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ABSTRACT

Difficulties inherent in the reading of mathematics at secondary and college levels are discussed. Special emphasis is placed on the reading of arithmetic numerals, literal numbers, operational symbols, and expressions of relationships, as well as the reading of technical vocabularies and specialized meanings of general words. While each mathematical field has its own symbolization and terminology or shares those with others, they may already be or may become at the same time obstacles toward efficient reading in that content field. To seek remediation and, ultimately, prevention of mathematical reading inefficiencies, a Group Informal Reading Inventory, a visual-auditory-kinesthetic-tactile approach, and a Directed Reading Activity (DRA) are recommended. In addition, a complete DRA lesson (including readiness, guided silent reading, questions, oral rereading, and application) featuring a specific algebraic problem is presented in order to demonstrate its use in a classroom setting. References are given. (Author/DE)

READING IN MATHEMATICS

by Horst G. Taschow, Ph.D.

A language within a language! A language which needs more skill and knowledge in decoding than in computation!

The reading in the languages of arithmetic, algebra, geometry, and trigonometry demands mastery of both the verbal and the mathematical symbols. While the language in these mathematical fields with their subdivisions is concise, the reading is precise, particular, and critical. In addition, each mathematical field is spiced with its own distinct vocabulary and system of symbolization.

Verbal Symbols And Mathematical Symbols:

Consider, for example, the following rule written in verbal symbols: "The diagonal of a square equals the square root of twice the area, " and the same rule written in its special shorthand method or formula $D = \sqrt{2A}$. Or consider the following system of symbolization $A = \pi R^2 - \pi r^2$ which, conversely, the student needs to translate into verbal symbols.

While the student must be competent to read both, words and formula, he is also expected to develop concepts from verbal as well as from mathematical symbols. However, the latter only represents meanings which are much more complex than the meaning of a single word. To synthesize the meanings of any mathematical symbolization, the student needs to recognize, attach meaning from background experience, evaluate critically, and interpret each mathematical sign in its relationship to and its interrelationship within the whole symbolization. Without mastering this complex reading process, the student is vexed even if he retreats to memorization. While he may have memorized the rule that a division by a fraction is equivalent to multiplication by the fraction inverted, he may find it difficult if not impossible to apply this rule. For instance, the student is first asked to read one of the principles of algebraic processes, such as

"If a trinomial has the form $x^2 + ax + b$ and is factorable into two binomial factors, the first term of each factor will be x; the second term of the binomials will be two numers whose procut is b and whose sum is equal to a, which is the coefficient of the middle term of the trinomial,"

and then in accordance with this principle he is asked to factor $x^2 + 10x + 24$. Is the student able to do what he has read?

Essential Student Reading Abilities in Mathematics:

What are the specific reading abilities a student in mathematics should have? If a student wants to read - reading means thinking and understanding - the above cited principle, he needs full command of technical algebraic terms (binomial, trinomial), literal numbers, exponents, and operational symbols (x^2+ax+b), arithmetic terms (sum, coefficient, product), and the understanding of nontechnical words (first, second, and middle). When, in addition, a solution is required, the student is challenged to evidence competence in the process of factoring, in forming relationship, and in using symbols of grouping.

Effective reading in the language of the different mathematical disciplines requires understanding of arithmetic numerals, literal numbers, operational symbols, and expressions of relationships.

Arithmetic numerals are found in arithmetic, algebra, and geometry. Wherever these directed or non-directed numerals are encountered, the student must be able to read them correctly. But reading numerals makes a greater demand upon the student's attention than do words. In reading a 7 digit numeral the reader may make as many as 5 fixation pauses (6). In general, straightforward reading may break down when a numeral of more than one or two digits is found in context. Student's bewilderment may also increase with the reading of positive and negative numbers. In elementary school, he was taught that larger numbers cannot be subtracted from smaller ones; in secondary school and college, he is taught to do so. The 'apparent' mathematical dilemma may procure negative psychological effects which may be enhanced only by embarrassing the student to read +2 and -6 and then to know what the difference between the positive and the negative numbers is.

The reading of literal numbers of the alphabetical symbols from a,b,c to x,y,z, may pose another hurdle in the student's endeavor to read. Hogben (2) said that the literal numbers are the nouns within the mathematical language. But only at times, not always! In the expression n^a the symbol a is an exponent or an adverb telling that n is to be used as a factor a times. In the expression n_a the symbol a is a subscript or an adjective describing some particular value of n. Therefore a student must be taught visual discrimination to spot at sight the position the literal number takes in space. Students may indeed benefit from having acquired a meaningful algebraic sightword vocabulary which is augmented systematically and consistently in accordance with the requirements in the particular subject matter.

To make nouns work, verbs are needed. The operational symbols are the real workers of the mathematical sentence. Consequently, students must be skilled in reading them. Concrete experiences with them rather than memorization of definitions, principles, and generalizations about them will result in meaningful decoding and applying.

The ability to read expressions of relationships, such as $D = \sqrt{2A}$, "goes straight to the heart of all applied mathematics, showing the formula and equation as the story of a rule or law which certain events in nature follow or approximate"(8). Reading expressions of relationships demands the mature reader who has surpassed the beginning stage of literal comprehension. Recognition and recall no longer suffice. The reader must ascend to inferrential comprehension, to evaluation, and appreciation. Indeed, the student must be in full command of the basic reading skills if he is to understand expressions of relationships. In addition, the student needs to see that the structure of a formula is not just a line of ordinary print. A formula is a design which when read in the context makes a greater demand on the eye and produces a greater regression and fixation frequency than reading scientific prose or algebra narrative(7).

<u>The Mathematical Buildingstones</u>:

To strut to the heights of Olympus, the buildingstones must be set in a systematic, direct, and planned fashion or teaching approach. Technical vocabularies and their special concepts, the use of specialized meanings for general words, and the varied mathematical symbols must not be left to incidental learning. Research by Gray and Holmes (1) has shown that a small amount of direct and planned teaching of terms and concepts produced dramatic results both in comprehension of those terms and in superior achievement in content field knowledge.

To gain, secure, and maintain efficient reading comprehension, the student needs to get acquainted with and needs to learn to understand these buildingstones.

When the eyes have transmitted the stimuli to the retina, responses of recognition occur which may or may not be verbalized or pronounced. Accurate visual and auditory discrimination in an applied left to right sequence will aid in the acquisition of new technical terms of verbal and mathematical symbols. Sheer recognition of terms and symbols without the ability to pronounce them may restrain comprehension and later interpretation and evaluation. When, however, the visual and auditory sense require additional help, writing the word or symbol (kinesthetic movement) will provide a third sensory attack on new or possibly forgotten terms and symbols. Students ought to be taught as well as encouraged to use the multi-sensory approach instead of being content with but one stereotyped approach. If these three senses do need further support and reinforcement, the tactile or touch approach should be employed; i.e., touch and outline terms and/or symbols by tracing them with two fingers.

In the light of these considerations which may already be or may become an obstacle to efficient and effective reading in the language of mathematics, how can a teacher or instructor in this field find out who of the students is plagued by such barriers, and what kind of help can be extended to those students in need?

Informal Reading Inventory in Mathematics:

At any instructional level, secondary school or college, the teacher can discover the students reading efficiency or inefficiency in the reading of mathematics by asking the students to read silently an assigned problem in the textbook. Since the purpose is to learn of the student's mathematical reading abilities of verbal and mathematical symbols, no computation is necessary at this time. Instead, after the reading of the problem is completed by the students, answers are given in writing to the following questions which have been mimeographed or written on the chalkboard(3):

1. What does the problem ask for?
2. What information does the problem give?
3. What kind of mathematical computation should be used to get the answer?
4. In what order should these mathematical computations be done?
5. How can the answer be verified?

The results of this Group Informal Reading Inventory in the reading of mathematics yield two answers: Those students who are able to read the problems and think through them and those who are not able to do so. Lack of satisfactory performance to answer the questions may be linked to student's inability to apply basic reading skills which, as the writer found, is not uncommon among a great number of college students. In this case mathematics instructors would need the help of a reading specialist who is professionally trained to take care of the more deep seated reading disabilities. On the other hand, if student reading disability results from lack of mathematical background, i.e., lack of setting purpose, meaning of technical vocabulary, and mathematical symbols, the mathematics instructor is best equipped to assist, aid, and help the student. For the teaching of reading of terms and symbols in this content field is an integral part of the required learning in mathematics and belongs to the instructor's charge.

While the suggestion of the Group Informal Reading Inventory (5) answers the first part of the above question, the following Directed Reading Activity in algebra answers the second part of the cited question.

The Directed Reading Activity in Algebra:

Further assistance to instructor and student is the Directed Reading Activity or DRA in this specific subject matter(4). Little or nothing is known of this teaching instrument by Mathematics instructors. Yet, readiness, silent guided reading, questions, oral re-reading, and application - the five steps of the DRA - apply to Mathematics in the same manner as to almost any other subject matter in high school and in college. Classroom and group application of the DRA in Mathematics assures the instructor that all students according to their own capability get not only a chance to participate but become motivated towards positive and active participation.

The following example demonstrates the application of the five steps of the DRA in a specific algebraic lesson: Factor the binomial $8a^2x^2 + 4a^3x$.

The first step is the consideration of student readiness. This involves asking such questions as: Are the students ready to decode words, numbers, and symbols in order to solve the algebraic process? Are the students psychologically ready to attack the problem or does the problem threaten the students?

What does the instructor expect the students to be ready for? Is each student ready to launch successfully into the process of 'know-what, why and how'? How does the instructor know that his students are indeed ready for the task? Although many more questions could be submitted, the maxim to be learned is: If the student is educationally and psychologically not ready to engage himself in the task, he cannot do it and the instructor cannot expect him to handle it successfully.

Readiness in the above cited example necessitates theoretical and practical cognizance of words, numbers, and symbols as well as their intricate algebraic relationship and interrelationship. The cognitive process for which the student must be ready involves comprehension and retention of the factual infomations disseminated by the verbal and non-verbal phrasing of the problem. Meaning derived from background experiences needs to be attached to and concepts formed of such words as: factor as a noun and a verb, binomial as to polynomial and monomial, the mathematical connotations of the common words inspection, trial, resolving or separating, quantity, squared and cubed, and a and x. To perceive relationship and interrelationship of the factual information logical ideas are inferred from context meaning and conclusions are drawn through inductive thinking which culminate in critical thinking and evaluation of the results.

While some students make every effort to enter such cognitive process, others need to be prompted into it. It is therefore suggested to engage all students in an active class discussion in which they demonstrate their actual working knowledge of factoring binomials. It is paramount, however, that all students are participating in the suggested dialogue. The better students may be challenged to perceive relationships between and among terms and symbols, while the weaker ones concern themselves with factual details. Here the better students can assist in establishing sequence and comparison in the algebraic process of literal comprehension. This unfolding process embraces all students in accordance with their own capability. It is a process of sharing knowledge, of mutual give and take in which the instructor's knowledge provides indirect structure and guidelines and in which he is the resource person to supplement missing information, to explain what is not understood and to assist in the exploration of the mathematical background.

Understanding of the cognitive process leads to formulating and setting general and specific purposes which are directly related to the algebraic problem. The task is no longer just an instructor's assignment to be done for the instructor, but is permeated by the student's personal and active involvement. He has a purpose for doing it and he is ready for it. Psychologically it can be said that the teacher-set extrinsic motivation turned into student-desired instrinsic motivation supported by teleological behavior(4).

The readiness period therefore fulfills a threefold task: (1) to introduce new algebraic terms and concepts, (2) to explore and supplement the student's algebraic background, and (3) to set purposes. Student failure to perform successfully is invited when the development of readiness is incomplete or when student readiness for the task is assumed.

The second part of the DRA is the guided silent reading. The silent reading is said to be guided because it is based on the understanding attained in the readiness period. The instructor then asks the students to read silently to themselves the algebraic problem. During the silent reading the instructor observes the students read. While in this cited example the silent reading is short, close observation of the silent reading is recommended in order to detect any possible disturbing student behavior. A careful associational type of silent reading is suggested for this and any other mathematical reading whose main purpose is the critical evaluation as well as application of the algebraic materials. The reasons for associational reading are twofold: (1) Since students are expected to solve the factoring of the binomial, they need to read critically. (2) Since they are expected to transfer attained knowledge to similar problems, they need to draw inferences and generalizations from the specific presented algebraic materials.

The guided silent reading is followed by the third part or the questions. Fact, vocabulary, and inference questions are probing students algebraic competency relevant to the problem. Numbers and symbols of the binomials may be discussed in fact and vocabulary questions, while inference questions are directed to discuss relationships and interrelationships of numbers and symbols within the binomials.

When doubt or disagreement is voiced in discussions or when any pertinent information needs to be emphasized, oral re-reading as the fourth part of the Directed Reading Activity is recommended.

Oral re-reading is exercised only whenever a specific goal-directed need arises. In the cited sample, a specific goal-directed question is: Which is the term of the polynomial that contains the same monomial factor?

When oral re-reading is not necessary, students proceed to factor the binomial. The work being in progress, the instructor has ample time to observe the working habits of the students. Since not all students factor with uniform accuracy and rate, it is suggested that additional purposeful work is always available for those who finish the task ahead of the others. In fact, for the instructor to gain insight into how accurate and how fast his students work, a simple table showing possible combinations of accuracy and rate is suggested:

Rate	Accuracy: High(h)	Average(a)	Low(l)
High(H)	Hh	Ha	Hl
Average(A)	Ah	Aa	Al
Low(L)	Lh	La	Ll

Table shows possible relationships between Rate and Accuracy. Rate indicates how fast the work is completed. Accuracy indicates the correct responses within the time of completion.

Application of the studied problem is the fifth or the last part of the DRA. Application of the algebraic principle of factoring a binomial is readily applied in accordance with the proficiency demonstrated by the students.

As described in this sample lesson, Readiness, Guided Silent Reading, Questions, Oral Re-reading, and Application, are the five elements of the Directed Reading Activity. They are not five separate entities which should be presented in isolation by the instructor. Instead, as evidenced by the writer, each part follows the other linked sequentially, logically, and psychologically as night follows the day. Each preceding part is related to the succeeding one and all parts are interrelated to the extent that the DRA as a whole contributes an active and constructive development in breadth and depth of (1) student's background experiences, (2) his general and specific vocabulary repertoire, and (3) his literal and inferrential algebraic comprehension. Student success or student failure, if not determined by, is to a high degree embedded in and grows out of this effective teaching instrument of the DRA.

Summary:

This article presented difficulties inherent in the reading of mathematics at secondary and college levels. Special emphasis was placed on the reading of arithmetic numerals, literal numbers, operational symbols, and expressions of relationships, as well as the reading of technical vocabularies and specialized meanings of general words. While each mathematical field has its own symbolization and terminology or shares those with others, they may already be or may become at the same time obstacles toward efficient reading in this content field. To seek remediation and, ultimately, prevention of mathematical reading inefficiencies, a Group Informal Reading Inventory, a visual-auditory-kinethetic-tactile appraoch, and a DRA have been recommended. In addition, a complete Directed Reading Activity lesson featuring a specific algebraic problem demonstrates its use within a classroom setting. Each instrument in itself and all together will serve the purpose to assist students toward greater reading comprehension in general and specific fields of mathematics.

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