and (5) crystalizes his learning experience around ideas that he understands, generalizations that he perceives and verbalizes, and skills that he masters (p. 20).

The U.S.O.E. Conference on Low Achievers in Mathematics (132) considered the need for the development of curricular materials appropriate for low achievers. Their recommendations provide the framework for the following guidelines to curriculum development.

1. Identify appropriate aims for instruction

The U.S.O.E. Report emphasized the importance of developing

mathematical understandings essential for vocational competence (p. 10). Particular attention was paid to the potential of cooperative work-study programs which focus upon the achievement of skills demanded by families of jobs (p. 11).

Mallory (70) outlined the following criteria for selecting course material.

1. Every topic selected must definitely and positively contribute to the social, home or community needs of the pupil.
2. Every topic must be of such a nature and presented in such a way as to interest the pupil and convince him of its value to him; that is, it must be intrinsically interesting and of evident worth.
3. The material must be concrete to him and within his ability to succeed with it.
4. The material must be of such a type and presented in such a way as to develop desirable habits, attitudes, and appreciations.

In 1970, the National Council of Teachers of Mathematics recognized the need for identifying minimal competencies in mathematics for all students. The Council charged a group of educators concerned with teaching low achievers to draw up a list of basic mathematical competencies, skills and attitudes essential for enlightened citizenship in contemporary society.

## 2. Multidisciplinary input

Whitehead, in his book Aims of Education, has observed that "You may not divide the seamless coat of learning...there is one subject matter for education, and that is life, in all of its manifestations." In another place, Whitehead then protests that "scraps of gibberish" are being taught "in the name of algebra" (p. 18-22). The point is clearly made, not only for low achievers, but perhaps more importantly for them, that what is taught in mathematics classes must bear firm relationships to what is taught in other classes, and what goes on "out there" in the world. Such an approach to curriculum building, which appears to be revitalized today, suggests that persons representing a wide variety of backgrounds must participate in the decision-making concerning curriculum building. The U.S.O.E. Report urged that "National leadership should initiate action to recruit a team of mathematicians, teachers, psychologists and other experts to develop outlines of the mathematical content of courses for the low achievers" (p. 10). Other curriculum groups have gone even farther beyond these constraints, and attempted to involve

community personnel (parents, youth workers, etc.) in the development of materials.\*

### 3. Maximizing success

The low achiever has, by definition, experienced difficulty in learning mathematics. Hence, one of his greatest needs is to experience success. The Experiences in Mathematical Ideas Project (EMI) has said this well, in describing their materials.

Success breeds success. Yet too many mathematics classes offer many students little else but experiences of failure. EMI materials are designed to provide activities that will give even the slower students successful encounters with mathematics. This is done by breaking down concepts into very small components and devising simple tasks that will carefully lead the student to a grasp of the basic ideas. The teacher using EMI materials will have to resist the urge to move too quickly through the experiences because they look "too easy". (159, p. x).

### 4. Individualized instruction

The U.S.O.E. Report (132) recommended that a variety of learning materials should be provided to meet the differing rates of learning. Attention should be directed toward their flexibility and ease of use in achieving the objectives of the program for the low achievers (p. 10).

### 5. Change of pace

Again, essentially by definition, the low achiever is able to give his attention to mathematics (i.e. that with which he has had repeated failure experiences) for only a short period of time. Teachers are aware, however, that for activities which have in themselves rewards of success or accomplishment, such as tinkering with an old car, or playing a musical instrument, this same student's span of attention may assume a remarkable growth!

Many curriculum projects, such as Experiences in Mathematical Ideas, have recognized the need for accounting for this characteristic of some students. Again, the EMI Project has stated it well.

A change is as good as a rest. Perhaps more than any other subject in the curriculum, mathematics poses the risk of becoming routine and dull in the classroom. The prospects are particularly bleak for the youngster who day after day faces the same kind of tasks that have repeatedly meant failure for him in the past. EMI

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\*This work is being done at present by the Curriculum Division of the Board of Education of the City of Chicago, but details are not yet available.

materials attempt to approach familiar mathematics from a fresh point of view. At the same time, the ultimate objective is to develop a more complete understanding of the fundamental mathematical concepts included in the set of units. (p. xi)

#### 6. Use of varied approaches to learning

People appear to differ in their learning aptitudes. Some persons are verbally oriented, some are helped more by pictures. By and large, a helpful approach to learning appears to be to provide a variety of stimuli to the student. Presumably, persons with certain aptitudes, say verbal, will respond better to certain of the "instructional treatments" than to others.

One extensive project, called "Program for Learning in Accordance with Needs" (PLAN), is a system of individualized instruction which uses currently available instructional materials. In this system, units are developed which prescribe various learning activities by which the student may accomplish assigned objectives. Of particular interest is the recognition by the system developers that individuals differ in respect to their learning styles. For example, some youngsters may prefer to learn, and can learn better, through auditory means while others are more disposed to a reading approach (114).

#### 7. Provision for evaluation

In order to maximize the usefulness of a set of curricular materials, it is essential that the developers obtain feedback concerning the effectiveness of those materials. The U.S.O.E. Report stressed that such evaluation should be conducted in a variety of schools with different types of low achievers (p. 11). Furthermore, provision for evaluation should be built into the over-all administrative scheme of operation at an early stage of the curriculum development process. In general, attempts to evaluate a project after completion of writing and field trials are destined to less than optimal usefulness.

#### Administrative Guidelines

The U.S.O.E. Report also emphasized the importance of sound administrative policies and procedures in implementing new curriculum patterns for low achievers, as follows:

##### 1. Key role of the school administrator

- initiate and provide staff and community orientation programs
- see that teachers of low achievers obtain due recognition for their important work

--involve non-school persons and agencies in development of programs to prepare low achievers for useful role in society

--interpret program to community (p. 5)

2. Provision of adequate course offerings at every grade level

Needs for appropriate curricular materials for low achievers exist at elementary, junior and senior high school, as well as at the junior college levels. Furthermore, Brain (95) has the encouraging observation that:

Research into the psychology of learning indicates that students of low ability can learn much more than they are learning at present. Interestingly, there is evidence that students of low ability can learn mathematical concepts and processes at age 16 to 18 that they were not able to learn when they were younger. (p. 34)

3. Use of appropriate grouping procedures

The U.S.O.E. Report encouraged the use of grouping for purposes of instruction, by gearing instruction toward the characteristics of each group (p. 5).

4. Provision of assistance to teachers

Teaching low achievers is time-consuming and demanding. Extra effort must be spent in such activities as developing and using varieties of special audio-visual and manipulative learning materials.

The U.S.O.E. Report recommended that administration provide for teachers of low achievers para-professional support such as teacher-aides (p, 86). As facilities such as instructional materials centers and mathematics resource laboratories become more common, administrators may well contemplate the prospect of providing teachers of special groups with released time for consultative, tutorial and conference sessions with selected groups of students. As Stenguist has noted (129),

One of the important functions of a board of superintendents is so to arrange working conditions that the rest of the personnel are provided with time and opportunity to carry on diagnostic education for all types of pupils. (p. 503)

5. Involvement of parents and community in school programs

Without the support of parents and community, the work of the school is virtually doomed to failure. Many projects for low achievers

have attempted to bridge the gap by providing for parent involvement in the programs. This involvement ranges from "mathematics for parents" courses, to literacy programs, to block meetings in the community at which problems and issues relating to the school are considered.

#### 6. Selection of teachers for low achieving classes

Even a cursory reading of the prior sections of this paper points to the high level of competencies--warmth, insight, imagination, resourcefulness, knowledge of psychology, firm grasp of mathematics--required by the teacher of low achievers. The responsibility of the administrator is a great one, as he assigns personnel to these critical staff positions.

As the U.S.O.E. Report points out, administrators could do a greater service to the cause of the low achiever by assuming responsibility for and taking initiative in, informing teacher education colleges of the needs and requirements of teachers for such positions (p. 87).

#### 7. In-service education programs

Shulte and Wells (127) have outlined activities for school principals as they facilitate the professional growth of their mathematics staff.

- Establish a library, accessible to all teachers
- Arrange for attendance of teachers at appropriate in-service activities
- Help to ensure that the staff is effective in implementing new programs
- Ensures that teachers who are effectively using innovative practices receive recognition (p. 143)

Shulte and Wells also offer specific suggestions for the conduct of in-service programs. Interested readers should refer directly to the source for more details here, as elsewhere in this paper, but the following points were particularly salient:

- Use of video tape for recording demonstration lessons.  
"All teachers benefit from seeing master teachers in action."
- Keep in mind that the demonstration class is primarily a showcase of excellent technique, not a sample of what takes place in a classroom day after day. (p. 143)
- The in-service presentations should not be too theoretical; they must present ideas that teachers can apply in the

classroom in the immediate future. (p. 144)

--If manipulative aids are discussed, teachers should have these aids in their hands and the session may be conducted much the same as if the teachers were the students. It may be wise to facilitate this practice in using aids by grouping teachers in twos and threes. (p. 144)

--At the secondary school level, mathematics teachers are more often specialists in the subject, so the emphasis need not be so much on content of new materials as on techniques of teaching and on relating techniques to content. Without such emphasis, secondary teachers can often manage to use modern materials in a traditional fashion. (p. 145)

--Some schools have adopted a "Buddy" system for new teachers where an experienced teacher is assigned to a new teacher, to help him or her adjust to the school and to the teaching situation. (p. 146)

#### 8. Provision of adequate guidance facilities

Many students who are having difficulty with mathematics also experience difficulties with other subjects, and may encounter social and emotional problems as well. The proper identification, analysis and treatment of this host of difficulties requires the team efforts of teachers, counselors, psychologists, social workers and other concerned specialists.

#### 9. Promotion of an investigative point of view

It must be stated that, recognizing all that is being done to attack the problem of low achievement, it still remains that little is known concerning basic issues. As programs and practices are implemented, school personnel are to be encouraged to seek answers to such questions as:

--What are the causes of learning difficulties in mathematics?

--How are particular learning difficulties dealt with?

--At what developmental stage of the child are particular difficulties more successfully dealt with?

--Which methods or modes of instruction are best for students with particular difficulties?

--What factors in the home, community and school appear

to foster or hinder mathematics achievement?

--What teacher characteristics relate best to high pupil achievement?

--What classroom administrative techniques relate best to high pupil achievement?

Again, Brain (95) in the U.S.O.E. states:

He (the administrator) must enable the teachers to develop and use new materials for low achievers. Improvization and innovation must be encouraged. (p. 37)

#### D. Summary

The identification of teaching resources for low achievers depends upon the analysis of student characteristics, teaching strategies, and mathematics curriculum content. We have looked at each of these areas in detail.

As we have seen, social and emotional problems of low achievers are often reflected in low motivation, short interest spans, antisocial behavior, high absence rates and goals based only on the immediate future. These characteristics suggest that appropriate resource materials should be constructed in small units with provisions for individualized work, and should emphasize success and self-worth - possibly by including an incentive or reward system.

Considerations of possible teaching strategies indicate that the use of a wide variety of learning aids is useful with low achievers. Computational aids such as tables and desk calculators allow us to avoid an area which has usually caused previous failure on the part of the low achiever. Other laboratory devices provide activity learning which is useful for improving and maintaining motivation levels of low achievers. An activity-learning approach provides an excellent vehicle for promoting the following teaching procedures that are particularly effective with low achievers.

1. Employ repetition through spiraling.
2. Take small segments of material.
3. Use language which the class is likely to understand.
4. Dramatize the material.
5. Individualize the problem assignments.

6. Pay attention to reading the problems.
7. Provide variety within the class period.
8. Use concrete approaches to the subject matter.
9. Provide activities.
10. Hold frequent reviews.
11. Use praise freely.
12. Display good student work.
13. Build assignments which lead to success.
14. Use diagnostic testing before and after teaching.
15. Measure the student against himself.
16. Correlate mathematics with other subjects whenever possible.
17. Establish consistent classroom management policies.
18. Use audio-visual techniques when possible.
19. Try supervised study rather than homework.
20. Grade work the day it is turned in.
21. Do not insist on verbalization.
22. Allow the use of calculators or tables.

Our final area, curriculum considerations, suggests that resource materials should have clear objectives which have some relationship to the immediate needs of the learner. (Although we usually think of needs in practical terms, we must not overlook recreational needs where mathematics puzzles and games may be worthwhile.) Content material should develop desirable habits, attitude, and appreciations as well. To do this, materials must combine low difficulty levels with high mathematical standards.

The resource materials in the bibliographies which follow suggest ways in which each of these standards can be met. They also affirm that we can teach good mathematics in interesting ways to the low achiever.

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160. National Council of Teachers of Mathematics, Experiences in Mathematical Ideas, Vol II. Washington, D.C.: National Council of Teachers of Mathematics, 1970. SE 010 741. (EDRS availability in process.)
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III. Annotated Bibliography of Mathematics Topics and Content Approaches Suitable for Low Achievers in Articles Appearing in The Arithmetic Teacher and The Mathematics Teacher

PRIMARY GRADES

Andrews, E. E., and Nelson, L. Doyal, "Beginning Number Experiences and Structured Materials," The Arithmetic Teacher, 10:330-33, 1963.

The authors make a plea to present computational procedures through meaningful problem situations so as to avoid pupils merely learning computation per se. A question is raised concerning the use of Cuisenaire rods.

Bachrach, Beatrice, "Do Your First Graders Measure Up? (A Report of a Unit with Disadvantaged Learners,)" The Arithmetic Teacher, 16:537-38, 1969.

A brief description of a unit on measurement which was used with disadvantaged first graders in Chicago.

Bridges, Raymond B., Jr., "Easily Made Arithmetic Aids," The Arithmetic Teacher, 10:507-8, 1963.

Details for constructing two simple devices for illustrating elementary number concepts. The first is a "block box" of wood and the second consists of stringing one inch dowels of varying lengths of string.

Fisher, Alan A., "The Peg Board--A Useful Aid in Teaching Mathematics," The Arithmetic Teacher, 8:186-88, 1961.

The author presents ideas on how to use the peg-board as an aid in teaching perimeter and area, place value, commutative law for multiplication, squares of numbers, equal ratios, coordinate axes and degrees in a polygon.

Harshman, Hardwick W., Wells, David W., and Joseph N. Payne, "Manipulative Materials and Arithmetic Achievement in Grade 1," The Arithmetic Teacher, 9:188-92, 1962.

The conclusions of this study along with the authors' recommendations, should be of particular interest to teacher educators and all those interested in research.

Liedtke, W. and T. E. Kieren, "Geoboard Geometry for Preschool Children," The Arithmetic Teacher, 17:123-26, 1970.

This article has some specific suggestions and questions which could be of value to teachers working with very young children on a geoboard. These suggestions and questions are in the areas of familiar shapes, plane figures, and segments.

163. Zimmerman, Joseph, Central Iowa Low Achiever Mathematics Project -- Low Achiever Motivational Project. Des Moines, Iowa: Central Iowa Low Achiever Mathematics Project, 1968.  
ED 025 433. MF and HC available, 145p.

Maloney, John P., "Arithmetic at the Primary Level," The Arithmetic Teacher, 4:112-18, 1957.

The article discusses some of the philosophy and techniques used at the Boston Curriculum Center for Arithmetic in teaching primary students.

McKillip, William D., "'Patterns' -- A Mathematics Unit for Three- and Four-Year Olds," The Arithmetic Teacher, 17:15-18, 1970.

The work described in this article could be used for slow learners at the primary level. The article describes how number concepts may be introduced by making use of blocks or tiles of two colors which are arranged to form various patterns.

Stern, Catherine, "The Concrete Devices of Structural Arithmetic," The Arithmetic Teacher, 5:119-30, 1958.

The teaching of structural arithmetic is discussed with explanations of how various models and devices can be used to allow students to discover the structure of our number system and its interrelationships.

University of Illinois Committee on School Mathematics, "Arithmetic with Frames," The Arithmetic Teacher, 4:119-24, 1957.

This article gives some ideas of the U.I.C.S.M. project staff concerning the informal teaching of some fundamental ideas in mathematics at the elementary level.

## INTERMEDIATE GRADES (AGES 9 - 11)

### Probability and Statistics

Burt, Bruce, "Drawing Conclusions from Samples, (An Activity for the Low Achiever,)" The Arithmetic Teacher, 16:539-41, 1969.

The author presents appropriate sampling activities which can be used in the intermediate grades. He also gives a short list of questions which teachers of probability at this level might find useful.

Grass, Benjamin A., "Statistics Made Simple," The Arithmetic Teacher, 12:196-8, 1965.

The author tells how his students made use of some simple statistics in his fourth grade class to determine whether some students had extra sensory perception.

Wilkinson, Jack D., and Owen Nelson, "Probability and Statistics--Trial Teaching in Sixth Grade," The Arithmetic Teacher, 13:100-6, 1966.

The article describes an experiment, and the authors make a number of recommendations concerning their experiment, which some teachers will find applicable to teaching low achievers.

### Measurement

Smart, James R., and John L. Marks, "Mathematics of Measurement," The Arithmetic Teacher, 13:283-87, 1966.

The article discusses some of the fundamental mathematics of measurement. In addition, there are some examples which illustrate methods for teaching these topics.

### Calculation

Mayer, Louise A., "The Scarbacus or Scarsdale Abacus," The Arithmetic Teacher, 2:159, 1955.

This one-page article describes a student-made counting device.

Travers, Kenneth J., "Computation: Low Achiever's Stumbling Block or Stepping Stone," The Arithmetic Teacher, 16:523-8, 1969.

The author presents a plea to de-emphasize computation as well as a brief history of the mathematics laboratory movement in England. The article includes a discussion of the use of calculators, and computers for low achievers.

Winthrop, Henry, "Arithmetical Brain-Teasers for the Young," The Arithmetic Teacher, 14:42-3, 1967.

A number of brain teasers appropriate for elementary pupils (as well as others) are enumerated.

Yates, William E. "The Trachtenberg System as a Motivational Device," The Arithmetic Teacher, 13:677-8, 1966.

Yates justifies the use of the Trachtenberg speed system of mathematics as a motivational device for slow learners as well as for all other ability levels.

### Arithmetic

Green, Roberta, "A Color-Coded Method of Teaching Basic Arithmetic Concepts and Procedures," The Arithmetic Teacher, 17:231-3, 1970.

The author describes a method which she found helpful in teaching a fifth grade boy severely retarded in arithmetic achievement. The procedure described helped the boy to understand and master basic arithmetic concepts and processes.

Haines, Margaret, "Modular Arithmetic," The Arithmetic Teacher, 9:122-6 1962.

A fairly simple introduction to modular arithmetic is given. Some useful arithmetic checks on addition, subtraction, multiplication and division are explained using modulo 9 arithmetic.

Le Ganke, Lucille, "Let's Use Our Checkers and Checkerboards to Teach Number Bases," The Arithmetic Teacher, 14:573-5, 1967.

Various aspects of teaching number bases by using a checkerboard and checkers are presented. The material might be used to motivate students.

Livingstone, Isobel L., "Live Models in Arithmetic?" The Arithmetic Teacher, 17:81-2, 1970.

The author describes a method which she has found useful in the intermediate grades for putting some "life" in her arithmetic classes by getting the students involved.

Lyda, W. J., and Margaret D. Taylor, "Facilitating an Understanding of the Decimal Numeration System through Modular Arithmetic," The Arithmetic Teacher, 11:101-3, 1964.

The article describes lessons taught by the authors to determine whether changes occur in understanding the decimal numeration system when pupils receive instructions in modular arithmetic.

Prielipp, Robert W., "Finger Reckoning," The Mathematics Teacher, 61: 42-3, 1968.

The reader is introduced to how to multiply by using his fingers. The author refers to Dantzing and Richardson for additional reading on this topic. (c.f. the list of references at the end of the article).

### Geometry

Fennell, Francis M., "Multiplication Football," The Arithmetic Teacher, 17:236-7, 1970.

A very simple motivational device is described which the author found useful in reviewing basic multiplication facts to intermediate grade children. The game could be modified for use at other levels.

Froelich, Effie, "An Investigation Leading to the Pythagorean Property," The Arithmetic Teacher, 14:500-4, 1967.

Froelich describes some intuitive approaches for obtaining background notions related to the Pythagorean theorem. No formal proofs are required.

Smith, Lewis B., "Pegboard Geometry," The Arithmetic Teacher, 12:271-4, 1965.

Smith describes some activities which can be performed by making use of a geoboard. The activities are designed in part to help develop the student's intuition concerning geometric shapes.

Teegarden, Donald O., "Geometry via T-Board," The Arithmetic Teacher, 16:485-7, 1969.

The author describes the use of a so-called "T-Board" which he successfully used to teach perimeter and area concepts to fifth and sixth grade classes.

Wells, Peter, "Creating Mathematics with a Geoboard," The Arithmetic Teacher, 17:347-9, 1970.

Suggestions for use of the geoboard with lower grade children are given. The author suggests some possible discussion questions for use with the geoboard.

### Percentage

Hauck, Eldon, "Concrete Materials for Teaching Percentage," The Arithmetic Teacher, 1:9-12, 1954.

The author discusses two difficulties in teaching percentage. He explains how he makes use of a "percentage box" to help in overcoming these difficulties.

## Recreation

Cook, Nancy, "Fraction Bingo," The Arithmetic Teacher, 17:237-9, 1970.

A game is described which was used to help fourth, fifth, and sixth graders to better understand fractions and discover how to find other names for a fractional number.

Dohler, Dora, "The Role of Games, Puzzles and Riddles in Elementary Math," The Arithmetic Teacher, 10:450, 1963.

The article is a description of games, puzzles, and riddles which are appropriate for elementary pupils.

Hunt, Martin H., "Arithmetic Card Games," The Arithmetic Teacher, 15:736-8, 1968.

Directions on how to play five card games whose purpose is to enhance the pupils' facility with number facts are presented. The games are war, fish, number trick, number scrabble, and number rummy. Some of these games might also be appropriate for primary children.

Jennings, Thomas J., "The Magic Box," The Arithmetic Teacher, 12:377-8, 1965.

The author suggests a way to teach elementary pupils about number patterns. The magic box referred to is what some textbook writers call a function machine.

Parker, Helen, "Seesaw Game," The Arithmetic Teacher, 10:449, 1963.

The "Seesaw Game" is used to help students develop facility in the four basic operations. Careful selection of content could make the game appropriate for almost any grade level.

Ruderman, Harry D., "The Greatest -- A Game," The Arithmetic Teacher, 17:80-81, 1970.

The author describes a game which involves the random selection of numbers and is appropriate for children in the elementary grades. Four variations of the game are also included.

Willerding, Margaret F., "Codes for Boys and Girls," The Arithmetic Teacher, 2:23-4, 1955.

This article describes how ciphers can be used in the elementary classroom as a worthwhile activity. Two examples of ciphers are given although their "answers" are not included.

Winick, David F., "'Arithmecode' Puzzle," The Arithmetic Teacher,  
15:178-9, 1968.

The article is a description of a decoding exercise which involves the correct solution to a number of mathematics problems. The example given could be modified for use at almost any difficulty level.

### Problem-Solving

Schaeffer, Anne W. and Albert H. Mauthe, "Problem Solving with Enthusiasm--the Mathematics Laboratory," *The Arithmetic Teacher*, 17:7-14, 1970.

The authors describe some of their experiences with a mathematics laboratory in the fifth grade. They include ten problems (well documented) which they use with children in the intermediate grades. They also include some children's reactions to the laboratory.

## JUNIOR HIGH

### Fractions

Archbold, John C., "Measuring with Maps," The Arithmetic Teacher, 14:393-5, 1967.

The article contains ideas for utilizing maps and bulletin boards to help in teaching certain mathematics concepts.

Braunfeld, Peter and Martin Wolfe, "Fractions for Low Achievers," The Arithmetic Teacher, 13:647-55, 1966.

A brief overview of U.I.C.S.M.'s "stretcher-shrinker" ideas for teaching fractions to culturally disadvantaged seventh graders.

Junge, Charlotte W., "A Game of Fractions," The Arithmetic Teacher, 13:494, 1966.

Describes a fraction game which can be played by two, three, or four children.

Zytkowski, Richard Thomas, "A Game with Fraction Numbers," The Arithmetic Teacher, 17:82-3, 1970.

The article describes an unusual way in which students can acquire practice in the basic operations with fractions. The author says the approach can also be used to teach ratio and sequence of numbers.

### Integers

Fremont, Herbert, "Pipe Cleaners and Loops -- Discovering How to Add and Subtract Directed Numbers," The Arithmetic Teacher, 13:568-72, 1966.

A novel way of helping students acquire facility in adding and subtracting signed numbers is explained using pipe cleaners.

### Geometry

Hart, Alice G. "The Angle Mirror Indoors," The Arithmetic Teacher, 17:419-23, 1970.

Describes a device for use in laboratory activities involving measurement.

Liedtke, Werner, "What Can You Do with a Geoboard," The Arithmetic Teacher, 16:491-3, 1969.

Liedtke suggests a number of pegboard activities in the areas of graphs, distance, perimeter, area, multiplication division, fractions and networks.

Woodby, Lauren G., "The Angle Mirror Outdoors," The Arithmetic Teacher, 17:298-300, 1970.

The article describes how to construct an angle mirror as well as how to use it. Some suggestions are given on how the angle mirror might be used by the students outdoors.

### Patterns

Edmonds, George F., "Discovering Patterns in Addition," The Arithmetic Teacher, 16:245-8, 1969.

Edmonds explains how he used a guided discovery approach to help primary teachers in Nairobi to find certain number patterns in addition. The patterns developed could be of interest to teachers of grades 3-8 and perhaps even beyond.

Peterson, John A., and Joseph Hashisaki, "Patterns in Arithmetic," The Arithmetic Teacher, 13:209-12, 1968.

The authors present quite a few number and word patterns which almost any junior or senior high teacher could find useful as supplementary or interest material.

Tinti, Robert, "Mathematics Through Cardboard Carpentry (A Unit for Low Achievers)," The Arithmetic Teacher, 17:209-10, 1970.

The article sites some of the mathematical value of such a unit for low achievers. A summary of six advantages resulting from the unit of study are listed. NOTE: For further information on materials and tools used in cardboard carpentry, write to:

Cardboard Carpentry  
Educational Development Center, Inc.  
55 Chapel Street  
Newton, Massachusetts 02160

Wahl, M. Stoessel, "'We Made It and It Works!' the Classroom Construction of Sundials," The Arithmetic Teacher, 17:301-4, 1970.

Rather detailed instructions are given for constructing a sundial. This activity provides a good opportunity for teaching some elementary geometry of the circle.

White, Donald E., "An Approach to Modern Mathematics through Pascal's Triangle," The Arithmetic Teacher, 10:441-45, 1963.

The article tells how eighth grade students were introduced to Pascal's triangle via coin-flipping exercises.

## Algebra

Biggs, Edith, "Trial and Experiment," The Arithmetic Teacher, 17: 26-32, 1970.

Miss Biggs gives quite a number of suggestions for teachers on how they might initiate activity learning in the areas of time, area, number patterns, along with some others, in their classroom. Some helpful hints are also given for the teacher who wishes to introduce activity learning in their classroom.

Getty, G. A., "Uncle Ed Juggles with Figures," The Mathematics Teacher, 48:143-46, 1955.

The article describes a mathematical "stunt" performed with two arrays of digits. Probably of most interest to junior and senior high students.

Travers, Kenneth J., "Through Clouds of Failure into Orbit," The Arithmetic Teacher, 15:591-98, 1968.

The article discusses the need for teaching mathematics to all students, and appeals for a change in approach. There is also a section on teaching the concept of variable.

## Number Theory

Davies, Robert A., "Low Achiever Lesson in Primes," The Arithmetic Teacher, 16:529-32, 1969.

The author presents an unusual way, based upon the U.I.C.S.M. stretcher and shrinker concept, of having low achievers "discover" the primes between 1 and 100.

Loftus, Sonja, "Fibonacci Numbers: Fun and Fundamentals for the Slow Learner," The Arithmetic Teacher, 17:204-8, 1970.

The author describes how she used Fibonacci and Lucas numbers to stimulate interest in her seventh grade low-ability arithmetic class. Some specific teaching suggestions are given for this topic.

Morton, R. L., "Divisibility by 7, 11, 13, and Greater Primes," The Mathematics Teacher, 61:370-4, 1968.

This article presents the mechanics of and the rationale for the divisibility tests described in the title of the article.

## Application

"A Job Survey as Class Motivation in General Math," The Mathematics Teacher, 43:318-20, 1950.

The article presents a description of how students were led to conduct a job survey, and how it affected their work during the year.

Bragdon, Wallace M., "Rapid Mental Calculations," The Arithmetic Teacher, 12:369-70, 1965.

The author presents methods for multiplying whole numbers between 1 and 100 mentally.

Gibb, E. Glenadine, "Mathematics in Baseball," The Mathematics Teacher, 45:35-6, 1952.

Provides suggestions as to how mathematics related to baseball can be used to stimulate the interest of intermediate grade students.

Holtan, Boyd, "Motivation and General Math Students," The Mathematics Teacher, 57:20-5, 1964.

Description of a project to improve mathematics achievement by presenting material in contexts which were interesting to general mathematics students.

Pyatt, Gladys, "Field Project in Junior High Math," The Mathematics Teacher, 38:327, 1945.

Teachers interested in doing field work with their classes will find this article valuable.

#### Recreation

Bradfield, Donald L. "Sparkling Interest in the Mathematics Classroom," The Arithmetic Teacher, 17:239-42, 1970.

The author makes a point for introducing comedy, mystery and drama into the mathematics classroom via certain types of problems. Several examples of problems are given which the author feels will accomplish the above. Answers are given in some cases.

Cappon, John, Sr., "Easy Construction of Magic Squares for Classroom Use," The Arithmetic Teacher, 12:100-5, 1965.

Teachers will find this helpful in preparing magic squares. See other articles in this section for use of these squares.

Condron, Bernadine F., "Game to Review Basic Properties and Vocabulary," The Arithmetic Teacher, 12:227-8, 1965.

Some of the vocabulary or properties reviewed include the following: addend, factor, reciprocal, identity element, distributive and commutative properties.

Hewitt, Frances, "4 x 4 Magic Squares," The Arithmetic Teacher, 9:392-95, 1962.

The title fairly well describes the article. A brief bibliography is given as well as a magic square whose numbers are congruent, modulo 8.

Janicki, George, "Bizz-Buzz Game in Arithmetic," The Arithmetic Teacher, 3:28, 1956.

The author describes a very simple game which he used with 5th, 6th and 7th grade students to test their knowledge of the multiplication tables.

Janicki, George, "Cross Figure Puzzle," The Arithmetic Teacher, 3:16, 1956.

This is a cross word puzzle involving numerals instead of words.

McCombs, Wayne E., "Four-by-Four Magic Square for the New Year," The Arithmetic Teacher, 17:79-80, 1970.

The author describes a way to construct four-by-four magic squares that will total any desired number.

Nies, Ruth H., "Classroom Experiences with Recreational Arithmetic," The Arithmetic Teacher, 3:90-3, 1956.

The article gives a description of a number of games and puzzles which she found useful with her sixth grade class.

Ruderman, Harry D., "Nu-Tic Tac Toe," The Arithmetic Teacher, 12:571-2, 1965.

The article gives directions for playing a modified version of the Tic Tac Toe. Rules can be learned, according to the author-inventor, in less than one minute.

Timmons, R. A., "Tic Tac Toe--A Mathematical Game for Grades 4-9," The Arithmetic Teacher, 14:506-8, 1967.

A modification of a familiar game is explained. The modified tic tac toe can be played by a whole class and involves some knowledge of how points are plotted on a rectangular coordinate system.

Willerding, Margaret F., "Cross-Number Puzzles," The Arithmetic Teacher, 4:221-223, 226, 1957.

These are cross word puzzles which involves numerals instead of words. The first puzzle is appropriate for primary grades, the second one for intermediate and the third for junior high school.

Williams, Russell, "Bingtac," The Arithmetic Teacher, 16:310-11, 1969.

A description of the equipment and rules needed to play this game is given. The author used the game at the junior high level.

## HIGH SCHOOL

### Fundamental Operations

"Concerning the Teaching of 2nd Track Math," The Mathematics Teacher, 44:547-41+, 1951.

The author lists some assumed characteristics of 2nd track pupils. He states a philosophy and describes the idea characteristics of the teacher. Suggestions on methods, classroom management and homework are given.

### Algebra

Greenholz, Sarah, "Successful Practices in Teaching Mathematics to Low Achievers in Senior High School," The Mathematics Teacher, 60:329-35, 1967.

This article reiterates some of the things Greenholz mentions in her article in the December, 1964 The Mathematics Teacher. A number of appropriate problems for low achievers in high school as well as some specific teaching techniques are enumerated.

Niechin, Arlene and Robert Brower, "The Abacus--A New Use for an Old Tool," The Arithmetic Teacher, 6:314-16, 1959.

The authors describe how they made use of the abacus in their sixth grade class. Several activities with the abacus are explained as well as some benefits reaped by the students.

### Geometry

Bauer, Marie L., "Projects for Plane Geometry," The Mathematics Teacher, 44:235-9, 1951.

The article contains a large number of topics suitable for projects or written or oral reports. Many of the topics are appropriate for low achievers and could help to stimulate their interest in math. A bibliography is also included.

### Applications

Bell, Clifford, "What Every Teacher Should Know about the Uses of Mathematics," The Mathematics Teacher, 56:302-6, 1963.

The article presents a summary of what mathematics courses will be necessary or helpful in field of higher learning which students

may choose after their high school graduation. The article should be of particular interest to teachers and counselors.

Lankford, Francis G., Jr., "Practical Math is Challenging to Students," The Mathematics Teacher, 48:401-5, 1955.

The author presents a case for the use of practical mathematics to stimulate students.

Morse, Jarvis M., "Math Teachers Train Thrifty Citizens," The Mathematics Teacher, 43:328-31, 1950.

This article explores many possible areas for the use of mathematics in finance and buying.

Myers, Sheldon S., ed., "Drawing a Circle with a Carpenter's Square," The Mathematics Teacher, 45:367, 1952.

The problem presented in this very short article would be an appropriate laboratory exercise.

Parker, Jean, "The Use of Puzzles in Teaching Mathematics," The Mathematics Teacher, 48:218-27, 1955.

A list of several factors justifying the use of puzzles in the classroom. Quite a number of puzzles and games are presented for use primarily at the high school level. A rather extensive bibliography is presented.

Peck, Lyman C., ed., "Pharmaceutical Arithmetic in the Home," The Mathematics Teacher, 46:438-39 plus, 1953.

The enterprising teacher could do quite a bit with the material of this section.

Stephenson, E. C., "Math in Retail Business," The Mathematics Teacher, 46:18-22, 1953.

This article is dated by now. In it the author is stressing the importance of computational skills for high school graduates. A number of statistics concerning the J. L. Hudson Company of Detroit, Michigan are given.

Weidemann, Charles, "Speed of Travel in Covered Wagon Days," The Mathematics Teacher, 46:101-2, 1953.

Use of "historical approach" in presenting a topic in an interest-context.

## Recreation

Adkins, Bryce E., "Adapting Magic Squares to Classroom Use," The Arithmetic Teacher, 10:498-502, 1963.

The author presents two ways of adapting magic squares for classroom use. For the reader who may not have mastered these patterns.

Carnahan, Walter, "If a Cricket Chirps One Hundred Times a Minute, How Fast Does an Ant Run?" The Mathematics Teacher, 45:58, 1952.

A very simple little problem which could be used as an attention getter at almost any level from junior high on up.

Gorts, Jeannie, "Magic Square Patterns," The Arithmetic Teacher, 16:314-16, 1969.

The author describes some magic square patterns which are useful in construction of magic squares.

Keepes, Bruce D., "Logic in the Construction of Magic Squares," The Arithmetic Teacher, 12:560-2, 1965.

Keepes makes use of some applications of logic and of our numeration system in constructing magic squares.

Schiele, Gerald D., "A Three-by-Five Card Plus an Opaque Projection Plus an Ice Pick," The Arithmetic Teacher, 16:533-35, 1969.

The author describes a moral teaching technique which he found useful in teaching remedial mathematics.

Andree, Richard V., "Cryptography as a Branch of Mathematics," The Mathematics Teacher, 45:503-9, 1952.

This article describes how ciphers could be used to advantage in the mathematics classroom especially as a motivating device in general mathematics classes. Examples of ciphers are given as well as a rather extensive bibliography to those interested in this topic.

