DOCUMENT RESUME

ED 043 675 TM 000 140

AUTHOR	Tucker, Ledyard R.
TITLE	Relations Between Multidimensional Scaling and Three-Mode Factor Analysis.
INSTITUTION	Illinois Univ., Urbana. Dept. of Psychology.
SPONS AGENCY	Office of Naval Research, Washington, D.C. Personnel and Training Research Programs Office.
PUB DATE	Jul 70
NOTE	44p.
EDRS PRICE	EDRS Price MF-$0.25 HC-$2.30
DESCRIPTORS	*Factor Analysis, *Individual Differences, Interest Tests, *Models, *Psychometrics, Statistical Analysis
IDENTIFIERS	*Multidimensional Scaling, Space Relations

ABSTRACT

Two lines of psychometric interest are combined: a) multidimensional scaling and, b) factor analysis. This is achieved by employing three-mode factor analysis of scalar product matrices, one for each subject. Two of the modes are the group of objects scaled and the third is the sample of subjects. Resulting from this are, an object space, a person space and a system for changing weights given to dimensions and of angles between dimensions in the object space for individuals located at different places in the person space. The development is illustrated with data from an adjective similarity study. (Author/LR)

RELATIONS BETWEEN MULTIDIMENSIONAL SCALING AND THREE-MODE FACTOR ANALYSIS

Ledyard R Tucker

July, 1970

The research of this report was supported by the Personnel and Training Research Programs Office of the Office of Naval Research under contract US NAVY/00014-67-A-0305-0003, NR 150-304.

Project on Techniques for Investigation of Structure
of Individual Differences in Psychological Phenomena
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Unclassified
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Psychology Department University of Illinois Champaign, Illinois	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE
RELATIONS BETWEEN MULTIDIMENSIONAL SCALING AND THREE-MODE FACTOR ANALYSIS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Technical

5. AUTHOR(S) *(First name, middle initial, last name)*
Ledyard R Tucker

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
July, 1970	42	12

8a. CONTRACT OR GRANT NO. US NAVY/00014-67-A-0305-0003 b. PROJECT NO. NR 150-304 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT
This document has been approved for public release and sale; its distribution is unlimited

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Office of Naval Research Personnel and Training Branch

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Unclassified
Security Classification

A-31408

Unclassified
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Multidimensional Scaling Factor Analysis Individual Differences Person Space Object Space						

DD FORM 1 NOV 65 1473 (BACK)
S/N 0101-807-6821

Unclassified
Security Classification

A-31409

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ABSTRACT

A combination is achieved of two lines of psychometric interest: a) multidimensional scaling and b) factor analysis. This is accomplished with the use of three-mode factor analysis of scalar product matrices, one for each subject. Two of the modes are the groups of objects scaled and the third mode is the sample of subjects. Results are an object space, a person space, and a system for changing weights given to dimensions and of angles between dimensions in the object space for individuals located at different places in the person space. The development is illustrated with data from an adjective similarity study.

One line of recent development in quantitative psychology involves creation of models which incorporate description of individual behavior with description of the variety of individuals with respect to this behavior. Multidimensional scaling of individual responses in comparisons among objects in a group of objects is a particular example. A separate scaling experiment could be conducted for each individual in a sample of subjects so as to obtain measures of dissimilarity between objects or pairs of objects. Such measures of dissimilarity frequently are taken as distances between points for the objects in a space which represents the responses of the subject to the objects. Tucker and Messick (1963) presented a model for investigating the variety of such spaces for a sample of individuals. An example of the application of this model to color vision data was given by Helm and Tucker (1962). Tucker and Messick emphasized the description of the subjects by the establishment of a person space. The multidimensional scaling space implied for selected "idealized individuals" could be determined by subsequent analysis and used as an aid in interpretation of the person space. In contrast, Horan (1969) has presented a model which involves a common multidimensional space for the objects and which is utilized, in theory, differentially by the various subjects in a sample by different weighting of the dimensions by the various subjects. Horan did not develop a procedure for the description of individual subjects. Carroll and Chang (1969) developed a model similar to that of Horan but included a person space. The present report concerns a model which involves both a person space and a common object space and for which individual parameters may effect changes not only in the weights given to the dimensions of the object space but also in the angles between these dimensions. This model utilizes Tucker's (1966) recent development of three-mode factor analysis.

A point of some interest is that the present model has wider applicability than to only multidimensional scaling. It may be used whenever each object in a group of objects is represented as a vector in an object space, eminating from an origin to a terminal point. Input data to analysis by the model are the scalar products of the pairs of vectors for each subject. Such scalar products may be obtained by some one of a variety of procedures, including direct ratings as in the data used in an example to be presented. For multidimensional scaling when measures of interpoint distances in a Euclidean space are available, these distances may be converted to scalar products of vectors eminating from the centroid of the objects by the procedure given by Torgerson (1958, see pages 257-258) where d_{jk} represents the distance between points for objects j and k and b^{*}_{jk} represents the scalar product between vectors for these objects. Such a conversion from interpoint distances to scalar products is to be accomplished separately for each subject. When the measures are comparative interpoint distances for each subject, a solution must be made for the additive constant for each subject and then the scalar products computed. In other cases, such as when only the rank order of the interpoint distances is known, one of the nonmetric scaling procedures like the one by Kruskal (1964) could be used to scale the data for each subject so as to obtain interpoint distances and the scalar products between vectors for the objects.

Consider that there are n objects in a multidimensional scaling experiment. Note that the word "object" is used to designate individual words, phrases, shapes, colored chips, etc. that are being scaled in an experiment. Subscripts j and $j' = 1, 2, \ldots, n$ will be used to designate these objects. Let data be for N individuals. A subscript $i = 1, 2, \ldots, N$ will be used to designate these individuals. The scalar product between the vectors for objects j and j' for individual i is designated by $x_{jj'i}$. These scalar products may be assembled into matrices in several different ways. One way is to define a matrix X_i for each individual having a row and a column for each object. Each of these matrices will be symmetric with each scalar product appearing twice in symmetric locations. The diagonal entries will be the squares of the lengths of the vectors. Another way is to define a matrix $\underset{\sim}{X}$ having a row for every combination of j and j', thus having n^2 rows, and a column for each individual. The matrix X_i for each individual is strung out into a column vector to form a column of the matrix $\underset{\sim}{X}$. Both of these forms will be used.

Multidimensional scaling for each individual would result in a matrix A_i of coordinates of the points for the objects on dimensions for the individual. Let there be Q_i dimensions in the multidimensional scaling space for individual i. Matrix A_i would be $n \times Q_i$ with entries a_{jq_i} where q_i is used as a subscript index for dimensions for individual i. Matrix A_i is related to X_i by

$$X_i = A_i A_i' \tag{1}$$

and may be obtained from X_i by any of several matrix factoring techniques. Its actual determination for each individual is only of theoretical concern in the present context. Consider a supermatrix formed by adjoining horizontally the X_i matrices for all individuals, thus forming $(A_1, A_2, \ldots, A_i, \ldots, A_N)$ which will be of order $n \times \sum_i Q_i$. Let the rank of this super matrix be P. Then matrices B, of order $n \times P$ and rank P, and $(W_1, W_2, \ldots, W_i, \ldots, W_N)$, of order $P \times \sum_i Q_i$ and rank P, exist such that

$$(A_1, A_2, \ldots, A_i, \ldots, A_N) =$$

$$B\,(W_1, W_2, \ldots, W_i, \ldots, W_N) \quad . \tag{2}$$

Since $(A_1, A_2, \ldots, A_i, \ldots, A_N)$ has n rows, its rank can be no greater than n; thus

$$0 < P \leq n \tag{3}$$

where the possibility of P equalling zero is discarded as being trivial. Note that there is a row of B for each object. The matrix B contains all of the information about the objects and can be considered to represent a common scaling space for all individuals. The matrices W_i for the individuals form transformations of the common scaling space to the individual scaling space.

Some relations for each individual based on the common scaling space and the individual transformations are of interest. From equation (2)

$$A_i = B W_i \tag{4}$$

which when substituted in equation (1) yields

$$X_i = B W_i W_i' B' \tag{5}$$

and, upon the definition of the $P \times P$, symmetric matrix H_i as

$$H_i = W_i W_i' \tag{6}$$

becomes

$$X_i = B H_i B' \tag{7}$$

Note that the rank of matrix H_i is the lesser of P or Q_i. Equation (7) gives an important relation of the individual scalar products matrix X_i to the common scaling space matrix B and the matrix H_i which contains information about the use of the common dimensions by the individual. The foregoing development has separated parameters for the objects from parameters for the individuals. Individuals may be conceived as making different use of the various common dimensions and of involving different relations among the common dimensions. Each matrix H_i characterizes each individual.

Further interesting results may be obtained for each individual by defining a diagonal matrix D_i containing the square roots of the diagonal entries in the matrix H_i

$$D_i^2 = \mathrm{Diag}\,(H_i) \quad . \tag{8}$$

Let

$$F_i = B\,D_i \tag{9}$$

and

$$\Phi_i = D_i^{-1} H_i D_i^{-1} \quad . \tag{10}$$

Note that Φ_i is $P \times P$, symmetric, and has unit diagonal entries. From equations (6) and (10), Φ_i is positive, semi-definite. Thus, the off-diagonal entries in Φ_i may be considered to be cosines of angles between the common dimensions for the individual. For any individual whose judgments were based on the same relations among the dimensions as existed for the common scaling space the matrix Φ_i would be an identity. Individuals who altered the relations among the common dimensions would be characterized by Φ_i matrices that were not identities. Substitution of equations (9) and (10) into equation (7) yields

$$X_i = F_i \Phi_i F_i' \quad . \tag{11}$$

Thus, the entries in the F_i matrix are the coordinates of points for the objects standardized for the individuals' use of the dimensions. Entries in D_i may be conceived as representing the weights given the dimensions by the individual. Parameters for each individual, then, include weights which he might use in his reactions to each common scaling dimension and the cosines of the angles representing the relations between the common dimensions which would characterize the individual's system of reactions.

A second aspect of the complete model is to be considered next. This aspect concerns development of a person space for description of the variety of individuals in a sample. While the matrices H_i contain parameters for the individuals, these matrices may not form the most compact basis for description of the sample of individuals. A more compact basis may be developed from the matrix $\underline{X}$ which has a row for each pair of objects and a column for each individual. Consider that $\underline{X}$ is of rank M which is less than either N or n^2. This presumes that the responses of the individuals may be described in a space having fewer dimensions than the number of individuals or the number of paired objects. Such a situation may or may not be the case for any particular body of observations; and when it is not the case, as is probably true for the majority of observed bodies of data, a reduced rank description may or may not be an adequate approximation to the observations. A model is being considered here when the rank of $\underline{X}$ is less than its order.

In this case matrices $\underline{C}$ and Z can be developed such that $\underline{C}$ has n^2 rows for the pairs of objects and M columns while Z has M rows and N columns for the individuals and so that

$$\underline{X} = \underline{C}\,Z \qquad . \tag{12}$$

Entries in matrix $\underline{C}$ may be designated as $c_{jj'm}$, and entries in matrix Z may be designated as z_{mi}. The c's, however may be recorded in M matrices C_m, one for each column of $\underline{C}$, with a row and a column for each object. An alternative form of equation (12) is

$$X_i = \sum_{m=1}^{M} C_m z_{mi} \qquad . \tag{13}$$

Note in equation (12) for any particular individual that the column of $\underline{X}$ is a weighted sum of the columns of $\underline{C}$, the weights being the z_{mi}'s for that individual. In equation (13) each of the columns of $\underline{C}$ is recorded as a matrix C_m which is multiplied by the scalar z_{mi}. These weighted matrices, $C_m z_{mi}$, are summed over all dimensions m. This type of paired statements of relations will be used several times in the following development.

Since the matrix Z has a row order equal to the rank of $\underline{X}$, it also has a rank of M and the product $Z\,Z'$ is non-singular and the matrix $\ddot{Z}$ may be defined as

$$\ddot{Z} = Z'\,(Z\,Z')^{-1} \tag{14}$$

with entries $\ddot{z}_{im}$. Then, from equation (12)

$$\underline{C} = \underline{X}\,\ddot{Z} \tag{15}$$

or

$$C_m = \sum_{i=1}^{N} X_i\,\ddot{z}_{im} \quad . \tag{16}$$

Since C_m is a weighted sum of symmetric matrices X_i , C_m is symmetric also. Substitution from equation (7) into equation (16) yields

$$C_m = \sum_{i=1}^{N} BH_iB'\ddot{z}_{im} = B\left(\sum_{i=1}^{N} H_i\ddot{z}_{im}\right)B' \quad . \tag{17}$$

A matrix G_m may be defined as

$$G_m = \sum_{i=1}^{N} H_i\ddot{z}_{im} \tag{18}$$

where G_m is $P \times P$ and symmetric since each H_i is $P \times P$ and symmetric. Substitution of equation (18) into the last term of equation (17) yields

$$C_m = BG_mB' \quad , \tag{19}$$

a result that will be used subsequently.

An alternative definition to that of equation (18) involves the matrix $\underline{G}$, which has a row for each pair of common scale dimensions, p and p' , and a column for each person dimension m , and the matrix $\underline{H}$, which also has a row for each pair of common scale dimensions and a column for each individual. Then equation (18) may be written as

$$\underline{G} = \underline{H}\,\ddot{Z} \qquad (20)$$

A consistent relation is for

$$\underline{H} = \underline{G}\,Z \qquad (21)$$

or

$$H_{\cdot i} = \sum_{m=1}^{M} G_m z_{mi} \qquad (22)$$

from which and equation (7)

$$X_{\cdot i} = \sum_{m=1}^{M} B\, G_m B' z_{mi} \qquad (23)$$

The same result can be obtained by substitution from equation (19) into equation (13). Equation (23) may be written in terms of the elements of the matrices as

$$x_{jj'i} = \sum_{p=1}^{P} \sum_{p'=1}^{P} \sum_{m=1}^{M} b_{jp}\, b_{j'p'}\, z_{mi}\, g_{pp'm} \qquad (24)$$

This equation is of considerable interest: it is a special case of the three-mode factor analysis model developed by Tucker (1966). A consequence is that many of the propositions and methods of analysis for three-mode factor analysis may be utilized in the present context. In this adaptation of the three-mode factor analysis theory, a first point is that two of the modes are identical, the objects form two of the modes while the individuals form the third mode. One simplification results, the matrix B is the factor matrix for the two object modes. Another point of interest is that the matrices H_i for the individuals are dependent on the core matrix G of the three-mode factor analysis and the factor matrix Z among individuals as per equations (21) and (22). Thus the factor matrix Z among individuals provides a basic description of the responses of the individuals.

An important topic involves possible transformations of the matrices in the model of equation (23). Consider square, non-singular matrices T and U of orders P and M so that

$$B\,T = B^t \tag{25}$$

and

$$U\,Z = Z^u \tag{26}$$

where B^t and Z^u are the transformed matrices B and Z. Inverse transformations are to be applied to the core matrix G so that

$$\sum_{m=1}^{M} T^{-1} G_m (T')^{-1} u^{mm'} = G^{tu}_{m'} \tag{27}$$

where $u^{mm'}$ is the mm' entry in U^{-1} and $G^{tu}_{m'}$ is the m' matrix of the transformed core matrix. The result of these transformations is that

$$\sum_{m'=1}^{M} B^t G^{tu}_{m'} (B^t)' z^u_{m'i} = \sum_{m=1}^{M} B G_m B' z_{mi} = X_i \tag{28}$$

so that these transformations do not change the form of the model nor the representation of the scalar products. These transformations are analogous to rotation of axes in factor analysis. A major difference is that there is no necessary equivalent interpretation of orthonormal transformations as representing uncorrelated factors and oblique transformations as representing correlated factors. The matrices T and U may or may not be orthonormal without affecting a difference in the interpretation as to correlations among the factors.

The transformations can be carried through to the matrices H_i of individual parameters for the scaling space. The transformed H_i matrices may be designated by H^t_i and defined by

$$H^t_i = \sum_{m'=1}^{M} G^{tu}_{m'} z^u_{m'i} \quad . \tag{29}$$

Then the scalar products matrices for the individuals are

$$X_i = B^t H_i^t (B^t)' \quad . \tag{30}$$

Matrices D_i^t , F_i^t , and Φ_i^t may be defined analogously to equations (8), (9), and (10). Equation (11) then applies to the transformed matrices.

A logical consequence of the possibility of transformations is that the object space of matrix B and the person space of matrix Z are identifiable from the scalar product matrices X_i but particular dimensions of these spaces are not identifiable. Some practical matters will be discussed subsequently as related to data analysis; however, some further conditions are necessary in order to establish complete identifiability for reference matrices B , Z , and G . Once such reference matrices are established uniquely, other solutions may be selected within the possibilities of transformations.

Since matrices T and U are non-singular, inverse transformations are possible from equations (25), (26), and (27) so that when B^t , Z^u , and G^{tu} are given along with T and U , matrices B , Z , and G can be determined. When any particular solution is given with matrices B^t , Z^u , and G^{tu} , matrices T and U may be determined such that

$$B' B = I \quad ; \tag{31}$$

$$Z Z' = I \quad ; \tag{32}$$

$$\sum_{m=1}^{M} G_m G_m' = \sum_{m=1}^{M} G_m^2 = \Delta^2 \quad ; \tag{33}$$

$$\underline{G}' \, \underline{G} = \Gamma^2 \tag{34}$$

where Δ^2 and Γ^2 are diagonal matrices. Proof of this possibility depends upon expressing the super matrix $(X_1 , X_2 , \ldots, X_i , \ldots, X_N)$ and the matrix $\underline{X}$ in basic form (see Horst, 1963, pages 364-382) such that

$$(X_1 , X_2 , \ldots, X_i , \ldots, X_N) =$$

$$B \, \Delta \, (L_1 , L_2 , \ldots, L_i \ldots, L_N) \tag{35}$$

$$\underline{X} = \underline{Y} \, \Gamma \, Z \tag{36}$$

where Δ and Γ are diagonal matrices, $(L_1 , L_2 , \ldots, L_i , \ldots, L_N)$ is a $P \times n N$ section of an orthonormal matrix, and $\underline{Y}$ is an $n^2 \times M$ section of an orthonormal matrix. Note that matrices B and Z are sections of orthonormal matrices as specified in equations (31) and (32). To obtain equation (7) from equation (35) define

$$\Delta \, L_i = H_i \, B' \tag{37}$$

so that, using equation (31)

$$H_i = \Delta L_i B \quad . \tag{38}$$

Further algebraic manipulations involving equations (14), (18), (31), and (32) yield equation (33). To obtain equation (12) from equation (36) define

$$\underline{Y} \Gamma = \underline{C} \quad . \tag{39}$$

Further algebraic manipulations involving equations (19) and (31) yield equation (34). Equations (31) through (34) specify a useful standard form.

Possible data analysis procedures will be discussed in conjunction with an example which will illustrate also a number of the features of the model. This example utilizes judgmental data on the relations between personality adjectives[1]. Each subject judged each pair of adjectives on a nine point scale running from -4 through 0 to +4. A rating of -4 was to mean that the two adjectives in a pair were opposite in meaning while a rating of +4 was to mean that the two adjectives in a pair were identical in meaning. A rating of 0 was to mean that the two adjectives were completely independent in meaning. Intermediate ratings were to mean gradations of tendencies toward oppositeness or similarity. The twelve adjectives

1. The questionnaire was constructed by Mr. Teddy Dielman while participating in a special seminar conducted by the author at the University of Hawaii. Mr. Dielman collected the data on the 22 subjects at the University of Hawaii.

listed in Table 1 were used in the study and the questionnaire included all 66 possible pairs of adjectives. Data from 87 subjects were used in the analysis, 22 graduate and undergraduate students at the University of Hawaii and 65 undergraduate students at the University of Illinois.

The ratings were divided by 4 so that the adjusted ratings ranged from -1 to +1 . A possible psychological model represents each adjective as a unit length vector with the adjusted ratings of similarity between a pair of adjectives being interpreted as the cosine of the angle between the two vectors for the two adjectives. Thus, a matrix X_i can be constructed from the ratings by each subject. This matrix will contain the adjusted ratings by the subject. This model has an intuitive appeal in that the following relations exist.

1) Two adjectives judged opposite in meaning by an adjusted rating of -1 are represented by oppositely directed vectors.
2) Two adjectives judged to be unrelated by an adjusted rating of 0 are represented by vectors at right angles so as to involve unrelated directions in the space.
3) Two adjectives judged to be identical by an adjusted rating of +1 are represented by identical vectors.

Ratings between -1 , 0 , and +1 , represent partial degrees of relatedness which may be taken to be equivalent to cosines of angles in the ranges of 180° , 90° , and 0° , respectively.

The ratings by each subject were recorded in a 12 x 12 matrix X_i with each rating recorded in the two, symmetrically located cells for the pair of adjectives. Values of +1 were recorded in the diagonal cells for the squares of the lengths of the vectors. Analysis followed Tucker's (1966, pp 297-298) method I which utilizes an Eckart-Young (1936) type approximation for each mode of the data matrix. Since, in the present case, two of the modes are identical, being the adjectives, the analyses for these modes were identical and required only one run through the computer. The super matrix $(X_1, X_2, \ldots, X_i, \ldots, X_N)$ was formed, the product $(X_1, X_2, \ldots, X_i, \ldots, X_N)(X_1, X_2, \ldots, X_i \ldots, X_N)'$ was computed, and characteristic roots and vectors of this product were obtained. The plot in Figure 1 of root size against root number was inspected to determine the number of dimensions to retain for the object (adjective) space. Roots 1 and 2 appear quite distinct from the remaining roots while roots 4 through 12 are quite close to a straight line. Root 3 is slightly above the straight line drawn through roots 4 through 12. A decision was made to retain three dimensions in the object space; that is, the number P of dimensions in the object space was set equal to 3. Thus, the matrix B, given in Table 1, contained the first three characteristic vectors. This matrix is in the standard form defined in equation (31).

Matrix $\underline{X}$ was formed by recording the entries in each matrix X_i as a column vector. Thus, matrix $\underline{X}$ had 144 (= 12 squared) rows and 87 columns, one for each subject. Since the number of columns was less than the number of rows, the product $\underline{X}'\underline{X}$ was computed and the characteristic roots and vectors of this product were determined. Figure 2 presents the plot of root size against root number. The points on this plot appear to lie on a hyperbolic shaped curve so that there is no clear break to aid in deciding on the number of dimensions to retain for the person space. The first root is considerably larger than any of the remaining roots; however, the differences between successive roots 2, 3, 4, and 5 are somewhat larger than the differences between successive roots 5 onward. From this observation, a 4 dimensional person space was selected for use in this example; that is, the number M of dimensions for the person space was set at 4. Matrix Z' contained the first four characteristic vectors. This matrix is in the standard form defined in equation (32), in subsequent processing it was multiplied by $\sqrt{N}$, which equals $\sqrt{87}$ in this study, so that the sum of squares of entries in each column became equal to N. Figure 3 presents the dimensional plots, or scatter plots, for pairs of dimensions of the adjusted matrix Z. Matrix $\underline{C}$ was computed by

$$\underline{C} = \underline{X}\,Z' \qquad , \qquad (40)$$

a relation that may be derived from equations (12) and (32) and which is consistent with an Eckart-Young resolution of the matrix $\underline{X}$.

An alternative procedure for determination of the person space is useful when the number of rows of $\underline{X}$ is less than the number of columns; that is, $n^2 < N$. In this procedure the matrix product $\underline{X}\,\underline{X}'$ is formed and its characteristic roots and vectors are obtained. Note that the non-zero roots are identical for $\underline{X}'\,\underline{X}$ and $\underline{X}\,\underline{X}'$, the one of these matrices having a larger order having more zero roots. Thus the root size against root number plot is identical for the alternative procedures and the decision as to number of dimensions to retain remains the same. Once the value of M is settled, a matrix $\underline{Y}$ may be formed containing the first M characteristic vectors of $\underline{X}\,\underline{X}'$. Let Λ be a diagonal matrix containing the M selected characteristic roots. Then

$$\underline{C} = \underline{Y}\,\Lambda^{1/2} \tag{41}$$

and

$$Z = \Lambda^{-1/2}\underline{Y}'\,\underline{X} \tag{42}$$

where $\Lambda^{1/2}$ is diagonal containing the square roots of the characteristic roots and $\Lambda^{-1/2}$ is diagonal containing the reciprocals of the square roots of the characteristic roots.

Once the matrices B, Z, and $\underline{C}$ are determined the core matrix may be computed. Matrices C_m are formed from the columns of $\underline{C}$, one matrix for each column of $\underline{C}$. As per the discussion following equations (12) and (16), each C_m matrix for the example was 12×12, symmetric with a row and a column for each of the 12 adjectives. Matrices G_m are computed by

$$G_m = B'\,C_m\,B \tag{43}$$

which may be derived from equations (19) and (31). The matrices G_m for the example are given in Table 2. They are 3×3, symmetric with a row and a column for each dimension in the object space.

Transformations of the axes of the object space and of the person space are considered next. The specific form of these transformations depends upon the nature of the domain of phenomena being investigated, the design of the study, and relations observed in the data and the results obtained in matrices B , Z , and G . The only general principle is that the transformations should aid in the interpretation of the results. These interpretations are dependent not only upon the results obtained from the study but also upon the design of the study and upon knowledge of the domain of phenomena being studied. Rotation to simple structure is not always appropriate and should be explicitly justified as meaningful when employed by an investigator. Use of a varimax rotation (Kaiser, 1958) should not be an automatic, reflex reaction for rotation of axes in factor analytic type analyses. In the adjective similarity study the adjectives were selected so as to, possibly, be representable in a two dimensional space having dimensions related to strength and to calmness. There were six pairs of more or less opposite meaning adjectives, two pairs for strength, two pairs for calmness, one pair for a combination of strength and calmness, and one pair for a combination of strength and the negative of calmness. Thus, if the hypotheses made during the selection of the adjectives were correct, the adjective vectors should radiate like spokes of a wheel in a two dimensional space. This is not the type of configuration for a simple structure. A different principle was required for determination of a transformation in the object space.

The design for the adjective similarity study was almost successful in terms of a two dimensional space. There were two large roots as shown in Figure 1 for the object space. However, a small third dimension appeared to exist. In order to define the transformation of axes in the three dimensional space a decision was made to define three "conceptual" adjectives each of which might be considered as pure for each of the transformed dimensions. For a strength dimension, which had an intuitive appeal for a meaningful dimension, the centroid of four adjective vectors was obtained: 7, strong; 8, courageous; -1, weak reflected; and -2, cowardly reflected. This vector is the first row of matrix B_c given in Table 3. The two positive vectors were nearly co-linear while the negative vectors were nearly oppositely directed from the positive vectors so that with the reflections the four vectors formed a relatively tight cluster. Calmness appealed as a second dimension and was represented by a conceptual vector determined as the centroid of three adjective vectors: 3, serene; 6, calm; -12, excitable reflected. Adjective 9, nervous, appeared as not being sufficiently opposite to 3, serene, and 6, calm, to be included in the cluster. The second row of matrix B_c in Table 3 is the conceptual vector for calmness. A problem remained as to the nature of the third dimension and the definition of an appropriate conceptual vector. After much study a decision was made to use adjective 4, self-conscious, to define the third conceptual vector. This adjective has the highest loading on the unexpected third dimension. Thus, the third row of matrix B_c is the vector for adjective 4, self-conscious. Matrix B_c can be considered to be a vertical extension of matrix B so that equation (25) can be used to yield

$$B_c T = B_c^t \tag{44}$$

where B_c^t is the transformed matrix B_c. In the present scheme for developing T, B_c^t is to be a diagonal matrix.

Before completing the development of matrix T the possibility of a transformation of the axes in the person space was studied. Inspection of the dimension plots for the subject space revealed no particular clustering of the individuals other than that they were all positive on the first dimension, see Figure 3. No other principle was appearant for a transformation. The first dimension would, then, represent a general dimension among the individuals and the other three dimensions would be interpreted as representing deviant perceptions of the similarity relations among the adjectives. As noted previously, a decision was made to scale the Z matrix so that the sum of squares of the person coordinates on each dimension would equal N. This scaling should make the person coefficients independent of the sample size except for sampling variance. The entries in the characteristic vector of a person space tend to be inversely related to the sample size since the sum of squares are restricted to unity. The preceding decisions resulted in the transformation matrix U being defined as

$$U = \sqrt{N}\, I \quad . \tag{45}$$

(Note that this is not a general definition of U.) This rescaling of the person coefficients results in an inverse rescaling of the core matrices

$$G_m^u = \frac{1}{\sqrt{N}} G_m \quad . \tag{46}$$

One other decision was made which affected the transformation matrix T for the object space: the diagonal entries in the transformed core matrix G_1^{tu}, for the general dimension among individuals, should be unity. This may be accomplished defining a diagonal matrix S such that

$$S^2 = \mathrm{Diag}(B_c\, G_1^u\, B_c') \quad . \tag{47}$$

Then the matrix T is defined such that

$$T^{-1} = S^{-1} B_c \quad . \tag{48}$$

Table 3 presents the computations for the Adjective Similarity Study. The transformed object space matrix B^t is given at the right in Table 1 and the transformed core matrix is given at the right in Table 2.

Inspection of the transformed matrix B^t for the adjectives indicates some interesting relations. As expected, the first dimension, A, may be characterized as strength with the adjectives 7, strong, and 8, courageous having high positive coefficients and the adjectives 1, weak, and 2, cowardly having negative loadings high in absolute value. These adjectives have trivial loadings on the other two dimensions. The second dimension, B, may be characterized as calmness with adjectives 6, calm, and 3, serene having high positive loadings and the adjective 12, excitable, having a negative loading high in absolute value. These adjectives have trivial loadings on dimensions A and C. The third dimension, C, is characterized by a high positive loading for the adjective 4, self-conscious. Interesting descriptions are given for the other adjectives. Adjective 5, retiring, is slightly weak, high on calmness and on self-consciousness. Adjective 9, nervous, is negative on calmness and positive on self-consciousness. Adjective 10, self-confident, is positive on strength and calmness while being negative on self-consciousness. Adjective 11, aggressive, is positive on strength, negative on calmness, and slightly negative on self-consciousness. These descriptions of the adjectives appear quite reasonable.

Interpretation of the core matrix and the person space dimensions may best be accomplished by the use of "conceptual" individuals located at various, but systematic, points in the person space. Table 4 presents results for seven conceptual individuals. The points for these conceptual individuals are indicated on the dimension plots of Figure 3 by open circles. The first conceptual individual is located at a point having a unit coordinate on the first person dimension and a zero coordinate on each of the other three dimensions. This conceptual individual may be thought of as representing the general dimension among the individuals. The weights used by this individual for the three dimensions of the object space are all unity. Dimensions A and B are almost orthogonal while dimension C has negative cosines of angles with the other two object dimensions. Thus, the self-conscious dimension is considered to be related to weakness (the negative of strength) and to excitability (the negative of calmness). Conceptual individuals 2+ and 2- have unit coordinates on dimension 1 but are contrasted by positive and negative coordinates on dimension 2 of the person space. The comparison of these two conceptual individuals indicates a small change in the weight for object dimension A, a larger change in the weight given dimension B, and no change in the weight for dimension C. Major contrasts occur, however, in the cosines of the angles between the dimensions of the object space. Conceptual individuals 3+ and 3- present a contrast associated with the third dimension in the person space. Conceptual individual 3+ gives higher weights to all three dimensions, which indicates that he gave more extreme ratings, while keeping the three dimensions of the object space nearly orthogonal. Conceptual individual 3-, in contrast, gave lower weights to all three dimensions and used the dimensions of the object space as quite oblique, especially dimensions A and C for which the cosine of the angle was -.90. Thus, for this conceptual individual, the self-conscious dimension was almost opposite to the strength dimension. Conceptual individuals 4+ and 4- present a contrast associated with the fourth dimension in the person space. The main effect of this dimension in the person space is a change in the weight given to dimension C of the object space plus some change in the weight given to dimension A. The preceding comparisons of conceptual individuals indicate that different locations in the person space are associated with both the changes in weights given to the object space dimensions and changes in the angles between the object space dimensions.

The Adjective Similarity Study illustrated one method of analysis. Other methods are possible for developing the standard form matrices B , Z , and G . There are other procedures possible, also, for developing the transformations. One possibility that may be appropriate for some studies is to make the core matrix as simple as possible. A point of special note is that, if every matrix $G^{tu}_{m'}$ can be made diagonal, all matrices H^{t}_{i} must be diagonal and the variation from individual to individual would occur only in variation in the weights applied to the several transformed object dimensions. This would correspond to the model utilized by Horan (1969) and by Carroll and Chang (1969). When not every matrix $G^{tu}_{m'}$ can be made diagonal the differences between individuals will involve changes in the angles between the dimensions of the object space as well as possible changes in the weights for the object space dimensions. The present model allows for each of these cases.

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TABLE 1

Adjective Similarity Study

Matrices for the Object Space

		Standard Form Matrix B_1 Dimensions				Transformed Matrix B^t Dimensions		
No.	Adjectives	1	2	3	No.	A	B	C
1	Weak	-.307	-.237	-.181	1	-.829	.082	.051
2	Cowardly	-.346	-.175	-.202	2	-.834	-.073	.036
3	Serene	.245	-.397	.105	3	.003	.970	.066
4	Self-conscious	-.263	-.030	.622	4	.000	.000	.885
5	Retiring	-.066	-.356	.499	5	-.146	.727	.703
6	Calm	.312	-.377	.075	6	.104	.987	-.016
7	Strong	.329	.218	.306	7	.925	.011	.082
8	Courageous	.331	.214	.239	8	.878	-.001	.005
9	Nervous	-.355	.232	.340	9	-.043	-.640	.554
10	Self-confident	.389	.022	-.081	10	.542	.298	-.347
11	Aggressive	.131	.445	-.027	11	.654	-.681	-.230
12	Excitable	-.207	.373	.012	12	.106	-.854	.050

TABLE 2

Adjective Similarity Study

Core Matrices

Standard Form Matrices G_m

m = 1

Dimensions	1	2	3
1	45.284	-.195	.005
2	-.195	35.422	-.068
3	.005	-.068	10.715

m = 2

Dimensions	1	2	3
1	-3.003	5.382	-2.153
2	5.382	4.649	-.714
3	-2.153	-.714	-1.502

m = 3

Dimensions	1	2	3
1	-4.809	-1.343	2.771
2	-1.343	4.070	2.445
3	2.771	2.445	3.779

m = 4

Dimensions	1	2	3
1	-.553	-.938	-2.656
2	-.938	-.735	-.016
3	-2.656	-.016	1.202

Transformed Matrices G_m^{tu}

m = 1

Dimensions	1	2	3
1	1.000	.143	-.360
2	.143	1.000	-.292
3	-.360	-.292	1.000

m = 2

Dimensions	1	2	3
1	.022	-.151	-.103
2	-.151	-.073	.073
3	-.103	.073	.004

m = 3

Dimensions	1	2	3
1	.049	-.074	.241
2	-.074	.065	.024
3	.241	.024	.016

m = 4

Dimensions	1	2	3
1	-.081	-.014	-.013
2	-.014	-.004	-.051
3	-.013	-.051	.175

TABLE 3

Adjective Similarity Study

Computation of Transformation in the Object Space

Matrix B_c : Centroid Vectors of Selected Groups of Objects

Group	Selected Objects	Mean Loading on Original Dimensions 1	2	3
A	-1 , -2 , +7 , +8	.328	.211	.232
B	+3 , +6 , -12	.255	-.382	.056
C	+4	-.263	-.030	.622

Product Matrix: $B_c G_1^u B_c'$

Transformed Dimension	Transformed Dimension A	B	C	$S_m = \sqrt{\text{diag}}$
A	.751	.116	-.276	.867
B	.116	.878	-.242	.937
C	-.276	-.242	.782	.885

Matrix $T^{-1} = S^{-1} B_c$

Transformed Dimension	Original Dimension 1	2	3
A	.379	.244	.268
B	.272	-.408	.060
C	-.297	-.034	.703

Transformation Matrix T

Original Dimension	Transformed Dimension A	B	C
1	1.471	.931	-.639
2	1.078	-1.785	-.259
3	.672	.308	1.140

TABLE 4

Adjective Similarity Study

Transformed Object Space Parameters for Conceptual Individuals

Conceptual Individual Number	Coordinates, z_{mi} on Person Space Dimensions				Weights, d_p^t, for Transformed Dimensions			Cosines, $\phi_{pp'}^t$, between Pairs of Transformed Dimensions		
	1	2	3	4	A	B	C	A-B	A-C	B-C
1	1	0	0	0	1.00	1.00	1.00	.14	-.36	-.29
2+	1	2	0	0	1.02	.92	1.00	-.17	-.55	-.16
2-	1	-2	0	0	.98	1.07	1.00	.43	-.16	-.41
3+	1	0	2	0	1.05	1.06	1.02	.00	.11	-.23
3-	1	0	-2	0	.95	.93	.98	.33	-.90	-.37
4+	1	0	0	2	.92	1.00	1.16	.13	-.36	-.34
4-	1	0	0	-2	1.08	1.00	.81	.16	-.38	-.23

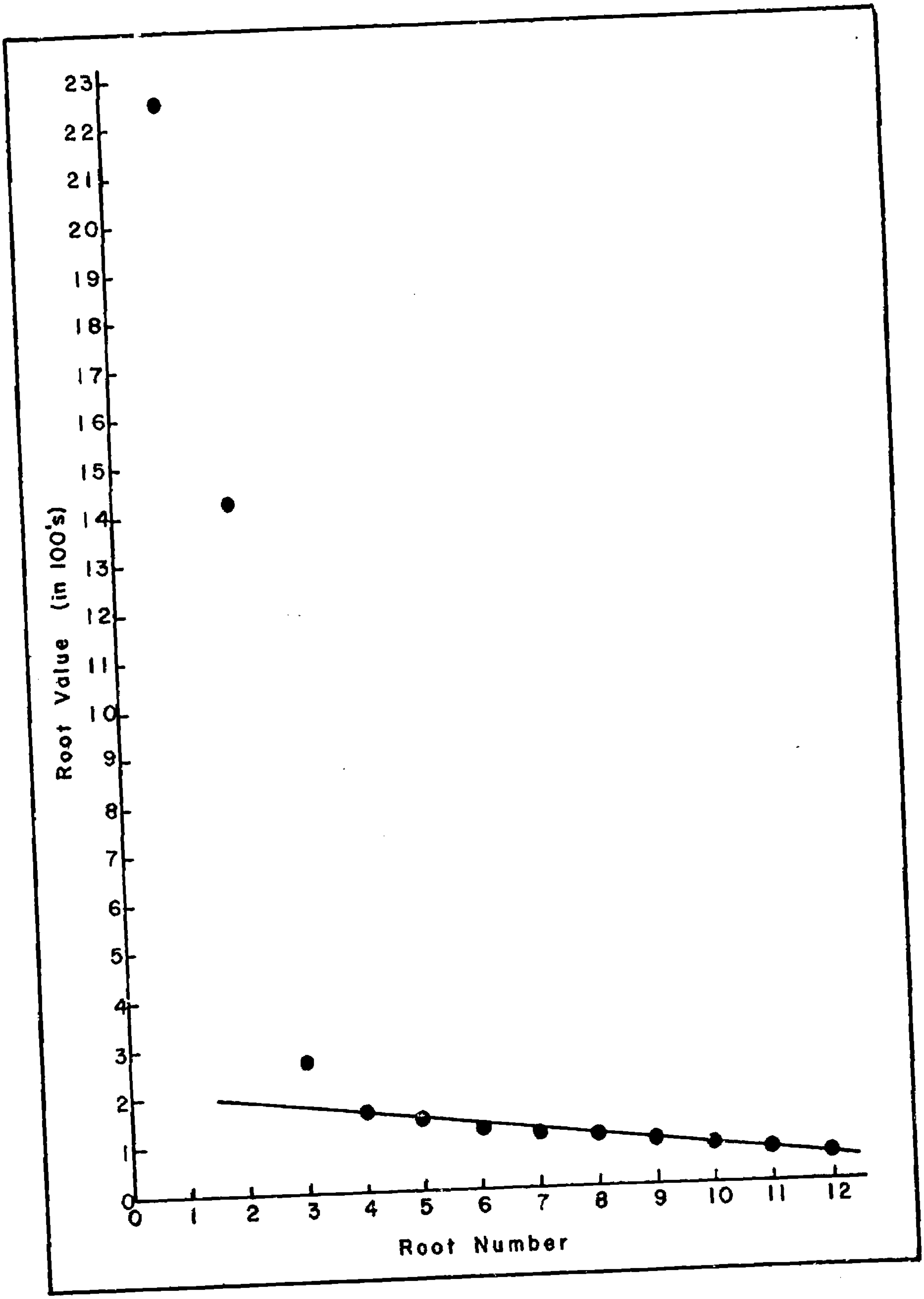

Figure 1

Adjective Similarity Study
Roots δ for Object Space

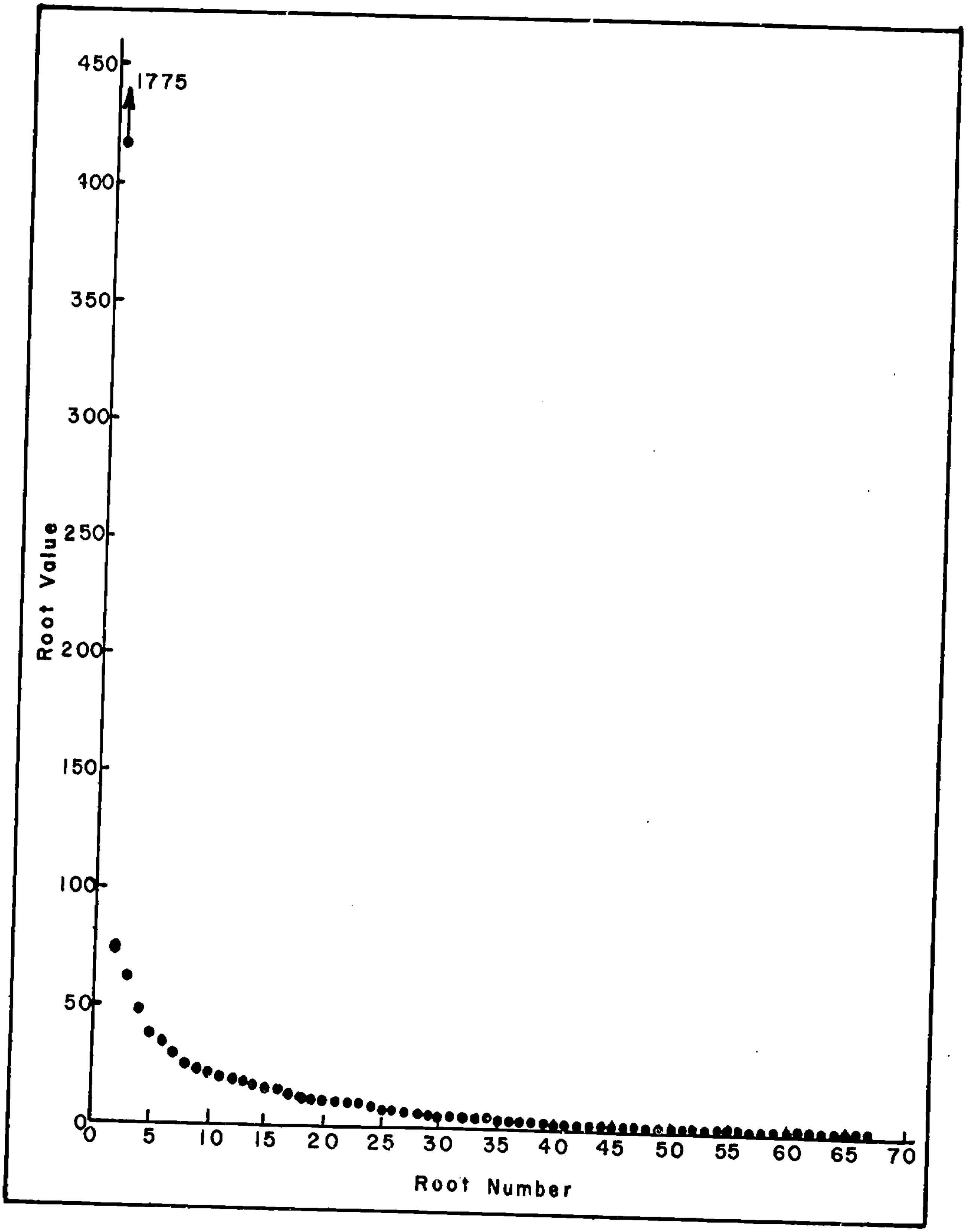

Figure 2

Adjective Similarity Study
Roots $\tilde{\gamma}$ for Person Space

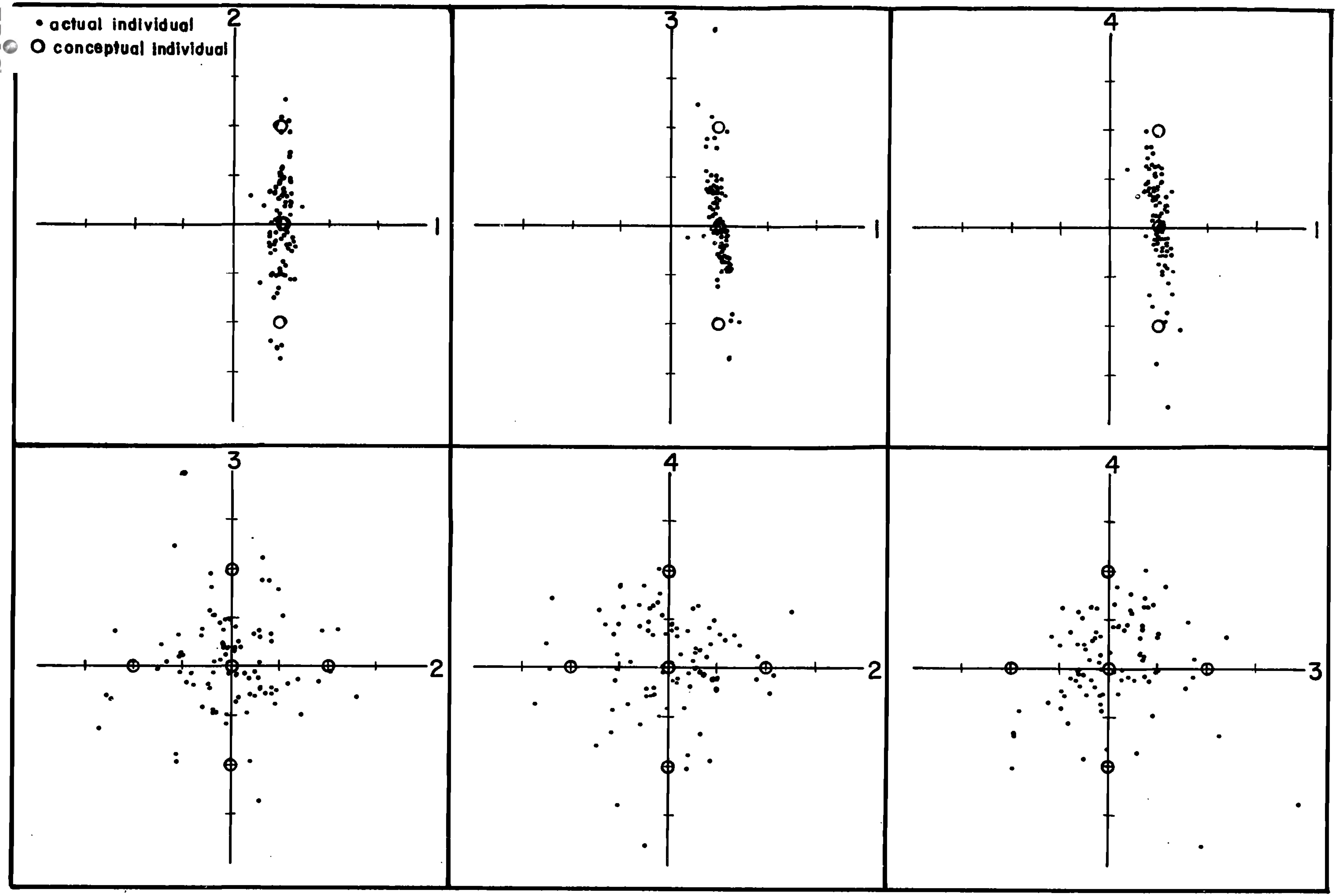

Figure 3

Adjective Similarity Study
Dimension Plots for Person Space

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