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ABSTRACT

This booklet augments earlier reports describing projects in mathematics for the educationally disadvantaged in New York State. Actual lesson preparation and classroom implementation are stressed. Projects represented include Mathematics Pilot Program, Individually Prescribed Instruction, and Plus Program. Other lessons concern cognition and learning, discovery learning, symbiotic learning and mathematics skills instruction. (Author/CT)

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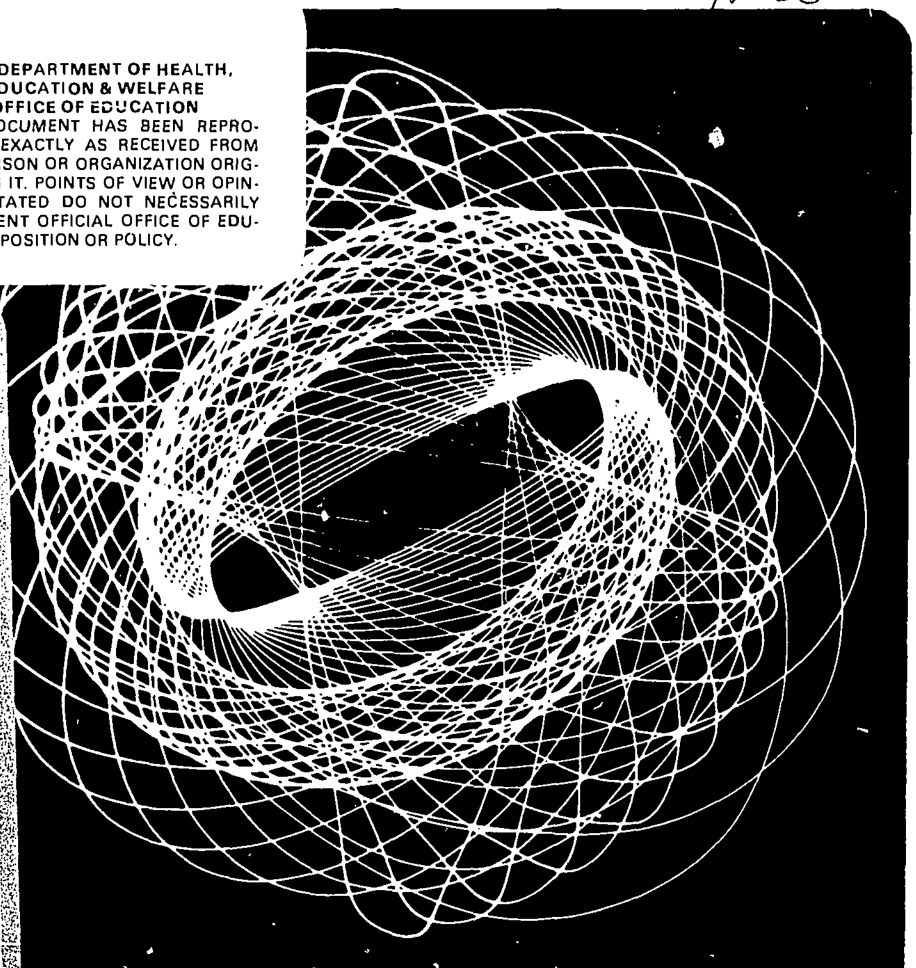

Mathematics Education

Descriptive Lessons for Problem Learners

The University of the State of New York/The State Education Department
Division of Education for the Disadvantaged/Albany, New York 12224

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PREFACE

Over five years have passed since the enactment of the Elementary and Secondary Education Act of 1965. Its purpose still remains: "...to provide financial assistance to local educational agencies serving areas with concentrations of children from low-income families to expand and improve their educational programs by various means which contribute particularly to meeting the special educational needs of educationally deprived children."

In the past, publications have been written to give those interested in mathematics education for the disadvantaged a brief overview of projects throughout the state, *25 Mathematics Education Programs, Volumes 1 & 2,* and background in educational and organizational procedures in working with the disadvantaged, *Mathematics Education and the Educationally Disadvantaged.*

This publication stresses the actual lesson preparation and/or implementation of several projects throughout the State. They are offered as examples as to what takes place in the classroom setting.

The publication was originally planned by Melvin Mendelsohn, then an Associate in Mathematics Education. This booklet was prepared and edited by Lynn A. Richbart, Associate in Mathematics Education under the supervision of Frank Hawthorne, Chief, Bureau of Mathematics Education and by William G. Brenton, Associate in Education of the Disadvantaged.

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A VIEW OF COGNITION AND LEARNING

Robert R. Kirsch
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Introduction

In recent years many reports, guideline papers, researches, and speeches have concerned themselves with descriptions and definitions of the educationally disadvantaged child. While there has not emerged a definition as concise and exact as the definition of, let us say, the set of all even integers between 1 and 2, there are several major areas of consensus.

Most reports agree that the lack of verbal stimulation in the home results in the child's lack of adequate language facility in coping with the educative process. Evidence of this weakness is reported by teachers in many ways; short attention span, an inability to cope with abstractions, physical reactions (acting out) to stress situations, lack of response.

Although mathematics is often considered to be nonverbal in development, the *teaching* of mathematics is usually highly verbal, and often far more abstract than the so-called verbal areas. As educators concerned with the mathematics learnings of disadvantaged children, we must concern ourselves with their listening skills, their methods of conceptualization, and their skills in verbalization. Those who teach mathematics, and in particular, those who teach mathematics to young children, must concern themselves with verbalization as an integral part of the child's mathematical development. In particular, this concern applies to teaching "modern" mathematics to disadvantaged children. For while it is true that the "modern" programs are logical, conceptual, and developmental, they are also highly verbal; dependent on verbal exposition of content and concept as an aid to the child's mathematical development.

Two other areas of consensus in the description of the disadvantaged child, both again related to his environment, are a lack of structure in his activities and thinking, and a marked "present-time" temporal orientation. The two are not distinct; they are intimately interrelated.

The organization of mathematics and mathematical thinking is highly dependent on temporal cognition in that the development of number concept is based on past experience and projected to future activity. How often has the mathematics teacher, even in a child's earliest mathematical experiences, said: "if ... then;" a temporal conjecture.

Much of the child's temporal sense and organizational structure are developed in the home at an early age. Imagine, if you will, the parallel occurrences in the middle class home and the disadvantaged home when the mother is involved in talking with someone else and the small child interrupts. Generally, in the middle class home the mother will stop, tell the child not to interrupt, and follow through with "...and when I finish we will etc., etc." As a contrast, observers of the culture of poverty report that in this situation the child is silenced, often physically, with no verbal interaction, or at best, with an authoritarian word or two. Here is an example of the temporal base, the "*if* you wait now *then* we shall..." as contrasted to the dead ended "shush." Teachers of mathematics must look for new techniques to use in coping with these differences in structure and temporal orientation.

Many of the characteristics of the disadvantaged child, as described by teachers, sociologists, and psychologists, have direct pertinence to the attitudes of teachers and their techniques of instruction. Frequently we find that acceptable methodology is in direct conflict with the characteristics we describe:

1) Children do not have adequate language....
   "Tell me how you arrived at that answer."
2) These young people lack the motivation to learn....
   "Don't you want to get a good mark on the test?"
3) These children have low self-concept....
   "Why don't you understand what I am saying."
4) These children have a fear of being challenged....
   "Let's go over what we've learned. When I call on you, answer quickly."
5) These children have many feelings of guilt and shame...
   "Raise your hand if you have the right answer."
6) They usually show apathy and a lack of response....
   "You must have *some* answer to my question."
7) They generally have a low standard of conduct....
   "How do you expect to learn if you don't behave?"

In isolated programs across the Nation, there is evidence of action which deals directly with the mathematical learning of disadvantaged children. At a recent conference which launched an experimental program in calculator-assisted mathematics for nonachieving students, the following instructions for teachers were related to the learning characteristics of the children:

1) Establish good rapport with the student.... make him understand that you are sincerely interested in his welfare.
2) Easy success is of key importance... build the student's self image.
3) Keep units of work short.... be aware of the difference in temporal sense.
4) Keep activities varied during any single class session.
5) Use imaginative materials and techniques.... try to find motivations.
6) Use concrete, manipulative materials as often as possible .... avoid overabstraction.
7) Provide a number of learning opportunities in the same lesson through seeing, hearing, manipulating and dramatizing.
8) Keep instructions simple and direct.... avoid oververbalizing.
9) Never put the student "on the spot".... know what may embarrass him and avoid it.
10) Foster verbalization... do not force verbalization.
11) Incorrect answers often reveal more than correct ones.... be receptive and willing to discuss the rationale for an incorrect answer.

As teachers and administrators concerned with the education of disadvantaged children, it is important that we be willing to try the untried, accept failure (ours or the students'), and try again.

For those of us concerned with the teaching of mathematics, and with the concurrent *learning* of mathematics by children, the message is clear. We must determine the specific learning characteristics of disadvantaged, and devise programs and methodology which will capitalize on the *strong* points of these characteristics.

Implied here is a challenge to the researcher, the learning theorist, and the master teacher to spell out the "learning styles" of groups of children with common characteristics and to provide, for all teachers, information to help them in the important task of teaching mathematics.

MATHEMATICS PILOT PROGRAM

Harold E. Switts, Assistant Supervising Principal
and
Joseph Horan, Coordinator
Washingtonville Central School
Washingtonville, N. Y.

The Mathematics Pilot Program emerged after much debate and deliberation on the part of the school administration and selected teachers who saw the need for such a program. Basic guidelines for the project were developed and an inservice program geared to accent the need, was offered to all teachers of grades 1-8. The underlying purpose of this inservice program was to: (a) gain clearer understanding of the modern mathematics program and (b) outline our thinking as to identification of educationally disadvantaged students. After three months of intensive programming, it became evident that educationally disadvantaged students seemed to be those:

a) who had a poor self-concept

b) whose environment produced little stimuli to make him educationally adaptable

c) who feared challenge and were willing to get by rather than achieve

d) who had a mistrust of school and most of its affairs

e) who had a negative attitude toward grades

f) who lacked the ability to associate learning experiences

g) who lacked abstract concepts and failed even rote learning

Past experience had demonstrated that remedial effort in isolation with these students was not particularly productive. It was obvious that two other factors must also play a role in educating these students, namely, the regular classroom teacher and the parent. Therefore, a three-pronged program was suggested.

STUDENTS — The student selected for the program would be involved daily in a resource center wherein he could investigate and grasp mathematical concepts through multimedia materials.

PARENTS — The involvement of the parents was approached from several aspects since it was learned that many parents of educationally disadvantaged students often feel ashamed or inadequate. Therefore, the approaches used were:

a) An open house meeting wherein all the parents of these students had the opportunity to understand the general aims of the program and view the equipment used.

b) Each parent was invited to visit the specific group his child attended. In this way, they could understand the "smallness" of the group, its strong impact, and its workability.

c) Private interviews were held for those parents who requested it so that no parent would feel put upon in a group setting.

d) Adult evening classes were held in which the parents were taught mathematical concepts, and specific problems of a general nature were discussed. In this way, the parent was better equipped to understand, coordinate, and cooperate in responding to his child's specific needs. They possessed firsthand knowledge.

e) Several members of the community (who also had children in the program) were selected as liaison aides. The parents were introduced to these people and were able to contact them for specific help.

TEACHERS — It was felt that to be effective, the classroom teachers who had these students in the regular classroom should have first hand knowledge of the program. They were invited to view and use the materials in the resource center and have the opportunity to see their students participating in the program. They were also expected to keep pertinent data that might help the resource teacher to better determine the growth of the students involved. Therefore, the term, "continual workshop" was used to inform the classroom teacher and keep the teacher actively involved in the project.

The resource teacher and the classroom teacher met periodically to discuss the specific and general attitudes of the student involved. One important factor was school attendance. We discovered that this program increased the attendance percentage for those who had previously been frequently out of school.

PROCEDURE — Since most of the teachers were originally involved in the planning stages, selection of children was not a major problem. Each teacher referred students according to a guideline which contained specific data:

a) Reading and arithmetic levels according to the New York State Tests.
b) Standardization test results from Iowa Test of Basic Skills, STAT and STEP.
c) Individual as well as group IQ scores.
d) A stanine report in the area of mathematics. Students were chosen from the 1-3 stanines.
e) Each teacher added his personal comments concerning each student and specific areas of weakness observed.

From the 200 students referred in grades 1-8, 85 were chosen. Each student was given the California Achievement Test to establish a comparison or consistency, and develop a diagnostic profile. Students in grade one were given an ITA mathematics test, using the California Achievement Test as a guideline, since they were involved with an ITA reading program. This test was devised by the teachers involved in the program. When this information was gathered, the students were grouped according to their specific needs and difficulties. This necessitated mixing age and grade levels which seemed to enhance the tutorial concept. Group size ranged from 4 to 8 students.

Since this program was geared basically to a resource center, no formal textbook approach was used. All mathematical concepts were to be taught through the use of multimedia materials and manipulative devices. Investigation and self-discovery coupled with discussion was used to bring out the inner curiosity of each student.

LESSON ILLUSTRATION — At the outset of the week, the teacher would discuss a mathematical concept with a general discussion by the group of their thoughts about the concept. This information was pooled and the teacher carefully noted the different responses of each student. Throughout the week, the students would then be taught the concept through:

a) individual filmstrip viewing that would expand and clarify the concept;
b) listening to tape recording, made by the teacher, that approached the concept from many viewpoints;
c) actual investigation and construction of the concept by the students;
d) physical involvement with objects to reinforce the concept.

Students set up examples of the concept being taught and tried to determine how it related to other concepts. Gradually but persistently, students sought to design and solve pupil constructed problems.

At the end of the week, the concept was reviewed by the teacher through the use of the flannel board and physical models. Each student offered his new viewpoint on the subject. Students would often play the role of teacher and present the concept to the rest of the group.

A TYPICAL LESSON ON TEACHING — "WHAT IS A FRACTION?"

The teacher would start off with a general discussion of "What is a fraction?" pooling the resources of the students. The general definition, "It's part of a whole," would then be discussed from the aspect: (a) What is meant by a whole? (b) What is meant by a part? Illustrations would be asked of students such as:

1. whole group being broken up into smaller ones;
2. students putting objects on the table and then dividing them into parts;
3. a ruler to show its subdivisions;
4. using different objects in the room to get over concept that whole does not have to be the same size;
5. using every day objects such as egg cartons, jars, measuring cups, etc.

The next approach would be to show filmstrips to the group. Here the concentration would be on, "How a fraction gets its name such as 1/3, 2/6, etc." The numerator and denominator concept would thus be reinforced. Following this, students would then construct posters or develop examples of fractions from everyday use.

Use of the tape recorder, with each student having his own earphones, enables each child to work independently on the concept of fractions. This listening device would help him concentrate on following direction. A taped lesson might ask students to construct visual displays of specific fractions in terms of their meaning.

The "game technique" was used whereby the group was divided into teams and each team sought out the *best* expression or illustration of specific fractions. Often, teams designed problems to be solved by other teams. Scoring reinforced the concept of fractions — i.e. — total points needed to win (whole amount) and adding each point gained (fractional part of whole).

At the end of the week, students would try to see relationships that exist among fractions, and discover ways to use the understanding in daily living—cooking, mechanics, etc. This was a "Now that we have learned this what can we do with it" approach.

Goals To Be Accomplished Through the Continual Teachers Workshop and the Evening Classes for Parents

Since the prime goal of the project was to reduce student deficiency in mathematics, we needed to design a procedure which would alert us to changes in the student when he was outside the small group. The classroom teacher had constant use of the resource center and was exposed to the procedures and materials being used in teaching the mathematical concepts to the students. She then had a good picture of her students when dealing with them in regular class. The teacher's awareness of the student's progress in the program acted to guide the teacher in everyday dealings with the student. Of prime importance were the student's attitude and behavior. The classroom teacher was asked to interpret the student's actions and share the interpretations with the resource teacher. The classroom teacher and the resource teacher discussed the needs of these students in terms of behavior, attitude, and skill development to coordinate their programs.

The evening classes involved the parents and had a twofold mission: (1) to grasp the mathematical concepts that were being taught to their child; (2) to understand his special needs and difficulties. The evening program produced interest on the part of the parents. They became interested and concerned for they were learning in the same fashion as their child. The multimedia equipment was used with the parent group and its functions and purposes were discussed in order to develop an understanding of the benefit their child was to gain through the use of such equipment.

In addition, the child could discuss his problems at home with a knowledgeable parent who understood and could assist the child's learning. Understanding general areas of weaknesses and why students failed also give the parent insight and tended to improve the parent-child relationship.

Since no child in the program received a grade in mathematics, both the parent and the classroom teacher had to be involved. They had to know where the child was, mathematically, and this called for coordination. Therefore, the resource person, classroom teacher, and parent met frequently to review the progress of the child. This continual, "open line communication" or "the three-pronged approach," assured the child that he was a most acceptable human being but, like most people, occasionally needed assistance. The child, knowing his parent and teacher were involved, began to understand and appreciate the fact that he could be deficient in certain studies and still be an adequate person.

THINKING LEADS TO DISCOVERY

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The West Islip Summer Clinic is a combination of reading and mathematics progress for children who require assistance to become more proficient in these areas. The mathematics program is built around two sessions of intensive, direct, and small group instructional periods. The child attends these sessions for approximately 6 weeks.

A host of mathematical innovations has been devised and a major portion of each period involves the use of mathematical games for developing generalizations to be utilized later in learning mathematics.

The teachers in the clinic have found that the mathematical games are highly motivational. The children like games — games are "fun". If games then are associated by contiguity, mathematics, too, may be considered fun. The games tend to change the attitudes of the students, and it is generally assured that one learns more easily that which one likes.

There appear to be three major transfer theories in vogue today. Thorndike's theory of transfer by identical elements states that training in one activity influences another activity only insofar as the two have elements or aspects in common. Judd's theory of transfer by generalization states that transfer is due to generalizations or to general principles built up in one activity and utilized in the other. The transposability theory of the Gestaltists says that a pattern is composed of identical components and that the whole pattern, rather than its parts, is what is important in transfer. Many psychology texts which discuss transfer agree that we can neither completely reject nor completely accept any one of the theories. Studies have shown each theory valid in specific instances in which it was applied.

If we accepted only the identical elements theory, we could not make much of a case for using games to teach mathematics, as there are very few elements common to both a game situation and an abstract mathematical situation.

Judd's generalization theory and the Gestaltist transposability theory seem to be more in harmony with the idea of using games to teach mathematical concepts. Children should be able to use the generalizations, general principles, or patterns built up in game activities in abstract mathematical activities.

Inherent in the use of games in teaching arithmetic or mathematics is the assumption that there is a transfer from the game situation to relevant experiences within the curriculum. The transfer of learning theory which is accepted, determines the extent to which one will utilize games for instruction.

As teachers, we must be sure we have a definite idea of what generalization we wish the children to acquire. We then can choose certain games, the playing of which leads to those generalizations we desire.

There are very few supporting research studies in the area of transfer of training in mathematical game situations. Perhaps the reason for this scarcity of research evidence is that there are so many hard-to-control variables (such as teaching methods; previous experiences of students; and student attitudes toward teachers, mathematics, and learning in general). There are, however, a number of writers who offer opinions on the discovery method of teaching, using games (in general) in teaching, and using specific games in teaching mathematics.

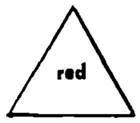
Example of a Partial Lesson Description

This is one of the game exercises used with various groups of children in the clinic.

Every child in the group is provided with a transparent page protector with a white backing sheet and a red, blue, and gold grease pencil. During the period, each child may be responsible for as many as 75 responses using the protector and the grease pencil. When a child has marked his answer, he holds up the protectors so that the teacher can see the response. A tissue will clear off responses so the children can continue.

The children face the blackboard initially and discuss triangles, squares, and circles of different colors (red, blue, yellow). The teacher then puts a red triangle on the blackboard and asks the pupils to draw something on their protectors that has one thing different from the drawing on the blackboard. The children respond very quickly. Some produce a blue or gold triangle, others a red square or a red circle. The answers are all discussed as to the reasons why each was chosen. Other teacher beginnings are also performed. The teacher has an opportunity to get a response from each child and note those children who demonstrate difficulty in these tasks.

Later, the teacher will progress to problems such as:

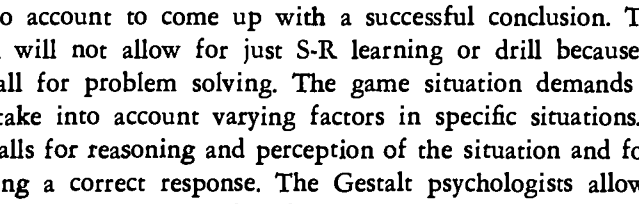


One thing different ....
Two things different ....

Size as a dimension is added later to find things that are three things different or differentiate from each other in three ways.

More thinking and abstracting areas are later developed in an exercise such as this:

Teacher: Draw from those shapes and colors that we are using for this game, something that is one thing different from the object I draw then another, that is one thing different from what you drew, and two things different from what I have drawn. Examples could include:



Many other types of games are developed and utilized.

The game situation calls for insight. The learner must take several facts into account to come up with a successful conclusion. The game situation will not allow for just S-R learning or drill because the elements call for problem solving. The game situation demands that the learner take into account varying factors in specific situations. In general, it calls for reasoning and perception of the situation and for insight in creating a correct response. The Gestalt psychologists allow for insight, thinking, meaning, and understanding rather than just the pairing of stimulus and response.

It should be stated at this point that no study which we have investigated indicates specific factors that imply a transfer which accrues directly from the game situation. If interest, fun, or competition are factors of transfer, then the children do accrue benefits from the games.

The children become interested in the game situation. They are active and interested participants in something that they do not consider school-oriented.

Since it appears that the "discovery approach" (which may involve using games) is being used in more and more classrooms today, we can expect in the future more research on the effectiveness of this technique, hopefully pointing out under what specific conditions it is effective.

SYMBIOTIC LEARNING AND MATHEMATICS SKILLS INSTRUCTION

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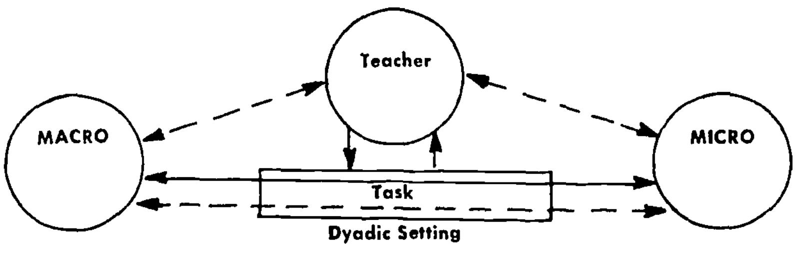
The symbiotic learning model was conceived by the authors in October 1967, and subsequently developed to a usable state in the Guilderland Central School District.

This learning model, which will be described shortly, grew from the widely held conviction that the most effective learning conditions were to be found in the one-to-one, teacher-pupil interaction situation. Equally cognizant of the presumed practical impossibility of attaining this one-to-one, teacher-pupil ratio, a way was sought through an analysis of the components of the learning process to approximate this ideal. The resulting synthesis, which we've called the symbiotic learning model, includes the following essential components:

1. The principles of operant conditioning
2. The social unit of the dyad, or pair
3. The content area of basic math skills
4. The manifestation of 1, 2, and 3 in the form of simple tasks and games

The following diagram should help in the description of the symbiotic model.



The terms macro and micro refer to an older child (i.e., a fifth grader) and a younger child (i.e., a first grader) respectively.[1] The social unit in this model for school learning is the dyad composed of a macro and a micro. It's important to note that the teacher is not directly involved in the dyad. The square, labeled "task," is a relatively simple task or game which is the focus of activity in the dyad. These tasks are so designed that they impose a structure of conditions and contingencies which manifest such principles of operant conditioning as immediate reinforcement, schedules of reinforcement, pacing, prompting, cuing, and fading.

The solid line which passes through the task, and which connects the macro and micro represents that part of the interaction which bears upon the learning of the specific content. The similar but broken line represents those behaviors in the interaction of a more social nature, such as smiling, nodding, and small talk.[2]

The solid arrows between the teacher and the task indicate the teacher's role of controlling the learning through the insertion of particular tasks, and in retrieving certain tasks which are designed to be an actual record of the micro's learning progress.

The broken diagonal arrows between the teacher and the macro and micro indicate that although the teacher is not directly involved in the learning transaction, he is in a position to observe and guide the dyadic interaction.

It may be apparent by now that what this model represents is a unique case of programed instruction. The tasks and games are the format containing and extending the content step-by-step. The dyad provides the vehicle for maintaining the operant activity and supplying some of the reinforcement, prompting, and so forth, and, among other things, assuring that the conditions and contingencies of the tasks are observed. It is important to bear in mind that in this model, as in the case of teaching, to paraphrase McLuhan, the material is the message. The macro is not a teacher, not a tutor. He is a participant in a task or game. Since he has neither the knowledge nor the training to make judgments concerning the learning process, he is not only not asked to,

[1]Bogus erudition (English & English), such as "macro" and "micro" has been deliberately used in describing this model in order to forestall the prejudgment by some that what is being described is nothing but the use of "student tutors," "cross-age helpers," etc. These terms are used here because they have been found to be generally helpful.

[2]It is obvious to the reader that what is being described here is analogous to the difference between Zusammenklingen and Freundschaft.

but is given as little opportunity to do so as possible. Ideally, his behavior is as highly controlled by the conditions and contingencies of the dyadic relationship and the task materials as the micro.

Assuming that the reader is sufficiently familiar with the general principles of programed instruction, further discussion of operant conditioning as an internal component of this model will be dispensed with, in order to address what might be called an external, or a molar problem of programed instruction.

During the 1950's, primarily due to Skinner, the advent of the teaching machine and programed instruction held the promise of revolutionizing classroom practices. More than a decade has passed, however, and not only has this promise remained unfulfilled, but there is little to indicate that programed instruction, or teaching machines, have had the significant impact so eagerly anticipated on the schools. Many of the principles of programed instruction have been incorporated into the educational jargon, and are also given substantial emphasis in the more recent educational psychology textbooks, but they seem to have taken the form of hortatory remarks of what the teacher should do, which may be helpful, but which by no means can be construed to be a significant technological breakthrough in the educational practices of the schools.

This proposal contends that the relative failure of teaching machines and programed instruction has not been due to any breakdown in the principles of operant conditioning, but rather in the attempt at direct application which not only minimized, but virtually eliminated, the most powerful conditions and contingencies of reinforcement, social interaction.

The second component of our model, the dyad, not only restores these conditions but does so at very little expense of the use of operant conditioning principles.

An essential point in viewing these two components — operant principles of learning, and use of the dyad — is that the operant principles are not imposed upon the dyad, they *are* the principles of dyadic interaction. The materials do not supply the operant principles, they simply set the conditions and contingencies. The dyadic interaction guided by the tasks are the power of the model for learning. The content contained in the tasks is almost irrelevant to this effective interaction. Consequently, the applicability of this model to a considerable portion of the school learning encountered in the primary grades, is limited only to the teacher's skill in translating the content of his program, in this case basic math skills, into simple tasks and games appropriate to this dyadic interaction.

The teacher's talents and judgmental abilities are potentially maximized. The teacher using the traditional methods of instruction is operating as a foreman; the teacher involved in symbiotics is operating as an executive. That is to say, the teacher is not caught up in his own performance, but rather is viewing the performances of others and is, therefore, able to make individual and precise judgments based on these observations.

Via the paradigm, we have spoken of one dyad. Now let us view symbiotic learning from the point of view of full class involvement.

A typical symbiotic session may consist of a fifth grade and a first grade. Since class size is normally no more than 30 at the elementary level, at any given symbiotic session two teachers would be involved with not 30 individuals, as in a regular class situation, but rather with 15 dyads each — dyads which are to a great degree autonomous, self-motivating, closed units. Where formerly the teacher concentrated on teaching, he is now concentrating on learning.

Individualization of instruction for every child has been achieved, not in the unattainable traditional sense, but rather in a synthesis of the powerful components of operant conditioning, dyadic interaction, and precise teacher control in the form of task-game materials.

When one of these sessions is in operation a perceptive educator begins to see the actualization of the tenets of learning theory so long sequestered in texts or lecturers' notes. Phrases like "immediate reinforcement," "small step acquisition," "prescriptive teaching," "starting where the child is," "learning to attend," take on the real meaning of all the qualities of the one-to-one situation enhanced by childhood communication.

At this point, it is important to remember that symbiotic learning is not a "child as tutor" system. Such an approach attempts to make the older child a surrogate teacher, an improbable hope at best, while symbiotic learning leaves the teaching aspects of the relationship to the materials which are under the direct control of the teacher.

In order to catch the flavor of symbiotic learning and better understand its operation, let me now take you to a symbiotic learning session at Altamont Elementary School. The time is 2:25 p.m. The day is Wednesday. The place, Miss Gladys Campbell's fifth grade classroom. Fourteen fifth graders are seated at desks, next to each is a first grader. They have been in session since 2:15 p.m. As you enter the room your first sensory perception is auditory. The room hums with the even modulation of an electrogenerator.

A teacher moves among the dyads, stopping to jot down a note, observe a task-game in action, make a comment. Occasionally she sits down and "sits in" on a game, adjusting its use without actually giving instructions. Each dyad is interacting through a task-game. However, not all are doing the same thing. Closer observation reveals that even when the game format is the same the content may vary.

We stop to watch Sean and David. Sean is an active little blonde inclined to be defensively silly. He and David are playing a Parcheesi-like game. The moves on the board are determined by turning over a card from a face-down deck. Sean has learned what a number contains but cannot readily associate this knowledge with the number's name. As they play, David repeatedly says the name of the number that has been turned over. Occasionally he will ask Sean if he knows a particular number's name. Every time it is his turn, Sean moves his man the number of moves called for. He is intent on the game. He smiles often, scrunches his face when David draws a high number.

Sean is beginning to understand the relationships that exist between things and numbers. More importantly he's learning to attend to a specific task. David and Sean seem to flourish in a state of harmonious altercation.

We move on to another dyad. It's somewhat less talkative than the one we just left. We move close to see what is happening. On the desk in front of a heavyset mac and a fidgety mic is a green gameboard with a tan strip on top. Counting the tan strip there are 4 number lines on this board. In front of Mike is a small deck of orange cards. He turns one over. It reads "6 + 1." He goes to the tan number line, points to the 6, and then says merely "one more," and places the orange card on one of the green 7's below.

Kate then turns a card. Hers are yellow. The card reads "4 + 1." She looks puzzled for awhile. Then she goes to the tan number line and without speaking moves to the 4, hesitates, and puts the yellow card on one of the green 5's. She looks at Mike. He smiles, and turns another card. They're playing Coverall. The player who gets the most cards on the board is the winner. Mike seems to talk more when he's not being watched. As we move toward another dyad his voice becomes more audible. Mike is retarded. His mic, Kate, comes from a large rural family of little means.

There's no gameboard on the next desk. In front of a black 6-year-old is a line of 10 blue cards. A similar line stretches in front of his mac, Pat, a slight, shy blonde. Some of the blue cards have yellow cards on top of them. Between Bobby and Pat is a yellow deck of cards. Bobby turns over one of these cards. It has a "5" on it. He scans his hand. Then with excited quickness puts it on top of one of the blue cards. The card he placed it on reads "$1 + \square = 6$." Calmly Pat draws a yellow card and returns it. She can't make a play. Now, Bobby is really up. He draws his feet under him and draws another card. Pat and Bobby are playing "Box + 1." The object is to fill the box with the yellow number cards. Bobby previously learned his + 1 combinations. Now he's learning to view them in another fashion. Bobby, usually a bit unruly, becomes a riveted competitor when challenged.

Just in front of Pat and Bobby are Jane and Karen. In front of each of them is a row of cards with the numbers from 1 through 10 on them. Jane has two large dice in her hands. She gives them a roll. The side that comes up on the first die is divided by a diagonal line cutting the square face into two right triangles. In one of the triangles there is a red 7, in the other a green 2. The second die comes up *1*. Close observation shows that all the faces of this die are ones. "I'll take the 2, 2 and 1 are 3." Jane turns over the card in front of her that shows the 3.

"Karen," she says, "it's your turn."

Karen rolls. "The idea is to get them turned over," Jane unsolicitedly contributes. She gives us one of her candy-head smiles and returns to the game.

Karen is really starting to catch on. She and Jane are a great team. Karen has cerebral palsy, her family is poor, her intelligence limited, she has a speech defect. She's as happy as a clam.

Over to our right a plump, grinning, towhead, fumblingly counts out eight chips and hands them to his mac. "Look, I only have these many left."

Kurt has spotted us.

Bret turns over the next card and reads, "Take two chips."

"Give me two more, Kurt."

Kurt's face reminds you of a polished apple. He counts them out, "1, 2," scintillas of saliva are noticeable at the corners of his mouth.

We move to our left.

"Take two chips," reads Bret.

The game's called "Give and Take"; the kids call it "Chips."

Kurt's teacher thought he should have initially been retained in kindergarten. Recently, much to her pleasure, she's changed her mind.

From the two boys we move to two girls. They sit close together. Before them is a board divided into squares. In each square is a number. The mic of the dyad turns over a yellow card. It reads "2 + 5." Judy picks up a red marker and places it on a square that shows a "7."

Bobby, a vibrant brunette, conveys her approval. Then she turns a card and follows the same procedure. Her markers are green. The board is labeled on top, "Bingo-Board + 2."

"She is good, you know." Betty never misses a chance.

Judy was very shy when the year began. Once she cried because she had to leave her classroom.

An about-face and two steps brings us to Virginia and Lisa. They're playing the " + 1 Coverall" game the same as Kate and Mike are. Virginia is beginning to want to play the game, to be rewarded, to gain Lisa's approval. She's beginning to want to meet the challenge that life presents. The content is being learned, but more importantly, she's beginning to relate to people. Virginia is black. She's been retained. Her parents are apple pickers.

Just off to the right is another black first grader, Clinton. Clint is Virginia's half-brother. However, he seems to have little in common with her. They rarely speak to one another and never really interact. Yet Clint is as animated as Boboo Bear. Stephen, his mate, is white. He's a bit of an itch himself. The task-game they are playing deals with number names. The game board has three intertwining trails starting from one circle and culminating at another. Each trail is divided into squares in which are the numbers 1 through 10. Clint picks up a spinner that has a grip handle and gives it a twirl. The arrow points to a "5." Clint blurts out the name of the symbol. His southern drawl accentuates the "i" and trails off on the "v." He moves one of his men up the trail to a square containing a "5." Then he drops the spinner on the floor and crawls under the desk to retrieve it. Clint's like that.

Way over near the door is another dyad. Both members are small and quiet. Their outstanding feature is their inconspicuousness.

Thomas, a willing little lad with sad blue eyes, has just lost a tooth.

Joseph and he are engaged in a variation of the ancient game of Kalah, the moves being controlled by a deck of cards which show "Numbers + 1" on them. Joe will talk to us when the session is over. He has excluded himself from the lives of his peers, why or how he does not understand. Self-exiled and lonely he worries about Thomas liking him, since he feels no one can.

We have seen 9 of the 15 dyads operating in the room.

As we leave the room you have more questions than when we first entered. You wonder what went on before today and what will go on after today. And you wonder how was this brought about.

At the point in time we observed the symbiotic session, it has been in operation for over 2 months. Each mic being first exposed to talk-games which were designed to evaluate his present degree of prelearning skills. They had then progressed. For some, the skills come easily. Subsequently, the content of their task-game was constantly changed. Others seemed roadblocked at a particular stage. These mics had their task-games changed often. The approaches becoming progressively more reductive until their difficulties were found and dispelled. Then the changes were made to maintain motivation and allow the ever important overlearning to take place.

At this writing the session has been in existence 3 more months. They meet Monday, Tuesday, and Wednesday every week from 2:15 to 2:45 p.m.

Half of Miss Campbell's fifth grade goes to Mrs. Steven's first grade and half of Mrs. Steven's first grade goes to Miss Campbell's fifth grade.

This means that the scene we have just witnessed was repeating itself in the first grade classroom.

Let us now briefly view the mechanics of symbiotic learning as they relate to the teacher.

What would the teacher do after the symbiotic learning session we just observed?

When she was ready to prepare for the next symbiotic session, the teacher would refer to notes and comments she had jotted down while observing the dyads, with these and her recollections of their past.

Kate might read:

Date	Task-Game	
12/2	+ 1 Coverall	Going well. This game suits both their needs.

While a note on the Bobby and Pat dyad might say:

Date	Task-Game	
12/2	Box 1 +	Bobby knows all of his +1 combinations without any help. Move on to something a bit more challenging.

These comments contain two considerations, appropriateness and enjoyment in successful achievement.

Mike and Kate's task-game was continued the following session since it contained the appropriate content and was motivating and understandable to the dyad.

Bobby and Pat's task-game was changed. It has served its purpose. Bobby no longer needed it. The comment implies that he enjoyed the game and felt successful, indicating that later when the content is changed, this task-game format will be used again.

In summary, symbiotic learning is a system of instruction which capitalizes on the natural interaction of two children of dissimilar ages, the principles of operant conditioning and the experience of the teacher.

The quotation for those who are taking notes is:

Symbiotic Learning = dyads + materials + teacher judgment

INDIVIDUALLY PRESCRIBED INSTRUCTION

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In any classroom there is evidence of a wide range of differences in children's learning abilities. Educators attempt to provide for these differences by constantly seeking new and better ways to individualize instruction — that is, to assess each student's progress, and to provide him with a program of studies specifically geared to his learning needs.

Recently, in response to this recognized need for more highly individualized teaching procedures, a very promising program in mathematics has been developed. This program is entitled "Individually Prescribed Instruction" — or, popularly, I.P.I.

I.P.I. is a specific system of individualized instruction which includes a continuum of behavioral objectives correlated with diagnostic instruments and instructional materials in mathematics. Teachers use this special ungraded body of materials in conjunction with various teaching techniques and patterns of classroom organization. Also, supportive services of teacher aides, who perform specific nonteaching functions, are an integral part of this program.

The I.P.I. project began in 1963. It was developed at the Learning Research and Development Center, University of Pittsburgh, and is being field tested by Research for Better Schools, Inc., a Title IV educational laboratory located in Philadelphia, Pa.

In September 1967, a pilot I.P.I. math project was launched in two classrooms in F. E. Bellows School, in the Rye Neck School District, to determine if the use of I.P.I. methods and materials would be effective in providing for more individualized instruction. It became evident that with the implementation of the program there was a significant increase in the amount of individual help a teacher was able to give each child; children were being taught at their own instructional level; student motivation was extremely high; teachers were enthusiastic. For these reasons, it was decided to expand the I.P.I. math program to all classes in grades 1-6 in September 1969, involving 800 children (grade 4-6 classes are at Daniel Warren School, Rye Neck).

To implement this expanded program, it was necessary that the principal attend a 3-week training session in Philadelphia to learn I.P.I. procedures. Teachers were then trained by the principal at a 2-week inservice workshop, and they became conversant with the theory, goals, techniques, and materials of the I.P.I. program in mathematics. Substitute teachers, student teachers, and aides also attended this summer workshop.

Just as I.P.I. has grown in Rye Neck, so it has across the country. During the 1968-69 school year, 92 schools were involved in the field testing program. In 1969-70, more than 150 schools will be part of the project.

At present, participating schools find that I.P.I. is an expensive program to implement. This is due to a number of factors, including costs of teacher training, storage areas, materials, ($12 per pupil, 1969-70) and salaries of teacher aides. It is hoped that as I.P.I. project development proceeds, charges will diminish.

The I.P.I. Mathematics Continuum for Kindergarten Through Grade 6 is organized into 14 broad topics, called "areas." They are: numeration, place value, addition, subtraction, multiplication, division, combination of processes, fractions, money, time, systems of measurement, geometry, special topics, and applications. Each area has a possible range of seven levels (A-G) of increasing complexity, requiring a growing competency. These levels approximate traditional grade levels, A for grade 1, B for grade 2, etc.

Each level for each area contains carefully sequenced sets of behavioral objectives or unit skills the student must master at that point in the continuum. There are over 400 unit skills currently specified in the I.P.I. Mathematics Continuum. For each of these unit skills there is a corresponding work booklet. In an I.P.I. school, it is necessary that a materials center be established to store all of the booklets and diagnostic instruments which must be available to all classes, since it is physically impossible for teachers to keep such a wide range of materials in the classrooms.

I.P.I. Placement tests are administered to each child entering the program. Results of these placement tests indicate the general areas of strengths and weaknesses of a student and an individual placement profile is developed for each child.

Pretests are then administered to determine which specific objectives within a particular unit the student must learn. Based on these profiles and pretests, teachers write individual prescriptions, indicating work to be accomplished, and specifying certain instructional techniques and settings for each child, which will help him to achieve the behavioral objectives as sequenced in the curriculum.

Periodically, curriculum-embedded tests, which are short tests geared to assess a child's progress towards a particular objective, are given.

After work in appropriate skill booklets and/or other materials is completed, a posttest for a particular skill is given to determine the pupil's mastery of the unit.

During a typical day in the implementation of I.P.I., a teacher will perform some or all of the following functions for each child in her class:

1. Diagnose pupil strengths and weaknesses, by results of placement tests or pretests or from other background data.
2. Write a prescription which may include:
   a) work pages
   b) use of manipulative devices
   c) group instruction
   d) tutoring (teacher or peer)
3. Analyze student progress through a study of:
   a) work completed
   b) time spent
   c) results of curriculum-embedded tests
4. Write next prescription.

Aides are employed to do all the clerical work, leaving for teachers the actual teaching, evaluation of the work, and prescription writing.

The following are duties of teacher aides:

1. Organize I.P.I. instructional materials center.
2. Maintain a current file of individual student I.P.I. records.
3. Keep an up-to-date inventory, and order additional materials as necessary.
4. Score and record results of pupils' daily work.

A ratio of 1 aide per 100 pupils is recommended for I.P.I. math programs.

Teachers and aides have weekly planning sessions to discuss procedures and implementation of the program. These sessions are generally held after school, or perhaps during lunch hour.

A significant strength of I.P.I. lies in the fact that it helps teachers to develop an organizational procedure for individualization to take place in a self-contained, heterogeneously grouped class. All too often, plans for individualizing instruction turn out to be simply a regrouping of children into homogeneous levels to facilitate total group instruction. When this occurs, some educators rationalize that as long as the range of ability in each group is narrowed, it is not necessary to individualize further. Or, if teachers are truly trying to individualize, the amount of work involved in developing appropriate materials for each child becomes insurmountable.

It is important to note that I.P.I. math programs can be successfully administered in schools utilizing various types of organizational procedures, such as nongraded, Joplin plan, or team teaching arrangements.

However, implementation of I.P.I. in Rye Neck is based on the philosophy that the needs of all children are best met in the framework of a heterogeneously grouped classroom environment. Within this framework, using specific I.P.I. techniques, procedures, diagnostic instruments, and materials, teachers are able to provide a meaningful program for each child, and individualization becomes a manageable process.

Of course, I.P.I. also provides extra work for teachers, since each child's work must be perused each day, and this obviously takes more time than correcting a set of 25 papers completed by all the children in the class. Teachers must also change their way of teaching, the way they use their time, and the way they manage their classrooms. No longer can they spend much of the day lecturing to the total class, nor can they give blanket assignments to all children in mathematics. With I.P.I., faced with the reality that no two children are working in the same unit, it is difficult (if not impossible) for teachers to find a topic to discuss with the whole class at one time, and to feel comfortable about it. Hopefully, the days are gone when the only thing to be seen upon entering a classroom is a quiet class with the teacher talking at the front of the room, or with all children sitting with books open to the same page. How many children are these teachers really reaching?

On a typical morning, in the I.P.I. program at Rye Neck, classes begin with a planning session, with children and teacher discussing general plans for the day. Individual I.P.I. math folders are distributed, containing each child's work in his current unit. Some children may be able to proceed without help from the teacher; others will need specific instruction in a particular skill; some children will be assigned to use tapes or worksheets previously prepared by the teacher. The teacher moves about the room, helping children as needed, or she may sit at a table and call children up to her. If children cannot proceed without the help of the teacher, they use their time to work on another subject, perhaps creative writing, or spelling, or a reading assignment. Each child has his own reading materials which may include a basal reader and workbook, as well as a library book. Most classrooms have science corners, where children can go to work on a science lesson or to organize an experiment. Thus concurrently during a morning, children in a classroom are working at various tasks in various subjects.

From time to time children may leave the room to take their I.P.I. folders to be corrected by the aides, or to get additional material from the I.P.I. center, as indicated in their current prescription. They may return to their I.P.I. work later if their folder has been returned to the classroom.

Sometimes a math prescription will call for several children working together with their teacher, or for the use of concrete manipulative materials, which are located in every classroom.

At times individual work stops, and the teacher brings the group together for a lesson in another subject (other than math) as she deems it necessary. All of this has previously been discussed in the planning session.

It is a pleasure for this principal to walk into classroom after classroom, and see children working at different tasks, very often no two doing the same thing, while the teacher is working individually with one student at a time, on math, or reading, or writing. It is also a pleasure to hear teachers say that although they work harder with I.P.I., they could never go back to the "old way" of teaching mathematics.

PLUS PROGRAM: MATHEMATICS INSTRUCTION FOR THE EDUCATIONALLY DISADVANTAGED

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The PLUS Program is a multifaceted project. The segments of the program are remedial mathematics and reading, pupil personnel services, primary enrichment, field trips, and after school instruction. The remedial mathematics section initiated instruction during March 1966. During the 1969-70 school year, 36 full- and part-time teachers staffed 35 positions. In addition to maintaining a full remedial mathematics program, many teachers acted as resource personnel in materials and ideas for the regular staff.

The School

An effective remedial program was initiated by two teachers in one of the project's largest elementary schools (K-6) located in the city of Buffalo target area. The building services over one thousand children from the immediate community. Both teachers have space equivalent to a classroom with suitable instructional materials available. They acted as resource personnel for the regular staff, demonstrating new materials and techniques upon request.

The basic program consisted of small group instruction; six pupils per period; seven 30-45 minute periods daily. The children are selected on the basis of standardized achievement testing, plus principal and classroom teacher recommendation. The length of participation in remedial instruction is not fixed. The pupils remain in the program until the special teacher and the classroom teacher agree that the child can perform adequately in the classroom. It was in this pleasant, relaxed setting that the following lesson occurred.

Lesson Description

The teacher, Mrs. L. has been with the program since its inception. A fully qualified teacher, she has repeatedly demonstrated sensitivity and skill in working with her pupils.

The fifth grade children present for the lesson were Charlene, Juanita, Christopher, George, and Michael. They were representative of pupils receiving instruction in the school.

The lesson started when Mrs. L. met the children in their room. As they quietly traversed the halls, they discussed different uses of the numbers they saw along the way. The lesson started with a discussion of the date, April 1, April Fools' Day. Each pupil had a chance to solve an example on the board dealing with division as the inverse of multiplication. This was a review of previously learned concepts that were used as a foundation for the lesson topic, "averages."

Pencils, paper, and erasers were distributed so the children could make their own computations when necessary. The class defined "average" in terms of pupil age. The teacher asked a key question, "How would I find the average?" The pupils responded: "Add ages." "Divide by five." Juanita worked the problem at the board while the remainder of the class worked around a table. The informal atmosphere in the learning situation facilitated a continual "give and take" between pupils and teacher. A question received an immediate response.

After reaching an agreed-upon answer, it was compared with the largest and smallest number. The children discovered that the average is greater than the smallest number and less than the largest number. Formal spoken logic and terminology were stressed when appropriate. It was interesting to note that the use of formal language by the children appeared natural.

The class used library books as a source of activity. "How many books have you read in the last 3 months?" "How do you find the average?" The children constructed hypothetical examples using actual library books. Errors in the children's work were discussed in terms of their prior knowledge base. The emphasis remained upon pupil discovery.

From library books the lesson quickly moved to find the average cost of packages of flower seeds. The process was discussed and the class proceeded to solve examples involving the average cost of flower seed packages.

When asked how he would find the average weight of the class, Michael answered that he would "Put them on a scale and weigh them." This was done. "How much does the class weigh altogether?" A chorus answered, "Three hundred seventy-five pounds." "How do we find the average?" The children responded, "Divide 375 by 5," before computing the answer.

As a summarizing activity, the pupils made up "average" problems to be solved by their peers. The children enjoyed the class. They worked on interesting examples at their own pace. The sensation of discovery and mastery maintained a high level of concentration over an extended period.

The lesson typified good teaching. The procedures were applicable to a regular as well as a remedial class. The high level of teacher-pupil interaction in a remedial situation was the key to its success. Children were motivated to participate in the remedial program by experiencing continuous success.

DESCRIPTION OF A LESSON GIVEN TO FIFTH GRADE LOW ACHIEVERS

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The following is a transcription of a lesson given to a group of 10 fifth grade children participating in the ESEA Title I Corrective Mathematics Program for the New York City nonpublic schools. For each group of 10 children the project provides two 1-hour instructional sessions per week.

The children serviced by this program come from poverty areas and are chosen on the basis of their retardation in mathematics. Therefore, there are certain considerations that must be met in order to satisfy their needs. Deficiencies revealed on a standardized test in arithmetical skills form the basis of the instructional program. Stress is placed on developing with the children mastery of basic operations and related verbal problems through the use of a variety of concrete materials and approaches leading to understanding. Observation, manipulation, and discovery by the children are basic to developing a positive emotional outlook as well as a keener number sense.

The objective of the observed lesson as stated in the teacher's lesson plan was threefold:

1. "to relate common fraction forms to decimal fraction forms for tenths."
2. "to extend the children's knowledge of place-value through tenths. (decimal form)"
3. "to provide practice in reading and recording decimal fractions."

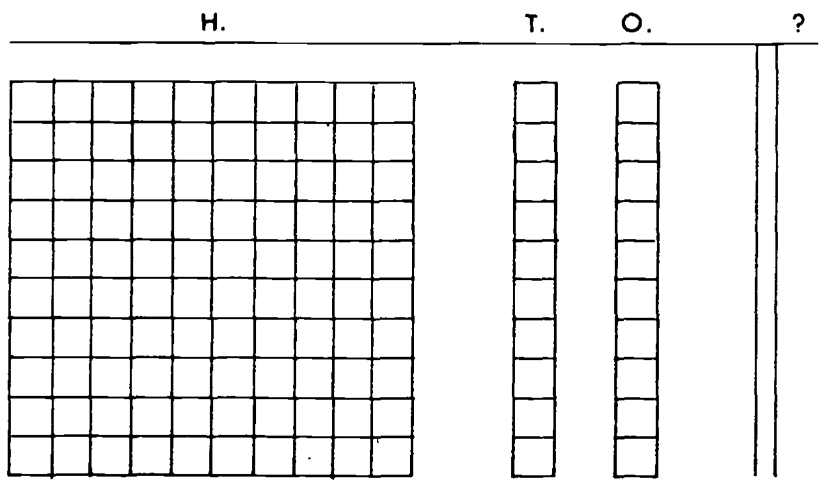
Through the use of money, (pennies, dimes, dollars), squared materials, (units, tens, hundreds), number lines, and chalkboard, the teacher was able to reach each child at his level of understanding.

The interest manifested by the children as they manipulated the materials and the social interplay among the children as they sought to solve problems, revealed the teacher's skill in training the children to work together efficiently and happily.

The children were brought into a room, new to them, and greeted by the teacher. She told them that they would use this room just for the day. (The room usually used by this group was not available because of an auditorium production.)

Nelson:	Yesterday we started something new.
Teacher:	You mean one of those mathematical games?
Nelson:	I keep on trying to do the same thing and don't get the right answer.
Teacher:	Suppose we save it for another time. We can work on it later. Better still, you can come back this afternoon and get all the steps in order. Then you can spend your 2-week vacation period working out the mathematical puzzles. Do you remember that we said a number has many different names. I would like you to be thinking of that all the time we are working this morning. Do you remember what we called this, Jaime? (Teacher held up a single 1-inch square from a set of squared materials.)
Jaime:	Ones.
Teacher:	Give me a whole sentence.
Osaris:	One cent is one-tenth of a dime. (Teacher writes on the blackboard, "one cent is 1/10 of a dime."
Teacher:	Read what we have so far, Lillian.
Lillian:	"One penny is 1/10 of a dime."
Teacher:	(Showed a dollar bill and a dime). Let's use these facts to compare the dollar and the dime. How many dimes make a dollar?
Sonya:	10 dimes
Teacher:	Therefore, how much greater is a dollar than a dime, Ricardo?
Ricardo:	10 times
Teacher:	What is 10 times, Ricardo? Give me a whole sentence.
Ricardo:	The dollar is 10 times the dime.

Teacher:	Why?
Edwin:	Because it takes 10 dimes to make 1 dollar.
Teacher:	What part of a dollar is a dime, Sonya?
Sonya:	10
Teacher:	Can you give it as a fraction?
Sonya:	Ten-tenths
Teacher:	If it's 10-tenths then you will have the dollar again.
Edwin:	One-tenth
Teacher:	Why is it one-tenth of a dollar, William?
William:	The dollar is more bigger.
Teacher:	How many times bigger is the dollar?
William:	Ten times. I know a dime is smaller than a dollar.
Teacher:	Children, Think! I want you to use a fraction. What part of a dollar is a dime?
Nelson:	It's one-tenth.
Teacher:	How shall I write it in a sentence on the blackboard?
Nelson:	(Teacher writes on the blackboard as Nelson dictates.) "1 dollar is 10 times 10 cents." "1 dime is 1/10 of 1 dollar."
Teacher:	How do you compare a dime with a dollar?
Edwin:	The dime is one-tenth of a dollar.
Teacher:	(Showed a flannel board with squared material displayed as follows:)



Teacher: How many hundred do you see, Lillian?

Lillian: One.

Teacher: How many tens are there?

Joseph: One.

Teacher: How many ones?

Shirley: One.

Nelson: That shows one hundred eleven.

Teacher: What does the question mark mean? (No reply.) We might be doing something with hundreds, tens, and ones today. We'll also be using that question mark. Nelson has informed us that the squared material shows the number "111". He is correct.

Teacher: (Distributed one unit square to each pupil and held up one unit square from her own set. She asked, "What is this?"

Shirley: A one.

Teacher: If I have this one unit square and decide to divide it into 10 equal parts, how could I do it?

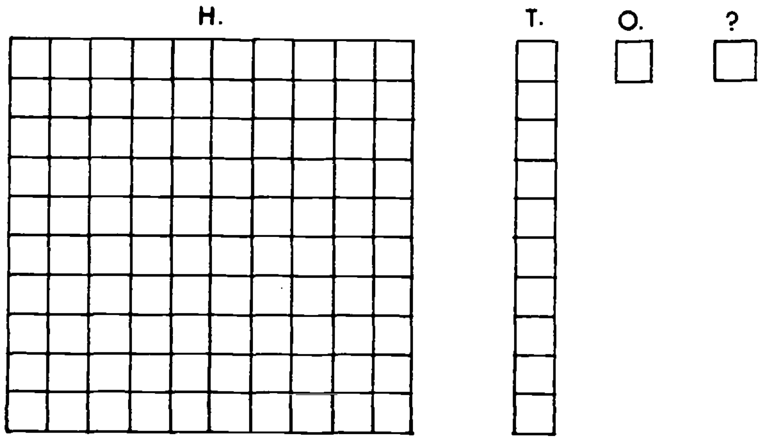
Nelson: You can't do it.

Teacher: Why do you say it can't be done?

Ricardo: It's too small.

Teacher:	I know why Ricardo says it can't be done. Because this (showing unit square) is one. And Ricardo thinks there's nothing smaller than one. How about a fraction? Can a fraction be smaller than one?
Joseph:	You could cut that in 10 pieces.
Teacher:	How about cutting it in half first? Nelson, cut your unit in half. (Nelson cuts a unit square in half.)
Teacher:	Now, how can you get 10 pieces? (After some prodding, reply came.)
Sonya:	You can cut the halves into smaller pieces.
Shirley:	Cut each half in five pieces.
Teacher:	Nelson will cut each half. What happens to the size of each piece? (Teacher shows 10 tiny pieces that she has previously mounted on a sheet of construction paper.)
Shirley:	They're smaller.
Teacher:	(Shows the unit square and one of the smaller pieces cut by Nelson) How much greater is the unit square than this little piece?
William:	One times greater.
Teacher:	Only one times greater?
William:	Oh, I mean 10 times bigger.
Teacher:	Ten times greater because it takes 10 little pieces to make one unit square. What should we call each little piece, Jaime?
Jaime:	One-tenth.
Teacher:	How many times greater is one unit than one-tenth?
Jaime:	Ten times.
Teacher:	How many of these tiny pieces does it take to make one unit?
Osaris:	Ten-tenths.
Teacher:	Is this one-tenth piece as big as the unit square?
Pupils:	(in chorus) NO!

Teacher: Where would you put this one-tenth on the flannel board? (There was discussion among Ricardo, Lillian, and Osaris. Ricardo tried to place the tenth under the tens. Lillian wanted to place it under the ones. Osaris finally placed it under the question mark.) The flannel board then looked as follows:



Teacher: What do you think that new place is called?

William: Tenths?

Teacher: (In a surprised voice) Very good, William! Why is he right, Lillian?

Sonya: (Calling out eagerly) Because you took one and cut it into tenths. (At this point the teacher removed the question mark from the flannel board and replaced it with the word, "tenths".)

Teacher: Suppose we take all of Nelson's 10 little pieces. Shall I place them all here? (Teacher points to the area on the flannel board under the tenths.)

Shirley: You have to change them all for one.

Teacher: Why?

Shirley: Because 10-tenths equals one.

Teacher: William, can you write the correct number under each part of the squared material on the flannel board?

William: (Wrote the number "1" under each of the four pieces on the flannel board.)

Teacher: Can you write that number on the blackboard, Edwin?

Edwin: (He wrote on the blackboard: "1,111, Several hands go up.)

Teacher: Is that what the flannel board shows, Shirley?

Shirley: It's one hundred eleven and this extra little piece.

Teacher: What did we decide to call that extra little piece?

Shirley: Tenths?

Teacher: Yes. This tenth is giving us trouble. How shall we write it? We need a new symbol to separate the tenths from the one. I'm going to show you a chart that I made last Tuesday. Do you remember how hard it rained last Tuesday? When I got home I found my basement flooded. After my family and I had cleaned up all the rainwater, I looked in the newspaper to see how much it had rained in other cities. And I made this chart. (Teacher showed following chart:)

City	*Number of Inches of Rainfall*
Atlantic City	2.0
Boston	1.9
Detroit	....
San Juan	.4
New York City	______

Teacher: Atlantic City is not too far away from New York City. Atlantic City had this much rainfall. (Teacher points to the chart.) How much rain fall was it? How many inches?

Osaris: Two point zero inches.

Teacher: How about Boston?

Pupil: One point nine inches.

Teacher: What do you think the one point nine means?

Sonya: (Calling out) One point nine-tenths?

Teacher: Even though you called out, Sonya, you're almost right. It's one and nine-tenths. And Atlantic City had two and no tenths inches of rainfall. How much rainfall did Detroit have?

Joseph: None?

Teacher: Very good! You people really know how to read this chart. New York City had two and seven tenths inches of rainfall. Who can come up and write that on the chart? Ricardo?

Ricardo: (Wrote "2.7" on the line opposite New York City.")

Teacher: Now I want you to think. What symbol on my chart separates the tenths from the ones?

Pupils: (Excitedly.) The point?

Teacher: Yes, we call that a decimal point. Now, how much rainfall did San Juan have?

Osaris: Point 4.

Teacher: What does point four mean?

Pupil: Four tenths?

Teacher: Yes! Of all the cities on my chart, which one had the smallest amount of rainfall?

Pupil: San Juan.

Teacher: Which city had the most rainfall?

Pupil: New York City.

Teacher: Who can read the whole chart? Would you try, Shirley?

Shirley: (Reading aloud from the chart.)

Atlantic City	Two and no tenths;
Boston	one and nine tenths;
Detroit	nothing;
San Juan	four tenths
New York City	two and seven tenths

Teacher: I wonder if anyone can tell me how much more rainfall there was in New York City than in Atlantic City.

Nelson: Seven-tenths.

Teacher: Very good! Seven-tenths inches. I see that some of you remember our work with fractions. Now let's go back to our squared material on the flannel board. Who can write this number correctly?

Nelson: (Strides over to the blackboard and places the decimal point in the right place.) "1,11.1"

Teacher: Erase the comma. (The number on the blackboard then appeared as "111.1") Who can read this number?

Osaris: One hundred eleven and one tenth.

Teacher: Let's compare all these ones. What part of the one hundred is the one ten?

Osaris: One ten.

Teacher: I think you mean one-tenth. (Through similar questions the teacher guided the children to see the hundred as ten times the one ten; the ten as ten times the one one and the one as ten times the tenth.)

Teacher: What place comes after the tenths? What do you think?

Lillian: Hundreds?

Teacher: Not hundreds. We already have a hundreds place on the chart. Come up and show it to us, Lillian. (Lillian points to the hundreds place.)

Edwin: Hundredths.

Teacher: Yes, why is that right?

Edwin: Because it's one-tenth of a tenth.

Teacher: Is one hundredth larger or smaller than one-tenth?

Sonya: Smaller.

Teacher: Now, I'd like to see if you can write some numbers using the decimal point. (She directed a pupil to distribute paper and presented the chart on page 39.)

Teacher: I want you to write the correct number for these. Watch the numbers with the stars in front. You will have to think a little harder about them.

8 tens	3 ones	6 tenths	=	83.6
1 ten	2 ones	5 tenths	=	12.5
9 tens	7 ones	1 tenth	=	97.1
*4 tens	6 ones	10 tenths	=	______
*$4 + \frac{5}{10}$			=	______

These numbers were written as dictated by pupils.

The children worked independently for a few moments as the teacher walked around and watched their work. Then she called upon individual children to dictate the way the numbers should be written and completed the right-hand side of her chart. (See above:) For 4 tens, 6 ones, 10 tenths, Nelson said forty six and ten tenths.

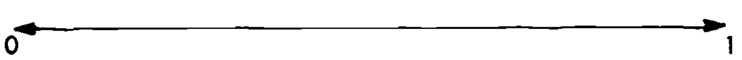
Teacher: What did you write, Sonya?

Sonya: Forty-seven because ten-tenths is the same as one.

Teacher: How did you write $4 + 5/10$?

Sonya: Four and five tenths. (4.5) (Teacher puts number on the chart.)

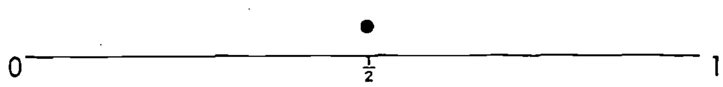
Teacher: You happen to be right. 5/10 can be written as .5 and I'm going to show you why right now. Do you remember when we worked with fractions how we used the number line?
(At this point 4-inch strips of oak tag were distributed to each child. An unmarked line was drawn across the center of the strip and the children were directed by the teacher to label it as shown.)



Teacher: Do you remember when Nelson had the one piece and he wanted to cut it into 10 equal pieces. What did he do first?

Pupil: Cut it in half.

Teacher: Right. So with your eye measure off one-half of the line and under that point write ½.



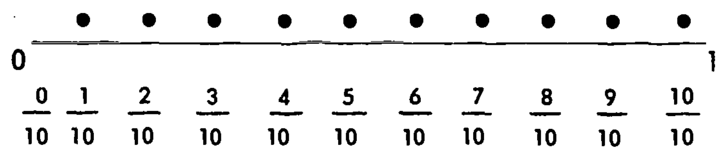
Teacher: Now divide each half into five equal parts. Into how many parts has the whole line been divided? (Some pupils had difficulty in making the five equal parts.)

Sonya: Ten parts.

Teacher: So what do we call each part?

Shirley: A tenth.

Teacher: Now label each part of your line beginning with no tenths.



Teacher: How many tenths is one-half?

Lillian: Five.

Teacher: What about one? One is how many tenths?

Pupil: Ten-tenths.

Teacher: I wonder if we can write the tenths another way using something we learned today. Who remembers how we wrote nine-tenths a little while ago?

Joseph: (At the blackboard — wrote ".9".)

Teacher: This time under your fractions write the number of tenths using the decimal point. (As the pupils wrote the decimal numbers at their seats the teacher wrote on display line:)

.0 .1 .2 .3 .... up to 1.0

Teacher: Which way do you like to show tenths now, as a fraction or as a decimal?

Pupils: (In chorus, the pupils gave varied answers. Some said, "Fractions"; some said, "decimals.")

Teacher: I noticed that some of you had trouble dividing the line into equal parts. I'm going to let you take your number lines home. On the other side you can draw another line and practice writing tenths in two different ways.

Critique of Lesson

The description of the lesson fails to show the spontaneity of the children, the warmth of the teacher's personality and the teacher's pervasive guidance of the group.

We recognize the existence of flaws in this lesson. A more effective approach might have been through the use of the children's understanding of our money system or the immediate relationship between 1/2 and 5/10.

However, the lesson was recorded verbatim in an effort to show how the effective teacher can, through persistent, patient questioning, accomplish her stated aim. The teacher's use of materials with these low achievers contributed to their attentive search for answers. Their responses throughout the lesson revealed a feeling of security, eagerness, and self-confidence. This we consider to be one of the major goals of this Corrective Mathematics Program.

Manuscript set in Linotype Garamond with Garamond bold heads.