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ABSTRACT

This curriculum guide is specifically designed for the slow learning students in grade 10. It is one of a series of course guides for grades 6-11. The intent of the curricular designers was to outline mathematical experiences which would be appropriate for the characteristics of these students. The areas of mathematical content included are: 1) numbers, operations, and algorithms, 2) geometry, 3) measurement, 4) graphing, 5) probability and statistics, 6) algebra, 7) logic. Each content area contains: 1) master charts for grades 6-11, 2) grade level chart for grade 10, 3) behavioral objectives for the area, 4) teacher commentary sheets, 5) student worksheets. A collection of recreational activities is included for student motivation. (RS)

MATHEMATICS FOR BASIC EDUCATION

GRADE 10

Baltimore County Public Schools

BALTIMORE COUNTY PUBLIC SCHOOLS

Mathematics for Basic Education

Grade 10

A Tentative Guide

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Towson, Maryland
September, 1967

FOREWORD

The last ten years have reflected a growing awareness and concern by mathematics educators for slow learners. Recent conferences sponsored by the National Council of Teachers of Mathematics and the School Mathematics Study Group have focused their attention upon the various aspects of the slow learner--economic, sociological, psychological, and pedagogical. Researchers are now able to present some of the preliminary findings of their work to curriculum specialists. Curriculum materials are being developed, but these are limited and have not yet been produced commercially.

The Office of Mathematics has been deeply involved in this pioneering stage. Staff members have participated in the recent NCTM conference, attended national conventions which have presented important addresses on the slow learner, and invited consultants for assistance and guidance. The summer workshops in 1963 and 1964 produced bulletins which presented guidelines and recommendations for the mathematics education of the slow learner. The summer workshop in 1966 produced a resource manual of activities--developmental, recreational, and computational--as a first stage in creating a mathematics program for the slow learner. The efforts of this 1967 summer workshop have been directed toward providing structure and continuity of the desired behavioral outcomes as they relate to mathematical concepts and skills

This publication is specifically designed for the slow learning students in Grade 10. It is one of a series of tentative course guides in Mathematics for Basic Education, Grades 6-11. It will presume teacher evaluation in terms of student reaction and behavioral responses. The teacher is urged to study carefully the philosophy of behavioral objectives as stated in action words and its implications for instruction and assessment. These ideas are stated in the following sections of the Introduction.

The Board of Education and Superintendent of Schools wish to express their appreciation to the curriculum committee and to all mathematics teachers of Baltimore County whose effort made possible the development of this curriculum publication. The staff wishes to acknowledge the valuable contributions of Dr. Henry H. Walbesser, University of Maryland, and Dr. Max A. Sobel, Montclair State College.

Special commendation is due Mrs. Betty V. Hagedorn, Mrs. Barbara S. Parks, and Miss Martha Ann Lynch for their careful and painstaking effort in the production of this bulletin.

William S. Sartorius
Superintendent of Schools

Towson, Maryland
September, 1967

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INTRODUCTION

A. PHILOSOPHY

The rapid trends toward greater automation, use of computers, and the increased technological skills demanded of workers have dramatically reduced the market for unskilled and semi-skilled laborers. Twenty percent of our population consists of persons, who, according to their academic talents, are termed "slow learners." It becomes apparent that young people of limited ability, who are potential unskilled and semi-skilled workers, must be prepared for a useful place in our society. Unless these youths are taught salable skills, they must be supported by tax money. This situation could lead to a society composed of one segment which is overworked to contribute tax dollars and services, and another segment which is unemployed and consumes the wealth, yet produces nothing.

The magnitude and urgency of this problem demand that schools develop appropriate educational opportunities for slow learning students throughout their school experiences. The schools are thus faced with the problem of training students for jobs and services which may be outmoded by the time they enter the business world. Equally disturbing is the fact that no one can foretell the many new jobs and products which will be created for which no training has been provided. It is generally conceded that the service occupations hold the greatest promise of employment for the slow learner. Functional competence in mathematics is essential for all persons entering these service occupations. Industry is retaining many semi-skilled and unskilled workers who have been displaced by automation through extensive retraining. Reports indicate that greater success is obtained in retraining workers who have more mathematics background than those who do not. Furthermore, training in mathematics provides youth with broader choices of vocational employment. It is imperative that the student be given a sound foundation in mathematics if he is to function effectively as a producer and consumer, and a citizen in his community.

It is axiomatic that the slow learner should be educated in his own right and to the maximum of his ability. Any adaptation of an academically oriented program must surely fail. A program of mathematics for the slow learner should be based upon the latest developments and research in learning theory, an appropriate selection and reorganization of mathematical topics, and the inclusion of new materials as well as new techniques for presenting mathematical concepts and developing skills. Proper pacing of these concepts and skills must underlie the entire structure. All the human resources of the educational system - the mathematics teacher, the principal, the mathematics supervisor, resource teachers, the guidance counselor, and other specialists - must be brought to bear on this problem.

Probably the most important factor in the success of a mathematics program for the slow learner is the teacher. Such a teacher should be prepared psychologically to teach students of limited ability. This implies an acceptance of the student for what he is, and an awareness of the operational level of the student. Furthermore, the teacher should have such characteristics as emotional maturity, a broad background of mathematics and a curiosity for more, a liking for young people, patience, and above all, a sense of humor. Such a teacher can do much to enhance the usually poor self-image of the slow learner, and convince the student that he is indeed a person worthy of dignity and respect in this society.

B. IDENTIFICATION OF THE SLOW LEARNER

The most obvious characteristic of the slow learner is his inability to keep pace with those students who are average in their rate of academic growth. However, other psychological, social, cultural, and physical factors may be considered in identifying these students. The following criteria, which are divided into two categories, may be used to form a basis for the selection of the students who may be classified as slow learners. The two criteria - measurable and traits - should receive equal consideration when the student is being identified.

Measurable Criteria

1. I.Q. Range 75 - 90 resulting from at least two group tests or an individual test.
2. Percentiles on group tests of mental ability and achievement ranging from 0 - 19 (approximately two or more years below grade level in reading comprehension and arithmetic.)
3. Teacher grades - consistently below average, as indicated by "ability" C's and D's as well as E's.

Traits Criteria

1. Limited academic interest
2. Difficulties in planning and in carrying out work without supervision
3. Limited creativity and intellectual curiosity
4. Indications of short attention span
5. Severe limitations in the ability to communicate orally or in writing.

C. BEHAVIORAL OBJECTIVES

An integral part of any collection of instructional materials is a statement of the objectives. This bulletin is no exception. The objectives stated here are stated in behavioral terms. That is, each objective is stated in terms of the desired student behaviors.

To clarify, consider the following example of a behavioral objective which is taken from the Grade 7 geometry section of this bulletin.

> The student should be able to construct a drawing of a quadrilateral using a straightedge.

The characteristics of this objective are that it tells who is to perform, how he is to perform, and what constitutes an acceptable performance.

To assess the acquisition of the above stated behavior it is only necessary to give a student paper, pencil, and straightedge and instruct him to make a drawing of a quadrilateral. In response, the student can either make such a drawing or he cannot. In any event, it is possible to decide whether or not the stated objective has been realized. Any well stated behavioral objective should point clearly to the type of performance task necessary to assess its attainment.

The clarity of a behavioral objective such as the one stated above is in clear contrast to the vagueness of comparable objectives which state that the student should "understand the concept of quadrilateral" or that the teacher should "develop the concept of quadrilateral." These and other objectives such as "developing appreciations and attitudes" do not lend themselves well to evaluation. Indeed, the assessment of these qualities have always posed difficulties for researchers.

Behavioral scientists such as Jean Piaget and Robert Gagne have asserted that true learning involves a change on the part of the learner so that he no longer reacts as he did before. His whole being views similar situations in a new light. If our instructional program is to effect such changes in slow learning students, then the objectives such be so constructed that they specifically state the desired behavioral responses which are observable and hence can be assessed.

D. ACTION WORDS USED IN STATING BEHAVIORAL OBJECTIVES

The action words which are used to construct behavorial objectives are:

1. IDENTIFY

The student selects by pointing to, touching, picking up, or circling the correct object or class name. This class of performances also includes identifying object properties such as rough, smooth, straight, curved.

e.g. The student should be able to identify the prime numbers from a given set containing prime and composite numbers.

2. DISTINGUISH

The student identifies objects or events which are potentially confusable. This is a more difficult identification.

e.g. The student should be able to distinguish between ordered pairs such as (a, b) and (b, a).

3. CONSTRUCT

The student generates a construction using instruments, a freehand drawing, or by building a model.

e.g. The student should be able to construct a copy of an angle given a straight edge and a compass.

The student constructs an answer or example. The teacher is concerned only with the student's ability to construct the answer or example, not the method or procedure he uses in arriving at the solution.

e.g. The student should be able to construct the product of a fraction and a whole number.

4. NAME

The student supplies the correct name for a class of objects or events orally or in written form.

e.g. The student should be able to name the associative property of addition in the set of whole numbers.

The student names the correct solution to a problem. This is different from construct in that an immediate response is expected. In this sense Name is used in relation to the basic arithmetic facts which students should commit to memory.

e.g. The student should be able to name the addition facts through 9.

5. ORDER

The student arranges or classifies two or more objects or events in proper order in accordance with a stated category. This word is used when the student arranges something from largest to smallest, most to least, or fastest to slowest.

e.g. The student should be able to order a set of whole numbers from largest to smallest.

6. DESCRIBE

The student states all the necessary categories or properties relevant to the description of a designated situation. The student's description must be stated so clearly that any other individual could use the description to do a task, identify an object, or perform an operation. The description is mostly verbal, however a model, hand motions, or a written example could be used to aid in the description.
The teacher must be willing to accept more than one response. For example, the student might describe something in terms of his surroundings, by using example or by stating a definition. The description may include color, size, shape, etc.

e.g. The student should be able to describe sample spaces as ordered arrangements, listing all possible outcomes.

7. STATE A PRINCIPLE OR RULE

The student makes a verbal statement which conveys a rule or principle. This is more limiting than describing in that only one basic response is acceptable. Students may use their own words in stating the rule. For example, when asked the question, "How do you find the area of a square?" A student may respond, "To find the area of a square, measure the length of a side and multiply this number by itself, " or "$A = S^2$"-- both are acceptable answers. Any formula, theorem, or definition is a statement of a rule.

e.g. The student should be able to state the principle that the circumference of a circle equals pi times diameter, ($C = \pi D$).

8. APPLY THE RULE

The student uses a rule or principle to derive an answer to a question. The question is stated in such a way that the student must employ a rational process to arrive at the solution. Students might not be able to state the rule, however, he may still be able to apply it.

e.g. The student should be able to apply the principle of casting

nines to check addition
problems involving whole
numbers.

9. DEMONSTRATE

The student shows a procedure or test for the application of a rule or principle. The teacher wants the student to show how he arrived at an answer, not just the answer alone. This usually involves some action, other than verbal, on the part of the student

e.g. The student should be able to demonstrate a procedure for finding the least common multiple of a given pair of numbers.

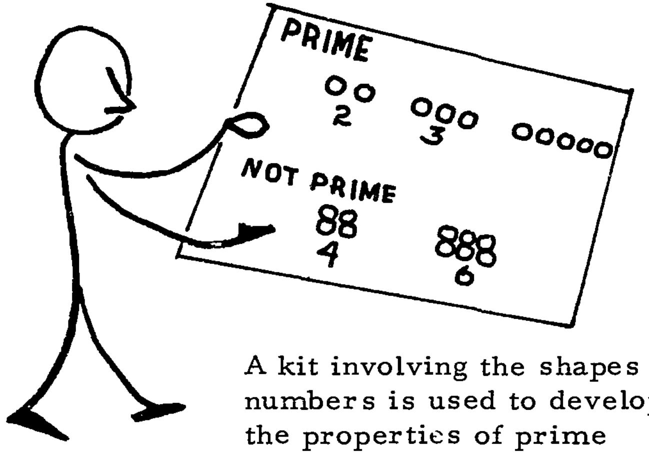
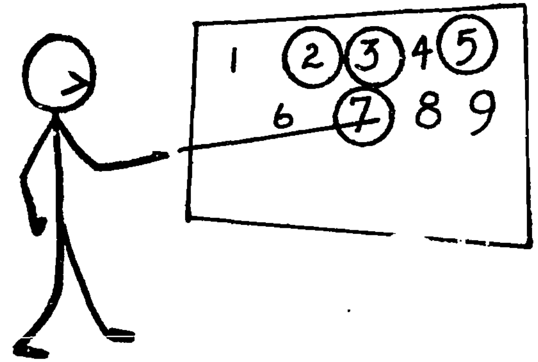
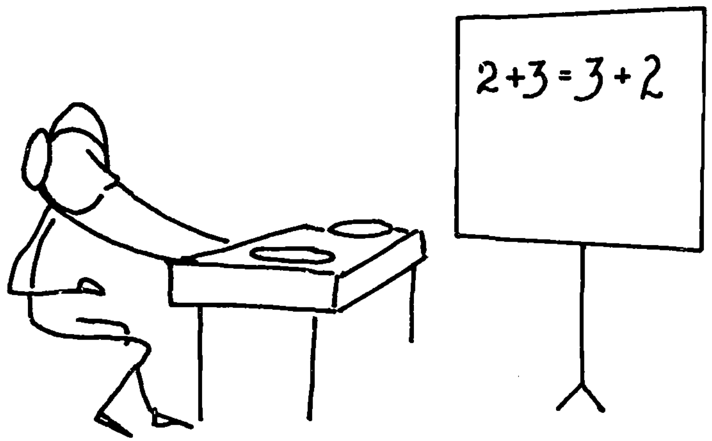
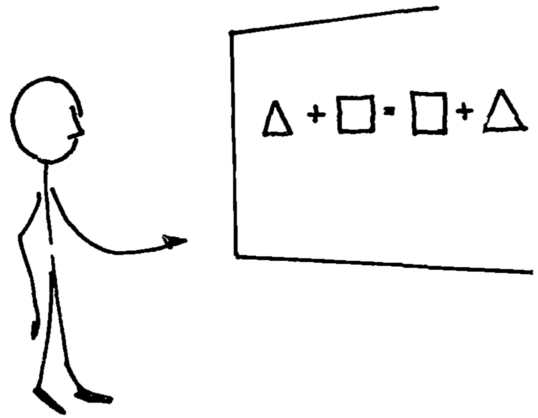
10. INTERPRET

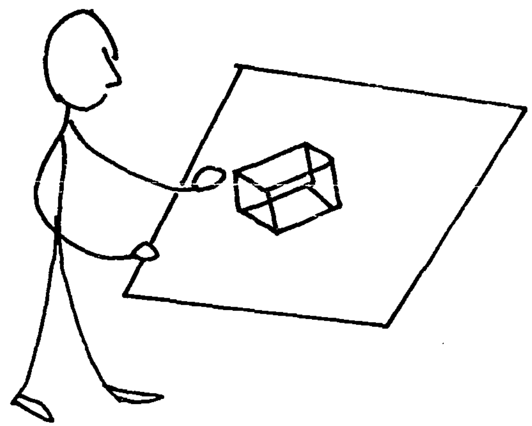
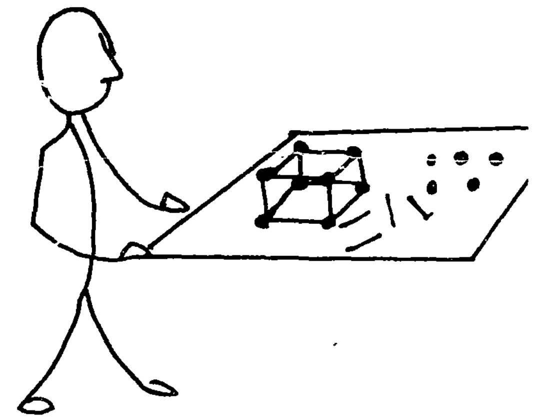
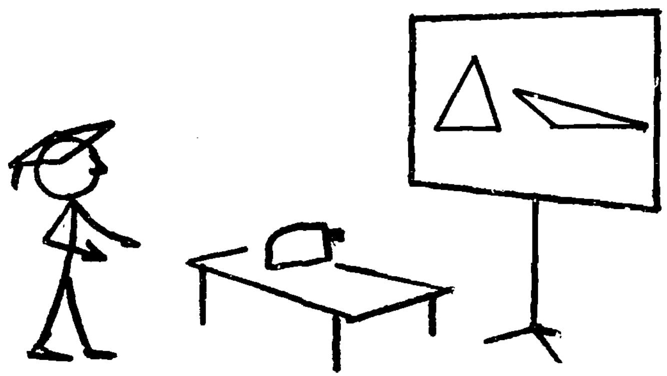
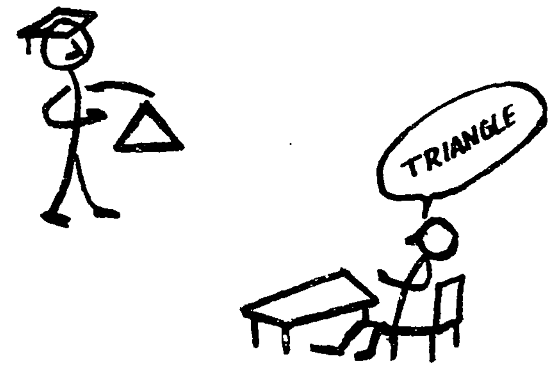
The student uses several rules or principles to draw a conclusion, or identifies objects and/or events in terms of their consequences. This constitutes a high level of learning since the student must see various relationships in order to arrive at the desired conclusion.

e.g. The student should be able to interpret the principles of angle measure by measuring and then classifying angles as right, acute, obtuse, straight, complementary and supplementary.

E. OBJECTIVES - INSTRUCTION - ASSESSMENT

In each of the instructional activities included in this grade the focal point of most comments is the student. First the objectives of the activities are stated in terms of the desired behavioral outcomes on the part of the student. Secondly, the lessons are devoted primarily to student activities. Finally, the suggested assessment procedures indicate ways in which the student shows whether or not he has acquired the desired behavior. In effect, there should be a one-to-one correspondence between the set of objectives, the set of learning activities, and the set of assessment items. The following examples should clarify the relationship.

OBJECTIVE	INSTRUCTION	ASSESSMENT
1. The student should be able to identify a prime number less than 25.	 A kit involving the shapes of numbers is used to develop the properties of prime numbers.	 Students are to circle the numbers which are prime.
2. The student should be able to distinguish among the commutative, associative and distributive properties of addition.	 A tape and filmstrip are used to present the properties.	 Students are given examples on the board and instructed to identify each property.

OBJECTIVE	INSTRUCTION	ASSESSMENT
3. The student should be able to construct a model of a cube given appropriate materials.	 Given straws and string make a model of a cube.	 Use toothpicks and gumdrops to build a model in the shape of a cube
4. The student should be able to <u>name</u> a triangle.	 Filmstrip is used initially to present the name triangle.	 A model is used to assess the student's ability to name a triangle.

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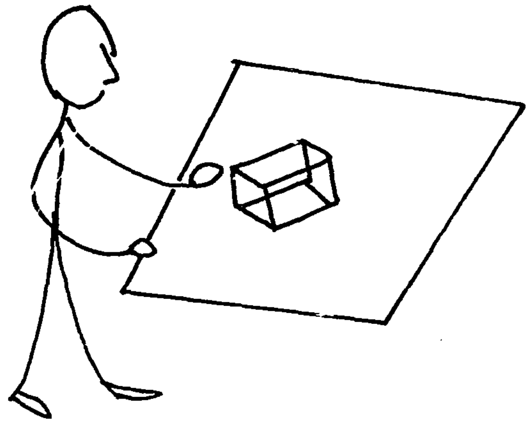
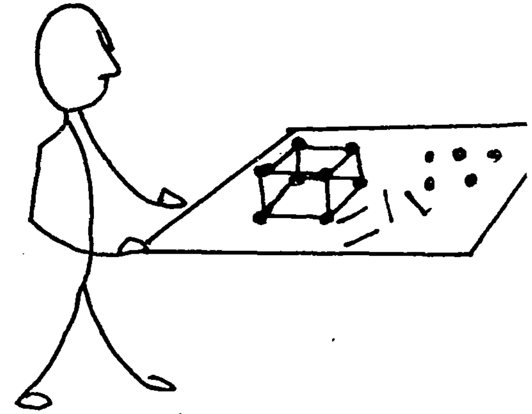
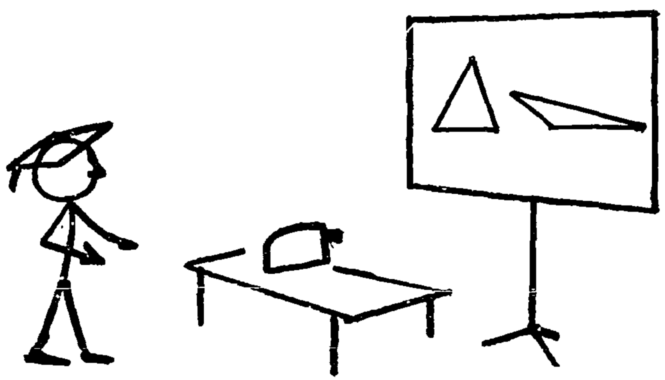
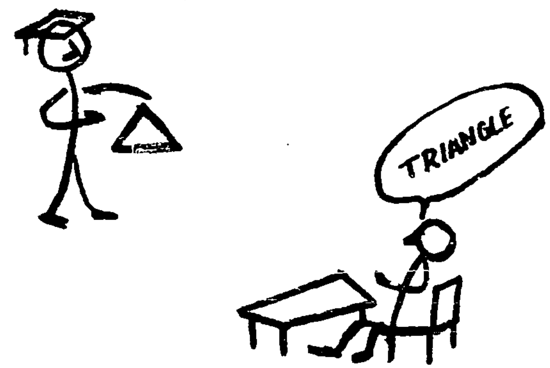
OBJECTIVE	INSTRUCTION	ASSESSMENT
3. The student should be able to construct a model of a cube given appropriate materials.	 Given straws and string make a model of a cube.	 Use toothpicks and gumdrops to build a model in the shape of a cube.
4. The student should be able to name a triangle.	 Filmstrip is used initially to present the name triangle.	 A model is used to assess the student's ability to name a triangle.

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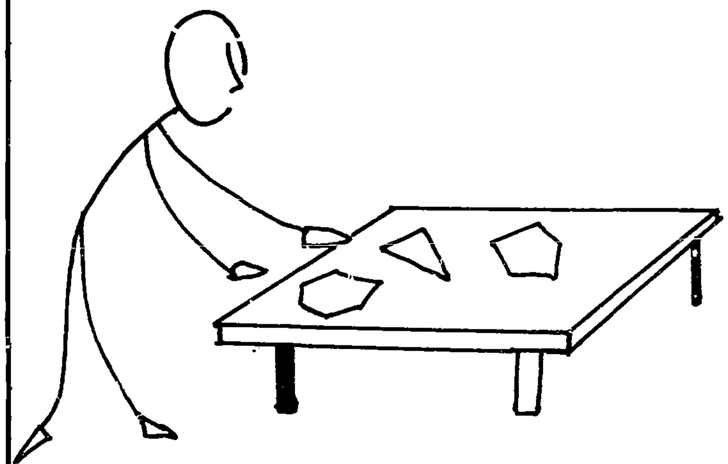
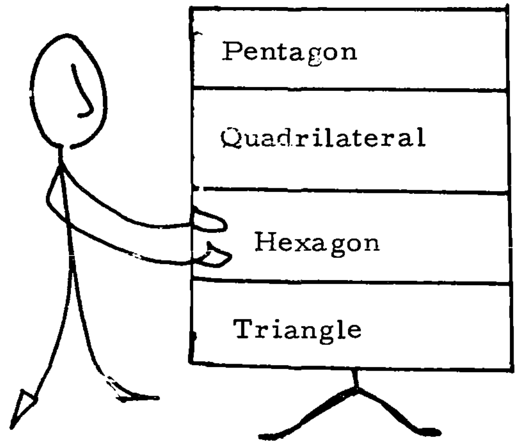
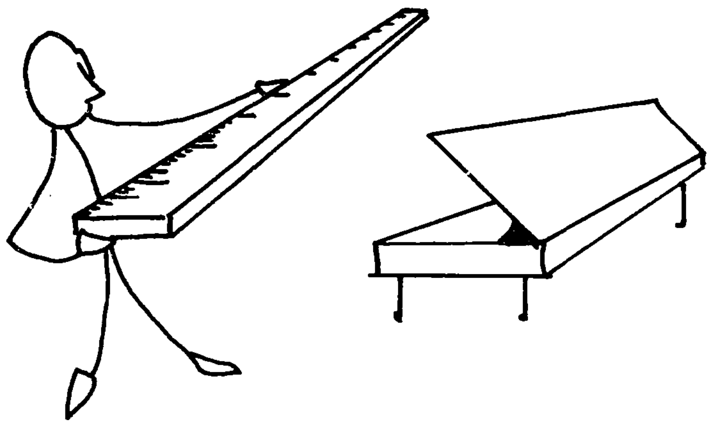
OBJECTIVE	INSTRUCTION	ASSESSMENT
5. The student should be able to order polygons in terms of increasing number of sides.	 Use models to order polygons in terms of increasing number of sides.	 Use the flannel board to order polygc in terms of increasing number of sid
6. The student should be able to describe a foot.	 Using kits students are introduced to the foot as a unit of measure.	 Student may describe a foot by a hand motion.

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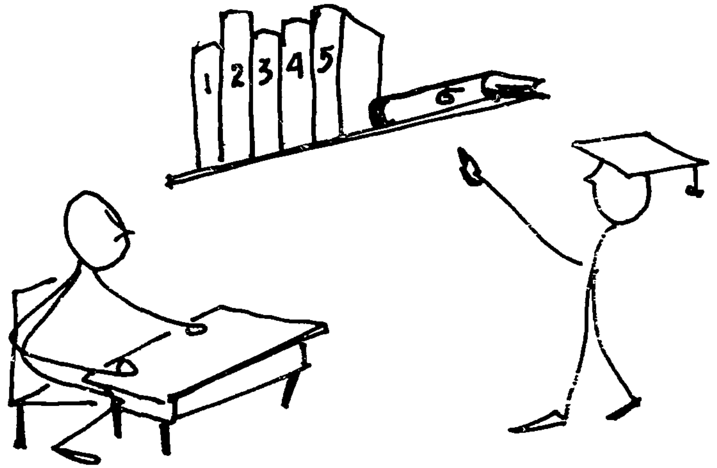
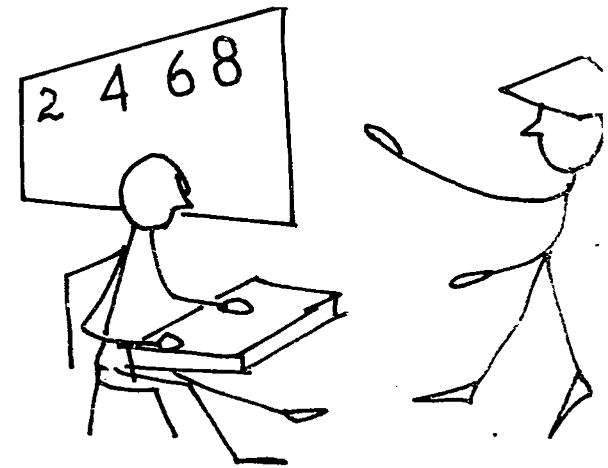
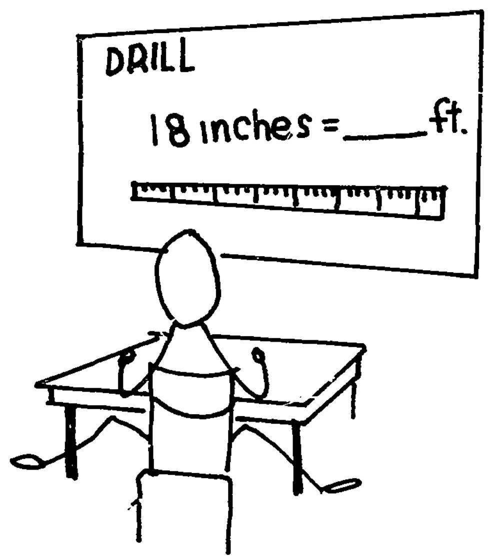
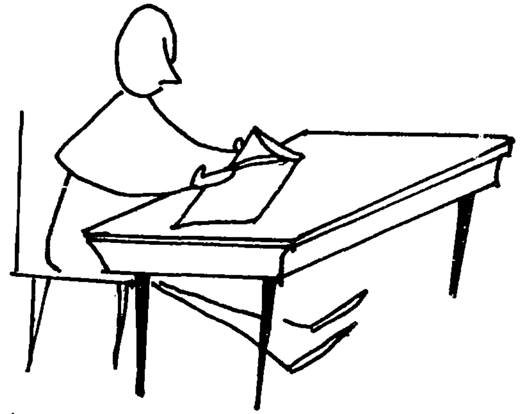
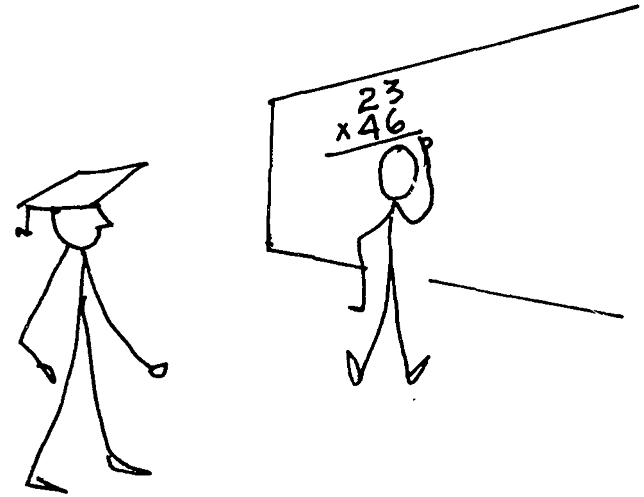
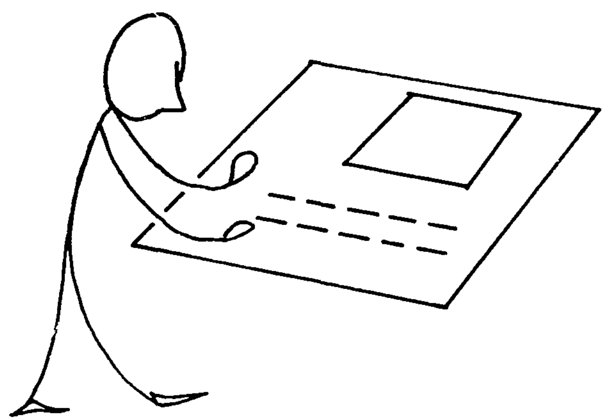
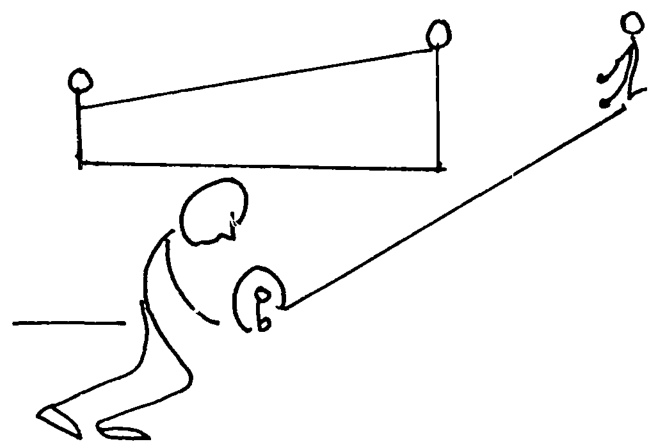
OBJECTIVE	INSTRUCTION	ASSESSMENT
7. The student should be able to state the principle that events which are certain to happen have a probability of 1.	 Six math books on a shelf are used to illustrate what is the probability of picking a math book if blindfolded.	 Student states "The probability of selecting an even number is certain to happen, so it has a probability of 1."
8. The student should be able to apply the principle that 12 inches equals 1 foot to convert a measure expressed in one unit to the other unit.	 A drill on converting inches to feet is used to introduce the topic.	 A written test is used for assessment purposes.

OBJECTIVE	INSTRUCTION	ASSESSMENT
9. The student should be able to demon - strate a procedure for constructing the product of two whole numbers.	 Student uses book to work problems at his seat.	 Students show work at the board.
10. The student should be able to interpret the rules for finding the areas of rectangles and computing in order to solve related problems.	 Students are asked to find the number of tiles necessary to cover the sheet of paper, and then compute to check their results.	 Students measure the dimensions of the tennis court, then compute to find its area.

F. THE BANDED APPROACH

Teachers who participated in the experimental program last year developed a method of teaching which seems to be effective with the slow learner. The key to the success of this method was that a variety of mathematical topics were incorporated into each lesson. Naturally, small group instruction, individual lab work, extensive use of audio-visual aids, games and the like provided the variety of activities within the lesson which is necessary to change the pace when working with this ability student. Thus, the student was exposed to a program of instruction which provided a variety of activities as well as a variety of mathematical content within a given period. This method of teaching will be referred to as the "banded approach."

To elaborate further the "banded approach" is a flexible way of organizing instructional activities in the class period. Normally the lesson is divided into three bands, although sometimes it may be divided into one, two, or even four bands depending on the nature of the activity. All bands are not necessarily concerned with the same mathematical topic. For example, a unit on Geometry might be taught along with related activities on Fundamental Operations. Thus, the unit on Geometry is split into smaller parcels and presented over a longer period of time rather than being presented as a two week concentrated unit. Since basic students typically have short attention spans, the material must be presented within a smaller time interval. Thus, the major portion of the lesson might be presented during a 25 minute segment since this seems to be about the maximum length of time these students can concentrate on any one activity.

Description of Bands

Band I is usually a short activity of about 5 - 10 minutes duration. For example, students may review their addition facts using the Math Builder, have an oral number puzzle, or complete a number pattern. The variety of activities which might be used is numerous.

Band II usually contains the major topic for the day. It is about 25 minutes in length. For this activity, specific behavioral objectives are stated. Students are exposed to instructional activities which are designed to enable them to acquire the desired behaviors. Assessment procedures might also be employed here to determine whether students have acquired some of the behaviors specified in the objectives. Remaining objectives may be assessed in other bands of subsequent lessons.

Band III is usually a short activity of about 5 - 10 minutes. This band can be handled two ways. First, all the students might begin work at the same time on a class activity. Secondly, as each student completes his work in Band II he begins some planned individual or small group activity. For example, after a student has completed his work from Band II, he may go to a specified place in the room and pick up an interesting puzzle or game, work on one of the SRA kits, or listen to a tape at the listening post. This approach keeps students actively involved in learning activities rather than just waiting for the class to finish an assignment. Thus, a more efficient use of the student's time is made.

The teacher should realize that the above descriptions indicate a general outline of what constitutes a banded approach. Flexibility is the key. Teachers should vary the number of bands as well as the length of time devoted to each depending upon what is being presented.

To illustrate how this approach could be implemented with your students, a sample two week unit is included in section H at the end of the Introduction. This unit contains:

1. A block plan indicating the topics to be presented each day.
2. Detailed lesson plans indicating the materials to be used, the behavioral objectives, suggested methods for presentation, student work sheets, and assessment items.
3. A series of inventory tests designed to indicate areas of difficulty.

This two week unit should be taught near the beginning of the school year. It is hoped that this unit will provide a model from which the teacher can create other units utilizing the same approach.

G. HOW TO USE

This guid rovides a structured program of instruction for the slow learner in mathei atics grades 6 - 11. Suggestions for implementing the program are included. The teacher is urged to read this section carefully. Familiarity with the materials included and suggestions for their use should be of great assistance in determining the best program of instruction for basic students.

The guide is divided into the following major areas of mathematical competency: Fundamental Operations, Geometry, Measurement, Graphing, Algebra, Probability and Statistics, and Logic. Recreation is the last section of the book.

Each of the areas of mathematical competency contains the following items:

MASTER CHARTS

These charts give an overview of the mathematical content and the behaviors students are to acquire in grades 6 - 11. The teacher can use these charts to get a picture of the total mathematics program for the slow learning student. Furthermore, the teacher can see which behaviors the students should have acquired prior to entering this grade, which behaviors will be developed during this grade, as well as those to be developed later.

GRADE LEVEL CHART

These charts are identical to the master charts except they contain only the information for a specific grade. They can be used to get an overview of those behaviors which should be acquired by the student during the school year.

LIST OF BEHAVIORAL OBJECTIVES

A list of behavioral objectives for this grade should enable the teacher to interpret the details omitted in the chart. The teacher can use these objectives when planning lessons, since they state precisely what is expected of the student. The teacher should realize that these objectives should not necessarily be taught in the order they are presented, rather objectives from several areas might be used in order to present a logical development of the topic. However, by the end of the year the students should be able to exhibit all the behaviors mentioned.

Also included in this section are references to the student activities which have been developed. These activities have been specifically designed to bring about the desired behavioral changes indicated in the objectives. This should assist the teacher in identifying the type of activity which might be used when developing a particular topic.

STUDENT ACTIVITIES

This section contains a series of suggested activities.

For each activity a Teacher Commentary printed on yellow paper is included. This commentary indicates the title of the activity, the unit, the behavioral objectives, necessary materials, a procedure for implementation and suggested assessment items. Student work sheets are printed on white paper and immediately follow the Teacher Commentary. The teacher can reproduce these work sheets by taking the master copy out of the guide and making a thermal spirit master. The spirit master can then be used to run off copies for the students. Be sure to place the original copy back in the guide so it can be used again at a later date.

If color is desired it may be added by using colored masters before duplication.

The Student Activities section also includes references to:

1. Kits. These kits are effective devices for use in small groups or with individual students. Students perform various experiments and as a result of this experimentation, are lead to generalizations. The teacher is supplied with all the necessary instructions for constructing the kit as well as the accompanying student work sheets. It is suggested that the teacher use student help in the construction of the kits.

2. Tapes. Some tapes and their related student work sheets are included. These tapes cover a variety of topics on each grade level. It is suggested that these tapes be used with small groups of students using listening posts, rather than as a class activity.
3. Programed Instruction. Several programed booklets are included in the guide. These can be used with individual students or small groups for remedial purposes or when the teacher feels additional development might be necessary. The programed booklets can be reproduced using the same procedures outlined for the student work sheets. Again, it is suggested that student help be employed in assembling the programed booklets.
4. Films. Several films are included in the Teacher Commentary. These films can be obtained from the Baltimore County Central Film Library. They can be used with the banded approach since the average running time is between 10 - 15 minutes.

RECREATIONAL ACTIVITIES

The Recreation section of the guide is significantly different from the sections dealing with mathematical competencies. The activities described in this section are designed to develop a positive attitude towards mathematics. There are no behavioral objectives specified in this section. Games and puzzles play an important role in the teaching of mathematics. These activities are to be used throughout the year for motivational purposes. When using the banded approach, the recreational activities are used extensively since they help provide the variety which is necessary to the success of this method of teaching.

In the period of time allocated to produce this guide it was impossible to create activities in each of the areas. Therefore, provisions were made to supplement this guide as other activities are developed. Teachers are requested to send activities which they have found to be successful to The Office of Mathematics so that they may be added to this guide.

4. A SAMPLE UNIT USING THE BANDED APPROACH

SAMPLE UNIT OF BANDED LESSONS - Grade 10

OUTLINE OF TOPICS

LESSON	BAND I	BAND II	BAND III
1	Inventory test-operations with whole numbers	Inventory test-operations with whole numbers	Introduction to modular arithmetic
2	Drill-open and closed sentences	Chart for twelve hour clock	Chart for twelve hour clock
3	Drill-open and closed sentences	Inventory test discussed	Closure and commutativity using twelve hour clock
4	Puzzle-missing digits	Associativity using twelve hour clock	Equivalent sentences
5	Math Builder	Additive identity and additive inverse using twelve hour clock	Explanation of use of SRA Kit
6	Drill-equivalent sentences	Order of operations	SRA Kit used
7	Chart for mod 7	Order of operations	Math Builder
8	SRA-Inventory test	Farmer Friendly	Properties of seven hour clock
9	Additive inverse in mod 7	Subtraction in mod 7	Puzzle-pouring
10	Puzzle-hidden messages	Faces	Faces
11	Drill-commutativity	Properties of faces	SRA Kit used
12	Puzzle-missing digits	Quiz-properties of operations	Quiz-properties of operations

LESSON 1 - BAND 1

INVENTORY TEST ON OPERATIONS WITH WHOLE NUMBERS

Teacher Commentary

I. Unit: Algebra

II. Materials: Duplicated tests

III. Procedure:

A. Give the following directions:

1. The test will determine how well students remember certain work that they had last year.

2. No grades will be given on the test since this is an inventory test. It will enable the teacher and student to see what areas need additional work.

3. Sufficient time will be given for students to complete all parts of the test that they are able to do.

4. If students are unable to answer certain questions, they should skip them.

B. Test should be checked as quickly as possible since they will be returned to students on the third day.

C. Solution to test:

1. e	6. c	11. a
2. d	7. b	12. d
3. c	8. b	13. d
4. a	9. d	14. a
5. b	10. b	15. c

LESSON 1 - BAND 2

INTRODUCTION TO CLOCK ARITHMETIC

Teacher Commentary

I. Unit: Algebra

II. Materials:

A. Model of a clock face

B. Chalkboard

III. Procedure:

A. Place the following examples on the board.

4	8	2	9	10	11	7	8	
+ 5	+ 3	+ 6	+ 7	+ 9	+ 2	+ 8	+ 11	
9	11	8	4	7	1	3	7	Answers

B. Students should give answers orally. Do not place the answers to the problems on the board initially. Then say that you would agree with some of the answers, but disagree with others. Put in the answers shown above.

C. Ask students whether they can think of any place where these answers would be true. If they have difficulty recognizing that these answers would be true on a clock, direct the attention of the students to the classroom clock or make a drawing of a clock face on the board.

D. Using the clock face drawn on the board as a guide, do several more examples orally with the students.

SURVEY TEST

OPERATIONS WITH WHOLE NUMBERS

Name ____________________

Class ________________

Multiple Choice - Circle the letter which represents the best answer for each of the following:

1. An example of the commutative property of addition is:
   a. $2 \cdot 3 = 3 \cdot 2$
   b. $(2 + 3) + 4 = 2 + (3 + 4)$
   c. $(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$
   d. $2 \cdot (3 + 4) = 2 \cdot 3 + 2 \cdot 4$
   e. $2 + 3 = 3 + 2$

2. An example of the distributive property of multiplication over addition if:
   a. $2 \cdot 3 = 3 \cdot 2$
   b. $(2 + 3) + 4 = 2 + (3 + 4)$
   c. $(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$
   d. $2 \cdot (3 + 4) = 2 \cdot 3 + 2 \cdot 4$
   e. $2 + 3 = 3 + 2$

3. An example of the associative property of multiplication is:
   a. $2 \cdot 3 = 3 \cdot 2$
   b. $(2 + 3) + 4 = 2 + (3 + 4)$
   c. $(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$
   d. $2 \cdot (3 + 4) = 2 \cdot 3 + 2 \cdot 4$
   e. $2 + 3 = 3 + 2$

4. An example of the commutative property of multiplication is:

   a. $2 \cdot 3 = 3 \cdot 2$

   b. $(2 + 3) + 4 = 2 + (3 + 4)$

   c. $(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$

   d. $2 \cdot (3 + 4) = 2 \cdot 3 + 2 \cdot 4$

   e. $2 + 3 = 3 + 2$

5. An example of the associative property of addition is:

   a. $2 \cdot 3 = 3 \cdot 2$

   b. $(2 + 3) + 4 = 2 + (3 + 4)$

   c. $(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$

   d. $2 \cdot (3 + 4) = 2 \cdot 3 + 2 \cdot 4$

   e. $2 + 3 = 3 + 2$

6. Since any number multiplied by <u>1</u> (one) gives us the same number for an answer, we call <u>1</u> (one):

   a. the inverse element for addition

   b. the identity element for addition

   c. the identity element for multiplication

   d. the inverse element for multiplication

7. Since any number added to <u>0</u> (zero) gives us the same number for an answer, we call <u>0</u> (zero)

   a. the inverse element for addition

   b. The identity element for addition

   c. the identity element for multiplication

   d. the inverse element for multiplication

8. $174 + 637 + 78 =$

   a. 899

   b. 889

   c. 789

   d. 1591

9. 2137 - 2038 =

a. 199

b. 89

c. 189

d. 99

10. 428 x 967 =

a. 412, 876

b. 413, 876

c. 413, 866

d. 44, 084

11. 1161 ÷ 27 =

a. 43

b. 421

c. 403

d. 313

12. 6 + 3 x 4 - 2 =

a. 12

b. 34

c. 18

d. 16

13. 32 ÷ 2 · 2 ÷ 4 + 4 =

a. 8

b. 6

c. 4

d. 12

14. (2 · 7) + 6 + (8 ÷ 2)

a. 24

b. 21

c. 14

d. 18

15. 3 · (8 + 5) + 4 =

a. 27

b. 33

c. 43

d. 51

LESSON 2 - BAND 1

DRILL: OPEN AND CLOSED SENTENCES

Teacher Commentary

I. Unit: Logic

II. Objectives: The student should be able to:

A. Name and identify open and closed sentences

B. Distinguish between open and closed sentences

III. Materials:

Chalkboard

IV. Procedure:

A. Drill sentences are to be answered orally by the students.

B. Those sentences in which truth value cannot be determined should generate a discussion of open and closed sentences.

C. Drill:

Are the following sentences True or False?

1. Abraham Lincoln was a president of the United States.
2. Mickey Mantle plays for the Orioles.
3. She is a tennis player.
4. He is six feet tall.
5. Balboa discovered America.
6. $5a = 25$
7. $10 + 2 > 8$
8. $3 + 7 < 4 + b$
9. $9 + 7 < 16$
10. $20 - 4 = 13 + 3$

D. Solution:

1.	True	6.	Open
2.	False	7.	True
3.	Open	8.	Open
4.	Open	9.	False
5.	False	10.	True

LESSON 2 - BAND 2

CLOCK ARITHMETIC

Teacher Commentary

I. Unit: ..gebra

II. Activity: Construction of a chart of all possible addition combinations using the twelve hour clock and identification of certain patterns in the chart.

III. Materials:

A. Model of clockface

B. Work sheet entitled, "The Twelve Hour Clock"

C. Chalkboard

IV. Procedure:

A. Review addition problems of the type done the previous day.

B. Ask the students how many different combinations or pairings of the numbers in the set of elements on the twelve hour clock can be made. (Many pairings can be made.)

C. Tell the students that you are going to replace the twelve on the clock with a zero. Ask what members will now be included in the set of elements on the twelve hour clock.
$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

D. Distribute the work sheet, "The Twelve Hour Clock." Do several examples with the students to show them how to fill in the chart.

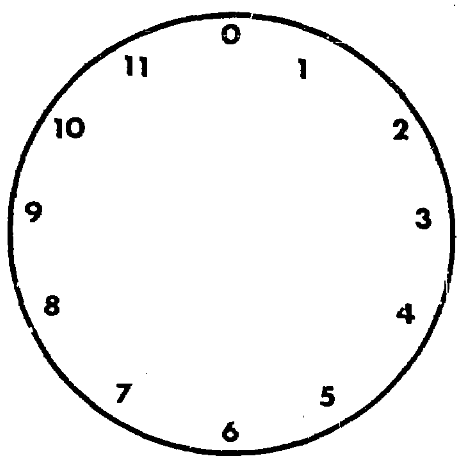
E. Have the students complete the chart for classwork.

F. Ask the students to look for patterns in the chart. (Same numbers on diagonals, and other patterns.)

THE TWELVE HOUR CLOCK

+	0	1	2	3	4	5	6	7	8	9	10	11
0												
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												

Directions: Use the clock below to help you to fill in the chart.



LESSON 3 - BAND 1

DRILL: IDENTIFICATION OF OPEN AND CLOSED SENTENCES

Teacher Commentary

I. Unit: Algebra

II. Objectives: Student should be able to:

A. Name and identify open and closed sentences

B. Distinguish between open and closed sentences

III. Materials: Chalkboard

IV. Procedure:

A. Have students complete the following drill.

B. Drill:

If a sentence is closed write the word True or False.
If a sentence is open, write the word Open.

1. Baltimore is the largest city in the U. S.

2. $8 + x = 15$

3. $12 + 5 + 6 > 10 + 9 + 2$

4. He plays baseball very well.

5. $1\frac{1}{2} + 2 + 3\frac{1}{2} = 1 + 4 + 2$

6. $8 + 4 \times 2 = 12 \times 2$

C. Solution:

1. False	4. Open
2. Open	5. True
3. True	6. False

LESSON 3 - BAND 2

DISCUSSION OF SURVEY TEST

Teacher Commentary

I. Unit: Algebra

II. Materials:

A. Corrected Survey Test from Lesson 1

B. Teacher key to test

III. Procedure:

A. Return corrected Survey Test to students. Permit students to discuss their tests individually with each other. Do not answer any questions during this time, but explain you will cover all questions later.

B. Describe class results by problem analysis--have the number of each problem listed on board and the total incorrect responses for each problem. This will show the students where they need additional help.

C. Present the correct solution for each problem. On those problems that a large percentage of the class failed to answer currectly the teacher should plan to spend additional time in teaching these skills.

D. Answer any individual questions.

LESSON 3 - BAND 3

CLOCK ARITHMETIC

Teacher Commentary

I. Unit: Algebra

II. Objectives: The student should be able to:

A. Name and identify closure and commutativity as properties of the operation

B. Construct an example of each property

C. State and apply the principle of each property

D. Distinguish among the properties

III. Materials:

A. Work sheet from previous day, "The Twelve Hour Clock"

B. Chalkboard

IV. Procedure:

A. Referring to the completed charts, ask students to name the members of the set of numbers that we are using in this system, {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}. Place this set on the board.

B. Ask students to inspect all sums on the chart. See if there are any sums present that are not represented by the same member of the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}.

C. Ask students if any of the sums had more than one answer.

D. Place the following statement on the board.

A mathematical system has the <u>property of closure</u> under addition if:

1. All sums are in the set of elements in the system and

2. Each pair of numbers, when added, has only one answer.

E. Ask the students to examine the clock system in relation to this definition. Then ask if this system satisfies the conditions in the definition. (Yes)

F. Place several examples of the commutative property on the board such as:

$$2 + 3 \stackrel{?}{=} 3 + 2 \qquad\qquad 11 + 4 \stackrel{?}{=} 4 + 11$$

$$5 = 5 \qquad\qquad 3 = 3$$

Have the students supply the answers to the examples. Ask them if they think that the property will always hold under the operation of addition. Have students identify the commutative property.

G. Refer them back to the work sheet and have them place a light "X" on each of the additions performed as illustrated in the chart (see item I below). When sufficient examples have been illustrated, students should be able to discover a pattern to the X's --a mirror image appears on both sides of a diagonal drawn from the top left to the lower right corner.

H. Explain to them that if an exact mirror image appears in a mathematical system, then we know that the system is always commutative. This is a test to see if an operation is commutative.

I. The chart to illustrate the commutatively check should be similar to the following:

+	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	$\cancel{2}$	3	4	5	6	7	8	9	10	11
1	1	2	3	4	5	6	$\cancel{7}$	8	9	10	11	0
2	$\cancel{2}$	3	4	$\cancel{5}$	6	7	8	9	10	11	0	1
3	3	4	$\cancel{5}$	6	7	$\cancel{8}$	9	10	11	0	1	2
4	4	5	6	7	8	9	10	11	$\cancel{0}$	1	2	$\cancel{3}$
5	5	6	7	$\cancel{8}$	9	10	11	0	1	2	3	4
6	6	$\cancel{7}$	8	9	10	11	0	1	2	$\cancel{3}$	4	5
7	7	8	9	10	11	0	1	2	3	4	5	6
8	8	9	10	11	$\cancel{0}$	1	2	3	4	5	$\cancel{6}$	7
9	9	10	11	0	1	2	$\cancel{3}$	4	5	6	7	8
10	10	11	0	1	2	3	4	5	$\cancel{6}$	7	8	9
11	11	0	1	2	$\cancel{3}$	4	5	6	7	8	9	10

LESSON 4 - BAND 1

SUBTRACTION: MISSING DIGITS

Teacher Commentary

I. Unit: Fundamental Operations

II. Materials: Chalkboard

III. Procedure:

A. Place the following subtraction problem on the board:

$$\begin{array}{r} 4x53 \\ -\ x3x7 \\ \hline 176x \end{array}$$

B. Have students find missing digits.

C. Solution:

$$\begin{array}{r} 4153 \\ -\ 2387 \\ \hline 1766 \end{array}$$

LESSON 4 - BAND 2

CLOCK ARITHMETIC

Teacher Commentary

I. Unit: Algebra

II. Objectives: The student should be able to

A. Name and identify associativity as a property of the operation

B. Construct examples of the property

C. State the principle of associativity

III. Materials:

A. Work sheet "The Twelve Hour Clock"

B. Chalkboard

IV. Procedure:

A. Place the following example of the associative property on the board and ask the students how they would check it to see if it was true. The students should be prompted into using a format similar to the following:

$$(4 + 11) + 6 \stackrel{?}{=} 4 + (11 + 6)$$
$$3 + 6 \stackrel{?}{=} 4 + 5$$
$$9 = 9$$

B. Ask the students if they know the property illustrated by the above example.

C. Have students work several more examples using the same procedure and have them discover that many different combinations are possible.

D. Tell students that since it would be difficult to check all possible combinations, we will assume that the property holds for all combinations.

E. Have students state the property in their own words.

F. Tell students that when they have finished stating the associative property they are to immediately begin the work sheet entitled, "Equivalent Phrases." See Band 3.

LESSON 4 - BAND 3

EQUIVALENT PHRASES

Teacher Commentary

I. Unit: Logic

II. Objectives: The student should be able to:

A. Identify equivalent quantifiers

B. Distinguish between equivalent and non-equivalent quantifiers

III. Materials: Work sheet "Equivalent Phrases"

IV. Procedure:

A. Give students pairs of statements and ask if the pairs say the same thing.

B. Allow sufficient time to complete work sheet, "Equivalent Phrases."

C. Discuss the work sheet with students.

D. Solution:

1. equivalent	6. equivalent
2. equivalent	7. equivalent
3. non-equivalent	8. equivalent
4. non-equivalent	9. non-equivalent
5. non-equivalent	10. non-equivalent

EQUIVALENT PHRASES

Tell whether the following pairs of phrases or sentences are equivalent or non-equivalent:

1. Washington, D. C.; capital of the United States.
2. $5 + 18 + 6$; $18 + 6 + 5$
3. $3(8 \div 4)$; $(3 \cdot 8) \div (3 \cdot 4)$
4. All students in biology are tenth graders; some students in biology are tenth graders.
5. $8 \cdot (3 + 6)$; $8 + (3 + 6)$
6. $6(5 + 7)$; $6 \cdot 7 + 6 \cdot 5$
7. $8 - (6 - 1)$; $(8 - 6) - 1$
8. Some teachers are not nice; not all teachers are nice.
9. $(6 + 5)(6 + 2)$; $6 + (5 \cdot 2)$
10. No boy is six feet tall; not all of the boys are six feet tall.

LESSON 5 - BAND 1

MATH BUILDER

Teacher Commentary

I. Unit: Fundamental Operations

II. Materials:

A. Math Builder

B. Filmstrip AR-FX 14

III. Procedure:

A. Preview the filmstrip.

B. Have the students do part A orally, using the full-frame setting with the answers masked.

C. If a problem is done incorrectly, use the "pause" button to give the students an opportunity to see the problem for a longer period of time.

D. Set the variable speed control at approximately 35. This will vary depending upon the ability of the student.

E. If these multiplication facts are too difficult, refer to filmstrip AR-FX 13. Substitute this filmstrip for Band 3 of lesson 7.

LESSON 5 - BAND 2

CLOCK ARITHMETIC

Teacher Commentary

I. Unit: Algebra

II. Objectives: The student should be able to:

A. Name and identify the additive identity

B. Construct an example illustrating the additive identity

C. State and apply the principle that when a given number is added to the additive identity, the answer is the given number

D. Distinguish between the additive identity and additive inverses

E. Name and identify the additive inverse for each element in the system

F. State and apply the principle that the sum of a number and its additive inverse is zero

G. Construct an example of the addition of an element and its additive inverse

III. Materials:

A. Work sheet, "Twelve Hour Clock"

B. Chalkboard

IV. Procedure:

A. Use the same chart the students used previously.

B. Ask students what is meant by an additive identity. If they can describe it by example or state the definition, ask them to identify the additive identity using the chart.

C. Have students recognize the identity element in the chart. The students should notice that the row that contains the identity is exactly the same as the row at the top of the chart.

D. Introduce the additive inverse by placing the following examples on the board. The students should use their chart to aid them in finding the solutions.

$7 + \boxed{} = 0$	$4 + \boxed{} = 0$
$2 + \boxed{} = 0$	$6 + \boxed{} = 0$
$11 + \boxed{} = 0$	$9 + \boxed{} = 0$

E. The students should then define the additive inverse of any number as the number to be added to the given number to obtain the identity element. Students should then construct and complete the following chart.

Number	Additive Inverse	Sum
0	0	0
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		

F. Solution:

Number	Additive Inverse	Sum
0	0	0
1	11	0
2	10	0
3	9	0
4	8	0
5	7	0
6	6	0
7	5	0
8	4	0
9	3	0
10	2	0
11	1	0

The sum in each case is zero.

LESSON 5 - BAND 3

SRA KIT

Teacher Commentary

I. Materials: SRA Computational Skills Development Kit

II. Procedure:

At this time the teacher should explain the use of the SRA kit, since students will be expected to work with the kit throughout the year.

A. This kit is not to be used for evaluation purposes. The students are to work on their own areas of weakness and proceed at their own rates. When students have completed their regular class work, they should not wait for other students to finish, but should go to the kit and begin work.

B. How to use the kit.

1. Diagnostic tests: There are sixteen different diagnostic tests, one for each of sixteen different skill areas. The tests are designed to indicate student strengths and weaknesses in addition, subtraction, multiplication and division.

2. Exercise cards: There are four color groups of exercise cards - blue for whole numbers, yellow for fractional numerals, aqua for decimal numerals, and tan for percents. The exercise sets, one for each problem on the diagnostic test, provide practice for the student if it is needed.

3. Reference cards: There are sixteen reference cards, one for each of the sixteen skill areas.

4. Student record books: There is one student record book per student. Each book has one survey test to help find student strengths and weaknesses in computation. The results indicate which diagnostic tests to take.

The student record book also contains sixteen progress tests. A high score on these indicates whether the student has mastered the specific areas of difficiency indicated. A low score indicates more work is needed in developing competency in that particular skill.

On pages 28-30 of the record book, the student can keep a record of how he is doing.

5. How to use exercise cards: Each card has a symbol along the edge that sticks out of the box. The symbol stands for the operation included on the card (+, -, x, or ÷). Next to the operation symbol is a large black numeral. These numerals help the student locate the set of exercises he will use. The mistakes on the diagnostic tests indicate which exercise cards he should use. If for example a student misses Problem 5 on the test "Subtraction with Fractional Numerals," he will use the yellow card with a "-" symbol and a large black "5" along the edge. If each side of the card has a "5" on it, then he should work the exercises on both sides of the card.

When a student is working exercises on the card, he places it on top of a sheet of paper. The card must be in either a-position of b-position. After writing his answers in the slots, he turns the card end over end on top of the paper, until it is in either c-position or d-position. By comparing his answers with those on the card, he can find out whether he has done the exercises correctly.

LESSON 6 - BAND 1

ORAL DRILL: EQUIVALENT PHRASES

Teacher Commentary

I. Unit: Logic

II. Objective: The student should be able to:

Construct equivalent phrases and sentences

III. Procedure:

Have each student write down a sentence and then ask another student to give in reply a sentence equivalent to the given one, such as:

"Some math problems are difficult."

Reply: "At least one math problem is difficult."

LESSON 6 - BAND 2

ORDER OF OPERATIONS

Teacher Commentary

I. Unit: Algebra

II. Materials:

A. Chalkboard

B. Work sheet, "Order of Operations"

III. Procedure:

A. Place several problems on the board illustrating the types or problems appearing on the work sheet.

B. Discuss with the student a method of doing each of these problems. If students need additional problems, they should be given at this time.

C. Distribute work sheets to the students.

D. Solutions:

1. 17
2. 19
3. 33
4. 68
5. 24
6. 86
7. 24
8. 18
9. 6
10. 19
11. 39
12. 28
13. 22
14. 0
15. 67

ORDER OF OPERATIONS

Name ______________________

Class ______________________

1. $4 \cdot 3 + 5 =$

2. $3 \cdot 5 + 4 =$

3. $3 + 5 \cdot 6 =$

4. $4 \cdot 7 + 5 \cdot 8 =$

5. $7 + 3 \cdot 4 + 5 =$

6. $7 \cdot 3 \cdot 4 + 2 =$

7. $8 \cdot 7 \div 4 + 5 \cdot 2 =$

8. $6 \div 2 + 5 \cdot 3 =$

9. $24 \div 6 + 8 \div 2 \div 2 =$

10. $(15 + 5) \div 4 + 7 \cdot 2 =$

11. $12 + 8 \div 2 \cdot 6 + 3 =$

12. $48 \div (6 \div 2) + 4 \cdot 3 =$

13. $2 + 2 + 2 \cdot 3 + 2 \cdot 3 \cdot 2 =$

14. $(4 - 4) \div 12 \div 3 \div 2 =$

15. $3 + (8 - 4) + 8 \cdot 9 - 12 =$

LESSON 6 - BAND 3

SRA KIT

Teacher Commentary

I. Unit: Fundamental Operations

II. Materials:

SRA student booklet, "Student Record Book"

III. Procedure:

A. As students finish work sheet from Band 2, have them go to the rear of the room and get the Survey Test, Part I, numbers 1-12 in their student record books.

B. If students do not finish Survey Test, tell them that additional time will be allotted in another lesson.

LESSON 7 - BAND 1

THE SEVEN HOUR CLOCK

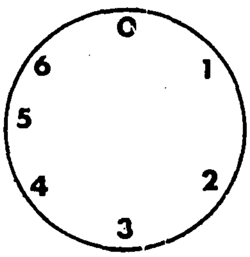
Teacher Commentary

I. Unit: Algebra

II. Materials: Chalkboard

III. Procedure:

A. Ask for a volunteer to make a drawing on the board of a seven hour clock. It should look something like this:



B. Have students solve several addition problems using the seven hour clock such as the following:

$$\begin{array}{r}4\\+\underline{2}\\6\end{array}\qquad\begin{array}{r}5\\+\underline{3}\\1\end{array}\qquad\begin{array}{r}0\\+\underline{6}\\6\end{array}\qquad\begin{array}{r}3\\+\underline{6}\\2\end{array}\qquad\begin{array}{r}5\\+\underline{5}\\3\end{array}$$

C. Students should then construct and complete an array illustrating addition in the seven hour system. The completed chart should look like the following:

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

LESSON 7 - BAND 2

ORDER OF OPERATIONS

Teacher Commentary

I. Unit: Algebra

II. Materials: Corrected work sheets, "Order of Operations"

III. Procedure:

Discuss the answers to the problems from the work sheet, "Order of Operations." Identify areas of difficulty indicated by the results on the work sheets.

LESSON 7 - BAND 3

MATH BUILDER

Teacher Commentary

I. Unit: Fundamental Operations

II. Materials:

A. Math Builder

B. Filmstrip AR-FX 14

C. Pencil and paper

III. Procedure:

A. Have students number from one to twenty on their paper.

B. Show part B of the filmstrip AR-FX 14 using the full frame projection.

C. Set the variable speed control at approximately 35. This setting may vary according to the ability of the student.

D. To check, call on individual students.

E. If time remains, allow students to complete part C of the filmstrip.

LESSON 8 - BAND 1

SRA KIT

Teacher Commentary

I. Unit: Fundamental Operations

II. Materials: SRA Computational Skills Development Kit

III. Procedure:

A. Give all students sufficient time to complete Survey Test-Part I, numbers 1-12.

B. Tell students that when they have completed the Survey Test, they may pick up an interesting puzzle at the back of the room. This activity, labeled Band 2, can be completed by students who finish the Survey Test before the other students.

LESSON 8 - BAND 2

A FARMER'S PROBLEM

Teacher Commentary

I. Unit: Logic

II. Materials: Work sheet, "A Farmer's Problem"

III. Procedure:

A. Distribute work sheet, "A Farmer's Problem."

B. Give students a few minutes to consider the problem. Then discuss the solution.

C. Solution:

Farmer Friendly first takes the duck across, leaving the fox with the corn. Then he returns and takes the corn across, picks up the duck and brings it back. Then he drops off the duck, picks up the fox and corsses the river. After leaving the fox, he returns to pick up the duck.

A FARMER'S PROBLEM

Farmer Friendly had three possessions: a fox, a duck, and a bag of corn. He had to take his three possessions to the other side of the river in a small boat. He could take only one of his possessions in the boat at a time. If he left the fox with the duck, the fox would eat the duck. If he left the duck and the corn together, the duck would eat the corn. How can Farmer Friendly get all three of his possessions to the other side safely?

LESSON 8 - BAND 3

PROPERTIES OF SEVEN HOUR CLOCK

Teacher Commentary

I. Unit: Algebra

II. Materials:

A. Chart constructed by students, "Seven Hour Clock"

B. Chalkboard

III. Procedure:

Using the chart that students have constructed for the seven hour clock, have students work selected problems to see if the properties of closure, associativity, commutativity, and the identity hold for this system. Use the same procedures that were used with the twelve hour clock.

LESSON 9 - BAND 1

ADDITIVE INVERSES FOR THE SEVEN HOUR CLOCK

Teacher Commentary

I. Unit: Algebra

II. Activity: Drill on identifying and naming additive inverses of elements of the seven hour clock.

III. Materials:

Chart constructed by students, "Seven Hour Clock"

IV. Procedure:

A. Place the following chart on the board and have the students complete it.

B. Complete the following chart:

Element	Additive Inverse	Sum
0	0	0
1		
2		
3		
4		
5		
6		

C. Solutions:

Element	Additive Inverse	Sum
0	0	0
1	6	0
2	5	0
3	4	0
4	3	0
5	2	0
6	1	0

LESSON 9 - BAND 2

SUBTRACTING BY ADDING THE ADDITIVE INVERSE

Teacher Commentary

I. Unit: Algebra

II. Objectives: Student should be able to:

A. Name and identify the operation of subtraction

B. Apply the principle that $a - b = a + (-b)$ where a and b are elements of the seven hour clock

C. Construct an example of subtraction

III. Materials:

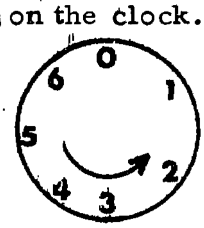
A. Chart from preceding drill

B. Drawing of seven hour clock

IV. Procedure:

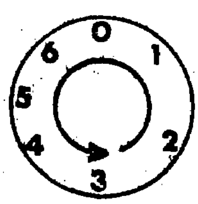
A. Review addition as a clockwise movement on the clock.

B. Ask students how they might show a subtraction such as $5 - 3 = \square$ on the clock. (Counter-clockwise motion as in diagram)



$5 - 3 = \boxed{2}$

Then do $2 - 6 = \square$ as:



$2 - 6 = \boxed{3}$

C. Place the following examples on the board and have students fill in the squares.

Column I		Column II	
5 + 2 =	0	5 - 5 =	0
1 + 6 =	0	1 - 1 =	0
3 + 4 =	0	3 - 3 =	0
4 + 1 =	5	4 - 6 =	5
2 + 3 =	5	2 - 4 =	5
6 + 0 =	6	6 - 0 =	6
0 + 5 =	5	0 - 2 =	5

D. Ask students whether they see any similarities between the two columns. With some cues the students should notice that the first number in columns I and II are the same and that the last numbers in columns I and II are the same.

E. Ask students whether they see any relationship between the two remaining columns connected by the dotted line. If they do not see that considering each pair of numbers separately, the number in one column is the additive inverse of the element in the second column, refer them to the drill.

F. Emphasize the fact that the operation of addition is being performed in one case and subtraction in the other, but the answers are the same.

G. Have students try to describe by example, using the chart, that when a second element is subtracted from a first element, the same answer may be derived by adding the additive inverse of the second element to the first element. Since this is difficult to verbalize, have the student express this relationship symbolically as: $a - b = a + {}^{-}b$ where ${}^{-}b$ is the additive inverse of b.

LESSON 9 - BAND 3

A POUR PROBLEM

Teacher Commentary

I. Unit: Logic

II. Procedure:

A. Have students try to solve the following puzzle which is written on the board:

A man has three containers. One container holds 8 gallons. Another container holds 5 gallons. The third container holds 3 gallons. How can he measure exactly four gallons using only the three containers?

B. Solution:

Fill the 5 gallon container. Fill the three gallon container from the 5 gallon container, leaving two gallons in the 5 gallon container. Pour those two gallons into the 8 gallon container. Empty three gallon container. Repeat the process, thus leaving four gallons in the 8 gallon container.

LESSON 10 - BAND 1

HIDDEN MESSAGE

Teacher Commentary

A Recreational Activity on Order of Operations

I. Materials: Student work sheet

II. Procedure:

A. Hand out student work sheets.

B. Have students decode the hidden message.

C. Solution: MINIMATH

A HIDDEN MESSAGE

Secret messages are sometimes in code. Often the code will have numbers or letters in it. Today you will figure out a message by solving some problems.

I. The answers to each example stands for a letter.

1. $6 \cdot 2 + 1 =$ 13
2. $3^2 =$ ____
3. $2 + 6 \cdot 2 =$ ____
4. $9 \div 3 + 6 =$ ____
5. $6 + 2 + 5 \cdot 1 =$ ____
6. $16 - (8 + 7) =$ ____
7. $4 \times 4 + 4 =$ ____
8. $(4 + 2) \div 3 + 2 \cdot 3 =$ ____

II. Look at your first answer (13), and find that number on the decoding chart. Right under this number you will find the letter it stands for (M). Write this letter (M) in space 1 of the hidden message. Do the same thing for each of your answers, and you will see the message.

DECODER

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Hidden Message

This message tells a story. It describes a mathematics class which does little work.

M ___ ___ ___ ___ ___ ___ ___
1 2 3 4 5 6 7 8

LESSON 10 - BAND 2

FACES

Teacher Commentary

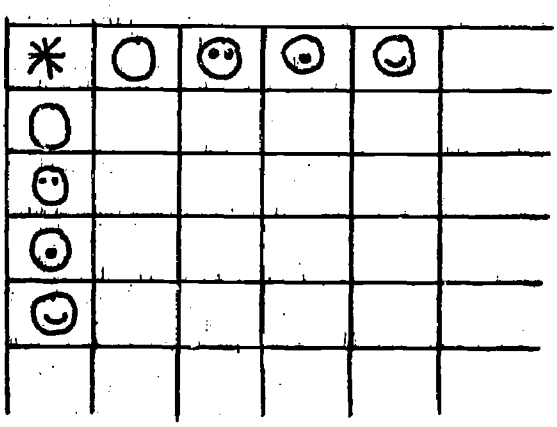
I. Unit: Algebra

II. Materials: Chalkboard or overhead projector

III. Procedure:

A. It is possible to have mathematical systems with properties such as closure, commutativity, and the others examined in clock arithmetic even though the system contains no numbers.

B. Place the chart below on the board or overhead projector and have students make a copy of the chart. Make sure that they leave space below and to the right since elements will be added as the chart progresses.

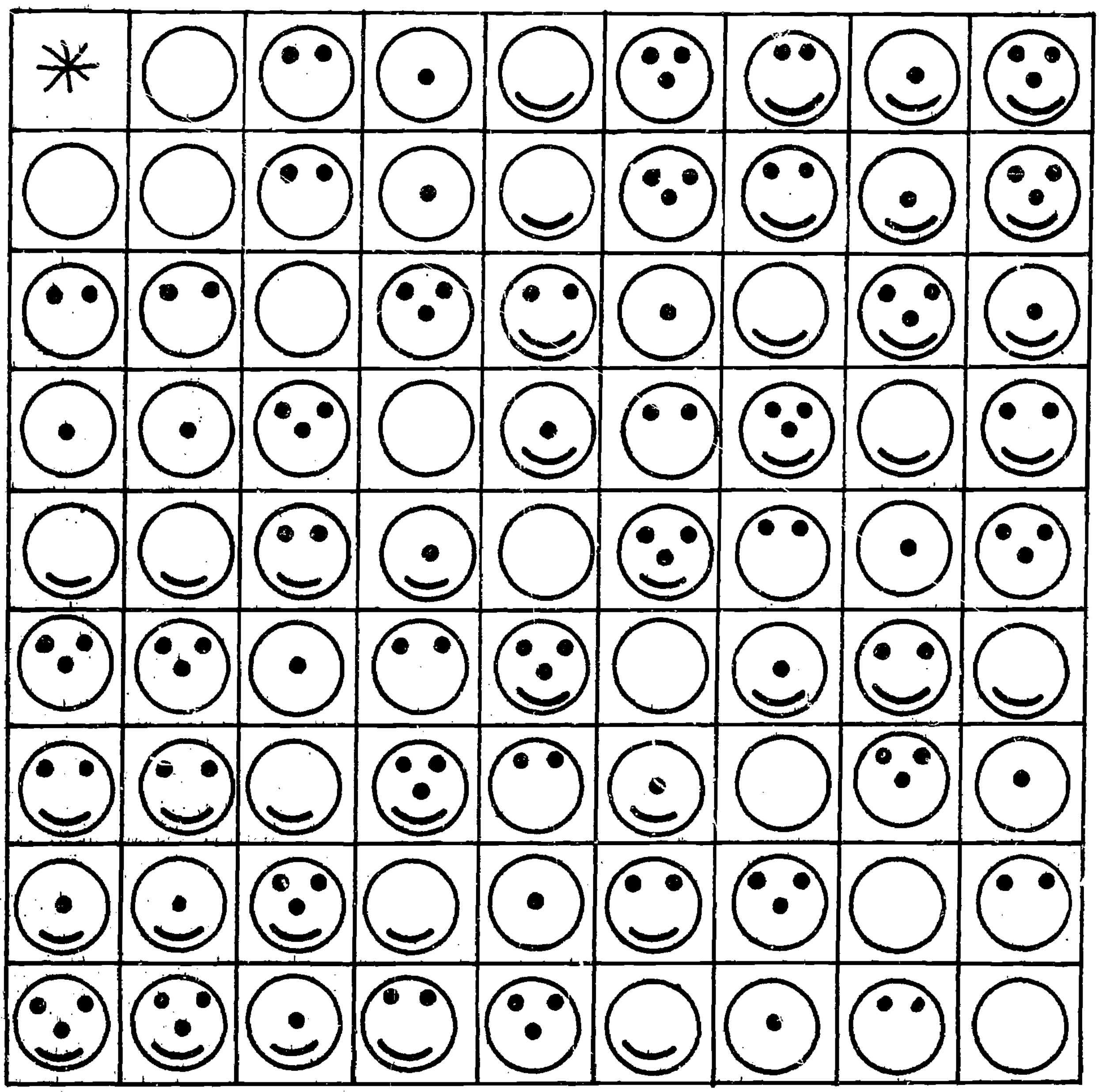


C. The operation (*) will be defined as mentally placing the element in the left column over the element in the top row. If any of the features match, they disappear. For example: * = and * = .

D. Have the student begin to fill in the chart. When they reach the following example * = , ask them why this element causes a problem. They should state that the system will not have closure since the element ( ), was not included in the original set of elements. This element should therefore be added to top row and the left column of the chart.

E. As the students fill in the chart, they will find that they need to add the following elements (, ,). Thus, the set of elements is { , , , , , , }. After all elements have been added, let students complete the chart themselves.

F. The completed chart will be:



LESSON 11 - BAND 1

DRILL: COMMUTATIVITY

Teacher Commentary

I. Unit: Algebra

II. Materials: Chalkboard

III. Procedure:

A. Place the following on the chalkboard:

Tell whether the following operations are commutative or non-commutative.

1. Putting on left glove and right glove.
2. Putting on shoe and sock.
3. Putting on a parachute and jumping out of an airplane.
4. Seeing lightning and hearing thunder.
5. $(7 - 2) + 3 \stackrel{?}{=} 3 + (7 - 2)$

B. Solution:

1. Commutative
2. Non-commutative
3. Non-commutative
4. Non-commutative
5. Commutative

LESSON 11 - BAND 2

PROPERTIES OF FACES

Teacher Commentary

I. Unit: Algebra

II. Materials:

A. Chart on faces completed the previous day

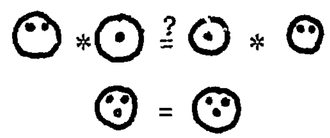
B. Chalkboard

III. Procedure:

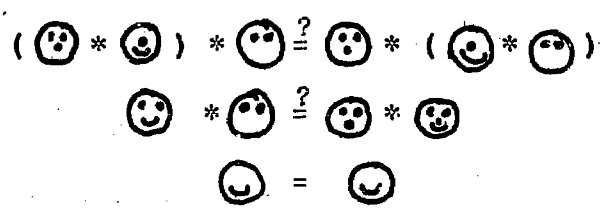
A. Using the chart on faces, discuss closure, commutativity, identity and inverses.

B. In relation to closure, have students demonstrate that the answer to each is a member of the set.

C. Have the students construct examples to show that commutativity holds such as:



D. Have students do several examples to illustrate associativity in the following manner:



E. Students should identify () as the identity element and identify each element as its own inverse under the operation *.

LESSON 11 - BAND 3

SRA KIT

Teacher Commentary

I. Unit: Fundamental Operations

II. Materials: SRA Computational Skills Development Kit

III. Procedure:

When students finish discussion, they should take materials from the SRA Kit at the rear of the room. The student should be directed to those activities from the kit which would strengthen his compentencies (according to results of the Survey Test, Part I, 1-12).

LESSON 12 - BAND 1

MULTIPLICATION: MISSING DIGITS

Teacher Commentary

I. Unit: Fundamental Operations

II. Materials: Chalkboard

III. Procedure:

A. Place the following multiplication problem on the board:

$$\begin{array}{r} x3x \\ \underline{x8} \\ 3x96 \\ \underline{2xx2} \\ xxxx6 \end{array}$$

B. Have students find missing digits.

C. Solution:

$$\begin{array}{r} 437 \\ \underline{68} \\ 3496 \\ \underline{2622} \\ 29716 \end{array}$$

LESSON 12 - BAND 2

QUIZ ON PROPERTIES

Teacher Commentary

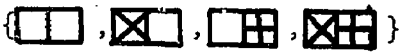
I. Unit: Algebra

II. Activity: Test on properties of operations

III. Materials: Duplicated tests, "Properties of Operations"

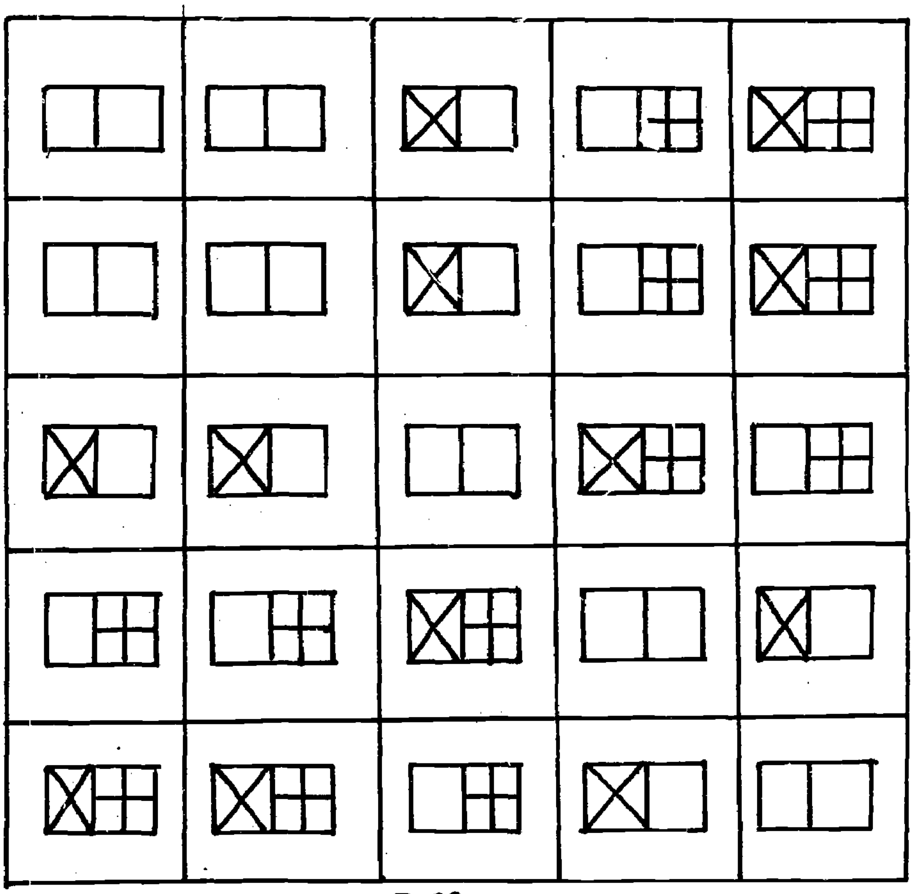
IV. Procedure:

A. Distribute the test and give the following directions orally:

1. The operation for the mathematical system below will be exactly the same as the operation with faces. The outline of the figure, (), will always remain. The set of elements in the system will be: 

2. Fill out the chart below. Then check the system for closure associativity, commutativity, identity element and inverse elements. If the example given for associativity works, assume that the system is associative.

B. Solution:

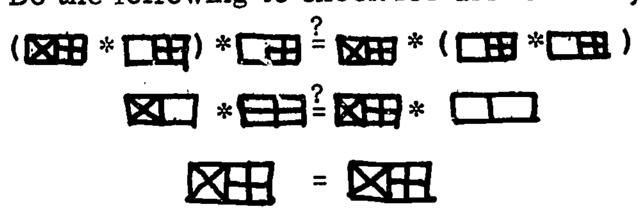


Answer to the questions:

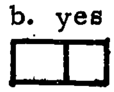
1. Yes

2. Yes (Draw diagonal and look for reflections)

3. a. Do the following to check for associativity.



b. yes

4. 

5. Write the inverse of each element.

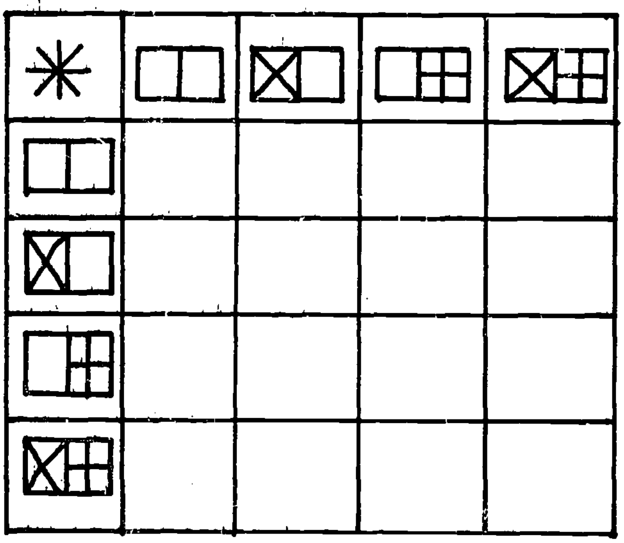
Element		Inverse	Element		Inverse

QUIZ

PROPERTIES OF OPERATIONS

Name ____________________

Sample:

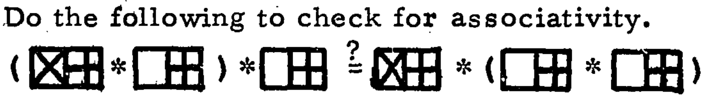


Answer the following questions:

1. Does the system has closure? (Circle one) Yes No
2. Does the system have commutativity? Yes No

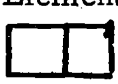
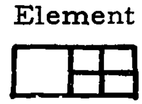
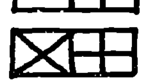
   Show on your chart how you checked for commutativity.

3. a. Do the following to check for associativity.

   b. Is the system associative? Yes No

4. What is the identity element?
5. Write the inverse of each element.

Element	Inverse	Element	Inverse
	-		-
	-		-

RESOURCE MATERIALS

A. Books

Adler, Irving. The Giant Golden Book of Mathematics. New York: Golden Press. 1958

Bergamini, David. Mathematics, Life Science Library. New York, N. Y.: Time, Inc. 1963

Heddens, James M. Today's Mathematics. Chicago: Science Research Associates, Inc. 1963

Highland, H. J. The How and Why Wonder Book of Mathematics. New York: Wonder Books. 1963

Hughes, Toni. How to Make Shapes in Space. New York: E. P. Dutton and Co., Inc. 1955

Johnson, Pauline. Creating With Paper. Washington: University of Washington Press. 1958

Morris, Dennis and Topfer, Henry. Advancing in Mathematics, Grade 7. Chicago: Science Research Associates, Inc. 1963

Morris, Dennis and Topfer, Henry. Advancing in Mathematics, Grade 8. Chicago: Science Research Associates, Inc. 1963

Northrop, Eugene P. Riddles in Mathematics. New York: D. Van Nostrand Co., Inc. 1944

Wirtz, Robert; Botel, Morton and Nunley, B. G. Discovery in Elementary School Mathematics. Chicago: Encyclopaedia Britannica Press, Inc. 1963

Young, Mary. Singing Windows. New York: Abingdon Press. 1962

B. Pamphlets and Periodicals

Amir-Mo-Az. Ruler, Compasses and Fun. New York: Ginn and Co. 1966

Bazdon, Jack and Murtin, Mark. Cross Number Puzzle Boxes. Chicago: Science Research Associates, Inc. 1966

Criflinski, Henry. Modern Mathematics, Ditto Workbooks. Washington, D. C.: Hayes School Publishing Co. 1964

Herrick, Marian C. Modern Mathematics for Achievement. New York: Houghton Mifflin Co. 1966

Johnson, Donovan. Paper Folding for the Mathematics Class. Washington, D. C.: National Council of Teachers of Mathematics. 1957

Johnson, Donovan A. and Glenn, William H. Topology: The Rubber-Sheet Geometry. Atlanta: Webster Publishing Co. 1960

Larson, Harold. Enrichment Program for Arithmetic Grades 3-8: Elmsford, New York: Harper and Row Publishers. 1963

Murray, William D. and Rigney, Francis. Paper Folding for Beginners. New York: Dover Publications, Inc. 1960

Potter, Mary and Mallory, Virgil. Education in Mathematics for the Slow Learner. Washington, D. C.: National Council of Teachers of Mathematics. 1958

Proctor, Charles and Johnson, Patricia. Computational Developmental Skills Kit. Chicago: Science Research Associates, Inc. 1965

School Mathematics Study Group. Conference on Mathematics Education for Below Average Achiever. Pasadena, California: Vroman Co. 1964

Topics in Mathematics for Elementary School Teachers. Washington, D. C.: National Council of Teachers of Mathematics. 1964

Wirtz, Robert and Botel, Morton. Math Workshop, Levels A-F. Chicago: Encyclopaedia Britannica Press, Inc. 1961

Woodby, Lauren. The Low Achiever in Mathematics. Washington, D. C.: U. S. Office of Education. 1964

C. Games

Milton Bradley Co., Springfield, Mass.
"Primary Peg Board #474X"
Pegs #472X or #475X

Edmund Scientific, Barrington, New Jersey 08007
"Dr. Nim" ($2.98)
"Probability Kit" ($4.00)
"Soma" ($2.00)

Ideal Supply Co., 11315 Watertown Plank Road, Milwaukee, Wis. 53201
"Geometric Wire Forms and Patterns #794" ($3.00)

Kohner Bros., Inc., 155 Wooster Street, New York, N.Y. 10012
"Euclid" ($1.00)
"Hexed" ($1.00)
"Hi-Q" ($1.00)
"Kwazy Quilt" ($1.00)
"Pythagoras" ($1.00)
"Tormentor" ($1.00)
"Voodoo" ($1.00)

Krypto Corporation, 2 Pine Street, San Francisco, California 94111
"Krypto" ($3.95)

Parker Bros., Inc., P. O. Box 900, Salem, Mass.
"Take Twelve" ($3.00)

Science Research Associates, 259 East Erie Street, Chicago, Illinois 60611
"Equations" ($3.00)
"Cross Number Puzzles" ($22.75)

D. Films

Baltimore County Central Film Library

Probability, McGraw Hill Book Co.

Mean, Median and Mode, McGraw Hill Book Co.

NUMBERS, OPERATIONS AND ALGORITHMS

NUMBERS, OPERATIONS, AND ALGORITHMS

I. Master Chart - Grades Six through Eleven

II. Grade Ten Chart and Behavioral Objectives

- A. Whole Numbers
- B. Decimal Numerals
- C. Percent by Ratio and Proportion
- D. Square Root
- E. Computing Devices

III. Activities

UNIT WHOLE NUMBERS GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Numerals to Millions	6	6	6						6	6
Numerals to Billions	7	7	7						7	7
Rounding to Millions	6	6	6				6			6
Rounding to Billions	7	7	7				7			7
Expanded Notation to Millions	6	6	6			8				
Expanded Notation to Billions	7	7	7			8	7			
Renaming Numbers	6	6	6			8	6			
Vocabulary	6	6								6
Place Value to Millions	6	6								
Place Value to Billions	7	7								
Denominate Numbers		6	6		6	7	6			
Verbal Problems			6, 7, 8	6, 7, 8				6, 7, 8		
Betweenness			6, 7		6					
Symbol(s)	6, 7, 8	6, 7, 8		6, 7, 8						
Number Sentences	6	6	6		6		6			
Vertical Form Addition			6, 7, 8	6, 7, 8						
Using Number Line	6		6	6	6		6			
Basic Facts	6	6	6							

FO-2

UNIT WHOLE NUMBERS GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Number Patterns			6	6	6		6			
Estimation			7	7	7		7			
Casting Out Nines	8		8		8		8			
Closure	8				6	9				
Commutative Property	8	6	6		6	9	6			8
Associative Property	8	6	6		6	9	6			8
Identity Element	9	6	6		6	7	6			
Inverse Operations		7	6		7	10	6			
Vertical Form Subtraction			6, 7, 8	6, 7, 8						
Checking			6		6					
Role of Zero			6		6	7	6			
Order of Operations			6, 8		6, 8	10	6, 8			
Vertical Form Multiplication			6, 7, 8	6, 7, 8						
Factoring		6	6							
Divisors	6	6	6							
Rules for Divisibility						6, 7, 8	6, 7, 8			
Prime Numbers	7	7	7							7
Composite Numbers	7	7	7		7					

FO-3

UNIT WHOLE NUMBERS GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Prime Factorization	7	7	7	7	7					7
Multiples	7	7		7	7					8
Power	7	7	7	7	7				8	
Base	7	7	7		7					
Exponent	7	7	7		7					
Distributive Property	9	9	9			10	6			
Division			6, 7, 8	6, 7, 8						
Remainders	6	6			6					
Role of One			6		6	7	6			

PRELIMINARY TOPICS OF FRACTIONAL NUMBERS

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning of Fractions	6	6	6		6	8				
Numerator and Denominator	6	6			6					
Symbols (Fraction Bar)	6	6			6	6				
Number Line	6, 7, 8	6, 7, 8	6, 7, 8	7, 8	8					
Comparing Fractions			6	6					6	
Divisibility Rules		6, 7, 8	6, 7, 8			6, 7, 8	6, 7, 8, 9			6, 7, 8
Fractional Names for One	6	6	6	6	6	8	7			
Greatest Common Factor	7, 8, 9	7, 8, 9	7, 8, 9	7, 8	7, 8, 9					7
Simplifying Fractions	6, 7, 8	6, 7, 8	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8			
Renaming Fractions in Higher Terms	6, 7, 8	6, 7, 8	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8			
Mixed Form	6, 7	6, 7	6, 7	6, 7	6, 7					
Rename Mixed Form as Fractions			6, 7, 8	6, 7, 8	6, 7, 8	8	6, 7, 8			
Rename Fractions as Mixed Form			6, 7, 8	6, 7, 8	6, 7, 8	8	6, 7, 8			
Equivalent Fractions	6, 7	6, 7	6, 7	6, 7		6, 7	6, 7, 8			6, 7
Nonequivalent Fractions		6, 7, 8	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8		6, 7, 8	

FO-5

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Nine

Multiplication of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Expressing a Whole Number as a Fraction	6	6	6		6	6	6			
Fraction Times Whole Number, Meaning	6, 7	6, 7	6	7	7					
Fraction Times Whole Number	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Fraction Times Fraction, Meaning	6, 7	6, 7	6, 7	7						
Fraction Times Fraction	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Mixed Form Times Whole Number	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Mixed Form Times Fraction	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Mixed Form Times Mixed Form	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Closure Property					7	8, 9				
Commutative Property	7	6, 7	7	6, 7, 8, 9		7, 8				
Associative Property	8	8	8	8		8, 9	9			8, 9
Distributive Property	9	9	9			9	9			9
Identity Element	6	6	6	6	6	6	6			
Estimation	6	6	6	6, 7, 8, 9						
Translating Verbal Problems into Number Sentences	6, 7 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9			6, 7, 8, 9		

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Nine

Division of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning	6	6	6, 7	6, 7	7					
Reciprocals	7	7	7, 8	8	8		7			
Complex Fractions	7	7	7, 8, 9	7, 8, 9	8, 9	9	7, 8, 9			
Fraction Divided by a Whole Number	7	7	7, 8, 9	7, 8, 9	8, 9	9	7, 8, 9			
Whole Number Divided by a Fraction	7	7	7, 8, 9	7, 8, 9	8, 9		7, 8, 9			
Fraction Divided by a Fraction	7	7	7, 8, 9	7, 8, 9	8, 9	9	7, 8, 9			
Mixed Form Divided by a Whole Number	8	8	8, 9	8, 9	9		8, 9			
Whole Number Divided by a Mixed Form	8	8	8, 9	8, 9	9		8, 9			
Mixed Form Divided by a Fraction	8	8	8, 9	8, 9	9		8, 9			
Fraction Divided by a Mixed Form	8	8	8, 9	8, 9	9		8, 9			
Mixed Form Divided by a Mixed Form	9	9	9	9	9	9	9			
Closure Property					8	8				
Non-Commutativity		7, 8	7, 8	8, 9						
Non-Associativity	9	9	9							
Identity Element	8	8		8						
Inverse Operation	8	8	8	8	9		9			
Translating Verbal Problems into Number Sentences	7, 8, 9	7, 8, 9	7, 8, 9	7, 8, 9	7, 8, 9			7, 8, 9		

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Nine

Addition of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning of Addition of Fractions	6	6	6	6	7					
Addition of Fractions, Like Denominators	6	6	6	6		6	6, 7			
Least Common Multiple	6	6	6, 7, 8, 9	6, 7, 8, 9	7	6, 7, 8, 9	6, 7, 8, 9			6, 7, 8, 9
Addition of Fractions, Unlike Denominators	7, 8	7, 8	7, 8	7, 8, 9			7, 8, 9			
Whole Number Plus Fraction	6	6	6	6	6, 7		6, 7			
Whole Number Plus Mixed Form	6	6	6	6	6, 7		6, 7			
Fraction Plus Mixed Form, Like Denominators	6	6	6	6	6, 7	7	6, 7			
Mixed Form Plus Fraction, Unlike Denominators	7	7	7	7			7			
Mixed Form Plus Mixed Form, Like Denominators	6	6	6, 7	6, 7	6, 7	7	6, 7			
Mixed Form Plus Mixed Form, Unlike Denominators	7	7	7	7	7		7			
Miscellaneous Problems of Adding Fraction Expressions			8	8, 9		9	8, 9			
Closure Property	7	7				6, 7, 8, 9				
Commutative Property	7	6, 7	6, 7	6, 8, 9		7, 8	6, 7, 8			8, 9
Associative Property	8	8	8	8, 9		8	8, 9			8, 9
Identity Element	6	6	6	6	6	6	6			
Estimation	6	6	6	6, 7, 8, 9	6		6, 8, 9			
Translating Verbal Problems into Number Sentences	6, 7 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9			6, 7, 8, 9		

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UNIT FRACTIONAL NUMBERS GRADE(S) Six through Ten

Subtraction of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Relationship of Addition and Subtraction, Meaning	6	6	6	6	6					
Subtraction, Like Denominators	6	6	6	6	6	7	6, 7			
Subtraction, Unlike Denominators	7	7	7	7, 8	7	8	7, 8			
Fractions from Mixed Form, Like Denom., No Renaming	6	6	6	6, 7	6	7	6, 7			
Fractions from Whole Numbers	6	6	6	6, 7	6	7	6, 7			
Renaming Whole Numbers as Mixed Forms	6	6	6	6	6	7	6, 7			
Fractions from Mixed Forms, Like Denom., Renaming	6	6	6	6, 7, 8	7	8	6, 7, 8			
Fractions from Mixed Forms, Unlike Denom., No Renaming	7	7	7	7	7	8	7, 8			
Fractions from Mixed Forms, Unlike Denom., Renaming	7	7	7	7	7	8	7, 8			7, 8
Whole Number from Mixed Forms	6	6	6	6	6	7	6, 7			
Mixed Forms from Mixed Forms, No Renaming	6	6	6	6, 7	6	7	6, 7			
Mixed Form from Whole Number	6	6	6	6	6	7	6, 7			
Mixed Forms from Mixed Form, Like Denom., Renaming	6	6	6	6, 7, 8	7	8	6, 7, 8			6, 7, 8
Mixed Form from Mixed Form, Unlike Denom., No Renaming	7	7	7	7, 8	7	8	7, 8			
Mixed Form from Mixed Form, Unlike Denom., Renaming	7	7	7	7, 8	7	8	7, 8			7, 8
Closure Property		6	7	6, 7, 8		7, 8				
Non-Commutativity	7	6	7, 8							
Non-Associativity	7	6		6, 7, 8						

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Ten

Subtraction of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Identity Element		6	6		6	6, 7, 8	6			
Inverse Operation	8	8		8	9	10	9			
Translating Verbal Problem to Number Sentences	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9					

UNIT DECIMAL NUMERALS GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Place Value	6, 7, 8	6, 7, 8				6, 7, 8				6, 7, 8
Numbers to a Million	6	6	6						6	
Numbers to a Billion	7, 8	7, 8	7, 8						7, 8	
Number Patterns				6, 7, 8		6, 7, 8	6, 7, 8			
Equivalent Powers of Ten						6, 7, 8				
Expanded Notation			6, 7, 8							
Decimal Equivalence	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8						
Annexing Zeros	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9							
Number Line	6, 7	6, 7	6, 7			6, 7			6, 7	
Comparing Numbers			6, 7, 8						6, 7, 8	
Betweenness	6, 7, 8, 9		6, 7, 8, 9							
Rounded Numbers	6, 7, 8, 9	6, 7, 8, 9					6, 7, 8, 9			
Decimal Simplifying Numerals	6, 7, 8	6, 7, 8		6, 7, 8						
Equivalence Decimal Charts			8	8						
Column Form Addition			6, 7, 8, 9	6, 7, 8, 9						
Arrange Addends in Column Form			6, 7, 8, 9			6, 7, 8, 9				
Estimating Sums			8	8	8					
Subtraction			6, 7, 8, 9	6, 7, 8, 9						

UNIT DECIMAL NUMERALS GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Division of a Decimal by a Decimal			10	10						
Quotients Rounded			10	10	10					
Decimal Equivalents	10	10	10	10						

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UNIT PERCENT BY RATIO AND PROPORTION GRADE(S) Eight through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning of Ratio	8	8	8	8	8					
Meaning of Rate Pair	8	8	8	8	8					
Simplifying a Rate Pair	8	8	8			8	8			
Translating Verbal Problems into Rate Pairs		8	8	8	8					
Meaning of Percent by Rate Pairs		8	8	8	8					
Changing Percents in Verbal Problems to Rate Pairs		8		8						
Equivalent Rate Pairs						9	9			
Proportion	9	9	9	9						
Solving Multiplication Equation	9	9	9			9	9, 10			
Translating Verbal Problems into Equivalent Rate Pairs		9	9	9	9					
Solving Proportions		9	9			9	9, 10			
Finding What Percent One Number is of Another		9	9				9, 10			
Translating Verbal Problems into Equivalent Rate Pairs		9	9	9	9					
Finding a Percent of a Number		10	10			10	10			10
Finding a Number When a Percent of it is Known		10	10			10	10			10
Translating Verbal Problems into Equal Rate Pairs		10	10	10	10					10

FO-13

UNIT SQUARE ROOT GRADE(S) Seven through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Square Numbers	7	7	7	7	7					7
Vocabulary	7, 8	7, 8								8
Symbol	8	8		8						8
Expansion of Powers	8	8	8	8	8					8
Use of Square Root Table	9	9	9	9	9					
Extracting Square Roots			10, 11	10, 11	10, 11		10, 11			

UNIT COMPUTING DEVICES GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Vocabulary	10	10								10
Calculator			10	10	10					
Addition, Subtraction, Multiplication, Division			10	10						
Accuracy			10		10					
Combined Operations			10	10				10		

UNIT SLIDE RULE GRADE(S) Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Vocabulary	11	11								11
Construction of Slide Rules			11	11	11					
Reading Scales			11		11					
Multiplication			11	11						
Division			11	11						
Combined Operations			11	11	11			11		
Estimating Solutions			11	11			11			
Square Root			11	11						
Squaring			11	11						

UNIT WHOLE NUMBERS GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Inverse Operations						10				
Order of Operations						10				
Distributive						10				

FO-17

NUMBERS, OPERATIONS, AND ALGORITHMS
WHOLE NUMBERS - Grade 10

Note: In the following list of objectives, we shall agree to use phrases such as "a three digit number" to mean "a number named by a numeral containing three digit symbols."

Page

The Inverse Operations of Addition and Subtraction

The student should be able to:

1. State the principle that addition and subtraction are inverse operations

Order of Operations for Multiplication and Addition

The student should be able to:

1. State the principle of order for multiplication and addition

Distributive Principle

The student should be able to:

1. State the distributive principle of multiplication over addition

The Inverse Operations of Multiplication and Division

The student should be able to:

1. State the principle that multiplication and division are inverse operations

Order of Operations in Expressions Using Symbols of Grouping

The student should be able to:

1. State the principle of order of operations in expressions involving parentheses and all four fundamental operations

UNIT DECIMAL NUMERALS GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Division of a Decimal by a Decimal			10	10						
Quotients Rounded			10	10	10					
Decimal Equivalents	10	10	10	10						

FO-19

NUMBERS, OPERATIONS, AND ALGORITHMS
DECIMAL NUMERALS - Grade 10

Division of a Decimal by a Decimal | Page

The student should be able to:

1. Demonstrate how to construct quotients using a three digit divisor containing a thousandths digit and a dividend which contains a ten-thousandths digit with no remainders

2. Construct quotients using a three digit divisor containing a thousandths digit, and a dividend which contains a ten-thousandths digit, no remainder

Quotients Rounded to the Nearest Thousandth

The student should be able to:

1. Demonstrate how to construct a quotient to the nearest thousandth by obtaining a ten-thousandths digit in the quotient, then rounding to the nearest thousandth

2. Construct a quotient to the nearest thousandth

3. Describe how to find a quotient to the nearest thousandth

Decimal Equivalents

The student should be able to:

1. Name and identify any fractional numeral with a whole number as the numerator and a denominator of 7, 11, or 13

2. Demonstrate how to use division to construct decimal numerals for given fractions

3. Construct a decimal numeral when given a fraction

Operations with Decimals

The student should be able to:

1. Construct the sums, differences, products and quotients of decimals in problems which have practical application

UNIT PERCENT BY RATIO AND PROPORTION GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Solving Multiplication Equations							10			
Solving Proportions							10			
Finding What Percent One Number is of Another							10			
Finding a Percent of a Number		10	10			10	10			10
Finding a Number When a Percent of it is Known		10	10			10	10			10
Translating Verbal Problems into Equal Rate Pairs		10	10	10	10					10

FO-21

Page

Solving Multiplication Equations

(Listed below are some representative examples. Note that the coefficient of the variable may be a fraction or a number in mixed form.)

a. $5x = 6$

b. $3\frac{1}{3}x = 150$

c. $\frac{1}{4}x = 49$

d. $\frac{1}{2}x = 16$

e. $4\frac{1}{2}x = 15$

f. $33\frac{1}{3}x = 90$

The student should be able to:

1. Apply the principle of the multiplicative inverse to construct solutions for multiplication equations

Solving Proportions as Equivalent Rate Pairs

(Listed below are some representative examples. Note that the coefficients are whole numbers and fractions, and that the solutions progress in degree of difficulty.)

$(2, 5) = (x, 100)$

$(\frac{1}{2}, 2) = (10, x)$

$(x, 40) = (4\frac{1}{2}, 100)$

$(10, 3\frac{1}{3}) = (x, 15)$

$(3, x) = (20, 100)$

The student should be able to:

1. Apply the following principles to solve a proportion
   a. Use cross multiplication to construct a multiplication equation
   b. Solve the multiplication equation

Finding What Percent One Number is of Another

FO-36

(Listed below are some representative examples. Note that this list is a review of work completed in grade nine. The problems should be solved using rate pairs.)

a. 5 is what percent of 10?

b. What % of 30 is 10?

Page

c. 6 is what % of 2?

d. 10 is what % of 10?

e. What % of 7 is 15?

The student should be able to:

1. Apply the following principles to find what percent one number is of another

   a. Construct a proportion with an unknown term

   b. Use cross multiplication to construct a multiplication equation

   c. Solving the multiplication equation

Finding a Percent of a Number FO-36

(Listed below are some representative examples. Note that the problems increase in the degree of difficulty. These problems should be solved by the use of rate pairs.)

a. What is 5% of 20?

b. Find 45% of 102.

c. Find $3\frac{1}{2}$% of \$62.

d. What is 5.2% of 60?

e. What is 20% of 60?

f. Find $33\frac{1}{3}$% of 66.

g. What is $66\frac{2}{3}$% of 380?

h. Find $\frac{1}{4}$% of 20.

The student should be able to:

1. Identify problems that require finding the percent of a number

2. Demonstrate a procedure for finding the percent of a number

3. State the following principles for finding the percent of a number:

   a. Write a proportion with an unknown term

   b. Use cross multiplication to construct an equation involving multiplication

   c. Solve the equation

4. Apply these principles for constructing a percent of a number

5. Distinguish between problems which require finding what percent one number is of another and those which require finding the percent of a number

Finding a Number When a Percent of it is Known	Page
(Listed below are some representative examples. Note the increase in the degree of difficulty. These problems should be solved by use of rate pairs.)	FO-36

a. 5% of what number is 2?

b. 6% of what number is 50?

c. 60 is $4\frac{1}{2}$ % of what number?

d. 120 is 200% of what number?

e. 66 is $66\frac{2}{3}$ % of what number?

f. $4\frac{1}{2}$ % of what number is 80?

g. 5.2% of what number is 50?

h. 500 is 100% of what number?

The student should be able to:

1. Identify problems which require finding a number when a percent of it is known
2. Demonstrate a procedure for finding a number when a percent of it is known
3. State the following principle for finding a number when a percent of it is known by:
   a. Writing a proportion with an unknown term
   b. Use cross multiplication to construct an equation involving multiplication
   c. Solve the equation
4. Apply these principles to construct a number when a percent of it is known
5. Distinguish among problems which require finding a percent one number is of another; a percent of a number; and a number when a percent of it is known

Translating Verbal Percent Problems Into Sentences Involving Equal Rate Pairs

The student should be able to:

1. Identify verbal problems which involve percents
2. Demonstrate a procedure for writing a proportion from the information contained in the verbal problem

Page

3. Construct a proportion from the information contained in a verbal problem

4. Describe a procedure for writing a proportion from the information in a verbal problem

5. Distinguish among verbal problems which requi e finding: what percent or number is of another; a percent of a number; and a number when a percent of it is known

UNIT SQUARE ROOT GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Extracting Square Root			10	10	10		10			

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NUMBERS, OPERATIONS, AND ALGORITHMS
SQUARE ROOT - Grade 10

Extracting Square Roots	Page
The student should be able to:	
1. Describe Newton's Method of finding the square root using only square numbers	
2. Demonstrate how to use Newton's Method of square root using only square numbers	FO-38
3. Construct the square root of any square number by Newton's Method	FO-38
4. Apply the principle of Newton's Method with square numbers	

UNIT COMPUTING DEVICES GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Vocabulary	10	10								10
Calculator			10	10	10					
Addition, Subtraction, Multiplication, Division			10	10						
Accuracy			10		10					
Combined Operations			10	10				10		

FO-28

NUMBERS, OPERATIONS, AND ALGORITHMS
COMPUTING DEVICES - Grade 10

Digital and Analog Computers	Page
The student should be able to:	
1. Name and identify the terms computing device, calculator, analog computer, and digital computer	FO-30
2. Distinguish between analog and digital computers	FO-30
3. Describe a computing device as a machine for performing operations in a number system	
Fundamental Operations	
The student should be able to:	
1. Demonstrate how to use a desk calculator or simple homemade computer to find sums, differences, products, and quotients	FO-30
2. Construct sums, differences, products, and quotients using desk calculators or simple home-made computers	FO-30
Accuracy	
The student should be able to:	
1. Demonstrate how calculators round products and quotients by comparison to manual calculations	
2. Describe the accuracy of products and quotients on calculators	
Combined Operations	
The student should be able to:	
1. Demonstrate the most efficient procedure for doing combined operations using calculators	
2. Construct solutions of problems having combined operations using calculators	
3. Interpret problems involving combined operations to determine the most efficient procedure for solution using calculators	

HOMEMADE COMPUTERS

Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

A. Name and identify the terms: computing device; analog computer; and digital computer

B. Distinguish between analog and digital computers

C. Demonstrate how to use a simple homemade computer to construct sums, differences, products, and quotients

D. Construct sums, differences, products, and quotients using a simple homemade computer.

III. Materials:

A. Plywood ($\frac{1}{4}$") or corrugated cardboard or masonite, heavy cardboard, small round head screws, paste

B. Scissors

C. Student work sheets entitled, "Building A Digital Computer" and "Building An Analog Computer"

IV. Procedure:

A. Give students a multiplication problem of two ten-digit numbers. After a short time, ask why they aren't finished.

Explain: A desk calculating machine can do this problem in 30 seconds. Why can't you? A giant electronic computing machine could do this problem 250 times in the time it takes sound to cross the room--some of them even faster.

Have students tell what they know about computing machines. For later times, have them bring clippings and articles on them.

Ask - Just what, basically, does a computing machine do? Bring out that it performs operations in a mathematical system. It is usually a number system, and it may add, subtract, multiply, divide, raise to powers, take square roots, or various combinations of these.

B. Have students do a set of problems of mixed types (addition, subtraction, etc.), with the specific direction to go as fast as possible. When finished, compare answers and note discrepancies.

Explain: Speed is important, but so is accuracy.

Have students do another set of problems, but this time with the specific direction to work for accuracy above anything, especially disregarding speed. Time this. Point out that accuracy without speed is not very satisfactory either.

Have students do a third set of problems with the direction to go as fast as possible, while still being accurate.

Explain: Speed and accuracy are both important and we should always work for both.

Ask - Does thinking about speed and accuracy in computations show you something about the value of computing machines?
Bring out that this is their purpose: To do computations very rapidly, and to do it without error.

Ask - How are computing machines important in our lives today?
Bring out that they have made it possible to do calculations that one man could not do in a lifetime, and that this has made it possible to fire rockets past the moon and into orbit around the earth. It is now possible to design many things better, such as cars, atomic piles, etc., and that experiments in medicine, agriculture, space travel, and so on can be designed and carried out better.

C. Explain: There are many kinds of computing machines, but they fall into two classes. These are called "analog" and "digital."

A digital computer is one which uses digits. It works on the principle of counting. It does not give in-between values. Most digital computers do all calculations by adding.

An analog computer works by the principle of measurement. Its name comes from the word "analogy."

Discuss the meaning of this word, and point out that analog computers give results by a physical analogy with the mathematical situation.

Explain: We shall see more clearly the difference between digital and analog computers after we make some simple computers.

D. Distribute the work sheet entitled, "Building A Digital Computer." Discuss the directions carefully and have students use the pattern supplied to make a simple two-digit digital computer. Heavy cardboard and other supplies should be provided for. This will take several days. You might want to limit the construction to smaller groups of students interested in building models. You will probably have to help the students number the discs.

Have students display their computers, and discuss the fact that digits are used and that no in-between values can be obtained. Demonstrate how to use the computer by showing how to dial in the numbers and read the sum.

Give students a few problems to do on their computers. Have them demonstrate how to find sums.

Divide the class into two teams. Have one team solve problems using computers while the other team does them by hand. Compare speed and accuracy.

Ask - Can the device be used to do problems which have units and tenths, rather than tens and units?

Bring out that problems which have just two digits with corresponding place value can be done with this device--that the decimal point can be regarded to be wherever we please.

Ask - Can you imagine how to make a digital computer similar to this with which you could add more than two-digit numbers whose sum is less than 99?

Bring out that extra wheels could be added. (An outside project might be assigned to those interested, to design, or design and build one.)

Ask - Could we subtract with this device?

Bring out that we would turn the wheel in the opposite direction for subtraction.

Ask - Could we multiply with this device?

Bring out that by doing repeated additions we could multiply.

Ask - Could we divide with this device?

Bring out that it can be done by doing repeated subtractions.

Have students do a few exercises in these other operations.

Discuss other kinds of digital computers, and have students look for them. Examples are desk calculators, cash registers, pocket adding machines, etc. (Note: the mileage indicator on an automobile speedometer is <u>not</u> a digital computer. It is an analog computer.)

Have students look for articles in magazines or newspapers on electronic computing machines. Discuss.

E. Distribute the work sheet entitled, "Building An Analog Computer (Addition and Subtraction)." Have students make an addition and subtraction analog computer for natural numbers, using the pattern provided.

Give them some problems to solve with it. Have them demonstrate their computers for the class.

Discuss the fact that this device is based on measurement, and that in-between values can be estimated. The basic principle is thus different from that of the digital computer.

Divide the class into two teams. Have one team solve problems using the computers, while the other team does them longhand. Compare speed and accuracy.

Interested students might make an addition and subtraction analog computer for signed numbers (primarily integers) similar to the one for natural numbers.

F. Suggested Assessment Procedures:

1. In written drills the following items could be used on different days.

   a. A machine which performs operations in a number system is called a ______________________.

   b. A computer which works by counting is called

      ____

   c. A computer which works by measuring is called

      ____

   d. Match each term with the best identifying phrase.

A	B
Digital Computer	Performs operations in a number system
Computing Device	Works by counting
Analog Computer	Works by measuring

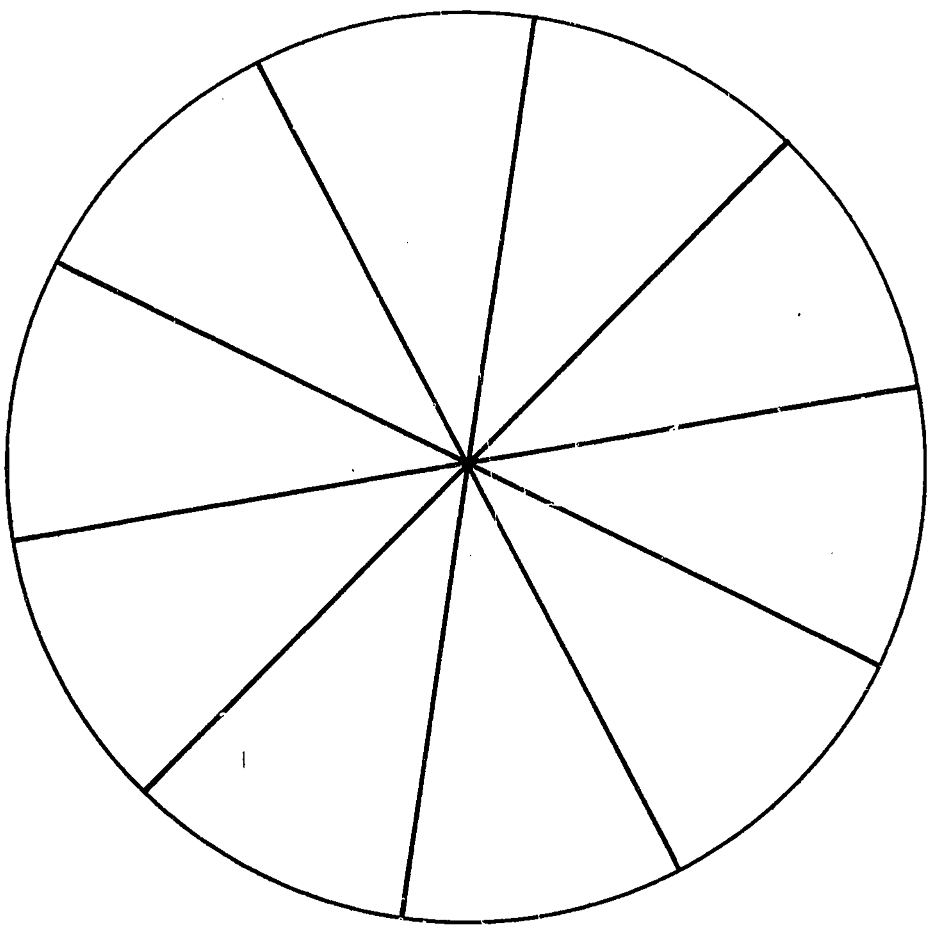
2. Give students a series of problems to solve with their computers.

3. Have students show you how to use their computers.

BUILDING A DIGITAL COMPUTER

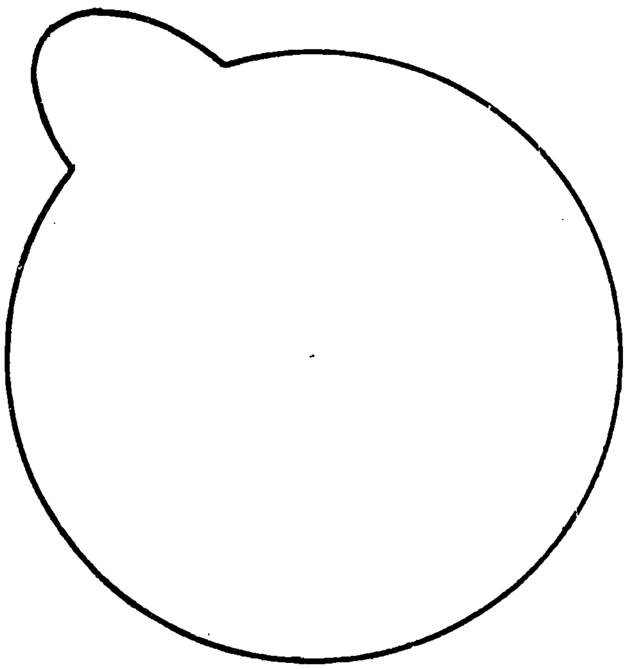
DISC 1

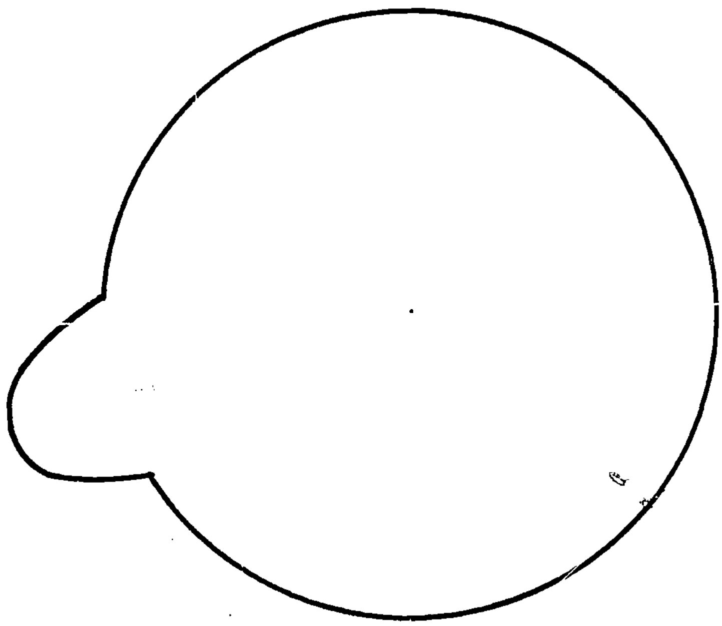
Paste this sheet on a piece of cardboard and cut it out.



BUILDING A DIGITAL COMPUTER
DISC 2

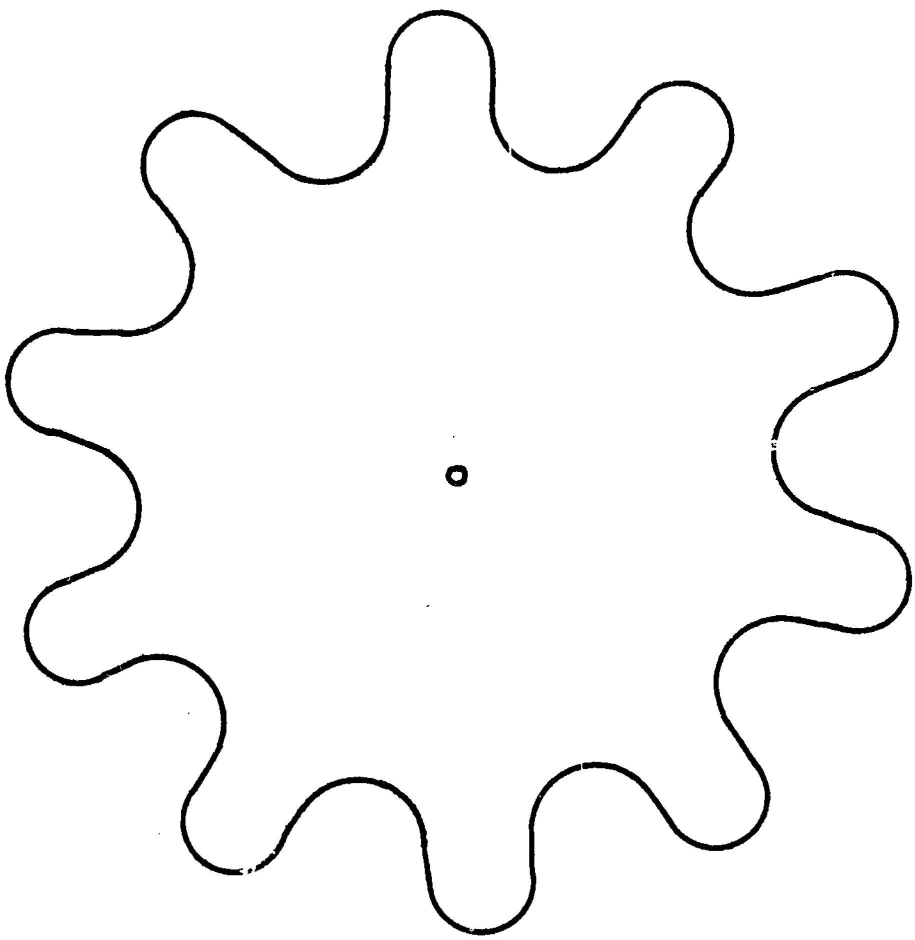
Paste this sheet on a piece of heavy cardboard and cut both discs out. Paste the discs together.





BUILDING A DIGITAL COMPUTER
DISC 3

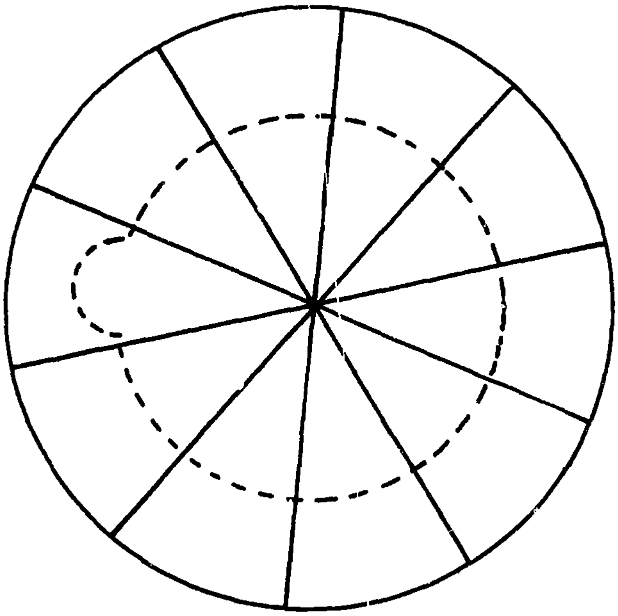
Paste this sheet on a piece of cardboard and cut it out.



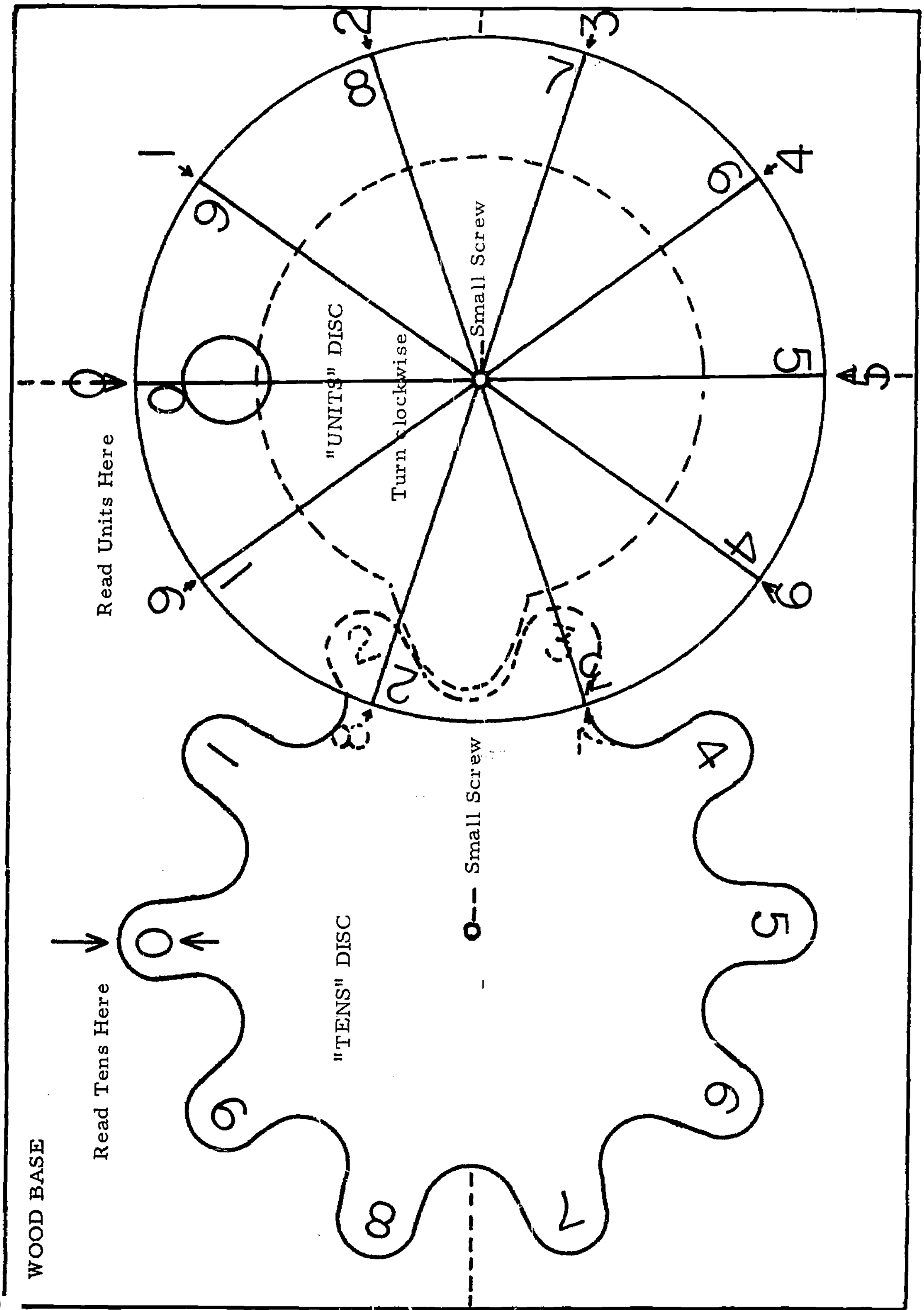
BUILDING A DIGITAL COMPUTER

Directions for Assembling

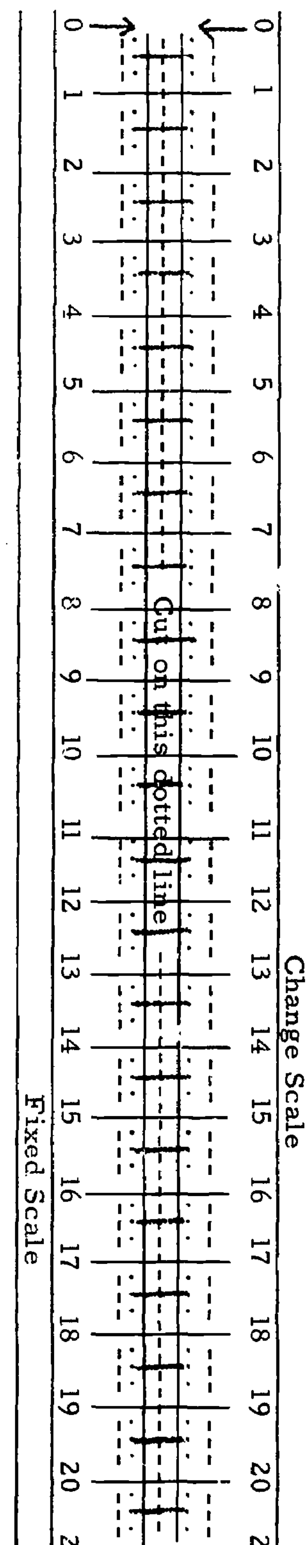
1. Paste Disc 2 on the back of Disc 1 as shown. Allow paste to dry.



2. Assemble the discs as shown on the next page.
   Allow at least $\frac{1}{8}$ inch between the cog on Disc 2 and the cut out section of Disc 3.
   Fasten the discs to the wood base with the round head screws.
   The units disc should be loose enough to allow free movement.
   The tens disc will need to be tighter to prevent easy movement as the units disc touches it.
   Number the discs and wood base exactly as shown on the next page.
   Do this after assembling.
   Cut a small hole in the units disc for a dial.


WOOD BASE
Read Tens Here
"TENS" DISC
Small Screw
Read Units Here
"UNITS" DISC
Turn clockwise
Small Screw

BUILDING AN ANALOG COMPUTER
(Addition and Subtraction)



To make an addition-subtraction computer cut out the complete scale at the right and paste on a heavy cardboard or tag board. After the paste has dried cut along dotted line separating the Change Scale from the Fixed Scale. The Fixed Scale should remain still while the Change Scale is moved along the Fixed Scale.

To add two numbers move the Change Scale to the right until zero is over the first addend on the Fixed Scale. Now read the sum in the Fixed Scale under the second addend.

For example, suppose you want to add 7 and 9. Move the Change Scale so that the zero mark is over the numeral 7 on the Fixed Scale. Read the sum in the Fixed Scale under the numeral 9 on the Change Scale.

To subtract move the Change Scale to the right until the zero is over the subtrahend in the Fixed Scale. Read the difference in the Change Scale over the minuend in the Fixed Scale.

For example, if you wish to subtract 5 from 8 move the Change Scale to the right until zero is over the 5 in the Fixed Scale. Find 8 in the Fixed Scale and read the number (3) above 8 for the difference.

NOMOGRAPH
PERCENT

Teacher Commentary

I. Unit: Percent by Ratio and Proportion

II. Objectives: The student should be able to:

Demonstrate how to construct the following using the nomograph

1. What percent one number is of another
2. A percent of a number
3. A number when a percent of it is known

III. Materials:

A. Student work sheet "Nomograph"

B. A twelve inch ruler

IV. Procedure:

A. Distribute the materials to each student.

B. Discuss the three scales A, B and C.

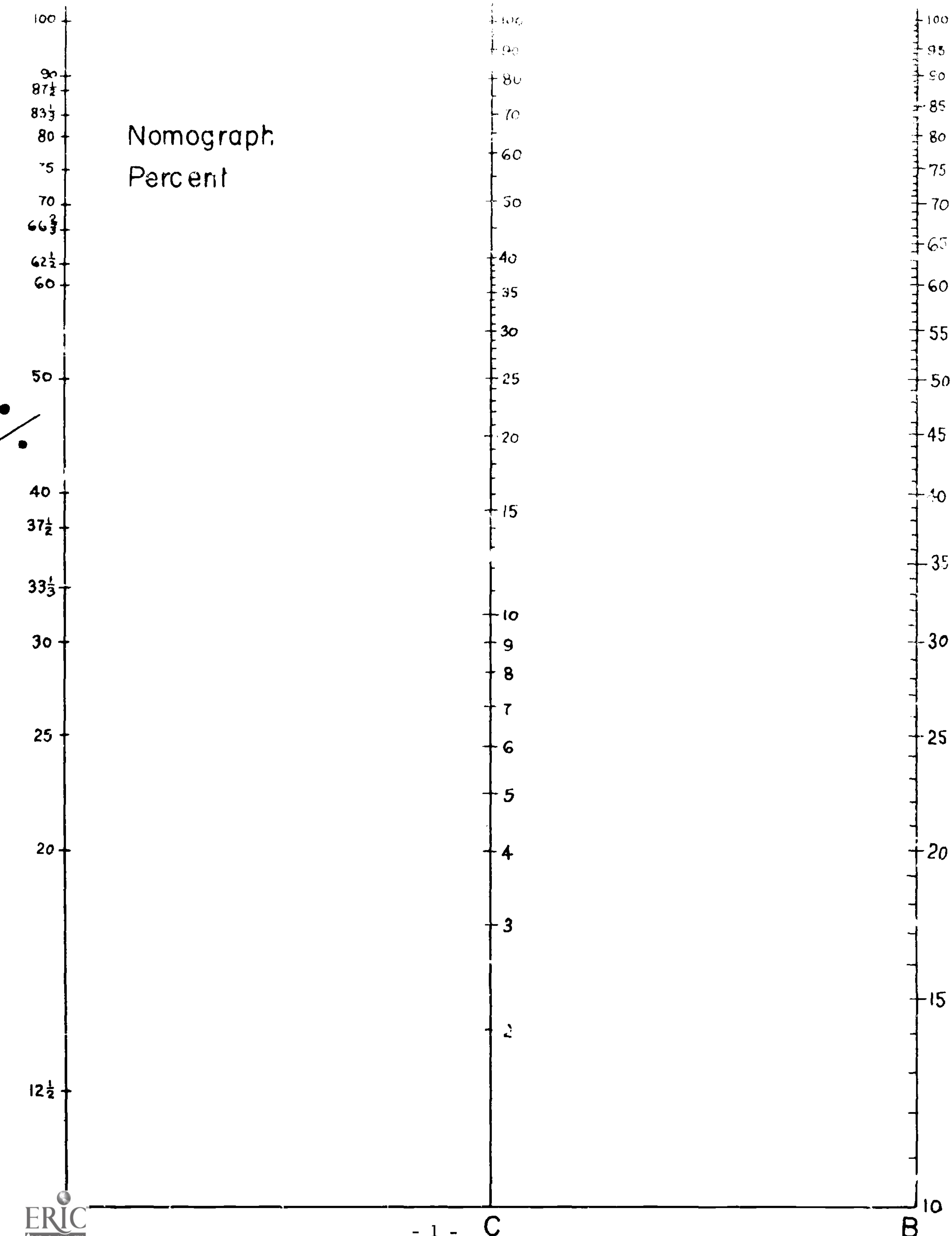
1. Scale A begins with ten and ends with one hundred. All numbers on this scale represent percent.
2. Scale B begins with ten and ends with one hundred.
3. Scale C begins with one and ends with one hundred.
4. Locate points on the scales and have students identify them.
5. Have students locate points on the scales.

C. In order to find a percent of a number (25% of 40), locate 25% on scale A and 40 on scale B. The line joining these two points will cross scale C at a point that represents the percent of a number (10). Choose problems so that the answers will be found on the nomograph.

D. In order to find what percent one number is of another (10 is ____% of 40), locate 10 on scale C and 40 on scale B. The line joining these two points will cross scale A at a point that represents what percent one number is of another (25%). Choose problems so that the answers will be found on the nomograph.

E. In order to find a number when a percent of it is known (25% of ____ is 10), locate 25% on scale A and 10 on scale C. The line joining these two points will cross scale B at a point that represents a number when a percent of it is known (40). Choose problems so that the answers will be found on the nomograph.

F. After the student has completed some written exercises, the nomograph may be used to check the results.


Nomograph
Percent
%
C
B

SQUARE

I. Unit: Fundamental ...

II. Objectives: The student should

A. Demonstrate how to use ... square root using ...

B. Construct the square ... Newton's Method

III. Materials: Student work ... root" and "Chart for ...

IV. Procedure:

A. This is an activity which ... square root by using a ... It is assumed that the co... been taught and reviewed ... that students approach this ... at square root is.

B. Distribute several copies of ... chart to each student.

C. Introduce these flow charts by discuss... computer solves problems, that is, the computer ... an input, follows directions to do various things ... input, and finally arrives at an answer. Tell the...s that ...ey ... to be human computers.

D. Tell the students that you wi... to find the square root of some large number (square number) such as 1...4. Have the students put this number below the space marked Number.

1. Explain that this process starts with a guess. Have students propose guesses. Choose a guess which is very poor, such as 10. Have the students put this guess below the space marked

Guess, opposite the Start sign.

[illegible] then follow the arrow and read the [illegible] that they should divide their [illegible] into [illegible] given. Have them attempt to [illegible] before you explain it.

3. In finding the qu[illegible]ested, ignore any remainder and put the whole [illegible] quotient into the chart marked Quotients. [illegible] sure all students perform the [illegible] ignore the remainder. For the example given above, 1024 ÷ 10 gives 102 for the quotient without remainder.

4. Again ask students to follow the arrows and see if they can do as the directions request. This means they must compare the quotient of step 3 above with the guess of step 1 above. This quotient must be either less than, greater than, or equal to the guess. If the quotient equals the guess, the direction indicates that this quotient should be written in the space marked Stop Sq. Root is ____, and the process is finished. In the example given, the quotient is greater than the guess, so the student should proceed to the direction to average the quotient and the guess.

5. At this point it will probably be necessary to review the averaging process before averaging the quotient and guess. In finding this average, ignore any fractional part of the average and put the whole number part of the average below the space marked Average. For the example given 10 + 102 = 112 and 112 ÷ 2 = 56.

6. Have the students now follow the direction to consider the average from step 5 above to be their new guess, and start the entire process again by following the directions under step 3 above.

7. Have students keep repeating steps [illegible] the quotient found in step [illegible] above equals the new guess [illegible] in step 6 above [illegible] the square root will be known.

Repeat this process with the entire class until enough competence has been [illegible] permit students to work problems independently.

1. When several square roots have been found by the class and teacher together, propose a square number, but this time ask students to use their own first guess.

2. Do several square roots with students using their own guesses.

F. Distribute the second work sheet entitled, "Chart for Square Root."

1. Explain that this chart is more compact and hence requires fewer pages.

2. Have students perform several square roots, using the flow chart as a guide, but using the vertical chart to record solutions.

3. Below is an example of a square root found by recording in the vertical chart, with headings taken from the flow chart.

Number	Guess	Quotient	Comparison $\left\{\begin{matrix} < \\ > \\ = \end{matrix}\right\}$	Average	Square Root
1024	10	102	>	56	
	56	18	<	37	
	37	27	<	32	
	32	32	=		32

G. Suggested assessment activities

1. Prepare a slip of paper for each student. On the paper place a square number and a number to be used as a first guess.

   a. Have students complete a flow chart for their number.

   b. Have students use a vertical chart for the same number as in 1 above.

2. Have students find square roots of square numbers at the chalkboard.

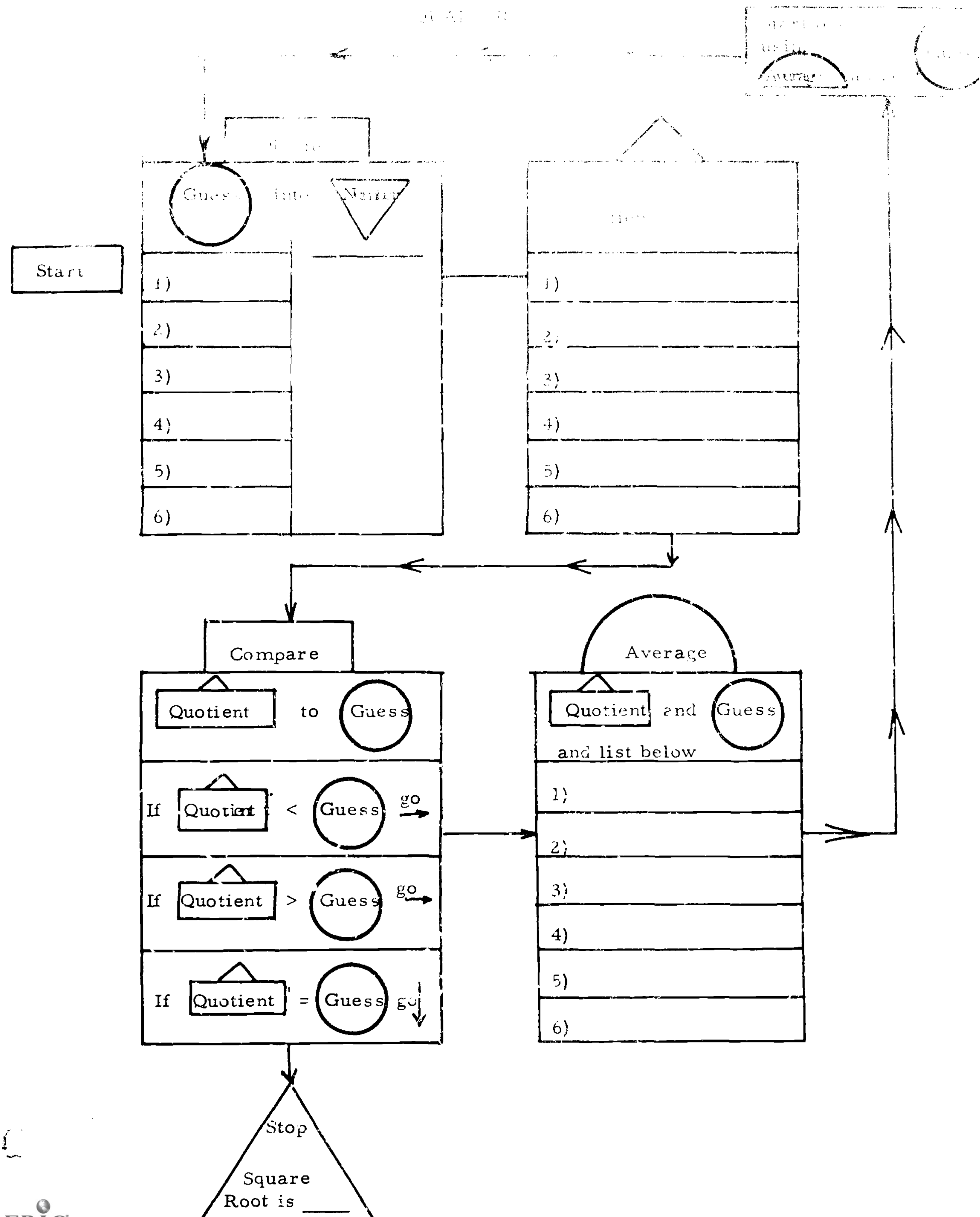

Start
1)
2)
3)
4)
5)
6)
Compare
Quotient
to
Guess
If
Quotient
<
Guess
go
If
Quotient
>
Guess
go
If
Quotient
=
Guess
Average
Quotient
and
Guess
and list below
Stop
Square
Root is ____

CHART FOR SQUARE RO

Number	Guess	Quotient	Comparison $\left\{\begin{smallmatrix}<\\>\\=\end{smallmatrix}\right\}$	Average	Square Root

GEOMETRY

GEOMETRY

UNIT GEOMETRY GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	[illegible]
Point	6	6		6						
Line	6	6		5	6					
Plane	6	6		7	6					
Closed Path	6	6		6	6					
Segment	6, 7	6, 7	7	6, 7	6					
Congruent Segments	9	9	9		9					
Ray	6, 7	6, 7		6	6					
Angles	6, 7	6, 7	7	6	6					
Vertex	[illegible]	7			7					
Right Angles	6	6	9	6	6					
Acute Angles	9	9		9	9					
Obtuse Angles	9	9		9	9					
Straight Angles	9	9		9	9					
Vertical Angles	9	9		9	9	9	9			
Supplementary Angles	9	9		9	9					
Complementary Angles	9	9		9	9					
Congruent Angles	9	9	9	9	9					
Triangles	6, 7	6, 7	10	6	6					

UNIT GEOMETRY GRADE(S) Seven through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	[illegible]	[illegible]
Equilateral Triangle	8	8	8	8	8					
Isosceles Triangle	8	8	8	8	8					
Scalene Triangle	8	8	8	8	8					
Right Triangle	9	9	9	9	9					
Acute Triangle	9	9		9	9					
Obtuse Triangle	9	9		9	9					
Perpendicular Lines	9	9	9	9	9					
Parallel Lines	7	7	9	7	7					
Transversal	10	10		10	10					
Corresponding Angles	10	10		10		10	10, 11			
Midpoint	7	7	7		7					
Partitioning a Segment			11							
Quadrilaterals	7	7		7	7					
Trapezoid	7	7		7	7					
Parallelogram	7	7	10	7	7	10	10, 11			
Rectangles	7	7	10	7	7	10	10, 11			
Square	7	7	10	7	7	10	10, 11			
Rhombus	7	7	10	7	7	10	10, 11			
Polygon	8	8		8	8					

UNIT GEOMETRY GRADE(S) Eight through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Pentagon	8	8	9	8	8				8	8
Hexagon	8	8	8	8	8				8	8
Octagon	8	8	8, 9	8	8				8	8
Congruent Triangles	10	10			10	10	10			
Similar Triangles	10	10			10	10	10			
Corresponding Sides of Similar Triangles						10	10, 11	10, 11		
Circle	6	6	7	6	6					
Radius	6	6	7	6	6	7				
Diameter	6	6	7	6	6	7				
Chord	7	7	7	7	7					
Tangent	8	8	10	8	8	10				8
Secant	8	8		8	8					8
Central Angle	10	10	10	10	10					10
Inscribed Angle	10	10	10	10	10	10				10
Ellipse	10	10	10	10	10	10				
Angle Bisector	9	9	9	9	9	9				
Sum of Interior Angles of Triangles						9	9			
45°			9	9						
60°			9	9						

UNIT GEOMETRY GRADE(S) Seven through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
30°			9	9						
Median Triangle	7	7	7	7	7	7				
Altitude of Triangle	9	9	9	9	9					
Cube	7	7		7	7					7
Rectangular Solid	7	7		7	7					
Pyramid	8	8		8	8					
Cone	8	8		8	8					
Cylinder	8	8		8	8					
Sphere	8	8		8	8					8
Line of Symmetry	8	8		8	8					
Sum of Interior Angles of Quadrilaterals						9	9			
Sin	11	11				11	11	11		11
Cos	11	11				11	11	11		11
Tan	11	11				11	11	11		11
Trig Tables	11	11		11				11		
Other Polyhedrons	11	11		9, 10						
Pythagorean Theorem						10	10			
Region	7	7		7	7					
Sum of Interior Angles of a Polygon						8	8			

UNIT GEOMETRY GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Triangles			10							
Transversal	10	10		10	10					
Corresponding $\angle$'s	10	10		10		10	10			
Parallelogram			10			10	10			
Rectangles			10			10				
Square			10			10				
Rhombus			10			10				
Congruent Δ's	10	10			10	10	10			
Similar Δ's	10	10			10	10	10			
Corresponding Sides of Similar Δ's						10	10	10		
Tangent			10			10				
Central $\angle$'s	10	10	10	10	10					10
Inscribed $\angle$'s	10	10	10	10	10	10				10
Ellipse	10	10	10	10	10					
Other Polyhedrons				10						
Pythagorean Theorem						10	10			
Sum of the Interior Angles of a Polygon						10	10			

GEOMETRY - Grade 10

Triangles	Page
The student should be able to:	
1. Demonstrate the construction of a triangle by SAS, ASA, or SSS	GE-13

Transversal

The student should be able to:

1. Name and identify transversals
2. Construct a drawing of a transversal using straightedge or freehand sketch
3. Describe a transversal by definition

Corresponding Angles

The student should be able to:

1. Name and identify corresponding angles
2. Construct a drawing of corresponding angles using straightedge
3. State the principles that:
   a. If two parallel lines are intersected by a transversal, then the corresponding angles are congruent
   b. If two lines are intersected by a transversal such that the corresponding angles are congruent, then the lines are parallel
4. Apply the principle by demonstrating how to test whether or not two lines are parallel and to solve related problems

Sum of the Interior Angles of Polygons

The student should be able to:

1. State the principle that the sum of the measures of the interior angles is (n - 2) 180 when n is the number of sides of the polygon — GE-14
2. Apply the principle to solve related problems — GE-12

Parallelogram

The student should be able to:

1. Demonstrate the construction of a parallelogram using straightedge and compass

2. State the following principles:

   a. If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram

   b. If a quadrilateral is a parallelogram, then the opposite sides are parallel

3. Apply these principles to solve related problems

Rectangles

The student should be able to:

1. Demonstrate the construction of rectangles using a compass and a straightedge or protractor

2. State the following principles:

   a. If a quadrilateral has four right angles, then it is a rectangle

   b. If a quadrilateral is a rectangle, then it has four right angles

Square

The student should be able to:

1. Demonstrate the construction of a square by straightedge and a compass or protractor

2. State the principle that:

   a. A square is a rectangle with all sides congruent

Rhombus

The student should be able to:

1. Demonstrate the construction of a rhombus using straightedge and compass or protractor

2. State the principle that the diagonals of a rhombus are perpendicular

Congruent Triangles | Page

The student should be able to:

1. Name and identify the figure and symbol for congruent triangles
2. Describe a method of determining whether two triangles are congruent
3. State the principle that:
   a. If two triangles have two sides and the included angle of one respectively congruent to two sides and the included angle of another, then the triangles are congurent (SAS)
   b. If two triangles have two angles and the included side of one repsectively congruent to two angles and the included side of the other, then the triangles are congruent (ASA)
   c. If two triangles have three sides of one respectively congruent to three sides of the other, then the two triangles are congruent (SSS)
4. Apply the principle to solve related problems

Similar Triangles

The student should be able to:

1. Name and identify the figure and symbol for similar triangles
2. Describe and demonstrate a method of constructing a triangle similar to a given triangle
3. State the principle that if three angles of one triangle respectively congruent to three angles of another triangle, then the triangles are similar
4. Apply the principle to solve related problems

Corresponding Sides of Similar Triangles

The student should be able to:

1. State the principle that corresponding sides of similar triangles are proportional
2. Apply the principle to solve related problems
3. Interpret problems involving corresponding sides according to the rules and principles necessary to a solution

Tangent

Page

The student should be able to:

1. State the principle that the tangent to the circle is perpendicular to the radius of the circle drawn to the point of contact

2. Demonstrate a method of constructing a tangent to a circle from a point on the circle using compass and straightedge or protractor

Central Angles

The student should be able to:

1. Name and identify a central angle

2. Construct a drawing of a central angle using straightedge or freehand sketch, given a circle

3. Demonstrate the construction of a central angle using straightedge, given a circle

4. State the definition of a central angle

5. Distinguish between central and inscribed angles

Inscribed Angle

The student should be able to:

1. Name and identify an inscribed angle

2. Construct a drawing of an inscribed angle using straightedge or freehand sketch

3. Demonstrate the construction of an inscribed angle using a straightedge

4. Describe an inscribed angle as one in which the vertex is on the circle and whose sides contain chords

5. State the principle that an angle inscribed in a semi-circle is a right angle

6. Distinguish between inscribed angles and central angles

	Page

Ellipse

The student should be able to:

1. Name and identify an ellipse
2. Construct a drawing of an ellipse by freehand sketch
3. Demonstrate the construction of an ellipse by use of string and thumb tacks
4. Describe an ellipse in terms of his surroundings

Other Polyhedrons

The student should be able to:

1. Construct models of other polyhedrons using available materials — GE-15

Pythagorean Theorem

The student should be able to:

1. State the principle that the square of the longest side of a right triangle is equal to the sum of the squares of the other two sides
   (i.e. $a^2 + b^2 = c^2$)
2. Apply the principle and its converse to solve related problems

FORMULAS

Teacher Commentary

I. Unit: Geometry

II. Objectives: The student should be able to:

A. State the principle that the sum of the measures of the interior angles is (n-2) 180, where n is the number of sides in the polygon.

B. Apply the above principle in similar problem-solving situations.

III. Materials:

A. Protractors

B. Rulers

C. Work sheet, "Formulas" (3 pages)

D. Pencils

E. Extra plain paper

IV. Procedure:

A. Have students follow directions on the work sheet and work figures 1, 2, 3, and 4. Students should place results in the third column of the table, "Sum of Measures Chart." Because of the size of the polygon and the difficulty of their measuring accurately with a protractor, the sums the students construct may vary a few degrees. It will, therefore, be necessary for the teacher to correct these before going on.

B. Upon examining their results students may discover that the formula for finding the sum of the measures of the interior angles of a polygon is (n-2) 180, where n is the number of sides in the polygon.

C. Have the students apply this formula to the data in their charts to see if the formula holds true in all cases tested.

D. Assessment - Ask the students to construct polygons containing 7, 8, and 9 sides, measure their interior angles, and continue to fill in the chart. Students shall state the principle illustrated by the formula and apply it to the new data in their charts.

FORMULAS

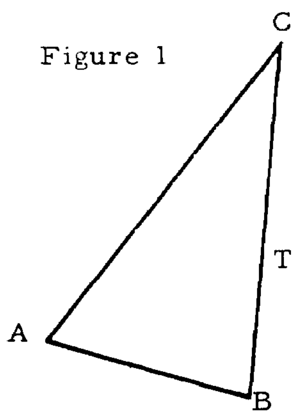
A. Using a protractor, measure the interior angles in the polygons below. Write your answers in the blank spaces.

B. Add the measures of the interior angles and put your total in the blank spaces.

C. When you have finished filling in the blanks next to the polygons, transfer your sums to the chart on the next page.

D. Look for a pattern in the third column of your chart.

Figure 1



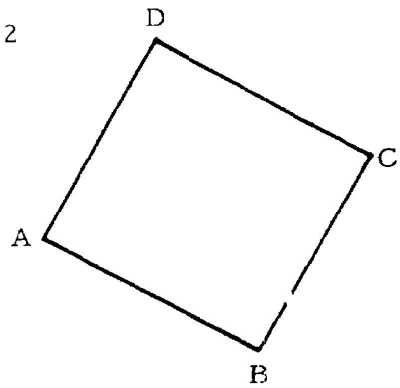
∠ A = _____°

∠ B = _____°

∠ C = _____°

Total = _____°

Figure 2



∠ A = _____°

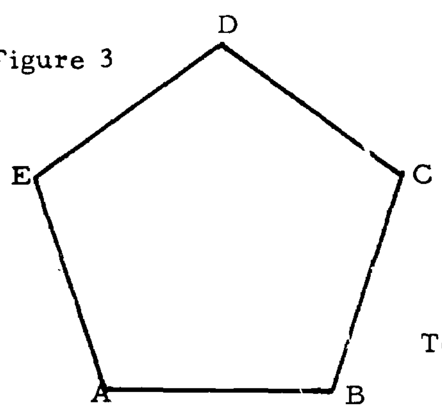
∠ B = _____°

∠ C = _____°

∠ D = _____°

Total = _____°

Figure 3



∠ A = _____°

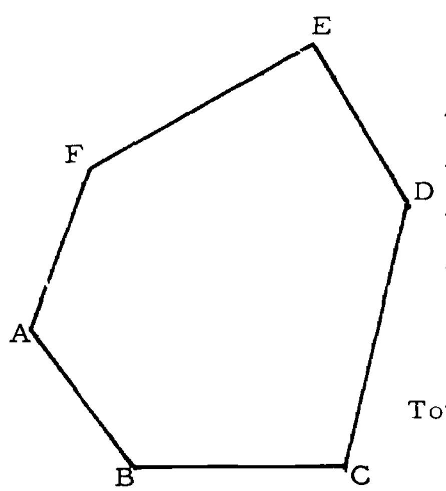
∠ B = _____°

∠ C = _____°

∠ D = _____°

∠ E = _____°

Total = _____°



∠A = _____°

∠B = _____°

∠C = _____°

∠D = _____°

∠E = _____°

∠F = _____°

Total = _____°

Sum of Measures Char

Figure	Number of Sides	Sum of Measures
1	3	
2	4	
3	5	
4	6	
5	7	
6	8	
7	9	

A. Construct a 7-sided polygon. Label the interior angles with capital letters.

Figure 5

$\angle A$ = ________°

$\angle B$ = ________°

$\angle C$ = ________°

$\angle D$ = ________°

$\angle E$ = ________°

$\angle F$ = ________°

$\angle G$ = ________°

Total ________°

B. Construct an 8-sided polygon. Label the interior angles with capital letters.

Figure 6

$\angle A$ = ________°

$\angle B$ = ________°

$\angle C$ = ________°

$\angle D$ = ________°

$\angle E$ = ________°

$\angle F$ = ________°

$\angle G$ = ________°

$\angle H$ = ________°

Total ________°

C. Construct a 9-sided polygon. Label the interior angles with capital letters.

Figure 7

$\angle A$ = _______ °

$\angle B$ = _______ °

$\angle C$ = _______ °

$\angle D$ = _______ °

$\angle E$ = _______ °

$\angle F$ = _______ °

$\angle G$ = _______ °

$\angle H$ = _______ °

$\angle I$ = _______ °

Total _______ °

D. What is the formula for finding the sum of the interior angles of a polygon?

E. Using the formula, what is the sum of the interior angles of a polygon having 22 sides?

TRIANGULAR SHAPES

Teacher Commentary

I. Unit: Geometry

II. Objectives: The student should be able to:

Demonstrate the construction of triangles using the ASA method

III. Materials:

A. Work sheets entitled, "Triangular Shapes I" and "Triangular Shapes II"

B. Scissors, protractor, ruler

C. One sheet of notebook paper

IV. Procedure:

A. Distribute materials. Placing the materials in a shoe box might make the distribution easier.

B. Allow students to begin on their own with little or no direction. They should raise their hand for you to check their work before they cut out the triangle. Checking can be easily accomplished by having a cardboard model cut to the actual size of their drawing. (This is a good place to stop, when using the banded approach if the teacher wants to make a two day lesson.)

C. Distribute work sheet II entitled, "Triangular Shapes II." Allow the students to use their rulers and protractors to see how many of the questions they can answer.

D. Below are the solutions plus some suggested concepts which can be used for enrichment.

1. Solutions:

7. a. equal

   b. altitude

8. a. equal

   b. isosceles

9. a. equal

   b. isosceles

10. a. 180°

    b. 180°

11. a. at least 12 triangles

    b. at least 3 rectangles (including 1 square)

    c. at least 11 trapezoids

    d. at least 19 quadrilaterals

    e. at least 3 pentagons

2. Additional concepts to be developed from the figure:

    1. $\overline{EF}$ the altitude to the base of an isosceles triangle bisects the base $\overline{BD}$, bisects the vertex angle $\overline{BED}$, and is perpendicular to the base.

    2. $\overline{EF}$ forms two equal, adjacent, supplementary angles.

    3. $\overline{EG}$ joins the midpoints of $\overline{BA}$ and $\overline{AC}$ respectively.

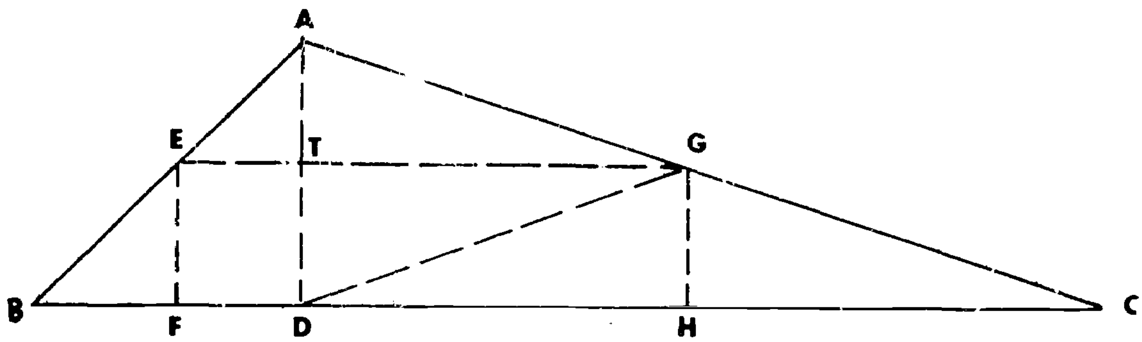
    4. $\overline{MEG}$ is equal to $\frac{1}{2}$ the measure of $\overline{BC}$.

    5. $\overline{EG}$ is parallel to $\overline{BC}$.

    6. $\overline{DG}$ is a median to the hypotenuse of right triangle $\overline{ADC}$.

    7. $\overline{MDG}$ is equal to $\frac{1}{2}$ $\overline{MAC}$.

TRIANGULAR SHAPES I



1. Measure line segment $\overline{BC}$ and draw it on the blank paper. Label the end points B, C like the drawing.

2. Measure $\angle B$ using the protractor and construct it at the end of the line segment $\overline{BC}$ at B. Also measure $\angle C$ and construct it on $\overline{BC}$ at C.

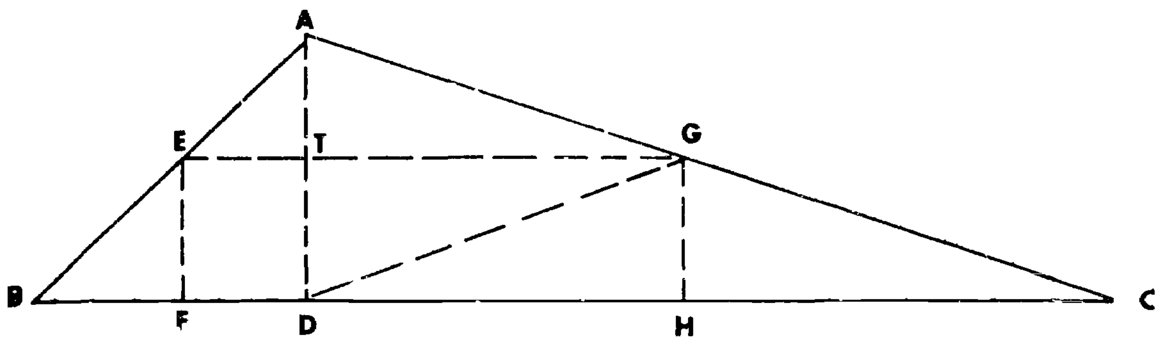
3. Extend the sides of $\angle$'s B and C until they intersect at point A.

4. You may check your drawing by using the protractor to see if $\angle A$ on your drawing and $\angle A$ on this work sheet have the same measure.

5. Raise your hand and the teacher will check your work.

6. Cut out your triangle <u>after</u> it has been checked. Stop here until you receive further directions.

TRIANGULAR SHAPES II



7. Fold point B over the segment $\overline{BC}$ so that the crease passes through A. This determines point D on $\overline{BC}$.

   a. Measure $\angle ADC$ and $\angle ADB$. What do you know about the size of these two angles? ____________________

   b. AD is the ________________ of triangle ABC to side BC.

8. Place point B on point D and fold, forming crease EF.

   a. Measure segment $\overline{BE}$ and segment $\overline{ED}$. What do you notice about their lengths? ____________________

   b. What kind of a triangle is triangle BED? ________________

9. Place point C on point D and fold, forming crease GH.

   a. Measure segment $\overline{DG}$ and segment $\overline{GC}$. What do you notice about their lengths? ____________________

   b. What kind of a triangle is triangle DGC? ________________

10. Place point A on point D and fold, forming crease EG.

   a. When all three angles A, B, and C are folded over to point D, they appear to form an angle of how many degrees? __________

b. The sum of the measures of the angles of a triangle is equal to how many degrees? ________________

11. Look at the figure to see if you can answer the following:

a. There are how many triangles in the figure? ________________

b. There are how many rectangles in the figure? ________________

c. There are how many trapezoids in the figure? ________________

d. There are how many quadrilaterals in the figure? ________________

e. There are how many pentagons in the figure? ________________

CONSTRUCTION OF THE FIVE REGULAR POLYHEDRONS

Teacher Commentary

I. Unit: Geometry

II. Objectives: The student should be able to:

Construct the five regular polyhedrons: cube, tetrahedron, octahedron, dodecahedron, icosahedron.

III. Materials:

A. Tagboard

B. Tape

C. Scissors

D. Pin

IV. Procedure:

A. The models which follow have been arranged according to the number of sides. Therefore, start with the cube and proceed with the others in order.

B. Have the students trace the patterns on tagboard. This may best be done as follows:

1. Place the sheet containing the pattern on the tagboard.

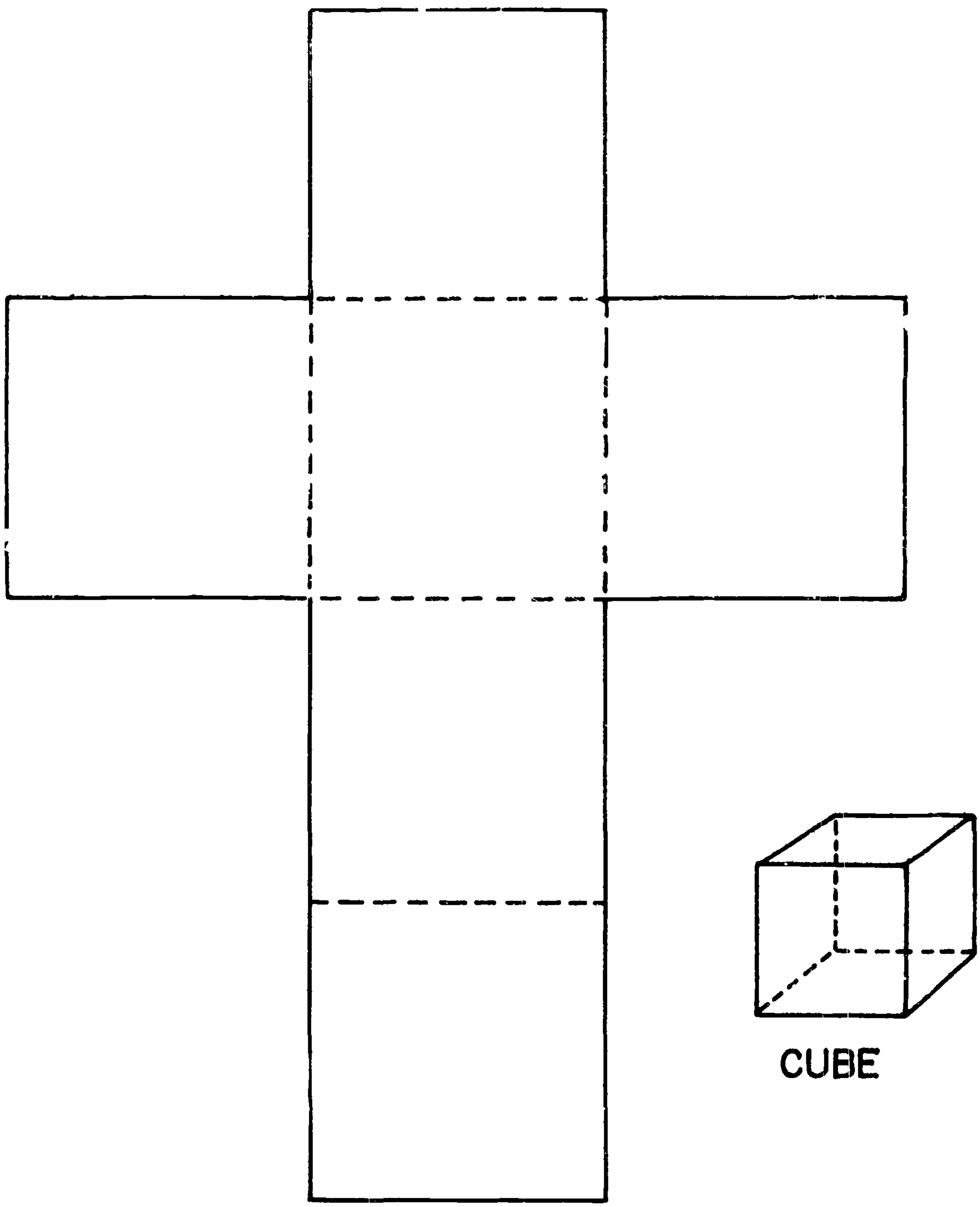
2. Press the pin through each vertex.

3. Connect the pin marks with the appropriate solid and dotted lines.

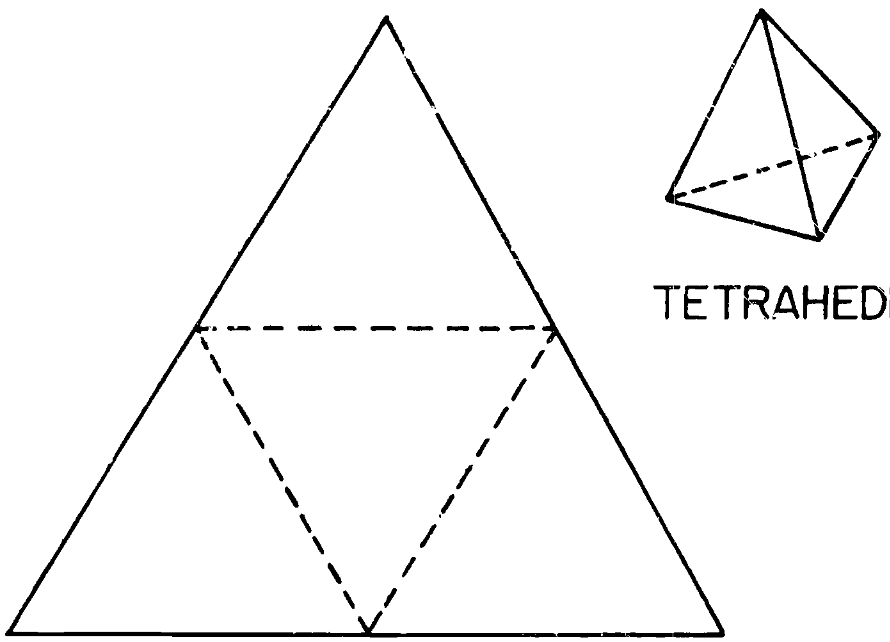
C. Cut out along the solid lines.

D. Crease and fold along the dotted lines.

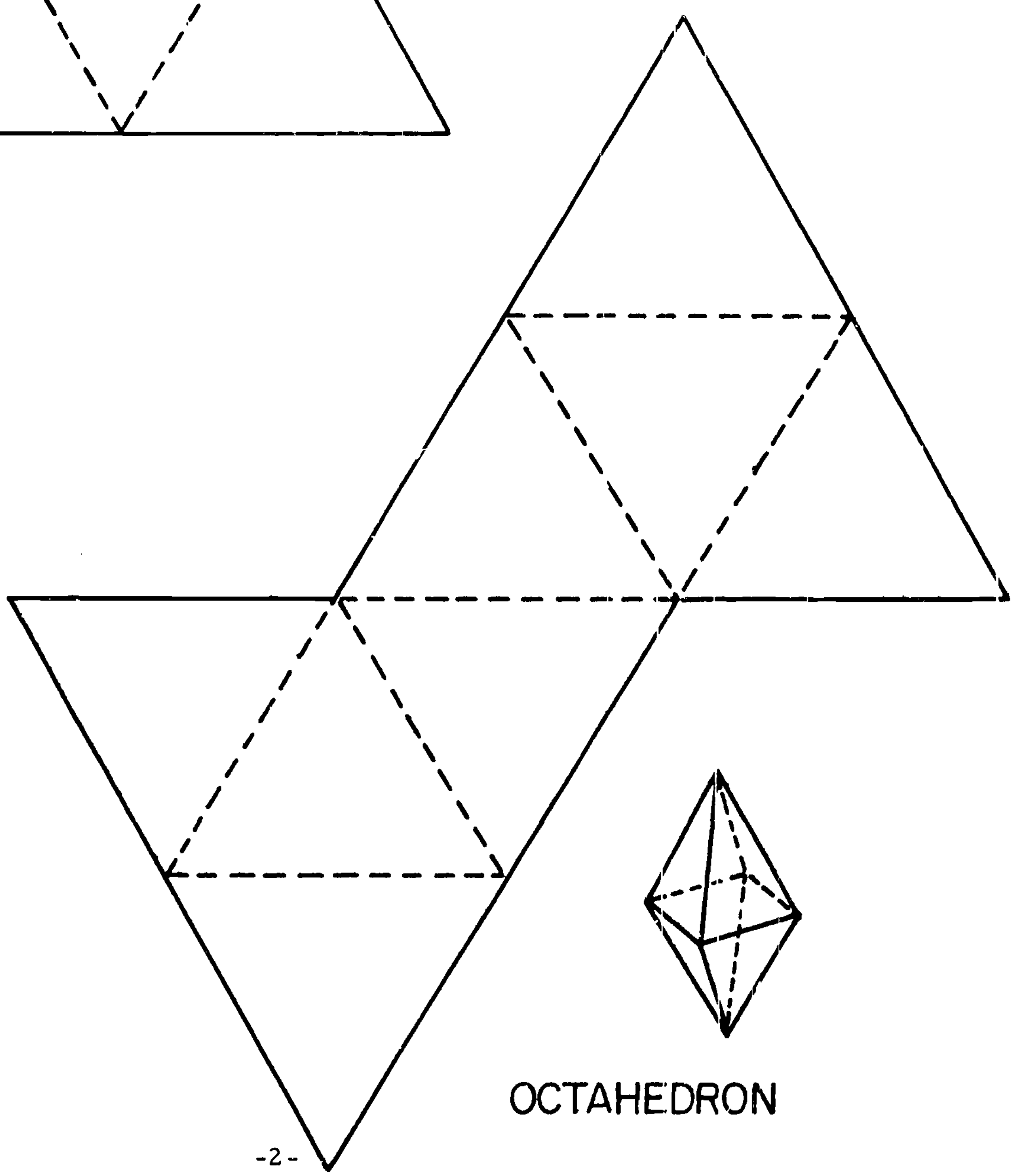
E. Tape the solidly lined edges together from behind.



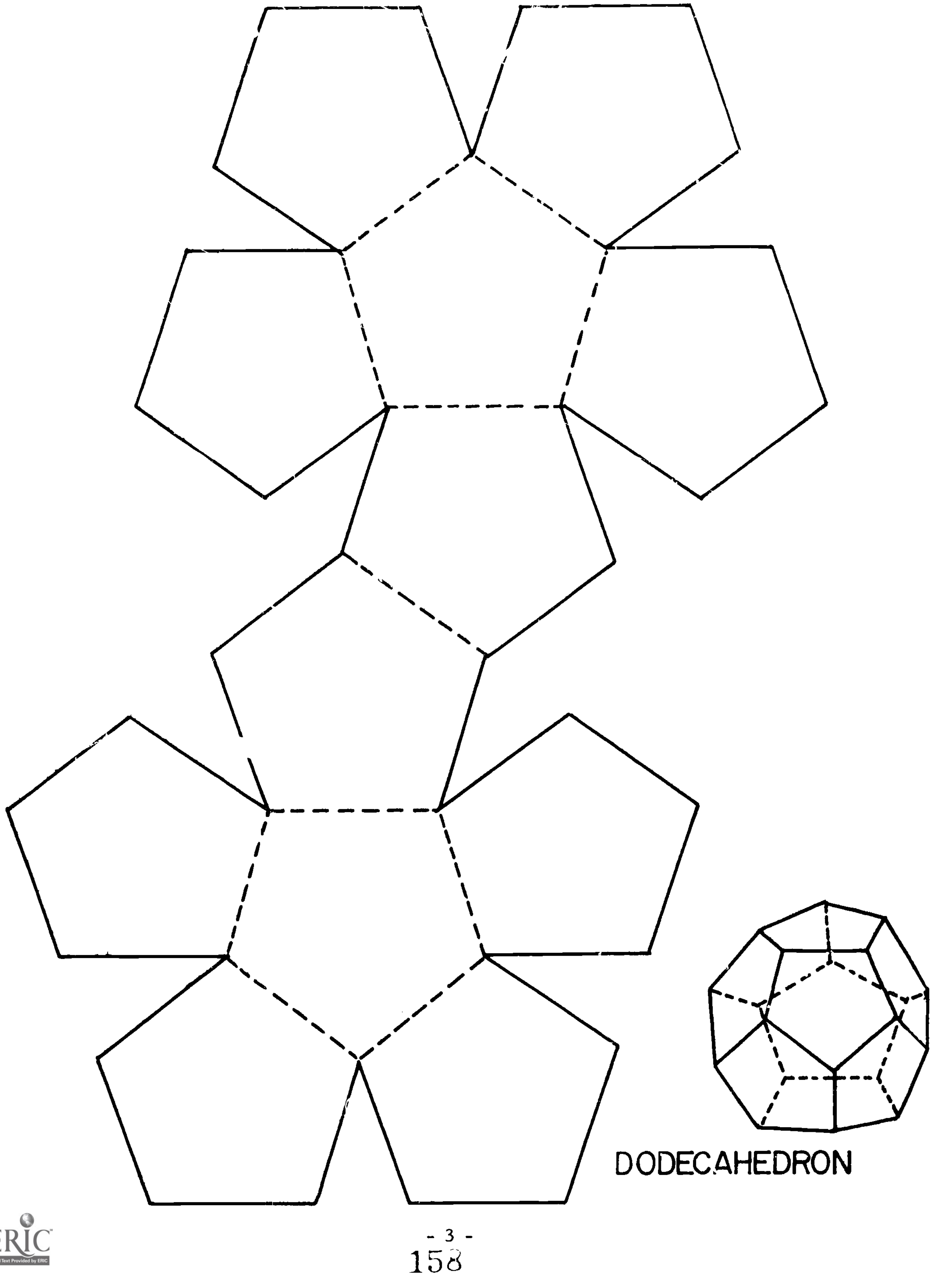
CUBE



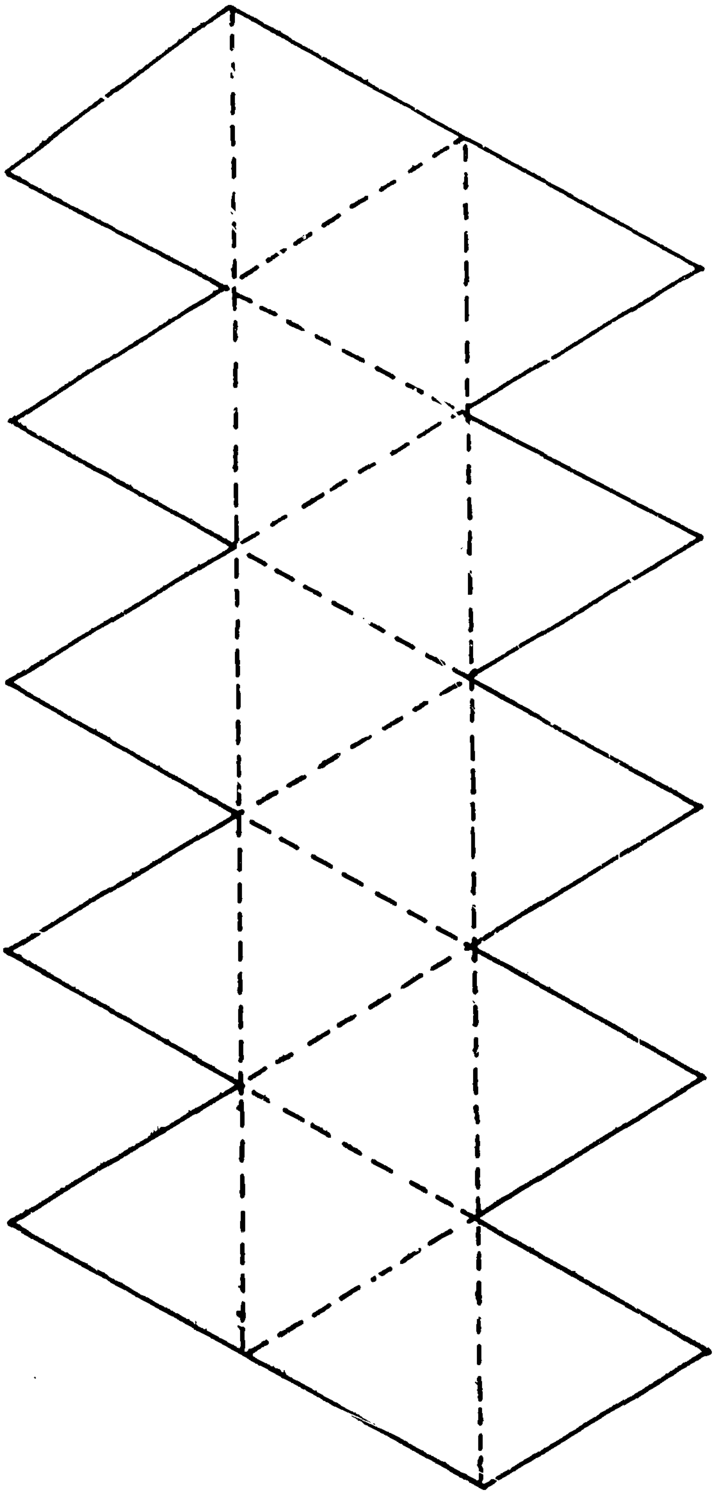
TETRAHEDRON

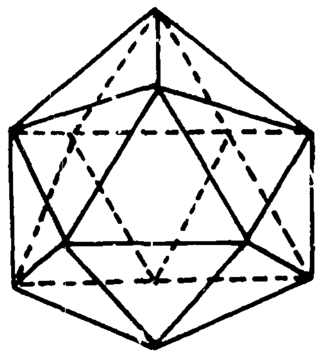


OCTAHEDRON



DODECAHEDRON





ICOSAHEDRON

MEASUREMENT

MEASUREMENT

I. Master Chart - Grades Six through Eleven

II. Grade Ten Chart

III. Behavioral Objectives

IV. Activities

UNIT MEASUREMENT GRADE(S) Seven through Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Area	7	7	7			7				
Square Inch	7	7		7	7	7				
Square Foot	7	7		7	7	7	7			
Square Yard	7	7		7	7	7	7			
Area of Square	7	7	7	7		7	7	7		7
Area of Rectangle	7	7	7	7		7	7	7		7
Area of Triangle	8	8	8		8	8	8			8
Area of Parallelogram	8	8	8		8	8	8			8
Area of Circle	8	8	8		8	8	8			8
Area of Other Polygons			9		9	9	9	9		
Acre	9	9	9		9	9				
Surface Area	9	9	9			9				
Total Surface Area of Cube	9	9			9	9	9			
Lateral Surface Area of Cube	9	9			9	9	9			
Total Surface Area of Box	9	9	9			9	9			
Lateral Surface Area of Box	9	9	9			9	9			
Total Surface Area of Cylinder	9	9	9			9	9			
Lateral Surface Area of Cylinder	9	9	9			9	9			

UNIT MEASUREMENT GRADE(S) Six through Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Inch	6	6	6	6	6	6	6		6	
1/2 Inch	6	6	6	6	6		6		6	
1/4 Inch	7	7	7	7					7	
1/8 Inch	8	8	8							8
1/16 Inch	9	9	9							9
Foot	6	6	6	6	6	6	6		6	
Yard	6	6	6	6	6	6	6		6	8
Meter	8	8	8	8	8				8	8
Centimeter	8	8	8			8	8		8	9
Millimeter	9	9	9			9	9		9	9
Mile	7	7				7	7			
Perimeter	7	7	7			7	7			
Perimeter Square	7	7	7	7		7	7	7		
Perimeter Triangle	7	7	7	7		7	7	7		
Perimeter Rectangle	7	7	7	7		7	7	7		
Perimeter Polygons			8	8		8	8	8		
Circumference	8	8	8			8				
Pi	8	8			8	8	8			

UNIT MEASUREMENT GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTINGUISHING
Micrometer	10	10	10							
Caliper	10	10	10							
Volume	10	10	10			10				
Cubic Inch	10	10		10	10					
Cubic Foot	10	10		10	10	10	10			
Cubic Yard	10	10		10	10	10	10			
Volume of Cube	10	10	10			10	10	10		
Volume of Rectangular Solid	10	10	10			10	10	10		
Volume of Cylinder	10	10	10			10	10	10		
Angular Measurement	9	9				9				
Degree	9	9			9	9	9	9		
Protractor	9	9	9	9	9	9	9	9		
Central Angle	10	10	10	10		10	10			10
Inscribed Angle	10	10	10	10		10	10			10
Corresponding Angles	10	10		10		10	10	10		
Ounce	6	6	6			6	6	6	6	
Pound	6	6	6			6	6	6	6	
Pint	6	6	6			6	6	6	6	

UNIT MEASUREMENT GRADE(S) Six through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTINGUISH
Quart	6	6	6			6	6	6	6	
Fluid Ounce	10	10	10			10	10	10	10	
Gallon	6	6	6			6	6	6	6	
Cup	10	10	10			10	10	10	10	
Degree Fahrinheit	6	6	6			6	6	6	6	
Degree Centigrade	7	7	7			7	7	7	7	
Similar Triangles (Ratio, Proportion)	10	10	10		10	10	10	10		
Scale Drawing			10	10	10	10		10		
Trig Tables	11	11		11	11					
Pythagorean	10	10	10			10	10	10		

UNIT MEASUREMENT GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Micrometer	10	10	10							
Caliper	10	10	10							
Volume	10	10	10			10				
Cubic Inch	10	10		10	10					
Cubic Foot	10	10		10	10					
Cubic Yard	10	10		10	10					
Volume of Cube	10	10	10			10	10	10		
Volume of Rectangular Solid	10	10	10			10	10	10		
Volume of Cylinder	10	10	10			10	10	10		
Central ∠	10	10	10	10		10	10			10
Inscribed ∠	10	10	10	10		10	10			10
Corresponding ∠'s	10	10		10		10	10	10		
Fluid Ounce	10	10	10			10	10	10	10	
Cup	10	10	10			10	10	10	10	
Similar △'s (Ratio, Proportion)	10	10	10		10	10	10	10		
Scale Drawing			10	10	10	10		10		
Pythagorean	10	10	10			10	10	10		

Page

Micrometer

The student should be able to:

1. Name and identify the micrometer M-22
2. Demonstrate how to measure an object using the micrometer M-22

Caliper

The student should be able to:

1. Name and identify the caliper
2. Demonstrate how to measure an object by using the caliper

Volume

The student should be able to:

1. Name and identify the volume of a solid object as a number
2. Demonstrate how to measure the volume by using various solids
3. State the principle that the volume of an object is the number of congruent solids that can be placed in the object

Cubic Inch, Cubic Foot, Cubic Yard

The student should be able to:

1. Name and identify the following cubic units of measure: cubic inch, cubic foot, cubic yard
2. Construct drawings of models of the cubic units of measure
3. Describe each unit as a cube having a given measure, inch, foot, yard, as each of its three dimensions or in terms of surroundings

Volume of Cube, Box, Cylinder

The student should be able to:

1. Name and identify the volume of a cube, box, and cylinder

2. Demonstrate how to measure the volume by using the standard cubic units

3. State the principles:

   a. The volume of a cube $= s^3$

   b. The volume of a box $= l \cdot w \cdot h$

   c. The volume of a cylinder $= \pi r^2 h$

4. Apply the principles by finding the volumes of various objects when given their dimensions

5. Interpret the principles by solving practical problems

Central and Inscribed Angles

The student should be able to:

1. Name and identify central and inscribed angles

2. Demonstrate how to measure a central angle and an inscribed angle by using a protractor

3. Construct drawings of inscribed and central angles using a protractor

4. State the principles:

   a. The measure of an arc is equal to the measure of its central angle

   b. The measure of an inscribed angle is one-half the number of degrees in the arc it cuts off

5. Apply the principle by solving related problems

6. Distinguish between central and inscribed angles

Corresponding Angles

The student should be able to:

1. Name and identify corresponding angles

2. Construct a drawing of corresponding angles

3. State the principle that if two parallel lines are cut by a third, then the corresponding angles formed are equal

Page

4. Apply the principle to determine the measure of one corresponding angle given the measure of the other

5. Interpret the principle by determining the measures of the other seven angles when given parallel lines cut by a transversal and the measure of one angle

Fluid Ounce and Cup

The student should be able to:

1. Name and identify the fluid ounce and the cup

2. Demonstrate a fluid ounce by pouring liquids

3. State the principle that there are eight fluid ounces in a cup

4. Apply the principle by converting cups to ounces and ounces to cups

5. Interpret the principle by solving applied problems

6. Order ounces and cups in relation to pints, quarts, gallons

Similar Triangles

The student should be able to:

1. Name and identify similar triangles

2. Demonstrate how to test whether two triangles are similar by measuring the angles and sides of triangles

3. Describe similar triangles as triangles having the same shape

4. State the principles:

   a. The measure of corresponding angles of similar triangles are equal

   b. The measure of corresponding sides of similar triangles are proportional

5. Apply the principles by determining whether or not triangles are similar given their angles measures or sides

6. Interpret the principle to solve applied problems

Scale Drawing

The student should be able to:

		Page
1.	Demonstrate that objects drawn to scale are similar in shape	
2.	Construct scale drawings to enlarge and reduce a given drawing	M-11
3.	Describe scale drawings as a drawing which has the same shape as the object represented	M-11
4.	State the principle that all scale drawings are similar to the object represented	
5.	Interpret the principle by determining the size of an object given the scale of a drawing or given the object and scale determine the size of the object in a drawing	M-11

Pythagorean Theorem

The student should be able to:

1.	Name and identify Pythagorean Theorem	M-20
2.	Demonstrate the Pythagorean Theorem by showing that the area obtained by squaring the hypotenuse is equal to the sum of the areas obtained by squaring the other two sides	M-20
3.	State the principle: $a^2 + b^2 = c^2$ for every right triangle where c represents the hypotenuse	M-20
4.	Apply the principle by finding the missing sides of right triangles or given the sides determine if the angle is a right triangle	M-20
5.	Interpret the principle to solve applied problems	M-20

SCALE DRAWING

Teacher Commentary

I. Unit: Measurement, Graphing

II. Objectives: The student should be able to:

A. Describe a scale drawing as a drawing which has the same shape as the object represented.

B. Construct scale drawings to enlarge or reduce a given drawing.

C. Interpret the principle by determining the size of an object given the scale of a drawing or given the object and scale, determine the size of the drawing of the object.

III. Materials:

A. Overhead or opaque projector and transparencies

B. Graph paper

C. Work sheets

IV. Procedure:

A. Draw a picture on the board and have students sketch it. The desired result would be for the pictures to differ from the original drawing. A comparison of students' results would indicate the need for a method to make a more accurate copy of a drawing.

B. Using an overhead projector, project a transparency of the simple drawing on a screen with a grid overlay superimposed on it. Have another grid prepared on the chalkboard approximately the same size as the projected image. Referring to the projected image, use the chalkboard grid to teach the techniques of copying. Be sure to refer to "Key points" in the squares in developing the techniques. Scale will be introduced after students are proficient in making drawings.

C. Discuss the different uses of scale drawings. Students may mention blueprints, road maps, and reproductions of drawings in textbooks.

D. Have the students make a copy of the drawings of a ship on the work sheet, "Scale Drawings."

E. Additional exercises in copying a figure may be given. Procedure for copying a drawing is as follows:

1. On a work sheet, have a simple drawing.

2. Show the same drawing on a work sheet with a grid over it.

3. Distribute a grid the same size.

4. The teacher should start the drawing so that all students start on the same grid coordinates. Have students complete the drawing.

5. On future copies, have pupils perpare their own grids.

6. Introduce scale at this point. Since a $\frac{1}{4}$" grid was used with the ship drawing and the same size grid used in the copy, the scale is $\frac{1}{4}" = \frac{1}{4}"$ or a ratio of 1 to 1.

F. Ask the students how they might enlarge a copy of a drawing. Using the overhead projector, show pupils a drawing which has been enlarged. Use ratios such as 1:2, 1:3, 1:4. A sample work sheet dealing with enlargement of a drawing is the one entitled, "Swan."

G. Have pupils enlarge a drawing using the following procedure:

1. Use work sheet entitled, "The Leaf."

2. Have pupils make the enlargement to a scale of 1:2.

H. Ask the students, "How could we make the drawing smaller than the original one?" Show pupils some drawings that have been reduced. These reductions are in ratios of 2:1 and 3:1. See the work sheet entitled, "The Elephant."

I. Have students reduce a drawing. The procedure is indicated on the scale drawing of the fish.

1. Place the drawing on the board.

2. Place a one foot square grid over the drawing.

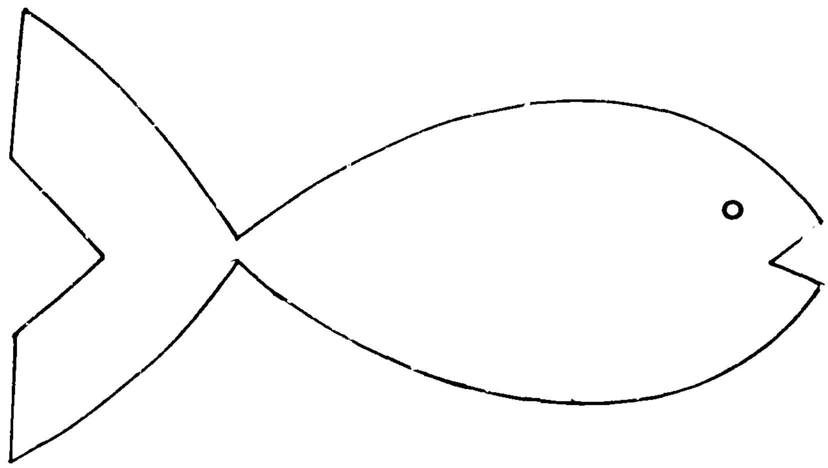
3. Have pupils draw a $\frac{1}{2}$ inch square grid on their papers.

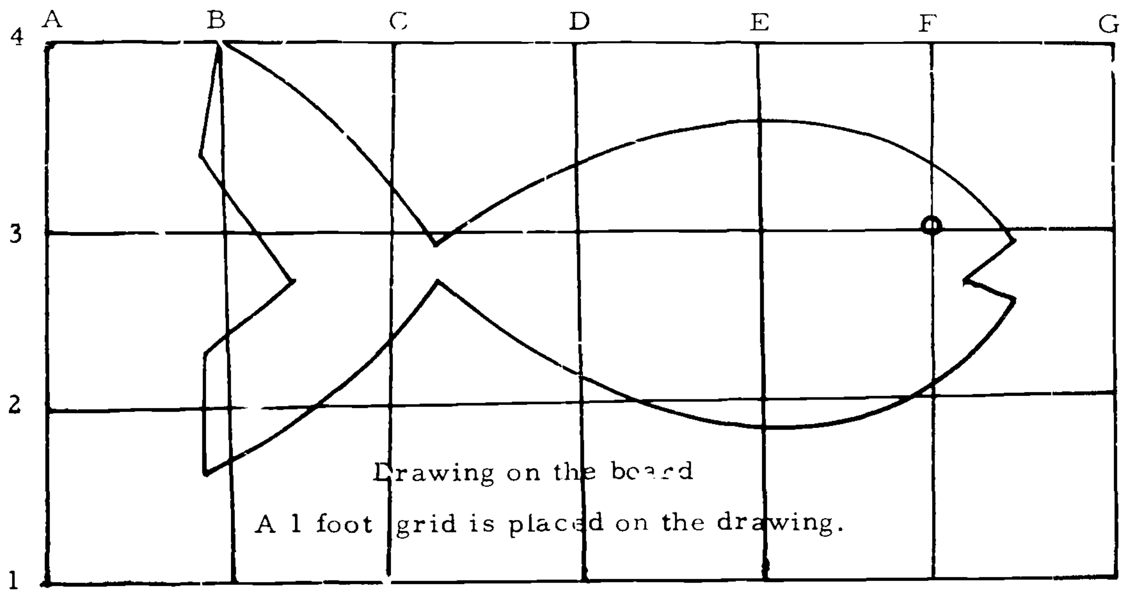
4. Students should then copy the drawing using the techniques presented earlier.

5. Emphasize that since $\frac{1}{2}$ inch squares on the paper represent one foot squares on the board, the scale is $\frac{1}{2}$ " on the copy = 12 inches on the original. We could write this 1" = 24".

6. Give more practice in reducing scale drawings and in determining scales.

ILLUSTRATION FOR SCALE DRAWINGS

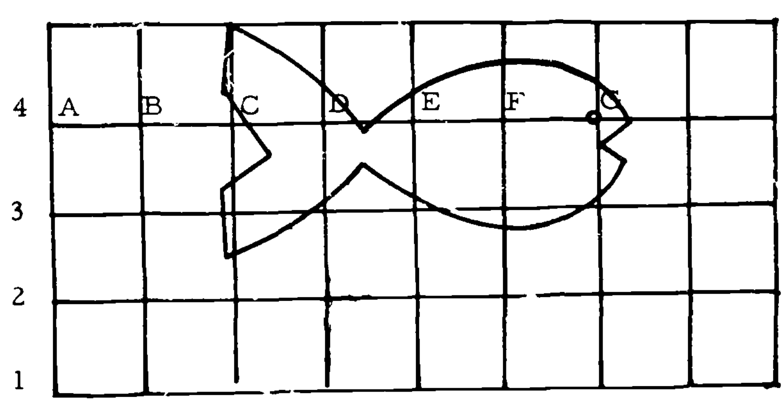




Drawing on the board

A 1 foot grid is placed on the drawing.

pupil has drawn a $\frac{1}{2}$ inch grid to reduce the given drawing



J. After reviewing the procedure for graphing points in a plane, give students a set of coordinates which will produce some pictures. Following this, give students a set of coordinates in which all of the original coordinates have been doubled. A variety of these may be used involving various ratios. Use work sheets entitled, "Manny the Martian" and "Wanda the Witch."

K. Have students start with an initial set of coordinates, determine their own multiples, and construct the new coordinates and drawings from them.

L. Using copies of road maps that may be obtained from any gasoline station, have students compute distances from the vicinity of the school to other places, or between any two points on the map. Try to get maps with different scales, Baltimore, Maryland, Pennsylvania, Eastern United States.

M. Assessment: Have students circle the correct letter.

1. If a scale drawing is made of an object, then the drawing and the object will always have the same:

   a. size

   b. shape

   c. shape and size

2. If the scale drawing is an enlargement in the ratio of 6:1 and the drawing is 18 inches long, then the object's length is:

   a. 24 inches

   b. 108 inches

   c. 12 inches

   d. 3 inches

3. If an object is 24 feet long and the scale is 2 inches = 1 foot, then the length of the drawing will be:

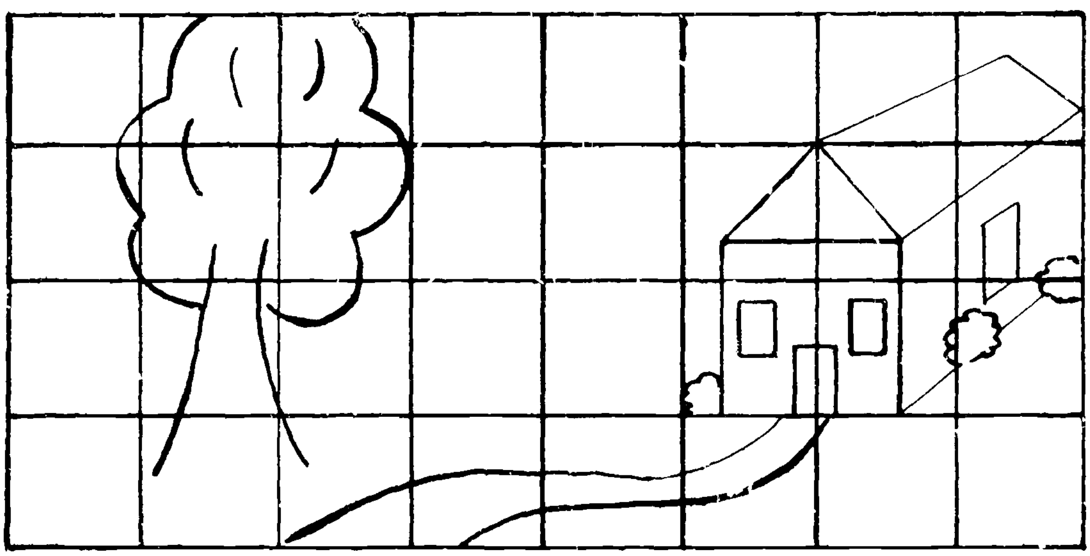
   a. 22 inches

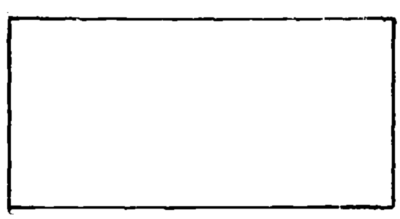
   b. 12 inches

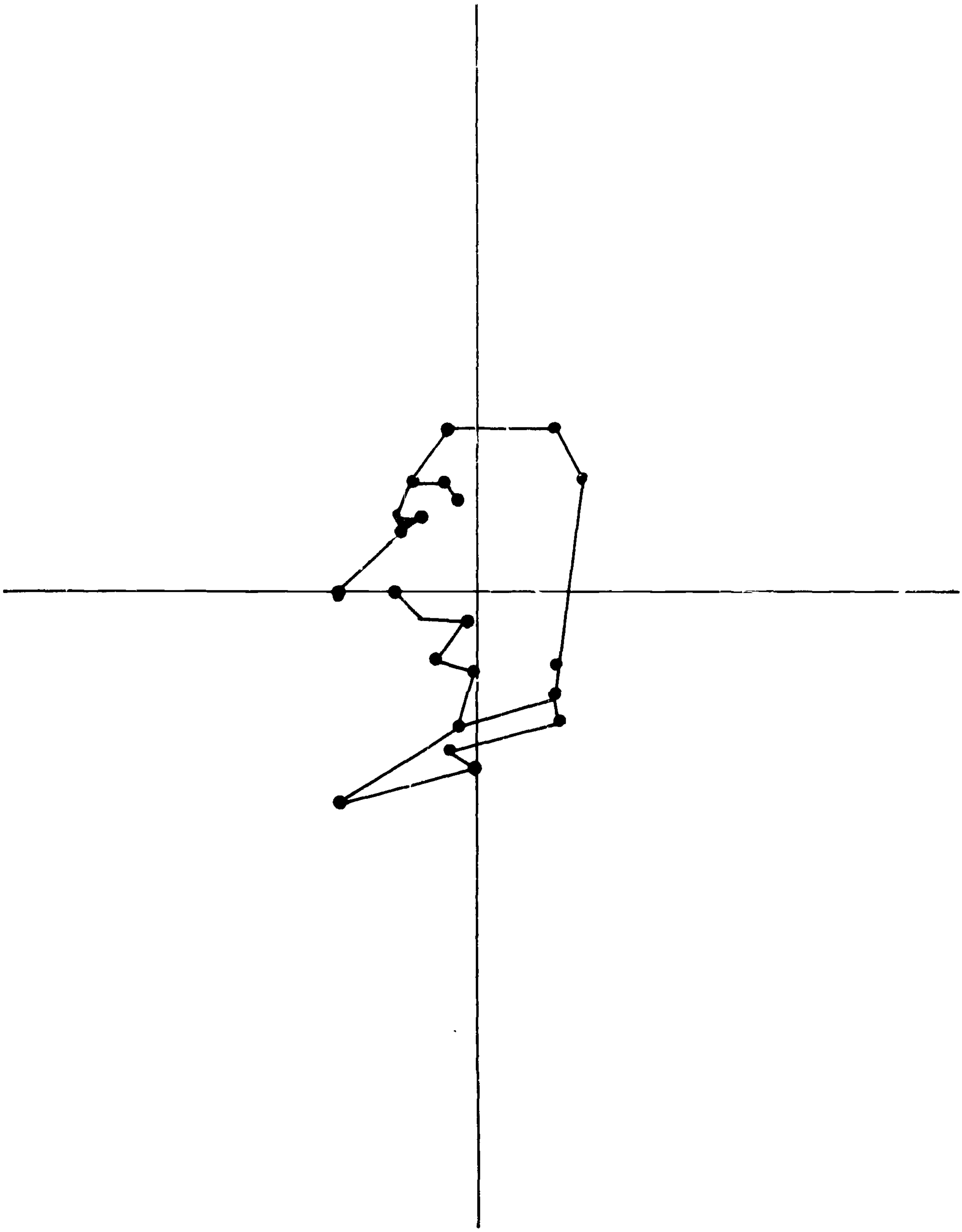
   c. 48 inches

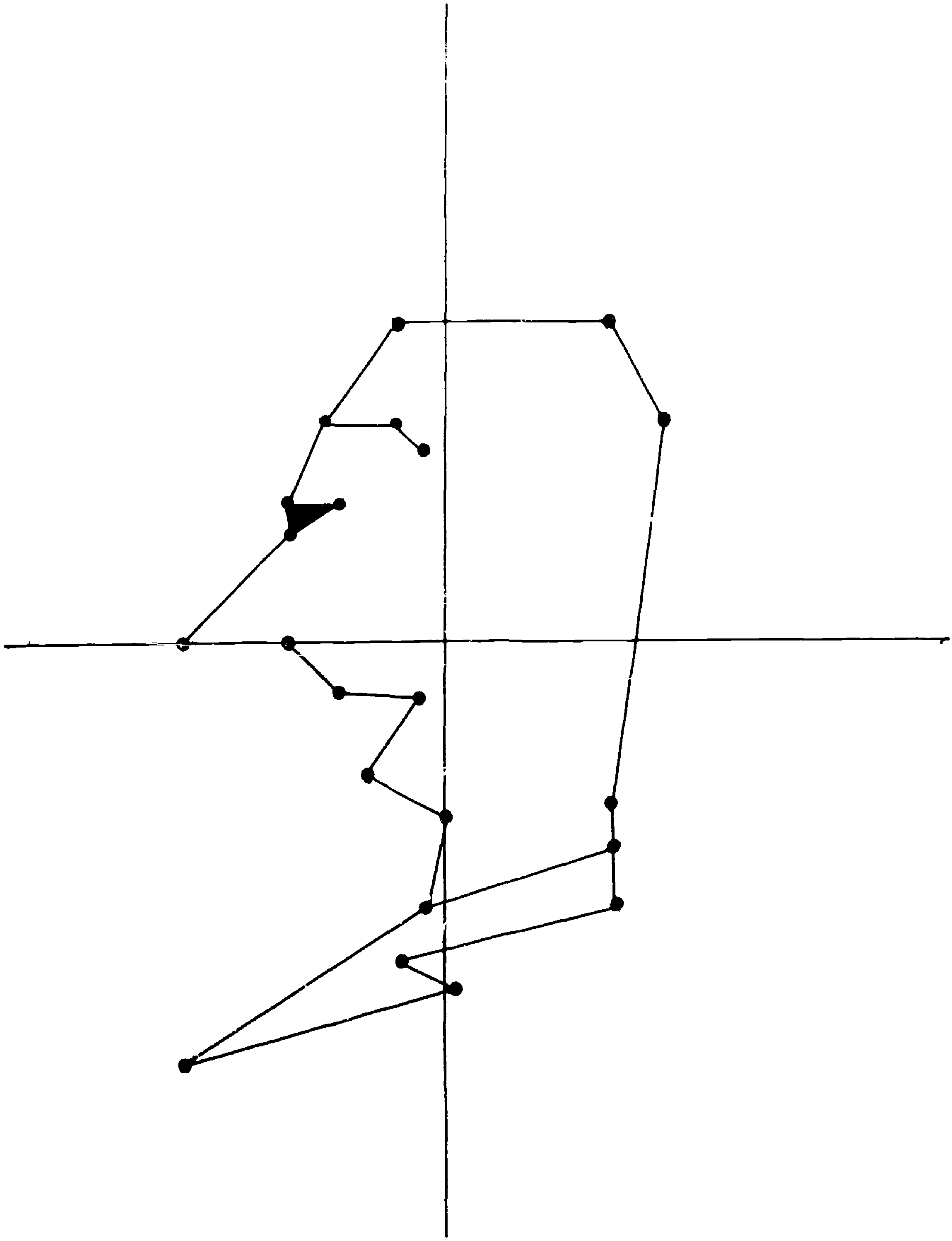
   d. 40 inches

4. Given the following drawing, reduce it in the ratio of 1:3 by constructing a grid. Use the given rectangle in which to construct your grid.

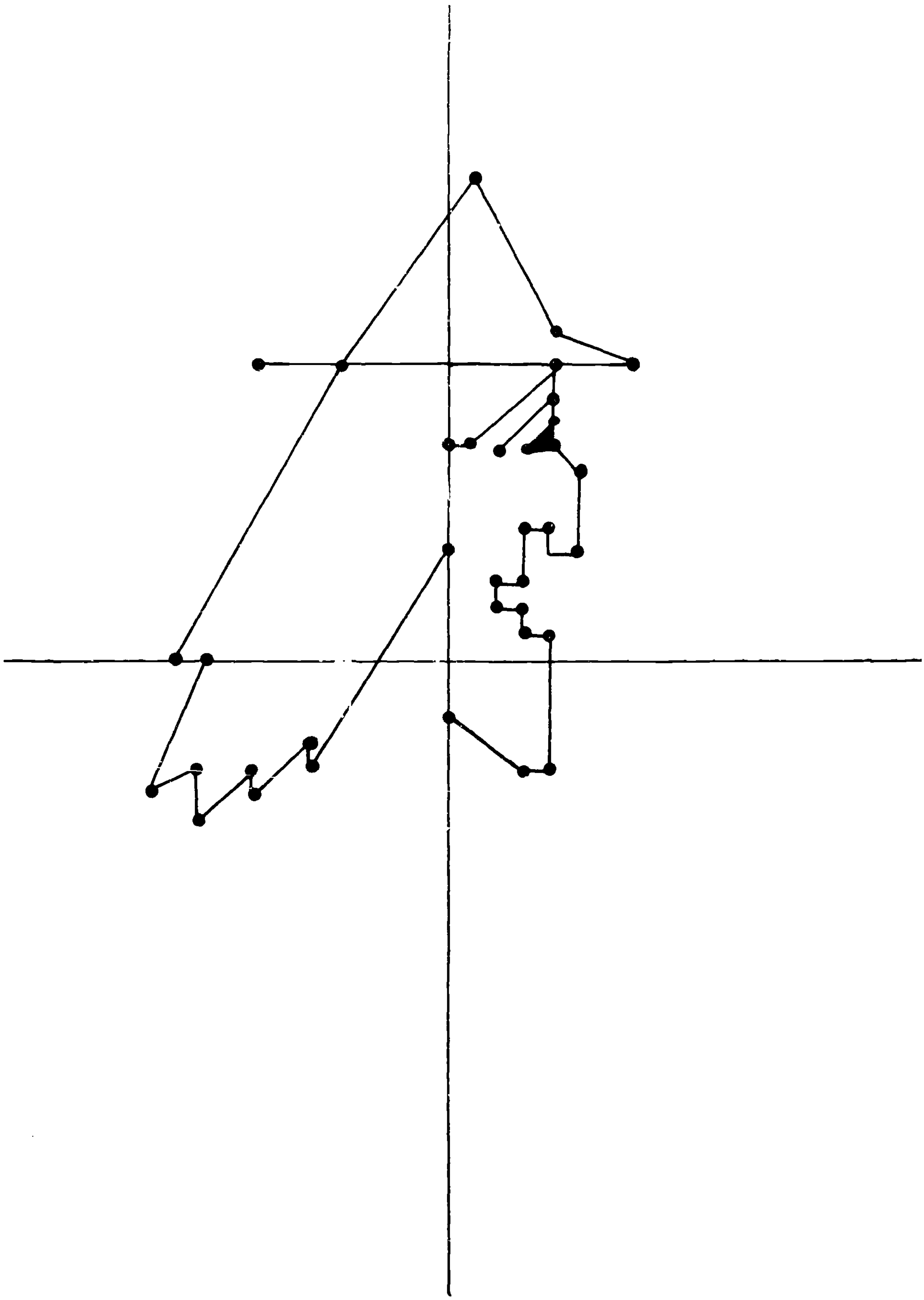




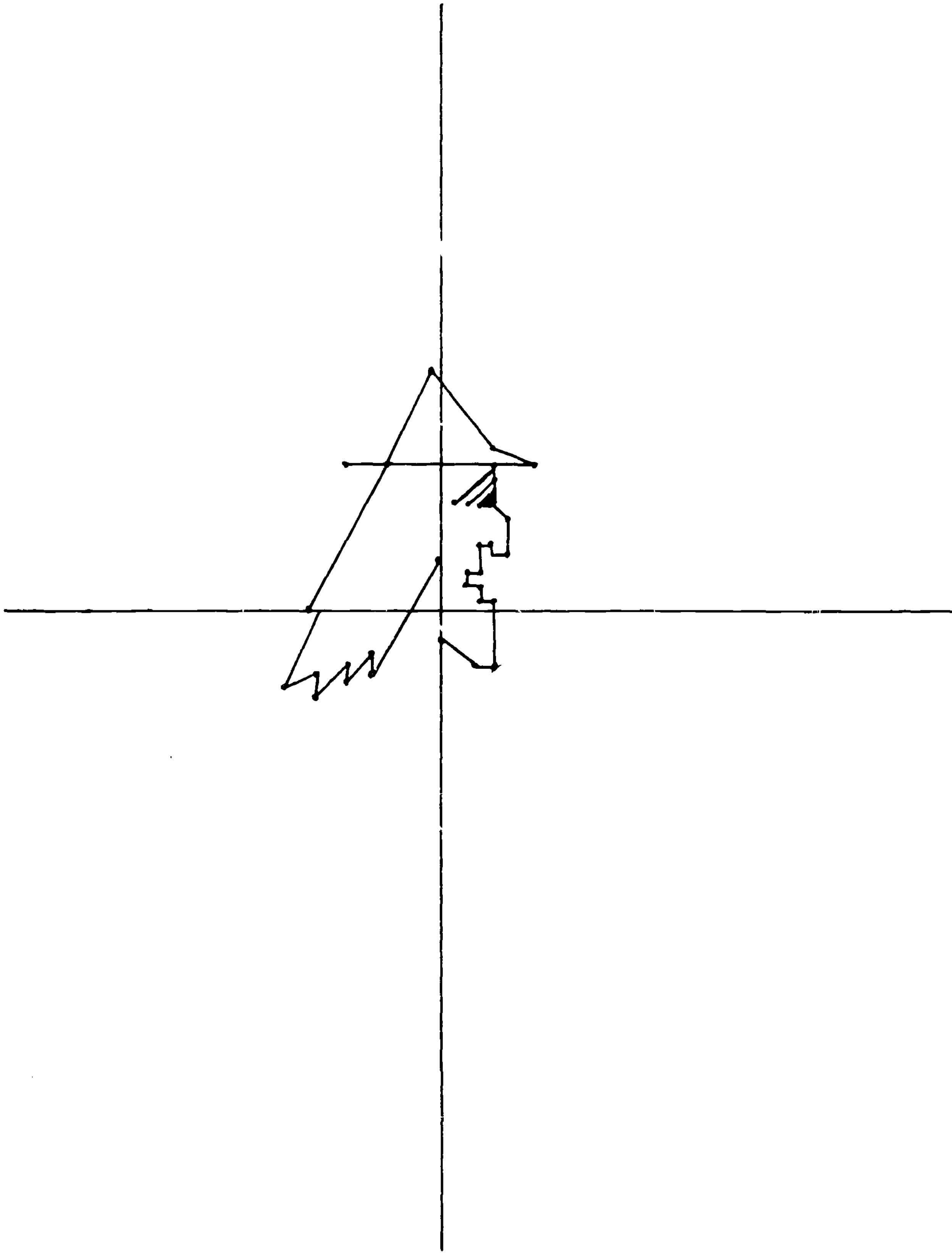




M-17



M-18



M-19

MANNY THE MARTIAN - PART I

Directions: Plot the following points and connect them.

$(-1, +6)$

$(+3, +6)$

$(+4, +4)$

$(+3, -3)$

$(+3, -5)$

$(-1, -6)$

$(0, -6\frac{1}{2})$

$(-5, -8)$

$(-\frac{1}{2}, -5)$

$(0, -3)$

$(-1\frac{1}{2}, -2\frac{1}{2})$

$(-\frac{1}{2}, -1)$

$(-2, -1)$

$(-3, 0)$

$(-5, 0)$

$(-3, +2)$

$(-2, +2\frac{1}{2})$

$(-3, +2\frac{1}{2})$

$(-2\frac{1}{2}, +4)$

Draw a line segment between $(-3, +2)$ and $(-3, +2\frac{1}{2})$.

Color in the enclosed area.

Plot points:

$(-1, +4)$

$(-\frac{1}{2}, +3\frac{1}{2})$

Plot point $(+3, -4)$

Draw a line segment between $(-\frac{1}{2}, -5)$ and $(+3, -4)$.

MANNY THE MARTIAN - PART II

Directions: Plot the following points and connect them.

(-2, +12)

(+6, +12)

(+8, +8)

(+6, -6)

(+6, -10)

(-2, -12)

(0, -13)

(-10, -16)

(-1, -10)

(0, -6)

(-3, -5)

(-1, -2)

(-4, -2)

(-6, 0)

(-10, 0)

(-6, +4)

(-4, +5)

(-6, +5)

(-5, +8)

Draw a line segment between (-6, +4) and (-6, +5).

Color in the enclosed area.

Plot points:

(-2, +8)

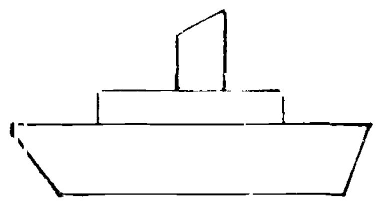
(-1, +7)

Plot point (+6, -8)

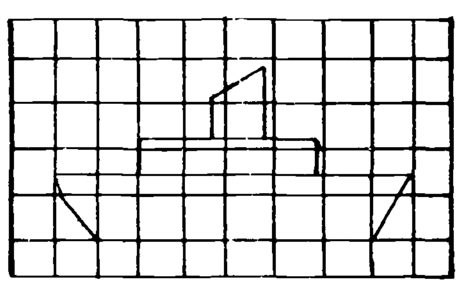
Draw a line segment between (-1, -10) and (+6, -8).

SCALE DRAWINGS

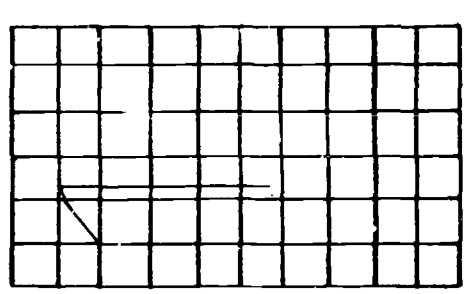
Here is a drawing. Can you copy it exactly?



You can, if you draw a grid like this!

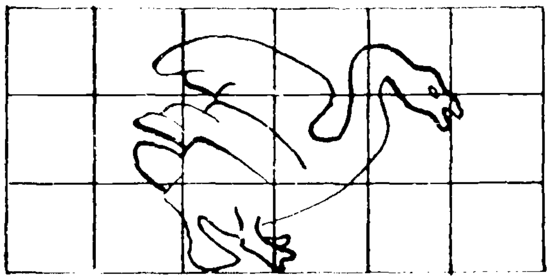


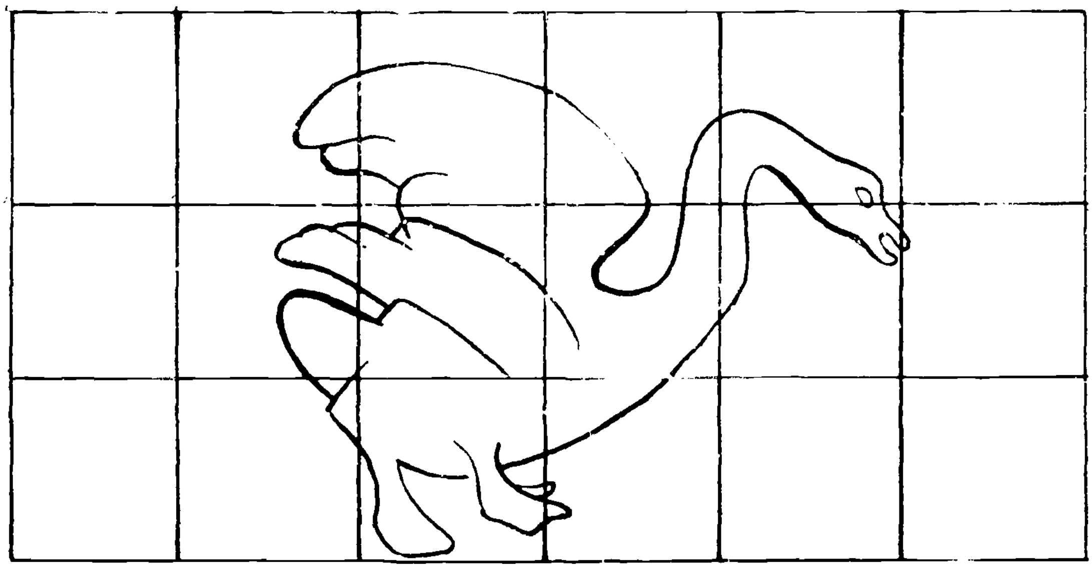
Now you should be able to copy the ship. Do this by drawing block by block on the grid below. The drawing has been started for you.



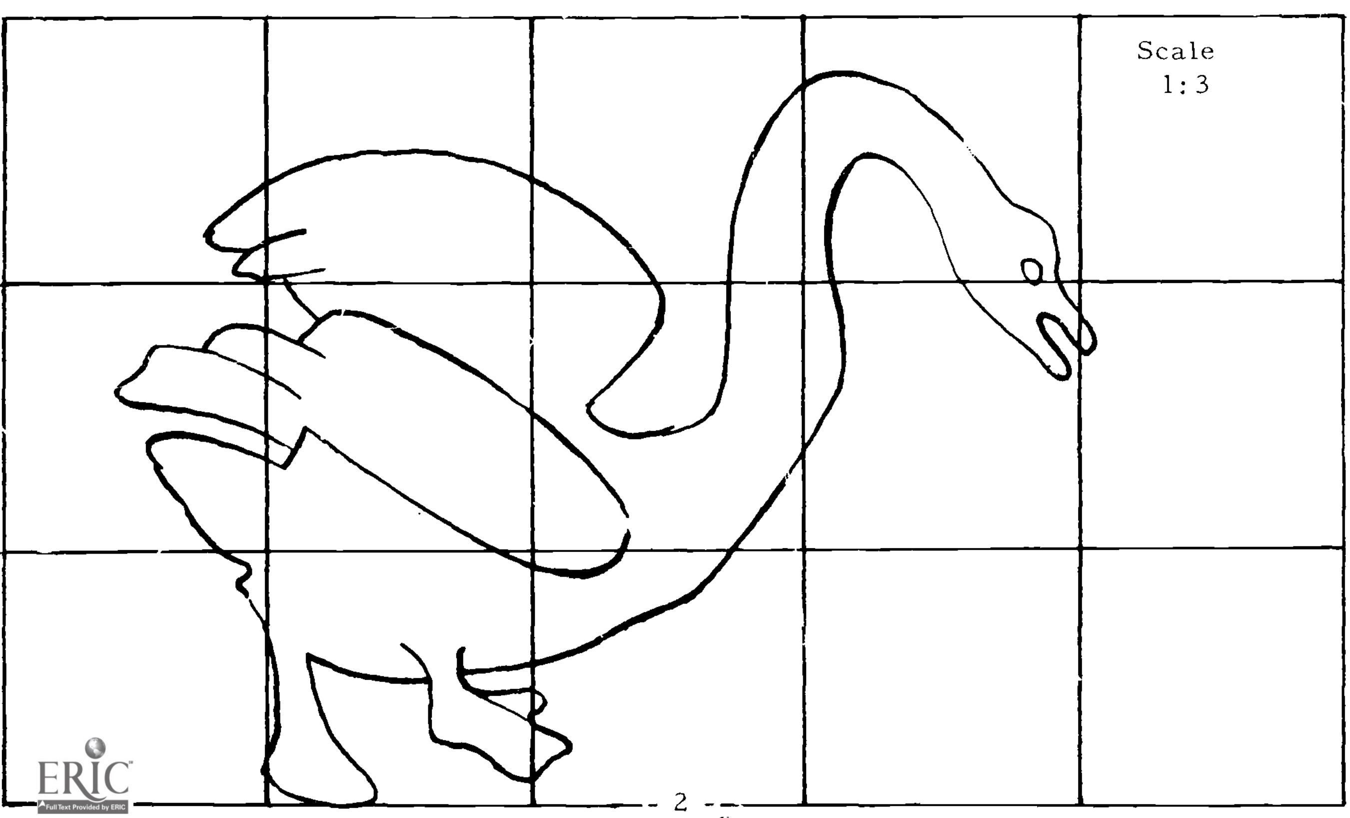
SWAN

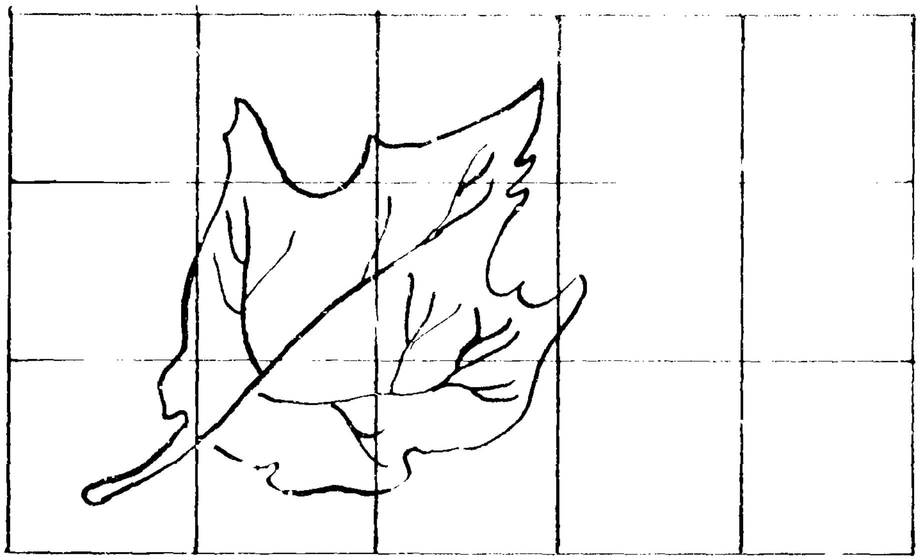
ENLARGING SCALE OF WINGS





Scale
1:2





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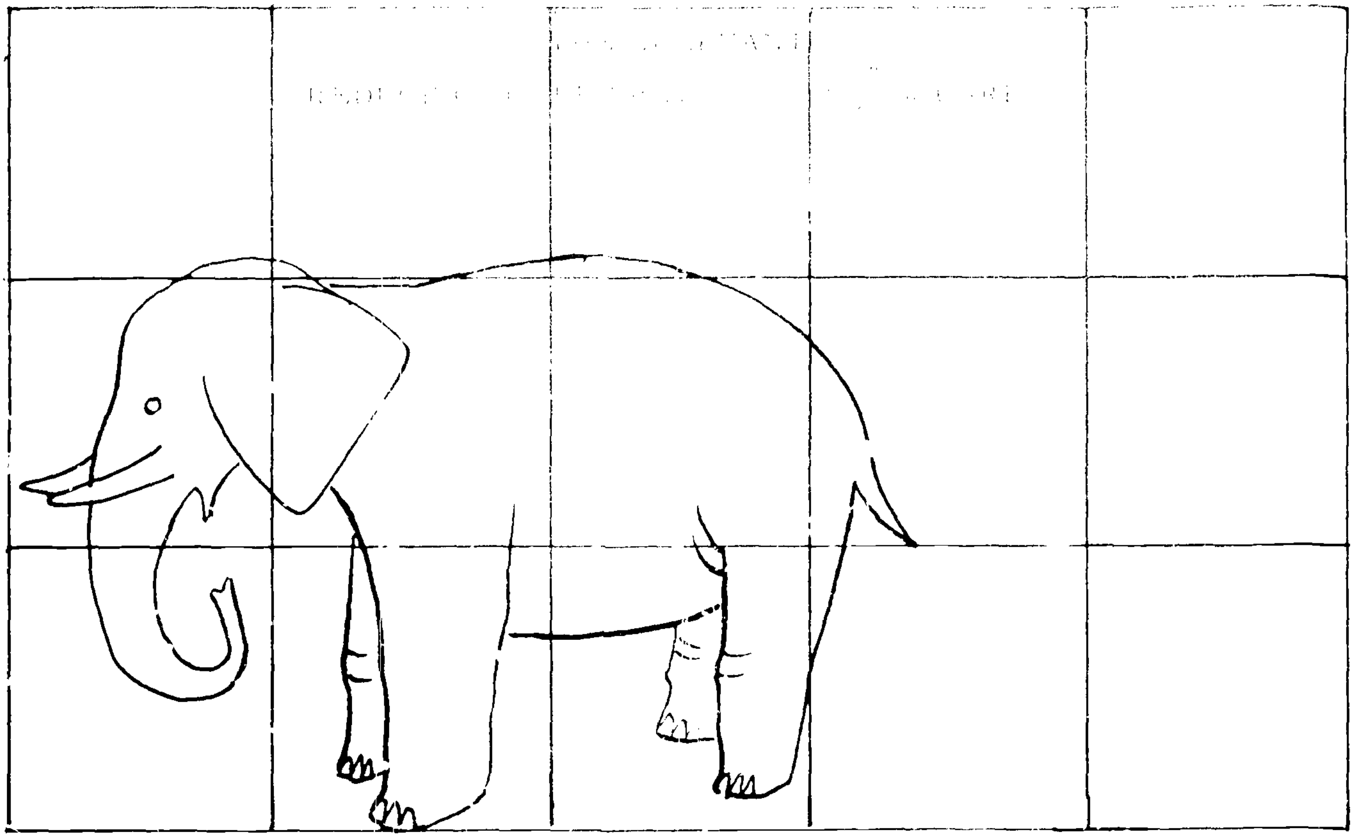
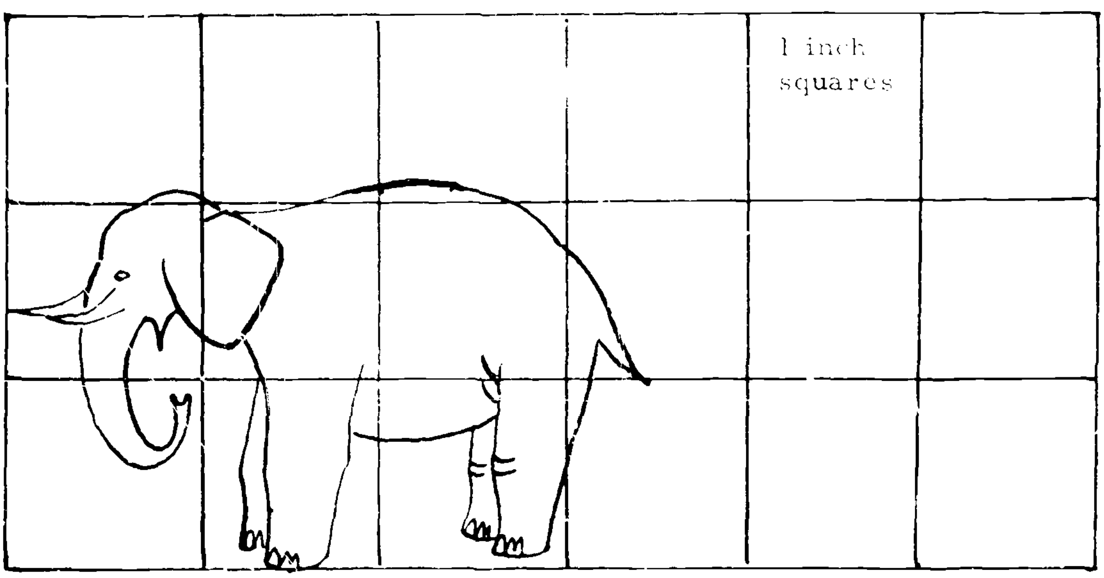
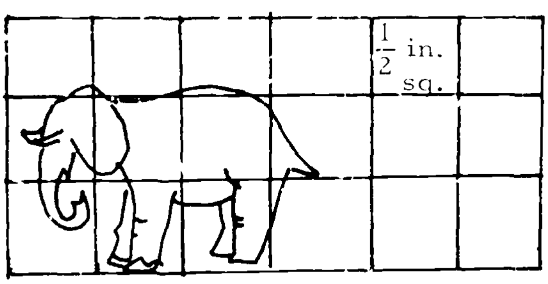



1 inch
squares


$\frac{1}{2}$ in.
sq.

WANDA THE WITCH - PART I

Directions: Plot the following points and connect them.

(0, +4)
(-5, -4)
(-5, -3)
(-7, -5)
(-7, -4)
(-9, -6)
(-9, -4)
(-11, -5)
(-9, 0)
(-10, 0)
(-4, +11)
(+1, +18)
(+4, +12)
(+7, +11)
(-7, +11)

Lift pencil

(0, +4)
(0, -3)
(+1, +8)
(+4, +11)
(+4, +8)
(+5, +7)
(+5, +4)
(+4, +4)
(+4, +5)
(+3, +5)
(+3, +3)
(+2, +3)
(+2, +2)
(+3, +2)
(+3, +1)
(+4, +1)
(+4, -4)
(+3, -4)
(0, -2)

Lift pencil

(+3, +8)
(+4, +9)

Construct a line segment between (+3, +8) and (+4, +9).

Color in the enclosed area.

(+4, +10)
(+2, +8)

WANDA THE WITCH – PART II

[illegible]: Plot the fol[illegible]ing points and [illegible] them.

[illegible] $(+3\frac{1}{2}, +3\frac{1}{2})$

[illegible] $(+2\frac{1}{2}, +2)$

[illegible]

THE PYTHAGOREAN THEOREM

Teacher Commentary

I. Unit: Measurement

II. Objectives: The student should be able to:

A. Name and identify the Pythagorean Theorem

B. Demonstrate the Pythagorean Theorem by showing the area obtained by squaring the hypotenuse is equal to the sum of the areas obtained by squaring the other two sides.

C. State the principle: $a^2 + b^2 = c^2$ for every right triangle where c represents the hypotenuse.

D. Apply the principle by finding the missing sides of right triangles or given the sides, determine if the triangle is a right triangle.

E. Interpret the principle to solve applied problems.

III. Materials:

A. Work sheet

B. Chalkboard

IV. Procedure:

A. Give out the attached work sheet.

B. Have the students construct a chart similar to the following and put the same chart on the chalkboard or on a transparency.

The Pythagorean Theorem

Triangles	Area of I	Area of II	Area of III
A	9	16	25
B	36	64	100
C	25	144	169
D	64	225	289
E	49	576	625
F	81	144	225

C. Describe the procedure for filling out the chart and do the first triangle with the students. Have students complete the chart. Answers from B to F should be supplied by the students.

D. Ask students if they see any relationship between the areas of I and II and the area of III for each triangle.

E. Students may see that the sum of the areas of I and II in each case is equal to the area of III.

F. The longest side in each case should be identified as the hypotenuse.

G. Give the hypotenuse the designation of side c and the other two sides a and b, the relationship established from the chart should be:

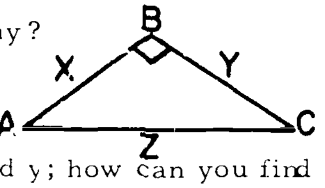
$$c^2 = a^2 + b^2$$

This should be identified as the Pythagorean Theorem.

H. Give the students practice in applying this principle to solve related problems. Have the students make use of tables of square root or use the Newtonian method of determining square root to solve these problems. Restrict the problems to those involving perfect squares.

I. Evaluation - Questions of the following types may be given.

1. Given a triangle similar to the one below:



   a. Is this a right triangle? (Yes) Why? ($\angle$ B shown as right angle)

   b. Identify the hypotenuse. (z)

   c. If you are given the lengths of x and y; how can you find the length of z? ($z^2 = x^2 + y^2$)

   d. If you are given the lengths of z and x; how can you find the length of y? ($y^2 = z^2 - x^2$)

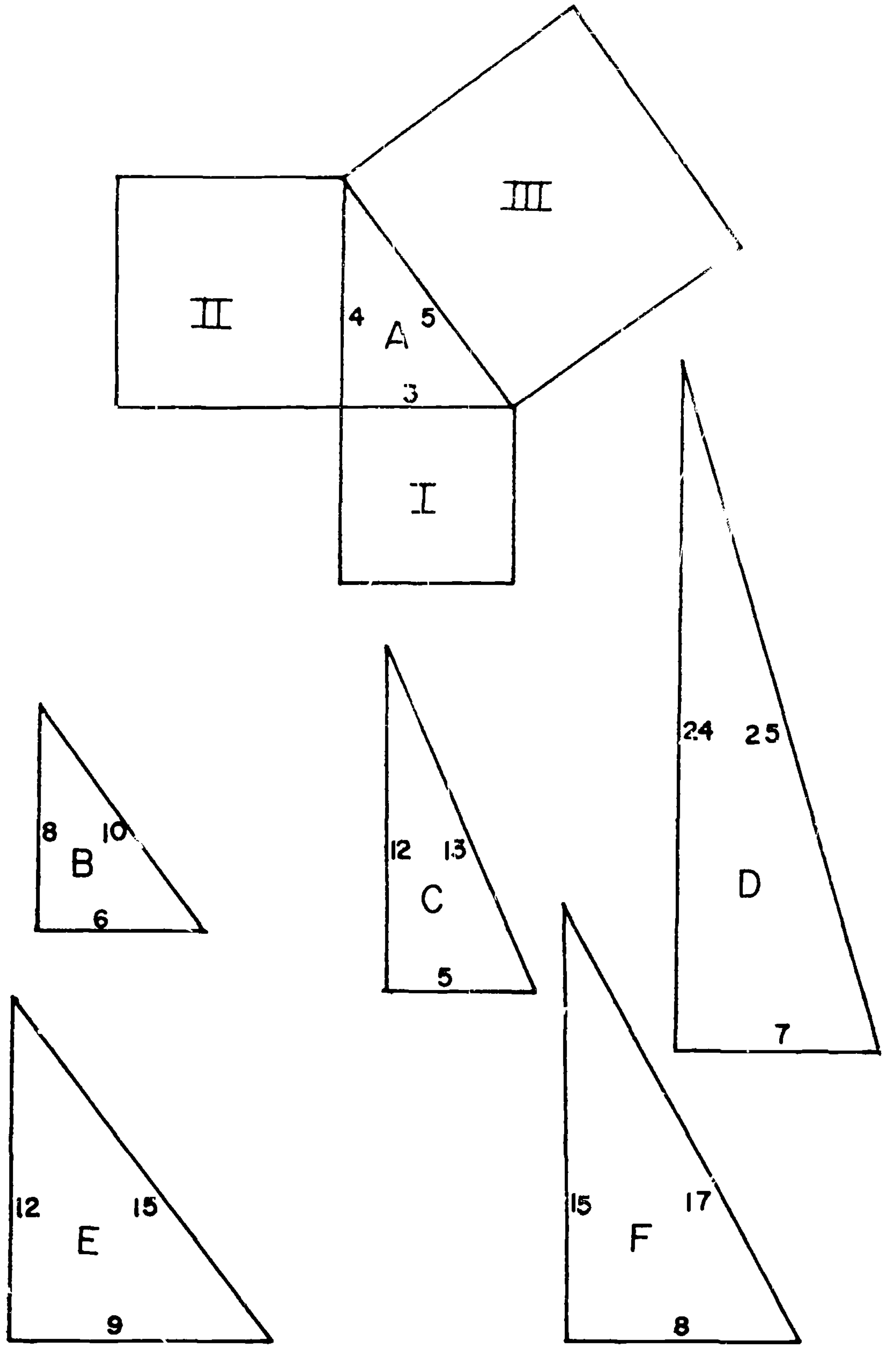
   e. What is the relationship $z^2 = x^2 + y^2$ called? (The Pythagorean Theorem)

2. The lengths of sides of right triangle that may be expressed as triplets of integers are:

a. 20 - 21 - 29	f. 13 - 84 - 85
b. 40 - 9 - 41	g. 16 - 63 - 65
c. 11 - 60 - 61	h. 48 - 55 - 73
d. 12 - 35 - 37	i. 39 - 80 - 89
e. 28 - 45 - 53	j. 36 - 77 - 85

Tables may be used to find the lengths of sides in the preceding cases. Problems may be constructed using these triplets to test student acquisition of the behaviors.

THE PYTHAGOREAN THEOREM



THE MICROMETER AND CALIPER

Teacher Commentary

I. Unit: Measurement

II. Objectives: The student should be able to:

A. Name and identify the micrometer and caliper

B. Demonstrate how to measure an object using the micrometer

III. Materials:

A. Micrometers

B. Calipers

C. Various objects to be measured

IV. Procedure:

A. The use of micrometers is not a subject that should be pursued in depth. There are, however, a number of interesting activities that may be performed as a class or in small groups which will give students practice in the use of micrometers and calipers. Emphasis should be placed on the micrometer's role as a precision instrument and the need for care in the use of micrometers. Micrometers with ratchet attachments are best for use in inexperienced hands since they make it possible to measure more accurately and reduce the probability of damage to the micrometers.

B. Demonstrate the use of the micrometer calipers in measuring a variety of small objects. Measure with calipers by using a ruler to determine the distance between the tips of the calipers.

C. Ask students to guess how high a stack of 1,000,000 \$1.00 bills would go if the bills were placed one on top of the other. By careful questioning, the use of the micrometer to measure the thickness of a single dollar bill may be identified as a method of approaching the problem. The answer, about .004 inches, is then multiplied by 1,000,000. (Answer $\approx$ 333 ft.)

D. Have students put the measures of several objects in ascending or descending order after determining measure by use of the micrometer or caliper.

E. Have students measure a hair and determine how many could be put side by side in a space of 1 inch. Related problems could also be given.

F. Bring feeler gauges in and check accuracy of the micrometers and the gauges by comparing the indicated thickness with the measured thickness.

G. Evaluation - Have students make a variety of measurements of given objects using the micrometers and calipers. Test them on manipulation of the micrometers and calipers as well as proficiency in reading the scale.

Since this activity, due to limitations on materials, must continue over a considerable period of time in small groups in order to give all students an opportunity to practice, evaluation by the teacher can be done on an individual basis when time is available.

GRAPHING

GRAPHING

UNIT GRAPHING GRADE(S) Seven thro' Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISH
Pictograph	7	7		7	7		7	7		
Bar Graph	7	7		7	7		7	7		
Line Graph	7	7		7	7		7	7		
Circle Graph	7	7			7		7	7		
Number Line	8	8		8	8	8	8			
Graphing Equalities	8	8		8		8				
Cartesian Products	8	8		8						
Graphing Ordered Pairs in First Quadrant	8	8	8	8	8					[illegible]
Coordinate Axes	9	9	9	9	9					[illegible]
Graphing Ordered Pairs	9	9	9	9	9	9	9			
Quadrants	9	9			9	9	9			
Table of Values	9	9	9	9	9			10		
Graphing Formulas	9	9		9	9			10		
Graphing Linear Equations and Inequations	10	10		10	10					[illegible]
Slope	10	10			10	10	10			
Graphing Simultaneous Equations	11	11		11	11	11	11			
Graphing Quadratic Equations and Inequations	11	11		11	11					[illegible]

UNIT GRAPHING GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Table of Values								10		
Graphing Formulas				10						
Graphing Linear Equations and Inequations	10	10		10	10					10
Slope	10	10			10	10	10			

Table of Values

The student should be able to:

1. Interpret the table of values by constructing a formula from it

Graphing Formulas

The student should be able to:

1. Construct a table of values from a graph
2. Construct a formula from a graph

Graphing Linear Equations and Inequalities

The student should be able to:

1. Name and identify the graph of linear equations and inequalities — GR-6
2. Construct graphs of linear equations and inequalities — GR-6
3. Describe the graph of:
   a. A linear equation as a line — GR-6
   b. An inequality as a half-plane
4. Distinguish between a graph of a linear equation and a graph of an inequality

Slope

The student should be able to:

1. Name and identify the slope of a line given a graph
2. Describe the slope of a line as the amount of slant
3. State the principles that:
   a. If the two variables increase or decrease in the same way, then the line is rising and the slope is positive

Page

b. If the two variables increase or decrease in opposite ways, then the line is falling and the slope is negative

c. Slope $= \dfrac{\text{Vertical Change}}{\text{Horizontal Change}} = \dfrac{\text{Change in y}}{\text{Change in x}}$

4. Apply the principles by:

   a. Determining the slope given a table of values

   b. Determining the slope given the graph of the line

   c. Describe the graph given the slope

ORDERED PAIRS UNDER INVESTIGATION

Teacher Commentary

I. Unit: Graphing

II. Objectives: The student should be able to:

A. Name and identify graph of linear equations

B. Describe the graph of a linear equation as a line

C. Construct a graph of a linear equation

III. Materials:

A. Work sheets

B. Ruler

C. Graph paper

IV. Procedure:

A. Distribute student work sheets entitled, "Ordered Pairs Under Investigation"

B. Have the students read the story.

C. Discuss the story to make sure that students understand the relationship between the two numbers. Then discuss the formula used.

D. Work sheet "Roger's Number Game" should be done with the class. Be careful not to move too quickly through these questions.

E. The students should complete the work sheet, "The Thermometer Ant - #1."

F. Assess student progress by having them complete, 'The Thermometer Ant - #2."

ORDERED PAIRS UNDER INVESTIGATION

John and Roger were playing a game with numbers. John said, "I am thinking of a number. If you add 2 to the number the sum is 9. What is the number?"

Roger guessed 7. Was he correct?

Roger then picked two numbers. He said, "I am thinking of two numbers less than 10. If you multiply the first number by 2 and add 1 to the sum, you will get the second number."

Roger was thinking of the numbers 2 and 5.

John guessed 4 and 9.

They soon decided that both answers were correct.

We can show that both sets of numbers work in the following manner. If we let f stand for the first number and s stand for the second number, then two times the first number plus one is equal to the second number. A short way of writing this would be:

$$2f + 1 = s$$

Roger was thinking of the numbers 2 and 5.

Since: $2f + 1 = s$

Then: $2(2) + 1 = 5$

John guessed the numbers 4 and 9.

Since: $2f + 1 = s$

Then: $2(4) + 1 = 9$

Therefore, both pairs of numbers fit Roger's clue.

John then decided that any time he knew the first number, he could find a second number that would fit Roger's clue. For example, if the first number is 5, we can find the second number by using this formula:

$$s = 2f + 1$$

$$s = 2(5) + 1$$

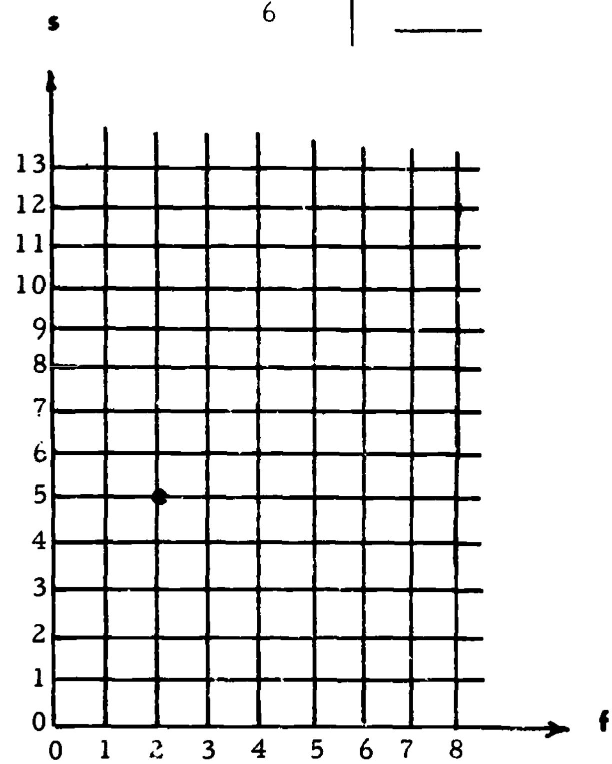
$$s = 11$$

ROGER'S NUMBER GAME

1. Use the formula $s = 2f + 1$ to complete the chart at the right.

f	s
1	____
$1\frac{1}{2}$	____
2	5
3	____
4	9
5	11
6	____

2. On the graph to the right, the order pair (2, 5) has been plotted. Plot the other points from the chart in exercise 1.



3. On the graph above, draw a line connecting the points (2, 5) and (6, 13). Look at the points between (2, 5) and (6, 13). Do these points lie on the line? ______________

4. Consider the ordered pair $(\frac{1}{2}, 2)$.

   a. Does the ordered pair make the sentence true? ________

   b. Plot the point named by the ordered pair.

   c. Does this point lie on the line? __________

5. Consider the ordered pair (1, 2).

   a. Does this ordered pair make the formula true? _______

   b. Plot the point named by this ordered pair.

   c. Does this point lie on the line? __________

6. Write the ordered pairs from the list below which make the formula true. ____

   $(1, 4)$, $(2, 5)$, $(2\frac{1}{2}, 6)$, $(4\frac{1}{2}, 8)$, $(7, 15)$

7. Write the ordered pairs from the list in question 6 which lie on the line of your graph. ____________________________

8. What do you notice about your answers to questions 6 and 7?

   ____

9. Complete the following statements to express your conclusions:

   a. If an ordered pair makes our formula true, then it names a point on the ________________.

   b. If an ordered pair names a point on the line, then it makes the formula ______________.

THE THERMOMETER ANT - #1

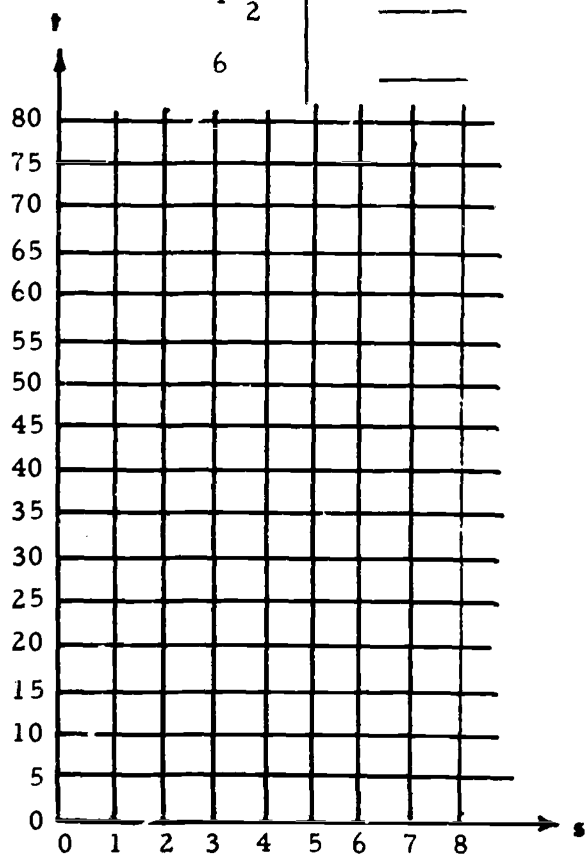
A certain type of ant, the Thermometer Ant, can be used to tell the temperature. If we know the speed of the ant, we can find the temperature by using the formula: $t = 11s + 9$, where t is the temperature and s is the speed in inches per minute.

If we know the speed is 1 inch per minute, what can you conclude about the temperature? ______________________________

1. Complete the table at the right, using the formula: $t = 11s + 9$

s	t
1	20
2	______
$2\frac{1}{2}$	______
3	______
$4\frac{1}{2}$	______
6	______

2. Draw the graph of the ordered pairs, (s, t) from the table in problem 1. Notice that a scale of 1 space = 5° has been used on the vertical axis.



3. Connect the points. Name the graph that you drew. ____________________

4. Choose any ordered pair which makes the formula true. Plot the point. Does this point lie on the line? ____________________

5. Can you find any ordered pair which makes the formula true and does not lie on the line? ____________________

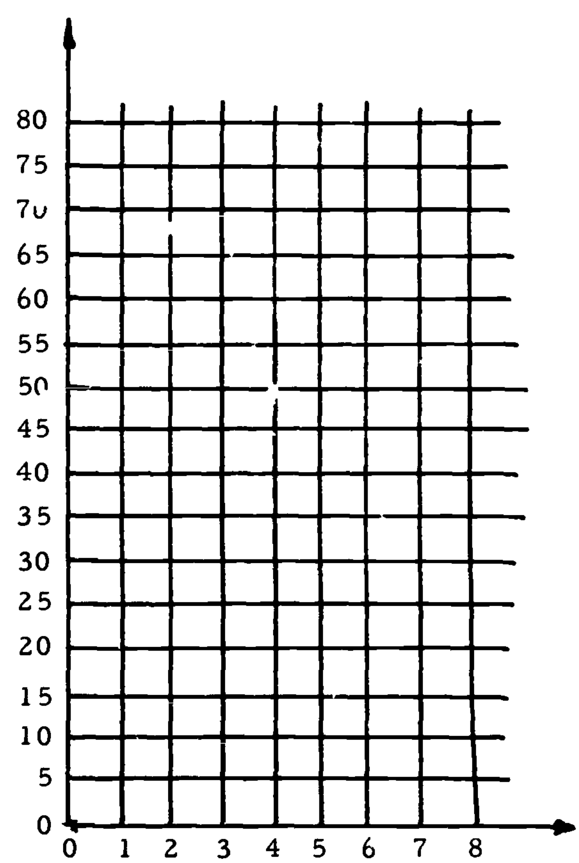
THE THERMOMETER ANT - #2

Suppose that the relationship between the speed of the Thermometer Ant and the temperature is given by the formula: $t = 12s - 6$

1. Complete the chart at the right using the formula.

s	t
5	____
4	42
$6\frac{1}{2}$	____
8	____
7	____

2. Graph the ordered pairs from the table above. Connect the points.



3. Name the graph of the formula $t = 12s - 6$.

4. Circle the ordered pairs from the list below which name points on the graph of $t = 12s - 6$.

   a. (10, 114)
   b. (2, 20)
   c. (3, 30)
   d. $(5\frac{1}{2}, 60)$
   e. $(9\frac{1}{2}, 106)$
   f. (12, 140)

5. Below are two more formulas. Complete the table of values and graph the ordered pairs on your graph paper.

   a. $a = 9 - b$

b	a
1	____
2	____
$3\frac{1}{2}$	____
4	____
6	____

   b. $y = 3x - 5$

x	y
2	____
4	____
$5\frac{1}{2}$	____
6	____

PROBABILITY AND STATISTICS

PROBABILITY AND STATISTICS

I. Master Chart - Grades Six through Eleven

II. Grade Ten Chart

III. Behavioral Objectives

IV. Activities

UNIT PROBABILITY GRADE(S) Eight through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Event	8	8	8							
Certainty - Uncertainty	8	8		8	8					
Sample Spaces - Ordered Arrangements	8	8		8	8					8
Equally and Unequally Likely Outcomes	8	8	8			8	8			8
Probability	8	8				10				
Ordered Arrangements (Permutations)	9	9		9	9	9				
Factorial	9	9		9	9		9			
Permutations of N things taken N at a time	9	9		9		9	9			
Permutations of N things taken R at a time	9	9		9		9	9			9
Tree Diagram	9	9		9	9		9			
Box Diagram	9	9		9	9		9			
Decreasing and Increasing Probability	10	10		10		10	10			
Probability of 0 and 1	10	10		10		10	10			
Independent and Dependent Events	10	10		10	10	10	10	10		10
Complementary Events	10	10		10		10	10			
Experimental Probability	10	10	10		10					
Theoretical Probability	10	10			10					10
Sample	11	11	11		11		11	11		11

UNIT PROBABILITY GRADE(S) Eight through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Circular Permutation	11	11		11	11	11	11			
Combination	11	11		11	11					11
Combinations of N things taken N at a time	11	11		11		11				
Combinations of N things taken R at a time	11	11		11	11	11	11			
Preduction	11	11		11	11	11	11			
Chances (Odds)	11	11		11		11	11			11

UNIT STATISTICS GRADE(S) Nine through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Data	9				9				9	
Range	9	9	9		9	9	9			9
Rank	9	9	9		9		9	9		9
Interval	9	9	9			9	9			9
Tally	9	9	9		9		9			
Frequency	9	9	9	9	9					9
Frequency Table	9	9		9	9	9		9		9
Mean	9	9				9	9	9		9
Median	9	9				9	9	9		9
Mode	9	9				9	9	9		9
Pictograph	9	9						9		
Circle Graph	9	9		9				9		
Bar Graph	9	9		9				9		
Line Graph	9	9		9				9		
Normal Curve	10	10		10	10			10		
Pascal's Triangle	10	10	10	10			10	10		
Percentile	10	10		10	10	10	10	10		10
Correlation	11	11			11		11	11		
Scattergram	11	11		11	11	11	11			11

UNIT PROBABILITY GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Probability						10				
Decreasing and Increasing Probability	10	10		10		10	10			
Probability of 0 and 1	10	10		10		10	10			
Independent and Dependent Events	10	10		10	10	10	10	10		10
Complementary Events	10	10		10		10	10			
Experimental Probability	10	10	10		10					
Theoretical Probability	10	10			10					10

UNIT STATISTICS GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Normal Curve	10	10		10	10			10		
Pascal's Triangle	10	10	10	10			10	10		
Percentile	10	10		10	10	10	10	10		10

PROBABILITY - Grade 10

	Page
Probability	
The student should be able to:	
1. State the principle: $\text{Probability} = \dfrac{\text{Total numbers of successes or failures}}{\text{Total number of possible outcomes}}$	P-16 P-20
Decreasing and Increasing Probability	
The student should be able to:	
1. Name and identify decreasing and increasing probability	
2. Construct examples of decreasing and increasing probability	
3. State the principle that the possibility of decreasing and increasing probability varies as the number of possible outcomes vary	
4. Apply the principle by calculating the probability of outcomes through experimentation	
Probability of Zero and One	
The student should be able to:	
1. Name and identify zero and one as probabilities	P-16 P-20
2. Construct examples of events which have probabilities of zero or one	P-16
3. State the principles:	
a. If an event has a probability of 1, it must occur	P-16 P-20
b. If an event has a probability of zero, it cannot occur	
4. Apply the principles by determining a probability of zero or one from given information	
Independent and Dependent Events	
The student should be able to:	
1. Name and identify independent and dependent events	

2. Construct examples of dependent and independent events

3. Describe:

   a. Dependent events as events whose outcomes are determined by previous events

   b. Independent events as events whose outcomes are not affected by previous events

4. State the principles that:

   a. The probability of dependent events changes as events occur

   b. The probability of independent events does not change as events occur

5. Apply the principles by constructing the probability of different events

6. Interpret the principles by solving related problems

7. Distinguish between independent and dependent events

Complementary Events

The student should be able to:

1. Name and identify complementary events

2. Construct examples of complementary events

3. State the principle that the probability of the success plus the probability of its complement must equal one

4. Apply the principle by finding the probability of complementary events when the probability of successes is known

Experimental Probability

The student should be able to:

1. Name and identify experiment probability

2. Demonstrate how to experimentally determine the probability of an event whose outcomes are not equally likely and whose outcomes are equally likely P-16

3. Describe experimental probability as the probability of outcomes which occurs as a result of direct experimentation

Theoretical Probability

Page

The student should be able to:

1. Name and identify theoretical probability
2. Describe theoretical probability in terms of that which should occur if the event occurs infinite number of times
3. Distinguish between theoretical probability and experimental probability

STATISTICS - Grade 10

Normal Curve	**Page**
The student should be able to:	
1. Name and identify the normal curve	P-21
2. Construct the normal curve by a freehand sketch	P-21
3. Describe the normal curve in terms of its appearance	
4. Interpret the normal curve in the following ways:	
a. It pertains to an infinite population	
b. The curve is symmetrical	
c. The mean, median, and mode coincide	
Pascal's Triangle	
The student should be able to:	
1. Name and identify Pascal's Triangle	P-12
2. Demonstrate how to construct Pascal's Triangle	P-12
3. Construct Pascal's Triangle	P-12
4. Apply the principle of Pascal's Triangle to solve binomial probability problems	
5. Interpret Pascal's Triangle by discovering various number patterns and by showing its relationship to the normal curve	
Percentile	
The student should be able to:	
1. Name and identify percentile	
2. Construct percentile ranks for a given set of data	
3. Describe percentile as the position of a score in terms of the percentage of scores below that score	
4. State the principle that percentile rank = $\frac{\text{Number of scores below given score}}{\text{Total number of scores}} \times 100$	
5. Apply the principle by solving related problems	

6. Interpret the principle by finding the rank, score, or percentile rank when given one of these from a list of data

7. Distinguish between percentile and percent

PASCAL'S TRIANGLE

Teacher Commentary

I. Unit: Statistics

II. Objectives: The student should be able to:

A. Name and identify Pascal's Triangle

B. Demonstrate how to construct Pascal's Triangle

C. Construct Pascal's Triangle

III. Materials:

A. Student work sheets entitled "Pascal's Triangle - Numbers One, Two, and Three," "Pascal's Triangle - Totals," and "Three Questions About Pascal's Triangle"

B. Coins (6 per team)

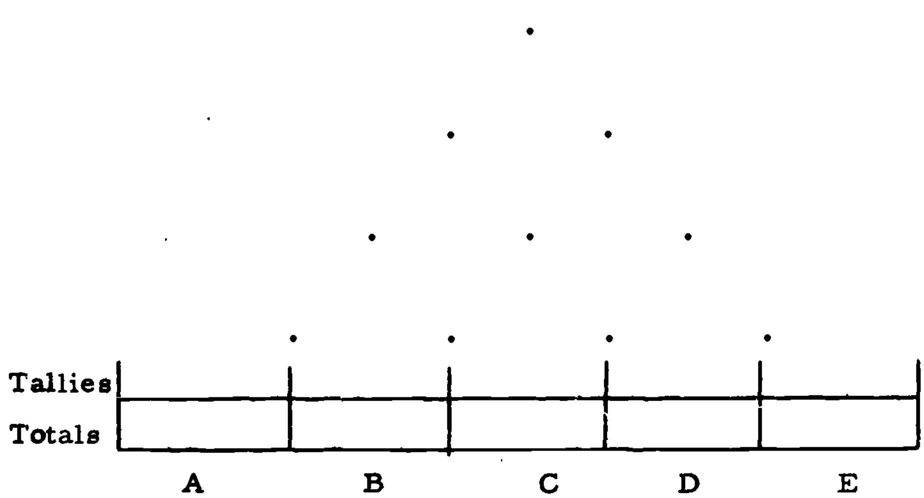
C. Desk padding (newspapers)

IV. Procedure:

A. This activity is an experimental method for constructing Pascal's Triangle. It should be presented before Pascal's Triangle is formally introduced.

B. Introduce this activity as an experimental way to determine a set of numbers needed in calculating probabilities. Tell them the numbers can be theoretically determined, but that they are to find them experimentally.

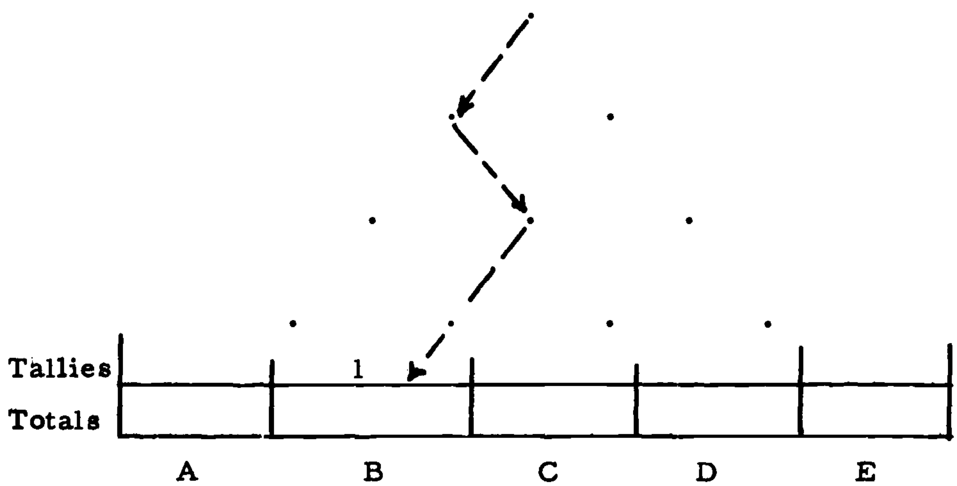
C. To demonstrate how the game is played, place the drawing below on the board or overhead projector.



1. The teacher should toss four coins. Suppose three heads and 1 tail are thrown. This should be written H T H H.

2. In this game we define heads to mean down and to the left, and tails to mean down and to the right.

3. Starting at the top dot, move according to the results of the coin toss. For example, if we tossed H T H H we would move down left, down right, down left, down left. We would then land in Slot B. Place a mark in the Tally above Slot B as shown below.



The order makes no difference, i.e. H T H H = T H H H = H H T H = H H H T

4. Perform several more tosses to illustrate the procedure.

5. Students will play the game using various number of coins and various number of dots.

6. Divide the class into groups of two students each. Distribute 6 coins and work sheets entitled "Pascal's Triangle - Number One" and five copies of "Pascal's Triangle - Totals."

7. Using the top figure on work sheet number one, have students toss two coins.

   a. Four tosses are to be made. The students should trace their path according to the rules and enter a tally in the chart depending on which slot they enter.

   b. Since there are four tosses made, there should be four tallies recorded.

8. Student should then toss three coins eight times using the figure at the bottom of work sheet number one.

9. Using "Pascal's Triangle - Number Two" have students:

   a. Toss 4 coins 16 times using top figure.

   b. Toss 5 coins 32 times using bottom figure.

10. Using "Pascal's Triangle - Number Three" students should t[illegible] 6 coins 64 times.

D. The students are now ready to gather the results from each group.

   1. Place a copy of "Pascal's Triangle Totals" on the board. Starting with the top figure on "Pascal's Triangle - Number One," have each group give their number of tallies in each slot.

   2. Have students copy these totals on the total sheet.

   3. Each team should then compute the totals of all the teams and find the average for each slot by dividing the totals by the number of teams.

   4. Use the same procedure for the rest of the experiments. Use a different "Totals" chart for each experiment.

E. The final averages found above should represent lines three through seven of Pascal's Triangle, as shown below:

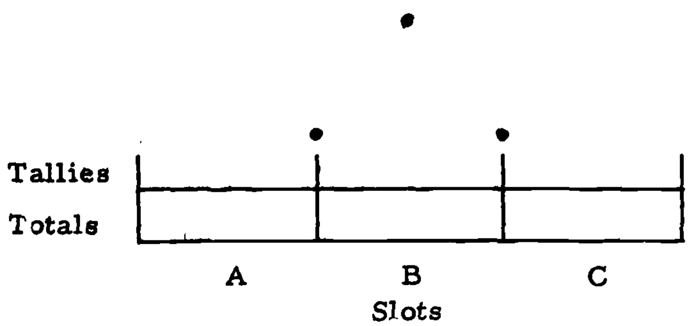
```
                    1
                 1     1
              1     2     1  ←———————— First figure used
           1     3     3     1
        1     4     6     4     1
     1     5    10    10     5     1
  1     6    15    20    15     6     1  ←—— Last figure used
```

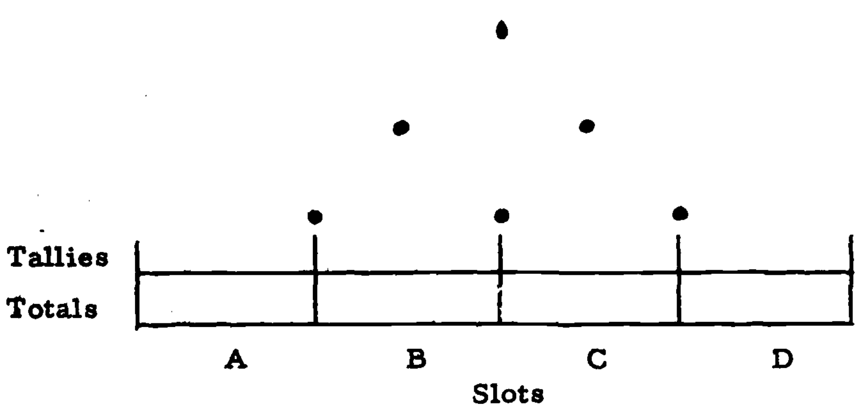
Discuss the name "Pascal's Triangle," using biographical references to Pascal.

F. This lesson should be followed with lessons to develop the patterns found in Pascal's Triangle, to relate this triangle to the normal curve, and to use this triangle to solve binomial probability problems.

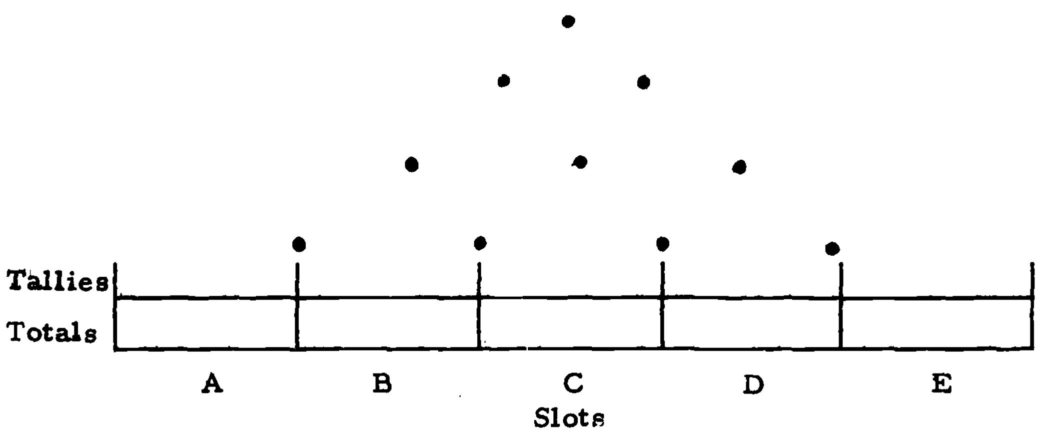
G. To assess this activity, have the work sheet entitled "Three Questions about Pascal's Triangle" completed.

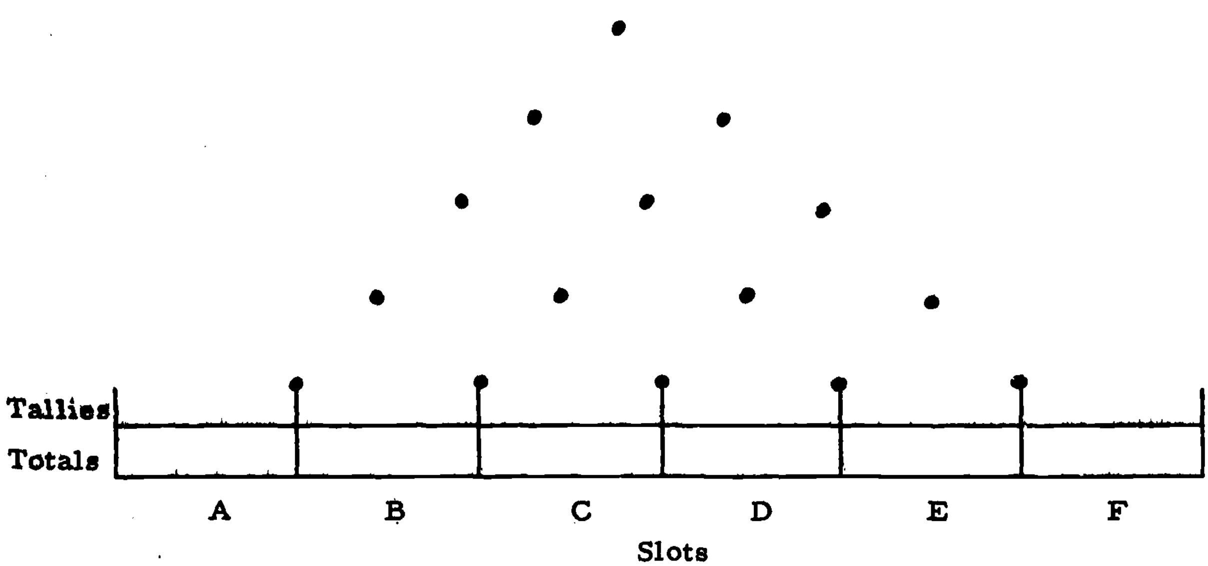
PASCAL'S TRIANGLE - NUMBER ONE



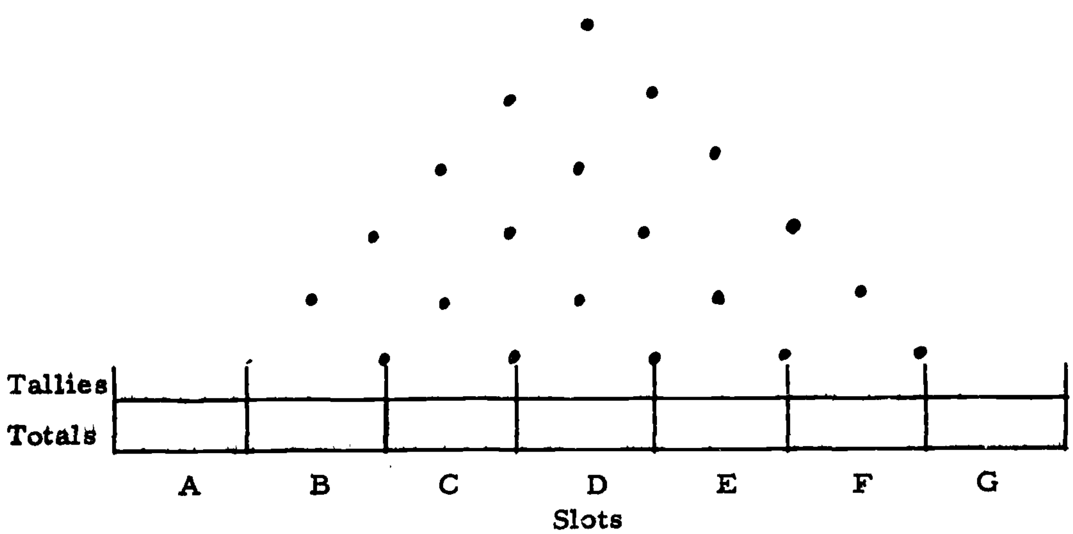


PASCAL'S TRIANGLE - NUMBER TWO





PASCAL'S TRIANGLE - NUMBER THREE



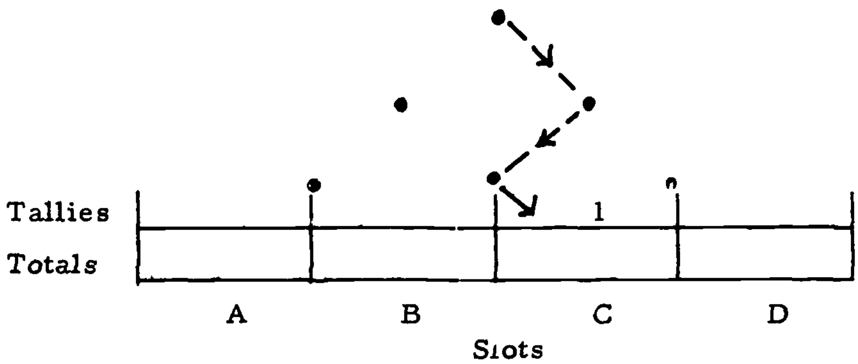
PASCAL'S TRIANGLE TOTALS

SLOTS

Team Numbers	A	B	C	D	E	F	G
Total							
Average							

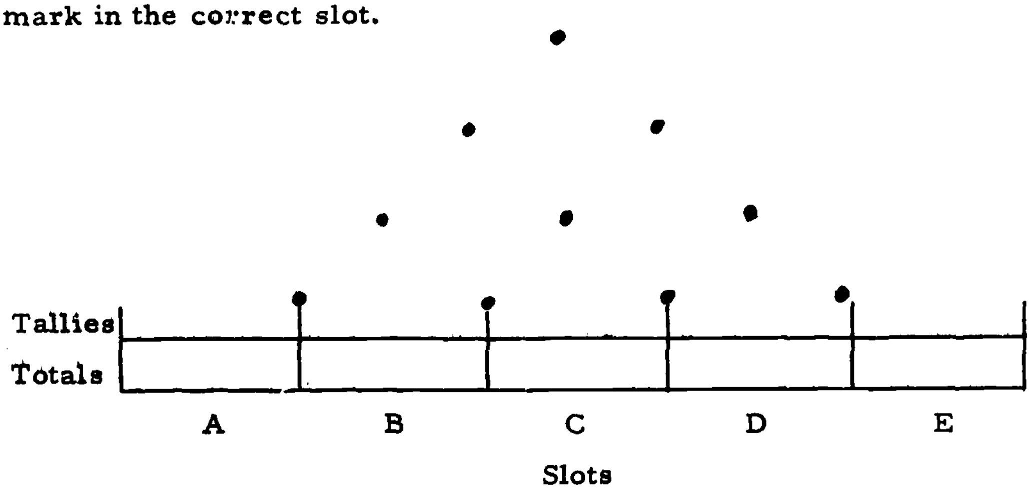
THREE QUESTIONS ABOUT PASCAL'S TRIANGLE

1.



What must have been the result of the toss to make the path shown above? ________ Heads and ________ Tails

2. Trace the path below for a toss that came up H, T, H, H. Put a tally mark in the correct slot.



3. Why did we use the averages of all the teams for our results instead of just one team's results? ______________________________

KARTS-A-GO-GO

Teacher Commentary

I. Unit: Probability

II. Objectives: The student should be able to:

A. State and apply the principle that probability is the total number of successes or failures divided by the total number of possible outcomes.

B. State the principle that the probability of an event is a number from 0 to 1.

C. State and apply the principles that if an event has probability 1 it must occur and that if an event has probability 0, it cannot occur.

D. Demonstrate how to experimentally determine the probability of an event whose outcomes are not equally likely.

E. Construct examples of events which have probabilities of zero or one.

III. Materials:

A. Work sheets entitled, "Karts-A-Go-Go," "Probability Fractions," "Special Probabilities," and "An Unknown Probability"

B. Pencil

C. Dice (two)

D. Deck of cards

E. Thumbtacks, one for each student

F. Toothpaste tube caps, have students bring in their own

IV. Procedure:

A. Hand the work sheet entitled, "Karts-A-Go-Go" to students and ask them what they think the title suggests. If they mention go-karts, you might want to carry on a brief discussion about them.

B. Ask the students to read the first paragraph silently. Ask-- "What did the shop class do?", "How was the go-kart driver to be chosen?"

C. Have the students complete the exploratory exercise individually. They could write their answers directly on the work sheet.

D. Discuss their answers to the exploratory exercise and proceed to the introduction of the word "probability." They may have heard of it before and if so, let them discuss it. Perhaps you might mention that the study was begun as a result of interest in games of chance. Students might also be interested in modern applications such as in industry or medicine.

E. Proceed to the work sheet entitled, "Probability Fractions" in the same fashion as before. Let the students read the introduction silently. Discuss the introduction.

F. Follow with the Exploratory Exercise 2 which should be done individually.

G. Have the class read the follow up to Exploratory Exercise 2 and discuss it carefully with them. See how many got most answers correct. Emphasize that each event had the same chance of happening. Point out that in this case the fraction can be formed with a denominator which is the total number of events that could happen and a numerator of the number of ways the favorable event could happen. Don't bother simplifying probability fractions at this time.

H. Let the students complete the practice exercises on their own and check the results. In exercise 4, you will want to be sure that they understand what a die is. It might be interesting to illustrate some of these problem situations by actually performing the experiment, i.e. toss coins, roll blocks, or draw marbles from the box.

I. Proceed with the work sheet entitled, "Special Probabilities." Let the students complete Exploratory Exercise 3 individually and read the follow up. Discuss the material and have them complete the practice exercises.

J. Have the class read the introduction on the work sheet "An Unknown Probability" and discuss it with them to make sure it is clear.

K. Distribute a thumbtack to each student and let him toss it 100 times keeping a record on the chart. Help the students combine their results. See if they can suggest a way of determining an estimate of the probabilities. Point out that the resulting fractions should add to 1.

L. Assessment - Have the students complete a written test or conduct oral interviews with them. Some suggested items follow.

Suggested Evaluation Items

1. What name is given to the study of chance events?

2. An Easter basket contains 12 eggs. Three are colored green, 5 are colored red, and 4 are colored blue. Jean is blindfolded and picks an egg from the basket.

   a. The probability that Jean picks a blue egg is

      (1) 4 (2) 4 out of 8 (3) $\frac{4}{12}$ (4) $\frac{1}{4}$

   b. The probability that Jean picks an egg which is not colored is

      (1) 1 (2) 0 (3) $\frac{1}{12}$ (4) $\frac{12}{12}$

3. Circle each number which could be a probability:

   5, $\frac{2}{3}$, $\frac{4}{5}$, $7\frac{1}{2}$, 6, 0, 1, 9, $\frac{3}{16}$

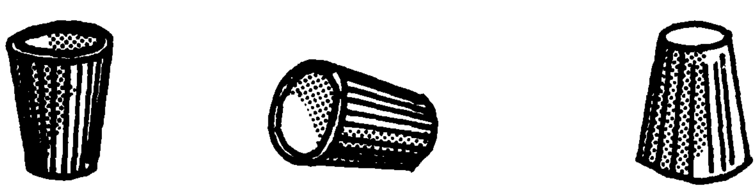
4. There are 52 cards in an ordinary deck of cards. Four of these are aces. If you draw a card from the deck, what is the probability that you will draw an ace?

5. What does it mean if an event has a 0 probability?

6. A die is one of a pair of dice. If a die is rolled, what is the probability that a 4 will face up? What is the probability that an odd number will face up?

7. The 26 letters of the alphabet are printed on cards. The cards are placed in a box. What is the probability that the letter Z will be picked? What is the probability that the letter will be in the word game?

8. Give the student a toothpaste tube cap. Ask him to try to determine an estimate of the probabilities of the ways it might land when it is tossed.



Here we would expect him to toss the cap about 100 times and keep a record of his results. Then he should be able to write an estimate of the probabilities involved based on his evidence.

9. Give an example of an event which has probability 0.
10. Give an example of an event which has probability 1.

KARTS-A-GO-GO

Lesson I - At the end of this lesson you should know about the kind of mathematics which deals with chance events.

The 8 boys in Tom's shop class have built a go-kart. They plan to race it at the go-kart track. The boys must choose a driver. Each boy writes his name on a strip of paper. He puts the paper into a hat. The teacher will pick a name from the hat. What are the chances that Tom's name will be picked? Let's see.

Exploratory Exercise 1

1. There are 8 strips of paper in the hat. How many of them have Tom's name on them?
2. Tom has one chance out of how many of being picked?
3. What are the chances that Tom will get to drive the go-kart?

Problems like the one above involve chance. The study of such problems is called probability.

PROBABILITY FRACTIONS

Lesson II - At the end of this lesson you should know how to write probabilities as fractions.

In Tom's homeroom class, there are 8 boys and 10 girls present. The teacher must choose a student for the go-kart race committee. All of the students put their names on cards. The cards are mixed and the teacher picks one. What might happen? Let's see.

Exploratory Exercise 2

1. There are 18 cards. How many cards have Tom's name on them?
2. Tom has one chance out of how many of being picked?
3. How many cards have girls names on them?
4. How many chances out of 18 are there that a girl will be picked?
5. How many cards have boys names on them?
6. How many chances out of 18 are there that a boy will be picked?

In the problem above there were 18 cards. Each student had the same chance of being picked. The probability or chance that Tom's name was picked is <u>1 out of 18</u>. We can also write this probability using the fraction $\frac{1}{18}$. There were 10 cards with girls names on them. The probability of a girl being picked was $\frac{10}{18}$. The probability of a boy being picked was $\frac{8}{18}$.

We can now see that probabilities can be written as fractions. When each event has the same chance of happening the denominator of the fraction is the total number of things that can happen. The numerator of the fraction is the number of ways for the event to happen.

Example: Suppose a bag contains 2 chocolate candies and 1 peppermint. What is the probability or chance of picking each kind of candy?

Since there are 3 candies you have 3 chances. This means 3 is the denominator of our fraction. Two of the candies are chocolate. We can use 2 as the numerator of the fraction. This means the probability or chance of picking a chocolate candy is $\frac{2}{3}$.

Exercises

1. If you toss a coin, what are the chances that the coin will come up heads? Write a fraction to show this probability.

2. A child's block has 6 letters on the sides. The sides are marked 3, 4, C, D, E, and F.

   a. If the block is rolled on the table, what is the probability that the letter C faces up? Write a fraction to show this.

   b. The probability that 3 faces up is:
   (1) $\frac{4}{6}$ (2) $\frac{1}{2}$ (3) $\frac{2}{6}$ (4) $\frac{1}{6}$

   c. The probability that a numeral faces up is:
   (1) $\frac{1}{2}$ (2) $\frac{2}{6}$ (3) $\frac{3}{6}$ (4) $\frac{4}{6}$

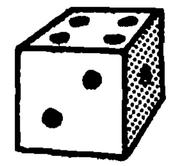
3. Three red marbles, two blue marbles, and five green marbles are placed in a box. A marble is picked from the box.

   a. The probability that it will be blue is:
   (1) $\frac{3}{10}$ (2) $\frac{2}{10}$ (3) $\frac{4}{5}$ (4) $\frac{1}{2}$

   b. The probability that it will be green is:
   (1) $\frac{3}{10}$ (2) $\frac{2}{5}$ (3) $\frac{1}{2}$ (4) $\frac{2}{10}$

   c. What is the probability that it will be red?

4. A die is one of a pair of dice, Suppose a die is rolled.



   a. What is the probability that a 6 will face up?
   b. What is the probability that a 2 will face up?
   c. What is the probability that an even number will face up?

SPECIAL PROBABILITIES

Lesson III - In this lesson you will learn about events with probabilities of 0 and 1.

Sometimes events have no chance of happening. Other events are sure to happen. What probabilities are given to such special events? Let's see.

Exploratory Exercise 3

1. Suppose you toss a coin that has two heads. What are the chances that it will land tails?
2. Think of a way to write the probability of the coin landing tails.
3. Suppose a bag contains 25 peppermint candies. You reach in the bag and pick a candy. What are the chances that it will be a peppermint candy?
4. Think of a way to write the probability of the candy being peppermint.

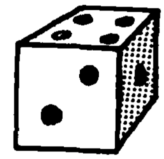
You should have said that there was no chance of the two-headed coin landing tails. When an event has no chance of happening it has probability 0. When the candy was picked from the bag it had to be peppermint. When an event is sure to happen it has probability 1.

We have seen that probabilities can be fractions. They can also be 0 or 1. Therefore, probabilities can be numbers from 0 to 1. A number like 6 could not be the probability of an event. Probabilities must be either fractions, 0 or 1.

Exercises

1. A die is one of a pair of dice.

   Suppose a die is rolled.

   a. What is the probability that a 10 will face up?

   b. What is the probability that an odd number will face up?

   c. What is the probability that some number from 1 to 6 will face up?

2. What is the probability that the day after Sunday will be Monday?

3. Which of the following numbers could be probabilities?

   a. 2 b. $\frac{1}{8}$ c. $\frac{2}{3}$ d. 0 e. 17

4. A die is rolled. What is the chance that a 17 will face up?

5. Make up an example of an event which has probability 0.

6. Make up an example of an event which has probability 1.

AN UNKNOWN PROBABILITY

Lesson IV - At the end of this lesson you should be able to find an unknown probability by experimenting.

In each of our problems, so far, we could figure out the probability. This was true because each event had the same chance of happening. Suppose we tried an experiment by tossing a thumbtack. The tack could land in one of two ways as shown.

Would the probability of each of these ways of landing be $\frac{1}{2}$?

Exploratory Exercise 5

1. Get a thumbtack from your teacher.
2. Toss it 100 times. Keep a record of the number of times the tack lands with its point up, on its side. Use a chart like the one below.

Tack lands point up	Tack lands on its side

3. With your teacher's help, combine your results with those of your classmates.
4. Write fractions for the probabilities as shown below.

   a. $\dfrac{\text{Number of times the tack lands up}}{\text{Total number of tosses of tack}}$

   b. $\dfrac{\text{Number of times tack lands on its sides}}{\text{Total number of tosses of tack}}$

PROBABILITY

Teacher Commentary for Film

I. Unit: Probability

II. Objectives: The student should be able to:

A. Name and identify the zero and one probabilities

B. State the principles:

1. If an event has a probability of 1, it must occur

2. If an event has a probability of zero, it cannot occur

3. $\text{Probability} = \dfrac{\text{total number of successes or failures}}{\text{total number of possible outcomes}}$

C. Apply these principles, given related information

III. Materials: Motion picture film - Probability. McGraw Hill Book Co.; Baltimore County Central Film Library

IV. Procedure:

A. This film may be used as an introduction to probability.

B. This film is 15 minutes long and may be used as a band in a lesson.

C. Motivate the students immediately before showing the film.

D. List the following guide questions on the board in order to insure that students know the reason for viewing the film.

Examples:

1. How is probability expressed?

2. What is the probability of the sun rising in the west tomorrow?

3. What is the probability that you will die?

E. Evaluate the ideas which the student learned from the film by checking the guide questions.

AN EXPERIMENTAL GRAPH OF THE NORMAL CURVE

Teacher Commentary

I. Unit: Statistics

II. Objectives: The student should be able to:

A. Name and identify the normal curve

B. Describe the normal curve in terms of its appearance

C. Construct bar graphs of statistical data

III. Materials:

A. Straight edges

B. Coins

C. Attached work sheet

IV. Procedure:

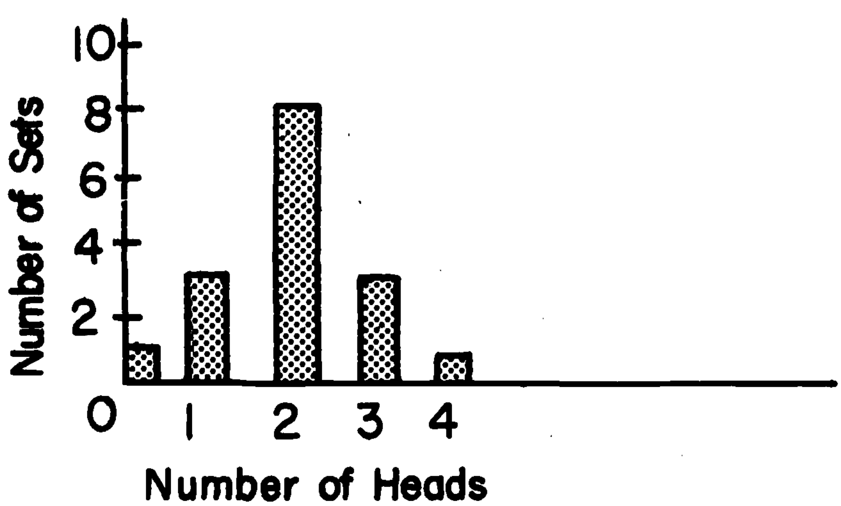
FIRST DAY

A. This activity produces a normal curve from recorded results in coin tossing. It takes two days.

B. Have students make 25 sets of four tosses each, recording their results in the table provided.

C. Have students count the number of sets of four tosses that have no heads, the number having one head, etc., recording their results in the table provided.

D. Have students make bar graphs of the chart formed in part C above, as illustrated below.



E. Discuss the general features of the normal curve; symmetry, bell-shape, infinite range. Point out that the experimental curves obtained merely give rough examples showing some of these features.

F. Have some student curves displayed, either on the board or overhead projector. Summarize the characteristics of normal curves from these examples to end this day's lesson. Collect student sheets.

SECOND DAY

A. From student sheets collected in the previous lesson, have prepared a composite chart of all the student frequency charts, as illustrated below.

Number of Heads	Number of Sets
0	50
1	150
2	325
3	150
4	50

B. Present this chart to the class, explaining how it was obtained.

C. Form a bar graph on the board using the information given on the composite chart. Explain why this graph should more closely approximate a normal curve than any of the individual student charts could.

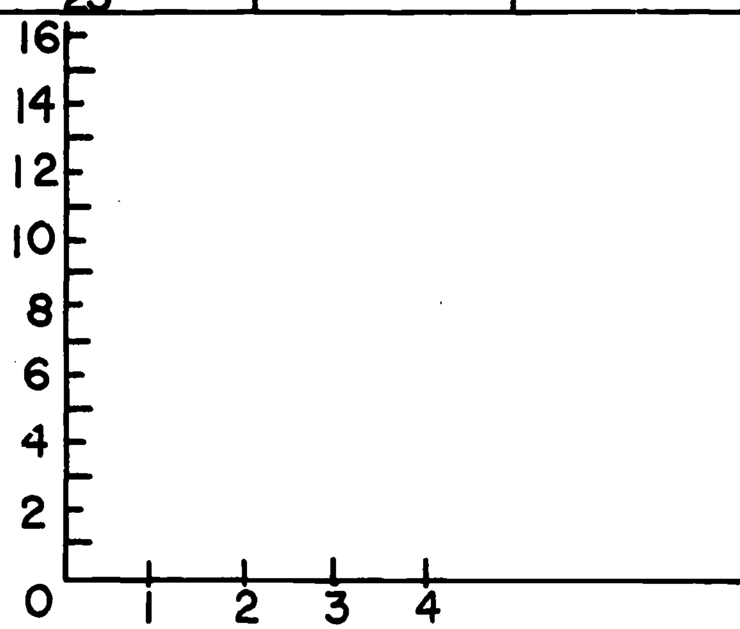
D. Review the characteristics of normal curves.

E. Return the student sheets, and have them make a graph using tails instead of heads. They should use the same chart tallying heads and tails that they made the previous day, but make a new frequency table using tails, and a new bar graph using tails. Collect this as an assessment of the success of this activity.

A GRAPH OF THE NORMAL CURVE

Set (4 tosses)	Heads	Tails
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		

Number of Heads	Number of Sets
0	
1	
2	
3	
4	



ALGEBRA

ALGEBRA

I. Master Chart - Grades Six through Eleven

II. Grade Ten Chart

III. Behavioral Objectives

IV. Activities

UNIT ALGEBRA GRADE(S) Six through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Set	6, 8	6, 8		6	6					6
Member	6, 7	6, 7			6					6
Types of Sets	6, 7	6, 7		6	6, 7					6, 7
Relationship Between Sets	6, 8, 9	6, 8, 9		6, 8	6, 8					6, 8
Methods of Describing Sets	6	6			6					6
Operations With Sets	6	6			6					6
Language of Algebra	6, 7, 8 9, 10	6, 7, 8, 9 10		9	7, 8, 9	7			9	6, 7, 8, 9
Symbols for Operations	6, 7, 8	6, 7, 8, 9	6	8, 9	9					6, 7, 8
Symbols for Grouping	6, 10	6, 10	6, 7	10		6, 7, 9, 10	6, 7, 9, 10		10	6, 10
Evaluating Algebraic Expressions			9, 10, 11	10, 11	9, 10, 11	9, 10, 11	9, 10, 11			
Number Line	8, 9	8, 9	9	8, 9		9	9		8, 9	9
Operations With Rationals	9, 10	9, 10	10	9	9	9, 10	9, 10			9
Similar Terms	10, 11	10, 11	10, 11		10, 11	10, 11	10, 11			10, 11
Open Sentences With One Operation			9	9		9	9			
Open Sentences With Combined Operations	9, 11	9, 11	9, 11		9, 11					
Inequalities With One Operation	9	9	9		9					9
Inequalities With Combined Operations	11	11			11	11	11			11
Rationals	9	9		9	9				9	9

UNIT ALGEBRA GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Coefficient	10	10								
Term	10	10								
Symbols of Grouping	10	10		10		10	10		10	10
Evaluating Algebraic Expressions (Brackets)			10			10				
Addition of Rationals	10	10	10			10	10			10
Multiplication of Rationals	10	10	10			10	10			10
Subtraction of Rationals (Inverse of +)	10	10	10			10	10			10
Division of Rationals (Inverse of x)	10	10	10			10	10			10
Operations with Similar Terms	10	10	10		10	10	10			10
Solution of Inequalities Involving Combined Operations	10	10	10		10	10	10			10

ALGEBRA - Grade 10

Page

Coefficient and Term

The student should be able to:

1. Name and identify coefficient and term

Symbols of Grouping - Brackets

The student should be able to:

1. Name and identify a bracket as a symbol of grouping
2. Construct a number phrase using brackets
3. State the principle that given an expression containing brackets, the operations within the brackets should be performed first
4. Apply the principle in evaluating problems containing brackets
5. Order operations in sequence involving brackets enclosing parentheses
6. Distinguish between brackets and parentheses

Evaluating Algebraic Expression

The student should be able to:

1. State and apply the principle that the value of an algebraic expression can be found if each variable is replaced by a constant
2. Demonstrate how to use the above principle

Formula Derivation

The student should be able to:

1. Describe a procedure for constructing a formula given a relationship in tabular or verbal form
2. Construct a formula given a set of verbal facts or relationships
3. Demonstrate how to devise a formula from a table of values

Page

Addition of Two Positive Rational Numbers

The student should be able to:

1. Name and identify the addition of two positive rational numbers
2. State the principle that the sum of two positive rational numbers is a rational number
3. Apply the above principle in various problems
4. Demonstrate the above principle

Addition of Two Negative Rational Numbers

The student should be able to:

1. Name and identify the addition of two negative rational numbers
2. State the principle that the sum of two negative rational numbers is a negative rational number
3. Apply the principle to solve related problems
4. Demonstrate how to use the above principle

Addition of a Positive Rational Number and a Negative Rational Number

The student should be able to:

1. Name and identify the addition of one positive rational number and one negative rational number
2. Describe that the sum of two rational numbers having unlike signs may be found by subtracting the "smaller" from the "larger" and giving the answer the sign of the "larger " or through specific examples
3. Apply the principle to solve related problems
4. Demonstrate how to use the principle

Properties of Addition of Rational Numbers

(The statements below will apply to the following properties: closure, associativity, commutativity, and identity element.)

The student should be able to:

1. Name and identify each property

Page

2. Construct an example of each property
3. State and apply the principle of each property
4. Distinguish among the properties

Additive Inverse of Rational Numbers

The student should be able to:

1. Name and identify the additive inverse for each rational number
2. State the principle that the sum of a number and its additive inverse is zero
3. Apply the principle to solve related problems
4. Construct an example of the principle

Multiplication of Rational Numbers

The student should be able to:

1. Name and identify the operation of multiplication involving rational numbers

Multiplication of Two Positive Rational Numbers

The student should be able to:

1. State and apply the principle that the product of two positive rational numbers is a positive rational number
2. Demonstrate how to use the principle to solve related problems

Multiplication of a Positive Rational Number and a Negative Rational Number

The student should be able to:

1. State and apply the principle that the product of a positive rational number and a negative rational is negative
2. Demonstrate how to use the principle to solve related problems

Page

Multiplication of Two Negative Rationals

The student should be able to:

1. State and apply the principle that the product of two negative rational numbers is a positive rational number

Properties of the Multiplication of Rational Numbers

(The statements below apply to the following properties: closure, associativity, commutativity, and identity element)

The student should be able to:

1. Name and identify each property
2. Construct an example of each property
3. State and apply the principle of each property
4. Distinguish among the properties

Multiplicative Inverse For Rational Numbers

The student should be able to:

1. Name and identify the multiplicative inverse for every rational number except zero
2. State and apply the principle that the product of a rational number and its multiplicative inverse equals one
3. Construct an example illustrating the principle
4. Distinguish between inverse element and identity element

Distributive Property of Multiplication over Addition

The student should be able to:

1. Name and identify the distributive property
2. Construct an example of the distributive property
3. Apply the principle to solve related problems
4. Distinguish among the distributive property and the associative properties of addition and multiplication

Multiplicative Property of Zero

The student should be able to:

1. State and apply the principle that the product of any rational number and zero is zero
2. Construct an example of the principle
3. Distinguish between multiplication of a rational number and zero and multiplication of a rational number and one

Subtraction as The Inverse Operation of Addition

The student should be able to:

1. Name and identify the operation of subtraction using rational numbers
2. Apply the principle that $a-b = a + (-b)$ when a and b are rational numbers
3. Construct an example of subtraction of rational numbers
4. Distinguish subtraction from addition

Division as The Inverse Operation of Multiplication

The student should be able to:

1. Name and identify the operation of division using rational numbers
2. Apply the principle that $a \div b = a \cdot \frac{1}{b}$ when a, b are rational numbers, $b \neq 0$
3. Construct an example using the principle
4. Distinguish between the operations of multiplication and division

Operations with Similar Terms

The student should be able to:

1. Name and identify similar terms
2. Describe similar terms with respect to variable and exponent
3. Demonstrate the operations of addition and subtraction using similar terms

Page

4. State the principle that similar terms can be added and subtracted in same manner as rational numbers

5. Apply the principle of similar terms in equations involving one variable

6. Distinguish between similar and non-similar terms

Open Sentences Involving Combined Operations

(The following combinations of operations will be considered with respect to open sentences: multiplication and subtraction, multiplication and division; division and addition, division and subtraction; addition and division, subtraction and division)

The student should be able to:

1. Name and identify the operations involved

2. Demonstrate a procedure for determining the solution for various equations A-10

3. Describe the method used in the solution of an equation

Solution of Inequalities Involving Combined Operations

The student should be able to:

1. Name and identify open sentences involving inequality

2. Demonstrate a procedure for solving inequalities

3. Describe the procedure for solving equations involving these operations

4. Distinguish inequalities for equalities

NOMOGRAPH
ALGEBRA
Teacher Commentary

I. Unit: Algebra

II. Objectives: The student should be able to:

Demonstrate how to construct the solution of a multiplication equation using the nomograph

III. Materials:

A. Student work sheet "Nomograph"

B. A twelve inch ruler

IV. Procedure:

A. Distribute the materials to each student.

B. Discuss the three scales A, X and C.

1. Scale A, which represents the numerical coefficient, begins with one and ends with one hundred. Units one thru ten are divided into fourths. Units ten thru twenty are divided into halfs.

2. Scale X, which represents the solution to a multiplication equation, begins with one and ends with one hundred.

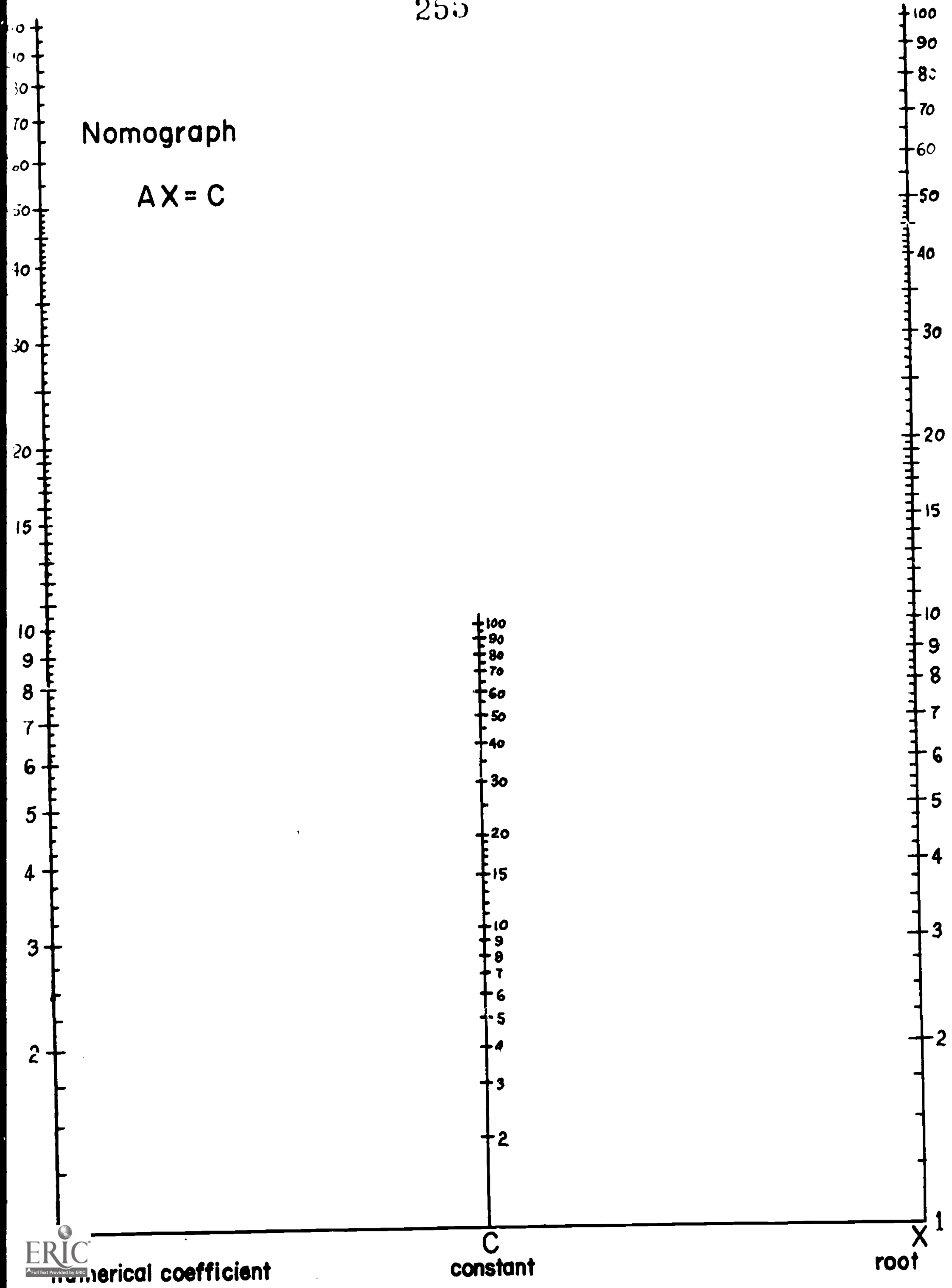
3. Scale C, which represents the constant, begins with one and ends with one hundred.

4. Locate points on the scales and have students identify them. On scale X, students should be able to identify points like $3\frac{1}{3}$, $8\frac{1}{2}$, and $13\frac{3}{4}$.

5. Have students locate points on the scales.

C. In order to solve an equation like $2\frac{1}{2}x = 10$, locate $2\frac{1}{2}$ on scale A and 10 on scale B. The line joining these two points will cross scale X at a point that represents the root of the equation (4).

D. After the student has completed some written exercises, the nomograph may be used to check the results.


Nomograph
AX = C
numerical coefficient
C
constant
X
root

LOGIC

LOGIC

UNIT LOGIC GRADE(S) Ten and Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Equivalent Phrases and Sentences	10	10		10						8, 10
Assumptions	10	10		11	10					10
"If - Then" Statements	10, 11	10, 11		10				10		10, 11
Converse	11	11		11	11	11		11		11
Inverse	11	11		11	11	11		11		11
Circular Reasoning (Doubletalk)										11
Valid Arguments					10, 11					10, 11
Non-Valid Arguments					10, 11					10, 11
Syllogism (Chain of Logic)										11
Indirect Proof						11	11			
Venn Diagrams	10	10						10		
Open Sentences	8, 10	8, 10		8, 10	8, 10					8, 10
Closed Sentences	8, 10	8, 10		8, 10	8, 10					8, 10
Deductive Reasoning					11					11
Inductive Reasoning					11					11
Counterexample				10	10	10	10			

UNIT LOGIC GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON- STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN- GUISHING
Assumptions and Sentences	10	10			10					10
"If-Then" Statements	10	10		10			10	10		
Venn Diagrams	10	10		10				10		
Open Sentences	10	10		10	10					10
Closed Sentences	10	10		10	10					10
Counter Example				10	10	10	10			
Arguments: Valid					10					10
Arguments: Non-Valid					10					10

Page

Quantifiers

The student should be able to:

1. Name and identify quantifiers in sentences
2. Construct sentences using quantifiers
3. Distinguish among the quantifiers in sentences

Assumptions

The student should be able to:

1. Name and identify an assumption
2. Describe an assumption as something which we assume to be true
3. Distinguish an assumption from a conclusion

Venn Diagrams

The student should be able to:

1. Name and identify Venn diagrams
2. Construct Venn diagrams illustrating logical relationships or quantifiers
3. Interpret a Venn diagram for its meaning

Open and Closed Sentences

The student should be able to:

1. Name and identify open and closed verbal sentences
2. Construct open and closed sentences
3. Describe open sentences as sentences whose truth cannot be determined until a replacement from the replacement set is made for the variable
4. Describe closed sentences as those whose truth value may be determined
5. Distinguish between open and closed sentences

"If-Then" Sentences

Page

The student should be able to:

1. Name and identify an "if... then" statement

2. Construct an "if...then" statement, including being able to rewrite a statement in "if... then" form

3. Apply the principle in deductive reasoning that a true assumption followed by a false conclusion yields on invalid argument

4. Interpret an "if...then" statement for its truth value

Counter Example

The student should be able to:

1. Describe a counter example as an example which proves a statement false

2. Construct a counter example to disprove a conclusion

Arguments: Valid and Invalid

The student should be able to:

1. Describe a valid argument as a series of statements that are logically correct

2. Describe an invalid argument as a series of statements that are not logically correct

3. Distinguish valid from invalid arguments

RECREATIONAL ACTIVITIES

RECREATIONAL ACTIVITIES

The units contained in this section are supplementary activities for recreation. These may be used to provide variety throughout the year as well as in the daily lesson.

CROSS NUMBER PUZZLE ON FRACTIONS

Teacher Commentary

A Recreational Activity on Operations With Fractions

I. Materials: Attached puzzle to be duplicated

II. Procedure:

A. Distribute attached puzzle.

B. Since this is a long puzzle, it may be best to do it in parts over a number of days. Some of the parts will take teacher directed review.

C. Solution:

	2	4			1	5
1	2		1	8		4
1	4		1	1	2	
		6		8		
	1	2	1		3	2
4		2	6		1	4
2	1			1	2	

CROSS NUMBER PUZZLE ON FRACTIONS

	1	2			3	4
5			6	7		8
9			10		11	
		12		13		
	14		15		16	17
18		19			20	
21	22			23		

Horizontal

1. The L. C. D. of $\frac{7}{8}$ and $\frac{5}{6}$.
2. Change $\frac{5}{8}$ to number of 24ths.
5. Change $\frac{36}{3}$ to a whole number.
6. $6\frac{2}{\;}$ $5\frac{1}{4} + 6\frac{5}{6} + 2\frac{1}{4}$.
8. $7\frac{1}{2} - 3\frac{1}{2}$.
9. $8\frac{2}{5} \times 1\frac{2}{3}$.
10. $\frac{1}{4}$ of what number is 28?
12. $46\frac{1}{2} - 7\frac{3}{4}$.
13. Subtract $7\frac{1}{3}$ from $13\frac{1}{4}$ and add $2\frac{1}{12}$.
14. Add: $27\frac{5}{8}$; $32\frac{1}{3}$; $16\frac{5}{24}$; and $44\frac{5}{6}$.

Vertical

1. Divide 14 by $\frac{1}{16}$.
2. Change $\frac{1}{4}$ to number of 16ths.
3. $\frac{1}{2} + \frac{1}{6} + \frac{1}{3}$.
4. Find $\frac{3}{4}$ of 72.
5. $9\frac{1}{6} \times 1\frac{1}{5}$.
6. Divide $3\frac{3}{10}$ by $\frac{3}{10}$.
7. Add: $148\frac{3}{4}$; $210\frac{2}{3}$; $56\frac{1}{6}$; and $402\frac{5}{12}$.
11. Change $\frac{16}{8}$ to a whole number.
12. Multiply the difference of $120\frac{4}{5}$ and $27\frac{1}{2}$ by $6\frac{2}{3}$.
14. $\frac{2}{5} \times 1\frac{1}{2} \times 1\frac{2}{3}$.

Horizontal

16. Multiply $13\frac{5}{7}$ by $2\frac{1}{3}$.

18. Divide $29\frac{1}{2}$ by $7\frac{3}{8}$.

19. $19\frac{1}{2}$ is $\frac{3}{4}$ of what number?

20. $5\frac{3}{5} \times 1\frac{1}{2}$ divided by $\frac{3}{5}$.

21. Simplify:

$$\frac{12\frac{3}{5} \times 2\frac{1}{2}}{1\frac{1}{2}}$$

23. Divide the product of $17\frac{1}{2}$ and $\frac{2}{5}$ by $\frac{7}{12}$.

Vertical

15. 12 is $\frac{3}{4}$ of what number?

16. Take $\frac{4}{5}$ of 390.

17. L. C. D. of $\frac{1}{8}$; $\frac{1}{3}$; $\frac{3}{4}$; and $\frac{5}{12}$.

18. $\frac{3}{4}$ of what number is $31\frac{1}{2}$?

22. $1\frac{1}{5} \times 2\frac{1}{2}$ divided by 3.

23. Simplify:

$$\frac{5\frac{1}{2} - 2\frac{1}{4}}{1\frac{3}{4} + 1\frac{1}{2}}.$$

DECIMAL PATTERNS

Teacher Commentary

A Recreational Activity on Converting Fractions to Repeating Decimals

I. Materials:

   A. Student work sheets entitled, "Can You Find a Pattern?"

   B. Overhead projectuals

II. Procedure:

   A. This material may be used as a guide exercise at the beginning or end of a period. It is designed to be used during several periods rather than covered during one lesson.

   B. Distribute student work sheets.

   C. Using an overhead projectual of the work sheet discuss the title, the fractions shown, and a method for changing them to their decimal equivalents.

   D. The table should be filled in by dividing to the point where the students recognize a pattern.

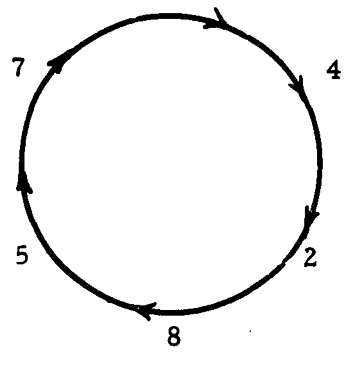
   E. The rest of the table should then be completed by using the pattern and then checked by dividing.

   F. Solutions:

   Table 1 = $.22222\overline{2}$, $.33333\overline{3}$, $.44444\overline{4}$, $.55555\overline{5}$, $.66666\overline{6}$, $.77777\overline{7}$, $.88888\overline{8}$

   Table 2 = $.18181\overline{8}$, $.27272\overline{7}$, $.36363\overline{6}$, $.45454\overline{5}$, $.63636\overline{3}$, $.72727\overline{2}$, $.81818\overline{1}$, $.90909\overline{0}$

   Table 3 = $.28571\overline{4}$, $.42857\overline{1}$, $.57142\overline{8}$, $.71428\overline{5}$, $.85714\overline{2}$

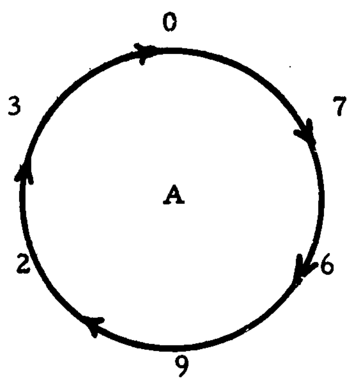


Note: In the decimal equivalents for sevenths, the same digits reappear in a different sequence. The digits for the decimal equivalent of $\frac{1}{7}$ have been arranged clockwise about the circle below. To obtain the decimal equivalent for $\frac{2}{7}$ the digit in the circular pattern with the next greater value appears in the tenths place and the remaining digits of the recurring decimal are read clockwise from the circle.

Table 4	Column 1	$.23076\overline{9}$, $.30769\overline{2}$, $.69230\overline{7}$, $.76923\overline{0}$, $.92307\overline{6}$
	Column 2	$.38461\overline{5}$, $.46153\overline{8}$, $.53846\overline{1}$, $.61538\overline{4}$, $.84615\overline{3}$

To find the decimal equivalent for $\frac{3}{13}$, find the circular pattern which contains the digit with the next greater value, and continue clockwise about that circle. For instance, the next larger digit is 2 found in circle A. The digits following are 30769 thus $\frac{3}{13} = .23076\overline{9}$.

In finding the decimal equivalent for $\frac{4}{13}$, we observe that each circle contains the next larger digit, 3. Hence we must examine each sequence of digits beginning with 3 to find the next larger decimal. In this case we have $.30769\overline{2}$ and $.38461\overline{5}$. Since $.30769\overline{2}$ is the next larger decimal we select this as the equivalent for $\frac{4}{13}$. A similar investigation will have to be performed for $\frac{8}{13}$.



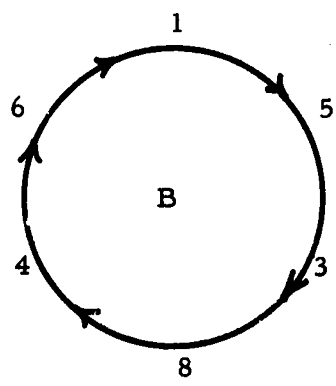


Table 1

CAN YOU FIND A PATTERN ?

$\frac{1}{9} = .11111\overline{1}$

$\frac{2}{9} =$

$\frac{3}{9} =$

$\frac{4}{9} =$

$\frac{5}{9} =$

$\frac{6}{9} =$

$\frac{7}{9} =$

$\frac{8}{9} =$

Table 2

CAN YOU FIND A PATTERN ?

$\frac{1}{11}$ = $.0909\overline{09}$

$\frac{2}{11}$ =

$\frac{3}{11}$ =

$\frac{4}{11}$ =

$\frac{5}{11}$ =

$\frac{6}{11}$ =

$\frac{7}{11}$ =

$\frac{8}{11}$ =

$\frac{9}{11}$ =

$\frac{10}{11}$ =

Table 3

CAN YOU FIND A PATTERN ?

$\frac{1}{7}$ = $.\overline{142857}$

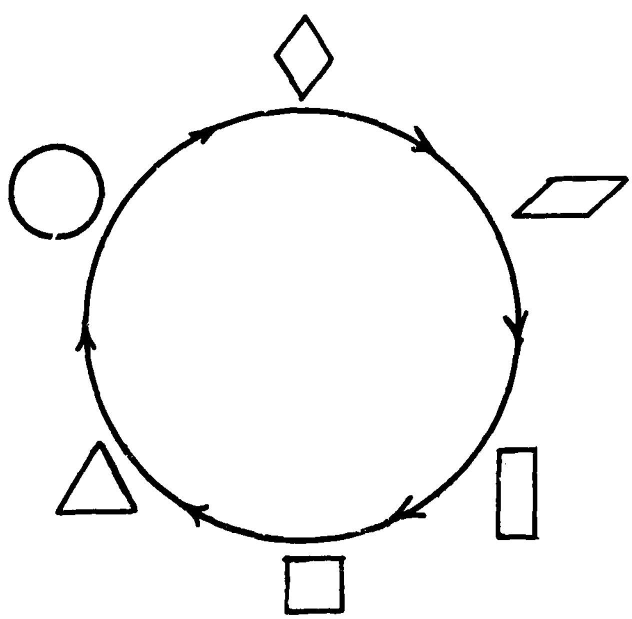
$\frac{2}{7}$ =

$\frac{3}{7}$ =

$\frac{4}{7}$ =

$\frac{5}{7}$ =

$\frac{6}{7}$ =

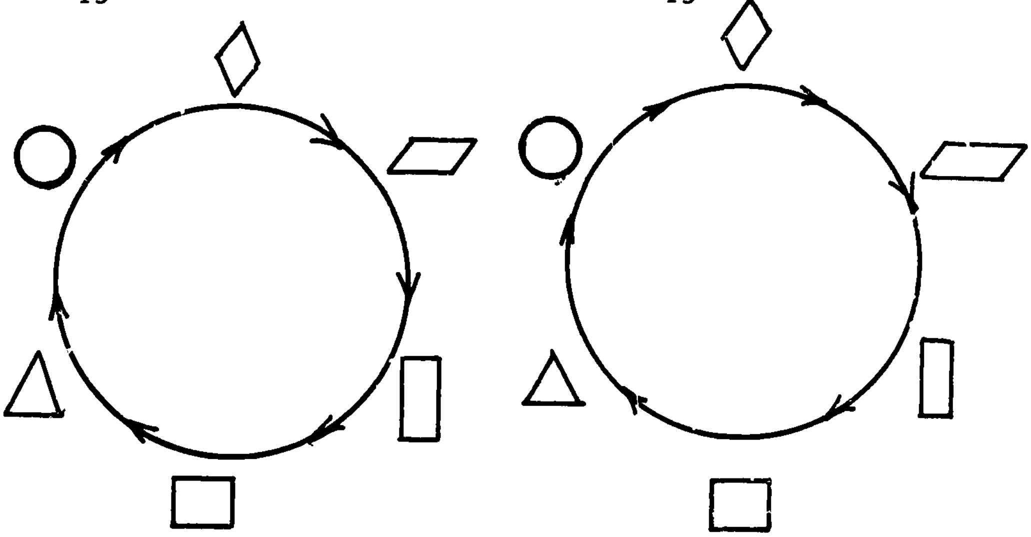


Hint: Fill in the frames with the numerals you used in example 1.

Table 4

CAN YOU FIND A PATTERN ?

$\frac{1}{13} = .07692\overline{3}$	$\frac{2}{13} = 15384\overline{6}$
$\frac{3}{13} =$	$\frac{5}{13} =$
$\frac{4}{13} =$	$\frac{6}{13} =$
$\frac{9}{13} =$	$\frac{7}{13} =$
$\frac{10}{13} =$	$\frac{8}{13} =$
$\frac{12}{13} =$	$\frac{11}{13} =$



Hint: Fill in the frames for circle A with numerals for $\frac{1}{13}$.

Fill in the frames for circle B with numerals for $\frac{2}{13}$.

PERCENT CROSS NUMBER PUZZLE

Teacher Commentary

A Recreational Activity on Percent

I. Materials: Attached sheet to be duplicated

II. Procedure:

A. Have students complete attached cross number puzzle.

B. Solution:

[1] 4	[2] 3		[3] 4	[4] 5	[5] 2
[6] 5	6			[7] 3	2
	[8] 8	7		2	
[10] 7			[11] 3	0	[12] 5
[13] 8	[14] 0		[15] 5		0
	[16] 9	4		[17] 5	

PERCENT CROSS NUMBER PUZZLE

Directions: Fill in the blanks with the proper answer to each problem.

1	2	■	3	4	5
6		■	■	7	
■	8		■		■
10	■	■	11		12
13	14	■	15	■	
■	16		■	17	■

Horizontal

1. 50% of 86 = what number?
3. 80% of 565 = what number?
6. 7 = $12\frac{1}{2}$% of what number?
7. A bicycle costs $48. It was sold at a $33\frac{1}{3}$% loss. Find the selling price.
8. A grade of 75% was increased by 12%. What is the new grade?
11. A boat reduced $16\frac{2}{3}$% from $366 is sold for how much?
13. What number decreased by $37\frac{1}{2}$% of itself is 50?
16. During a chess tournament Jim won 47 out of 50 games. What % of the games played did Jim win?
17. $\frac{1}{2}$% of 1000 = what number?

Vertical

1. 30 = $66\frac{2}{3}$% of what number?
2. 320 increased by 15% is what number?
4. A $5800 salary is decreased by 10%. Find the new salary.
5. 22 is what % of 100?
10. Sam learned 39 out of 50 words. What % did he learn?
11. A team won 13 ball games and lost 7. What % did it lose?
12. $\frac{1}{3}$% of 15000 = what number?
14. Write 9% as a decimal fractional number.

THE MELANCHOLIA MAGIC SQUARE

Teacher Commentary

A Recreational Activity on Addition of Whole Numbers

I. Materials: Student work sheets

II. Procedure:

A. Read the story of the Melancholia Magic Square to the students.

B. Distribute work sheets.

C. Check squares with students after work sheets have been completed.

D. Story of the Magic Square

This magic square appears in the famous 1514 engraving "Melancholia" by Dürer. The actual creator is unknown. For extra motivation a picture of the engraving could be shown to the students. It can be found is most sets of encyclopedia.

The properties of the Melancholia Magic Square go beyond those of the ordinary magic square. A partial list of these properties is shown below. The work sheet can be extended to include the properties beyond number 10 in the list below.

E. Properties of the Melancholia Magic Square:

1. All rows add up to 34.
2. All columns add up to 34.
3. The diagonals add up to 34.
4. The four corners add up to 34.
5. The four center squares add up to 34.
6. All opposite pairs of center squares add up to 34.
7. All opposite pairs of slanting squares add up to 34.
8. All corner sets of four squares add up to 34.
9. The sum of the top 8 squares and the sum of the bottom 8 squares add up to 68.

10. The sum of the 8 squares on the left and the sum of the 8 squares on the right add up to 68.

11. The date of the painting (1514) appears in the lower center squares.

12. The sum of the squares in alternate rows and alternate columns add up to 68.

13. The sum of the squares of the numbers in the top 8 squares and the sum of the squares of the numbers in the bottom 8 squares add up to 68.

14. The sum of the squares of the numbers in alternate rows and alternate columns add up to 748.

15. The sum of the numbers in the diagonals and the sum of the numbers not in the diagonals are equal. (68)

16. The sum of the squares of the numbers in the diagonals and the sum of the squares of the numbers not in the diagonals are equal. (748)

17. The sum of the cubes of the numbers in the diagonals and the sum of the cubes of the numbers not in the diagonals are equal. (9248)

18. The squares of the numbers in the opposite slanting squares (#7) are equal. (374)

19. The cubes of the numbers in the opposite pairs of slanting squares (#7) are equal. (4624)

20. By adding each pair of numbers vertically or horizontally, the following patterns are discovered.

Horizontal	21	13	13	21		Vertical	
	13	21	21	13		19	15
						15	19
						15	19
						19	15

THE MELANCHOLIA MAGIC SQUARE

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Place the correct sum in each circle.

1. Add each row:

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

2. Add each column:

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

3. Add each diagonal:

16			
	10		
		7	
			1

			13
		11	
	6		
4			

4. Add the 4 corners:

16			13
4			1

5. Add the 4 center squares:

	10	11	
	6	7	

6. Add the opposite pairs of center squares:

	3	2	
	15	14	

5			8
9			12

7. .dd the opposite pairs of slanting squares:

		2	
			8
9			
	15		

	3		
5			
			12
		14	

8. Add each corner set of 4 squares:

16	3		
5	10		
		7	12
		14	1

		2	13
		11	8
9	6		
4	15		

9. Add the 8 numbers in the top half:

16	3	2	13
5	10	11	8

10. Add the 8 numbers in the bottom half:

9	6	7	12
4	15	14	1

11. Add the 8 numbers on the left side:

16	3		
5	10		
9	6		
4	15		

12. Add the 8 numbers on the right side:

		2	13
		11	8
		7	12
		14	1

POLYHEDRON MODELS

Teacher Commentary

A Recreational Activity on the Protractor, Straightedge, and Compass

I. Materials:

Compasses, rulers, protractors, scissors, Tester's Cement, large sheets of manila tag board, enamel paints, overhead projector, and prepared transparencies.

II. Procedure:

A. One might use the need for center pieces at an upcoming school affair, decorating the school's Christmas tree, or crystal building for science classes as motivation for this activity. However, in all cases, view this activity as an appreciation lesson.

B. Use the overhead projector to project the transparency of the polyhedron (net) on screen.

C. Give the following directions to the students:

1. On a large sheet of manila tag board, reproduce the pattern using a radius of 3 inches for the side of each of the equilateral triangles appearing in the net.

2. Cut along all outside lines with scissors; score all solid lines ___________ and all dashed lines ------- with point of your compasses. Fold the pattern along these lines away from you.

3. Score lines marked ____.____.____. on reverse side of the pattern. Fold the pattern along these lines toward you.

4. Match sides which have been marked with the same letters.

5. Paint the finished product.

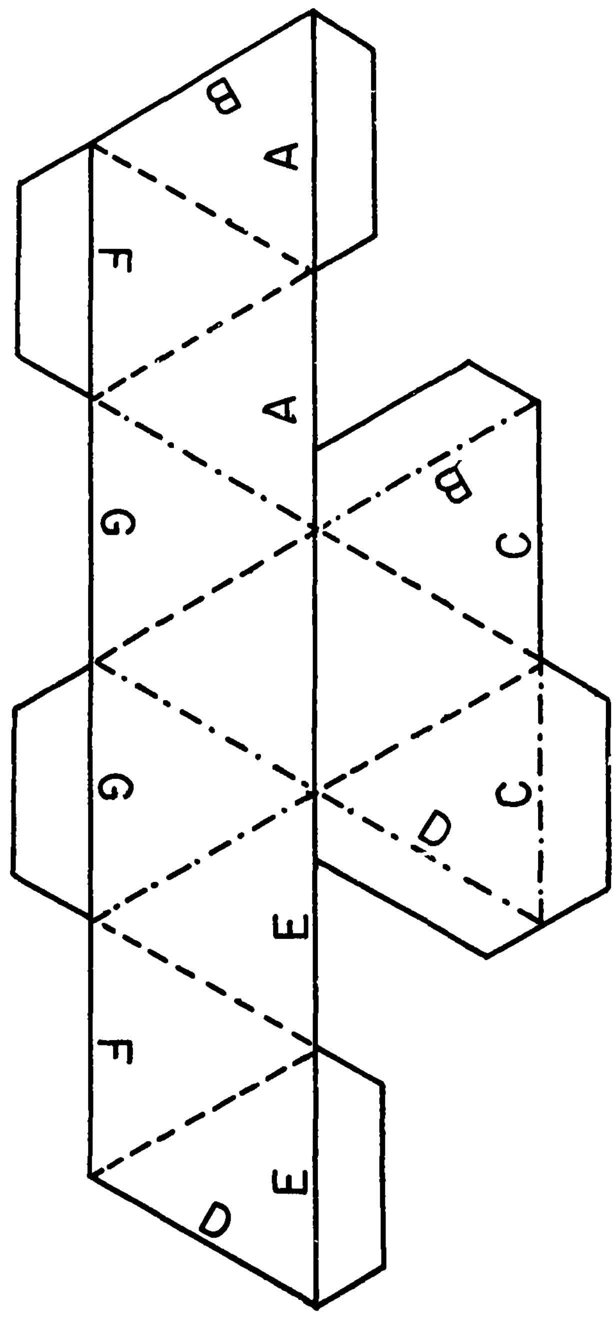
Additional patterns may be found in the following source books:

Cundy and Rollett. Mathematical Models. Oxford University Press, N. Y.

Steinhaus, H. Mathematical Snapshots. Oxford University Press, N. Y.

Multi-Sensory Aids in the Teaching of Mathematics (The 18th Yearbook of the National Council of Teachers of Mathematics. Washington, D. C., 1945)

COMPOSITE POLYHEDRON



——— ——— ——— Fold convex

(Fold pattern along lines away from you)

———·———·——— Fold concave

(Fold pattern along lines toward you)

Polyhedron formed by mount_ng equal tetrahedrons on each face of a tetrahedron.

EDGES, FACES, AND VERTICES

Teacher Commentary

A Recreational Activity on Parts of Polyhedrons

I. Materials:

A. Models of 5 regular polyhedrons

B. Models of some other simple polyhedrons

C. Refer to - Recreational: Polyhedrons - Models

D. Refer to - Developmental: Constructing Polyhedrons

II. Procedure:

A. This activity is to be used after the students have had time to construct models of polyhedrons.

B. Distribute the story, "Edges, Faces, and Vertices" and Work Sheet #1.

C. Have students read the story silently. Then have one student read it aloud.

D. Review the story to make sure students understand the problem.

E. Use the models of polyhedrons to identify faces, edges, and vertices.

F. Point to these three parts of a polyhedron, one at a time, and have students name the part.

G. Students should be instructed to work individually or in groups of 2, and collect data to complete columns V, F, and E on their work sheet. Label all of the models to correspond with the names on the work sheet. Place models at different stations in the room. Allow 3 or 4 students to work at each station, then move to the next station.

H. After all students have collected the data, use the overhead projector to project a chart like theirs. Have students fill in the columns. This will serve as a check for the collected data for each student.

I. Have students complete Work Sheet 1. Hopefully they will come up with the rule: $V + F = E + 2$.

J. Give each student an opportunity to demonstrate how to test the rule by using other models. For example, a rectangular solid, a pentagonal prism, a square pyramid, or a triangular prism. Have about six different models labeled and placed around the room. Have each student test the rule using any two models.

K. Have the students apply the rule by completing Work Sheet 2. Be sure to help the student begin this work sheet. You may want to use other models.

L. Since this is a laboratory lesson, the most successful method of evaluation is by interviewing each student individually. Suggested questions for the assessment are given on the next page. The number of each question corresponds to the number of the objectives.

M. Have a model with which the student has never worked.

1. Tell the student to point to a vertex; an edge; a face.

2. Point to some vertex, face, or edge and have the student name the part.

3. Have the student write the rule or tell the rule which gives the relationship between the number of edges, faces, and vertices of a polyhedron.

4. Have the student test the rule using the model. He should count the number of edges, faces, and vertices and substitute these numbers into the formula.

5. Give the student data similar to that on Work Sheet #2. Have him complete each row.

Polyhedron	V	F	E
Octahedron	6		12
Square Prism	8	6	

EDGES, FACES, AND VERTICES

The students in the math class finished making their models. Jean said, "All of these models have edges and faces and vertices. We are always looking for patterns with numbers. Is there any pattern we should look for here?"

In order to answer Jean's question we will have to do an experiment. First we will review the terms:

A FACE of a model is any one of its sides.

An EDGE of a model is a line segment formed where two faces (sides) meet.

A VERTEX is a corner of a model. (If you have more than one vertex, they are called vertices.)

Let's look at each model and count the edges, faces, and vertices of each one. We will put the numbers on a chart. Then we will look for patterns.

EDGES, FACES, AND VERTICES - WORK SHEET 1

Polyhedron	V Number of Vertices	F Number of Faces	E Number of Edges	V + F
Tetrahedron				
Hexahedron				
Octahedron				
Dodecahedron				
Icosahedron				

1. Look at the data which you have collected. Be on the look out for patterns throughout this exercise.

2. Find the sum of V and F for each model which you observed. Enter this sum in the V + F column.

3. Look at the V + F column and the E column. Can you find a pattern between the numbers in these columns? ______________________

4. What pattern do you see? ______________________

5. State a rule which tells this relationship. ______________________

EDGES, FACES, AND VERTICES - WORK SHEET 2

Directions: Fill in the blanks.

Polyhedron	V	F	E
Dodecahedron	20	12	
Equilateral Triangular Prism	6	5	
Hexagonal Prism	12		18
Hexahedron		6	12
Icosahedron	12		30
Octagonal Prism		10	24
Octahedron	6		12
Square Prism	8	6	
Square Pyramid	5		8
Tetrahedron		4	6
Triangular Pyramid	4	4	

ROLLING ALONG

Teacher Commentary

A Recreational Activity on Constructing Curves of Constant Width

I. Materials:

A. Student work sheet entitled, "Rolling Along"

B. Masking tape

C. Rulers for each pair of students

D. Compasses for each pair of students

E. Scissors for each pair of students

II. Procedure:

A. The students will work in pairs to complete the constructions and perform the experiments.

B. Have the class read the introductory paragraph. Discuss this in order to make sure that they have the correct ideas about the problem.

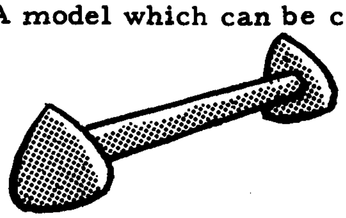
C. Let the class complete the work on Exploratory Exercise 1. Make sure they are supplied with the necessary materials. Some might need help with the constructions.

D. When the curve has been cut out, have one student from each pair cut two one-foot strips of tape.

E. The strips should be taped to the desk so that they are parallel and two inches apart. Some students may require help.

F. Students should roll the cutouts between the two strips and notice if the figure is always touching the strips. Some groups may need help.

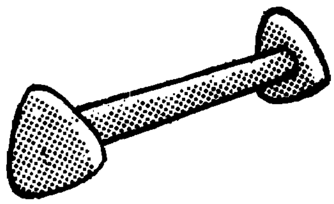
G. A model which can be constructed is illustrated below. You can use dowels if available to fasten several cutouts together. The resulting model will roll as long as the dowel is glued near the center of the cutout.



Note: Do not try to use the dowel as an axle since the cutouts will not operate as wheels. The model can be placed under a large book or sheet of cardboard to demonstrate its rolling properties.

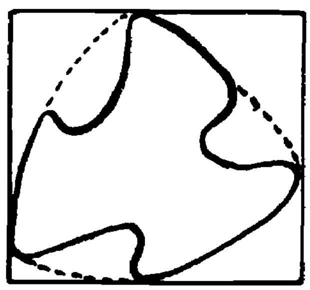
H. Discuss the findings and the definition of a curve of constant width.

I. Proceed with Exploratory Exercise 2. The students will probably need more help with this exercise than the previous one. A model similar to the one on the preceding page can be constructed.



Discuss the results of this exercise reemphasizing the definition of a curve of constant width.

J. If the class is interested in pursuing the square hole drill you can have them construct a model from the curve of Exploratory Exercise 1. Such a model is illustrated below.



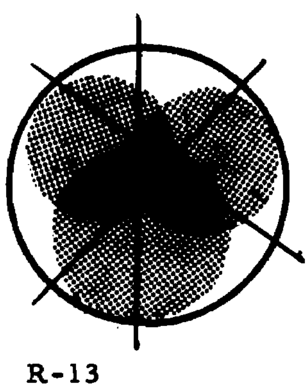
The points would serve as cutting edges. Such a drill was first developed in 1914.

K. An oral activity may be used to evaluate this lesson.

1. Ask students to describe in their own words, what a curve of constant width is.

2. Give students cutouts of various curves. Have them demonstrate whether or not they are curves of constant width.

L. Below is an example of another curve of constant width.



Bibliography

Gardner, Martin. "Mathematical Games" Scientific American, Vol. 208 (February, 1963) pp. 148 - 156.

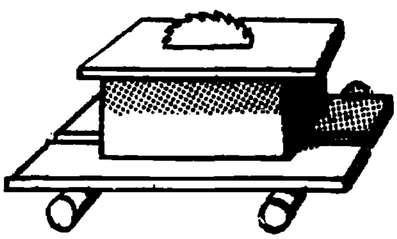
Northrop, Eugene P. Riddles in Mathematics. Princeton, New Jersey: D. Van Nestrand Co., Inc. 1944.

Redemecher, Hans and Tosplits, Otto. The Enjoyment of Mathematics. Princeton, New Jersey: Princeton University Press. 1957.

ROLLING ALONG

Mr. Wood, the shop teacher, needs help. He has to move a heavy saw to a new room. He has asked some of his students to help him.

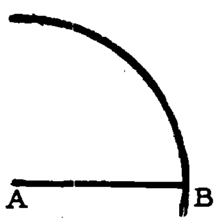
The saw is so heavy that the boys are afraid that it will break the axle on a cart. They decide to use planks and rollers. Jack asks an interesting question. "Can we use rollers when the shape is not circular?" What do you think?



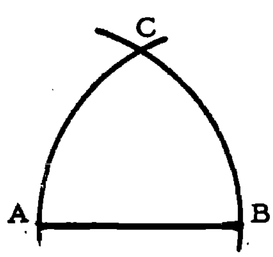
Exploratory Exercise 1

1. On heavy tagboard, draw a two inch line segment. Label the end points A and B.

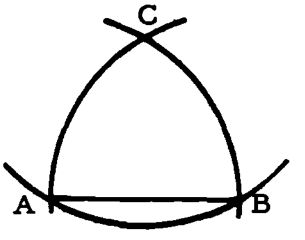
2. Set your compass opening at two inches.
3. Place the compass tip at point A and draw an arc through B.



4. Place the compass tip at B and draw an arc through A which intersects the first arc. Label the intersection of the arcs C.



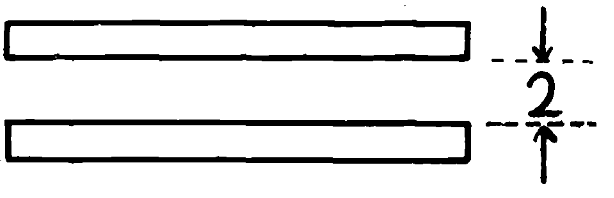
5. Place the compass tip at C and draw an arc through both A and B.



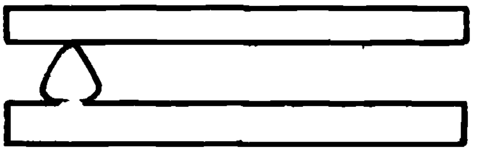
6. Cut out the figure you have drawn.

7. Cut two strips of tape. Each should be one foot long.

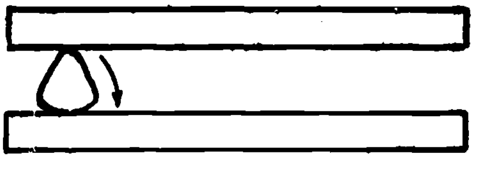
8. Tape the strips to your desk top so that they are parallel and two inches apart.



9. Place your cutout figure between the two strips.



10. Roll the cutout along between the two strips. Does the figure always touch the strips?



You have probably made a surprising discover. The shop class could make rollers which are not circular. They could make rollers in the shape of your figure.

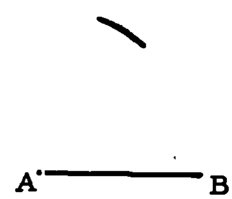
Such rollers would also work.

Figures like a circle and the one you constructed are called curves of constant width. A curve of constant width can be rolled between two parallel lines. It will always touch the two lines. Can we find another curve of constant width? Let's see.

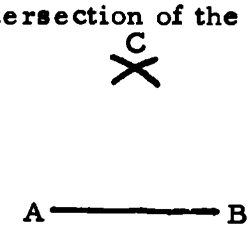
Exploratory Exercise 2

1. On heavy tagboard draw a two inch line segment. Label the end points A and B.

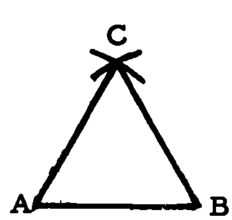
2. Set your compass opening at two inches.
3. Place the compass tip at A and draw an arc as shown.



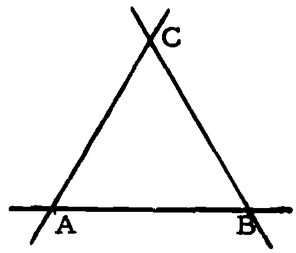
4. Place the compass tip at B and draw an arc which intersects the first arc. Label the intersection of the arcs C.



5. Use your ruler to connect A with C and B with C.

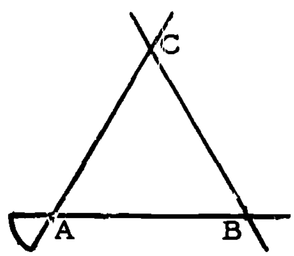


6. Using your ruler, extend each segment $\frac{1}{2}$ inch as shown.

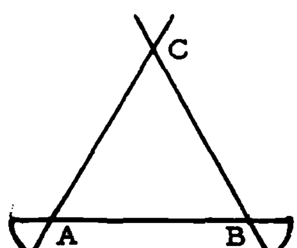


7. Set your compass opening at $\frac{1}{2}$ inch.

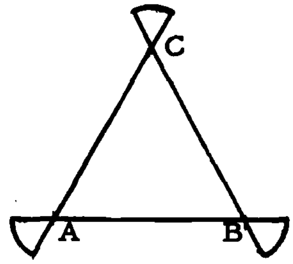
8. Place the compass tip at A and draw an arc such as the one shown.



9. Place the compass tip at B and draw an arc like the one at A.

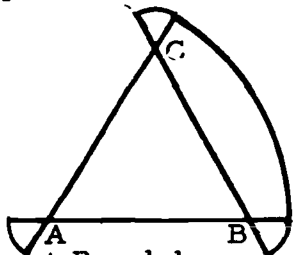


10. Place the compass tip at C and draw an arc like the ones at A and B.

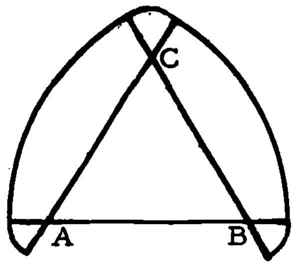


11. Set your compass opening at $2\frac{1}{2}$ inches.

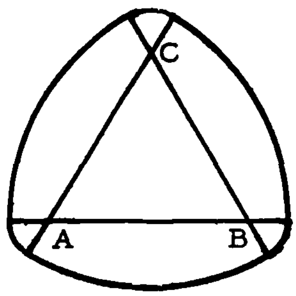
12. Place the compass tip at A and draw an arc as the one shown.



13. Place the compass tip at B and draw an arc like the one formed from A.



14. Place the compass tip at C and draw an arc like the ones from A and B.



15. Cut out your figure.

16. Use the strips of tape from Exploratory Exercise 1 to see if this figure is a curve of constant width.

You should have found another curve of constant width. There are many other types. An interesting application can be made of the curve you found in Exploratory Exericse 1. Using this curve it is possible to construct a drill which will drill a square hole.

HIDDEN MESSAGE

Teacher Commentary

A Recreational Activity on Operations With Signed Numbers

I. Materials: Student work sheets

II. Procedure:

A. Distribute a work sheet to each student.

B. Each student is to decode the hidden message.

C. Solution:

Sorry About That

A HIDDEN MESSAGE

Secret messages are sometimes in code. Often the code will have numbers or letters in it. Today you will figure out a message by solving some examples.

I. Each solution represents a letter.

1. $(-1)(-4)$
2. $(5)(-2)(0)(2)$
3. $-3 + 6$
4. $5 - 2$
5. $12 + 4 - 6$
6. $6 - 19$
7. $(6)(-2)$
8. $(6) + (-6)$
9. $(-12) \div (-2)$
10. $2 - 12 + 15$
11. $(-1) + (-1) + (+7)$
12. $(-2) + (-4)$
13. $2(-6) + (-1)$
14. $(-25) \div (-5)$

II. Look at your first answer (4) and find that number in the decoding chart. Right under this number you will find the letter it stands for (S). Write the letter (S) in space 1 in the hidden message. Do the same thing for all of your answers, and you will see the hidden message.

DECODER

-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
A	B	C	D	E	F	G	H	I	J	K	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

HIDDEN MESSAGE

<u>Hint!</u> This message is a saying of a television star.

S
___1 ___2 ___3 ___4 ___5   ___6 ___7 ___8 ___9 ___10   ___11 ___12 ___13 ___14

GEOMETRY VOCABULARY

Teacher Commentary

A Recreational Activity on Geometric Terms

I. Materials: Work sheet

II. Procedure:

A. Distribute work sheet.

B. Allow students to compare papers to find the ones they were unable to do.

C. With papers turned over, try to see how many words they can spell. Do not push for 100% proficiency.

GEOMETRY VOCABULARY

Try to find as many of these words in the chart as you can. Some words are written across the row while others go down the column. Two words are written diagonally. Circle the words when you find them.

triangle	hexagon	compass	bisect	obtuse
square	octagon	parallel	vertical	right
rectangle	circle	angle	horizontal	degree
pentagon	protractor	perpendicular	acute	polyhedron

R	S	P	O	L	Y	H	E	D	R	O	N	N	O	C	P
S	T	Q	A	R	C	P	R	O	T	R	A	C	T	O	R
A	C	I	R	C	L	E	B	R	R	S	T	U	F	M	I
T	C	F	O	T	S	N	A	Q	A	X	L	M	E	P	G
B	A	U	B	U	B	T	J	P	W	G	D	H	V	A	H
U	V	Y	T	R	I	A	N	G	L	E	H	W	I	S	T
V	B	V	U	E	K	G	Y	Q	V	X	V	U	T	S	S
E	I	Y	S	C	X	O	W	P	A	R	A	L	L	E	L
R	S	W	E	T	C	N	C	O	N	H	G	J	F	R	J
T	E	Z	B	A	B	Z	D	T	E	I	K	U	G	H	F
I	C	X	Y	N	A	I	G	M	A	C	T	I	S	D	C
C	T	Z	L	G	J	R	C	S	L	G	J	K	Q	D	B
A	W	M	Y	L	K	F	M	K	L	M	O	I	U	E	A
L	Z	L	H	E	X	A	G	O	N	D	H	N	A	E	O
C	X	H	O	R	I	Z	O	N	T	A	L	N	R	F	G
O	N	E	Z	N	O	D	P	D	E	G	R	E	E	P	P
Q	P	E	R	P	E	N	D	I	C	U	L	A	R	Q	E

FINDING AREA BY PROBABILITY

Teacher Commentary

A Recreational Activity on Areas and Probability

I. Materials:

A. A cork board. Cork bulletin boards can be used. The experiment can actually be left as a bulletin board after the unit.

B. 3 darts

C. A blindfold

D. Tag board shapes showing areas of:

1. square
2. rectangle
3. triangle
4. circle
5. irregular figure with known area - see sketch
6. parallelogram
7. irregular shapes of unknown area - smooth curves see sketch

II. Procedure:

A. Setting up the experiment

1. Have the pupils measure the length and width of a small bulletin board in the room. These are approximately 2' x 3'. Use the exact measurements. In the examples following we will use 2' x 3' as our measurements.
2. Have the pupils compute the area of the bulletin board.
3. Staple 1' x 1' square anywhere on the bulletin board. Do not tell the pupils the area of the square.

B. Determining the area of the square by probability methods

1. Blindfold a pupil and direct him toward the bulletin board at a distance of 5 paces. Be sure to change the position of the square after each child has been blindfolded so that he will not consciously try to aim at the original position of the square.

2. Each pupil has a turn of throwing 3 darts. If the dart hits outside of the bulletin board he throws again.

3. After the last pupil finishes throwing the darts list the following data on the board:

   a. Total number of hits on the target. This is the sum of the hits on the corkboard including those which hit the square.

   b. Total number of hits on the square.

4. Place the following proportion on the board:

$$\frac{\text{Total number of throws}}{\text{Area of the corkboard}} = \frac{\text{Number of hits on the square}}{\text{x (area of the square)}}$$

   Example: $\frac{60}{6} = \frac{10}{x}$ Solving the proportion for x we have $x = 1$.

5. Measure the sides of the square and have the pupils compute its area (1 square foot). Compare the results obtained by this method with the area found by probability. Discuss the reliability of the experiment.

C. Additional experiments with familiar figures

1. Place other familiar figures on the corkboard. The rectangle, triangle, parallelogram, circle, and trapezoid can be included.

2. Repeat the experiment

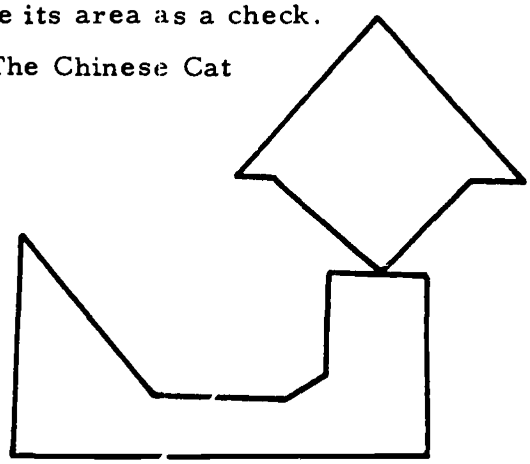
3. Check the experimental area against the computed area

D. Experiments with other polygonal shapes of known areas.

1. Begin with the Chinese Cat figure and find its area by the probability method.

2. Rearrange the Chinese Cat figure to form a square and then compute its area as a check.

Example: The Chinese Cat



How to draw the Chinese Cat

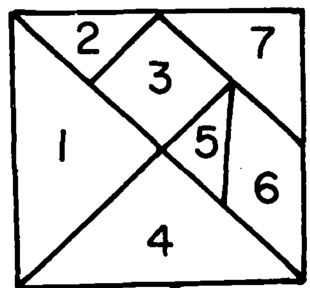
a. Draw a square 11" x 11"

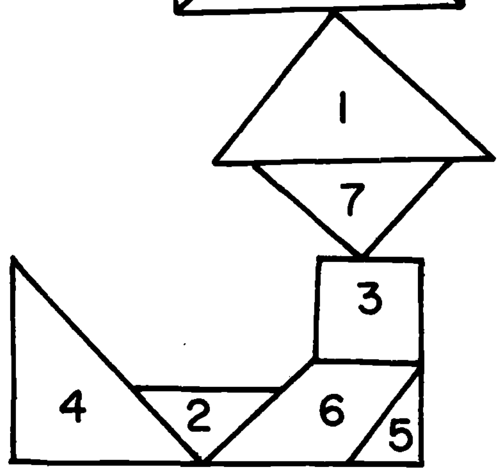
b. Divide the square as shown
This is a tangram.

c. Number the pieces

d. Cut on the lines

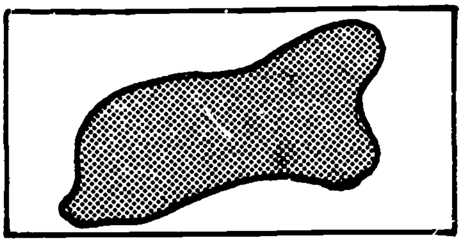
e. Put the numbers in the positions shown below to make the cat.





E. Experiment with non-polygonal shapes with unknown areas.

Example:



1. Place non-polygonal shape with rounded curves, of an unknown area on the corkboard.
2. Ask pupils how they might determine the area of the figure.
3. Perform the experiment.
4. Discuss the results.

PERIMETER-AREA-VOLUME CROSS NUMBER PUZZLE

Teacher Commentary

A Recreational Activity on Perimeter, Area, and Volume

I. Materials: Student work sheet

II. Procedure:

A. Distribute copies of the attached sheet.

B. Review the formulas needed to complete this puzzle before students begin work.

C. Solution:

3	6		1	2	2	5
2	2		2	4	6	5
1	5	4		2	1	
		1	1	4		
	1	6		4	3	2
1	7	2	8		1	5
7	8	5	4		5	0

PERIMETER-AREA-VOLUME CROSS NUMBER PUZZLE

Directions: Fill in the blanks. Use decimals for all fractions.

1	2	■	3	4	5	6
7		■	8			
9		10	■	11		■
■	■	12	13		■	■
■	14		■	15	16	17
18			19	■	20	
21				■	22	

Horizontal

1. Perimeter of a 9" square in number of inches.
3. Area in number of square inches of a square with side $3\frac{1}{2}$ inches.
7. Circumference in number of inches of a circle having a diameter of 7". (Use pi equal 22/7.)
8. Area in number of sq. ft. of a rectangle that is 17" by $14\frac{1}{2}$".
9. The diameter of a circle is 14". Find area in sq. in. (Use pi equal 22/7.)
11. Perimeter in number of inches of a triangle 7 in. on a side.

Vertical

1. Each side of a triangle is 107 in. How many inches in the perimeter?
2. Number of sq. in. in a 25-in. square.
3. Number of inches in the perimeter of a 3-in. square.
4. Number of sq. ft. in a parallelogram with an altitude of 44 ft. and base of 551 ft.
5. Number of cu. in. in a metal plate 3 in. square and .29 in. thick.
6. Area in sq. in. of a parallelogram with an 11-in. base and 5-in. altitude.

Horizontal

12. [illegible] in sq. in. of a tri-a[illegible]e with a base of 19 in. and an altitude of 12 in.

14. Diameter in number of in. of a wheel having a radius of 8 in.

15. Volume in number of cu. in. of a rectangular solid 9 in. by 8 in. by 6 in.

18. Number of cu. in. in a cube 1 ft. on a side.

20. How many 2-in. squares will fit into a rectangle 10 in. long and 6 in. wide?

21. Number of cu. in. in a right cylinder 10 in. in diameter and 10 in. high. (Use 3.1416 for pi.)

22. Perimeter in number of inches of a rectangle 15 in. long and 10 in. wide.

Vertical

16. Number of cu. in. in a rectangular solid 9 in. long, 7 in. wide, and 5 in. high.

17. Number of cu. ft. in a pyramid with a base 10 ft. square and height of $7\frac{1}{2}$ ft.

18. Number of sq. yds. in a triangle with a base of 18 ft. and an altitude of 17 ft.

19. Number of cu. ft. in a rectangular solid 7 ft. long, 4 ft. wide, and 3 ft. deep.

CAN YOU EARN A $500 BONUS CHECK?

Teacher Commentary

A Recreational Activity on Finding Volume and Surface Area of Rectangular Solids

I. Materials:

A. Student work sheet "Can You Earn a $500 Bonus Check?"

B. Pencil and paper

II. Procedure:

A. Introduce the lesson by telling the class about the policy of issuing bonuses for exceptional work or profitable suggestions in industry. For example, Bendix Corporation used to make radios for Ford automobiles. A person who now teaches in Baltimore County used to work at Bendix. His job was to drill holes in the casing for the radio. The complete set of holes required three separate operations, involving three men. He realized the same job could be done in two operations. He dropped his suggestion in the suggestion box, and the next day it was taken into consideration. Soon the suggestion was adopted and he was awarded $500.

B. Distribute the work sheet. Make the students feel as if each of them is "Mr. Smarts." Have them realize that there is not only value in offering a suggestion, but the way it is presented on paper is of equal importance. ("The boss may throw it in the trash if it is not neat.")

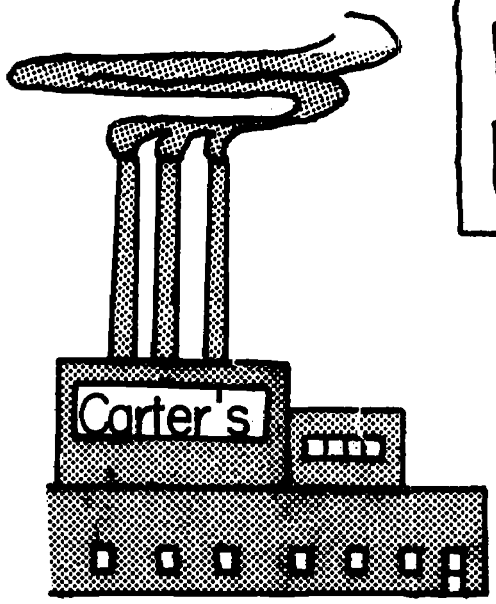
C. When the students find the surface area and volume of the five cartons, they may feel they have discovered the key $500 idea. They will probably see that all volumes are the same, whereas the surface areas differ, and conclude that the $500 idea is to use the carton with the least surface area. When the student reaches this conclusion, it may spark the lesson by telling him he has earned $250 of the bonus, but there is still another idea worth an additional $250. A comparison of dimensions and areas will suggest the carton with the smallest surface area to be used.

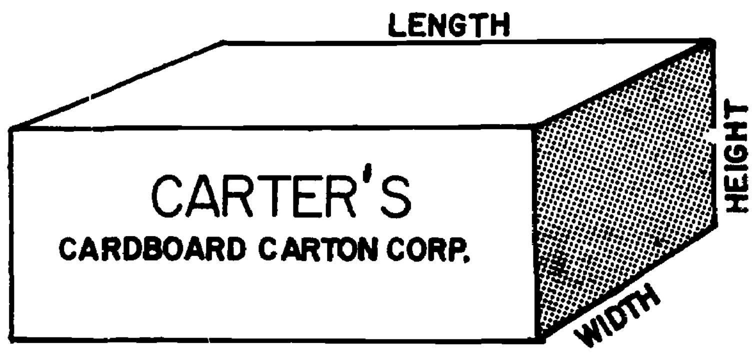
Solutions:

length	width	height	surface area square inches	volume cubic inches
36	12	4	1248	1728
24	12	6	1008	1728
27	16	4	1208	1728
16	12	9	888	1728
27	8	8	992	1728

The $500 answer is that a cube 12 x 12 x 12 would be the most efficient. Its surface area is 864 sq. in. and volume is also 1728 cu. in. Of course mention should be made of the fact that the product would restrict the size and shape of the carton.

Can You Earn A $500 Bonus Check?





Mr. Smarts works at a box factory. He makes cardboard boxes. The factory makes many different types of boxes, but all the boxes hold the same amount of goods. Mr. Smarts figured out a way to make a box that held just as much but used less cardboard. The owner of the factory gave Mr. Smarts a $500 bonus for his idea. Now the factory saves $500 a year on cardboard because they use less of it.

See if you can earn a bonus like Mr. Smarts. Here is what he did. He first made a list of all cartons, and their sizes made in the factory. Then he made this chart.

	Length	Width	Height	Square Inches of Cardboard Needed	Volume in Cubic Inches
Box A	36"	12"	4"		
Box B	24"	12"	6"		
Box C	27"	16"	4"		
Box D	16"	12"	9"		
Box E	27"	8"	8"		

When he had filled in his chart, Mr. Smarts noticed a way to use much less cardboard by making only one size box. See if you can find out which box he suggested that earned him his $500 bonus check. This box is not on the chart because it has never before been made at this factory.

HIDDEN WORD PUZZLE

Teacher Commentary

A Recreational Activity on Vocabulary Words From Algebra

I. Materials: Student work sheet with square array of letters

II. Procedure:

A. Give each student a duplicated sheet.

B. Have him find as many hidden math words as he can.

C. Solution:

I		E	X	P	O	N	E	N	T					
D		Q		I	N	F	I	N	I	T	E	S	E	T
E		U				I							N	T
N	R	A	T	I	O	N	A	L	N	U	M	B	E	R
T		T				I						S	G	
I	C	I		S	E	T					B	C	A	P
T		O			T	E	R	M		U		L	T	O
Y		N	N		E	S			S	C		U	I	S
E				S		E	Q	U	A	L		T	V	I
L			A		T	T				O		I	E	T
E		B				A				S		O		I
M	E	M	B	E	R		N			U		N		V
E					F	A	C	T	O	R		S		E
N	V	A	R	I	A	B	L	E		E		E		
T	C	O	M	M	U	T	A	T	I	V	I	T	Y	

LANGUAGE OF ALGEBRA PUZZLE

See if you can find words from algebra in the puzzle below. Some words run across; some run down. A few words go diagonally. Altogether there are 20 words. How many can you find?

I	Q	E	X	P	O	N	E	N	T	A	R	P	E	B
D	A	Q	U	I	N	F	I	N	I	T	E	S	E	T
E	R	U	D	O	F	I	P	L	O	R	I	W	N	T
N	R	A	T	I	O	N	A	L	N	U	M	B	E	R
T	L	T	O	M	U	I	V	A	L	C	E	S	G	P
I	C	I	S	S	E	T	F	E	N	G	B	O	A	P
T	R	O	M	A	T	E	R	M	B	U	A	L	T	O
Y	C	N	N	Q	E	S	G	U	S	C	L	U	I	S
E	M	A	R	S	P	E	Q	U	A	L	O	T	V	I
L	U	P	A	V	T	T	D	E	F	O	D	I	E	T
E	R	B	L	E	R	A	M	O	X	S	T	O	K	I
M	E	M	B	E	R	L	N	K	E	U	R	N	S	V
E	T	T	A	N	F	A	C	T	O	R	T	S	O	E
N	V	A	R	I	A	B	L	E	R	E	K	E	D	G
T	C	O	M	M	U	T	A	T	I	V	I	T	Y	O

WORD MAZE

Teacher Commentary

A Recreational Activity on Vocabulary Words
From Algebra and Geometry

I. Materials: Work sheet with maze

II. Procedure:

A. Give each student a work sheet

B. Have each student try to find the ten math words hidden in the maze

C. You may give the words to be found or you may allow students to find them without hints.

Hidden Words:

1. angle
2. ellipse
3. parallel
4. pi
5. ratio
6. ray
7. set
8. similar
9. tangent
10. term

WORD MAZE

There are ten math words hidden in the large square below. Can you find them? Here is what to do.

You may start in any square and move in any direction to the square next to it. You may continue to move one square in any direction until a word is made. You may <u>not</u> enter the same square twice while spelling a word.

T	L	E	Y	O
N	G	N	A	I
E	M	R	L	T
S	A	L	E	L
P	I	M	I	S

SCRAMBLED WORDS

Teacher Commentary

A Recreational Activity on Measurement

I. Materials: Work sheet of scrambled words

II. Procedure:

A. Hand out student work sheets.

B. Have students unscramble words.

C. Solution:

1. caliper
2. volume
3. cube
4. cylinder
5. micrometer

SCRAMBLED WORDS

Unscramble the letters to form measurement words:

1. repical
2. muvole
3. buce
4. redlincy
5. errocimmet

SCRAMBLED WORDS

Teacher Commentary

A Recreational Activity on Vocabulary Words for Probability

I. Materials: Work sheets of scrambled words

II. Procedure:

A. Hand out work sheets to students.

B. Have students unscramble words.

C. Solution:

1. event
2. outcome
3. probability
4. factorial
5. arrangements

SCRAMBLED WORDS

Unscramble the letters to form probability words:

1. veent

   ecomtuo

3. labiprobity
4. actorfail
5. ranmenargets

SCRAMBLED WORDS

Teacher Commentary

A Recreational Activity on Vocabulary for Geometry

I. Materials: Work sheet of scrambled words

II. Procedure:

A. Give each student a work sheet.

B. Have each student unscramble the letters to form vocabulary words.

C. Solution:

1. triangle
2. square
3. rectangle
4. rhombus
5. tangent
6. ellipse

SCRAMBLED WORDS

Unscramble the letters to form math words:

1. glaterin
2. rsueqa
3. artclegne
4. hursbom
5. ttaegnn
6. leleisp