DOCUMENT RESUME

ED 044 711 CG 005 884

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TITLE	The Application of Operations Research in Mental Health Systems.
INSTITUTION	American Psychological Association, Washington, D.C.; Illinois Univ., Champaign.
PUB DATE	4 Sep 70
NOTE	24p.; Paper presented at the American Psychological Association Convention in Miami Beach, Florida, September 3-8, 1970
EDRS PRICE	EDRS Price MF-$0.25 HC-$1.30
DESCRIPTORS	*Cost Effectiveness, Mental Health Clinics, *Mental Health Programs, Models, Program Design, *Systems Analysis, *Systems Approach, Systems Concepts

ABSTRACT

Operations research is defined as the application of scientific or quantitative methods to problems involving the operations of a system, which are directed toward obtaining optimal solutions. The goal, as applied to mental health systems, is the establishment of a "best" improvement path which can be implemented with minimum uncertainty. The uncertainty can be minimized by utilizing a model, a representation, of the system which can be manipulated, torn apart, reconstructed, all without risk to the real system. The concepts involved in this "model" approach are viewed as the basis for solving a wide range of problems encountered in designing a community mental health system. Three examples are given: (1) the development of an expansion path for community services, which concerns the effective distribution of resources; (2) a network analysis which focuses on how many separate cycles a client goes through at an agency, their duration and resultantly, the incumbent demand for a comprehensive information system; and (3) an agency clustering/problem clustering model which focuses on clients' problem and condition sequences, and the implications for more efficient and orderly service delivery. (TL)

THE APPLICATION OF OPERATIONS RESEARCH IN MENTAL HEALTH SYSTEMS

Presented by:

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September 4, 1970

The Application of Operations Research In Mental Health Systems

Operations Research is the application of techniques which can be advantageously utilized in solving problems encountered in human organizations and systems. It is the application of scientific or quantitative methods to problems involving the operations of a system, which are directed toward obtaining optimal solutions; that is, best decisions. It is somewhat difficult to describe how operations research can be effectively utilized in community mental health in general terms. For this reason the approach taken in this paper is to present some actual examples; and in this context, to also describe certain difficulties which must be overcome before widespread use of the methodologies will be a reality. This tack is taken with misgivings, however, since a comprehensive treatment of the subject is necessarily sacrificed.

Some concepts and terminology are needed at the outset-specifically, a brief description of the nature of "systems" and "models," will be useful for what follows. The essence of a "system" is interaction between the entities or elements which are defined to make up the system. These elements may be anything we describe--people, money, materials, services, or whatever. A boundary is drawn to include certain elements, and conversely to exclude others, and this characteristic is also a fundamental one. But the essential concept is the interrelationships which exist. A story may serve to illustrate these points:

> It happened one time that a circus train stopped in a small city to water the elephants. The elephants were arranged in typical circus fashion, in order of

> size, trunk to tail; from the largest down to the littlest baby elephant who clung tightly to the tail of his immediate predecessor with his short trunk. It was necessary to lead this procession across the tracks to a lake. But a large freight train bore down from the opposite direction just as the trainer had coaxed all the elephants, save the tinniest, out of danger. The freight train hit the smallest and last in line knocking it at least 50 yards down the track, killing it instantly. The circus owner who happened to be nearby ran up and after a glance sighed in relief "Thank heavens it was only a small one and not worth very much." But the trainer who had a better opportunity to survey the situation, quickly dispelled this momentary good fortune when he observed - "Yes, but it tore the guts out of all the rest of them."

Now for the lesson of this "parable." The relationships between elements of mental health systems are not well understood, nor are the system boundaries, and it is difficult to predict major interactions. A word of caution may seem to be in order relative to innovative, wide sweeping changes, but it is in just this connection that operations research may be utilized most advantageously. It is suggested that systematic analyses of resource redeployments, service delivery reconfigurations, reallocation of functions and so forth, will aid in minimizing the risk of serious errors. The scientific method, which is the basic approach of operations research, can contribute much to establishing a "best" improvement path for the mental health system, which can be implemented with minimum uncertainty.

So much for the nature of systems--now let us consider how to describe a system in a form which serves our purposes. Our primary purpose is to experiment--this is fundamental to the scientific method (and the Operations Research approach as well). But we must avoid the "trial and disaster" type of experimentation which results when the real world is haphazardly manipulated. For example, we might consider nailing shut the doors of state hospitals as a possible solution to the re-admission problem, but actually implementing this decision on a trial basis could have serious consequences. What is needed is a representation of the real world; that is, a <u>model</u>, which can be manipulated, torn apart, then put back together in all kinds of configurations, but without the cost or risk of actually testing theories using the real system.

To illustrate the development and use of a model consider the following example originally presented by Elmaghraby (3). The problem is to determine the area of an island in a lake of <u>known</u> area such as shown in the map in Figure 1. The island is large and of such irregular shape that it virtually defies direct measurement, at least at a reasonable cost. There are a number of possible solutions, however, all of which involve utilizing a <u>representation</u> of reality:

1. Take a steel plate, cut out the lake area and weigh it to obtain W_L. Then cut out the island and weigh this piece obtaining W_I. It is then easy to calculate

   A_I (Area of the Island) = A_L (Area of the Lake) * $\frac{W_I}{W_L}$

Figure 1

2. Lay a piece of screen with a fine mesh over the map. Count the number of squares in the lake area, N_L, and the number in the island area, N_I.

$$\text{Calculate } A_I = A_L * \frac{N_I}{N_I+N_L} .$$

3. Select from a table of random numbers many x and y coordinates of points which can then be plotted in the lake area, P_L, and the number falling in the island area, P_I.

$$\text{Calculate } A_I = A_L * \frac{P_I}{P_L+P_I} .$$

The types or representations illustrated in the first two solutions are analog models in that they substitute one property for another. The third is a functional or mathematical model which is generally more useful in working with complex systems problems primarily because it yields much more flexibility. The general form of such mathematical models can be expressed in the following equation:

$$P = f(U_i, C_j) ,$$

which says that the performance of the system is a function of uncontrolled (U_j) and controlled (C_j) variables. What is done is to try various sets of control variables in an attempt to optimize the performance measure. It is desirable, of course, to find some technique or algorithm which will yield an optimal solution more or less directly because the number of possible trials is usually enormous; but this is often difficult, especially when one must define fairly complex mathematical relationships in order to capture a reasonably accurate representation of the actual system.

By definition it is not possible to experiment with the uncontrolled variables, or to optimize relative to them. Thus, if a decision maker has no discretionary control over the elements (resources) of a system little can be done to improve its performance. This would seem to lead to the conclusion that the deployment of community resources must be accomplished in a systematic and controlled manner if the effectiveness of the system is to be enhanced, other than by chance.

There are other good reasons for building models--some of which occur almost as side benefits:

- models facilitate the formulation, communication, and discussion of hypotheses.
- they bring about a better understanding of system variables, their significance and interrelationships.
- models permit the time scale to be controlled. Real world processes occur over long periods of time but these intervals can be collapsed in a model.

Based on these concepts a wide range of problems encountered in designing a community mental health system can be addressed. A few examples will now be developed and particular attention will be focused on the limitations of the operations research or systems analysis approach.

A. The Development of an Expansion Path for Community Services

Perhaps the most important class of decisions in mental health systems are concerned with determining an effective distribution of resources. That is, answering the question of "where the next dollar is to be best spent." It is, of course, desirable to propagate successful programs, while on the other hand curtailing ineffective ones. Resource redeployments must account for changes in resource availabilities, since funds may be cut back, new sources of services may be discovered, technologies modified, etc. There are a number of techniques available for addressing this problem; but one linear programming, permits an exceptionally good insight into the inner mechanisms, and also the difficulties involved in modeling the resource allocation problem.

First, it is necessary to define some notation:

Let b_i = amount of resource i available (expressed in terms of man-hours of a particular skill type i, for example). These resources can undoubtedly be used in various ways to deliver different types of services.

x_j = level of activity j to be carried out. This is the unknown in the problem. We want to determine how many "units" of a particular type of client condition or problem are going to be handled.

a_{ij} = amount of resource i required per unit of activity j.

d_j = total estimated demand for activity j (a lower bound).

c_j = cost of carrying out one "unit" of activity j.

b_j = benefit accrued by carrying out one "unit" of activity j.

Then the problem can be stated as:

minimize $\sum_j c_j x_j$ or maximize $\sum_j b_j x_j$ (called the objective function)

Subject to constraints:

- on resource availability,

$$\sum_j a_{ij} x_j \leq b_i, \quad \text{(for all i)}$$

- on demand which must be satisfied,

$$x_j \geq d_j, \quad \text{(for all j)}$$

This is actually a contrived and very unrealistic model as will become obvious in the discussion that follows. From a mathematical standpoint, however, a relatively straightforward algorithm can be utilized to obtain optimal solutions to such formulations--optimal in the sense of maximizing benefits or minimizing cost. This mathematical technique is generally known as the "simplex method" and has been traditionally utilized in many operations research applications.

The main purpose of presenting this example, however, is that it illustrates the kinds of difficulties which arise in modeling complex systems such as community mental health. Before this model and solution technique can be used for "real-world" decision making it would be necessary to overcome problems of a technical nature as well as limitations on model definition.

First, two objective functions have been defined (maximize benefit and minimize cost) but the mathematical solution procedure cannot accommodate both simultaneously. While this is a technical consideration it has definitional implications. One objective or the other could, of course, be selected, but this leaves much to be desired since no one would be satisfied to save a lot of money if at the same time all considerations of successful outcomes were completely ignored. Likewise costs cannot be discounted in pursuing goals. One might attempt to find a way to combine both objectives into a single mathematical relationship, but combining costs and benefits is comparable to adding apples and oranges--the units are not compatible. This point represents one of the most severe limitations to the application of quantitative disciplines in the mental health field. There are no well defined, unified, measures of system performance! There are quantitative indicies, to be sure, but these are either not uniformly understood and accepted, or there is no suitable conversion factor between the various measures. Everyone agrees with the goal of establishing a more effective service delivery system, but try to define "effective service." How many pounds of "effective service" can you get for $2, or trade for 10 sq. ft. of space? The economists for some reason are much further ahead in deriving well-accepted and well-understood performance measures. This is probably the reason that they are more advanced in utilizing quantitative techniques and systems approaches. And this is <u>not</u> because they deal with simple systems. The stock market, for example, is a very complex environment considering the number of variables which influence prices; but what index in the mental health field has a universal meaning comparable to the Dow-Jones average? What performance measure in mental health has anywhere near the utility of GNP? In some sense the "correctness" of a measure is a moot question--whether the measure is understood, communicated and accepted is the crucial issue.

Getting back to the example problem, the model must be linear as the name <u>linear</u> programming implies. That is, all relationships must be expressed as first order equations.

We know that this is an approximation since many variables are surely stochastic and the relationships are undoubtedly very complex non-linear functions. So the question becomes, is the approximation sufficiently accurate? While this question can only be answered within the context of how the model is to be used, the limitation probably does not obviate using the model for many purposes.

The whole concept underlying the model is based upon the possibilities for trade-offs of resources between different activities. If there is not much flexibility here then we have a somewhat sterile problem. Stated differently, there may not be many alternative resource deployments which satisfy the constraints on the problem.

Demand, d_j, is difficult to estimate as defined in this problem, since in some sense it approaches infinity. Actually, demand is not being estimated in specifying the d_j's; rather cut-off points are being selected for the activities. This has the connotation of selecting a solution, then solving the problem.

There are other minor difficulties which could be mentioned, but these are sufficient to convey the fact that the problem is a formidable one. On the other hand, these difficulties are not unsurmountable and the approach is promising considering the information it is capable of providing.

The most exciting possibility in connection with the linear programming model is the ease with which one can parametrically investigate adding resources. Technical details aside, this capability is at the heart of the expansion planning problem. These techniques are generally categorized as "parametric programming," and their application is the way to proceed in connection with this model.

B. Network Analyses-Agency Pathways

Having demonstrated a type of problem which was plagued with numerous technical difficulties, consider now a type of analysis which is limited primarily by information system and organizational restrictions.

Let us postulate an information system which cuts across agency lines and is thereby capable of collecting comprehensive service delivery data on clients. Specifically, assume it is possible to obtain the beginning date and ending date (from which the duration of service delivery can be calculated), and resource point or agency involved in each instance a client receives a relevant service. Such information is very difficult to obtain since it involves crossing agency boundaries which means that alliances must be established, that coordinated and compatible data collection must occur, and that access to proprietary files be permitted. Few such information systems exist or are planned in the mental health field. This is a very confounding situation since community mental health is predicated on a coordinated and cooperative effort. The evaluation of community mental health programs relies heavily on the establishment of a comprehensive information system, which cuts across heretofore independent resource centers.

In any case, given sequences and durations of service deliveries, a network such as depicted in Figure 2 can be constructed. Associated with each agency node in this graph is the duration of service delivery (the time period during which the client is "under the auspices" of that agency). Utilizing this network the following parameters are calculated:

- Number of cycles or returns to the same agency, distinguished by number of nodes and cycle lengths in terms of time and/or number of nodes. For example, in Figure 2 we have 2 cycles, a two-node (1-2-1) and a four-node (2-1-3-4-2). The lengths of these cycles are determined by adding durations along these paths.
- Longest path-length, the longest unique agency chain which does not contain any cycles calculated in terms of time and/or number of nodes.

A pilot study utilizing this kind of analysis was recently undertaken by the Research and Evaluation Division of the Adolf Meyer Center, State of Illinois Department of Mental Health in conjunction with the Champaign, Illinois sub-zone. Sub-zone staff were asked to identify cases which they considered had the most "successful outcomes," and, conversely, to select cases which were judged "unsuccessful." Nine of each type were identified, the data necessary to construct graphs such as Figure 2 for all 18 cases were searched out, and the above network calculations performed.

The results of these calculations for this small sample are shown in Tables 1 and 2 and indicate that:

- more 2-node, 3-node, and 4-node cycles occurred with "unsuccessful" cases.

Figure 2. Network of Client Services

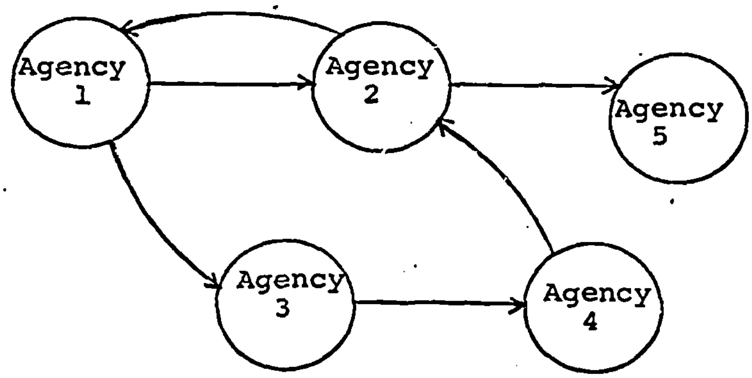

Table 1. Longest Pathway Analysis

Case # Unsuccessful	LP	Case # Successful	LP
1	.48*	10	.60
2	.81	11	1.00
3	.87	12	1.00
4	.33	13	1.00
5	.89	14	1.00
6	.54	15	.66
7	1.00	16	1.00
8	.25	17	1.00
9	.58	18	.63
TOTAL	5.75		7.89
Ave LP	.64		.88

*Longest path length is based on proportion of total time, rather than actual duration. The calculation utilizes actual agency sequences, without cycles, independent of start and end nodes.

Table 2. Cycle Analysis

Unsuccessful	4-Node	3-Node	2-Node
1	1.00 (1)	.48 (1)	.60* (2)**
2			.96 (1)
3	.68 (1)		
4			.33 (1)
5		.89 (1)	
6		.60 (1)	
7			
8			.83 (4)
9		.72 (1)	.26 (1)
TOTALS	1.68 (2)	2.69 (4)	2.98 (9)
Average Cycle Length	.84	.67	.33
Successful			
10		.38 (1)	.60 (1)
11			
12			
13			
14			
15	.24 (1)	.35 (2)	.35 (3)
16			
17			
18			.07 (1)
TOTALS	.24 (1)	.73 (3)	1.02 (5)
Average Cycle Length	.24	.26	.20

*Total cycle length based on proportion of total time, rather than actual duration.

**Number of cycles.

- the average cycle length was greater for "unsuccessful" cases.
- the cycle length for "successful" cases is independent of the number of nodes involved.
- the cycle length for "unsuccessful" cases is directly related to the number of nodes involved.

While it is not possible to provide any conclusive interpretation of these results, the greater frequency and duration of cycling exhibited by "unsuccessful" cases suggests that the same processes and referrals are tried over and over. Longer pathways in "successful" cases indicate that a greater variety of services are provided or that attempts at varied solutions rather than recourse to traditional modes, are more effective. Undoubtedly, we are begging the question in that staff implicitly identify "successful cases" as those that follow "successful paths." But even if this is the case, have we not found a useful measure of performance?

Additional studies along these lines are in progress. These studies will utilize a larger sample of cases and include additional variables, such as client socio-economic characteristics, in an attempt to yield at least partial answers to such questions as:

- Are services delivered in the proper sequence?
- Are the proper agency resources available?
- Are services comprehensive?

A high frequency of cycles, for example, would indicate negative responses to such questions. It will also be interesting to associate classes of clients with similar network parameters, and to identify agencies which are frequently involved in cycling.

The importance of an information system capable of monitoring and collecting historical client following data cannot be over-emphasized. Given time series network data it is possible to correlate changes in these statistics (i.e., mean number of cycles, standard deviation of path length, etc.) over time with system reconfigurations introduced at various points in time. For example, "Did consolidating certain agency functions diminish the average number of 2-node cycles?" or "What was the affect on path length after the point in time that community funding and/or local ownership of mental health programs was established?"

C. Agency Clustering/Problem Clustering

A third type of analysis utilizes much the same type of data as required in the previous network model. By accumulating problem or condition sequences across all clients served by the mental health system it is possible to construct the matrix $\{c_{ij}\}$, where each element c_{ij} = probability that problem j is scheduled for service delivery immediately following problem i. Constructing this matrix is accomplished by accumulating the number of occurrences of each problem or condition pair in the client following records. It is recognized that many such scheduled pairs occur on a purely chance basis, but this does not adversely affect the analysis when viewed in the aggregate.

Given the condition to condition transitional probability matrix $\{c_{ij}\}$ a number of interesting interpretations arise:

- large c_{ij} elements would indicate strongly dependent condition pairs (2-problem clusters).

- consider the possibility of a large c_{ij} but a small c_{ji}. This seemingly contradictory phenomena could contain very useful information. A cursory interpretation would be that the discovery of problem i usually leads directly to the discovery of problem j, but the reverse is not true. Another interpretation would be that problem i is a "higher priority" problem because the system tends to handle it first.

When the matrix $\{c_{ij}\}$ is raised to the second power ($\{c_{ij}\} * \{c_{ij}\}$) a new matrix $\{d_{ij}\}$ is obtained whose elements are defined as, d_{ij} = probability that problem j occurs in a triplet of problems in which problem i occurred first. Similarly, $\{c_{ij}\}^3$ provides information relative to 4-problem clusters, etc. This is useful data for purposes such as:

- reconfiguring service delivery centers to handle frequently occurring problem clusters.
- assessing client conditions in that it indicates problems to "look for" when other problems are discovered.

To illustrate let us use an over-simplified system containing 3 service delivery points, $S = (s_1, s_2, s_3)$, which handle 3 types of client conditions, $C = (c_1, c_2, c_3)$. Assume the client following records are available where each c_i | s_j pair indicates a particular condition, c_i, handled at a given service delivery center, s_j, arranged in chronological sequence.

client 1	c_1	s_1	c_3	s_1	c_3	s_3

client 2	c_2	s_3	c_1	s_1	c_3	s_2	c_2	s_1

client 3	c_1	s_2	c_1	s_1						
client 4	c_2	s_1	c_2	s_3	c_3	s_2				
client 5	c_1	s_3	c_3	s_3	c_2	s_2	c_1	s_1	c_3	s_3

From these records we could construct the matrix

$\{c_{ij}\}$ =

condition	condition 1	condition 2	condition 3
1	.2	0	.8
2	.5	.25	.25
3	0	.67	.33

A high dependency between condition 1 and condition 3 is observed ($c_{1,3} = .8$), but curiously the sequence 3,1 never occurs. A similar situation exists between problems 1 and 2. Such information raises interesting possibilities for combining services into comprehensive centers and at least partially explains the mechanisms whereby the system is uncovering problems.

The 3-problem cluster matrix, $\{d_{ij}\} = \begin{bmatrix} .04 & .54 & .42 \\ .23 & .23 & .54 \\ .33 & .39 & .28 \end{bmatrix}$, in this case.

Such an analysis can also be viewed from a service delivery standpoint by considering sequences of resource center contacts. In other words, a matrix $\{s_{ij}\}$ can be constructed where s_{ij} = probability that services will be delivered at center j immediately following service at center i. In the above example

$$\{s_{ij}\} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \text{service delivery point} \\ \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} .20 & .20 & .60 \\ 1 & 0 & 0 \\ .25 & .5 & .25 \end{bmatrix} \end{array}$$

Such a matrix can be useful in evaluating referral paths (agency 2 never refers anyone to any agency but agency 1, yet all other agencies refer to agency 2 with fairly high frequency). Probabilities along the diagonal are indicative of the tendency for service delivery points to retain cases (that is, there is a .25 probability that agency 3 will retain a case once it arrives at agency 3). $\{s_{ij}\}^2$ is again representative of 3-agency clusters, $\{s_{ij}\}^3$ yields information relative to 4-agency clusters, etc. It is also possible to incorporate time durations in constructing transitional probability matricies such as the above, which opens the possibilities for analyzing retention and maintenance rates.

Before leaving this problem type it is necessary to emphasize its dependence upon an information system which comprehends client movements between service delivery points. This was also the case with the network studies. It is conjectured that the evaluation of mental health delivery systems will be severely limited until such data are available.

While only three applications have been discussed, there are many other analytic procedures for addressing numerous decision making situations in mental health (see the selected bibliography). Many of these are directed toward operational or logistical problems, but the high payoff applications are concerned with the allocation of major resources. The measurement and evaluation of program plans and their translation into budgets is judged to be the area capable of supplying the kind of evidence needed to prove the value of operations research and systems analysis in mental health.

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