

DOCUMENT RESUME

ED 051 584

EA 003 591

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TITLE Minimizing the Spread in Per-Pupil Expenditures in School Finance Programs.  
REPORT NO P-4047  
PUB DATE Mar 69  
NOTE 23p.  
EDRS PRICE MF-\$0.65 HC-\$3.29  
DESCRIPTORS Cost Effectiveness, \*Educational Finance, Equalization Aid, \*Expenditure Per Student, Expenditures, \*Mathematical Models, \*Resource Allocations, \*State Aid

ABSTRACT

This paper summarizes a recent study of a mathematical approach to the allocation of State resources to local school districts. The purpose of this approach is to (1) assure the maximum utilization of resources available, (2) distribute State funds in accordance with the criteria of effectiveness imposed on the system, and (3) satisfy the budgetary and political constraints imposed on the system. The advantages of this approach are described and needs for further models are discussed. (JF)

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MINIMIZING THE SPREAD IN PER-PUPIL EXPENDITURES  
IN SCHOOL FINANCE PROGRAMS

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I. INTRODUCTION

In many states a crisis exists concerning the financial support of education. Growing teacher demands for higher salaries, along with the gradual erosion of the tax base and the increasing competition for the tax dollar from other municipal services, have placed many school districts, especially those in urban areas, in a financially precarious position. Some of these have conducted and/or sponsored research both on ways to modify their present support formulas to meet changing educational needs<sup>(1)</sup> and on the development of totally new schemes.<sup>(2)</sup> Most research in school finance, however, has been conducted with apparent disregard for any constraining conditions that might be placed upon the system, notably the limitations placed upon the state resources available for distribution. In addition, maximum use was not made of the available resources--from state, local, Federal, individual funds--because there was no established criterion of effectiveness on which to base the distribution of these resources. Clearly, methods must be derived for distributing state funds to local school districts which will (1) assure the maximum utilization of resources available, (2) distribute state funds in accordance with the criteria of effectiveness imposed upon the system (i.e., maximize state aid to each district), and (3) satisfy the budgetary and political constraints that posed upon the system. It appears that the most efficient ways to derive a method that will satisfy these complex requirements is through the application of techniques. This paper reports some of the major points of a recent study on a mathematical programming approach to the allocation of state resources to local school districts.

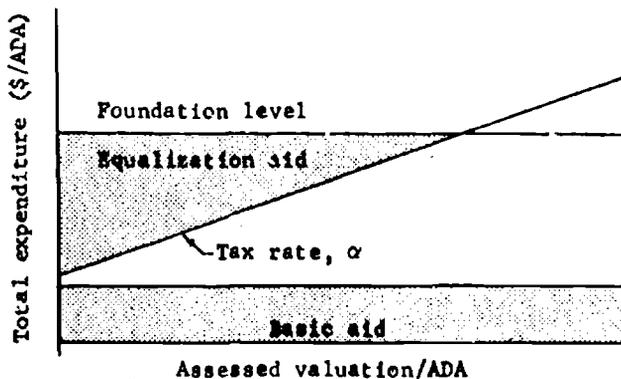
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## II. STATE SUPPORT PROGRAMS

In most states resources for education are distributed according to some type of formula. The most widely used formula is based upon the foundation-type state support program,<sup>(3)</sup> in which each student at each level (elementary and secondary) is guaranteed some minimum level of total expenditure corresponding to what educators consider necessary to support an "adequate" educational program. A major appeal of foundation-type programs is that they, in some measure, attempt to equalize opportunity for all students in a state by granting additional state funding to school districts with a poor tax base. School districts with a good tax base or high local ability to support education are allocated a fixed amount of funds (known as basic aid). Thus, all districts receive some state support, but those with local ability receive additional equalization aid, up to the minimum guaranteed level. The net result of this type of finance scheme is that the poorer districts have final total expenditures per student (in average daily attendance, ADA) of the foundation level, while the more affluent districts might have final total expenditures of up to two or three times this level.

The relationships among the facets of the foundation-type state support system are illustrated graphically in the sketch below:



The shaded areas represent state funds. The basic aid, as indicated earlier is a uniform amount, determined by the legislature; and the equalization aid is a variable amount given in addition to the basic aid brings the total expenditure--state funds plus local funds via the state mandated tax rate--to the foundation level.

The state mandated computational tax rate is a uniform tax rate considered by the legislature to reflect a minimum "effort" on the part of the school district to support its educational program. School districts are allowed to tax at rates greater than this amount but not less. The term "final total district expenditure" as used throughout this study indicates only the sum of the state funds and those local funds raised via the state mandated tax rate. The values to be used in the calculations always represent total expenditure per ADA.

The basic shortcomings of foundation-type support programs become apparent when the formula is applied. To illustrate, we have applied the formula to two hypothetical school districts. The results are shown in Table 1.

Table 1

EXAMPLE APPLICATION OF FOUNDATION-TYPE SUPPORT PROGRAM

Item	District 1	District 2
Assessed valuation, \$/ADA	100,000	300,000
State mandated computational tax, \$/\$10,000 of assessed valuation	25	25
Local funds provided (assessed valuation x mandated tax)	250	750
Basic aid, \$/ADA	125	125
Equalization aid, \$/ADA	225	---
Total expenditure, \$/ADA	600	875

Although both districts have the foundation level of funding, there is considerable inequity in the final total district expenditure. If the equality of educational opportunity is partly dependent upon a minimal spread in the final total district expenditure, as some educators suggest, then the foundation program as it is currently applied is not successful in meeting this objective.

Dissatisfaction with this aspect of the foundation program in California has recently been expressed in the form of a law suit filed in Los Angeles Superior Court by a group of parents and students living in the poverty areas of Los Angeles.<sup>(4)</sup> The main argument of the plaintiffs was that because of differences in wealth (assessed valuation per ADA), among the school districts in the state, there were marked differences in the quality of educational services, equipment, and other facilities. The plaintiffs contend that the situation constitutes a violation of the equal-protection clauses of the U.S. Constitution and California law.

Discussion of the legality of the state-support formula is beyond the scope of this study, however; we are concerned with a method for making state support programs more responsive to educational needs-- especially in lower-income districts. To this end we have developed a linear-programming model which could be used to optimize the distribution of state funds to local school districts, taking into account the constraints of the foundation-type support program. These include political constraints such as the maximum and minimum levels of state aid per ADA for each student and the percentage relationships of the state and local contributions to the total costs of the state support program; economic constraints, such as the total amounts of state funds available for distribution; and educational constraints, such as the minimum total expenditure considered necessary to provide an "adequate" educational program.

### III. DEVELOPING THE MODEL

#### DETERMINING THE OBJECTIVE FUNCTIONS OF STATE SUPPORT PROGRAMS

There are many effectiveness criteria which we shall call "objective functions," that could be used by educational planners in applying a linear-programming model. For example, the following objective functions might be appropriate here for a junior-college state support program:\*

1. Minimization of the state mandated computational tax rate
2. maximization of the minimum total level of district expenditure
3. minimization of total overall costs of the state support program
4. minimization of the percentage spread in final total district expenditure per ADA
5. minimization of the total state cost

#### FORMULATION OF THE CONSTRAINT SET

The constraint set of the state support model includes limitations which might be placed upon the support system. These limitations might concern the overall percentage relationships between state and local funds, budgetary limits on the amount of state funds

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\* The author has, in fact, used these objective functions in a study of the California junior-college state support program, with excellent results (see Ref. 5). The state support model in that study was also solved for the maximization of a specially derived objective function that allowed the model to distribute proportionately more state aid to the districts with the lowest assessed valuation. Three design criteria for state support programs were also explored: (1) all the final total district expenditures were required to be equal; (2) the percentage spread between the highest- and lowest-income districts was specified to be 10 percent; and (3) an optimal percentage spread (i.e., a spread in final total district expenditure that maximized the given objective function) was assumed to exist between the highest- and lowest-income districts.

available for distribution, the spread in final total district expenditure, etc. In addition, certain upper- and lower-bound constraints on some of the variables are necessary to restrict the problem to manageable size. The constraint set of the state support model developed here; then, includes the following equations and expressions.

The fiscal relationships at the school-district level are given by

$$A_i X + Y_i \geq F \quad (1)$$

where

$A_i$  = the assessed valuation in district  $i$ , modified and adjusted as necessary by the state to measure local ability to support education, \$/ADA

$X$  = the state mandated computational tax rate for the state support program

$Y_i$  = the total state aid to district  $i$  (basic aid + equalization), \$/ADA

$F$  = the foundation level for the state support program, \$/ADA

If a variable,  $E_i$ , representing the amount (in dollars per ADA expended by the school district  $i$  beyond the foundation level, is included in Expression (1), the inequality can be transformed into an equality:

$$A_i X + Y_i = F + E_i \quad (2)$$

At the state level, the total local costs or the total funds raised from local sources can be calculated by means of Eq. (3):

$$\sum A_i ADA_i X = T \quad (3)$$

$ADA_1$  = the average daily attendance in district 1

$T$  = the total local funds used by the program

The total state funds,  $S$ , used by the state support program are given by

$$\sum ADA_1 Y_1 = S \quad (4)$$

The total overall costs (state plus local),  $TT$ , of the state support program then equal

$$S + T = TT \quad (5)$$

or

$$\sum A_1 ADA_1 X + \sum ADA_1 Y_1 = TT \quad (6)$$

Expressions representing the percentage relationships between the local and state shares of the total overall funds used by the state support program can now be written as

$$T \leq \alpha_1 TT \quad (7)$$

$$T \geq \alpha_2 TT \quad (8)$$

$$S \leq \beta_1 TT \quad (9)$$

$$S \geq \beta_2 TT \quad (10)$$

where

$\alpha_1$  = the maximum-percentage local share of the total overall costs

$\alpha_2$  = the minimum-percentage local share of the total overall costs

$\beta_1$  = the maximum-percentage state share of the total overall costs

$\beta_2$  = the minimum-percentage state share of the total overall costs

The dollar spread in the final total district expenditure between the districts with the highest and the lowest local-support ability in the system is a very important design criterion. Many educators have stated that the equalization of final total district expenditures is a first approximation to equalization of education opportunity; (1) others feel that some spread is justifiable. Three design criteria dealing with the problem of the dollar spread in final total district expenditure were investigated in this study.\*

1. All districts in the system have the same final total district expenditure per ADA
2. The percentage spread in final total district expenditure is fixed
3. An "optional" percentage spread is used to maximize or minimize the particular objective function under consideration

The spread of the final district expenditure,  $E_1$ , can be controlled in the model by means of the following expression:

$$E_1 \leq \psi F \quad (11)$$

where  $\psi$  is the optimum maximum percentage of the foundation level that the poorest district's total expenditure can differ from that of the most affluent district.

In Expression (11), if  $\psi = 0$ , design criterion 1 is satisfied;  $\psi \geq 0$ , design criterion 2 is satisfied. To satisfy design criterion 3 an expression of the following type must be incorporated into the constraint set of the model:

$$E_1 \geq \pi F \quad (12)$$

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\*As stated earlier, the final total district expenditure does not include those funds raised by the local districts from a tax rate in excess of the state mandated computational tax.

where  $\eta$  is the optimal percentage spread to be determined and  $F$  is the optimal foundation level to be determined.

However, since both  $\eta$  and  $F$  are variables, Eq. (12) is nonlinear and therefore cannot be included as part of a linear constraint set of the model. Therefore, an assumption must be made concerning the foundation level,  $F$ . If  $F$  is specified by the legislature or the educational planner take some amount per ADA, then Eq. (12) becomes linear and can be included. The "optimal" percentage spread,  $\eta$ , in total district expenditure can then be calculated by the solution of the model under various objective functions. When the percentage spread is variable and the foundation level specified, the total district expenditure is given by

$$E_i \leq \eta \lambda \quad (13)$$

where

$\eta$  = the variable percentage spread in total district expenditure

$\lambda$  = the specified foundation level for the program

By the inclusion of Expression (13) into the constraint set of the model, the educational planner can calculate an "optimal" percentage spread; that is, the percentage spread that would be required to either maximize or minimize a particular objective function. The educational planner can also solve the model for the objective function of minimizing the percentage spread in final total district expenditure per ADA for all the districts in the state support program.

If the educational planners wish to include optional utilization of new resources that might be added to the state support system (e.g., student tuition or federal aid on an ADA basis), equations representing these "new" resources can easily be incorporated into the constraint set of the model. For example, if student tuition,  $Z$ , is to be a new resource, equations of the following type can be included in the constraint set of the model:

$$\sum ADA_1 Z = ST \quad (14)$$

where

Z = the tuition to be calculated for the system

ST = the total amount of student funds used

and

$$ST \leq \gamma_1 TT \quad (15)$$

$$ST \geq \gamma_2 TT$$

where

$\gamma_1$  = the maximum percentage of student funds to the total overall costs of the program

$\gamma_2$  = the minimum percentage of student funds to the total overall costs of the program

If one assumes that the optimum amount to be charged for tuition is no greater than a fixed percentage of the foundation level, an expression of the following form can be included in the constraint set:

$$Z \leq \delta_1 F \quad (16)$$

$$Z \geq \delta_2 F$$

where

$\delta_1$  = the maximum percentage of the foundation level to be charged for tuition

$\delta_2$  = minimum percentage of the foundation level to be charged for tuition

In addition, if the educational planner wishes to place absolute upper and lower bounds upon the tuition variable, even though the

constraint might be redundant in some cases, expressions of the following form can also be included in the model:

$$Z \leq \lambda_1 \quad (17)$$

$$Z \geq \lambda_2$$

where

$\lambda_1$  = the maximum tuition to be charged

$\lambda_2$  = the minimum tuition to be charged

Finally, the total amount of funds available from the state for distribution to local school districts can be represented by means of the following equations:

$$\sum A_1 A_1 X + \sum A_1 Y_1 = TT \quad (18)$$

$$\sum A_1 Y_1 = W + \sum A_1 Z \quad (19)$$

where

$W$  = the amount of funds available from the legislature

$\sum A_1 Z_1$  = the amount of new resources or student funds available to the system

$\sum A_1 Y_1$  = the total amount of funds distributed to the local school districts by the state

$TT$  = the total costs of the system

$\sum A_1 A_1 X$  = the total local costs of the program

Notice that the amount of state funds,  $S$ , available for distribution from the state is now given by

$$S = W + \sum ADA_i Z \quad (20)$$

where

$W$  = the total amount of funds appropriated by the legislature

$Z$  = the student tuition in the state support program

Since, from Eq. (4),  $\sum ADA_i Y_i = S$ , the use of Eq. (4) guarantees that all the funds available to the system from the state will be distributed to the local school districts. Equation (20) was included in the model to allow the researcher some flexibility in planning for the utilization of funds other than those from state or local sources.

To complete the constraint set, some of the variables and parameters in the model can have upper and lower bounds placed upon them. For example:

Minimum	Maximum
$X \leq$ tax rate	$X \geq$ tax rate
$F \leq$ foundation level	$F \geq$ foundation level
$Y_i \leq$ state aid to district i	$Y_i \geq$ state aid to district i
$Z \leq$ tuition	$Z \geq$ tuition
$W \leq$ state funds available from the legislature	$W \geq$ state funds available from the legislature

If the upper bound is set equal to the lower bound for a particular variable, the model treats this constraint as an equality (i.e., if  $W \leq \$39.9$  million and  $W \geq \$39.9$  million then  $W = \$39.9$  million).

In addition, alternate optimal solutions can be derived from the model by the parameterization of selected variables:

$\Delta F$  = foundation level

$\Delta W$  = state funds from the legislature

$\Delta Z$  = student tuition

$\Delta Y$  = minimum aid to district i

Parameterization allows for great flexibility in planning state support programs, since any of the political shortcomings of the model can usually be resolved by this method. Parameterization of some of the variables in the state support system can also test the economic sensitivity of the constraint set of the model, although a post-optimal sensitivity analysis of the dual solution and reduced costs can also be used for this purpose.

#### IV. AN EXAMPLE APPLICATION OF THE MODEL

To illustrate the use of this approach to educational-resource allocation, the model was applied to the foundation program for California junior colleges. Calculations were made for each of five different objective functions:

1. Minimization of the tax rate
2. Minimization of the foundation level
3. Minimization of costs to the state
4. Minimization of the percentage spread in the final total district expenditure per ADA
5. Minimization of the total overall costs

For each case, one of the following design criteria was selected:

1. Equalization of the final total district expenditure per ADA
2. A fixed percentage spread in the final total district expenditure per ADA
3. A variable percentage spread in the final total district expenditure per ADA

The illustrative results presented in this section are for the case in which the objective function was the minimization of the percentage spread in the final total district expenditure per ADA. (See Ref. 5 for solutions obtained for the other four objective functions.)

The characteristics of the sample of junior-college districts used in the model are given in Table 2.

Table 2

CHARACTERISTICS OF JUNIOR-COLLEGE DISTRICTS USED IN SAMPLE

Number of districts .....	55
Range of local-support ability* .....	14.096/1
Highest total expenditure per ADA, dollars.....	904.74
Spread in total district expenditure per ADA, percent.....	51
Total ADA.....	175,811
Total assessed valuation, dollars.....	27,336,409,135
State share of total costs, percent.....	37.089

Values of the important parameters and variables in the California junior-college items are given in Table 3, along with the values used as upper and lower bounds for these items in the model.

Table 3

CHARACTERISTICS OF CALIFORNIA JUNIOR-COLLEGE FOUNDATION PROGRAM

Item	Actual Value	Upper bound in model	Lower bound in model
Foundation level (F), \$/ADA	600	600	600
Tax rate (X), \$/\$10,000 of assessed valuation	25	50	5
Range of state aid to any district ( $Y_1$ ), \$/ADA	125-544	700	0 <sup>a</sup>
Total state costs, \$ millions	40,290	40,290	40.290 <sup>b</sup>
Total overall costs, \$ millions	108,631	---	---
Student tuition or other funding, \$/ADA	0	0	0
Spread in final district expenditure per ADA, percent	51	---	---
State share of total costs, percent	37.089	50.000	37.889

<sup>a</sup>The lower bound was parameterized in increments of \$5/ADA.

<sup>b</sup>This variable was parameterized in increments of \$1 million.

\* Ratio of local-support ability of most affluent to least affluent district.

SUMMARY OF MODEL AND VALUES USED\*

Minimize  $\Psi$

$$A_i X + Y_i \geq F + E_i$$

$$E_i < \Psi F$$

$$F = 600$$

$$\sum A_i ADA_i A = T$$

$$\sum ADA_i Y_i = S$$

$$S + T = IT$$

$$S \leq 0.500 IT$$

$$S \geq 0.365 IT$$

$$T \leq 0.635 IT$$

$$T \geq 0.500 IT$$

$$Z = 0$$

$$S = \sum ADA_i A + W$$

$$W = 40.29036 \text{ (parameterized in increments of \$1 million)}$$

$$X \leq 50$$

$$X \geq 5$$

$$Y_i \leq 0 \quad \text{(parameterized in increments of \$25/ADA)}$$

$$Y_i \geq 700$$

\* The model generated a constraint set with 233 rows (activities) and 116 columns (variables). The IBM 360 MPS linear-programming algorithm was used at the Central Computing Network, UCLA. The average execution time to solve the model was 2 minutes.

RESULTS OF THE CALCULATIONS

As stated above, the objective function used was the minimization of the percentage spread in final total district expenditure per ADA. While the actual spread was 31 percent, the model found the optimal value to be 24 percent--a difference of 27 percent.

Table 4 lists some of the actual and optimal values for other variables included in the model.

Table 4

COMPARISON OF ACTUAL DATA WITH MODEL RESULTS

Item	Actual	Optimal	Difference
Tax rate	0.25	0.23	-1.1
Local funds used, \$ millions	68.34	55.42	-2.9
State funds used, \$ millions	40.29	40.29	0
Student tuition	0	0	0
Total system costs, \$ millions	105.71	108.63	-2.9
Foundation level, \$/ADA	600	600	0
Minimum amount of state aid to any district, \$/ADA	125	0	-125
Maximum amount of state aid to any district, \$/ADA	544	574	+ 30

From Table 4 it is apparent that the solution of the model resulted in a lower tax rate, while the foundation level and the amount of a state funds used remained the same. The funds needed to compensate for the decrease in the tax rate were generated from the districts with high local-support ability. Some of the changes in state allocation among the districts in the state were the following:

- o The seven highest local-ability districts received less aid than before; the next three received more state aid.
- o A total of 48 districts received more state aid.

The total district expenditure determined by the model varied from \$600/ADA to \$746.46/ADA, while the actual range was from \$600/ADA to \$904/ADA. The amount of state aid distributed to any district in the system varied from none for the district with the greatest local ability to \$574/ADA for the district with the least. The actual range for this variable was from \$125 to \$544.

The minimum amount of state funds to be given to any district was parameterized in increments of \$25/ADA, and the total state funds available for distribution was parameterized in increments of \$1 million. Table 5 lists the alternate optimal solutions resulting from parameterization of certain initial variables in the state support program.

Table 5.

OPTIMAL SOLUTIONS RESULTING FROM PARAMETERIZATION

Item	Optimal Values				
Parameterization of Minimum State Aid Available					
	0	25	50	75	100
Minimum aid to each district, \$/ADA	0	25	50	75	100
Tax rate	23.93	23.95	23.94	24.04	24.29
Foundation level, \$/ADA	600	600	600	600	600
State funds used, \$ millions	40.29	40.29	40.29	40.29	40.29
Spread in final expenditure per ADA, percent	244	287	331	374	429

PARAMETERIZATION OF STATE FUNDS ALLOCATED

State funds allocated, \$ millions	40.29	41.29	42.29	43.29	44.29
Minimum aid to each district, \$/ADA	0	0	0	0	0
Tax rate	23.93	23.55	23.18	22.81	22.43
Foundation level, \$/ADA	600	600	600	600	600
Spread in final total expenditure per ADA, percent	244	224	205	185	165

LIMITATIONS

The primary limitation on the use of the model will be determined by the degree to which the distribution strategies meet the actual fiscal needs of the district. This limitation can be overcome by the inclusion of an equation of the following type into the constraint set:

$$ADA_1 Y_1 \geq N_1$$

where

$N_1$  = the minimum amount of state funds needed by the district to operate compensatory programs, state-mandated programs, etc.

$Y_1$  = the state allocation per ADA to district 1

$ADA_1$  = the average daily attendance in district 1

The measurement of  $N_1$  is critical to the success of the model in determining resource allocation. This value could represent, for example, some varying percentage of teacher-salary costs for each district. The percentage could be made variable according to the district's financial needs, its ability to support educational expenditures, and its current effort to support education. The final value of  $N_1$  for each district should also take into consideration the competition for the tax dollar from other municipal services. (5)

#### V. CONCLUDING REMARKS

The use of mathematical-programming techniques for determining the most efficient allocation of state funds to local school districts has many advantages, the most important of which are listed below:

A mathematical model can simulate the entire state support program in a constrained environment. All the available resources--state, local, and other (federal, student)--can be considered along with the constraints.

By use of an optimizing feature, one or a combination of variables in the system can be maximized or minimized subject to the constraints imposed on the system.

A post-optimal sensitivity analysis can be performed to give educational planners information as to the possible consequences of different values of the objective function, or of unit relaxations in the equations of the constraint set in the model. A sensitivity analysis can also give the range of values of the coefficients of the objective function for which the solution will remain optimum. In short, the sensitivity analysis can indicate directions for future research and can give the planner valuable information concerning the relative economic worth of the resources he has at his disposal.

Sophisticated objective functions can be developed to allocate state funds according to some agreed-upon priorities (e.g., allocating maximum amount to districts with the least local resources, or the highest ADA).

The financial need of each district can be considered in the model by placing a lower-bound constraint on the amount of state funds it can receive.

Advances in school finance systems, such as the use of correction factors<sup>(6)</sup> to allow for differing access to local resources, can be easily incorporated into the state-support model once they become available.

3)

With the use of computers and sophisticated mathematical techniques, it should be possible to devise state educational support programs that are fair and equitable and that at least ensure the equality of educational opportunity.

The tools and techniques of operations research can be used to redesign or improve the increasingly complex state educational finance systems, and this study has applied one of these tools, linear programming, to a resource-allocation problem in education. More work is needed in the formulation and development of state-support models, and it is to be hoped that educators will encourage the implementation and use of these models.

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