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ABSTRACT

The text contains a collection of essays to provide a background of factual information concerning the mathematical sciences in undergraduate education. It is intended for the nonmathematical scientist and the scientifically oriented layman. Contents include: (1) recommendations with regard to increasing the available faculty, strengthening the present faculties, improving the quality of undergraduate education, improving facilities and conditions of work, and problems of the underdeveloped colleges; (2) eight case histories of recent developments in mathematics education at specific institutions; (3) analyses of needs of clients of mathematics departments; (4) data on problems of staffing; and (5) special areas of concern such as problems of isolation, women in the mathematical sciences, the underdeveloped institutions, the changing background of the entering student, and promoting creativity in undergraduates. (Author/JG)

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The Mathematical Sciences: Undergraduate Education

A REPORT BY THE
PANEL ON UNDERGRADUATE EDUCATION IN MATHEMATICS
OF THE
COMMITTEE ON SUPPORT OF RESEARCH IN THE
MATHEMATICAL SCIENCES
OF THE
NATIONAL RESEARCH COUNCIL

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Foreword

The Committee on Support of Research in the Mathematical Sciences (COSRIMS) was appointed by the Division of Mathematical Sciences of the National Research Council at the instigation of the Committee on Science and Public Policy of the National Academy of Sciences. Our task was to assess the present status and the projected future needs, especially fiscal needs, of the mathematical sciences. It was clear to us from the very beginning of our work that our report would have to differ somewhat in structure from the corresponding reports for other disciplines that had already appeared.

Though mathematics provides the common language for all sciences, we realize that even scientific readers of our report, let alone nonscientists, may feel that they are not adequately informed about what mathematical research, especially modern mathematical research, consists of. Similarly, even professional mathematicians, or scientists who customarily use mathematics in their work, may be unaware of the manifold applications of mathematics in various sciences and technologies, especially the new applications influenced by the computer revolution.

To provide additional background of factual information concerning the mathematical sciences, we are supplementing our report with a collection of essays, written by distinguished authors on various topics in mathematics, in the applied mathematical sciences, and in the applications of mathematics. With three exceptions, which are reprints, these essays were written expressly for this collection. They are intended not only for the nonmathematical scientist but also for the scientifically oriented layman.

Mathematics pervades our whole educational system. As a matter of fact, we believe that the mathematical community has no obligations more important than those concerned with education, the most critical area being collegiate education. We have, therefore, included in our report questions of policy regarding higher education. Our Panel on Undergraduate Education has carried out an intensive study of this area; its report is presented in the present volume.

Simultaneously with our activities, the Conference Board of the Mathematical Sciences has been carrying out a survey of research and education in mathematics, and its Survey Committee has agreed to act as a fact-finding agency for our Committee. The Conference Board Survey Committee's report will contain a wealth of factual and statistical material pertaining to the matters discussed in our report. We take this opportunity to express our gratitude to the Survey Committee and to the Ford Foundation, which supported their work.

The activities of our Committee have been financed mainly by a grant from the National Science Foundation. This has been supplemented by smaller grants from the Sloan Foundation, the Conference Board of the Mathematical Sciences, the American Mathematical Society, the Association for Computing Machinery, the Association for Symbolic Logic, the Institute of Mathematical Statistics, the Mathematical Association of America, the National Council of Teachers of Mathematics, the Operations Research Society of America, and the Society for Industrial and Applied Mathematics. Columbia University has generously provided us with office space and many auxiliary services. To all these organizations we express our thanks.

We are deeply indebted to the authors of the essays, to the chairmen and members of our panels, and to the many other individuals who have contributed their time and expertise to our undertaking.

January 20, 1968

LIPMAN BERS
Chairman, Committee on Support of Research in the Mathematical Sciences

Preface

The creation by the Committee on Support of Research in the Mathematical Sciences (COSRIMS) of a Panel on Undergraduate Education recognized the unique and increasing importance of undergraduate mathematical education for all the sciences. Members of the Panel were selected to provide a cross section of institutions concerned with mathematics teaching at the college level. The Panel included representatives from three state universities, one large private university, two universities of science and technology, three liberal arts colleges, and one representative of industry. Among the colleges were one women's college and one predominantly Negro college. The Panel met three or four times a year beginning in December 1965. Most meetings were for several days. Members devoted considerable time between meetings to the work of the Panel. The analysis, comments, and recommendations presented in the following pages are the result of this discussion and effort.

Undergraduate education in the mathematical sciences must be considered in the context of the present rapid growth and development of mathematics. Traditional areas of mathematics, both pure and applied, have seen an extraordinary growth and deepening in research results in recent years. At the same time, mathematical research has broadened and diversified to a striking extent. A new and increasing emphasis on mathematics has appeared throughout science and engineering. No longer merely the language of those disciplines, mathematics has become increasingly important as a part of their theoretical substance. This has not only produced new

applications of existing mathematics but has also stimulated the creation of new mathematics. Recent work in computer science and in applied probability gives two examples of this. In addition, deep studies in pure mathematics have disclosed previously hidden interrelationships and have created abstract edifices whose future uses cannot yet be imagined.

In considering undergraduate education, one must keep in mind the traditional and honored place that teaching has in mathematics. Most research mathematicians view teaching as an essential part of their intellectual lives, and recent years have witnessed an increasingly organized concern with problems of curriculum and pedagogy. Various examples of this concern are mentioned in this report.

The increasingly rapid growth of the mathematical sciences has caused a simultaneous rapid growth in the demands on mathematicians for research, for consultation, and for teaching. The acutely increased demands on undergraduate teaching have been a special concern of the present Panel. These demands arise from two major developments. First, the percentage of students majoring in mathematics has increased during a period of rapid growth in the total number of students. Second, students in other disciplines are taking an increasing proportion of their courses in mathematics, and much of this in increasingly advanced areas of mathematics. This growth is described and documented below (see Chapters 2, 3, and 4). A reasonable estimate appears to be that the proportion of students majoring in mathematics has grown from 1.5 percent to 4.0 percent of all majors in the past ten years. Students at all levels are taking more advanced courses in mathematics, and the demands for teaching in the mathematical sciences will accelerate.

These increased demands have aggravated what is, in the Panel's opinion, the crucial problem in undergraduate mathematical education: the shortage of well-qualified teachers. The problem exists at almost all institutions. It is serious at institutions that do not serve as research centers, and it is critical at the least-well-developed institutions. Closely related to it is the problem of increasing the effectiveness of teachers. The Panel on Undergraduate Education has paid special attention to certain aspects of both the above problems—bringing high-quality staff to a broader spectrum of schools, updating and upgrading the college teacher's knowledge, and in general using teachers more efficiently and improving the conditions under which they work.

A program designed to maintain strength in the undergraduate

teaching of the mathematical sciences where such strength exists, and to develop it where it does not, has far-reaching national importance. Such strength is essential not only for the continued preeminence of American mathematical research, but also for the vigorous expansion of all science-related activities. Our recommendations are therefore aimed at both undergraduate mathematical science and its broader national implications.

It is a pleasure to acknowledge the many constructive suggestions and criticisms received from both COSRIMS and the National Academy's Committee on Science and Public Policy in connection with an earlier draft of the present report. Thanks are due to the Survey Committee of the Conference Board of the Mathematical Sciences and especially to its Chairman, Gail S. Young, and its Executive Director, John Jewett. Valuable suggestions concerning special needs of undergraduate colleges were formulated by a subpanel including Karl M. Folley of Wayne State University, Robert W. Murdock of Lindenwood College, and Billy J. Pettis of the University of North Carolina. We also wish to thank Columbia University and Dartmouth College for their help with meetings and for providing facilities. Finally, we are particularly grateful to Mrs. Lynnel M. Garabedian for the long hours of cheerful secretarial labor without which the report could not have come into existence.

THE PANEL ON UNDERGRADUATE EDUCATION

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Contents

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Recommendations

For convenience in reading, we have summarized our major recommendations under five headings. The first is concerned with ways of making available more high-quality staff to a broader range of institutions. The second has to do with ways of enhancing the effectiveness of existing staff. The third concerns ways of improving curriculum and teaching procedures. The fourth concerns improvements in environment necessary for the most effective use of a professional teaching staff. The fifth concerns the special problems of underdeveloped colleges and underprepared freshmen. In each case we state the recommendation and give brief reasons why the recommendation is made. We also furnish a reference to a place in the body of the report where more detailed discussion may be found. Wherever possible, we indicate the level of support recommended.

The recommendations presented here have been selected from a much longer list because of their importance. We feel, therefore, that all deserve the most serious consideration. We recognize, of course, that all the recommendations cannot be implemented simultaneously. We have therefore classified them according to their urgency and as to the manner in which we would like to see them implemented:

PLAN A For immediate implementation

PLAN B Immediate start but gradual implementation, or gradual expansion of existing programs

PLAN C Immediate experimentation, full implementation to be contingent on the success of the experiments

Group I: Increasing the Available Faculty

The United States has many extremely strong mathematics departments, but the majority of undergraduate students attend institutions with staffs not fully qualified in the mathematical sciences. In recent years, a critical shortage of new, well-educated staff members has hampered all but the best colleges and universities. The demand for instruction in the mathematical sciences is increasing so rapidly that the increased production of PhD's is not yet sufficient to relieve the shortage (see Chapter 4). Therefore, new means must be found to provide well-qualified faculty, particularly for the smaller and less-well-developed institutions.

Recommendation 1: Graduate Fellowships

In view of the continuing and deepening national shortage of doctorates in the mathematical sciences, support for a rapidly increasing number of graduate students is essential. Any attempt to slow the growth of, or even decrease, fellowship support in the mathematical sciences will have a disastrous effect on the ability of the mathematical sciences to provide the necessary high-quality undergraduate mathematical education to majors in all the fields that depend on mathematics, as well as to their own majors. The present proportion of doctorates in mathematical-sciences teaching is 53 percent, and an annual increase of 200 doctorates in the mathematical sciences, over the next five years, would be required in order to maintain that proportion for that time. In partial response to this need, we recommend that the number of federal fellowships and traineeships in the mathematical sciences be increased by 50 percent in the next five years. PLAN B.

DISCUSSION The case for this recommendation forms the substance of Chapter 4, Section 1 of this report. The key fact is that the problems of growth in the mathematical sciences are very different from those of the over-all nationwide growth in college teaching. As Allan M. Cartter has shown (Chapter 4, References 1, 2, and 4), the present over-all projections of doctorates in *all* fields lead to the conclusion that the academic job market will be saturated within five years. It is easily seen that if the demands for mathematical instruction were to grow at the average growth rate for all fields, the same saturation would result, with 100 percent of college teach-

ing appointments in the mathematical sciences being filled by doctorate holders by 1971. However, course enrollments in the mathematical sciences are projected to grow at almost three times the growth rate of college enrollments, with the inevitable consequence that the quality of the instructional staff will decrease unless there is a very great increase in doctorate production.

This rapid growth of the mathematical sciences is not caused by a single factor, and the conclusions are shown, in Chapter 4, Section 1, to be stable under a great number of alternative assumptions. Mathematics majors, physical-science and engineering majors, and elementary-education majors will *each* demand growth in the mathematical sciences that is 25–30 percent above average growth. The biological and social sciences, on the other hand, will demand twice this unusual growth, and so will the needs of computing.

Available predictions of numbers of doctorates in the mathematical sciences—which themselves presuppose increased support—lead to a *decreasing* proportion of doctorates in mathematical-science departments from the present 53 percent to about 45 percent. The increase in fellowships recommended above will, if paralleled by proportional increases in all other forms of assistance, only provide *half* the increase in doctorates needed to maintain the present 53 percent proportion for the academic years 1966–1971. Therefore, we consider our recommendation minimal.

Recommendation 2: Women Mathematicians

We urge a major national effort to enable more women mathematicians to continue their graduate education and to enable them to have second careers after raising families. We recommend (a) 100 special part-time graduate fellowships for women, (b) more opportunities for part-time employment in academic institutions, and (c) a relaxation of the nepotism rules. PLAN A.

DISCUSSION The "dropout" rate among women scientists in general, and mathematicians in particular, is disastrously high. To counteract this trend, we recommend special measures to capitalize upon the potential of women mathematicians who have interrupted their careers and wish to re-enter the profession. Programs are needed both for continuation of graduate work and for retraining after several years' absence.

These programs must be flexible and recognize the special prob-

lems of women in our society. The work may have to be part-time, may be either pre- or postdoctoral, may involve moving from institution to institution, and must allow for varying uses of the funds. The granting agency should be prepared for a large fraction of dropouts, remembering that the marginal cost of retraining is small. A new pool of people is being tapped, and costs should not be compared directly with those for males. Special encouragement could well be given to institutions whose programs enroll at least five such women, since mutual support and special attention by the university will enhance the probability of success.

Academic institutions should be encouraged to be more flexible in hiring women. They should follow government and industry in providing part-time opportunities to take advantage of this unused pool of mathematical talent. Finally, unless nepotism rules are relaxed, allowing a woman mathematician to teach at her husband's institution, we will continue to waste a national resource. (See Chapter 5, Section 2.)

Recommendation 3: Forgivable Loans

We recommend special financial aid for students from weaker or less-well-known colleges to enable them to start graduate work. Students from a list of approved non-PhD-granting institutions would be eligible for tuition scholarships, supplemented by forgivable loans, to cover their initial years at a graduate school. For each year of full-time teaching, 20 percent of the loan would be forgiven. We suggest 200 such awards per year. PLAN A.

DISCUSSION Students from less-well-known institutions are under a double handicap in competing for graduate fellowships. Their undergraduate preparation is often too weak to allow them to start graduate courses, and they may be forced to take undergraduate courses for the first postbaccalaureate year. This makes it difficult for them to win national awards, and graduate schools hesitate to use their own funds for such remedial work. They are also handicapped because professors recommending them may not be known by the graduate schools; hence it is difficult to evaluate their recommendations. While this group of students represents a "higher risk," it could be the source of a significant number of well-qualified new faculty members.

We envision the tuition scholarships and forgivable loans to cover

a two-year period, after which the student—if successful—should be eligible for the ordinary sources of graduate support.

Recommendation 4: Research Instructorships

We propose that funds be made available for research instructorships at smaller or less-well-developed institutions. Recipients would teach at the host institutions but would continue research at nearby universities. We recommend that this program cover about 10 percent of the new PhD's who go into teaching, or about 50 two-year awards per year. The host institution should pay two thirds of the academic-year salary, while the funding agency pays the remainder and provides a summer research stipend and a travel allowance. PLAN A.

DISCUSSION Research instructorships have been used extremely effectively by major institutions to provide new PhD's with opportunities to combine significant teaching experience with continued research. Recipients normally have reduced teaching loads during the academic year and are freed for research during the summer. Our recommendation assumes that the institutions at which the instructors teach need not be those at which they do research. Indeed, smaller institutions may provide them with more challenging and exciting teaching assignments. Our plan would bring well-qualified new faculty to less-well-developed institutions, while assuring the new PhD's that they can continue their research. (See Chapter 4, Section 6.)

Recommendation 5: Visiting Professors

We recommend experimentation with the idea of having established mathematicians spend a semester or a year at less-well-known institutions. The host institutions would pay salaries customary for men of that seniority at those institutions; the balance of the visitors' salaries and cost-of-living allowances would have to be paid by the funding agencies. We suggest five such appointments per year during the experimental stage. PLAN C.

DISCUSSION There is no substitute for a truly distinguished mathematician on a faculty. However, most mathematics departments cannot aspire to such a goal. The present recommendation, if suc-

cessful, would place distinguished mathematicians on more campuses temporarily. The period of the visit—a term or a year—is long enough to have a significant impact; and yet it does not remove the mathematician permanently from the university where he is contributing to research and graduate training. This plan would have the incidental benefit of increasing the awareness of each other's problems among both large and small institutions.

The success of this program requires that host institutions make sure that their own faculty members have ample opportunity to take advantage of visitors. It will also have to provide adequate office space and comfortable housing for visiting professors. (See Chapter 5, Section 3.)

Recommendation 6: Visiting Lecturers

We urge expansion of the existing programs of visiting lecturers. Some of these programs have recently been reduced in size; we request that the decisions to reduce these be reconsidered. PLAN B.

DISCUSSION Visiting-lecturer programs have been successfully run for a number of years by the Mathematical Association of America (MAA), the Society for Industrial and Applied Mathematics (SIAM), the Consultants' Bureau of the Committee on the Undergraduate Program in Mathematics (CUPM–MAA), and the Committee of Presidents of Statistical Societies (COPSS). These programs bring distinguished mathematicians to campuses all over the United States to deliver lectures of general interest and serve as consultants on educational problems. It is one of the least expensive ways of stimulating interest in the mathematical sciences and of spreading useful information.

Group II: Strengthening the Present Faculties

Many full-time and part-time professors in the mathematical sciences have had insufficient training for their jobs. We should provide them with opportunities for self-improvement. Even more critical is the problem of the well-trained new faculty member, perhaps with a recent PhD, who loses touch with current developments

in mathematics and with the ever-increasing range of applications of mathematics. The following recommendations are designed to strengthen mathematical preparation and to keep faculty members "mathematically alive." (See Chapter 4, especially Sections 3 and 6.)

Recommendation 7: Information on Existing Programs

We propose that a new program of public information be launched, to bring to the attention of faculty members the various opportunities for self-improvement. PLAN A.

DISCUSSION While many programs to strengthen present faculties exist, the necessary information often does not reach the faculty members who could best profit from them. Many programs are announced only to administrators, who may not effectively bring them to the attention of their various departments. Mailings to departments usually go only to chairmen, who may be reluctant to call opportunities for leaves to the attention of the members of their departments, thus avoiding the task of recruiting replacements. We feel that the existing programs would be greatly strengthened if a general information brochure were mailed to all full-time faculty members in the mathematical sciences.

Recommendation 8: Summer Institutes

We recommend that the present programs of summer institutes and summer seminars for college faculty members be considerably expanded. PLAN B.

DISCUSSION Large-scale summer institutes for high school mathematics teachers have been extremely successful in upgrading the qualifications of teachers. However, the number of institutes (and seminars) for college mathematics teachers has been much smaller. The summer is an ideal time for self-improvement, and the available evidence (see Chapter 4, Section 3) indicates that there is a considerable need for such improvement. Indeed, we argue that the institutes for secondary teachers cannot be fully effective until the qualifications of college teachers are significantly improved. While we are retraining a substantial number of our present high school

teachers, our teacher-training institutions are turning out thousands of new teachers who immediately need remedial training. (See Chapter 4, Section 6.)

Recommendation 9: Institutes in Applied Mathematics

We urge the establishment of new summer institutes in the applications of mathematics. Each such institute should discuss one or more of the major applications of mathematics, chosen from the physical, biological, or social sciences or from the use of computers in solving mathematical problems. PLAN A.

DISCUSSION We believe that the over-all shortage of mathematics professors is aggravated by shortages in special subject-matter areas, most notably in the applications of mathematics. This is particularly serious since the majority of the undergraduates who study mathematics do so for use in other disciplines. (See Chapter 4, Sections 1 and 2.) There is an immediate and continuing need to remedy this situation.

Recommendation 10: Science Faculty Fellowships

We recommend that the National Science Foundation's program of Science Faculty Fellowships be expanded to the point where, over a 25-year period, about one fourth of the PhD and one half of the non-PhD full-time faculty members in the mathematical sciences have opportunities to earn fellowships. This would amount to a gradual expansion to about 150 fellowships per year. PLAN B.

DISCUSSION Science Faculty Fellowships are the major means for refresher education of faculty members. Unfortunately, the number granted annually in mathematics is very small. It has averaged under 20 per year for PhD's. Although the number given for non-PhD's has been larger, many of these were in effect graduate fellowships for recent graduate students who took jobs before completing their graduate education. We recommend that this program grow to the point where it has a significant national impact in keeping faculty members mathematically alive. (See Chapter 4, Section 6, and Chapter 6, Section 3.)

Recommendation 11: Sabbatical Leaves

We urge college administrations to provide sabbatical leaves for faculty members in the mathematical sciences at least once every ten years. PLAN B.

DISCUSSION While research leaves are common at major universities, it is unfortunately not generally recognized that the nonresearch mathematician also needs to refresh himself periodically. He needs to spend time at a major mathematical center, both to keep abreast of developments in mathematics and to renew the stimulus he acquired in graduate school. (See Chapter 6, Section 3.)

Recommendation 12: Curriculum Development

We recommend to the profession and to college administrations that mathematicians be given released time, whenever appropriate, for curriculum improvement. We also recommend that more imaginative use be made of outside funds available for curriculum development. PLAN B.

DISCUSSION It is usually customary at major institutions to give time off for research. We feel that periodic relief from teaching for curricular planning at all institutions would yield a double dividend. First, it will help to strengthen undergraduate instruction. Second, imaginative curricular planning is a good means of keeping mathematicians up to date. (See Chapter 4, Section 4, and Chapter 6, Section 3.)

Recommendation 13: Travel Expenses

We recommend departmental grants of up to $300 per year to non-PhD-granting institutions, to be used to defray travel expenses over and above those usually granted by institutions. The recipients could either attend appropriate regional or state meetings (without the necessity of presenting papers) or use the grants to visit faculty members at other institutions with whom they share mathematical interests. We suggest 50 annual grants as an experiment. PLAN C.

DISCUSSION This is another inexpensive means for helping teachers to stay mathematically alive. We regret that present travel funds are

tied primarily to research grants and contracts, and that a group much needing assistance is thus excluded. (See Chapter 6, Section 3.)

Recommendation 14: An Expository Journal

We recommend the establishment of a high-quality expository journal in mathematics, whose content would be understandable to the vast majority of mathematical scientists. PLAN A.

DISCUSSION While there has been a great deal of discussion in the mathematical community about the need for such a journal, it still has not been established. We have in mind a journal analogous to *Physics Today,* which would keep mathematical scientists up to date on recent developments. Contributions to this journal should be by mathematicians who combine knowledge of the frontiers of mathematics with the knack of clear exposition. Each article should be written for mathematical scientists who are *not* expert in the field concerned. (See Chapter 6, Section 3.)

Group III: Improving the Quality of Undergraduate Education

Beyond improving the quality of the faculty as recommended in Groups I and II, we can strengthen undergraduate education by using organizational devices and modern hardware. These include computers and audiovisual aids, large lecture sections, special undergraduate research training, and regional centers for curriculum and course development.

Recommendation 15: Computers in Education

We endorse the recently published recommendations of the Panel on Computers in Higher Education of the President's Science Advisory Committee (Computers in Higher Education, *Report of the President's Science Advisory Committee, U.S. Government Printing Office, Washington, D.C., 1967), including the levels and schedules of implementation proposed.* PLAN A.

DISCUSSION We feel that during the next few years computer training and regular access to computers will become increasingly essen-

tial to undergraduates in almost every academic field. Special federal funds will clearly be needed to implement this, and special educational efforts will be required on three distinct levels: elementary introductory courses on computers and their capabilities for students generally, more detailed courses (for both faculty and students) on computer use in various fields, and education up through research training for specialists in computer science. In addition, extensive efforts will be needed to develop the specialized software (programs) necessary to make such large-scale use of computers possible. These matters are discussed in more detail in Chapter 3 and also in Chapter 4, Section 2, and Chapter 6, Section 1.

Recommendation 16: Large Lecture Sections

We recommend that elementary courses in the mathematical sciences increasingly use the large lecture section, taught by a top-flight faculty member and supplemented with small discussion groups led by graduate (or properly qualified upper-undergraduate) assistants.
PLAN B.

DISCUSSION This is an important device, and one too often neglected, for utilizing staff more efficiently and for bringing undergraduates into early contact with highly qualified faculty members (see Chapter 4, Section 5). This is primarily a recommendation to the mathematical-sciences teaching profession and to college administrations. However, lecture rooms adequate for large sections may, in many instances, have to be built with outside support. Such facility needs are discussed in more detail in Chapter 6, Section 1 and Reference 1.

We also urge further experimentation with all types of visual aids to strengthen lectures in mathematics. This includes both the development of new equipment and new materials for existing equipment. In particular, we would like to see much better and more varied films, as well as transparencies for overhead projectors, to enrich and improve mathematical lectures.

Recommendation 17: Promoting Creativity in Undergraduates

We recommend that mathematical-science departments provide opportunities wherever possible for undergraduate participation in

creative mathematical work. In particular, we suggest that more departments take advantage of existing programs supporting such participation, and that, where it is appropriate and possible to do so, project grantees in the mathematical sciences make budgetary provisions for undergraduate assistants. PLAN B.

DISCUSSION The Panel believes that undergraduates can make fruitful contributions in research projects, in programs of curriculum revision, in computation laboratories, and in programs specifically designed to foster undergraduate research activity. Such participation by undergraduates will contribute to their training and preparation for future work in mathematics and, at the same time, provide them with financial support for their studies. (See Chapter 5, Section 6.)

Recommendation 18: Teaching Centers

We recommend the establishment of three regional teaching centers, designed to aid in efficient curriculum and course development in the mathematical sciences, and perhaps in other sciences as well. PLAN C.

DISCUSSION Such a center is envisioned as a regional repository for course outlines, notes, reports, and pedagogical ideas and also as a regional center for communication, development, and experimentation with new devices and approaches to teaching (see Chapter 5, Section 5). This recommendation proposes an experimental program for which the seed funding might well be provided by a private foundation.

Group IV: Improving Facilities and Conditions of Work

In addition to the pressing need for faculty and student access to computers noted above in Group III, Recommendation 15, there are other important facility and service needs of the college teacher in the mathematical sciences. These include properly stocked separate departmental libraries and reading rooms, private offices for faculty, and adequate clerical services and equipment.

Recommendation 19: Separate Departmental Libraries

We recommend that any department offering graduate work in the mathematical sciences have a separate departmental library; for strictly undergraduate departments, a mathematics reading room should be considered minimal. PLAN A.

DISCUSSION This recommendation, discussed in more detail in Chapter 6, Section 2, is addressed primarily to college and university administrations. Departmental libraries for graduate work should be adequate in physical size, conveniently located, and comprehensively stocked with books and journals. Undergraduate mathematical reading rooms should include basic reference books and a number of well-selected journals.

Recommendation 20: Upgrading Weak Libraries

We recommend a federal program of 50 grants a year, of up to $1,000 each, for upgrading deficient mathematical libraries in colleges. PLAN A.

DISCUSSION A one-shot expenditure of $1,000 for the purchase of needed books in the mathematical sciences is a very inexpensive way of improvement. Even in colleges where instruction and course work in the mathematical sciences leave much to be desired, a determined student can learn much from well-selected modern books on mathematical subjects. The basic library list prepared by the Committee on the Undergraduate Program in Mathematics (see Chapter 6, Reference 2) makes excellent suggestions for a modest but balanced and up-to-date library in the mathematical sciences.

Recommendation 21: Office Space

We recommend that a private office with a sizable blackboard be provided as an essential requirement for a college teacher of the mathematical sciences. PLAN A.

DISCUSSION Whether he is engaged in research or in study and course planning, a college teacher of the mathematical sciences needs

a private office for undisturbed effort. This is a modest requirement, especially when compared to the laboratory needs of other sciences (see Chapter 6, Section 1). This recommendation is addressed primarily to college and university administrations.

Recommendation 22: Clerical Aid and Equipment

We recommend that undergraduate departments in the mathematical sciences be provided with one full-time secretary for each 1,000 student-course enrollments, and that they also be provided with mathematical typewriters and modern duplicating equipment. We further recommend that teachers of the mathematical sciences be provided with graduate or upperclass undergraduate assistants for correcting homework in elementary courses. PLAN A.

DISCUSSION The recommendations here are addressed to college and university administrations and are designed to relieve teachers of the mathematical sciences of time-consuming chores that interfere with optimal performance of primary jobs. See Chapter 6, Section 1, for further discussion.

Group V: Problems of the Underdeveloped Colleges

The above recommendations regarding faculty fellowships, summer institutes and stipends, working conditions, and special programs of visitors apply with special force to the seriously underdeveloped colleges. These recommendations will often require implementation in special ways at such colleges. The problems are compounded by the fact that entering freshmen often come from poor secondary schools and are ill-prepared for college-level work. They may need remedial precollege training (say in summer institutes) and may have to be allowed to take high school courses in college.

These problems are discussed in detail in Chapter 5, Sections 3 and 4, where a number of specific suggestions are made. We recognize that these are major national problems, going far beyond the mathematical sciences, that can be solved only by a concerted effort by federal, state, and local governments. We do not feel competent to propose a complete set of remedies; however, we must underscore the gravity of the situation.

2

A Quarter Century of Change: Eight Case Histories

The following chapters analyze a number of factors in undergraduate education in the mathematical sciences, calling attention to various problems and related statistical data.

In order to present some of these problems concretely, eight members of the Panel have prepared brief case histories of recent developments in mathematical education at their own institutions. Although no eight schools are typical of American higher education, our sample does consist of a representative variety of types:

Two state universities (New Mexico State and Virginia)
A private university (Harvard)
Two institutes of science and technology (MIT and RPI)
A Negro university (Texas Southern)
A men's liberal arts college (Dartmouth)
A women's liberal arts college (Mount Holyoke)

The case histories are given in alphabetical order.

While the institutions in this sample differ widely in their educational goals and in their clientele, they reveal many similar trends over the last quarter century. The following items occur repeatedly:

Significant increases in mathematics-course enrollments
Spectacular increases in mathematics majors
New undergraduate major programs in the mathematical sciences
New PhD programs in the mathematical sciences
Impact of improved high school curricula on beginning mathematics courses

Many new advanced courses
Undergraduates taking graduate courses
New courses for the social sciences, statistics, and computing
Significant increases in staff
Difficulties in recruiting new staff

1 Mathematics at Dartmouth 1939–1966

We present a brief comparison of the mathematics department in the last prewar year, 1939–1940, and in the recent academic year 1965–1966. In the prewar year, the department carried a heavy load of service courses and had only a small number of majors. Its work was exclusively on the undergraduate level. Today, the department offers a PhD in mathematics, has a vast increase in majors, and its program has expanded in both breadth and depth. Comparisons are made under four headings: Staff, Courses, Enrollments, and Facilities.

Staff

The staff in 1939 consisted of nine men with PhD degrees in mathematics, all at senior ranks. Since one man was Dean of the Faculty, there was a full-time equivalent of eight men. The standard teaching load was six courses per year for professors, with eight or even ten courses for junior staff members. Research leaves were very rare.

Currently, the staff consists of 20 men, evenly distributed through the ranks, 19 of whom have PhD degrees in mathematics. Since a regular policy of research leaves is in effect, and since two members have outside assignments, the full-time equivalent staff in a given year is 17—just over twice the 1939 figure. The standard teaching load is five semester courses per year, irrespective of rank.

The department has a long-standing reputation on the campus for fine undergraduate teaching. The current generation has tried to maintain this reputation, while making a significant increase in the research productivity of the staff.

Courses

A quick count of courses shows that there were 32 undergraduate courses in 1939, and that there are 33 now. This comparison is highly misleading, however.

First, the department has instituted a four-year honors program in mathematics. In the first two years this is open to any student who has demonstrated ability in mathematics and is interested in a challenging mathematics sequence. For the last two years there is an honors major, designed for men planning to go on for a PhD degree in a mathematical science. There are ten parallel "honors sections" of courses, which are substantially different from the corresponding regular sections.

The new PhD program started with a grant from the Carnegie Corporation; the 1965–1966 academic year was its fourth year of operation. Five PhD's were produced in 1966. Ten or more graduate courses are offered each year; many of these are open to undergraduates.

Even though the basic undergraduate courses have changed little in number, nearly two thirds of them have been drastically altered in content in the last 25 years. "Losses" during this period are the disappearance of precalculus courses, de-emphasis of classical geometry, and dropping of a sequence of courses in actuarial mathematics. The most significant improvements, in addition to the availability of honors work, are the strengthening of the offerings in algebra and analysis and new courses in logic, probability, topology, and statistics.

Courses aimed at physical-science and engineering students have been supplemented by courses designed for students in the biological and social sciences. The department also teaches an introduction to computing—as part of the freshman sequences—to 80 percent of the students. Improved high school mathematics curricula have substantially influenced course offerings. Not only have precalculus courses disappeared, but 20 percent of the students receive advanced placement in calculus. The emphasis in courses has shifted from technique-oriented courses to those that emphasize mathematical structure.

Enrollments

The total undergraduate enrollment at Dartmouth has risen by 25 percent during this period, to 3,000 students. However, in a change from the semester to the three-term, three-course system, each student elects 10 percent fewer courses. Hence, course enrollments in the college at large have increased less than 15 percent. During this same period, undergraduate mathematics enrollments increased from

about 1,000 to about 2,100 student-courses. Ninety percent of each entering class takes some college-level mathematics before graduating. A significant element of this is a one-year sequence, designed for students outside the physical sciences, which is completed by one third of the student body.

There were 13 senior mathematics majors at Dartmouth in the prewar year 1939. During the postwar decade, this number dropped to about five to ten per year. Then, for about five years it rose to between 10 and 20, while in the past five years it ranged from 20 to 40. It should be noted that, since Dartmouth offers an undergraduate major in engineering science, the number of majors is not so high there as at liberal arts colleges that do not have engineering programs.

More significant than the increase in the total number of majors is the fact that 10 or 15 majors are honors majors who hope to go on for a PhD degree in the mathematical sciences. A survey of six recently graduated classes indicates that 72 men have either obtained a PhD degree or are pursuing one. This is 12 per graduating class—and the number appears to be increasing. Honors majors take a variety of graduate courses while they are still undergraduates, and they write theses that occasionally contain publishable results.

In 1939, all courses were taught in sections of 25–30 students. Currently, three class sizes are in vogue. Freshman courses (other than honors courses) are taught in lecture sections of 120–150 men. Advanced courses often are of seminar size (about 10 men). Intermediate courses are still in sections of 25–30 students. One effect of the vast increase in honors courses has been that seminars are increasing dangerously in size. Courses first given in alternate years now have to be given every year or in two parallel sections.

Facilities

In 1939, the Mathematics Department was housed in one corner of Dartmouth Hall, the oldest building on campus. It shared ordinary classrooms with several other departments and had no lecture room. Faculty offices tended to be small, and none had a blackboard. Absolutely no secretarial aid or duplicating facility was available—the department typed its own letters and used commercial duplicating facilities.

Five years ago, the Bradley Mathematics Center was built, with a grant from the Sloan Foundation. It is a modern classroom–office

building. Classrooms vary from a 200-seat lecture hall specially designed for mathematics to small seminar rooms. It houses a departmental library and has a faculty lounge and individual offices for faculty members—with blackboards. Currently it also has office space for graduate students and a teletype input to the Dartmouth time-sharing computer system. There are three full-time secretaries and full duplicating facilities.

2 Mathematical Sciences at Harvard University

For many decades, instruction and research in the mathematical sciences at Harvard University have had worldwide recognition. Despite the success that has been achieved, the educational programs have not remained static; indeed, changes at all levels of education have occupied the serious attention of scholars of first rank.

Sources of Mathematical Instruction

Mathematical instruction and research at Harvard University are spread through several schools and a number of different departments within each school.

FACULTY OF ARTS AND SCIENCES The Department of Mathematics in the Faculty of Arts and Sciences devotes its efforts to abstract mathematics and encourages the teaching of applied mathematics in other departments and schools of the university. In addition to its excellent senior faculty, the Department is continually refreshed by the flow of excellent and enthusiastic young instructors through its Benjamin Peirce instructorships. Within the Faculty of Arts and Sciences, the Division of Engineering and Applied Physics also offers sequences of mathematics courses under the titles of "Engineering Sciences" and "Applied Mathematics." The "Applied Mathematics" list includes courses in computation and numerical analysis as well as courses in the more traditional fields of applied mathematics.

The Committee on Applied Mathematics and the Department of Statistics have parallel histories. Both offered an undergraduate concentration for the first time in 1964–1965; both have offered advanced degrees since 1957. Of course, advanced degrees in applied mathematics were awarded regularly even earlier.

The Department of Economics has several courses in mathematical economics and statistics. Both the Department of Social Relations and the Department of Psychology offer mathematical courses as well as statistical courses.

OTHER GRADUATE SCHOOLS Outside the Faculty of Arts and Sciences, the Graduate School of Business Administration has several faculty members whose training and research interests lie in theoretical statistics. The Business School joined with the Division of Engineering and Applied Physics, the Department of Economics, and the Department of Statistics to offer a set of programs in the general area of decision and control, beginning in the fall of 1966.

Turning to the medical sciences, the School of Public Health has a Department of Biostatistics with a large course offering and many students. The School of Medicine has more than one project in biomathematics.

The School of Education has recently developed a number of programs of training in mathematics proper, a Master of Education for Experienced Teachers, a Master of Arts in Teaching, a Master of Education in the Academic Year Institute. In addition, the School of Education offers a variety of programs in statistics and in tests and measurements, as well as courses in computation.

Mathematics for Undergraduates

By and large, undergraduates take their course work within the Faculty of Arts and Sciences, and therefore they take most of their mathematical courses in the Department of Mathematics, in the Division of Engineering and Applied Physics, and in the Department of Statistics. A continuing interest in the study of course offerings has been maintained throughout the years. Harvard has been particularly sensitive to the undergraduate's needs and preparation, especially in the period of increasingly available funds for the sciences. This is readily seen in the following changes in course offerings.

In 1919–1920, one main route led the new undergraduate from analytic geometry through advanced calculus. In 1939–1940, the same single route was available. In 1954–1955, the route had scarcely changed. But in 1956–1957, a new sequence was added for students who wanted to do four semesters' work in three, with extra

material added. For those who can stand the pace, it has been a great success.

In 1960–1961, a further two-semester alternative sequence was introduced. For students who wished a more applied flavor to their calculus, in 1956–1957 an engineering-sciences course was introduced as part of the basic sequence. In 1963–1964, a more algebraic approach to calculus was offered on an experimental basis. It may become the new intermediate course.

The advanced-placement student could begin with the second course of the basic sequence, or at a higher level if he were further prepared. The course offerings in the mathematical sciences offered in the Faculty of Arts and Sciences have changed as shown in Table 1.

TABLE 1 Number of Semester Courses Offered

YEAR		UNDER-GRADUATE	PARTLY GRADUATE	GRADUATE	TOTAL
1939	Mathematics Department	6	10	19	35
	Other departments	2	2	3	7
	TOTAL	8	12	22	42
1965	Mathematics Department	11	10	31	52
	Statistics Department	3	6	7	16
	Applied Mathematics	2	0	14	16
	Other departments	0	14	8	22
	TOTAL	16	30	60	106

The total number of different courses has more than doubled: from 42 in 1939–1940 to 106 in 1965–1966. In 1965–1966, 16 courses were offered in the Department of Statistics, a new department formed within the last decade. The number of mathematics courses has increased substantially in economics and in engineering sciences and applied mathematics. In 1965–1966, only half the mathematics courses offered were in the Department of Mathematics.

In this discussion, no attempt has been made to consider the many courses in the sciences with mathematics prerequisites, even though many of these courses could themselves be regarded as courses in mathematics and, indeed, in some places and at some times are so regarded: a few examples of such courses are thermodynamics and statistical mechanics, theory of relativity, mechanics, dynamics. On

the other hand, we have been more lenient in the social sciences, allowing more courses to be counted, so as to reflect the recent increased interest in statistics, mathematics, and computing in these fields. The point is that the amount of mathematics taught in a course in mechanics or in electricity and magnetism may be much more than that in a course in economics, but the mathematical content in courses in physics always has been large. What is especially interesting in the social sciences is the increasing interest and the newness of the mathematics used.

Without going into the fact that today's undergraduate courses are sometimes yesterday's graduate courses, we can also note the improvement in student preparation, first by the increase in advanced placement in mathematics:

YEAR	ONE-YEAR ADVANCED PLACEMENT IN MATHEMATICS OUT OF CLASS OF ABOUT 1,100
1950–1951	NONEXISTENT
1955–1956	29
1960–1961	163
1964–1965	251
1965–1966	264

Suitably prepared Harvard College undergraduates freely enter courses in mathematics listed as "Primarily for Graduates," and the usual experience is that the undergraduates do well in these courses. We find the following changes in the number of graduate mathematics courses taken by undergraduates during their college careers:

GRADUATING YEAR	TOTAL MAN-SEMESTERS OF UNDERGRADUATES IN MATHEMATICS COURSES LISTED AS "PRIMARILY FOR GRADUATES"	CLASS SIZE
1939–1940	37	900
1946–1947	102	1262
1954–1955	50	1013
1964–1965	262	1091

In the field of general education, the Department of Mathematics offers a one-semester course taken largely by undergraduates concentrating in fields outside the natural sciences. This nonprerequisite course emphasizes the creative aspect of mathematics and topics of historical significance.

The mathematics faculty has increased from 13 in 1939–1940 to 31 (of whom three were on leave) in 1965–1966. The number of undergraduates has increased from 2,603 to 4,836, or only about 86 percent.

Undergraduate Concentration in Mathematics

The enrollment in the Department of Mathematics as a field of concentration led to the following:

YEAR	NUMBER OF CONCENTRATORS	NUMBER OF BACHELOR'S DEGREES IN MATHEMATICS
1939–1940	67	22
1954–1955	118	17
1964–1965	232	71

This shows an enormous increase in the number of successful concentrators (majors). Beyond this, the enrollment in undergraduate courses in the mathematical sciences in man-semesters has increased as follows:

YEAR	MAN-SEMESTERS IN MATHEMATICAL COURSES
1939–1940	758
1954–1955	2,086
1964–1965	2,700

The amount of mathematics being taken by mathematics concentrators increased from 11.6 semester hours for the class of 1954 to 15.0 semester hours for the class of 1962.

In addition to his course work, the mathematics undergraduate honors student writes a thesis. A few of these lead to published papers, and some others lead the student toward a master's degree or a doctoral dissertation.

Effect of Federal Support of Research

The question frequently arises whether federal support of research has reduced the effectiveness of teaching or reduced interest in it. While discussion of this matter is difficult to document, the continuing active interest by eminent Harvard scholars in the needs of

undergraduates belies this fear. Furthermore, the teaching load for permanent faculty members in the Department of Mathematics has not been changed throughout the period, as nearly as one can determine, and certainly not in the postwar years. Finally, Harvard University, thus far, does not accept federal funds for payment of salaries of permanent members during the regular academic year. This policy is unusual among universities.

3 Undergraduate Mathematics at the Massachusetts Institute of Technology

At the Massachusetts Institute of Technology, most mathematics courses, both pure and applied, are taught in the Mathematics Department. Some courses in computer science, statistics, operations research, and mathematical models are taught in other departments (Civil Engineering, Electrical Engineering, Economics, Management, and Linguistics). Of course, a variety of theoretical courses in other fields have substantial mathematical content. The present report concerns only those courses in which the primary emphasis is on mathematics as such rather than on specific applications.

A notable feature of mathematics at MIT has been its growth over the past 40 years, both absolutely and in relation to other areas of research and instruction. This growth can be measured in several different ways: number of faculty, number of undergraduate majors, number of mathematics courses offered, enrollment in mathematics courses, and fraction of average student's time spent in mathematics courses. We document this growth below by presenting appropriate figures for the academic years 1919–1920, 1939–1940, 1956–1957, and 1965–1966. Among the more notable findings of this tabulation are the growth in undergraduate majors in comparison with other fields at MIT and the growth in the fraction of the average student's time spent in courses taught in the Mathematics Department.

Some Historical Comments

The first master's degree in mathematics was awarded in 1919, the first bachelor's degree in 1923, and the first PhD degree in 1927. The Mathematics Department was organized as a research department coordinate with the other science departments in 1932.

UNDERGRADUATE MAJORS	1919–1920	1939–1940	1956–1957	1965–1966
Undergraduate mathematics majors	0	23	103	326
Total number of undergraduates	2,987	2,379	3,688	3,755
Proportion of upperclass undergraduates majoring in mathematics	0%	1%	4%	12%

For purposes of comparison we give the number of majors in certain other departments for the same years:

	1919–1920	1939–1940	1956–1957	1965–1966
Chemistry	66	114	100	123
Physics	15	91	513	391
Aeronautics and Astronautics	2	183	127	193
Civil Engineering	255	80	182	90
Mechanical Engineering	472	372	154	161
Electrical Engineering	103	334	358	596

The following table gives the number of different subjects given as distinct semester courses by the Mathematics Department:

	1919–1920	1939–1940	1956–1957	1965–1966
Semester courses	25	37	49	74

The proportion of the average student's time spent in mathematics courses is given in the following table:

	1919–1920	1939–1940	1956–1957	1965–1966
Time spent in mathematics courses	12%	9%	11%	13%

In 1919, 1,748 of the 2,987 undergraduates were freshmen and sophomores, and only 480 were seniors. The postwar influx of lowerclassmen, together with the substantial amount of mathematics instruction given to lowerclassmen, accounts for the high percentage for that year.

DEGREES IN MATHEMATICS	1919–1920	1939–1940	1956–1957	1965–1966
BS	0	7	15	81
MS	1	5	12	2
PhD or ScD	0	4	7	26

Size of Faculty

Figures are given here for the professorial ranks (full, associate, and assistant), not including instructors or lecturers (of which there were 30 in 1965–1966).

	1919–1920	1939–1940	1956–1957	1965–1966
Mathematics faculty	10	17	25	41
Total MIT faculty	126	270	558	815
Percentage of total faculty	8%	6%	5%	5%

In addition, in 1965–1966 six predoctoral students were used for teaching. Of the 3,721 student-years spent in mathematics courses, approximately 180 (or 5 percent) were spent in classes with predoctoral graduate students as teachers.

Mathematics Instruction in Other Departments

All the foregoing figures refer to courses and students taught in the Mathematics Department. Mathematics courses are also offered, with the approval and encouragement of the Mathematics Department, in several other departments. Almost all these courses fall into the areas of computer science, statistics, and models for social science. An approximate count of such semester courses outside the Mathematics Department for the year 1965–1966 is as follows:

	COMPUTER SCIENCE	STATISTICS AND MODELS
Civil Engineering	3	5
Electrical Engineering	11	2
Economics	0	9
Management	3	8
Linguistics	0	2
	17	26

This tabulation includes those courses that, because of emphasis and content, could appropriately be given under the label of mathematics or of mathematical science.

Basic Undergraduate Program in Mathematics

In 1919, for the first time, MIT offered a course in calculus and analytic geometry as a part of the standard first-year program. In that year, the decision was also made henceforth to require trigonometry of entering students. Almost all students were required to take a common standard course in mathematics throughout their first two years as undergraduates. This requirement continued until 1965, when it was reduced to one year in the interest of allowing more flexibility and choice for students in their sophomore year.

Changes in the basic program since 1919 have included a gradually deepening and more accelerated treatment of calculus and a lessening emphasis on elementary drill. Theoretical material covered in the present first year includes much of the material covered in the first three years of calculus and advanced calculus in 1919.

Freshmen who entered in 1965 were offered a considerable choice of mathematics subjects for their sophomore programs. Most are expected to take a one-semester course in differential equations. Courses in advanced calculus, algebra, probability, statistics, and several other subjects were also available.

In the past few years, there has been a marked increase in the number of students offered advanced placement in mathematics upon entrance to MIT. In 1965, 341 students received advanced placement of one or more semesters.

In 1948, a special section of second-semester calculus was made available to students with more theoretical interest and sufficiently good first-semester grades. In 1960, this was extended to the first term; students for this first-term section are selected on the basis of their college board scores and their own preferences. Students who enter thus may take one of three paths at the present time: regular course, special theoretical section (about 90 do this), and advanced placement.

The Undergraduate Major in Mathematics

Research interests of the department faculty vary widely, with increasing strength in both pure and applied mathematics. The program for undergraduate majors reflects this breadth. There are two main paths that the student may follow: one with an emphasis in pure mathematics (including analysis, algebra, and geometry) and one with an emphasis in applied mathematics (including analysis,

algebra, and the methods of applied mathematics and continuum mechanics). Each path has a small core of required mathematics courses together with a group of largely unrestricted electives in mathematics and the other sciences.

4 Mount Holyoke College: 25 Years Ago and Today

Among liberal arts colleges for women, Mount Holyoke College enjoys a reputation for an unusually high proportion of students interested in science. Thus, the graduating class has usually exhibited an almost even division of majors into the humanities, social studies, and science. Mathematics is not required but has always served as one of the options in a distribution requirement in science. Majors in the physical sciences, and increasingly those in biological and social sciences, frequently complete essentially a mathematics major as well.

In the past 25 years the college enrollment shows an increase of 67 percent, from 1,027 to 1,715 undergraduates. Recently, the college changed from a two-semester five-course curriculum to a two-semester four-course plan. Thus each student currently elects 20 percent fewer courses, suggesting an over-all college net increase in course enrollments of less than 47 percent. During this same period, mathematics enrollments increased from 225 in 1940–1941 to 747 in 1965–1966. The number of mathematics majors in the class of 1941 was four; in the class of 1966 it was twenty.

Until the past decade, the typical mathematics major had one of the physical sciences as a supporting field, and a popular position for graduating seniors was that of engineering aide. In the 1960's, the secondary interests of majors are much more diversified, including philosophy and logic, economics and psychology, as well as the biological sciences. The proportion of majors entering graduate school in pure mathematics has decreased as more choose graduate programs in business administration, biostatistics, or computer science.

The field of secondary school teaching of mathematics is increasingly chosen by majors, frequently prefaced by graduate work in a Master of Arts in Teaching program. The college has been able to offer a head start to this group of students through a grant made by the Du Pont Company, which provides financial support for sum-

mer study in education for prospective secondary school teachers of a physical science, usually in the summer after the student's junior or senior year.

The department program in 1940–1941 included 13 different courses, ranging from a course in intermediate algebra to a course in complex-variable theory. The standard first-semester freshman course was either trigonometry and analytic geometry or college algebra and analytic geometry. The study of calculus was begun in the second semester of the freshman course and continued throughout the sophomore year. A semester course in theory of equations constituted the beginning of the major for juniors, together with a semester of advanced calculus. Advanced courses included three geometry courses, differential equations, higher algebra, probability, complex variable, and occasionally courses in higher analysis and number theory. Several of the advanced courses were offered in alternate years.

In the academic year 1965–1966, the department program again included only 13 different course offerings. The standard first course in mathematics, consisting of an introduction to the differential and integral calculus, is the first of a sequence of four semester courses, each of which is offered both semesters. This sequence includes an introduction to linear algebra and the theory and applications of ordinary differential equations. Advanced placement students generally begin with the second or third course of the sequence. Courses in abstract algebra and analysis following this sequence are consequently elected by sophomores in increasing numbers. Advanced courses include probability, geometry (projective or differential), number theory, and topics in algebra or analysis with alternate-year offerings in some cases. In addition, the department has for the past three years included a course in finite mathematics for students primarily interested in the social and biological sciences, and it offers in alternate years, jointly with the Philosophy Department, a course in the foundations of mathematics and logic.

It is interesting to note that, despite the increased number of mathematics students and the general trend to acceleration due to better high school background, the number of advanced courses has not increased. The press of students at the beginning levels (first four-course sequence) has been primarily responsible for the department's limited advanced-course offerings. This limitation has resulted in increased demand for independent work by advanced students. Fulfilling this demand has certainly not lightened the de-

partment staff load. Actually, with the present size of enrollment in the abstract algebra and analysis courses that follow the calculus sequence, honors groups at this level suggest themselves as a more advantageous approach than individual tutorials, as well as implying a better basis for senior honors work.

In conclusion, the comparison of 1940–1941 with 1965–1966 should include some statement concerning the mathematics staffs of these two years. The department in 1940–1941, admittedly overworked, consisted of three members, one of whom was Professor Antoni Zygmund (now at the University of Chicago). All members had PhD degrees and were seasoned mathematician-teachers. In 1965–1966, for the first time in 10 years, the staff (five members) all held doctorates. Two of these are relatively new to teaching, having recently completed their PhD work. During the last 10 years at least two of the five positions have been occupied by a succession of temporary staff members, the department frequently borrowing instructors from neighboring institutions to teach courses or sections.

5 Mathematics at New Mexico State University

In 1942, Dr. Earl Walden came to New Mexico College of Agriculture and the Mechanical Arts (then a school of about 500 students) and assumed the chairmanship of the Department of Mathematics. He was one of three faculty members to hold a PhD degree in the sciences and was assigned the typical teaching load of 21 semester hours. The mathematics staff at that time totaled four, and there were no graduate programs in any department of the College. Indeed, the Department of Mathematics was totally a service department to the School of Agriculture and the budding departments of engineering. The study of basic science was minimal: the College had but one physicist on the staff.

Dr. Walden's teaching load was reduced to 18 hours in the next semester so that he might give more time to his duties as department chairman. Perhaps this was an early presaging of the innovations that were to be brought about by this man in the next 24 years before his retirement.

The next major milestone in the progress, growth, and development of this small agricultural college came in 1945 when the first

captured V-2 rockets were brought to the nearby White Sands Proving Grounds. The influx of scientific personnel to White Sands and the demands of this fledgling rocket industry began the metamorphosis of this institution from an agricultural college to a university oriented toward the basic sciences. Dr. George Gardiner (the sole member of the Physics Department) recognized an opportunity at this time to make a contribution to rocket research and at the same time to develop a sound program in physics, mathematics, and engineering at the College. He conceived the idea of the Physical Science Laboratory, an organization within the College that would bid for contracts to do basic research in rocket design, guidance systems, and data reduction. As this project began to flourish, it served to bring well-trained personnel to the campus and provide part-time employment for many students (650 were so employed in 1964–1965).

It is quite natural that the influence of the factors mentioned above should bring about a demand for course offerings at the graduate level and corresponding demand for graduate degree offerings. In 1956, Dr. Walden was appointed the first dean of the Graduate School, and in 1960 the first PhD degree was awarded; significantly, that degree was in mathematics. Doctoral degrees are now offered in biology, chemistry, electrical engineering, civil engineering, mechanical engineering, mathematics, and physics. The following tables are indicative of the growth:

Faculty

	TOTAL	SCIENCE AND ENGINEERING	
1945–1946	78	15	(4 PhD's)
1950–1951	152	50	(15 PhD's)
1964–1965	285	152	(112 PhD's)

Undergraduate Enrollments

	AGRICULTURE	ARTS AND SCIENCES	BUSINESS ADMINISTRATION	ENGINEERING	TEACHER EDUCATION	TOTAL
1954–1955	331	581	189	804	127	2,032
1959–1960	420	1,087	212	1,232	472	3,423
1964–1965	586	1,041	511	1,259	679	4,853

Library Expenditures

1951–1952	$ 38,787
1964–1965	$277,000

The Department of Mathematics now graduates an average of eight PhD's a year, almost all of whom enter the teaching profession.

A Computer Center was established in the academic year 1964–1965, and a Statistics Laboratory was established in 1966–1967. In September 1966 the Department moved into a new research building—appropriately named Walden Hall—which provides 35 private offices for the senior staff and 56 semiprivate offices for graduate students. This building was built with a matching grant from the National Science Foundation.

At the undergraduate level, while course offerings have improved in both number and modernity, it must be admitted that the undergraduate curriculum suffers from the drains caused by the graduate program. Large lecture sections in all the basic courses are commonplace, with "laboratory-type" problem sessions manned by graduate students. The input of undergraduate students is healthy, many good students being supplied since 1952 through a cooperative work–study program with the White Sands Missile Range. There were 165 undergraduate mathematics majors in 1964–1965. (Students declare their majors in their junior year.) All told, about 3,000 students were enrolled in mathematics courses in 1964–1965. But the output is disappointing in that very few undergraduate mathematics majors continue in graduate school (though possibly some do return to study at the graduate level after a stint in industry). Precalculus courses are offered for no credit through the Adult Study Program; even so, the Department is at times forced to use graduate students (generally more experienced) to teach some service courses.

In order to emphasize undergraduate mathematical study, the Department (with some financial assistance from the National Science Foundation) established in 1965–1966 a tutorial course for just ten outstanding freshmen intending careers in mathematics. The course is taught by a senior staff member, and the students receive very handsome scholarships. This program will continue indefinitely.

The mathematics library—practically nonexistent in 1959–1960—is now quite good. The annual library budget for mathematics now exceeds $5,000.

In April 1966, the National Science Foundation awarded this Department a Science Development Grant of $700,000 for a three-year period, during which time the entire mathematics program can be expected to expand and improve at an even greater rate.

6 Mathematics at Rensselaer Polytechnic Institute

Recent History

Rensselaer Polytechnic Institute, founded in 1824, became in 1835 the first privately endowed educational institute in any English-speaking country to award degrees in science and engineering. As in other technologically oriented institutions, the major emphasis prior to World War II was the undergraduate engineering program. Although the university had achieved an excellent national reputation by 1945, a Rensselaer postwar planning committee recognized that a true polytechnic must also include a strong graduate school with the accompanying teaching and research programs. Expansion in this direction started in the 1950's.

The change in the distribution of student population during the past quarter century has been dramatic:

	UNDERGRADUATE	GRADUATE
1940	1,500	50
1965	3,300	950

Revisions in undergraduate curricula and the growth in graduate instruction have made considerable modifications in the function and load of the Department of Mathematics. The problems that have arisen in achieving this growth are by no means unique with Rensselaer. They will also be found in other universities with large science and engineering schools in which curricula have become strongly analytical. We have also experienced the difficulties that arise in the shift from an almost exclusively undergraduate student body to one containing a substantial number of graduate students. Finally, the financial problems involved in this growth are similar to those encountered by many privately endowed universities.

Department Load

Until the introduction of an undergraduate mathematics curriculum in 1956, the major task of the Department was supplying elementary and advanced courses needed by students in other majors. A few advanced degrees in mathematics had been given sporadically for many years. Now, the Department has 150 under-

graduate mathematics majors in the sophomore through senior years and 75 graduate students.

At the present time, some 16 percent of the total teaching load of the Institute is carried by the Mathematics Department. Although the average annual rate of increase of student population at Rensselaer during these past ten years has been 2.9 percent, the corresponding increase for registrants in mathematics courses has exceeded 5 percent per year. In 25 years, the total enrollment in mathematics courses has increased from 790 to 3,020. The larger share of this change has taken place in advanced undergraduate and graduate courses requiring additional talented staff with considerable educational background. The problem of meeting these staff needs in a market with very short supply has been, and will no doubt continue to be, a serious one.

Mathematics Programs

At the time that major changes were made in the Department in the 1950's, Rensselaer decided that the primary emphasis would be on the applications of mathematics. As a result, many of the more recently appointed members of the Department are applied mathematicians or men who work in those areas of pure mathematics that are directly related to applications. Both the undergraduate and graduate programs partially reflect this emphasis; and a special effort is made to include at the undergraduate level some of the special courses given at Rensselaer.

During the first two years, an undergraduate major takes the basic courses in calculus and differential equations offered to all students. A year course in analysis and a year course in modern algebra are required in the junior year. Two courses in mathematics are required in each semester of the senior year. These can be selected from a list that includes geometry, topology, intermediate differential equations, complex variables, computing and numerical analysis, operations research, probability and statistics, mathematical logic, theory of automata, and foundations of applied mathematics. Work in physics, chemistry, mechanics, the social sciences, and humanities is required of all mathematics majors. Properly qualified undergraduates may take graduate courses.

All undergraduate students at Rensselaer take the basic calculus sequence, and no distinction is made in sectioning on the basis of eventual major except for architectural students. A considerable

amount of experimentation is being carried out, however, according to the varying backgrounds of the entering freshmen.

The distribution within a freshman class of slightly over 1,000 is as follows. Approximately 50 students are given advanced placement and move directly into second-year work. Twenty-five freshmen, who display unusual ability but not necessarily superior background, are placed in an honors section. Approximately 150 students, in addition to the advanced placement group, enter with at least a year of calculus. They are invited to replace the standard three-semester calculus course (11 semester hours) with a one-year course (8 semester hours). Only 250 freshmen have had less than six weeks of calculus before entering Rensselaer. In addition, experimental sections are now being tried to see whether our present three-semester sequence in calculus can be changed to a three-semester program in which about 30 percent of the time normally devoted to calculus is replaced by linear algebra.

The Rensselaer Computing Laboratory is an all-Institute service but is administratively part of the Department of Mathematics. It carries a separate budget for equipment, supporting personnel, administration, and maintenance. Members of the Laboratory holding academic positions are carried on the Mathematics Department budget. The Laboratory has had several computers in the course of its history, running through an IBM 650, an IBM 1410, an IBM 360-model 30, and, since February 1966, an IBM 360-model 50. The computing facilities are used for research problems and course work for all departments at Rensselaer, as well as for a small portion of the administrative activities.

During the spring semester, the Mathematics Department offered courses in computer programming, computational methods, intermediate numerical analysis, a graduate course in numerical analysis, theory of automata, data processing, and advanced programming. Currently, 750 students are enrolled in these courses in the computing arts and sciences, nearly all of whom use the facility for homework problems. A number of courses in the Engineering School make use of the background thus acquired by their students, and assigned exercises are handled in the Laboratory. An extensive program in computer research has been initiated with the cooperation of members of the Electrical Engineering and Physics Departments. A graduate program of an interdisciplinary nature in computer sciences was introduced with an enrollment of ten students in the first year. This is expected to grow rapidly.

Facilities

The Department of Mathematics has undergone two changes in quarters during the last ten years as the faculty has increased. The present building not only houses the Department comfortably but has capabilities for expansion without additional work for the next five years. There are now 40 single offices for faculty and secretarial help, three large rooms for graduate assistants, an additional secretarial office, three seminar rooms, a reference room, and approximately 8,000 square feet devoted to the Computing Laboratory with associated offices and workrooms. Four large classrooms ranging in capacity from 80 to 175 are available in the building, although these are not for exclusive use by the Department.

The general Institute library has not been adequate, but a program of improvement in holdings and facilities has been initiated during the past year. The deficiencies have been partially alleviated by a departmental reference–reading room supported by alumni gifts as well as by the regular academic budget. The holdings are primarily books, with journal subscriptions carried by the main library. The reference room has been designed for faculty and students who need rapid access to noncirculating material for courses and research, and circulation is very restricted. This facility has been in operation for slightly over a year and receives heavy use from 8:30 a.m. until midnight.

7 The Mathematics Program at Texas Southern University

Texas Southern University, a developing institution predominantly for Negroes, is located in Houston, Texas. When Texas Southern opened as a state school in the fall of 1947, it had a mandate to develop programs leading to both bachelor's and master's degrees in mathematics. Through the spring of 1966, 56 Bachelor of Arts degrees in mathematics, 86 Bachelor of Science degrees in mathematics, and 32 Master's degrees in mathematics have been awarded.

Staffing

When Texas Southern University opened as a state school, the mathematics staff included one full-time professor and two part-time

professors. During the 1965–1966 academic year, the staff consisted of 19 people; of these, 16 were full-time mathematics teachers. The breakdown of the 1965–1966 staff, according to rank, is shown in Table 2.

At the present time, none of the mathematics staff holds the PhD degree in any of the mathematical sciences. In fact, since 1947, only three mathematics PhD's made what were anticipated to be long-time teaching commitments to the University. One of these is now acting president of the University, and each of the other two re-

TABLE 2 Number of 1965–1966 Staff by Rank

RANK	NUMBER
Professor	0
Associate professor	1
Assistant professor	5
Instructor	11
Graduate teaching assistant	2

TABLE 3 Highest Degree Earned by Members of 1965–1966 Staff

HIGHEST DEGREE	NUMBER
PhD in education	1
MS in mathematics	11
Litt. mathematics	1
MA in mathematics	3
AM in education	1

TABLE 4 Number of Semester or Quarter Hours Earned by the 1965–1966 Staff since Receiving Their Highest Degree

NUMBER OF HOURS	NUMBER OF STAFF
Above 100	2
50–99	2
25–49	1
11–24	3
Below 10	9

signed after less than two years of service. The highest degree earned by the 1965–1966 mathematics staff is shown in Table 3.

In order to keep mathematically alive or to earn additional degrees, many of the staff members have attended school since earning their highest degrees (Table 4).

Courses

Through the years there have been many changes in the types of courses required for all mathematics majors. Since 1947, each mathematics major has been required to take 30 hours (10 courses) in mathematics. Initially, college algebra, trigonometry, plane analytic geometry, solid analytic geometry, three calculus courses, advanced college algebra, and theory of equations formed the core program for all majors. Other requirements could be met by taking electives in applied statistics, mathematical statistics, advanced college algebra, and advanced calculus. In recent years, the core requirements have been changed to the extent that solid analytic geometry and theory of equations have been dropped completely. College algebra and trigonometry are required only if the student involved demonstrates, through a proficiency examination, that he does not have sufficient grasp of the subject matter normally covered in these courses. At the present time, the core mathematics courses for all mathematics majors are plane analytic geometry, differential and integral calculus, and advanced calculus. We are phasing out the first three courses in favor of a combined analytic geometry–calculus course occupying three semesters. This sequence will be open to all freshmen who are able to qualify. A student can then meet the other requirements by taking electives selected from the courses listed below. It is interesting to note that all the courses listed below were added to the curriculum in the last five years.

Topics in mathematics
College mathematics for nonscience majors
Introduction to digital computers
Foundations of geometry
Foundations of algebra
Concepts and structures in mathematics
Probability and statistics I and II
Introduction to modern algebra

Linear algebra
Mathematics internship (study–work with NASA)
Advanced computer programming

In 1961, the Mathematics Department initiated a Teaching of Mathematics Program. The degree earned by students in this program is the same as that for regular mathematics majors. In addition to the core courses for all mathematics majors, foundations of geometry and introduction to probability and statistics are required for students in the teaching program. Several years ago the teaching program was so popular that it made extensive demands upon the staff; but in recent years, students have been made aware of many new opportunities and are now electing not to go into the teaching program.

The catalog presently lists five undergraduate courses in applied mathematics that are taught outside the Mathematics Department. In 1963, the Sociology Department changed social statistics from a graduate course to an undergraduate–graduate course. The teaching of arithmetic and the business mathematics courses are offered through the Education and Business Departments, respectively. The Psychology Department offers an applied mathematics course for its majors. All the courses given outside the Mathematics Department are taught in the spirit of finite mathematics.

The course outlines in the Mathematics Department have been revised as far as possible in accordance with the recommendations of the Committee on Undergraduate Programs in Mathematics.

Remedial Mathematics

Before 1961, the Mathematics Department offered a sequence of two remedial courses for students who were unprepared for college mathematics. Many departments accepted these courses as sufficient work in mathematics for their students. In 1961, these two courses were replaced by a single noncredit course; and a sequence of two college mathematics courses was established for nonscience majors. Although this remedial work was given as a noncredit course, the State of Texas continued to finance it until the fall of 1964, at which time this course was dropped. Unfortunately, the closing of this particular remedial effort did not mean that the remedial problems were solved. New programs to meet our needs have been initiated.

Among these programs were several on-campus remedial programs sponsored jointly by Texas Southern University and an interested funding agency. During the academic year 1965–1966 and into the summer of 1966, three such programs were held on the campus. These programs served a total of approximately 500 students who were either high school graduates or high school juniors. We expect the number of remedial programs on the campus to increase, and we expect the need for such programs at the University to exist for many years to come.

8 The Mathematical Sciences at the University of Virginia

Throughout the past quarter century, and earlier as well, the teaching of the mathematical sciences at the University of Virginia has been confined almost entirely to the Department of Mathematics in the College and Graduate School of Arts and Sciences and the Department of Engineering Mathematics (now the Department of Applied Mathematics and Computer Science) in the School of Engineering. The principal exception to this has been the increased proliferation of elementary applied statistics courses taught in various schools and departments.

Here we shall confine ourselves to work in the departments directly responsible for mathematical sciences in Arts and Sciences and in Engineering and shall compare the situation in the academic year 1939–1940 with that in 1965–1966 with regard to staffing, courses, and facilities. Some idea of the general character and growth of the University as a whole over this period is afforded by the comparative tabulations, given in Table 5, of the degrees granted in all fields. During this period, the total undergraduate enrollment in the College of Arts and Sciences grew from 1,710 in 1939–1940 to 2,836 in 1965–1966.

Staff

For the academic year 1939–1940, the mathematics faculties were as follows, of whom five in Arts and Sciences and none in Engineering held the PhD degree:

1939–1940	ARTS AND SCIENCES	ENGINEERING
Professors	3	1
Associate professors	1	0
Assistant professors	0	0
Instructors	5	0
Service fellows	4	0
Service scholars	1	11

In the College of Arts and Sciences, professors and associate professors typically taught 9 to 12 class hours per week, while instructors taught 12 to 15. Service fellows were actually graduate students teaching part time (6 hours), and service scholars were undergraduate assistants who graded papers. (Nearly all staff members graded their own papers at this time.) In the Engineering School, teaching

TABLE 5 Degrees Granted by the University of Virginia

DEGREE	1940	1966
BACHELOR'S		
Arts	97 (2 math.)	371 (15 math.)
Science	47	13
Engineering	29	88
Law	121	205
Nursing	3	62
Education	22	35
Architecture	6	22
Commerce	48	64
MASTER'S		
Arts	31 (3 math.)	81 (5 math.)
Science	5	6
Engineering	0	24
Law	0	3
Education	0	96
Architecture	0	2
Business administration	0	67
Arts (teaching)	0	2
DOCTORATE		
Medicine	62	69
Philosophy (all fields)	26 (1 math.)	44 (5 math.)
Science (engineering)	0	9
Education	0	12

loads were typically considerably higher, and service scholars in Engineering usually held a bachelor's degree. They did most of the teaching of the first-year courses.

By 1965–1966, staffs in the mathematical sciences had grown to the following, of whom 18 in Arts and Sciences (i.e., all the permanent staff) and five in Engineering held the PhD degree:

1965–1966	ARTS AND SCIENCES	ENGINEERING
Professors	5	4
Associate professors	4	3
Assistant professors	9	4
Instructors	0	2
Junior instructors (full-time)	4	3
Junior instructors (part-time)	4	0
Assistants	8	8

In Arts and Sciences, all mathematics faculty members of at least assistant-professor rank typically teach six hours per week and have assistants to help grade papers in their lower-division undergraduate courses. They also usually participate in seminars in their special fields of interest. Junior instructors are themselves graduate students. They teach only lower-division undergraduate courses and usually grade papers for the classes they teach. For them the full-time load is nine hours and the part-time load is three or six hours. Assistants are usually upper-division undergraduate majors who grade papers.

Courses

Throughout the period of this study the Department of Mathematics in arts and sciences has continued to offer an active program through the PhD level, the PhD program itself having been revitalized around 1935. In addition, in the College of Arts and Sciences an elementary year's course in mathematics has always been a requirement for the bachelor's degree, whatever the major subject, while bachelor's degrees in engineering have always required mathematics at least through the calculus.

In 1939–1940, the typical first-year undergraduate courses (both in Arts and Sciences and in Engineering) consisted of trigonometry, college algebra, and analytical geometry. In addition, the Arts and Sciences Mathematics Department taught, as a service to

the School of Commerce, an introductory course in mathematics of finance. Introductory calculus was a second-year course. Beyond this, the Engineering School offered only differential equations and calculus drill, while Arts and Sciences offered some 15 more advanced mathematics courses, of which approximately 10 were open to qualified undergraduates. By 1965–1966, the Arts and Sciences Mathematics Department had dropped the teaching of trigonometry and college algebra, as well as the course in mathematics of finance, making the initial course introductory calculus and analytical geometry, with introductory probability a second-semester option. In addition, it had expanded its course offerings beyond the calculus to some 30 active courses, although not all of these are offered every year. During the past 25 years, the number of sections in elementary courses has increased considerably. In addition, the number of students in advanced courses has grown significantly, with an increase from 10 to 35 in complex variables being typical. In 1963, the Department of Engineering Mathematics became the Department of Applied Mathematics and Computer Science, the University having acquired its first computer (Burroughs 205) in 1960. This Department has launched a doctoral-level program, and its course listings now include over a dozen semester courses in applied mathematics and an additional 10 in computer science.

Facilities

In 1939–1940, the Arts and Sciences Mathematics Department, apart from classroom space, which it shared with a number of other departments, was housed in three offices and a tiny mathematics library in an old building. There was no Department secretary. Today, the Department still shares (greatly expanded) classroom space with other departments. For its permanent staff it has about a dozen offices (not all of them private, unfortunately) in a modern classroom and office building. This also houses a much-enlarged, though now again cramped, mathematics library and reading room. There is one full-time secretary plus extra secretarial help for typing manuscripts. The Department has its own duplicating equipment. It especially needs more private offices, a larger space for its library, and at least one common room for its graduate students and assistants.

The University's Computer Center has now retired the B–205 computer acquired in 1960 and has a modern B–5000 computer with

associated equipment. The Center has definite plans for a considerably more powerful computer installation by 1970; and the University is now in the early stages of plans to bring the Arts and Sciences Mathematics Department, the Department of Applied Mathematics and Computer Science, and the Computer Center together in a single Mathematical Sciences Building to be located in the complex of science buildings already partially built.

In the fall of 1966, the Arts and Sciences Mathematics Department experimented for the first time with a large lecture section (about 80 or 90 students) in the general first-year course. A senior professor lectured to the class two hours per week, and, for the third class hour, the class split into four discussion groups conducted by graduate-student assistants.

3

Clients of the Mathematician

1 Introduction

The mathematics department of a college or university serves a wide variety of clients. No other academic department except perhaps the English department has a clientele with such a wide range of needs.

First, the mathematics teacher trains future mathematicians either for academic work or for work in government or industry. Second, mathematicians have traditionally played a key role in the training of physical scientists and engineers. One of the largest groups of clients of the college mathematics teacher is the group of future primary and secondary school teachers. This is a task of continuing importance and increasing complexity. In recent years, mathematics has proved to be a useful tool for the biological and social sciences, and hence the mathematician is in the process of acquiring another large group of clients. It has also been recognized that some knowledge of mathematics forms a significant part of the general education of all college students. Indeed, mathematics has been a traditional part of liberal education for centuries.

In succeeding sections we shall consider these clients in more detail, pointing out their special needs and their impact on undergraduate mathematics teaching. We shall see, in each instance, that the undergraduate is being asked to take more mathematics than ever before. Much of this work was, in years past, considered to be

advanced in nature. At the same time, as other disciplines become more heavily analytic, graduate students from these fields are joining undergraduates in taking mathematics courses. Thus the growth in mathematics instruction will be faster than the growth of the college and university population. The education problem will be further complicated by demands that the teacher recognize that the motivations of his clients may be quite different from those of traditional mathematics students, and that appropriate accommodations must be made to a wide variety of backgrounds and interests.

2 Mathematicians for Colleges and Universities

As the case studies in Chapter 2 have indicated, the number of undergraduates majoring in mathematics has increased far more than has the general college population. For the country as a whole, according to Office of Education statistics,[1,*] we note that there were approximately 4,000 bachelor's degrees in the mathematical sciences in 1954–1955, whereas there were over 21,000 in 1964–1965. To be sure, only a small fraction of these continue into graduate work through the PhD degree and find employment in university teaching. Nevertheless, the number of PhD's in mathematics and statistics during these years has increased from 250 in 1954–1955 to 650 in 1964–1965, and close to 70 percent of these find employment in colleges and universities.

The level of the courses taken by the undergraduate major in mathematics has been raised considerably during this period, with a number of topics once considered as graduate subjects appearing as a regular part of the undergraduate curriculum. A hint of this is seen in the statistics given in a later part of this report (Chapter 5, Section 4) showing the rise in level of the first courses given on the college level.

Not only have these changes in curriculum taken place on a local level, but the profession itself has given serious study to the undergraduate mathematics curriculum through the efforts of the Committee on the Undergraduate Program in Mathematics (CUPM).* It

* Superscript numbers refer to the references at the end of the chapter.

is clear that the recommendations of the CUPM will not be universally adopted; but, nevertheless, it is expected that they will have significant effects on future curriculum planning. A small number of professional mathematicians will probably still receive their undergraduate education in other areas, but even in these cases the number of mathematics courses they take will be considerably larger than in the past.

It should be noted that teaching and research in mathematics have always been closely related and that most of the good research done today is by people who are also teachers. Furthermore, the research mathematician in industry and government frequently serves as a part-time teacher as well.

Classically, the research mathematician worked as an individual, albeit most happily in a community of scholars with similar interests. Nevertheless, we now find group efforts appearing more commonly. The basic tools of paper, pencils, and library are still fundamental for the mathematician; but the computing machine, ranging from the simple desk calculator to the advanced digital computer, is rapidly taking on greater importance. The computer and the whole area of computing sciences will be discussed later in this chapter.

3 Mathematicians for Government and Industry

Government and industry employ talent in the mathematical sciences all the way from high school and technical institute graduates concerned with keypunching and elementary programming up to research mathematicians in a great variety of significantly applicable branches of mathematics. We are concerned primarily with the college level and beyond and shall attempt to obtain a feeling for both the present situation and current trends.

* The Committee on the Undergraduate Program in Mathematics (CUPM) is a committee of the Mathematical Association of America and is supported in part by the National Science Foundation. The general purpose of this committee is to develop a broad program of improvement in the undergraduate mathematics curriculum of the nation's colleges and universities. A series of recommendations on programs for various groups of undergraduates have been published. These have been prepared by special panels of mathematicians with the advice of consultants from other fields wherever appropriate.

In 1960 (see reference 2, page 69), approximately 22,000 people were working as mathematicians in industry and government. About 65 percent of these, or 14,300, were trained in mathematics at least as far as the bachelor's degree; they include roughly 1,100 doctor's, 3,900 master's, and 9,300 bachelor's degrees in mathematics. A recent small CUPM study[3] of mathematics majors at liberal arts colleges and universities suggests that about 20 percent of all bachelors go into government and industry (mostly in computing). Perhaps 20 percent of all masters do likewise; and in recent years perhaps 15 percent of all doctorate holders in the mathematical sciences have gone into government and industry. If one supposes that half the staff hired over a ten-year period is still there at the end of that period (this is equivalent to an annual attrition of about 17 percent), then one might predict the following:

1961–1970	NEW DEGREES	INTO INDUSTRY AND GOVERNMENT	NET GROWTH
Bachelors	233,000	46,600	23,300
Masters	50,000	10,000	5,000
Doctors	7,500	1,100	550
		57,700	28,850

Thus, roughly speaking, slightly more than doubling of industrial and governmental mathematics staffs can be expected in the 1961–1970 period. This is in good agreement with the predictions of the Bureau of Labor Statistics.[4]

In recent years, the subjects most frequently reported by industrial mathematicians as lacking in undergraduate preparation were probability, statistics, linear algebra, and various aspects of computer science. These gaps in undergraduate mathematics are currently being narrowed, and the new crops of industrial and governmental mathematicians should be decidedly better prepared. At the same time, much remains to be done. The difficulty is partly one of staffing, and statistics and computer science present special problems. These matters are discussed in more detail in Chapter 4, especially in Sections 1 and 2.

4 The Physical Sciences and Engineering

In recent years, the amount of course work in the mathematical sciences taken by majors in the physical sciences and engineering

has continued to increase.[5,6] Hence, more mathematics will have to be taught to these potential users than ever before. Some of the implications will be traced in the following.

Physics is the science requiring the greatest mathematical preparation; its present demands are likely to be typical (in amount, if not detail) of future mathematical needs of other fields, since many of the recent advances in other natural sciences (and even now in some of the social sciences) have involved approaches first developed in attacking physical problems. Calculus in the freshman year is now needed in most undergraduate physics curricula. The undergraduate programs should involve a course in mathematics each semester; topics covered should include advanced calculus, ordinary and partial differential equations, complex variables, linear spaces and special functions, computer sciences, probability, and statistics. If present trends continue, an increasing proportion of students will properly elect, even as undergraduates, courses now considered as primarily at the graduate level: modern algebra (including group representations and applications), linear operators and spaces, real and complex variables, special functions, stochastic processes, and game theory.

The American Chemical Society publishes recommended curricula for undergraduate majors in chemistry. These programs cover a wide range of institutions, from the small liberal arts college to the technologically oriented university. Currently, a year of calculus is strongly recommended for all chemistry majors. In practice, much more mathematics is taken; and, in the more technically oriented programs, at least two years of mathematics is required. Normally, more such courses are elected. Computer work, which will be discussed below, is beginning to make a contribution to education in chemistry.

The numbers of students involved in the earth sciences and astronomy are considerably smaller than in the other two areas. Since the mathematical requirements for an undergraduate program in astronomy are very close to those in physics, students in the former area can be included with the latter for our purposes. Undergraduate enrollment in geology has fluctuated considerably in the past few years, showing a decline several years ago, but more recently there has been a reversal in the trend. Graduate enrollment has been increasing, however, and one should note that within geology there has been a marked increase in the analytic rather than the descriptive approach. Hence, the requirements for mathemati-

cal education among undergraduates in geology are increasing significantly.

Requirements for engineering students are in the process of undergoing very serious changes. Although in the past ten years the number of mathematics courses taken by these majors has increased, recently, fundamental changes have been proposed.[5] These will have significant strengthening effects on the mathematical education of engineers. It is interesting that the implementation of the new engineering curriculum will fit well with the general program in mathematics recently proposed by the CUPM.

The rapid growth of knowledge in mathematics, the physical sciences, the engineering sciences, and the social sciences—all of which will see use in the practice of engineering—makes it clear that the traditional four-year engineering program cannot hope to begin to educate professional engineers. Consequently, more time is required for the educational process. More work in each of these disciplines is asked for the undergraduate engineering student. A brief survey of the specific topics to be included shows that the rate of acquisition of the required knowledge must also be speeded up. The problem of efficiency of education will quickly become important, and this is just as true in mathematics as in other areas. The educational program for the embryonic professional mathematician presents precisely the same problem.

It appears that seven semester-length courses in mathematics will be required of all engineering students. These will include a two-semester course in calculus, a one-semester course each in linear algebra, in differential equations, in probability, in computing and numerical analysis, and in advanced calculus. Additional courses will be taken by many students as electives. These will undoubtedly range over several areas of mathematics. This program represents an increase in the total amount of time devoted to mathematics. On the other hand, the material to be covered has been increased by an even greater amount. Thus, much remains to be done in the way of detailed planning on each course.

5 Computer Science

The most rapidly expanding clientele of the mathematician consists of prospective computer users in general and computer scientists in particular. The vast impact of the high-speed digital computer will

have a dramatic effect on undergraduate mathematics education, which is only now beginning to be felt.

Both the Association for Computing Machinery[7] and the Mathematical Association of America[8] have recently published detailed recommendations concerning undergraduate programs in computer science. The impact of computer applications on undergraduate mathematics instruction has been the subject of symposia held as a feature of the Sixth Annual Computing Conference at Florida State University.

Most important, the President's Science Advisory Committee (PSAC) has recently produced a comprehensive report[9] containing a variety of recommendations on computers in higher education. We quote relevant passages from the recommendations (pages 4–6).

Approximately 35 percent of college undergraduates are enrolled in curricula in which they could make valuable use of computers in a substantial fraction of their courses. An additional 40 percent are in curricula for which introductory computing training would be very useful, and limited computer use should be part of several courses. The remaining 25 percent could make some use of computers in one or more courses during their college education, but computer training is not now important in their major studies.

In 1965 less than 5 percent of the total college enrollment, all located at a relatively few favored schools, had access to computing service adequate for these educational needs. However, it is practical to supply adequate computing service to nearly all colleges by around 1971–72.

We recommend that colleges and universities in cooperation with the Federal Government take steps to provide all students needing such facilities with computing service at least comparable in quality to that now available at the more pioneering schools.

We recommend an expanded faculty training program to provide adequate faculty competence in the use of computing in various disciplines.

We recommend that the Federal Government expand its support of both research and education in computer sciences.

We recommend that the Government agencies which support computing allow the schools to be free to apply the funds either to the purchase or rental of equipment and the support of staff, or to the purchase of service.

We recommend that universities and the Government cooperate in the immediate establishment of large central educational computing facilities capable of serving several institutions.

We recommend that NSF and the Office of Education jointly establish a group which is competent to investigate the use of computers in secondary schools and to give the schools access to past and present experience. Cooperation between secondary schools and universities, and particularly providing service to secondary schools from university centers, should be encouraged.

We endorse these recommendations, and we shall now consider their implications for new tasks for faculties in the mathematical sciences.

First, an introductory course in computing is needed for a large fraction of the undergraduate body. Second, a major fraction of the faculty will require an introduction to computers, and a significant minority of the faculty will need much more training to enable them to make serious use of computers. Such courses will vary widely, but the burden will, in most cases, fall upon the staff in the mathematical sciences.

In addition to the training of occasional users of computers (be they students or faculty members), there will be a critical need for a substantial increase in the number of computer scientists. Whole curricula will have to be worked out, on many more campuses than at present, for the training of specialists. These courses will again, in large part, have to be staffed by mathematical scientists.

Finally, there is the vast problem of providing software for hundreds of computing centers, to make the large-scale use of computers possible. The cost (both in money and in time) of developing appropriate time-sharing executive routines, simple general-purpose languages, and problem-oriented compilers, must be added to the cost of the computer equipment. Again, a large share of this burden must be assumed by the mathematical sciences.

6 The Biological, Management, and Social Sciences

The connections of mathematics with the biological, management, and social sciences are more recent developments than that with the physical sciences. The applications of statistical methods to these disciplines are older and will be considered later in this report. The more recently developed work might be best described as mathematical model making. Although still in its infancy, this approach appears to have an ever-increasing appeal.

The work in biology has ranged from application of the classical fields of differential and integral equations to mathematical logic, Boolean algebra, stochastic processes, and information theory. These techniques have been used in a wide range of problems from biochemistry to the interaction of species. Although this area of biology is still a very young science, present development is rapid.

The use of mathematical models in the management sciences is of even more recent vintage. Thanks to the popular press, the intelligent layman is well aware of the use of these techniques as aids to decision making in a significant number of areas in the government as well as in industrial management. Even the shift in terminology from "business administration" to "management science" is testimony of the fact that these areas are becoming more susceptible to quantitative analysis and mathematical model making. The trend has been accelerated by the existence of modern computers. This shift is reflected in the requirements in mathematics for undergraduates seeking professional education in these areas.

In similar fashion, social scientists have found that mathematics can be an effective tool in their disciplines. Statistical methods have been used for some time, and, again, the use of mathematical techniques in model making has proved unusually valuable.

The situation in all three of these areas appears closely akin to that of the early developments in mathematical physics. Perhaps the chief difference is the considerable shortening of the time scale and the large population of students who want to use mathematical techniques in these new areas of application.

These developments have significant implications for undergraduate mathematical curricula. The problem was studied in great depth by a CUPM panel, which consisted of mathematicians and experts in the various subject-matter fields. Their report[10] concludes that students planning graduate work in any of these areas should take two years of college mathematics in addition to the traditional training in statistics. They also recommend that a special curriculum is needed. We can do no better than to quote the Introduction to the report of the CUPM panel:

> The Committee on the Undergraduate Program in Mathematics recommends that departments of mathematics offer courses designed for the needs of students in the biological, management, and social sciences (BMSS).
>
> There is an increasing need for improved undergraduate mathematical training for future graduate students in the BMSS areas. The Social Science Research Council years ago recommended substantial undergraduate mathematical training for future social scientists. The following recommendations are in the same spirit as those made by the SSRC. There is also increasing use of advanced mathematics in many branches of biology and medicine. More recently, some graduate schools of business have been requiring more mathematical preparation of entering students, and have been introducing sophisticated mathematical courses as part of their own programs.

A survey of this Panel has shown that graduate schools in all BMSS areas would like to have more mathematical training for their entering students, but that very few of these students today have enough preparation, or preparation of the type needed. Since the exact need of BMSS students is hard to predict, they benefit most from preparation in considerable breadth in both classical and modern topics in mathematics. Of special importance are elementary analysis, probability and statistics, and linear algebra.

Since high speed computers are sure to play an increasingly important role in all BMSS areas, prospective graduate students should have the mathematical training necessary for data processing, and should have some experience with computers.

The problem is to provide this varied training in the limited time BMSS students have available for mathematics during their undergraduate years. It is, therefore, important to recognize that they need mathematics primarily as a language for scientific reasoning, and that they do not need as much training in detailed techniques as mathematics and physical science students. Nor is it reasonable to expend as much time on rigorous proofs for BMSS students as for mathematics majors. Also, more stress should be placed on applications which are of special interest in the biological and social sciences.

These reasons have convinced CUPM that special courses must be developed for BMSS students.

The Mathematical Social Science Board, funded by the National Science Foundation, was established in 1964 to encourage and improve the use of mathematics in research in social sciences. Its main activities have been research seminars and conferences for research workers and summer research training seminars for graduate students. While the areas of economics, psychology, sociology, and linguistics have participated heavily, as expected, the Board has found itself much more heavily engaged in programs for anthropologists, political scientists, and, most surprisingly, historians than it had anticipated. In the long run these surprises will mean more study of mathematics by undergraduates in these fields.

7 Teachers

Apparently, more than ever before in the history of U.S. education, professional mathematicians today recognize and are responding to the needs of an important class of clients—teachers. We refer here both to teachers of the elementary grades and to junior and senior high school mathematics teachers.

The mathematical training of teachers divides these clients naturally into two classes: preservice training and in-service train-

ing. This division arises through considerations such as previous training, age, experiences on which to build, time available for study, and motivation.

For preservice teachers, the Mathematical Association of America has recommended minimal course work in mathematics amounting to:

E: For elementary teachers-to-be, four semester courses covering the number system, topics in algebra, and topics in geometry.

J: For junior high school teachers, seven semester courses taken from the initial courses for S (below).

S: For senior high school mathematics teacher-trainees, eleven semester courses beginning at the calculus level and rounded out by a balance of the standard courses mathematics majors take (the emphasis on balance tending to prohibit early specialization).

Thus, the J and S curricula place no new hardships on most departments, though their full implementation will cause an increase in upperclass enrollments. But the E curriculum calls for four entirely new courses to be taught by mathematicians, and its full implementation on a nationwide scale will make violent demands on available manpower. It is well to note that several institutions have adopted these recommendations fully, while a great many others have made initial strides through partial implementation with plans to do more in the future.

For in-service teachers, there is much local activity in special summer courses, evening or weekend workshops, institutes, texts or self-study books, films, scholarships, and visiting speakers. Many commercial publishers of elementary and high school material respond in various ways to the needs of in-service teachers, and most curriculum reform committees turn their attention to the problem of these clients. But clients they are indeed, and the scurry of activity in their behalf has placed very real strains on the manpower pool of mathematics teachers.

8 Statistics

The statistical training of students is shared between mathematical-sciences departments and various other departments. The reason for this is that statistics is a mixture of mathematics and scientific methodology, where the latter is often subject-matter oriented. Nevertheless, the training of statisticians and statistical training of

students in general is an important task for the mathematical sciences.

There is a serious shortage of statisticians, particularly in government and industry. Therefore, we must expect that the demand for training statisticians will lead to a considerable increase in the clientele of the mathematician.

Statistical training poses a number of special problems for undergraduate education. These will be discussed at length in Chapter 4. We will limit ourselves at this time to stating that this is one area of increasing need, and an area in which relatively few mathematicians are qualified to serve the clientele.

9 General Education

The total enrollment in freshman mathematics courses clearly indicates that thousands of students elect mathematics each year even though their fields of specialization do not require mathematical training.

The impact of science and technology has become so significant in our daily life that the well-educated citizen requires a background in the liberal sciences as well as the liberal arts. It has long been recognized that mathematical literacy is an important goal of all liberal education. But in current education this training often stops at the secondary-school level. With the increasing quantification of many of the newer sciences, the impact of high-speed computers, and the general expansion of the use of the language of mathematics, it becomes increasingly important for the college graduate to have some postsecondary training in mathematics. Such training is currently provided by a variety of "general education" courses in mathematics. As civilization becomes more complex and the role of science expands, the clientele of these courses will increase by an order of magnitude.

10 Other Forms of Undergraduate Education

We recognize that a significant portion of undergraduate education in mathematics takes place at other than four-year colleges and universities. Junior colleges and community colleges are playing an

ever-increasing role in higher education in this country, and this can be expected to continue at a greater rate. A complete picture of undergraduate mathematics in the United States should include educational activities of the Department of Defense and in-service courses given by industrial organizations, but we have not attempted to cover these in this report.

It is important to note that four-year colleges and universities will have to incorporate, in their plans for the future, recognition of the fact that some of their clients may have received part of their mathematical education from these newer programs. Furthermore, they will have to be increasingly aware of the demand for future in-service education.

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4

Problems of Staffing

1 The College Teacher Shortage in the Mathematical Sciences

The decade of the 1950's produced a number of studies of college teacher supply and demand in various fields (see reference 1, pages 267–268 and reference 2, page 1). For the most part, these predicted more or less dire shortages and a consequent deterioration of faculty quality. In 1965, Cartter published critiques,[1,3,4] of the earlier studies and a new assessment of his own. He found that over the preceding dozen years faculty quality, as measured by highest earned degree, had in general slowly but steadily improved in colleges and universities both public and private. He concluded that for the aggregate of all fields, shortages of qualified faculty have not been as critical as had been predicted. At the same time he conceded that a major task of expansion still lies ahead for American colleges and universities, and that there are, and will continue to be, more or less severe shortages in some fields.

Cartter points out (see reference 2, page 2) that of the new faculty required each year the major portion is hired to meet enrollment expansion rather than to replace staff losses due to death, retirement, or net outflow to other professions. In fact, he argues persuasively (see reference 1, pages 270–273) that for the aggregate of fields the net loss of doctorates from teaching to nonteaching employment, retirement, or death should be placed at only approximately 2 percent of the PhD's presently in college and university teaching rather than at approximately 6 percent, a figure that had

been widely used in earlier studies. We shall give primary attention to an analysis and projection of enrollment expansions and their implication for faculty in the mathematical sciences.

Most of the problems in mathematics education revolve around providing a sufficient number of qualified college teachers for our nation's present and future undergraduates. We are indeed fortunate to have available to us Cartter's technique for analyzing the problem, and to be able to adapt this technique in order to show how difficult the staffing problems in the coming years are going to be.

Let us first be clear on the present situation in the four-year colleges and universities of the United States. The following 1965–1966 data under (a) are estimates from a stratified sample of 283 out of 1,069 such institutions in the United States, 91 percent of which provided completely usable returns.[5] Some earlier data from reference 6 are provided for comparison. The data under (b) and (c) are from reference 7.

	1960–1961	1965–1966
(a) Total undergraduate course enrollments in the mathematical sciences, fall semester	756,000	1,084,200
(b) Number of bachelor's degrees in mathematics	13,100	21,200
(c) Total undergraduate enrollments, same institutions, same semester	2,776,000	4,103,000

In the latter half of this section, we shall see that even though the index "mathematics courses per student" is essentially constant (about 0.27), the need for rapid growth in the number of doctorates in the mathematical sciences is severe. Majors in mathematics, and in the fields in which the mathematical sciences play an increasing role, are on the whole growing in number much more rapidly than the college population itself. A similar effect arises from the rapid spread in the use of computers.

Specifically, we shall show that the growth rate to be expected in mathematical-sciences enrollments over the 1966–1971 period is about three times the corresponding growth rate in college enrollments, with the result that the ratio of doctorate holders to total faculty in mathematical-science departments will inevitably de-

crease. The conclusion that "things are going to get worse before they get better" will turn out to be valid under a great variety of alternative circumstances—for example: any single factor contributing to the abnormal growth disappears; increases in elections of mathematical-science courses by nonmathematics majors take twice as long as predicted to materialize; or a different set of projections for the next 10 years turns out to be closer to the truth.

Let us now marshal our data.

Full-Time Mathematical-Sciences Faculty, Fall of 1965[5]

Doctorate holders	5,712
Masters	4,644
Bachelors	397
	10,753

Excluding graduate teaching assistants, part-time mathematical-sciences faculty form less than 8 percent of the full-time-equivalent staff and will not be considered further at present.

The distribution of the 5,712 full-time doctorate holders is far from uniform across different types of institutions. Universities, which in total carry about 45 percent of the undergraduate teaching load, have about 63 percent of the doctorate-holding faculty; liberal arts and teachers' colleges, with 49 percent of the teaching load, only command 30 percent of the doctorate-holding faculty. In fact, 75 percent of the private four-year liberal arts colleges have at most one doctorate holder on their mathematics staffs. Thus, the quality of mathematics instruction at many liberal arts colleges is open to some doubt.* However, even at universities, a considerable portion of the teaching of the more elementary courses tends to be done not by doctorate-holding faculty but by graduate assistants. In the median university, 40 percent of the freshman–sophomore load is carried by graduate assistants, and in 38 percent of the universities, at least half of the freshman–sophomore teaching load is carried in this way.[5]

* Other indications of the same doubt[6]: 89 percent of the faculty in four-year purely undergraduate colleges were unlisted in the 20-year index of *Mathematical Reviews*, with only 5 percent listed more than once. About 25 percent were hired from elementary or secondary schools, 62 percent of the teachers did not think of mathematics as a preferred career, and about 25 percent also engaged in paid work unconnected with mathematics.

Educational Trends Influencing Mathematical Enrollments

With this picture of the present situation as background, let us now consider the major educational trends that will seriously affect mathematical-science enrollments in the next five years.

TOTAL ENROLLMENTS Total enrollments in the colleges and universities will continue to rise significantly in this five-year period. We will use the 1965–1966 period as a base and predict to 1970–1971. For this interval, Cartter[3] predicts a total rise in enrollments for higher education of 35 percent, while the latest Office of Education projection[7] is 32 percent. The corresponding Office of Education projection for undergraduate enrollments in four-year colleges is 29 percent, and we shall use the latter figure. Thus we may expect to have to meet at least the proportional additional demand for mathematics instruction, regardless of any change in the relative popularity of the mathematical sciences. In the fall of 1965, as we have seen, 10,750 professors handled 1,084,000 course enrollments, so that an average figure of 0.01 professor per course enrollment will be used henceforth. Thus the normal growth of college enrollment will demand about 3,120 additional mathematical-science faculty. A number of other factors affecting mathematics enrollment are clearly visible, however, and will now be considered.

MATHEMATICS MAJORS The Office of Education figures (and projections)[7] indicate an increase from 21,200 bachelor's degrees in mathematics in 1966 to 40,900 in 1971. Of this total increase of 19,700, 6,150 may be attributed to the "normal" 29 percent growth, but there will be 13,550 additional mathematics majors above normal growth. On the average, a major takes a one-year course in each of his underclass years and two in each of his upperclass years. Thus 81,300 extra course enrollments, requiring 810 professors, will result.

PHYSICAL-SCIENCE MAJORS The Office of Education estimates[7] that the number of physical-science majors will rise from 18,000 in 1966 to 26,600 in 1971. Only 3,400 of this increase is above normal growth, and they, at a minimum of three one-year courses, will require about 100 additional mathematics professors. However, various recommendations by professional organizations of physical scientists indicate that we should expect a typical major to take at least one more

upperclass mathematics course than in the past. Here we find a need for 270 additional mathematicians. Thus it is estimated that the physical sciences will create a need for a net increase over the normal growth of 370 professors in the mathematical sciences.

ENGINEERING MAJORS The projected increase[7] in undergraduate engineering enrollments is at 21 percent instead of the "normal" figure of 29 percent, and this figure implies a decrease in required mathematics faculty of 70 below the normal figure given above under Total Enrollments. This is based on a typical five-semester mathematics sequence for engineers. However, it has been recommended that engineers in the future change from this five-semester mathematics sequence to a seven-semester sequence. Such a change would mean that the 44,000 engineering students in 1971 will require 440 additional mathematics professors. Thus it is estimated that engineering will create a need for an increase above normal growth of 370 mathematical-science professors.

BIOLOGICAL AND SOCIAL SCIENCE MAJORS Projections[7] of needs in the biological and the social sciences for mathematical-science instructors run above the normal growth rate (each at 53 percent instead of 29 percent for the 1966–1971 period). It is difficult to estimate the total mathematical-science program in these fields. However, in view of the new demands for mathematical training of students in these areas, it is a conservative estimate to say that each major by 1971 will take an additional year of study in the mathematical sciences. The projections for 1971 are 39,000 majors in the biological sciences, 133,000 in the social sciences, and 25,000 in psychology. To handle the increased mathematical training, 1,970 new mathematical-science professors would be required.

ELEMENTARY-EDUCATION MAJORS The Office of Education projects 130,000 education majors in 1971. While no separate projection is made for elementary teachers, these have in recent years been approximately two thirds of the figures in Table 19B of reference 7 on first degrees in education. Thus we may expect approximately 90,000 elementary-education majors in 1971. The Mathematical Association of America has recommended that in view of the upgrading of mathematics training in the grade schools, it will be essential for all future elementary teachers to take four semesters of college mathematics as compared with the one or two semesters that represent

typical current practice. We therefore expect an increase of at least 90,000 elections of underclass mathematics by such education students by 1971. Some 900 additional college professors will be required to teach these new courses.

COMPUTING The field of computing will perhaps put the greatest single strain on faculties in the mathematical sciences. Implementation of recently published recommendations[9] would make it possible by 1970 for every college student in junior colleges, colleges, and universities to receive a first introduction to computing. Let us assume that, in fact, half of the 5,500,000 undergraduates in four-year colleges in 1971 will receive a semester of such training, and half of these are not already included in the above list. (These figures are eminently reasonable when compared with the present distribution of majors.) A one-semester course would then provide a need for an equivalent 172,000 year-student capacity or 1,720 additional mathematical-science professors.

We have, in the above list, considered neither the increase in graduate work in mathematics, which is very costly in staff size, nor the tendency of liberal arts students as a whole to elect more mathematics than in the past, nor the extra staff required within the mathematics departments to handle mathematics majors over and above their course work, nor the problems of the two-year colleges. There will be further comments on some of these at the end of this section. Let us now summarize our computation:

AREA	NEW MATHEMATICAL-SCIENCE PROFESSORS NEEDED
Normal growth of enrollments	3,120
Mathematics majors	810
Physical-science majors	370
Engineering majors	370
Biological and social sciences majors	1,970
Flementary-education majors	900
Computing	1,720
	9,260

Notice that the total required growth is almost three times that for "normal" growth of total enrollments during the period in question. It is precisely this "abnormal" growth in the need for mathematical-science faculty that makes the problem so critical.

Estimate of the Production of PhD's

With these projections in hand, we may now apply the model developed by Cartter to see what the projected PhD production in the mathematical sciences will do to the quality of undergraduate instruction. We assume, with Cartter, that there is a net 2 percent attrition of previously granted doctorates from universities every year; however, we will assume that 70 percent of the production of new PhD's goes into teaching (as opposed to Cartter's reference value of one third), since this is the recent experience in mathematics. We further assume, and this is not so accurate as we would like, that the required growth of 9,260 professors is evenly spread over the five-year period. However, the error is a second-order effect. Now if $D(t)$ is the number of doctorates in mathematics teaching in academic year $(t-1)$ to t, and $P(t)$ the production of new PhD's in the same academic year, we obtain the recurrence equation,

$$D(t) = D(t-1) - 0.02D(t-1) + 0.7P(t-1).$$

We begin by using $D(1966) = 5{,}712$, and the Office of Education projections[7]:

YEAR	MATHEMATICS AND STATISTICS DOCTORATES
1965–1966	770
1966–1967	860
1967–1968	990
1968–1969	1,150
1969–1970	1,240

In using these numbers for $P(t)$ in the above recurrence equation, a slight correction is necessary. Of the 5,700 doctorate holders in mathematics teaching in 1965–1966, 5,000 hold doctorates in the mathematical sciences, 500 in mathematics education, and 200 in other disciplines. Thus, in any appraisal of the current situation, it is necessary to add 14 percent to the mathematical-science doctorate holders in order to obtain the total number of doctorate holders in mathematics teaching. We shall therefore add a corresponding 14 percent to $0.7P(t-1)$ in computing the supply of doctorate holders in teaching. This is, of course, only an approximation, but it is hoped that the very slow predicted growth of doctorate holders in education will be compensated in part by an increase of "other" PhD's (e.g., economics, engineering) going into teaching in the mathe-

matical sciences. We will thus use $0.7P(t-1) + 0.14[0.7P(t-1)]$, or $0.8P(t-1)$, in our recursion.

t	$D(t)$	$P(t-1)$
1966	5,712	770
1967	6,214	860
1968	6,778	990
1969	7,435	1,150
1970	8,187	1,240
1971	9,016	

We see that, of the total required growth of about 9,260 professors, the growth from 5,712 to 9,016 will produce only about 3,300 new PhD's, and that the percentage of doctorate holders on mathematical-science faculties, currently 53 percent, will decrease seriously, since only 36 percent of the new faculty over the period will have doctorates. In order to obtain the present 53 percent doctoral level in the summer of 1971, an additional 2,000 PhD's in the mathematical sciences above the Office of Education projections would have to be produced before that time.

It is instructive to compare this result with what would have happened if only the "normal" growth of 3,120 mathematical-science professors were necessary. In this case, the 3,300 new doctorate holders in mathematical-sciences teaching would cover the demand, all new staffing would essentially be by doctorate holders, and the happy saturation predicted by Cartter would clearly be in sight.

Alternatives to the Central Case

We shall now consider a number of variations in the assumptions underlying the central case that has just been described.

VARIATIONS OF A SINGLE PARAMETER Suppose there were no net 2 percent attrition. Then $D(1971) = 9{,}720$, and 43 percent of the newly hired staff would have doctorates.

Suppose you wish to count only doctorate holders in the mathematical sciences as doctoral additions to the staff. Then $D(1971) = 8{,}552$, and 30 percent of the newly hired staff would hold doctorates.

Suppose that any single factor in the above-normal demand for mathematical-science faculty were to disappear. If, for example, elementary computing courses were to be covered entirely outside

of the mathematical sciences, or if biological and social sciences departments taught their own additional mathematics, or if the number of mathematics majors were to revert to normal growth, the most favorable consequence would be that 45 percent (instead of 36 percent) of the required new faculty would have the doctorate.

Suppose one assumes that only half of the predicted additions in other majors taking mathematical-science courses actually occur (but still the same predicted growths in numbers of students). Then 6,660 instead of 9,260 new mathematical-science professors would be needed, and the new doctorate holders would cover 50 percent of the need—still no improvement in the present faculties.

A TEN-YEAR PROJECTION Another suggestive variant is that the increases in the elections of mathematical-science courses described in the above sections happen over a ten-year rather than a five-year period. Clearly the strain on the doctoral production will be lessened. How much? We will use the Office of Education ten-year projections and carry out the corresponding analysis.

1. Total enrollment growth at four-year institutions from 1965–1966 to 1975–1976 is predicted to be 56 percent. Thus normal growth will require 6,020 new mathematical-science faculty.
2. There will be 31,500 mathematical-science majors above normal growth, requiring 1,890 new mathematical-science faculty.
3. There will be 7,000 physical-sciences majors above normal growth, requiring 210 new mathematical scientists. In addition, one extra course for each major will require 350 additional faculty. Total: 560.
4. Engineering growth will be substantially below normal, implying a loss of 250 mathematical-science professors. One new course, on the other hand, will require 460 new professors. Net gain: 210.
5. An extra mathematical-science course for each of 266,000 majors in biological or social sciences or psychology will require 2,660 new professors.
6. A predicted total of 156,000 education majors, of whom roughly two thirds are elementary teachers, will require an additional 1,000 mathematical-science faculty.
7. A total four-year college enrollment of 6.4 million, one quarter of whom take a semester of computer science not previously counted, will require 2,000 new faculty in the mathematical sciences.

The total required new staff is 14,340 mathematical scientists over the ten-year period.

The Office of Education projections for doctorates in the mathematical sciences continue as follows:

1970–1971	1,310
1971–1972	1,430
1972–1973	1,770
1973–1974	2,000
1974–1975	2,090

If we apply $D(t) = 0.98D(t - 1) + 0.8P(t - 1)$, we obtain $D(1976) = 14{,}795$, so that $D(1976) - D(1966) = 9{,}083$. Under these suppositions, the percentage of doctorate holders among the newly hired staff in the five-year period to 1971 is now 46 percent (instead of 36 percent in the reference case). The percentage of doctorate holders among the newly hired staff over the full ten-year period is 63 percent, which is an improvement over the present 53 percent. In the last two years, 1974 and 1975, in fact, all the newly hired will hold doctorates, so that saturation would begin around 1974.

MAINTENANCE OF THE PRESENT FACULTY MIX Suppose the mathematical community were to insist on maintaining the present 53 percent mixture of doctorate holders in mathematical-sciences teaching. How much of the predicted growth would then have to take place in other departments? Since 3,300 new doctorate holders are 53 percent of 6,230, then 3,030 remaining new positions would remain to be filled. This could be accomplished, for instance, if all the new mathematics teaching to biological and social sciences majors and all new computing work were staffed by faculty who did not hold doctorates in the mathematical sciences. That some of this may already have happened is suggested by the fact, noted at the beginning of this section, that the ratio of course enrollments in courses given by undergraduate mathematical-science departments to the total undergraduate student body has not increased between 1960 and 1965.

AN ALTERNATIVE GROWTH PROJECTION Another prediction of growth in undergraduate and graduate study is contained in recent unpublished projections from the National Science Foundation. They predict (in thousands of majors):

	MATH	PHYS. SCI.	ENG.	BIO. SCI.	SOC. SCI. + PSYCH.	EDUC.
1965–1966	20.1	21.0	36.6	28.4	105.7	130.4
1970–1971	31.2	29.9	45.6	44.5	168.6	181.8

Corresponding predictions of mathematical-sciences doctorates are:

1965–1966	805
1966–1967	956
1967–1968	1,121
1968–1969	1,280
1969–1970	1,450

Notice the much smaller growth of mathematics majors and the much larger growth of mathematics PhD's!* These figures give a need for 8,840 (rather than 9,260) new mathematical-sciences faculty. Of these, 3,800 (rather than 3,300) or 43 percent (rather than 36 percent) would have doctorates.

Summary

Population forecasts are notoriously inaccurate. On the other hand, they can be helpful for appraising the general order of magnitude of effects. They also help us study the sensitivity of the output to the assumptions. We have tried to allow above for various changes in assumptions; we find again and again that large numbers of additional mathematics PhD's will be needed in addition to those required for normal growth, if quality is to be maintained. Two sensitive points in our analysis whose implications are not clear are (1) the effects of increased mathematics teaching in the secondary schools on the amount of mathematics that will be taught in the college and (2) the effects of the computer boom. We do not question at all that more and better mathematics is being taught in the secondary schools or that the computer will have an enormous impact in all fields and colleges. Could these factors, however, decrease the demand for mathematical scientists in the colleges?

We think it reasonable that more secondary-school mathematics will lead to more advanced mathematics enrollment in college and

* The difference in the 1965–1966 physical-sciences figures is due to a shift, relative to the Office of Education figures, of geography from a social to a physical science. There is also a difference in the definition of the "education" classification.

the need for more teacher training, but we are not at all sure of this. Similarly, we are not clear how computer training will develop and whether its relation to the mathematical sciences will be as direct as seems obvious now or whether it might be more roundabout. It could happen that substantially more people would be available for teaching mathematical sciences, more people such as economists or engineers coming through the computing door than we have counted on. Recall that the present number of doctorate holders who came from outside the mathematical sciences and mathematics education is only 200 out of 5,700. However, even if all the growth of enrollment in elementary computing were covered outside of the mathematical sciences, still only 44 percent (rather than 36 percent) of the required new faculty would have doctorates. To keep abreast of mathematical needs we need a regular continuing survey so that adjustments can be made as they are needed.

A different kind of objection may be raised to the above models—that a mathematics professor teaching underclass courses can perhaps handle more than 100 students, which is the current average. However, this effect is balanced by smaller class size in more advanced courses and by the need to give extra attention, beyond classwork, to mathematics majors, and the net requirement for about 9,300 new professors in 1966–1971 is substantially unchanged.

The above discussion has entirely omitted two-year institutions. The present guess[5] of full-time mathematics faculty in junior colleges is about 2,400, of whom only 3 or 4 percent have the doctorate. Enrollments in two-year colleges are expected[7] to increase at about the same rate as those at four-year colleges. Thus about 2,000 additional doctorate holders in the mathematical sciences would be needed, over the next five years, to bring the two-year colleges to the level of mathematics-staff quality common in the four-year colleges now.

2 Shortages in Specific Subject-Matter Areas

Special problems of staffing exist in several areas within the mathematical sciences, most notably in statistics, applied mathematics—both in the physical sciences and in such newer domains as the biomedical and social sciences—and computer science. In general, the Panel feels that undergraduate students need to be better acquainted

with the kinds of career opportunities and mathematically challenging work that these fields offer, and that the staff competent to provide training in depth in these fields needs to be provided on a much broader basis in colleges and universities today. A more detailed discussion of individual areas is given below.

Applications in the Physical Sciences

The traditional applications of mathematics in the physical sciences are more important today than ever before, since modern technology has become highly analytic in character and requires the use of more advanced mathematical methods. As has been previously noted, students who will work professionally in the physical sciences and their applications form a significant portion of the clientele of the mathematician. It is thus most important that the teacher have an understanding of these traditional uses of mathematics. In fact, as a mathematician, he can be well rewarded by seeing the interplay between the demands of application and the creation of new mathematics. The CUPM[10,11] has directed its attention to appropriate courses on the undergraduate level in the art of application. These are highly recommended in the education of future teachers of college mathematics.

Statistics

The growth of academic statistics during the twentieth century has been greatly stimulated by the needs of applied areas. Indeed, in colleges and universities today courses in elementary statistical practice in various fields—including economics, psychology, education, business administration, and commerce, as well as in many branches of biomedicine and engineering—are widely taught within the departments of the fields concerned, rather than in departments of mathematics or statistics. The Survey Committee of the Conference Board of the Mathematical Sciences[5] has studied the extent of the teaching of statistics and computer science outside departments of mathematical sciences. (Separate statistics and computer-science departments, where they exist, are included *among* departments of mathematical sciences.) The problem is most apparent in universities, where the percentages of outside departments teaching courses in statistics or computer science are as follows:

DEPARTMENT WHERE TAUGHT	PERCENTAGE OF UNIVERSITY DEPARTMENTS TEACHING COURSES IN	
	STATISTICS	COMPUTER SCIENCE
Biological sciences	15	2
Physical sciences	2	0
Engineering	26	53
Agriculture	23	0
Education	44	2
Business administration	61	16
Social sciences	58	2
Other	5	2

Undoubtedly there will continue to be a demand and thus a place for such elementary statistics courses oriented toward applications within given single fields. Statistical practice is, however, becoming increasingly sophisticated and hence dependent on deeper knowledge of mathematical statistics and its underlying probability theory than such elementary application-oriented courses can hope to supply. While there are undoubtedly many mathematics professors competent to teach basic undergraduate courses in probability theory, relatively few have any special training or interest in mathematical statistics or its applications. Furthermore, for reasons that are not entirely understood, the number of PhD's in mathematical statistics being produced in the United States today is relatively small (fewer than 10 percent of those being produced in the mathematical sciences generally, COSRIMS' Graduate Panel has estimated). At the same time, the United States has several universities with strong PhD programs in mathematical statistics that are by no means swamped with able students. There seems to be a genuine problem in attracting more first-rate men to do PhD work in the field.

One difficulty here may lie in the smaller opportunity for students to major in statistics at the undergraduate level. Through a questionnaire sent out by a member of this Panel it was found, for academic year 1965–1966, that of 71 responding institutions that offer PhD degrees in statistics, only approximately 30 offer undergraduate programs leading to majors in statistics, although a few others do have a "statistics options" within their undergraduate mathematics major programs. To improve the teaching of probability and statistics in institutions where the mathematical part of such teaching will be done largely in the department of mathematics, the Committee on the Undergraduate Program in Mathematics (CUPM) and

the Committee of Presidents of Statistical Societies (COPSS) have recently agreed to form a committee to review the mathematical curriculum in statistics and to develop programs appropriate for institutions in which there is demand for such courses.

In undergraduate statistics programs the emphasis is generally on mathematical statistics rather than on statistics oriented toward specific subject matters, and the problem arises of providing more extended programs of training in *applied* statistics. There is a wide demand in government and industry for statisticians who not only have sound backgrounds in mathematical statistics but also know something about computers, and most important, a great deal about the numerical statistics arising from a particular subject matter, be it health, vital statistics, employment, money and banking, or education. The people involved in these positions need not be talented mathematicians, though there are important mathematical aspects of their work; but several other disciplines are involved in their training. The American Statistical Association has asked the Social Science Research Council to create a committee to study the problems of the training and the supply of manpower for such positions. Whatever the eventual training program may turn out to be, it is certain that it will represent new demands on the academic mathematical statistician with specialized training. It is also certain that this manpower pool is in very short supply at present.

Mathematics for the Biological and Social Sciences

Increasingly, students going on to graduate work in any of these areas are expected to know a substantial amount of mathematics. However, the traditional mathematics sequence that was developed either for future mathematicians or for physical scientists is not ideally suited to students within these newer areas. The usual core mathematics sequence of the first two years chooses its applications primarily from the physical sciences and emphasizes those areas of mathematics that are of particular importance to the physical scientists. The biological and social scientists must learn a good deal of mathematics, but not necessarily in the same depth or with the same emphasis as the physical scientist (see, for example, references 12 and 13). This will require new types of courses for the new clientele, with emphasis on different types of applications.

Recommendations of the CUPM[13] in the biological, management, and social sciences were drawn up by a panel consisting of mathe-

maticians particularly knowledgeable in these fields, together with subject-matter experts from these fields. The Panel unanimously recommended a basic sequence of four semester courses to be taken in addition to work in statistics. These courses are specially designed to meet the needs of students outside the physical sciences and differ considerably from the usual course sequence.

These recommendations may be difficult to implement, partly because of the significant task involved in designing the courses, but mostly because of a lack of mathematicians sufficiently qualified and interested in teaching the recommended courses. Even if we succeeded in producing 10,000 new college professors of mathematics, there would be no automatic guarantee that these recommended courses could be properly staffed. The traditional training programs for college professors of mathematics normally ignore applications, especially applications outside the physical sciences.

Computer Science

All estimates made so far of the new demands placed on the mathematical sciences by the computer revolution are likely to prove too conservative. The coming of the high-speed computer will force a drastic change in mathematics education within the next decade, and, increasingly, computers will be used in connection with traditional mathematics courses.

It has been characteristic of mathematics instruction that the instructor has covered all the theory and then illustrated the theory with highly oversimplified examples. In any realistic application, the computations have been too difficult or excessively time-consuming or have required too much routine manipulation by the student. In modern mathematics instruction, much of this formal manipulation can be taken over by the computer. Since students of subjects involving mathematics are not primarily interested in mathematics for its own sake, use of computers in instruction should greatly enhance the motivation of such students by giving them a greater sense of the usefulness of mathematics.

There is already a need for experts in computer science, and this need will become increasingly apparent when, within the next decade, the computer becomes accessible to every undergraduate student. At that time, students will demand at least an introduction to elementary programming along with a little bit of numerical analysis. In large part, the job of training these experts will fall to

faculty members in the mathematical sciences. For the dissemination of elementary computer skills, the mathematics community hopefully will have an increasing reservoir of computer-oriented members entering the profession from the graduate schools. (See the more detailed discussion in Chapter 3, Section 5.)

3 Qualifications of Mathematics Professors

Let us first examine the educational qualifications of the current mathematics teaching staff. A recent survey[5] reveals that the number of current full-time college mathematics professors plus the number of full-time equivalents in part-time mathematics professors is somewhere around 11,000. About 46 percent of these people are employed by universities, the remaining 54 percent by other institutions. The mathematics staffs are divided roughly as follows:

	FULL-TIME	PART-TIME	GRADUATE ASSISTANTS
Universities (U)	48%	7%	45%
Liberal arts colleges (LAC)	78%	17%	4%
Teachers colleges (TC)	78%	6%	16%
Technological schools (TS)	66%	12%	22%

Within the full-time faculties in each kind of institution, the highest attained degrees are approximately as follows:

	DOCTORS			MASTERS			BACHELORS		
	MATH. SCI.	MATH. EDUC.	OTHER	MATH. SCI.	MATH. EDUC.	OTHER	MATH. SCI.	MATH. EDUC.	OTHER
U	73%	2%	1%	21%	1%	—	1%	—	—
LAC	27%	6%	3%	50%	5%	3%	4%	—	1%
TC	13%	9%	1%	51%	15%	3%	7%	—	—
TS	48%	4%	3%	35%	3%	3%	4%	—	—

The same percentages for part-time faculty are:

	DOCTORS			MASTERS			BACHELORS		
	MATH. SCI.	MATH. EDUC.	OTHER	MATH. SCI.	MATH. EDUC.	OTHER	MATH. SCI.	MATH. EDUC.	OTHER
U	33%	1%	4%	30%	7%	4%	13%	6%	1%
LAC	10%	1%	10%	37%	10%	16%	12%	1%	2%
TC	—	5%	5%	42%	18%	24%	5%	—	—
TS	27%	3%	11%	37%	5%	5%	8%	1%	2%

The greatest impact of graduate assistants is, of course, at the universities. Here are some facts about the teaching load borne by graduate assistants at those universities (about 98 percent of all) where they are employed. In the freshman–sophomore courses, graduate assistants carry

0–19	percent of the load in	22	percent of the universities.
20–39		22	
40–59		32	
60–79		15	
80–100		8	

In the median university, 40 percent of the freshman–sophomore teaching load is carried by graduate assistants. In 38 percent of the universities, at least half of the freshman–sophomore teaching load is carried by graduate assistants.

In only 19 percent of the universities is any part whatever of the junior–senior teaching load carried by graduate assistants. At only 3 percent of the universities does it exceed 10 percent of the junior–senior load.

From these tabulations we may infer that something like three quarters of all mathematics sections at the freshman and sophomore levels throughout the nation are handled by instructors who do not have doctorates in their specialty. This may be closely related to some of the criticisms leveled against mathematics teaching in colleges and universities, especially since most of the nonmathematical clients of mathematicians take all or at least a majority of their mathematical training in freshman and sophomore courses.

The problem is aggravated by the knowledge explosion within mathematics. A mathematics PhD, even from a first-rate graduate school, may be completely out of touch with present-day mathematics within 20 years after receiving his PhD degree. And even the fresh PhD has by necessity specialized, generally speaking, and is truly competent in but a certain small fraction of the mathematical specialties. As we have noted before, among the specialties missing in his background are likely to be areas of application, particularly applications in computer science and in the biological and social sciences, not to mention new applications in the physical sciences.

Given the changes taking place in the undergraduate curriculum, competent teachers are needed to present modern undergraduate courses in linear algebra, probability, multidimensional calculus, and the like; these topics are in many cases new to seasoned college staff members.

We conclude that, in addition to a need for drastically increasing the total teaching staff in mathematics, there is an urgent need for updating the qualifications of present teaching staffs.

4 Current Changes in the Teaching of Mathematics

Traditionally the college curriculum in mathematics has consisted of the sequence college algebra, trigonometry, analytic geometry, differential calculus, integral calculus, differential equations, theory of equations, and advanced calculus. As suggested by the case histories in Chapter 2 of this report, this structure, which comfortably regulated mathematics for many years, is undergoing considerable change.

The Committee on the Undergraduate Program in Mathematics (CUPM) has made recommendations[14] for the improvement of training of elementary and secondary teachers, recommendations that have been widely discussed and are now being implemented at many teacher-training institutions. As already noted, in reference 13, the CUPM Panel on the Biological, Management, and Social Sciences has proposed the first special mathematical curriculum for students in these areas. Specialists in these areas simply did not exist a generation ago but are now being turned out by our colleges in small but encouraging numbers. Additional publications of the CUPM include recommendations on the undergraduate mathematical program for engineers and physicists and for work in computing.

The work of the CUPM, as well as of other individuals and groups within the mathematical community concerned with curriculum reform, has been motivated by growing awareness of various criticisms of undergraduate mathematics education both from within and from the outside. Perhaps the commonest of these criticisms have been those listed below.

"Mathematicians orient their mathematics courses primarily for future mathematicians. In particular, mathematicians are either ignorant of the needs of various types of clientele or unsympathetic toward the teaching of applications."

"Future teachers of mathematics are often ignored."

"Mathematics courses are unnecessarily theoretical or stress rigor at the expense of concrete consequences and intuitive understand-

ing. Undergraduates receive little encouragement to develop originality and independence."

"Mathematics professors are insufficiently prepared or their preparation is out of date."

As already referred to in Section 2, in connection with statistics and computer science, the recent survey[10] attempted to find out the extent to which mathematics courses are taught outside of mathematics departments. A sample of colleges was asked whether the following courses are offered by departments outside the mathematical sciences: probability, statistics, logic, computer sciences, calculus, numerical analysis, and optimization techniques. For each of these topics, several schools responded that it was taught in a department outside the mathematics department. Local conditions may in some cases justify such practices—even dictate them—but these practices seem to be developing too widely to be ignored by mathematicians.

For example, it was learned that statistics courses are offered in departments of biological science, physical science, engineering, agriculture, education, business administration, social science, and others. Probability theory is offered somewhere in nearly all these types of departments. There was also considerable evidence that computer-science courses and some calculus-type courses are being taught outside of mathematics departments.

It was notable that some engineering schools teach *all* these courses, and, at some institutions, schools of business administration and social science departments teach almost all these courses.

A distinguished physicist was asked by the Panel to comment on what appeared to be typical criticisms of the way mathematicians relate to their students. Here is his reply:

Mathematics courses are becoming increasingly abstract, as they should. However, it is still necessary for the physicist to have some technical facility (e.g., calculation of multiple integrals, qualitative features of the solutions of differential equations, physical interpretation of common vector operators and their integral forms) in order that computational details do not interfere with the appreciation of central physical ideas which are being given theoretical expression. This facility is not well inculcated by the present mathematics programs; it is perhaps necessary for physicists to supplement the students' mathematical training. This is traditionally done at the senior–first-year graduate level in courses on "Methods of Mathematical Physics." However, it might now be necessary to offer supplementary mathematical training, beginning in the freshman year, which would present

those tools which are a required part of a physics major's training. The whole scheme of numerical methods, which are of basic use in machine computations today, might be relegated to such a course.

Much work is required in updating physics texts so as to be consistent with the mathematics being taught in the "better" math courses. The notation alone presents a difficulty.

Mathematicians have presented those topics which they feel are of importance, rather than those which the physicists think are the most useful. This has led to a growing lack of coordination between mathematics and physics training.

In particular, we should resist the temptation of physics departments to offer their own mathematics courses, and this is best done by mathematics departments meeting the problem "half way" but still stressing the essential mathematical concepts.

While the present stage of mathematics-curriculum development throughout the country is in a fluid state and thus not easily described, there is evidence of progress toward meeting these criticisms. There has been a veritable explosion in the writing of new textbooks, many written for courses of an entirely new type. There is evidence that the vast majority of colleges and universities has instituted at least some truly basic reforms in undergraduate mathematics curricula during the past decade. These reforms are greatly aided by the recommendations of the CUPM, by the consultants whom the CUPM sends to institutions that desire them, and by the visiting lecturers of the Mathematical Association of America, the Committee of Presidents of Statistical Societies, the Society for Industrial and Applied Mathematics, and others. It should also be noted that during the past decade university professors have taken a significant interest in the reform of secondary education, and also to some extent the reform of grade school education. During this period there has been more discourse between professors and teachers on mathematical education than ever before, and the discourse has been particularly fruitful on the secondary-school level.

It is important to realize that research developments within mathematics itself can have a profound and beneficial influence on the content and emphasis of undergraduate courses in mathematics. Many areas of pure and applied mathematics, both novel and classical, are currently undergoing extraordinary growth as research areas. New insights and approaches being obtained often yield valuable educational dividends.

Rapid change in mathematical research and mathematics cur-

ricula has sometimes contributed, at the undergraduate level, to lack of communication between college mathematics staffs and the various departments that use mathematics. We do not deny that many mathematics departments place insufficient emphasis on application. But some of the criticism appears to arise from the fact that mathematical training of professors in client-departments is out of date. They berate mathematics departments because their students are not familiar with the techniques that the professors learned when they were in college. But the students often know the same techniques in more modern guise, or they have learned more powerful techniques not known to their professors.

We therefore conclude that, although many of the criticisms leveled against college mathematics teaching are justified, we have already witnessed some significant improvement. But there is need for a great deal more work. We shall, under our recommendations, make some concrete proposals for the use of possible new funds to accelerate improvement.

5 Methods Used to Relieve the Shortage of Teachers

Recent years have seen significant experimentation with various mechanical means of relieving the shortage of teachers—most notably films, television, and programmed instruction. We have reviewed a variety of reports in these areas and must conclude that they have thus far not had any significant effect in relieving the shortage of teachers.

Educational media such as television, films, and programmed instruction have been utilized at the college level primarily for enrichment or for remedial purposes. Information about the current state of evaluation of these experiments is scanty. The Subcommittee on Educational T.V. of the Committee on Educational Media of the Mathematical Association of America (MAA), in a preliminary draft of a report on this topic, has emphasized the incompleteness of data to which they have had access, especially noting the omission of sources of broadcasted material. Their general findings indicate that televised courses appear to be less favorably received by college students and their instructors than by very young viewers and out-of-school adults. In particular, they report that student attitudes

seem more favorable toward televised instruction in science and other courses in which visual demonstrations seem to be more spectacular than in mathematics.

We must emphasize that we in mathematics have not yet taken full advantage of the skills of the motion-picture industry or of the broadcasting media in the way that the Bell System's fine series of science films (beginning with "Our Mr. Sun") has pointed. We have especially not tried this for an entire course in mathematics. These remarks do not criticize television or film courses in mathematics but rather suggest what has not yet been tried.

We would like to suggest that a major experiment, involving outstanding motion-picture or television directors and first-rate mathematicians, working full time for an extended period, using professional writers to produce several versions of a script, might be highly successful. This would have to be well funded, of course. Exciting illustrations would be required, suggesting physical meaning for abstract ideas. The team should be free to run many experiments and to discard most of the film actually shot. While such experimentation would require a generous budget, its results would be useful in hundreds of institutions. Until it is tried, we are in no position to evaluate the potential of filmed courses in mathematics.

The following quotation is from *Programmed Learning and Mathematical Education,* by Kenneth O. May of the Committee on Educational Media of the MAA (October 1964, Berkeley, California), a report concerned solely with programmed learning in mathematics at the college level:

> A teaching system for college mathematics must take into account the full range of objectives, many of which cannot be tested easily or objectively. It must reflect an apparent contradiction in mathematics itself, the fact that mathematics requires on the one hand accurate and even automatic application of existing rules, algorithms, and theories, and on the other hand, insight, imagination, originality, trial and error. These twin aspects are present at every level of mathematical education and practice. Our problem is to find ways to teach both in the context of increasing demands and a growing teacher shortage. The solution is not to narrow our goals to those compatible with some instructional device, but to experiment with a variety of devices without abandoning those that have proved themselves capable in the past.

The essence of May's report is that printed programs are useful only as auxiliary study aids, not in place of textbooks or teachers. We conclude that mathematics, with its stress on abstract reasoning,

is a poor subject for the use of programmed material. At the same time, vigorous experimentation in writing and using a variety of special-purpose auxiliary materials is urged. The suggestion is made that college mathematics texts might well incorporate some of the devices developed by the programming movement while maintaining the present exposition–problem pattern.

There is a device of a different sort, however, which has been highly successful at a number of institutions. This is the large lecture section taught by a senior (and hopefully outstanding) member of the faculty, with discussion sections handled either by junior members of the faculty or by trained graduate students. The patterns in which lecture courses are taught vary greatly from institution to institution and often depend on existing physical facilities and the preferences of local staffs. The following may be a typical example: A course previously given in six sections of 25 students is now organized into a single lecture section of 150 students. The three-semester-hour course meets twice a week with the lecturer, and then for its third meeting is divided into 10 discussion sections of 15 students each, which are handled by graduate assistants. Thus, instead of requiring six regular faculty members, each to take one section, a single experienced faculty member presents the lectures and is assisted by three graduate assistants. This is a clear saving of manpower and money, and preliminary local studies made at several institutions indicate that, since multiple underclass sections are often taught by inexperienced staff members, substituting a single experienced faculty member often actually leads to an improvement in the quality of the course.

We regret the fact that this device is not in use more generally; one reason for this is the lack of suitable lecture rooms, and we shall comment on this under the discussion of physical facilities. We should also warn that not all faculty members are successful with large lecture sections, and much depends on the attitude of graduate assistants.

In general we conclude that:

1. Mechanical devices, as presently conceived, are not likely to lead to a significant relief in the teacher shortage in the near future.

2. Much can be done to use existing staffs more efficiently.

3. Significant further experimentation with mechanical and visual aids (particularly films) seems indicated.

4. We shall need many thousands of new highly qualified mathematics professors.

6 Keeping Mathematics Teachers "Alive"

Many of the weaker mathematics departments think that their principal problem is attracting faculty members with strong preparation, preferably PhD's, from well-known universities. If they are fortunate enough to attract one or more such people, they feel that their worst problems are solved. In fact, such an acquisition may be only the beginning of some problems that are new to local administrations.

The new PhD will be anxious to speak to other experts in his specialty, since most mathematicians dislike working in isolation. If his new institution is not near one of the major mathematical centers, he will probably either leave after a short time or become mathematically inactive and lose touch with current research. These problems will be discussed in more detail in Chapter 5.

We now raise our final question concerning fundamental problems of staffing. Even if a smaller or weaker institution attracts a first-rate staff member, how does it keep him, and how does it keep him mathematically alive? A number of methods are now in use, many of which depend on outside funds for their continuation. Some of the most popular are:

1. Regular attendance at mathematical conferences
2. Visits to other campuses
3. Departmental seminars, with outside speakers
4. Attendance in courses or seminars at nearby universities
5. Attendance at summer or academic-year institutes
6. Granting of leaves for further study or research
7. Participation in curriculum reform

The first three of these are most easily arranged and require only modest expenditures. It is therefore discouraging to find how little most college mathematics departments are allowed to spend for such purposes. Small federal or foundation subsidies would reap significant benefits in this area.

Item 4 depends heavily on the geographic location of the institution, as does item 3 to some extent. The most effective means,

items 5 and 6, have serious drawbacks. They are expensive, and academic-year programs remove the faculty member from a campus where he is desperately needed. We note that the federal funds spent for items 5 and 6 are much too small. While vast sums are being spent on the retraining of secondary teachers, corresponding programs for college teachers are unrealistically small and inadequately financed. It is difficult for the faculty member who most needs retraining to obtain a fellowship for this purpose. We shall return to this problem in Chapter 6.

Two programs with possibilities for widespread impact on the improvement of undergraduate mathematical education are suggested here. The more ambitious of the two involves providing support for a sort of "lend-lease" program, in which a senior member of a department with a strong undergraduate as well as research program would visit for a semester or a year at a smaller college. The visitor would combine some teaching in the department with conducting a seminar for staff members. In this way, the introduction of new courses or improvement of existing programs would have a good chance of having continued effect after the visitor's departure. The most important ingredient of such a plan would, of course, be the enthusiasm and willingness of the visitor to participate, but funds for providing arrangements for his temporary replacement at the home institution would be needed. In implementing this program, one of the professional organizations, such as the Mathematical Association of America, might effectively serve as a clearinghouse for establishing communication between interested host institutions and potential sources of visiting professors.

The second major proposal is for setting up research instructorships for new PhD's, to be administered by the parent institution from which the research instructor would be selected, with, of course, cooperative arrangements with the visited smaller or weaker institution on whose faculty the candidate would serve. The research tie-in with the candidate's graduate department would provide the twofold advantage of continuing research efforts and opportunities for discussion of early teaching and curriculum questions. The visited department would, in turn, gain by the added contact, if only indirectly, with other members of the candidate's institution. It is suggested that team teaching in the visited department be considered, in which event, for example, a more experienced staff member of the faculty might help the new teacher in pointing out to prospective high school teachers the relevance of some more advanced

topics to the teaching of high school mathematics. In general, the young PhD might be more successful in offering a course at the junior–senior level than in offering one for beginning students.

An obvious dividend hoped for in the research instructorship plan is that of providing the young PhD with increased motivation for college teaching as a career.

Both these plans promise to make an impact on an entire department over a significant period of time. They are designed to strengthen the weak institution while "stealing" from the center of strength, where there should be less difficulty in staffing. Both plans depend heavily on the attitude of the mathematical community toward weaker institutions and on the availability of outside funds.

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5

Special Areas of Concern

1 Problems of Isolation

The individual mathematicians of a college department of mathematics need contact with other workers in the field. Mathematics flourishes only when it is communicated. In this field, as in the laboratory sciences, one needs to test one's ideas by experiment. Mathematical experiments, of course, require no elaborate equipment—only the sounding board provided by other minds.

The problem of isolation is serious for small departments of mathematics, especially those that are also geographicaly remote. Contact through printed material does not meet the problem. Where a department is small and isolated it may help to choose faculty members with closely related mathematical interests, so that they can at least talk professionally with one another, rather than to strive for the broader coverage given by a faculty with diverse interests. A teacher can also gain some mathematical stimulus and challenge from his own specially gifted students. But these measures do not really suffice.

The National Opinion Research Center (NORC) survey[1] reports that four out of five department spokesmen felt that members of their departments did not have enough to do with mathematics and mathematicians elsewhere and that greater communication with the off-campus world was called for. The two factors held most responsible for these inadequacies were heavy workload and limited funds. Of the various means suggested to remedy the situation, more contact with professional organizations and closer work with other academic

institutions and their faculties were frequently cited. The possibility of more visiting mathematicians was also noted. The survey reported that departments gave evidence of their attempts to compensate for lack of contact through increased membership in professional organizations, continued formal study, and use of the existing Visiting Lecturer Program of the Mathematical Association of America (MAA).

The Visiting Lecturer Program of the MAA has been of service in providing opportunity for a limited number of colleges to benefit from the presence of distinguished mathematicians who spend a day or two on campus giving talks on mathematics and meeting with students and faculty. The various regional meetings of the sections of the MAA help to meet the need for sharing mathematical ideas and problems of undergraduate mathematics teaching. The MAA sections usually meet once or twice yearly at centrally located institutions.

There is a growing tendency toward joint regional enterprises among neighboring institutions for the purpose of furthering mathematical research and discussing common problems of undergraduate mathematical instruction. Swarthmore College, Haverford College, Bryn Mawr, and the University of Pennsylvania have combined forces for such purposes. Another example is the Connecticut Valley group of colleges originating with Mount Holyoke, Amherst, Smith, and the University of Massachusetts, but now including several others—Williams, Wesleyan University, American International College, Trinity, and Connecticut College.

Now that the impact of recent developments in school mathematics is making itself felt more universally with entering college students evidencing better and more advanced mathematical training, the need for joint exchange of ideas with other college mathematics departments will be especially acute for isolated and small college staffs. This will, in turn, entail enlistment of creative mathematical minds to offer leadership in attacking the problem of continued education or even retraining of college teachers from this group of colleges. The Committee on the Undergraduate Program in Mathematics (CUPM) of the MAA, which has issued reports and a variety of recommendations for undergraduate mathematics programs, is at work on a series of recommendations for the qualification of college teachers at the various levels of college and university instruction. It seems clear that much of the reorganization and upgrading of such mathematics staffs can best be effected by joint efforts of individuals in this group either in academic-year or sum-

mer institutes, where they might share common problems and curriculum innovations as well as extend their mathematical backgrounds.

2 Women in the Mathematical Sciences

A general study[2] by the Carnegie Foundation for the Advancement of Teaching calls attention to women as an important source of additional talent for college teaching. This study finds that, during the period between the two World Wars, women consistently received 15 percent of all doctorates awarded, but that by the decade of the 1950's this percentage had sunk to about 10 percent. During the 1950's, women accounted for about 32 percent of all BA's and about 32 percent of all MA's, according to a study by Clarence B. Lindquist.[3]

For chemistry, mathematics, physics, and psychology over the 15-year period from 1948 to 1962, the Committee on the Undergraduate Program in Mathematics reports[4] the percentages of those obtaining various degrees who were women:

	BA	MA	PHD
Chemistry	19	13	5
Mathematics	29	19	5
Physics	5	4	2
Psychology	42	31	15

Thus, while the percentage of women among the BA's in mathematics (29 percent) is close to the percentage of women among the BA's in all fields, the percentage of women among the PhD's in mathematics (5 percent) is only about half the percentage of women among the PhD's in all fields. Physics and chemistry also have a low percentage of women PhD's. The BA and PhD totals for men and women during the period 1948–1962 reported in this same study suggest that fewer than one sixth as many women as men go on from a mathematics BA degree to get a mathematics PhD degree.

To utilize the potential of women mathematicians who have interrupted their careers during the child-bearing years and wish to re-enter the profession, retraining programs are needed. The colleges and universities should assume responsibility for initiating and structuring such opportunities for women. Support, both

private and federal, for strong action along these lines should be forthcoming; substantial returns on the investment seem assured. In constructing such programs, we must recognize the unusual needs of the prospective woman scholar—for example, in addition to scholarship funds, funds for responsible baby-sitters may be essential not only to obtain time for classes but also time for study. As a special case, the type of program involving part-time graduate study for women in a position to resume advanced-level mathematical work in teaching or industry and government recommends itself to the National Science Foundation.

The reasons for the decline of women PhD's in all fields, the especially low percentage of women among those obtaining PhD's in the physical sciences and mathematics, and the much smaller fraction of women than men who go on from BA to PhD degrees in mathematics are not entirely clear. Undoubtedly many of the women who stop at the BA in mathematics go into grade school or high school teaching, or marry and have families, with no further professional participation.

Proportionally many more women undergraduate mathematics majors than men are currently being dissuaded from graduate study by the tempting offers of high-salary jobs in industry and government. Thus the NORC survey[1] found that, while women in nonacademic mathematical work constituted the same percentage of the total work force that they did in college mathematics teaching, the women in the age bracket of 35 years or younger accounted for only 3 percent of the total mathematics college faculty of this age group, although they constituted about one sixth of the total work force in this age bracket. The same survey points out that female college teachers have disproportionate numbers in the older age brackets and relatively few in the younger age groups. Thus, of the group in the survey, 15 percent of the female teachers were 65 or older, compared with 4 percent of the male teachers who were in this age group, while only 5 percent of the female teachers were in the age group of 34 years or younger, compared with 21 percent of male teachers.

The Panel concludes that women represent a major source for increasing the number of college teachers of mathematics (and of the sciences). Ways to encourage women to consider academic careers in mathematics should be explored. In view of the existing shortage of qualified academic mathematicians, society needs to make every effort to encourage able women undergraduate majors

in mathematics to continue with graduate study and give serious thought to college teaching. Marriage and family raising need not mean interruption of professional careers in mathematics. There are young women who successfully combine the two, and in general this is easier in mathematics than in a laboratory science. One factor contributing to the successful combination has been the availability of part-time employment of women in the mathematical sciences by government and industry, a practice that might be more widely adopted by educational institutions.

Women are increasingly being urged to consider resuming professional work when their family responsibilities begin to taper off. More has been said than done about implementing these suggestions for those re-entering mathematical activities. A highly successful exception is the program at Rutgers University, under Ford Foundation and National Science Foundation support, to provide retraining for women interested in the computer field or in high school teaching. This program might serve as a model for other such ventures under both private and government support. In particular, for women planning to re-enter college or junior college teaching, opportunity for part-time graduate study as a preliminary is essential. Finally, relaxation and eventual removal of outdated nepotism rules enforced at many colleges and universities should be urged.

3 The Underdeveloped Institutions

Elsewhere in this report (Chapter 4, Section 3) it is noted that 40 percent of all liberal arts colleges have no staff members with mathematics doctorates, and 32 percent have only one such staff member. Among the institutions in which underdevelopment is most acute are most of the schools operated predominantly for Negroes, including many church schools as well as state institutions. Underdeveloped institutions are to be found in large numbers in all sections of the nation, but there is a heavy concentration of them in the South. One could hardly list all the factors influencing such a growth pattern for an institution of higher learning, yet quickly identifiable as causative factors are such outside forces as inadequate financial support for institutions and limited opportunities for their graduates and such inside forces as nonacademically inclined administrators, poorly trained and uninterested faculty members,

and students who are insufficiently motivated or severely limited in potential. Perhaps the most damaging factor has been the perpetuation of mediocrity by faculty members who have mistakenly adjudged themselves to be competent and even outstanding educators. These factors notwithstanding, it is significant to note that there is now strong motivation within these institutions to improve the quality of their educational activities. Determining how this goal can best be accomplished is the task at hand.

Since practically all these seriously underdeveloped institutions have programs in which mathematics is taught or used, and since these institutions are involved in preparing teachers for tomorrow, it is extremely important to the mathematical community that some of their problems be identified and understood, and that the nation as a whole rally to assist these institutions in their attempt to solve these problems. We shall discuss in depth two problems with broad implications related to mathematical instruction that are faced by underdeveloped institutions. We shall include some specific comments on these schools and then make some recommendations on how funds, federal or other, could be used to help solve some of their problems.

While the problems of staffing, physical facilities, and isolation considered in other sections of this report are particularly acute for the underdeveloped institutions, special consideration will be given here to understanding some of the attitudes and needs of teachers and students in these institutions.

Many of the teachers presently holding professional positions in mathematics in underdeveloped institutions received their formal training in first-class institutions, and upon graduation showed considerable promise. Later, however, they let themselves become mathematically stagnant through failure to continue with mathematical activities. Many other mathematics teachers in these institutions received all their formal training in underdeveloped institutions. Subsequent to this training, they were employed to teach and felt their training to be adequate. These judgments of adequacy were to a large extent rooted in limited training in places isolated from more advanced mathematical activities. For the most part, these people mean well and are respected and responsible teachers; but, in newer concepts, up-to-date exercises, and generalization, they just do not have much to teach.

What are we to do about these teachers? Only two alternatives present themselves. Either forget the present college teacher and

prepare better teachers for the next generation, or take steps to improve the competence of those already teaching, and ensure that the next generation of teachers does not have the same problem.

The latter is the only realistic alternative, because the present staffs are deeply involved in preparing the future generation of teachers. The cycle can be broken only by starting with the present staffs. But how do we start the retraining of teachers who (1) do not recognize the need for retraining, (2) cannot afford to leave their positions for any long period of time without substantial financial assistance, (3) have no replacements even if they could afford to leave, and (4) have a sense of insecurity and would reject help unless approached in a manner acceptable to them?

Probably the only effective way to start a good retraining program is to get these teachers to evaluate themselves and recognize their own limitations. This may be more easily said than done. Certainly an individual who has kept up with advances in mathematics can easily spot deficiencies in teachers in these underdeveloped institutions. But such a teacher may be blind to his own deficiencies. A few of his graduates have had successful careers in industry or government. He has had very few, if any, complaints from his present or former students and can probably truthfully boast that he has never had an opportunity in the classroom to teach all he knows. Actually, because of the press of his everyday teaching he has not had a chance to explore new or different ideas. He can also point with pride to a few of his graduates who have been admitted to first-class institutions for graduate training in mathematics. Little does he realize, however, that in almost all these cases his graduates are admitted on a conditional basis—the condition being that the student must take a year or so of additional undergraduate work to be able to compete with students from more-developed institutions. Many of these mathematics teachers can honestly see no need for any additional formal training for themselves. Somehow they must be brought to realize that, though their present efforts are productive, they can be made more productive through retraining.

What can be done to provide the training needed? We propose that the underdeveloped institutions provide released time during the summers. Released time as used here does not mean that the teachers are taken off the payroll, but rather that they are provided with income sufficient to cover their basic needs and responsibilities while they study. In the same vein, we believe that summer institutes, such as the present National Science Foundation institutes

for college teachers, are valuable and should be continued. But they are attractive only to teachers forced to go to school or to teachers who would otherwise be unemployed during the summer and need the stipend. Few teachers with financial responsibilities would freely assume more financial responsibilities to attend school for a stipend of $75.00 a week, when they can work and earn more money and have no additional expenses. A more realistic summer stipend for college teachers attending summer institutes is essential.

At the present time, there are several fine programs designed to help bring active college mathematics teachers into contact with the principal advances in mathematics. The total number of these, however, is certainly inadequate for the job to be done. And even in the existing few, many young, ambitious teachers with extraordinary potential are denied participation mainly because of two factors. First, many administrators, when asked to recommend faculty members for such programs, select their senior teachers to participate. Second, when applications are sent in directly by faculty members, these must usually be supported by written recommendations; and if the faculty members concerned have never had contact with well-known mathematicians who will recommend them, their chances for acceptance are poor. Somehow, the criteria for admittance to these programs must be broadened, and the funds available must be increased, to admit young faculty members who could benefit but are otherwise unknown.

In addition to specific course work for teachers, some aid must be given in helping to create a favorable mathematical atmosphere at the less-developed institutions. Among the various techniques that could be used is that of providing the services of an active mathematician to the campus. A one-, two-, or three-day lecture series is of questionable value unless the audience is doing research in, or at least is interested in, the research specialty of the lecturer. This is not to say that the various existing visiting-lecturer programs are not valuable, for they are and should be continued. But their effectiveness could certainly be enhanced by advance contact between the lecturer and the teachers and students who are to be his audience, to determine whether the majority of them have mathematical backgrounds appropriate to his lecture. Where they do not, he could make adjustments in his topics to fit the situation. He might also mail to the campus to be visited outlines of his intended presentations in time for the audience to become familiar with particular topics and terminologies. Finally, he might have detailed manu-

scripts prepared, to be passed out after his presentations. The various agencies supporting visiting-lecturer programs should be encouraged to finance necessary paper work and mailing expenses.

Lecturers are more effective, of course, when they are themselves broadly familiar with recent trends in the mathematical sciences and how they are affecting undergraduate education in the mathematical sciences. When the opportunity arises, lecturers should take time to discuss these trends with the appropriate administrators of institutions visited. In some cases, a few remarks from a visiting lecturer may do more toward improving the situation than would frequent remarks coming from the regular staff.

Much more desirable would be to have a visiting research mathematician on the campus for a period of not less than a year. One activity such a visiting professor could conduct would be a series of informal seminars involving both teachers and students. Since it is likely that the students and many teachers will never have seen the actions and reactions of a research mathematician at work, he should assume nothing and carefully build within his audience a desire to learn mathematics and to seek out information for themselves. After this, he could propose problems based on the materials covered and on which the students could work independently. Finally, individual members of the audience could write up their results and present them in a seminar atmosphere. In any event, faculty members at the host institution must have enough free time to be able to profit from the presence of the visitor.

Another method that could be used is to identify underdeveloped institutions located in the shadows of leading universities and to use faculty personnel from those universities to conduct specific courses for the teachers of the underdeveloped institutions, based on the interests of these teachers. The courses should have full graduate approval of the leading universities, and grades should be issued through those universities. Furthermore, administrators on the campuses of the developing institutions should be made aware of steps taken by faculty members to improve themselves and should be urged to reward the effort in both salary and promotion.

A few words of caution are in order here. Teachers should not be forced into any course. And, at least at the outset, a lecturing professor must be patient and deliberate in explaining details, involving the teachers in such a way that he always knows whether he is communicating with them. If the participating teachers are all Negroes and the major university involved is predominantly for whites, some

care should be taken in selecting the place where the course is to be offered. It may be that it would be better to teach the course on the campus of the developing institution in order to avoid aggravating the sensitivities and insecurities apparent in many of those teachers.

It is worth noting that most of the developing institutions predominantly for Negroes are located in the South, and that many Negro teachers from these institutions would attend major universities in their respective states if they would be fully accepted in these universities. Unfortunately, some professors are still not in favor of desegregated higher education. There are, however, many mathematics professors in the South who are concerned with the plight of the developing Negro institutions and who would make significant contributions toward alleviating some of these problems if they were asked to do so. Since there is no simple way to identify these people, we recommend that, through some large organization in the mathematical community, interested professors be urged to volunteer their help in creating an atmosphere favorable to mathematics in developing institutions.

Many things could be done in addition to formal classroom activities to help teachers at underdeveloped institutions. For example, a professor from an advanced institution might involve one or two of these teachers in some research or in developing new teaching materials. The possibilities here are as varied as the many people wishing to become involved.

In addition to the attention given to present teachers, special concern should be given to preparing the next generation of teachers for these underdeveloped schools. Many effective programs for preparing future college teachers are now in existence. The chances are, however, that the better teachers from these programs will seek out positions and be accepted in institutions that are already well advanced. More programs should be designed to provide reasonable assurance that the teachers produced will spend some time in some developing institution. One way to implement such a program would be to select several mathematics centers staffed by able mathematicians who are expert teachers; then, from a consortium of developing institutions, select bright young graduates with majors in mathematics and send them to one of the centers to work for the master's degree. The average time necessary to earn such a degree could conceivably be two years. During this time the student would be supported completely, with the stipulation that when he com-

pletes some phase of his formal training, which may include earning the PhD degree, he must work in a developing institution of his choice for a time equal to that for which he was supported, or else he must pay back the money invested in his education. Admitting 200 or more to such a program each year could put 100 or more well-trained young mathematics teachers in the classrooms of developing institutions each year.

Inadequately prepared teachers represent only one phase of our special problem. To create a favorable atmosphere for mathematics in a school, one must have in addition to qualified teachers some good mathematics students in attendance. Here the underdeveloped institutions differ greatly from the developed ones in that they do not get, and cannot expect to get, many students with proven potential for mathematics. Yet we must be mindful that some students who enter after demonstrating only limited mathematical potential later turn out to be good mathematics students and still later may become valuable assets to the mathematical community.

Ordinarily when students enter an institution they are assigned various courses in accordance with their performances on some standardized tests. Most of these tests, however, are geared to measure educational and cultural experiences. Since many of the students admitted at underdeveloped institutions are lacking in these experiences, they are often placed in remedial courses. It has become increasingly clear that a more reliable method for distinguishing between mathematical potential and educational experiences is urgently needed. Closely associated with this is the need to develop techniques for raising aspirations, promoting energetic performance, and stimulating motivation toward achievement in mathematical sciences, particularly among the culturally deprived.

One of the most difficult problems caused by admitting poorly qualified students to the underdeveloped institutions is that of finding suitable text materials. There are popular texts written for a wide range of students. Most of these, however, are not suited to many of the poorly prepared freshmen. It seems that suitable materials could be written only after taking into account (1) what experiences are lacking in students' backgrounds, (2) to what extent reading is a factor in learning to do mathematics and use mathematics, (3) what ~~up-to-date~~ problems from the mathematical sciences would be useful in reviewing certain basic skills, disseminating certain general-education information, and introducing new vocations, and, finally, (4) what vocabulary would be most effective in

communicating with these students. Since the answers to many of these questions will have regional variations, it may be necessary to prepare materials to be used in specific regions. We recommend that funds be made available for serious study of the problems presented by poorly qualified undergraduate students in the developing institutions, as well as for the support of writing teams to produce experimental materials to be used in teaching these students and perhaps to be developed in the process of working with such students.

To summarize, federal or other funds could be effectively used to improve undergraduate mathematical programs at underdeveloped institutions as follows:

1. By providing study grants to present faculty members who are interested in teaching up-to-date college-level courses but who are not adequately prepared to do so.
2. By bringing competent research people and perhaps some of their graduate students to developing institutions for periods of a year or more to help create a mathematically stimulating atmosphere.
3. By establishing mathematics centers for the specific purpose of training some of the next generation of mathematics teachers for the developing institutions.
4. By sponsoring basic research in better techniques for separating college potential from educational and cultural experiences.
5. By creating precollege centers for the purpose of raising aspirations and conditioning college-bound students who have college potential but who are victims of cultural deprivation.
6. By making scholarships available so that institutions that have recently improved very considerably in quality may attract more able students.
7. By providing released time to faculty for doing mathematical research, participating in seminars, developing new teaching materials, or working with groups of secondary-school teachers in improvement programs.
8. By providing students with funds necessary to work with mathematics teams during the summer and thus become more involved in mathematics.
9. By providing the services of experts in testing and learning so that new methods of measuring achievement and new methods of teaching may be developed.
10. By establishing a center through which information on fellow-

ships, new job opportunities, and other relevant matters can be collected and mailed directly to the members of the mathematical community. We are concerned about the teacher's knowing about opportunities to keep himself mathematically alive and having up-to-date information to use in his role as a counselor.

4 Changing Background of the Entering Student

The reform of school mathematics and science teaching has received much publicity. It is, therefore, worth pointing out that the entire reform movement started in mathematics, and that it started before Sputnik. After notable group efforts at a small number of institutions (e.g., the University of Illinois), there followed the report of the Commission of Mathematics of the College Entrance Examination Board in 1955. This blueprint for updating secondary-school mathematics served as a basis for future discussions of reform.

We must mention Professor J. R. Zacharias of MIT, who started the Physical Sciences Study Committee (PSSC) and who was a key figure in promoting recognition of curriculum reform in secondary-school science teaching as an important national problem. A major effort to implement the mathematics recommendations was carried out by the School Mathematics Study Group (SMSG), under the leadership of Professor E. G. Begle. The magnitude of this undertaking may be seen from the fact that over a period of years some 250 college and university mathematics teachers participated in the writing project—and the resulting experimental text materials have sold three million copies.

The changes brought into secondary-school mathematics curricula have had a tremendous effect in increasing the amount and the quality of mathematics taught in the nation. We shall document this below by showing how much the ability of freshmen and sophomores to take college mathematics courses earlier has increased. We shall not discuss the question of how much and what kinds of additional changes may be needed. However, the country is not likely to return to the curriculum that was standard a decade ago. We shall therefore examine recent changes in the backgrounds of entering college students.

The first step in reforming mathematics instruction was a weeding-out process. Most topics in traditional high school mathematics

are still important and were recast in a more modern framework and terminology. The time saved was used partly to introduce more modern topics and partly to move topics traditionally reserved for college mathematics down into the secondary schools. Throughout, the emphasis was shifted from rote learning of techniques to understanding of basic principles. Subsequently, the reform movement spread from the senior high school to junior high school and the elementary school.

Since college students enter with up to 12 years of training in mathematics, extensive improvements in this training program were bound to have radical effects on the preparation of entering students. There has been a systematic shifting of college mathematics courses to earlier years. We have seen in our case histories that in some schools a significant fraction of freshmen enter already knowing a semester or a year of calculus. The extent to which this is felt throughout the curriculum is shown by the following table. It shows estimated changes in enrollments in key courses over a five-year period at our universities:

COURSE	FRESHMEN	SOPHOMORES	JUNIORS-SENIORS
Calculus			
1960	49,000	49,100	500
1965	106,000	49,900	1,500
Differential Equations			
1960		7,600	8,200
1965		10,200	5,600
Advanced Calculus			
1960		50	8,900
1965		1,100	10,400
Probability			
1960	0	0	1,500
1965	1,200	600	3,400
Linear Algebra			
1960	0	0	2,400
1965	100	1,300	7,400

The following percentages of entering freshmen taking mathematics took courses at various levels in the fall of 1960 and 1965, respectively:

ENTERING FRESHMEN TAKING COURSES ON THE LEVEL:	AT UNIVERSITIES		AT ALL INSTITUTIONS	
	1960	1965	1960	1965
Below college algebra and trigonometry	17%	12%	25%	18%
Of college algebra and trigonometry	61%	52%	53%	49%
Of analytic geometry, calculus, or higher	22%	36%	22%	33%

These tables document both the significant increase in enrollments in mathematics courses and the shift of courses to earlier years, as well as the higher level of the incoming freshman.

The reforms are still in a state of flux. While many students now enter with a year of calculus, we still find that a large percentage of entering freshmen need a year or more of remedial mathematics before starting college-level work. Thus, while the average preparation has improved, the differences in the preparation of students at a given institution may be much greater than in the past. All these factors contribute to making heavier demands on the mathematics faculty. And so far we have felt only the effect of the high school revolution. The first group of students who studied a revised curriculum from first grade on is now in junior high school. They will find the accelerated high school sequence much too easy and will come to college with even stronger preparation. Throughout the next decade, therefore, we may expect continued revision of the undergraduate mathematics curriculum.

Entering-Freshman Problems

Grave problems still exist at the beginning undergraduate level because students frequently come from secondary schools that have not prepared them for collegiate mathematics. Sometimes the fault lies with the student, sometimes with the school, sometimes with no one. Placing the blame is irrelevant for our discussion.

The Panel agrees that the basic mathematics program in college starts at the calculus level, but the facts of secondary-school education are that a student may not have taken enough mathematics in high school, he may have been poorly motivated, or he may not have

realized the growing importance of mathematics to his career, as mathematics may have crept up on his field more rapidly than his guidance counselors realized. Furthermore, underprivileged groups may not have planned on going to college and now find that opportunities are available for which they are not prepared. What can be done about this lack of preparation?

1. We could set up summer schools across the nation to give ill-prepared graduating high school seniors an opportunity to study mathematics in these summer schools and to receive some financial support. This program would consist of remedial mathematics and new mathematics—new to the student. The reason for the remedial program is that if a student is far behind, he very likely has also had poor training in fundamentals or at best will have forgotten the thread of mathematics. We cannot teach very many mathematics courses to a student at one time, and so these summer institutes might be run in tandem with some other science or computing program.

2. The problem described above is by no means restricted to mathematics or even to the sciences generally. Many students who wish to go to college and who have the capability of making a go of it are ill-prepared for any except the weakest colleges. We need to provide realistic remedial programs that will bring the student forward at a sensible pace and slide him into the regular collegiate system at some point if he is capable and motivated for the work. We need to develop more special programs and to learn better how to bring the young adult up to the pace of his fellows. Junior colleges offer one way to do this, but often they do not pull the capable student forward at the pace necessary to bring him up to his fellows. One special program that has had considerable success in this direction is that of the College of Basic Studies at Boston University. Special teaching groups are used and classes of students are kept together through a two-year program, at the end of which the successful student becomes a regular junior at Boston University. We regard the design of such programs as well beyond the province of this Panel's effort, but we do recognize this important problem and we encourage others to study promising means for its alleviation.

3. In some state schools, although many entering resident students from the state are not trained in the required secondary-school mathematics, funds for remedial noncredit programs have been cut off from the college on the grounds that subjects being taught are sec-

ondary-school level and not collegiate. Since the state has already paid for such teaching in secondary schools, it refuses to pay for it again. Even in a mass-oriented educational system we must remember that the system is not dealing with an unresisting, unchanging piece of raw material that must take its knowledge in just the order that its molders choose. We have to be prepared to give students consideration when they are ready for it, not just when we are ready and feel that they should be. The colleges for which this problem is the greatest are those that need fiscal help and teaching help the most, and those that are likely to have the most students from deprived sectors of society. It is short-sighted not to give remedial help when it is needed and wanted. We recommend that where students are ready for college and otherwise admissible, public funds not be withheld for supporting remedial training merely because it is regarded as secondary-school material.

5 Demands on the College Teacher of Mathematics

The stereotype of the college teacher of mathematics spending his time in contemplative fashion pursuing his twofold task of conveying knowledge and scholarly research is hardly true today—if it was ever so in the past. The mathematics teacher is now called upon for a wide variety of activities, and the resulting pressures can frequently produce unhappy results and frustrations.

One of the major tasks is the development of new curricula and new teaching methods in undergraduate mathematics education. This problem demands a high order of intellectual effort. Knowledge has grown so rapidly that new and shorter paths must be built to concepts and insights of current importance. Proper preparation and planning cannot be mere scissors-and-paste work in choosing topics from existing texts. Fundamental rethinking and reworking of the subject matter are required.

Curriculum development has taken place at various levels. At the national level, considerable amounts of time, talent, and effort have gone into the work of the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America. A series of reports on many aspects of undergraduate education in mathematics have been distributed. These have served, and will serve, as points of departure for many specific programs on individual campuses. The preparation of these reports has involved the

serious efforts of a large number of people in the mathematical community, but the task of curriculum development is not thereby completed. There always remains the task of initiation and implementation at the level of the individual institution and the individual teacher.

In addition to these broad efforts in curriculum development, a number of special programs have been worked out in individual institutions in such less-traditional areas as mathematical statistics, computer sciences, and applied mathematics. In each of these cases, substantial effort has been required.

The number of institutions with graduate programs in the mathematical sciences has increased significantly during the past 25 years. These have involved significant efforts by teachers who might previously have been involved exclusively in undergraduate teaching. Graduate education without associated research activities is impossible. Consequently, the fact that many mathematicians today are actively engaged in research efforts is related to educational as well as scientific requirements.

Thus the teacher of mathematics today is engaged in a wide variety of activities beyond his teaching duties. These may well include carrying out research programs, participating in curriculum reform, advising students, working on general college committees, writing text material, working with local and national groups, and seeking support to carry out these activities. Hardly a teacher can escape from what might at first be thought of as "outside" duties, whether he be talking before the local Parent-Teachers Association or serving on a high-level national committee.

All these demands on time must be met within a 24-hour day and a seven-day week. The problems of reasonable allocation of time are not, of course, exclusively the burden of the professor of mathematics. They are present in most walks of life. Nevertheless, in view of the acute shortage of professional teachers of mathematics, the question of effective use of time cannot be ignored.

There is no universal prescription that will aid all individuals and all institutions. A number of individuals have found that research and effective curriculum and course planning require long periods of concentrated effort, which are not available in the normal academic schedule. For these it has been proposed that consideration be given to rearranging teaching responsibilities during the academic year. A simple first approximation would consist of individual teachers carrying double teaching loads during half of the year. The second part of the year might then be spent on working with ad-

vanced students and research or on curriculum and course development.

With few exceptions, most curriculum and course development is carried out by individual teachers. This may not be the most effective process; therefore exploration and careful study should be given to the development of *teaching centers* at a number of academic institutions. These would offer, among other things, the following: (a) a central file of course outlines, notes, reports, and good ideas developed by individual teachers; (b) a center of communication concerning new materials and approaches to teaching; (c) facilities for development and experimentation with new materials and approaches to teaching. Existence of such centers would lead to significantly greater efficiency in the teaching and learning process, both for institutions and for students.

The practice of sabbatical leaves is by no means universal. The multitude of pressures on teachers, previously described, makes the opportunity for concentrated work on one phase of teaching activity free from other duties almost a necessity rather than a duty. A number of excellent studies on the problem of sabbatical leave have been made in the past, and these indicate that the cost is not so large as might first be assumed. Nevertheless, many institutions would find the introduction of such a program to be a serious added cost, and means must be found to make sabbatical leaves more widely available.

Support for summer research on a broader scale would be of significant help. This is particularly true for younger teachers at institutions that are attempting to develop research programs. These institutions may leave no funds for summer work, and the individuals themselves have not yet reached the level of reputation at which they can acquire their own grants. Help at the early stages of their careers would do much to ensure their ability to achieve their potential as research mathematicians.

Special summer institutes can play a significant role in keeping the mathematician aware of current work in his profession, but reasonable stipends for participants must be available if such opportunities are to be used effectively.

6 Promoting Creativity in Undergraduates

Mathematics is notable as a field of research in which significant contributions are sometimes made at very early ages. It is also argued

that the years between 25 and 35 are the most productive in the life of many research mathematicians. It is important, therefore, that really talented upperclass undergraduates try their hand early at independent creative activity.

We must point here to a problem of terminology. We are talking about independent activity by undergraduates, in which they try their hand at solving interesting problems, but in which the importance of the results obtained in particular individual problems may not be very great. Such activity in other disciplines (especially the laboratory sciences) is called "undergraduate research" and is so recognized by the granting agencies. Mathematicians, on the other hand, tend to reserve the term "research" for original work at higher levels. They have, therefore, tended to cut themselves off from support for a valuable undergraduate activity, because of an objection to the name of the program.

While there have been significant published results by brilliant undergraduates, publishable results are not the main goal of an undergraduate research program in mathematics. Rather the goal is to encourage the ablest undergraduates to exercise their originality in mathematics, even though the results they obtain have, very likely, been previously known. The attempt to organize such a program is of fairly recent date, and federal support has been available for less than a decade.

In 1953, under a grant from the Ford Foundation, the Social Science Research Council initiated a program of grants for undergraduate research in the social sciences. Some of the projects were carried on during the regular year and others during the summer, under the direction of individual teachers. These research programs, carried on until 1957, were most successful.

The National Science Foundation took up and extended the idea of the undergraduate research program beyond the social sciences and has supported such undergraduate research at least since 1959. The following table gives a notion of the requests and grants by year for mathematics:

	1960	1961	1962	1963	1964	1965
Requests	23	23	73	74	62	82
Grants	13	16	37	57	34	31
Funds ($ thousands)	53	120	346	610	387	420
Participants	168	128	153	247	211	147

Mathematics has received about 6 percent of all the grants given.

The Mathematical Association of America has set up a Committee on Undergraduate Research Participation, and in the summer of 1961 a conference on undergraduate research in mathematics was held at Carleton College under the auspices of the National Science Foundation. The conference had 75 participants from 70 colleges, and the proceedings of the conference are available.[5] Five years have elapsed since the Carleton Conference, and, as the above table shows, much undergraduate research participation has taken place. If the idea of this "research" is itself worthwhile, then it would be sensible to have a convention about every five years to get teachers together to discuss accomplishments and reconsider goals. Whether undergraduates could profitably participate in such a conference is another matter.

Additional evidence for growing interest in creative mathematical work is furnished by the growth of the William Lowell Putnam Mathematical Competition. This is open to undergraduates throughout the United States and Canada and is designed to test the ingenuity and originality of students. Many students who have excelled in this competition have gone on to become first-rate research mathematicians.

The following table gives the number of schools, number of teams, and number of contestants who participated in the Putnam Competition for the years 1940, 1955, and 1963. There are three contestants on each team, and individuals also enter.

DATE OF EXAMINATION	NUMBER OF SCHOOLS	NUMBER OF TEAMS	NUMBER OF CONTESTANTS
March 1940	68	45	208
March 1955	63	58	356
December 1963	199	170	1,260

In addition to the Putnam Competition, regionally sponsored contests provide crucial incentives for undergraduates at underdeveloped institutions. They also help to identify talent at such institutions. We hope to see an expansion of the program of regional contests.

The newest development in independent undergraduate activity in mathematics has been the impact of the high-speed computer. The college student now has (or soon will have) at his fingertips a research tool of fantastic power. Indeed, the computer may make mathematics one of the "laboratory sciences."

These facts raise a fundamental question. Research activity in the mathematical sciences begins early, and, for many mathematicians, the period of maximum productivity is rather short. It may therefore be that undergraduate preparation for future mathematical research deserves significantly more support than is customary at present. Certainly, greater participation in undergraduate "research" programs, even though they are not research in the usual mathematical sense, would be valuable. So also would be the use of undergraduates to assist mature mathematical scientists in both research projects and curriculum-reform activities. Ultimately, we may have to consider the distinct possibility that, for some majors in the mathematical sciences, the junior and senior years are as immediately relevant to their future research as the beginning graduate years are to others, and appropriate modes of support may have to be developed.

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6

Support for the College Mathematics Teacher

1 Facilities

The need for adequate physical facilities has been recognized for the laboratory sciences. Many colleges and universities spend substantial sums of money, frequently subsidized by the federal government, building expensive laboratories and carefully stocking them. Nowadays the mathematical sciences also need special physical facilities. The largest single item of this kind is computers. The computer needs of colleges and universities have been noted elsewhere in this report, in Chapter 3, Section 5, and in Chapter 4, Section 2. In addition, there is a continuing need, especially in statistics, for desk calculators. Applied mathematicians sometimes make use of experimental apparatus for qualitative physical demonstrations. Increasing use is being made of demonstrations and lectures involving projectors and television screens and, most recently, computer-aided displays.

Research in the mathematical sciences depends on undisturbed individual effort to a much greater extent than does research in the traditional laboratory sciences. Thus the primary laboratory of the mathematician consists of his office and his library. This fact has not been sufficiently recognized. It is not uncommon to find a mathematician sharing a small office with several colleagues.

At a time when there is a significant shortage of qualified college mathematics teachers, it is essential that we make optimal use of those available. Much can be done to improve the working conditions of mathematicians and to provide them with supporting staff

to relieve them of chores that less-skilled people could do equally well.

Since a mathematician typically does much of his research in his office, a private office of at least 120 square feet should be considered a minimum requirement. And it must contain a good-sized blackboard. This is very inexpensive equipment compared to the physical facilities required by the laboratory scientist.

We have noted previously that much could be done to relieve the shortage of mathematicians if the ablest and best lecturers could be used to teach freshmen and sophomores in large lecture sections. This is often not possible because of the lack of a suitable lecture hall with adequate visual aids. A modern lecture room equipped with large blackboards and other visual aids (such as Viewgraphs) is a small expenditure compared with the expense of hiring extra mathematicians. Yet few universities and even fewer colleges have taken this forward-looking step. Many mathematics departments have all classrooms of a traditional size, designed for 25 to 40 students. In most cases, this is far from ideal for mathematics instruction. Mathematics departments should have available both modern large lecture halls and comfortable seminar rooms for advanced courses and conferences. The kinds of buildings and facilities especially appropriate for the mathematical sciences are the subject of a recent study,[1] written jointly by a mathematician and an architect.

Visiting lecturers frequently comment on the fact that all mathematicians at smaller colleges are forced to do a great deal of routine work. An institution may have a mathematics staff with one or two unfilled vacancies and yet refuse to hire adequate secretarial aid and otherwise help its mathematicians. University mathematics departments are apt to have more or less adequate secretarial aid, but college mathematics departments often have little or none. The Panel recommends, as a minimum, that undergraduate mathematics departments should have one full-time secretary for every 1,000 student-course enrollments in mathematics. Mathematical typewriters and modern duplicating machines are inexpensive ways to make mathematicians more effective. At a time when mathematicians are fighting a losing battle with ever-increasing student enrollments, it is absurd to force them to type out their own examinations and run antiquated duplicating machines.

Another inexpensive way of relieving mathematicians of unnecessary chores is to provide them with assistants who correct homework papers. At institutions with graduate students this is a standard

practice. However, many undergraduate mathematics departments have made excellent use of upperclass students in the grading of elementary courses. The expense of this is nominal, and the experience may be most valuable to mathematics majors. It would relieve mathematicians of one of the most painful and time-consuming chores in the teaching of elementary mathematics courses.

2 Libraries

We call attention to the results of an unpublished survey, by New Mexico State University, concerning mathematics departmental libraries. A questionnaire was sent to 115 colleges and universities, of which 90 replied. Of these institutions, 62 percent have PhD programs in mathematics, 26 percent have MA programs, and 12 percent have only BA programs. Of the departments replying, only 39 percent reported that they have mathematics departmental libraries. At the remaining institutions, the entire mathematics collection is in the central library. When asked their attitudes toward departmental libraries, over half of the departments reported that they considered departmental libraries essential, and the remainder replied that although they would not classify them as essential, they considered them highly desirable.

It is the Panel's opinion that departmental research libraries are essential for all departments offering graduate work in mathematics. It is equally important that mathematics departments that do not have departmental libraries have at least mathematics reading rooms, stocking basic reference works in mathematics as well as a small number of journals.

The Committee on the Undergraduate Program in Mathematics (CUPM) has prepared a basic library list[2] for small colleges, which represents an excellent inexpensive way of making sure that undergraduates are presented with a well-rounded list of reference works. We estimate that a representative selection of some 170 works from this list would cost approximately $1,500.

Information on the caliber of libraries from a survey (reference 3, page 20), conducted for the CUPM by the National Opinion Research Center reveals that there are many libraries with weak mathematics collections. It would therefore appear that where needed a federal subsidy of up to $1,000 per college toward the purchase of basic mathematics books could do much to strengthen the mathematics

libraries at undergraduate institutions. This would be an extremely inexpensive one-shot federal program that would do much to improve mathematics education.

Some journals are also required to ensure that undergraduates may become acquainted with current mathematical literature. In general, the *American Mathematical Monthly* is the most important journal for undergraduates, although the *Mathematics Teacher* is important for future teachers of mathematics. The *Mathematics Magazine* should also be mentioned here. A major lack in mathematical literature is that of a good expository journal in mathematics. A journal analogous to *Physics Today* would be a great contribution to mathematical education and to keeping mathematicians up to date about current developments. There is also need for a regularly updated list of recommended expository papers.

3 Other Support

It is crucial for professors at smaller institutions to spend a year, from time to time, at a more active center of mathematics. This is important both to update their mathematical knowledge and to keep them mathematically alive through discussions with research mathematicians. Unfortunately, such leaves are much easier to arrange for research mathematicians than for those mathematicians who may be most in need of them.

The research mathematician is likely to be at a university with a policy of sabbatical leaves, and he may be able to get his government contract to pay a fraction of his salary while on leave. A typical formula is to have a year's leave in the seventh year with half the funds coming from one's own institution and half from some federal grant. Various types of research fellowships make it possible for a mathematician to obtain research leaves even more frequently. These programs are certainly desirable, but they have no effect on the vast majority of college mathematics teachers.

It is regrettable that even some good universities do not have sabbatical-leave policies. And such policies are rare at undergraduate institutions. The professor whose PhD degree is 20 years old, and who has not been active in research, is, on the one hand, badly in need of a visit to a major research center and, on the other hand, has no means of financing such a year. His institution may not have a sabbatical-leave policy. He does not have a research contract through

which he could ask for federal funds to help support a sabbatical leave, and he is not in a position to apply for a regular research fellowship. Occasionally a man in this position succeeds in obtaining a research fellowship by manufacturing a research project of very little intrinsic importance. It would be much more honest and beneficial to recognize that a year of study, say every ten years, is crucial for any mathematician, whether he is currently pursuing mathematical research or not.

The only federal program that makes such fellowships available is the National Science Foundation's Science Faculty Fellowship Program. However, awards to PhD's in the mathematical sciences have averaged less than 20 per year, and many of the awards to non-PhD's were, in effect, simply graduate fellowships. The program should be increased to a level where a significant fraction of mathematics professors can count on Science Faculty Fellowships once in their careers. If we assume that half of the present 5,000 full-time non-PhD college professors should have such an opportunity, and about one fourth of the present PhD's, and if we spread the program out over a 25-year period, we would have to ask for 50 Science Faculty Fellowships per year for the next 25 years. If we assume that the cost per professor, including travel allowance and away-from-home living allowance, will amount to $16,000 per fellow, such a program would average about $2,400,000 per year.

A highly effective and inexpensive measure would be to make it easier for mathematicians to attend professional meetings and to visit colleagues at other institutions. A mathematician who is at an institution where he is the only one in his specialty (a very common situation at undergraduate colleges) could gain greatly by being able to visit a colleague at another institution once or twice a year. There are increasing numbers of meetings, both national and regional, at which first-rate expositions of current mathematical trends are the order of the day. Often it is only lack of travel funds that prevents mathematicians from attending these valuable meetings. If the federal government provided $300 traveling expenses per year for every full-time mathematics professor, the cost of the program would be only $3,000,000 per year. We recommend a modest experiment along these lines.

A critical question is the question of summer stipends. These too are available primarily for the research mathematician and are very hard to obtain for those who are not creating new mathematics but who would make excellent use of summers in refreshing themselves

mathematically or in designing new courses. These are likely to be just the mathematicians who are forced to take summer jobs, either doing routine teaching or working in some other way that does not contribute to their professional development. It should be made easier to obtain summer grants for study or curriculum development as well as for doing mathematical research. Since the federal government is now committed to spending considerable sums of money to create many more new college mathematics professors, it is time that it also make a substantial effort to ensure that these professors stay mathematically alive once they have left their graduate institutions.

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3. Patricia Collette, *Undergraduate Mathematics Teaching,* National Opinion Research Center, Chicago, Ill. (1963).