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ABSTRACT

The procedures and results of a national study to measure the familiarity of 1165 pre-calculus mathematical terms and 154 mathematical symbols are reported. Unique tests of 100 randomly selected mathematical terms as well as unique tests of 36 randomly selected mathematical symbols were generated by a computer. The familiarity of each term was based on the responses of approximately 350 students. The familiarity of each symbol was based on the responses of approximately 250 students. The student sample consisted of approximately 5,500 seventh and eighth graders from 36 schools. The schools were chosen nationwide by a proportionate stratified random sampling plan. Measures of the familiarity of mathematical terms and symbols were established. (Author/CT)

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Measuring Vocabulary and Symbol Familiarity in the Language of Mathematics

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A research study was conducted by the authors in 1969 to measure the familiarity of mathematical terms and symbols. The significance and implications of this study in readability research, as well as the procedures and results of the national study will form the content of this paper.

Readability formulas have been used widely to assess the ease with which readers will comprehend written materials. A readability formula is a prediction equation. Independent or predictor variables that have been used in readability formulas include vocabulary difficulty, average sentence length, number of personal pronouns, and many others (Klare, 1963 and Chall, 1958). The number and type of predictor variables have varied greatly. However, some measure of vocabulary difficulty has usually appeared among the predictor variables.

Vocabulary difficulty has often been defined by using a list of words having a certain level of familiarity or frequency. Vogel and Washburne (1928) used a count of the words not found on the Thorndike list of 10,000 words. Dale and Chall (1948) used the percentage of words not on the Dale list of 3,000 words.

Both the Thorndike list and the Dale list were based on either the frequency or the familiarity of words in general usage. Chall (1958) recommended that adaptations be made in word lists and the resulting vocabulary measures when readability formulas are used with materials containing a specialized vocabulary, as is the case for mathematics textbooks. Mathematical materials contain an assortment of words and symbols: words with generally accepted meanings; words with general meanings used often in specific contexts, such as "set"; words with a mathematical meaning different from another ordinary meaning, such as "field" and "plane"; words with only a mathematical meaning, such as "perimeter" and "subtraction"; and a complex symbol system including such symbols as the square root sign and the equal sign.

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Prior to research conducted in 1969, there did not exist a measure of the difficulty of terms and symbols specific to mathematics. In the absence of such a measure, mathematical words usually have been classified as difficult and mathematical symbols ignored in applying existing readability formulas to mathematical materials (Heddens & Smith, 1964; Smith and Heddens, 1964; F. Smith, 1969; Wiegand, 1967).

The importance of vocabulary measures in general readability formulas and the necessity of adapting such measures to mathematical vocabulary argue for the development of a mathematical word list. A difficulty measure of mathematical words and symbols seems a necessary step in developing readability formulas appropriate for mathematical materials.

Developing measures of the familiarity of mathematical terms and of mathematical symbols presents problems of measurement and sampling. The construct, vocabulary difficulty, must be given an operational definition in terms of instruments used to scale mathematical terms and mathematical symbols on the attribute of familiarity. Sampling procedures require a definition of exactly what constitutes a mathematical term. There are many instances in mathematics where a phrase, rather than a word, is used to name a concept. For example, consider the term "absolute value". Is it appropriate to test only the single words "absolute" and "value"? If "absolute value" is tested only as a phrase however, will it count as one word or two words when used in a readability word count? As another example, the word "degree" has different mathematical meanings, depending on the context in which it is used. Consider, for instance, "arc degree" and "degree of a term". Further, if a decision is made to measure familiarity by student response, techniques of sampling students as well as techniques of sampling mathematical terms and mathematical symbols must be determined.

Possible solutions to these problems of measurement and sampling were proposed in a national study conducted by the authors in the fall of 1969. The research proposed to develop measures of the familiarity of mathematical terms and mathematical symbols to seventh and eighth grade students in the United States. Seventh and eighth grade students were chosen as subjects for the following reasons: (a) the set of mathematical terms and symbols appropriate at these grade levels seems less restricted than at lower grade levels and also less diverse than would be found at higher grade levels where different content may be studied in different classes; (b) these two grades would usually be in a single school and therefore increase the efficiency of data collection.

Two measuring instruments were developed in the study; one for mathematical terms and one for mathematical symbols. The measuring instrument for the familiarity of mathematical terms directed the student to judge whether each term was familiar to him. A term was described as familiar if the student could remember a definition, give an example, or explain the term in his own words. The student then placed a check mark under either "know" or "do not know". The familiarity score for each term was defined as the percentage of students responding to the item who indicated by a check mark that they knew the term.

The measuring instrument for the familiarity of mathematical symbols directed the student to write something he knew about the symbol or to place a check mark under "do not know." The responses written by students were scored. The familiarity score for each symbol was defined as the percentage of students responding to the symbol who wrote something acceptable about the symbol.

The reason for developing different measuring instruments can be explained as follows: a) 1165 terms were tested; only 154 symbols were tested; b) An acceptable response for a symbol was usually a translation, such as "empty set" for the symbol "$\emptyset$". It may be more difficult for students to express adequately what they know about mathematical terms than about mathematical symbols. It would also seem to be more difficult for scorers to judge the acceptability of a student's explanation of terms. These practical considerations as well as the results of a preliminary study using different types of test directions, argued the appropriateness of the instruments used.

The sampling frame, that is, the list of mathematical terms to be tested, was compiled from pre-calculus mathematics textbooks. Only pre-calculus materials were considered since the primary interest was the readability of elementary and secondary school materials. The following terms were not considered part of the sampling frame: names of mathematicians, theorems, figures; abbreviations; and terms from science and business that do not have a special mathematical meaning. A phrase was omitted when the meaning of the phrase could be derived from knowledge of the words contained in the phrase, such as "greatest common factor" and "negative reciprocal". In other cases, whenever a phrase was included, such as "additive identity" each word was also included alone if it had a mathematical meaning written alone. "Additive" and "identity" were included in the frame. The phrase "absolute value" and the word "value" were part of the sampling frame. "Absolute" was not included.

The familiarity measure will probably be used to establish a readability formula for mathematical English. Some type of word count will at least be tested as a predictor variable. For the formula to be efficient,a predictor variable must be simple to measure and well-defined. The presence of phrases on a word list may pose a problem. If the phrase "numeration system" is unfamiliar, how many unfamiliar words are counted?

Therefore, it seemed appropriate to have the option of considering each word of the passage separately. "Word" is used here in a broad sense as a set of letters enclosed by spaces. The option of considering each such word separately requires that a measure of familiarity be found for each word. Therefore, the measure of familiarity was defined on a word, but a word that was often listed and tested in the context of a phrase. In each phrase one of the words is enclosed with asterisks (example: * acute * triangle). The word so designated is the word whose attribute of familiarity is to be measured; but in the context of the given phrase.

The sampling frame for mathematical terms included 1165 terms. A randomization program was used to have a computer generate unique tests of 100 randomly chosen mathematical terms. Approximately 5,000 tests were produced.

The sampling frame for mathematical symbols was also compiled from pre-calculus mathematics textbooks. Letters, digits, placeholders such as $\square$, and superimpositions of letters and symbols, such as $\not{s}$., were not included in the frame. In establishing the sampling frame for the symbols some context was usually necessary to show the position of the symbol, such as 2 in 5^2, or the use, such as ( ) in (2 + 4). The symbol actually being tested was printed in red to distinguish it from the surrounding context.

The sampling frame for mathematical symbols included 154 symbols. Nine tests were constructed, each containing 36 symbols. The symbols were randomly selected and assigned to at least two of the tests.

The selection of students to form the sample of seventh and eighth grade students in the United States was made by a sample design that may best be described as a proportionate stratified random sampling plan. Schools in the United States were stratified on the factors of geographic location, size of school, type of community-urban or rural, and type of school-public or private.

The percentage of the total population of students represented in each of the strata determined the sample size for each strata. A random selection of schools was made from each strata. The tests of terms and symbols were then randomly assigned to all the seventh and eighth grade students in the participating schools.

Judgment on the part of the researcher or school administration, and availability of schools did not enter into either the selection of schools or the selection of students within a school that would respond to any given test or item. Data from 36 schools were obtained, providing approximately 350 responses to each term and 250 responses to each symbol. The sampling techniques used in the study provide a precision of approximately 5%.

A measure of judged familiarity for 1165 mathematical terms and of evidenced familiarity for 154 symbols was established as a result of this study. Credibility checks built into the tests of terms indicated the error in measurement due to respondents' carelessness, confusion,

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or distortion. The estimated error was 2.5%. The stability of the familiarity scores for terms was checked in a test-retest situation. A product-moment correlation of 0.94 was computed between the percentage of familiarity on the test and the percentage of familiarity on the retest across 1165 terms. Approximately 5% of the tests for symbols were scored by two different scorers and a level of agreement of 96% was reached.

The frequency distribution of mathematical terms and of mathematical symbols according to intervals of familiarity is given in Table 1. The large number of terms known by less than 40% of the students tested is expected since the sampling frame included many terms beyond the level of seventh and eighth grade mathematics.

Pages 7 and 8 contain a list of mathematical terms with a judged familiarity score of 90%-100% and 80%-90% respectively. Page 9 contains a list of the mathematical symbols known by at least 70% of the students tested.

Results of the study have implications for the measurement of the vocabulary difficulty of mathematical materials. Evidence indicates that students distinguished between word forms quite precisely. Note a few examples selected by the use of random numbers from the list of 1165 terms.

bisect	27%	compute	57%
bisection	24%	computation	38%
bisector	28%		
proportion	46%	sum	98%
proportional	31%	summation	17%

equal	92%
equation	83%
equality	72%
equate	24%

Both the consistency of student responses and the differentiation they attach to the form in which mathematical concepts are written is evidenced in the data on page 10.

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TABLE 1

FREQUENCY DISTRIBUTION OF MATHEMATICAL TERMS AND SYMBOLS ACCORDING TO INTERVALS OF FAMILIARITY USING THE DATA OF THE NATIONAL STUDY

Interval of Familiarity Score	Number of Terms in National Study	Number of Symbols In National Study
90.000 - 99.999	79	7
80.000 - 89.999	88	6
70.000 - 79.999	118	8
60.000 - 69.999	95	5
50.000 - 59.999	107	4
40.000 - 49.999	117	11
30.000 - 39.999	137	16
20.000 - 29.999	136	7
10.000 - 19.999	146	17
0.000 - 9.999	142	72

List of Mathematical Terms Arranged

Alphabetically within Two Intervals of Judged-Familiarity

Terms JUDGED FAMILIAR by 90%-100% of the students tested.

ADD
ADDITION
ANGLE
ARITHMETIC
CIRCLE
COUNT
COUNTING
* DECIMAL * POINT
DIFFERENCE
DIVIDE
DIVIDEND
DIVISION
DIVISOR
EIGHT
EQUAL
* EVEN * NUMBER
FACTOR
FIGURE
FIVE
FOOT
FOUR
FRACTION
GREATER
GREATER THAN
GREATEST
HALF
HOUR
LENGTH
LESS
LESS THAN
LINE
MATHEMATICS
MEASURE
MEASUREMENT
MILLION
MULTIPLE
MULTIPLICATION
MULTIPLIER
MULTIPLY
NINE
NOT EQUAL TO
NUMBER
NUMBER * LINE *
NUMERAL
* ODD * NUMBER
ONE
* PARALLEL * LINES
PLUS
* PLUS * SIGN
PLUS * SIGN *
POINT
* PRIME * NUMBER
PROBLEM
PROBLEM * SOLVING *
PRODUCT
REMAINDER
* ROMAN * NUMERAL
* ROUND * OFF
* ROUNDED * NUMBER
RULER
SET
SEVEN
SIX
SOLVE
SQUARE
* STRAIGHT * LINE
SUBTRACT
SUBTRACTION
SUM
TEN
TENS
THREE
TOTAL
TRIANGLE
TWO
VALUE
* WHOLE * NUMBER
YARD
ZERO

Terms JUDGED FAMILIAR by 80% - 90% of the students tested.

AREA
AVERAGE
BASE
BASE-TEN
BILLION
BORROWING
CARRYING
CENTER
CHART
CHECK
* CLOCKWISE * MOTION
CLOSED
COLUMN
* COMMON * DENOMINATOR
COMPARE
COMPARING
COMPASS
CUBE
DECIMAL
* DECIMAL * NUMERAL
DECIMAL * SYSTEM *
DEFINITION
DIAGRAM
DIAMETER
* DIAMETER * OF A CIRCLE
DIGIT
DIRECTION
DISTANCE
DIVISIBLE
DOT
DOUBLE
EMPTY SET
* EQUAL * SETS
EQUATION
ESTIMATE
* FRACTIONAL * NUMERAL
GRAPH
GROUP
HEIGHT
HORIZONTAL
* IMPROPER * FRACTION
INDEX
LINE * SEGMENT *
LONG DIVISION
* LOWEST * TERMS
MATHEMATICAL
MINUS * SIGN *
* MIXED * NUMBER
* MIXED * NUMERAL
* MULTIPLE * OF A NUMBER
NUMBER * BASE *
NUMBER * SENTENCE *
NUMERATOR
OPPOSITE
PAIR
PARALLEL
* PLACE * VALUE
PLANE
* PROPER * FRACTIONS
QUART
RECTANGLE
* RIGHT * ANGLE
ROUNDING
SAMPLE
SECOND
SENTENCE
* SHORT * DIVISION
* SIDE * OF A TRIANGLE
SIGN
* SIMPLEST * FORM OF A FRACTION
* SIMPLEST * FRACTIONAL NUMBER
SOLVING
* SQUARE * YARD
SYMBOL
SYSTEM
TABLE
* TAKE AWAY * PROCESS
TERM
TESTING
TESTS
* TRUE * NUMBER SENTENCE
UNITS
* UNLIKE * FRACTIONS
* VERTICAL * LINE
WEIGHT
WIDTH
* WORD * PROBLEM
YEAR

List of Mathematical Symbols Arranged Within Three Intervals of Evidenced-Familiarity

Symbols KNOWN by 90%-100% of the students tested.

2 + 7	$3.42
8 − 3	6 ÷ 3
2 × 3	3%
2¢	

Symbols KNOWN by 80%-90% of the students tested.

$5\overline{)20}$	$\begin{array}{r} 8 \\ +\ 2 \\ \hline \end{array}$
VI	
4^2	$\frac{9}{3}$
II	

Symbols KNOWN by 70%-80% of the students tested

IV	2.46
	c > 3
1, 3, 5, . . .	a = 3
+ 5	90°
X	

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commutative	71%	associative	76%	distributive	67%
* commutative * property	72%	* associative * property	76%	* distributive * property	73%
* commutative * law	68%	* associative * law	63%	* distributive * law	63%
* commutative * principle	62%	* associative * priciple	63%	* distributive * rule	57%
commutativity	44%	associativity	39%	distributivity	38%

When using the Dale-Chall readability formula (Dale, 1948) counting unfamiliar words requires the use of rules set up for different word forms. Regular plurals of nouns and past-participle forms (adding "ing") of verbs on the familiar word list are counted as familiar. However, a noun formed by adding "er" to a familiar verb, such as "burn", is not familiar unless the form "burner" is on the familiar list. Similar rules may be established for mathematical words following investigations of mathematical word forms now being initiated.

Evidence from the national study also indicates that students respond discriminately to mathematical words as used in different contexts. Two examples were selected using random numbers.

square	94%	* degree * of an angle	77%
* square * yard	90%	degree	76%
* square * measure	78%	arc * degree *	48%
* square * root	64%	* degree * of an arc	45%
* square * of a number	62%	* degree * of a term	35%
		* degree * of an equation	33%
		* degree * of a polynomial	26%

"Degree" received a familiarity score of 76%, "equation" of 83%. However, "* degree * of an equation" had a familiarity score of 33%. A count of prepositions may be investigated as a factor accounting for the vocabulary difficulty caused by a phrase such as "degree of an equation."

In conclusion, a measure of the familiarity of terms and symbols specific to mathematics does now exist. The construct validity and predictive validity of the measuring instruments used must be investigated in future research. A major question still remains in readability research. Will a measure of vocabulary difficulty have predictive power in a readability formula?

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