DOCUMENT RESUME

ED 053 059 SP 004 907

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TITLE	A Model of Mathemagenic Behaviors as Intervening Variables in Classroom Communication.
INSTITUTION	Stanford Univ., Calif. Stanford Center for Research and Development in Teaching.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C.
REPORT NO	RM-30
BUREAU NO	BR-5-0252
PUB DATE	Apr 68
CONTRACT	OEC-6-10-078
NOTE	18p.
EDRS PRICE	EDRS Price MF-$0.65 HC-$3.29
DESCRIPTORS	*Interaction Process Analysis, *Student Behavior, *Teacher Behavior, *Verbal Communication, *Verbal Learning, Verbal Stimuli
IDENTIFIERS	*Mathemagenic Behaviors

ABSTRACT

This paper makes an informational analysis of teachers' verbal discourse. A behavioral analysis is also made in terms of "tacts" and "mands," as defined by Skinner. These behavioral components of a message may be quantified in terms of "originality," "redundancy," and "complexity." But the most important problem is to find what effects the teacher's verbal behavior has on pupil behavior. A conceptual model using student mathemagenic behaviors as intervening variables is described. This concept is useful in describing the kinds of hypotheses that need to be formulated to guide empirical study. Mathemagenic behaviors--defined by Rothkopf as behaviors that produce learning--are the critical empirical concept linking teacher and student behavior. Thus, using the simplified behavioral and mathematical conception of teacher behavior and mathemagenic behaviors as intervening variables, a research strategy is envisaged. The utility of this approach is suggested by current experimental work on verbal learning. These concepts can also be used to integrate the research of several investigators on classroom discourse. (Author/RT)

STANFORD CENTER FOR RESEARCH AND DEVELOPMENT

IN TEACHING

Research Memorandum No. 30

A Model of Mathemagenic Behaviors as Intervening Variables in Classroom Communication

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April 1968

A Model of Mathemagenic Behaviors
as Intervening Variables in Classroom
Communication

Rothkopf (1965) has introduced the construct of mathemagenic behaviors to account for results occurring in experiments analyzing learning from textual materials. "Mathemagenic" means behaviors that produce learning. Rothkopf argues persuasively that such behaviors must be present if stimuli such as textual material are to effect learning.

This paper extends this model to classroom discourse. However, two extensions of this concept are made: 1)Skinnerian concepts of verbal behavior are applied to the teacher's verbal behavior; and 2) information theory concepts are used to quantify the model.

Information Analysis of the Teacher's Verbal Behavior

A teacher's behavior is obviously an audiovisual stimulus. As a visual stimulus, it presumably is analyzable in terms of such dimensions as form and movement.[1] These aspects of the teacher's behavior may also be analyzed as non-verbal communications. Arbitrarily, to simplify the analysis and with no implications about the relative importance of verbal and non-verbal communications, the analysis presented here will be confined to the verbal components of the teacher's behavior.[2]

A teacher's verbal behavior as an auditory stimulus is a linear phenomenon. Words, sentences, and collections of sentences occur sequentially. These words are messages in the communication sense. The teacher is the transmitter of these messages, the student, the receptor.[3]

The feature of interest in this system is the connection between the emitted message and the received message. There are no known set of conditions, physical, physiological, or psychological, under which these two events are identical. The message received is a microreplica of the message transmitted.

This message has two prime characteristics, meaning and information. The message has meaning if and only if both sender and receiver share a common semantic, i.e., if they share the same set of denotative symbols for encoding and decoding. Meaning is, therefore, not transmitted. It may be described as

[1]Time-lapse photography shows clearly that this behavior is remarkably patterned.

[2]An interesting line of research is the relation between the verbal and non-verbal components of the teacher's behavior. We found a positive correlation ($r = .44$) between verbal and non-verbal reinforcers given by the teacher, but a correlation of only .15 between the number of negative verbal and non-verbal reinforcers. (McDonald and Allen, 1967).

[3]We ignore for simplicity's sake the converse situation where the student is the transmitter, the teacher the receptor. The two cases are conceptually but not empirically symmetrical.

the degree of correspondence between a sender's and a receiver's respective sets of denotative symbols. For example, $e = mc^2$ is meaningless unless I know the denotations of each symbol, the meaning of an equality relation, and the meaning of a multiplicative relation. Levels of meaning may be defined in terms of the numbers of connections among denotative symbols which may be arranged in a hierarchical structure. "Understanding" may be defined operationally as the number of such connections that the receiver can make.

Information, in contrast, is a characteristic of the transmitted message. The information in a message modifies the behavior of the receptor. For example, the message, "Add this column of figures," is likely to produce adding behaviors. In some messages the expected effect on behavior is either implicit as in, "DNA molecules are the building blocks of genetic structures," where the implicit message is either, "Remember this," or "Here's a new way of looking at inheritance think with these concepts." Or, it may be unconsciously disguised as in 'double-bind' messages with their invitations to approach while signaling to avoid. Thus messages may contain both data and demands or commands; that is, sets of instructions on what to do with the data.[1]

In consecutive discourse, as in explaining, the command aspects of the message may be mediated by explicit directions. For example, an explanation prefaced by a statement of purpose directs how the material to be presented is to be processed: "We will compare Soviet Russia's and the United States' interests in the Far East." In other cases, commands are mediated by questions interspersed with explanatory and descriptive sequences.

The command aspect of a message is a qualitative characteristic of messages. A quantitative characteristic of a message is its originality. Originality is inversely proportional to predictability, that is, the message is original to the degree that uncertainty characterizes the response of the receptor. The message "BA-BA-BA-BA-BA . . ." is highly predictable, hence unoriginal; little uncertainty is generated in the receptor. However, the message, "The economic cycle varies as a function of . . .", is highly original since it generates considerable uncertainty (for the naive receptor; if the receptor already possesses the information in that message, the transmission of the message creates a kind of resonance phenomenon).

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[1] "Demand" or "command" is used here with the meaning ordinarily conveyed in describing and writing computer programs.

It should be apparent that unoriginal messages do not produce behavior modification in the sense of learning. If the message is certain, the receptor learns nothing new. An unexpected event is one which is unlikely to occur, has zero probability of occurring, hence, when it does occur, requires a response. Stated in behavioristic terms, the occurrence of an unexpected event is a change in stimulation, the necessary condition of a response change.

The originality of a message can be quantified. If "H" is the amount of information in a message (its originality), then

$$H = -Nt \sum_{i=1}^{n} p_i \log_2 p_i$$

where _i_ represents a symbol in the message, p_i the probability of its occurrence and _Nt_ represents the number of elements in the message after _t_ seconds. In words, this equation states that

> the quantity of information transmitted by a message is the binary logarithm of the number of choices necessary to define the message without ambiguity.

The advantage of quantifying messages in this way is that it is possible to describe quantitatively the necessary conditions for producing behavior changes in situations such as teaching where the nominal stimulation is in large part connected discourse. The problem of studying the effect of teaching behavior of this kind may be defined as assessing the probability that a message transmitted by a teacher will be received by a student. Variations in behavior of a single teacher through time and differences between teachers may be quantified in terms of amount of information (i.e., originality) transmitted. Learning would be expected to be directly proportional to the amount of information conveyed in a fixed unit of time.[1] Although stimulus variation (defined here in terms of variation in information) is a necessary condition for behavior change, it is not a sufficient condition. Obviously, the student must respond in some way.

This analysis enables us to describe the relations between the teacher's verbal behavior and the responses of the student primarily in terms of one concept, the _originality_ of the message (and for some purposes in terms of related concepts such as _redundancy_ and _complexity_). However, before the use of this analysis can be described, it is necessary to analyze the teacher's verbal behavior using behavioral concepts.

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[1]To understand the full implication of this statement, recall that the unit of information may be defined operationally in many different ways. For example, it may be the number of discrete facts or the number of causal relations stated in a lesson.

Behavioral Analysis of the Teacher's Verbal Behavior

When a teacher utters meaningful words in connected discourse, what responses are cued off? It is overly simple to assume that each word uttered is paired with a response. It makes more sense to assume that collections of words are paired with specific responses. The form of such pairings, with either single words or groups of words as stimuli, is an empirical question. More importantly, to what responses are these responses likely to be linked?

The teacher's verbal behavior is a set of linear or sequential utterences of varying degrees of predictability. The first condition that must be satisfied is that these utterances be perceived as sensory stimulation. They must be effective rather than nominal stimuli. Thus, a set of attending and orienting responses must be made by the student.

Since the teacher's utterances occur sequentially, they must be scanned. Hence, scanning responses must be activated in the student. We are familiar with one class of such responses, namely the scanning responses used in reading. However, these responses are probably only roughly analogous to those used in listening. In the latter case, the receptor is heavily dependent on memory, whereas in reading one can go back over the material being read. Also in reading, one can skip ahead or skim, thus anticipating what is coming. But in listening, such anticipations occur only when the material is highly predictable, hence, less informative. Scanning in listening, however, does enable the receptor to detect nuance, inflection and emphasis, cues which enable him to punctuate the speaker's message and to organize its elements in a hierarchy of importance.

A set of decoding responses must be initiated. Teacher and student must have some common meanings a priori to the act of communicating; otherwise, they cannot decode each other's messages. These decoding responses include such subsets of responses as translation and comparison responses by which the receptor matches symbols in his repetoire of meanings with those emitted by the sender.

Two other kinds of responses are needed before learning can occur. Transformation responses are required to manipulate the input material into different forms. Storage responses, such as rehearsal and pairing familiar symbols with unfamiliar ones are necessary to retain the input or its transformation.

The relation among the concepts described here may be schematized in the following way:

$$P_s = f(B_{t.o} \times B_{s.m})$$

where P_s is student performance; $B_{t.o}$ is the observable behavior of the teacher; $B_{s.m}$ is the mathemagenic responses of the student.

P_s is some observable performance of a student assumed to be correlated with learning (learning is, as always, an intracranial event inferred from performance changes. $B_{t.o}$, the observable behavior of the teacher, are utterances and movements which can be described and counted reliably. $B_{s.m.}$ are, however, covert responses which must be defined in terms of experimental operations which may be correlated inferentially with them.

The teacher's verbal behavior is of three kinds. First, one kind of statement is semantic, a statement about meanings, such as definitions and explanations. A second kind of statement is syntactical, statements about signs and symbols, what they are and how they are to be used. The following categories used by Smith (n.d.) appear to describe semantic and syntactical statements: defining, describing, designating, stating, reporting, evaluating, opening, classifying, comparing and contrasting, conditional inferring, and explaining. Bellack's (1966) two categories of meaning, substantive and substantive- logical, also appear to comprise semantic and syntactical statements.[1] Such statements may also be subsumed under Skinner's (1957) tacts.

The third kind of statements uttered by teachers are statements which indicate how semantic and syntactical statements are to be processed. These statements are the pragmatics of communication because they define relations between teacher and student, or between student and the statement, or both. An instance of a statement defining the relation of teacher to student is, Teacher: "John, what do you think are the reasons for General De Gaulle's seeming anti-Americanism?" This question means, "Tell me what you think." Similarly, an instance of a statement describing the way the student is to relate to the content is, Teacher: "Compare the attitudes of the French people toward Americans with those of General de Gaulle." The direction to tell the teacher what the student sees as the result of the comparison process appears to be implicit here; such statements indicate both the relation of the student to the material and to the teacher. The context of the teacher's statement would suggest how many directions are probably in the statement. Both are made explicit by saying, "John, will you compare for us . . ."

Such statements are called mands by Skinner. Bellak classifies them in his instructional meanings and instructional-logical meanings categories. Smith would include most of these statements in the categories referred to earlier; some would also be classified in his 'directing and managing classroom' category.

[1]The lack of an unequivocal assertion of identity that you may detect here reflects the uncertainty caused by my not having coded a transcript with either of these category systems. I also defer, temporarily, to the judgment of the system's originators.

The stimuli-response relations in a mand may be diagrammed in the following way:

Figure 1

Diagram of Stimulus-Response Relations in a Request-type Mand (After Skinner, 1957)

Teacher

Class	What is your opinion?	Opinion	Very good!
S^D	R^V ↓ S^{DV} →	$S^{rein} + S^D$ ↑ R →	R^V ↓ $S^{rein^V} + S^D$
	What is your opinion?	Gives opinion	Very good!

Listener

The teacher, aware of the discriminative stimulus of the class, asks a question associated with this stimulus. This question acts as a stimulus (S^{DV}) to the listener, who answersit (R). This response is reinforcing to the speaker (S^{rein}) as well as being a verbal discriminative stimulus (S^D). This combined stimulus ($S^{rein} + S^D$) evokes the response, "Thank you" (R^V) which in this case is a reinforcing stimulus and a discriminative verbal operant.

The mand in this sequence is the speaker's, "What is your opinion?" This analysis assumes that asking this question is highly likely to evoke an opinion statement. This assumption is valid only under a limited set of conditions; in behavioral language, the stimulus has a low-to-moderate probability of evoking a response describing the listener's opinion.

One explanation for the relatively low predictability of the opinion-giving response may be that, in Skinner's terms, the verbal community has not positively reinforced opinion-giving as a response to the question, "What is your opinion?" Another explanation is that the listener has no opinion. A third explanation is that he does not want to give an opinion, so inhibits opinion-giving responses.

These three explanations are plausible even though the second is difficult to verify independently and the third, empirically. Still a fourth explanation is that responses intervening between the stimulus question, "What is your opinion?" and the response to the question have not been made. The listener must be attending to the question, must decode the symbol, "give opinion", process information about the topic, formulate the opinion, and express it. As Figure 1 makes clear, the last step in this sequence, expressing the opinion, is an overt, verbal response (R^V). The following response chain may be postulated:

SPEAKER	LISTENER					
"What is your opinion?"	attending	decode	sort	evaluate	formulate	Opinion "
S^{DV}	r	r	r	r	r	R^V

It is obvious that the probabilities of the chain's being formed is progressively smaller at each step; i.e., it is more likely that a person will listen, than decode, will listen and decode than listen, decode and sort, and so on successively through the chain. The probability is greater that the listener will hear the question than that he will give an opinion.[1] The more responses likely to intervene between stimulus and response, the less likely the occurence of the response as a simple pairing to the stimulus.

The responses described symbolically in the diagram as lower-case <u>r's</u> are <u>mathemagenic responses</u>. These are covert responses whose occurrences are the sufficient conditions for the occurrence of the response as a simple pairing to the stimulus. These responses lead directly to learning. They intervene between the independent variable, the teacher's verbal behavior, and the dependent variable, the student's response.

Although these responses are covert, can they be brought under stimulus control? Obviously asking, "What is your opinion?", does not of itself evoke attending responses. Nor does an attending response lead invariably to a decoding response; nor does a decoding response lead automatically to a sorting response. This response chain must be learned and conditioned to the question, "What is your opinion?" Each response element in this chain may be under direct stimulus control; that is, if the teacher says, "Note the question I am about to ask," the probability of attending responses occurring will increase. If the teacher says, "Recall what is meant by an opinion," the decoding response is more likely to occur. If he says, "Sort the facts," then, "Assess the unknowns," then, "Weigh the evidence," then, "Evaluate the potential influence of unknown factors," and so on until he says, "Formulate a judgment," it is likely that he will have elicited all the responses leading to the forming of an opinion.

Such control of the pupil's responses may be undertaken by the teacher. Excercising such control seems implicit in the teaching strategies advocated by investigators who like Taba (1964), spell out sequences of teacher-pupil interactions. However, much classroom discourse is elliptical. All these intervening steps are omitted either because the teacher does not know when they are required or because it is reasonable to assume that the chain of the pupil's

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[1]These statements represent my guesses about the probabilities. The empirical data may provide quite different and more interesting estimates.

responses has been formed and has been conditioned to the teacher's question, "What is your opinion?"

Thus, it is clear that this set of pupil responses may be hypothesized to intervene between the stimulus statements of the teacher and the verbal responses of the student. These responses are intervening variables in the sense of that term as used by Tolman(1936) and by MacCorquodale and Meehl (1948). That is, they are quantities obtained by manipulating empirical variables; they are abstractions from empirically defined relations among variables.

Returning to the function presented previously,

$$P_s = f(B_{t.o}\ B_{s.m})$$

we note that two classes of variables are both observable and quantifiable, P_s, the performance of the student, and $B_{t.o}$, the behavior of the teacher (specifically in this analysis, his verbal behavior). How is $B_{s.m}$ to be quantified? The general method is as follows: 1) we must describe independent variables which are likely to be correlated with classes of mathemagenic behaviors: i.e.,

$$B_{s.m} = f(I_1)$$

where I_1 is an observable variable; 2) statements are made about the connections between various classes of mathemagenic responses and observed behavior, i.e.,

$$P_s = f(B_{s.m_1}, B_{s.m_2} \ldots) \text{ where}$$

$$B_{s.m_1}, B_{s.m_2} \ldots = f(B_{t.o})$$

The first kind of relation is illustrated by such predictions as, "If the teacher utters mands calling for attention, the frequency of attending responses will increase as measured by an increase in such pupil performances as ability to recall the teacher's subsequent statements more accurately, (and so forth).[1] Or, "If the teacher calls for enumerating responses, the accuracy of the student's enumerating responses will increase." The general question for empirical study is, "What utterances of teachers elicit which kinds of mathemagenic responses in students?" This research includes also the analysis of how chains of responses are conditioned. It is obvious that until empirical relations of this character are identified, we can understand very little about the connections between teacher and pupil behavior.

[1] Carefully controlled experiments under laboratory-like conditions will be necessary to establish the validity of these hypotheses. It is also possible to do experimental studies in classrooms by carefully controlling the teacher's verbal behavior.

The second class of statements is illustrated by this hypothesis: "Pupils' understanding of principles (kind specified) is a direct function of the number of translation and comparison responses that they make." Or, "Pupils' ability to predict the consequences of an experimental manipulation is a direct function of the number of variable substitution responses they make."[1] The general question for study is, "What kinds of cognitive processes can be shown to be linked to specific pupil performances?"

When relations of these two classes have been established, it will be possible to make general statements linking categories of teacher verbal behavior to changes in pupil performance. Mathemagenic responses of pupils conceptualized as intervening variables in such relations can be used as a conceptual tool to guide research. By producing empirical relations of the first kind (those relating teacher verbal behavior to mathemagenic responses of pupils), we learn how to bring the mathemagenic responses under stimulus control. The second set of relations specifies how these mathemagenic responses control classes of observable pupil behavior where the latter are significant categories of change in pupils. Thus, by conceptualizing mathemagenic responses as intervening variables we may develop a research strategy which will enable us to state, eventually, relations between the teacher's discourse and pupil achievement.[2]

Supporting Empirical Evidence

This conceptual analysis would have little meaning if the mathemagenic responses could not be tied more precisely to empirical events. Several lines of evidence suggest that they can be so linked. First, Rothkopf's (1966,1965,1963) experimentation illustrates one phase of needed research and provides relevant empirical data. In one study (Rothkopf and Coke,1966), the response mode of the lea

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[1] I am inventing classes of mathemagenic responses throughout this paper. Eventually, a taxonomy of these behaviors will have to be developed. However, there is enough data at present to make some heuristic categories. The concept of translating responses is analogous to the transformation categories used by Newell et al. (1958) in their computer analogues of the problem-solving process. Inhelder and Piaget (1958) have used the ability to substitute variables mentally as the necessary condition for moving to the stage in which the child can think logically.

[2] For those who feel strongly that detailed analysis of teachers' verbal behavior is likely to miss its significant characteristics, I call attention to the fact that this kind of analysis has been extraordinarily useful in clinical work (Watzlawick, et al., 1967). Analyses of this kind by Bateson, et al., (1956) led to the formulation of the 'double-bind' hypothesis about the origin of schizophrenic behavior.

was varied in two ways; 1) by requiring or not requiring him to anticipate a response, 2) by presenting the stimulus eliciting the response at various intervals after its first presentation. The material being learned was a set of sentences about a fictional tribe and their gods. These are sentences similar to those found in textbooks and the verbal discourse of teachers. The measure of learning is the number of sentences correctly recalled, a task demand analogous to the task demands of listening to a teacher.

Rothkopf found that immediate repetition resulted in poor criterion performance. These results confirmed the findings of an earlier experiment (Rothkopf and Coke, 1963) and gave firmer support to the earlier results because sentences were presented to some subjects in their original form and in syntatical and semantic variations.

Rothkopf interpreted these results by hypothesizing that length of rehearsal interval was inversely related to the probability of eliciting relevant mathemagenic responses. The immediate rehearsal (in this type of experimental treatment where the subject supplies missing words in the sentence) causes him to link his response to the last word in the sentence rather than to the concept-name with which it should be linked. Hence, relevant mathemagenic responses do not occur.

These results led Rothkopf to theorize about the relation of test-like events to the elicitation of mathemagenic responses (Rothkopf, 1965). He performed an experiment (described in Rothkopf, 1965) which demonstrated that inserting questions in the reading material improved performance. This experimental work is directly relevant to the analysis of the questioning behavior of teachers which is a substantial proportion of their verbal discourse.

Teachers' verbal discourse may closely resemble that of textual material, but it is likely to differ when it does by including a higher frequency of <u>mands</u>. Or, perhaps we should state the general question, "How does the insertion of <u>mands</u> in textual material or verbal discourse affect learning?" Rothkopf (1965, p.212) found that a group instructed to read carefully and slowly because the material contained much detailed information did as well as the two groups exposed to test-like events. It should be possible to use Rothkopf's experimental technique to test the effects of different combinations of mands, test-like events, and content statements.

Millett (1967) has shown that teachers who emit mands to elicit <u>translation</u> behavior (stating ideas in one's own words) elicit more translation behaviors in students. Millett found that the amount of translation behavior emitted in classroom discourse was uncorrelated with scores on a translation test. Why the latter should be the criterion is not clear. A more appropriate criterion would be degree of understanding of the content: that is, translation responses elicited in the classroom should be regarded as intervening variables correlated with some other criterion pupil performance.

Rothkopf's work is a heuristic basis for using mathemagenic behaviors as intervening variables in the analysis of classroom discourse. His methodology can be developed to provide the studies which will yield the empirical relations between pupil performance and mathemagenic behaviors.

A second class of studies is beginning to yield evidence supporting the probability of being able to formulate relations between mathemagenic responses and significant classes of pupil achievement.[1] Rosenshine (1968) identified sets of most and least effective teachers on the basis of mean achievement test scores of their classes. The achievement tests had been taken by these teachers' students immediately after the teachers had delivered lectures developed from expository articles given to each of them. Rosenshine counted a large number of verbal and nonverbal behaviors of these teachers and assessed which among them differentiated the most from the least effective.

He found that "explaining links" distinguished between most and least successfu teachers, the former having significantly more such links in their lecturing behavior. Explaining links are those words in sentences which indicate causality ("because"), means-ends relation ("by"), and effects ("as a result of").

These words may be cues in themselves, implicit mands, which tell the student what concepts and relations are to be understood and remembered. Or, if the teacher has clearly indicated what is to be learned, for example, the "whys," the "hows," and "what happened," the explaining links may act as signals to alert the listener to the relevant information in these categories. In either case, the explaining links are probably eliciting mathemagenic responses; if the first speculation is valid, attending responses; if the second, classifying responses.

Rosenshine also found that gestures and right to left movements distinguished favorably the most from the least effective teachers. The function of gesturing is clearer since teachers demonstrate or elaborate an idea in gestures, or even by their idiosyncratic gestures which highlight it. Movement is probably correlated with change of thought or of emphasis (acting is an obvious highly

[1]These studies are being conducted in the Explaining Project directed by N.L.Gage, Stanford Center for Research and Development in Teaching, School of Education, Stanford University.

controlled form of this idea; why right-to-left movements is a discriminating variable is not clear unless such movements are correlated with handedness).

In a related study (Podlagar, et al. 1967), self-reports of attending behavior correlated significantly (r = .68,.47,.48) with mean achievement scores, as did rated "clarity of aims" and "clarity of presentation." These correlations are for teachers across topics, a fact which suggests that the teacher's behavior is sufficiently consistent to elicit similar relevant responses from students for each topic.

The categories rated suggest categories of correlated mathemagenic behaviors. "Clarity of aims" implies that the teacher specifies goals, a behavior which should control orienting responses of students. "Clarity of presentation" is an omnibus category which probably refers to the organizing and emphasizing behaviors which we would expect to be correlated with categorizing and structuring behaviors of students.

These studies suggest that the model presented here of mathemagenic behaviors as intervening variables is a heuristic conceptual and methodological tool. The critical studies to test the idea remain to be done, although how to do them seems quite clear.

Applying Information Theory Concepts

This paper has two lines of thought; 1) an information theory analysis of teacher-student discourse which in this paper is the minor theme; 2) a behavioral analysis of teacher-student discourse. The former was introduced to show how the latter might be quantified.

Four reasons support tieing these different kinds of analyses together. First, information theory concepts are few in number, "originality" or "predictability," "redundancy," and "complexity". A wide variety of seemingly disparate concepts and unrelated facts may be subsumed by these concepts. Second, some empirical data exist which suggest that the concepts are most likely to be useful. Third, if the teacher's verbal behavior can be quantified in a more sophisticated way and relations among variables may be stated as equations. Fourth, the nature of the quantification increases the likelihood of simulating the behavior by mechanical means, a development which will lead to increased analytic power and, hence, to greater understanding. These latter reasons will not be discussed here, since the sophistication of the mathematics will be apparent, and the development of simulation models is at present an implicit promise in the analytical tool being used.

In the first section of this paper, it was suggested that the amount of predictability or originality of a teacher's verbal behavior was correlated with learning since unpredictable behavior was a necessary condition for learning. Since the empirical referent of "unit of information" may be any category whose relative frequency, i.e., probability, may be established within definable boundaries, any item of verbal behavior may be described in terms of its originality by the quantity, H.

Two other characteristics of a teacher's verbal behavior may be similarly quantified, its <u>redundancy</u> and its <u>complexity</u>. The redundancy of a message is the complement of the message's <u>relative information.</u> The greatest amount of information is transmitted when the symbols in a message are equiprobable, that is, least predictable; hence, the message is most original. If all symbols in a message are equiprobable, then H reduces to

$$H_m = -\log_2(1/n)$$

where the H_m represents the amount of information in a message each of whose symbols is equally likely to occur. The ratio of H to H_m is called the <u>relative information</u> in the message. <u>Redundancy</u> is the complement of relative information $(1 - H/_{H_m})$. That is, the greater the <u>relative information</u> in a message, the less its <u>redundancy</u> and contrariwise. In less complicated terms, the more predictable the message, the less information it contains, hence, the more redundant it is. <u>Complexity</u> is similarly described as a function of the number of elements in a message.

Can these concepts be applied to the verbal behavior of a teacher? To compute any of these quantities, the following operations are needed. First, the element of discourse (N) must be defined. Such units as Smith's <u>episode</u> or closed units of discourse such as a class period or some micro-unit thereof may be arbitrarily selected as a counting element. Second, a unit of information (n) must be identified. Any of the categories used by Smith, Bellak, Gage, or Skinner may be used. When the relative frequency of these categories is estimated (p_i), we have the three quantities, N, n, and p_i, needed to determine quantitatively the <u>information</u>, <u>redundancy</u> and <u>complexity</u> of a message. Thus, the teacher's verbal behavior may be described quantitatively in terms of these three categories. If we use these quantitative concepts, it is no longer necessary to resort to qualitative concepts.

This mathematical analogy would be merely an interesting analogy if there were no empirical referents for it. However, Rothkopf and Coke (1963) have provided empirical data which suggest that these mathematical concepts are highly

In that experiment, predictability of the response requirement (PPR) was inversely related to the amount of recall; i.e., subjects learned more if the response to be learned was less predictable. These data are consistent with the information theory prediction that learning is a direct function of the unpredictability of the message.

But, there seems to be some inherent contradiction between the concepts of unpredictability and mands, the former a description of information in a message, the latter, a description of how the information is to be processed. This seeming incompatability is resolved if the following hypothesis is valid:

> The information in a message is positively correlated with the number of mands in it; hence, the more frequent the mands, the greater the information; hence, the greater the learning from the message.

This seemingly contradictory hypothesis is easily explained. Mands are highly unpredictable; that is, how we are to process verbal discourse is literally unknown until the speaker indicates what processing he wants to occur (the privilege of the speaker is to indicate what processing is to occur; this is how he indicates his message will be intelligible). Therefore, while mands generally guide behavior, and, specifically guide or control mathemagenic responses, their total frequency is an empirical measure of the unpredictability of a message; hence, the frequency of mands is a quantitative measure of the information of a message.

The problem of studying teachers' verbal discourse may be reduced, therefore, to an analysis of the variables of mand-content relations to learning. The first phase in this analysis is to specify what mathemagenic behaviors are most likely to elicited by combinations of mand-content relations. The second phase is to specify how the elicitation of mathemagenic responses will elicit significant categories of pupil behavior.

Ultimately, we should be able to specify patterns of mand-content relations that will produce specific categories of pupil change, change in conceptual, attitudinal, or skill learning.

The value of information-theory to this analysis is that it provides conceptual simplicity and mathematical specificity. Thus, it may be possible to delineate the characteristics of teacher discourse in terms of a few concepts, each of which may be described quantitatively and each of which may be related to the others in terms of equations.

Conclusion

This paper has made an informational analysis of teachers' verbal discourse. A behavioral analysis was also made in terms of <u>tacts</u> and <u>mands</u>. These behavioral components of the message may be quantified in terms of <u>originality</u>, <u>redundancy</u>, and <u>complexity</u>, information theory concepts.

But the most important problem is to find what effects the teacher's verbal behavior has on pupil behavior. A conceptual model using student mathemagenic behaviors as intervening variables was described. This concept is useful in describing the kinds of hypotheses that need to be formulated to guide empirical study. Mathemagenic behaviors are the critical empirical concept linking teacher and student behavior.

Thus, using the simplified behavioral and mathematical conception of teacher behavior and mathemagenic behaviors as intervening variables a research strategy is easily envisaged. The utility of this approach is suggested by current experimental work on verbal learning. These concepts can also be used to integrate the research of several investigators on classroom discourse.

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