DOCUMENT RESUME

ED 041 750 SE 009 202

AUTHOR Daw, Harold A.
TITLE An Experimental Introduction to Kinetic Theory.
INSTITUTION Commission on Coll. Physics, College Park, Md.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE 67
NOTE 96p.; Monograph written for the Conference on the New Instructional Materials in Physics (University of Washington, Seattle, 1965)

EDRS PRICE EDRS Price MF-$0.50 HC-$4.90
DESCRIPTORS *College Science, Energy, *Instructional Materials, *Kinetic Molecular Theory, *Laboratory Experiments, Motion, *Physics

ABSTRACT

This monograph was written for the Conference on the New Instructional Materials in Physics, held at the University of Washington in summer, 1965. It describes a series of experiments designed to demonstrate the behavior of a system of macroscopic particles whose interactions are of very short range. The apparatus for the experiments include a two-dimensional air table, pucks, a polaroid camera and a stroboscope. The experiments include (1) speed distribution of a system of like particles, (2) collision frequency, (3) equipartition of energy, (4) path of a particle, (5) gravitational separation, and (6) time and space averages. The appendix contains a number of photographs showing the motion of pucks on an air table. (LC)

GENERAL PREFACE

This monograph was written for the Conference on the New Instructional Materials in Physics, held at the University of Washington in the summer of 1965. The general purpose of the conference was to create effective ways of presenting physics to college students who are not preparing to become professional physicists. Such an audience might include prospective secondary school physics teachers, prospective practitioners of other sciences, and those who wish to learn physics as one component of a liberal education.

At the Conference some 40 physicists and 12 filmmakers and designers worked for periods ranging from four to nine weeks. The central task, certainly the one in which most physicists participated, was the writing of monographs.

Although there was no consensus on a single approach, many writers felt that their presentations ought to put more than the customary emphasis on physical insight and synthesis. Moreover, the treatment was to be "multi-level" --- that is, each monograph would consist of several sections arranged in increasing order of sophistication. Such papers, it was hoped, could be readily introduced into existing courses or provide the basis for new kinds of courses.

Monographs were written in four content areas: Forces and Fields, Quantum Mechanics, Thermal and Statistical Physics, and the Structure and Properties of Matter. Topic selections and general outlines were only loosely coordinated within each area in order to leave authors free to invent new approaches. In point of fact, however, a number of monographs do relate to others in complementary ways, a result of their authors' close, informal interaction.

Because of stringent time limitations, few of the monographs have been completed, and none has been extensively rewritten. Indeed, most writers feel that they are barely more than clean first drafts. Yet, because of the highly experimental nature of the undertaking, it is essential that these manuscripts be made available for careful review by other physicists and for trial use with students. Much effort, therefore, has gone into publishing them in a readable format intended to facilitate serious consideration.

So many people have contributed to the project that complete acknowledgement is not possible. The National Science Foundation supported the Conference. The staff of the Commission on College Physics, led by E. Leonard Jossem, and that of the University of Washington physics department, led by Ronald Geballe and Ernest M. Henley, carried the heavy burden of organization. Walter C. Michels, Lyman G. Parratt, and George M. Volkoff read and criticized manuscripts at a critical stage in the writing. Judith Bregman, Edward Gerjuoy, Ernest M. Henley, and Lawrence Wilets read manuscripts editorially. Martha Ellis and Margery Lang did the technical editing; Ann Widditsch supervised the initial typing and assembled the final drafts. James Grunbaum designed the format and, assisted in Seattle by Roselyn Pape, directed the art preparation. Richard A. Mould has helped in all phases of readying manuscripts for the printer. Finally, and crucially, Jay F. Wilson, of the D. Van Nostrand Company, served as Managing Editor. For the hard work and steadfast support of all these persons and many others, I am deeply grateful.

Edward D. Lambe
Chairman, Panel on the
New Instructional Materials
Commission on College Physics

C O N T E N T S

AN EXPERIMENTAL INTRODUCTION TO KINETIC THEORY

INTRODUCTION

While the early Greeks introduced the notion of atomicity of matter, and a number of others including Hooke, Newton, and Bernoulli used the idea of hard little spherical particles to explain gas properties, it was not until 1859 that Maxwell announced his famous law for the distribution of velocity for such particles. The understanding of the atomic properties of gases in explaining gas behavior occurred close to the present time. If one draws a line six inches long and labels one end "beginning of Egyptian history, about (4000 B.C.)" and labels the other end "today," the period in which the atomic and molecular behavior of gases has been understood would occupy only the last 1/10 inch of the line. A brief historical summary can be found in Introduction to the Kinetic Theory of Gases by James Jeans, Cambridge University Press, 1948.

Rather than trace the historical development of this understanding, we will consider a series of experiments which will demonstrate the behavior of a system of macroscopic particles whose interactions are of very short range. In other words, we will use particles which are large enough to handle in our hands and which interact by colliding into each other. Perhaps you will be surprised as you do these experiments to see how much of gas behavior is explained from the behavior of such a simple system. In order to do this, we will need to keep track of the physical parameters of the apparatus we use, i.e., mass of the particles, etc.

Before proceeding to experiments on our system of colliding hand-sized particles, it would be well to see the behavior we will be studying. A movie film has been prepared showing the nature of the motions to be studied. It is recommended that you see the film at this time before proceeding.*

The film contains the following scenes:

1. 19 pucks moving at random on the table.
2. Large and small pucks in equilibrium with each other.
3. Slow motion of large and small pucks in equilibrium.
4. Table tilted to the left showing the density gradient produced by the gravitational field.
5. Random walk of one puck (it contains a small light). The 19 others are present, but not visible.
6. An integrated random-walk pattern.

After you have viewed the film, discuss with your fellow viewers the features of the motions observed and record them in the space provided below.

Interesting Features of the Motions

____

____

____

____

____

Question:

How would you characterize quantitatively the activity you observe among the particles?

*This film was partially prepared but is not being made available. Films containing related material are available from The Ealing Corporation 2225 Mass. Ave., Cambridge, Mass. 02140.

DESCRIPTION OF THE APPARATUS USED IN THESE EXPERIMENTS

All of the following experiments are carried out with a two-dimensional air table. This is a perforated table which permits air to be blown through its surface from underneath. The air provides a near-frictionless surface on which to float discs of various sizes. (Ref. Amer. J. Phys. 31, 867 (1963). The walls of the table can be agitated to maintain the motion of the particles.*

The data are taken with a polaroid camera. The type 47 film (speed 3,000) is very satisfactory, when the light source is a small stroboscope (GR, type 1531A for example). The images will be small and distances will need to be estimated to a 50th of an inch, but this is not too difficult.

*Two dimensional tables are available from the Ealing Corporation, Cambridge, Mass.

EXPERIMENT 1 SPEED DISTRIBUTION OF A SYSTEM OF LIKE PARTICLES

In this experiment we wish to look at a system of particles on the table and measure the speed of each of the particles. Since the speed of the particles is the distance covered by each of the particles divided by the time during which the distance is traversed, we will need to determine the positions of the particles at one time and then the new positions a short time later. The most convenient way of doing this is to photograph the system with a camera and to illuminate the air table with a stroboscope. The experimental arrangement is shown in Fig. 1.1.

Some attention should be given to the setting of camera exposure time and stroboscope flash rate to give suitable pictures. Figure 1.2 is a picture which has suitable properties.

Since the speed varies with the table agitation, the values you select may be appreciably different from those used for Fig. 1.2. Before proceeding further, however, let us ask some questions which should clarify the reasons for selection of particular camera and stroboscopic settings.

Questions:

1. If we get a single exposure, what can we say about the speed? (This is a spatial distribution of the particles. They can each be located by giving their x and y coordinates.)
2. If we get two exposures, what can be said about the speeds?
3. What problems would 100 exposures raise?
4. Consider the relative merits of 3 versus 4 exposures.

Let us return to the experiment. Write in the box the number of exposures you wish to have.

Number of Exposures Desired [ ] 1

Now observe the gross mean speed of the particles; judge how long it takes a particle to travel half its diameter. This might be accomplished

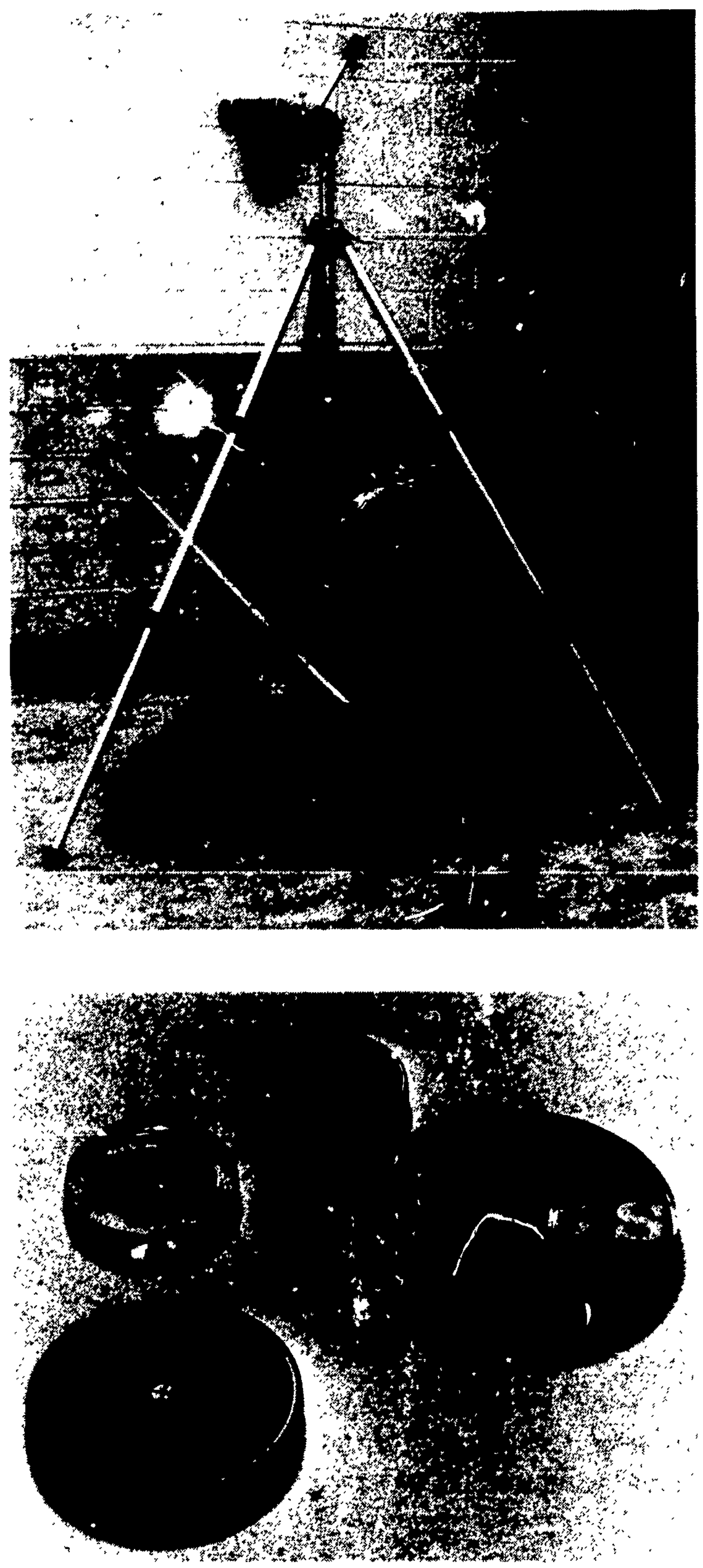


Fig. 1.1 The experimental arrangement for determining the speed distribution of a system of particles.

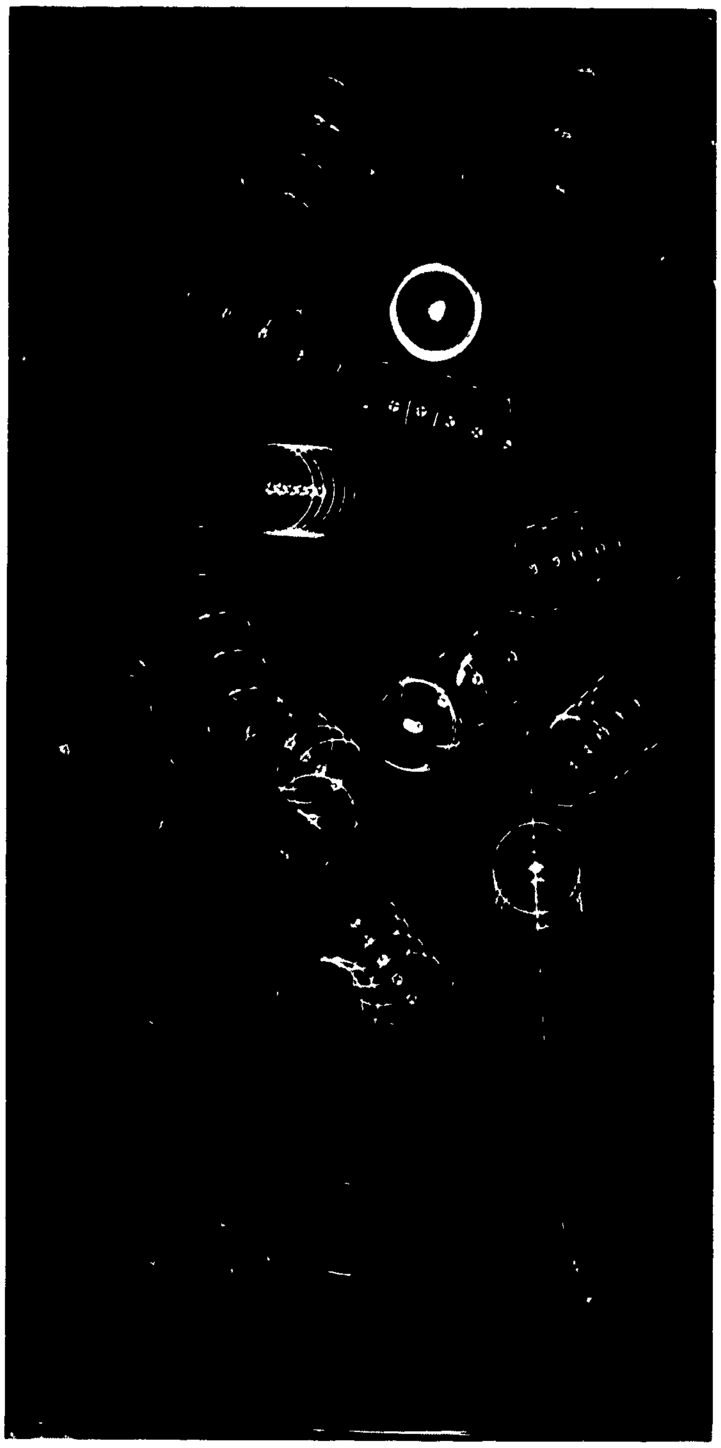


Fig. 1.2. Photograph of the air table taken with camera set at f#8. Shutter speed, ½ sec; stroboscopic flash rate, 12 per second. This figure is 40% smaller than the original.

by putting one particle on the table and pushing it at what seems to be an average speed and measuring how long it takes to cross the table. Enter in the box the time it takes for the particle to travel half its diameter.

Time for Particle to Travel 1/2 Diameter	sec	2

The shutter speed should be set at the product of the number of exposures desired times the time for an average particle to move one-half its diameter. From the numbers recorded in the boxes above, record the exposure time.

Exposure Time	sec	3

In this time the stroboscope must have flashed the correct number of times. It will do this if it flashes once every time an average particle moves one-half its diameter. The flash rate in flashes per second is just 1 over the time in Box 2 or the reciprocal of Box 2.

Flash Rate	/sec	4

Since most stroboscopes are read in revolutions per minute, you will probably have to multiply the number in Box 4 by 60. This will give the flash rate in rpm.

Flash Rate	rpm	5

There is a problem, however. The camera shutter and the stroboscope are not synchronized, and while the right time interval has been selected for the flashes, the proper number of flashes may not be recorded. This is clear by looking at Fig. 1.3.

If a small error has been made, either one more or one less flash than planned may occur. We suggest that

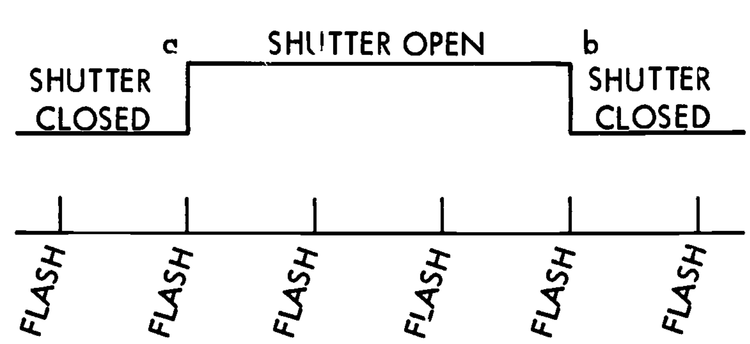


Fig. 1.3. This figure indicates that a small error in the time the shutter is open may include or exclude either or both flashes at a and b.

this is rare and such a photograph can be discarded if it appears not to be usable. After you have taken a few pictures (be sure the lights are off and only the stroboscope is on), it is suggested you adjust the exposure time and repetition rate to give the kind of pictures you wish. Always keep the camera exposure time equal to the product of the number of exposures desired and the reciprocal of the flash rate frequency.

When the camera and stroboscope are adjusted to give good photographs, take about 10 pictures with about 20 pucks on the table. Be sure to keep the same degree of table agitation, camera and stroboscope adjustments. Each photograph you have taken is an original. No one will have another exactly like it. It is your data, and yours alone. You will use it now to learn some interesting properties of the motion.

<u>Note</u>: For those students who do not have the facilities for making the pictures, suitable pictures are reproduced in an appendix to this manual. Photographs A.1 through A.23 may be used for the remainder of Experiment 1.

Take the pictures and label them. Using a scale marked in about 1/50th inches, measure each of the distances moved by the particles per flash (speed) in 50ths of inches/flash and record in Table 1.1.

Leave out the particles that underwent collision unless the speeds are unambiguous. In some cases you will be able to determine two speeds for pucks undergoing collision.

1.1 SPEED HISTOGRAM

Table 1.1. makes it clear that one cannot answer a question such as: With what speed do the particles move?

Table 1.1 Distance per flash (speed).

PICTURE NUMBER	1	2	3	4	5	5	7	8	9	10

Table 1.2 (Example) Distance per flash (speed).

DATA FROM FIG. 1.2 SPEED IN 50THS OF INCHES/FLASH			DATA FROM FIG. A.1 SPEED IN 50THS OF INCHES/FLASH	
1 *	15.8	10.5	13.0	6.9
[illegible]	13.1	23.6	7.3	4.5
22.6*	7		7.4	8.0
0 *	14.9		11.5	7.8
12.7*	3		6.0	2.5
13.8*	5.5		9.8	4.2
10 *	4.6		9.0	6.7
3.4*	8		6.5	25.0
8 *	28		8.3	3.0
6.9*	20			
14 *	29.1			

The * values have been plotted in Fig. 1.5. These measurements were made on the original.

In fact, very few move with the same speed, and if our measuring ability were sufficient, probably none moves with exactly the same speed as another. A whole distribution of speeds is present. It will be noted that very few of the particles move slowly. Also very few of them move very fast. How then are the speeds distributed among the particles? We now proceed to answer this question. Those students who have not had experience with histograms should examine Table 1.2 before continuing.

Since nearly every particle has a distinct speed, we would like to find not the individual particle speed itself, but rather how many particles have speeds in a certain interval, say between 5/50ths and 10/50ths of an inch per flash, and then between 10/50ths and 15/50ths inches per flash and so forth. These we will plot in Fig. 1.4.

This figure is a plot of the number of particles which have a speed in a certain speed interval versus the speed. Put on your own vertical and horizontal divisions. If you do not know how to do this, follow the example, Table 1.2.

It will be convenient to divide the horizontal axis into about as many (the nearest integral) intervals as two times the square root of the total number of speeds measured. The highest number on the horizontal scale should correspond to about the highest speed measured. Count the total number of particles having speeds between 0 and the first division on the horizontal scale. Always add half of the number of particles which have speeds exactly on the dividing line. Plot this number in Fig. 1.5 by drawing a line from 0 to the end of the first interval at the height corresponding to this number. Do the same thing for each succeeding interval, always adding half of the number of times the velocity falls exactly on the upper and lower dividing line. You now have a completed histogram. It should have a rather characteristic shape. If it jumps up and down and has no particular shape, the horizontal intervals were too fine. If it is simply one or two rectangles, the horizontal intervals were too coarse. It is doubtful either of these conditions occurred.

If you had made measurements on a large number of particles, the histogram would give a rather smooth curve with an uncertainty of about the square root of the number of entries in each column of the histogram. It will be left for a note to derive the theoretical shape of the curve, Cve^{-v^2/λ^2}. This equation was first derived by Maxwell in 1859. You can check whether or not your histogram gives a reasonable approximation to this curve if you plot it in Fig. 1.5. First, however, you must calculate the constants in the expression determined by Maxwell. The expression $C\,\lambda^2/2$ is equal to the area under the curve of the histogram. The average value of v^2 is λ^2. We can get the average value of v^2 from our histogram. Take the average speed for the first interval, square it, and multiply it by the num-

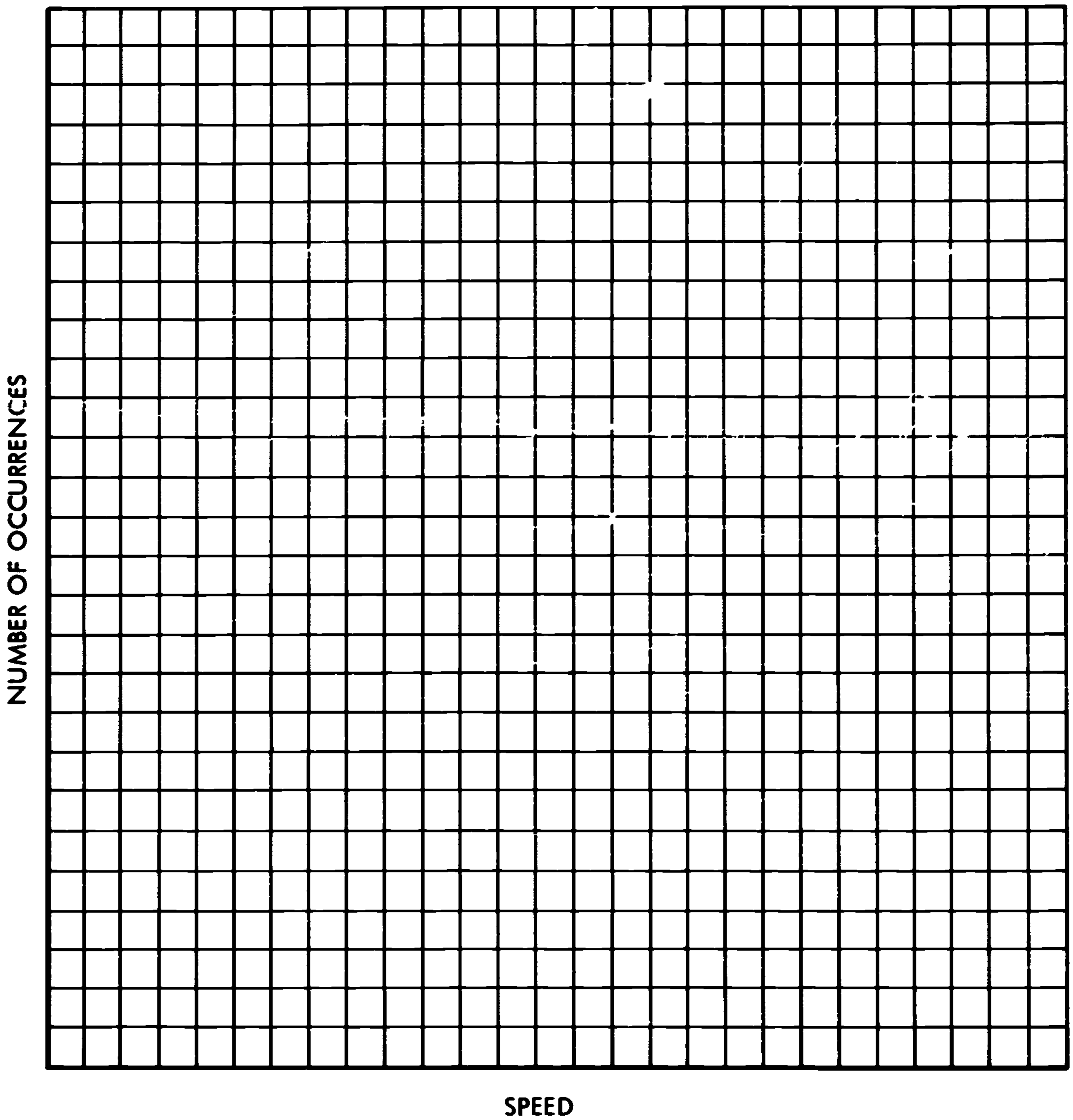


Fig. 1.4. Histogram of the speeds recorded in Table 1.1.

ber of occurrences for this interval. Add this result to the number of occurrences for the succeeding interval times the square of the average speed in this interval, and so on for all the intervals. The total value is now divided by the total of the number of velocities measured or the total number of occurrences. This permits one to determine λ^2. The area under the curve is calculated by multiplying the length of the individual intervals by the number of occurrences over these intervals and summing them together. When the area is determined, the value of C can be determined using the known value of λ^2.

We can now answer in an unambiguous fashion what is meant by the activity of the system so far as its speed distribution is concerned. We can give the curve shape ve^{-v^2/λ^2} and the value of the constant λ^2. Since the shape is always the same we may be satisfied to simply indicate the mean (speed squared). We will find that this quantity is related to the agitation rate of the table.

Table 1.3 Example (Fig. 1.2).

$$\lambda^2 = \frac{5188}{24} = 215$$

INTERVAL	AVERAGE SPEED	(AVERAGE SPEED)^2	NUMBER OF OCCURRENCES	PRODUCT
0–3	1.5	2	1	2
3–6	4.5	20	4	80
6–9	7.5	56	4	224
9–12	10.5	110	2	220
12–15	13.5	181	6	1086
15–18	16.5	273	2	546
18–21	19.5	380	1	380
21–24	22.5	510	2	1020
24–27	25.5	650	0	0
27–30	28.5	815	2	1630
			TOTAL 24	5188

Things to keep in mind:

The mass of each of our pucks is 40.3 g. The diameter of the pucks is 2.27 in. The camera and printing process reduces the puck diameter to .6 in or 3.8 times. The average of the speed squared is 8.0 $(cm/sec)^2 = (3.8)^2 \times 215/2500\ (2.54)^2$. The average kinetic energy per puck is 1/2 $\times$ 40.3 grams $\times$ 8.0 cm^2/sec^2 = 161 ergs.

1.2. VELOCITY SPACE

We have seen the particles distributed in space, i.e., each particle located at a particular x and y. Suppose our eyes responded not to positions but to velocities. What would our picture of the several pucks look like? To find out, we would need to know not only the speed of the particles, but also their direction of motion. From the pictures we have taken so far, however, we cannot tell which direction the particles move. While this ambiguity can be removed, most of the information we need can be obtained by simply determining the absolute value of the x and y velocity components.

In measuring the velocities, it will be most convenient to work in polar coordinates and measure the speed of the particles, and their directions (0° to 90°). (A drafting machine is ideal for this purpose.) Enter these in Table 1.4.

The values for the x and y components of velocity can be calculated from Table 1.4. $v_x = v \cos \varphi$, v_y

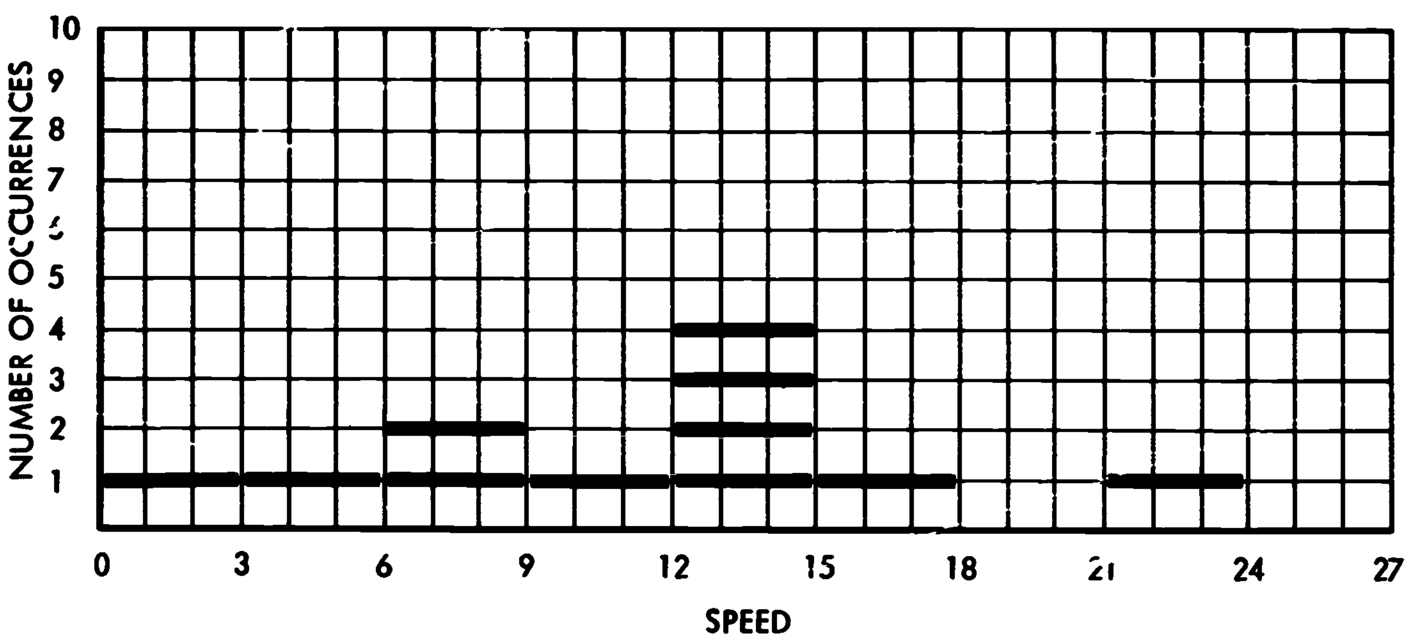


Fig. 1.5. Histogram (Fig. 1.2).

Table 1.4 The velocity components in polar coordinates.

PICTURE NUMBER	1		2		3		4		5	
	V	ANGLE	V	ANGLE	V	ANGLE	V	ANGLE	V	ANGLE

= v sin φ. Fill in Table 1.5 with the values of v_x and v_y calculated from Table 1.4.

When we know the velocities and positions of a system, we have all of the characteristics of the motion at a particular time. The evolution of the system as time progresses is one of the central problems of mechanics. Those of you who have thought of representations in more than three-dimensional space might consider mentally plotting the spacial configuration of particles in many-dimensional space (3N, where N is the number of particles). Each three-dimensional region in the space would correspond to a separate particle. The coordinates would be its 3-position coordinates. One can conceive of one point in the many-dimensional space whose projection on to any of these three-dimensional regions represents the position of the particular particle represented by that region. This 3N-dimensional space is called configuration space, since single points in it gives the entire spacial arrangement or configuration of the particles in the system. As the particles move, the point in configuration space moves along a continuous path.

Alternatively, we can consider a velocity space of 3N dimensions. The velocity picture for many particles is represented by a single point in this space. As the system moves in time, the arrangement of the velocities changes and the single point moves.

Finally, one might conceive of a 6N-dimensional space. Of these 6N dimensions, half (3N) correspond to position coordinates and half (3N) to velocity coordinates. The total arrangement of the system (velocity and position) is now represented by a

Table 1.5 Velocity components in rectangular coordinates.

PICTURE NUMBER	1		2		3		4		5	
	v_x	v_y	v_x	v_y	v_x	v_y	v_x	v_y	v_x	v_y

single point in this space. As time passes, the point moves in this multidimensional space. This space is called phase space. There are several powerful theorems about the possible ways in which points can move in phase space.

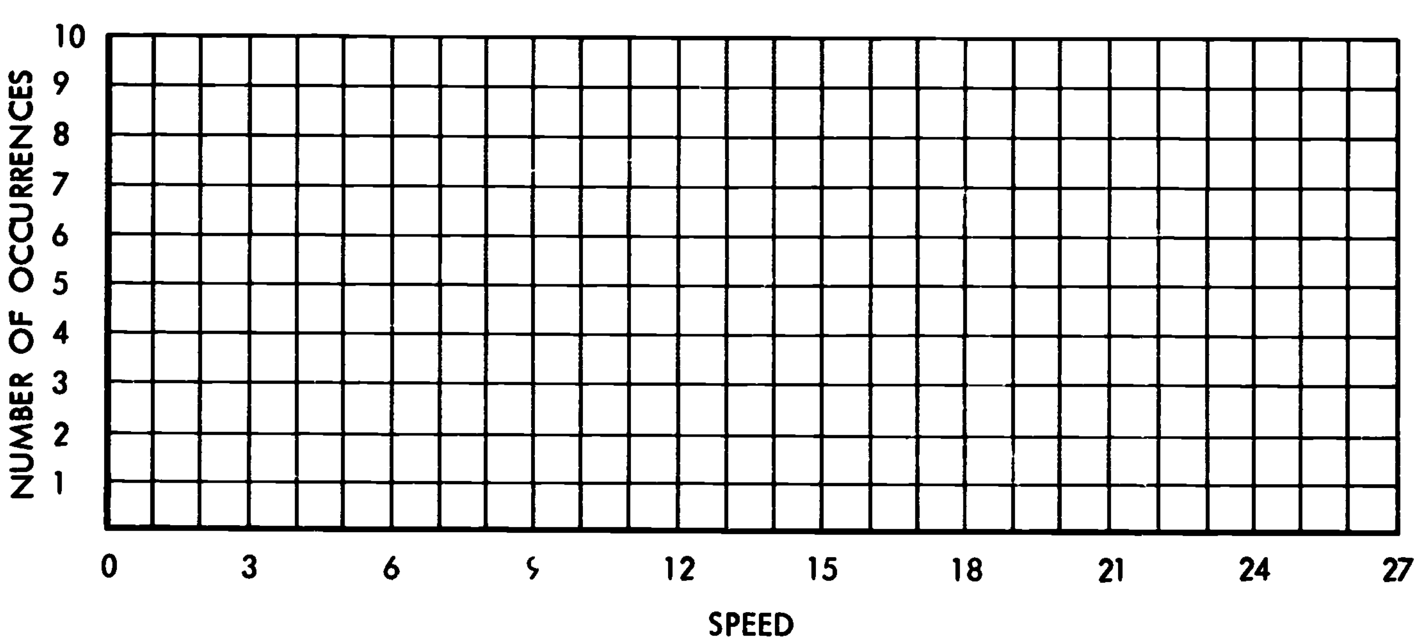


Fig. 1.6. Histogram (Fig. A.23). The student should complete Fig. 1.5 and do Fig. 1.6.

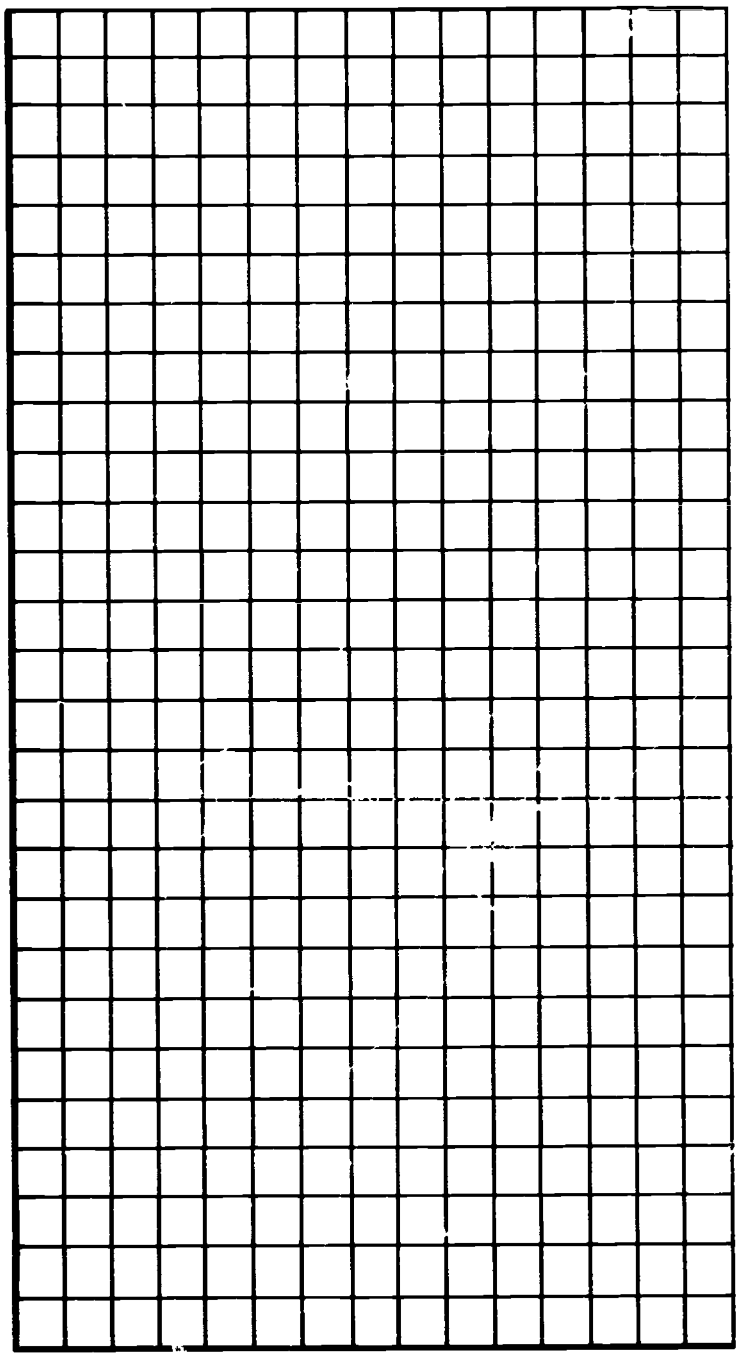
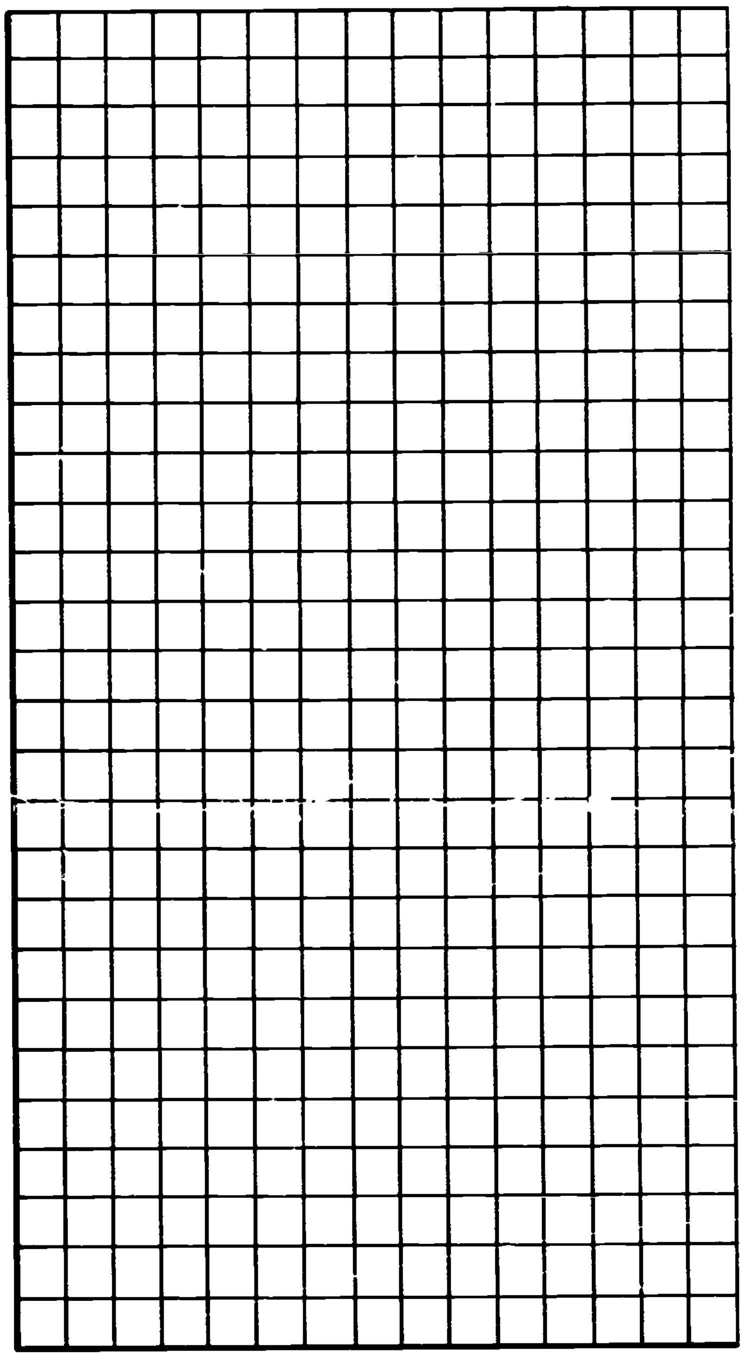


Fig. 1.7. Histogram for v_x .



Histogram for v_y .

1.3. MOMENTUM DISTRIBUTION

It will be interesting to plot the frequency distribution of one component of the velocity. This component of the velocity may be taken along any line. We will take the x and y directions only. The x and y components of velocity can be read directly from Table 1.5. Take the data and, for Fig. 1.7, plot a histogram for v_x and v_y separately. This should be done in the same fashion as the earlier histograms.

There are several things to be noted in these diagrams. They both have a similar shape. The shape is quite different from the speed distribution obtained in Fig. 1.5. The most probable velocity component is zero. The formula for this distribution is

$$C\, e^{-\frac{v_x^2}{\lambda^2}}$$

The constant C is related to the area under the curve A where $A = C\,\lambda\sqrt{\pi}/2$ and λ is the root of the mean of the square of the velocity, not a component of velocity.

NOTE:

If one were to represent the distribution of velocities in x and y coordinates and then in polar coordinates, the formulas look as follows: $\exp - \left(\frac{v_x^2}{\lambda^2} + \frac{v_y^2}{\lambda^2}\right) dv_x dv_y$ $\rightarrow v \exp - (v^2/\lambda^2)\, dv d\vartheta$. The speed distribution then is $v \exp - (v^2/\lambda^2)\, dv$ if there is no angle dependence. In a similar fashion, extension to three dimensions will give a speed distribution of $v^2 \exp - (v^2/\lambda^2)\, dv$.

1.4. VELOCITY DISTRIBUTIONS AS A FUNCTION OF AGITATION

Throughout the previous experiments the degree of agitation has been kept fixed. The previous experiments can be carried out with a different degree of agitation. If this is done, it is found that the general shape for the distribution curves remains the same. Only the value of the constant λ^2 changes. The degree of motion on the table is set by the agitation rate of the table. This agitation rate is 2.8/sec for Figs. 2, A.1 to A.59; 2.5/sec for Figs. A.60 to A.83, and 3.1/sec for Figs. A.84 to A.107. The amplitude of oscillation was constant: about 1 in. peak to peak.

EXPERIMENT 2 COLLISION FREQUENCY

2.1. WITH OTHER PARTICLES

With little additional effort we can re-examine the previous pictures and extract some additional important information. The total time the camera viewed the table is given by the product of the number of pictures made times the exposure time per picture. This time is sufficient to represent reasonably well the time development of the system. If one counts on the photographs the number of collisions which have occurred, one can determine the collision frequency (number of collisions/sec). There are, however, two types of collisions: collisions with other particles and collisions with walls. Count only the collisions with other particles. Enter this number in Box 1. Enter the total time (exposure per picture times the number of pictures examined) in Box 2. The quotient of these two numbers (number of collisions/total time) is the collision frequency. Enter it in Box 3.

☐	÷	☐	=	☐
Number of collisions with other particles		Total time exposure × number of pictures		Collision frequency (other particles)

2.2. WITH THE WALLS

The pictures will contain a number of collisions with the walls. The collision frequency with the walls can be determined in roughly the same manner as collision frequency with other particles was determined. Fill in the following boxes:

Collision Frequency with Other Particles

1		2		3
☐	÷	☐	=	☐
Number of collisions with the wall (counted from the photograph)		Total time exposure time/picture × number of pictures		Collision frequency (walls)

The average momentum change of a puck on collision with the wall is $2\,m\bar{v}_x$. The average velocity in the x direction can be calculated from the distribution function as follows

$$\bar{v} = \frac{\int_0^\infty v_x \exp(-v_x^2/\lambda^2)\,dv_x}{\int_0^\infty \exp(-v_x^2/\lambda^2)\,dv_x} = \frac{\lambda}{\sqrt{\pi}}.$$

One can also determine the rate at which pucks strike a unit length of the wall. This is simply the total wall length divided into the collision frequency with the walls. The force per unit length is then just the product of the average momentum change per puck times the collision frequency per unit length.

$$\frac{\boxed{} \times \boxed{}}{\boxed{}} = \boxed{}$$

Average momentum change per puck normal to a wall

Collision frequency (wall)

Force per length

Length of the wall

2.3. AS THE NUMBER OF PARTICLES IS CHANGED

One would expect the collision frequency to depend in some way on the number of pucks on the table. This dependence can be determined by putting different numbers of pucks on the table and measuring the collision frequencies again as was done in parts a and b. When very few pucks are on the table, collisions are much more apt to occur with the walls than with other pucks. When a large number of pucks are present, the reverse is true. We will have occasion to return to this point when we discuss mean free path.

EXPERIMENT 3 E Q U I P A R T I T I O N O F E N E R G Y

3.1. SPEED DISTRIBUTION

In the movie on Random Motion a system composed of two puck sizes was shown. Among the gross features you may have observed and noted on page 1 was that the larger pucks go slower (on the average) than the small pucks. We now proceed to examine in a quantitative way the velocity distribution for the large and small pucks. Place on the air table about 10 small pucks and 5 large pucks. Take about 10 photographs as was done in Experiment 1. (Be sure to keep the agitation constant.) Measure the speeds of the pucks and enter them in Table 3.1 and Table 3.2. Figures A.24 through A.59 are provided for this purpose.

Now plot histograms in Figs. 3.1 and 3.2.

The constant appearing in the distribution λ^2 is not the same for the two sizes of particles, but has an interesting relation to the masses of the particles. (The masses are proportional to the areas in the provided pictures.)

λ^2 small pucks = λ^2 large pucks =
m small pucks = 4m large pucks =

What is this relation?

If we had used energy in the exponent, we would have a shape given by $v \exp - \frac{1}{2} mv^2/E_0$ where E_0 is some energy set by the table agitation. The shape would then be the same for large and for small pucks.

Table 3.1 Speeds (small pucks).

PICTURE NUMBER										

Table 3.2 Speeds (large pucks).

PICTURE NUMBER										

Hopefully, the statistical fluctuations are not too large. If they are, more data will be needed. If you have taken and recorded your data properly, an interesting result is obtained. This means that on the average the energy associated with the larger pucks is the same as the energy associated with the small pucks. This is an example of a very important physical law which has wide consequences. The kinetic energy is equally distributed among the various particles. This means objects of all sizes have, on the average, the same kinetic energy when they are intimately inter-

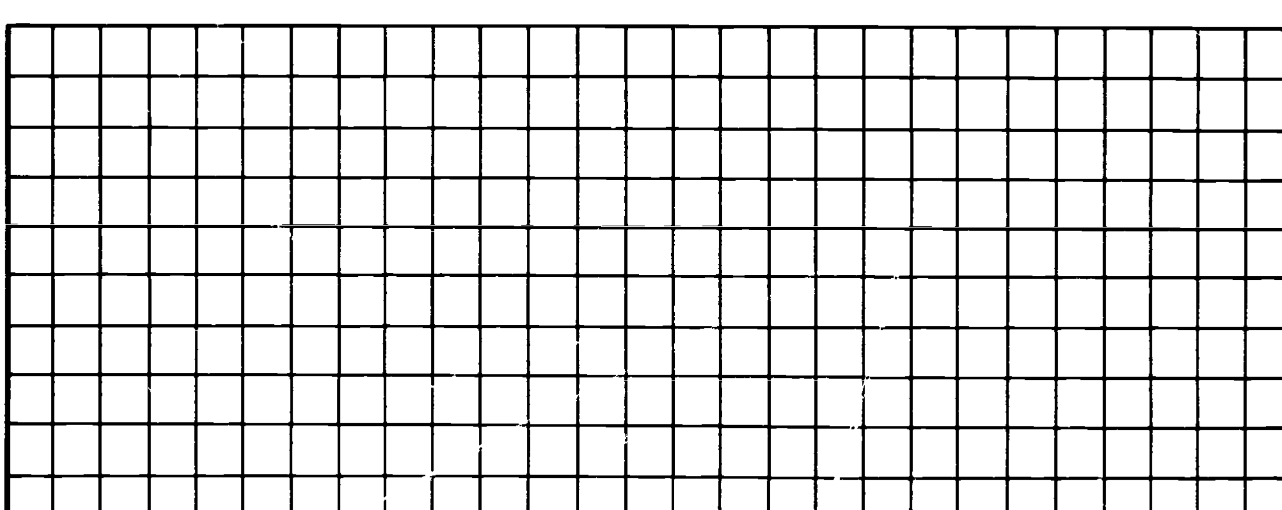


Fig. 3.1. Histogram (small puck) speeds.

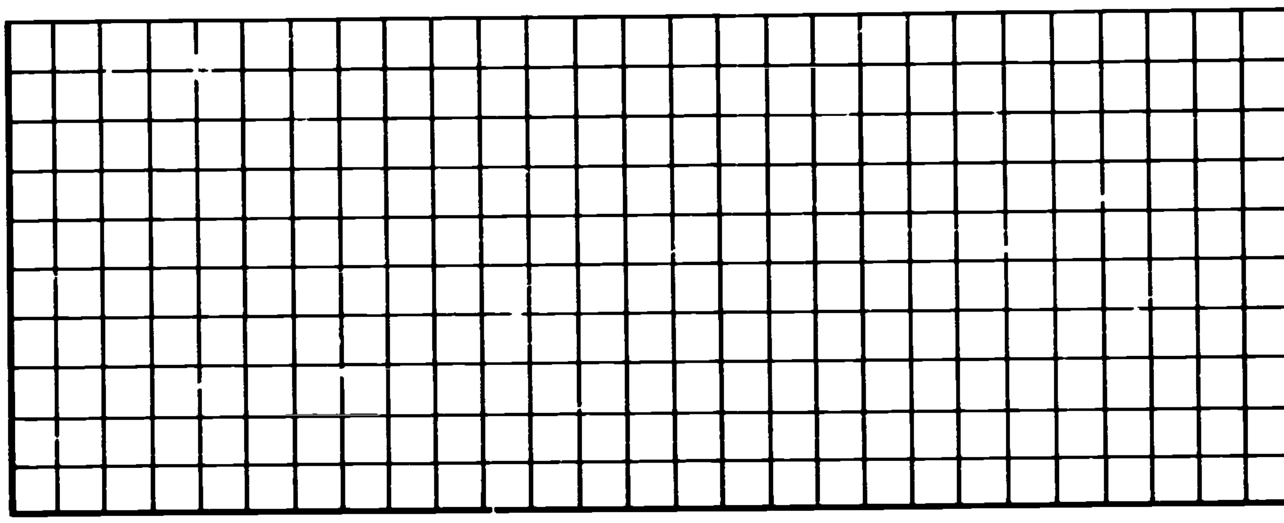


Fig. 3.2. Histogram (large puck) speeds.

acting with each other. It also means the average energy associated with motion in the x direction is equal to the average energy associated with motions in the y direction.

Since the kinetic energy is equally distributed between large and small interacting objects, the 1/2 mv^2 for gas atoms and molecules ought to be equal to the 1/2 mv^2 for pucks, etc. However, in the system we have been treating the pucks as intimately interacting with each other and with the table walls which are being agitated.

They are intimately interacting with the air. However the interaction due to air collisions on the pucks is too small to observe. If we could measure the properties of the motion of some sizable particles which are intimately interacting with the air and could measure their mean kinetic energy, and if we extrapolate what we have learned about equipartition of energy, we can then say something about the mean kinetic energy of the air atoms and molecules.

4.1. PATH LENGTH DISTRIBUTION

In the movie one of the pucks was specially identified so that one could get some appreciation of the complicated irregular motion of a puck. We wish to study this feature of the system motion next. The photographic technique which has been used so far is not adequate to follow one puck, since the images soon begin to overlap badly. We can change the photographic technique, however, to overcome this. One of the pucks can be fitted with a small light bulb and batteries. A very satisfactory arrangement is shown in Fig. 4.1.

With the light on in a darkened room only the center of this one puck is visible. If the camera is operated on bulb, a long exposure can be made of the path followed by this one puck. Figure 4.1. was made in this fashion.

The collisions are rather easily recognized. Take a ruler marked in 50ths of inches and measure the paths between collisions (or freepath length). Be sure to note which collisions occurred with the walls by placing a w after them. Record your measured data in Table 4.1.

These paths can now be used to form a histogram. However, since the collisions with the walls were not collisions with another puck, either discard both the path up to the wall and away from the wall, or assume a puck was responsible for the wall collision and use the separate pieces of the path. Now plot the histogram for the various paths of Table 4.1. in Fig. 4.2.

The shape of this distribution is represented by $e^{-x/\lambda}$. The constant λ is the average of mean path length. If you calculate the average path length by determining the total path and dividing by the number of collisions, you will have the value of λ. Determine this value for your data and place it in the box.

Average Free Path Length λ = $\boxed{}$

Since the puck on the average travels this distance free of collision, this distance is known as the mean free path.

The mean free path is an impor-

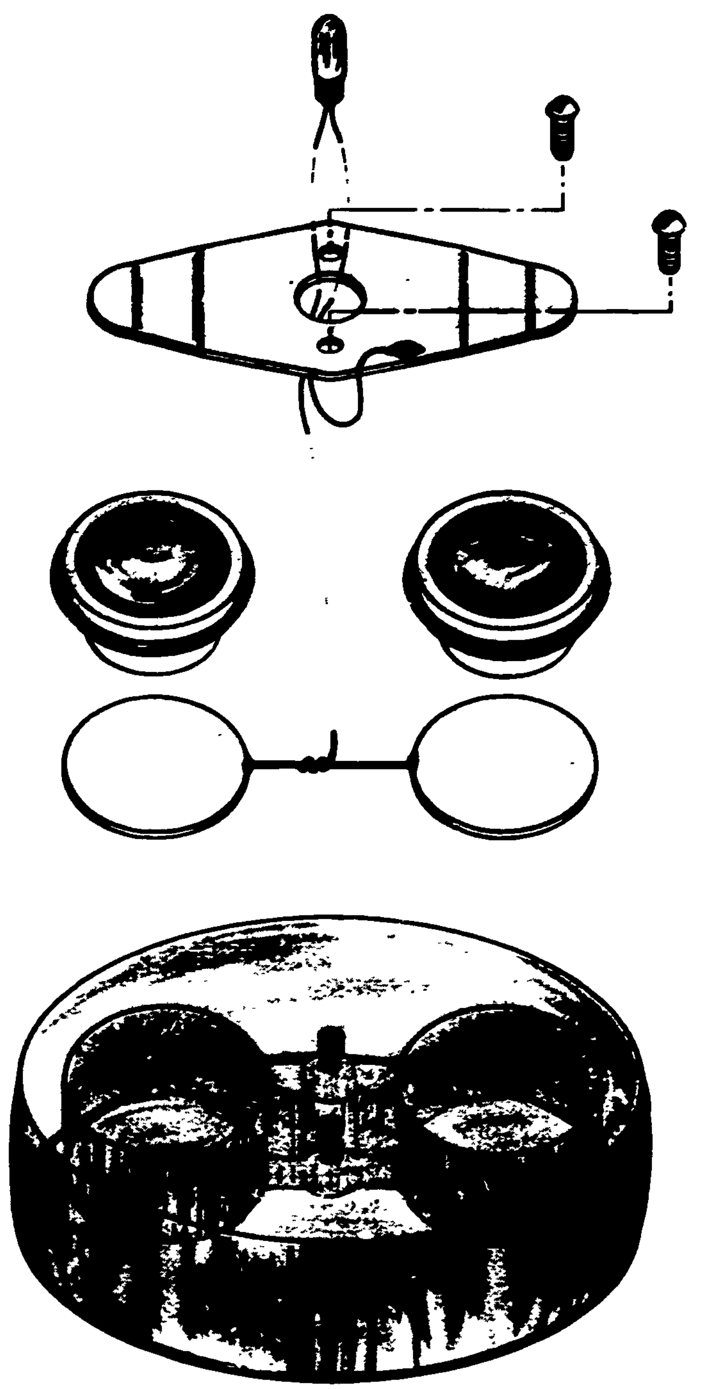


Fig. 4.1. Exploded view of a lighted puck.

PICTURE NUMBER											
DISTANCE BETWEEN COLLI-SIONS											

Fig. 4.2. Histogram for free paths.

tant concept for particle beams of all kinds (electron, proton, deuteron,

Table 4.1 Path lengths between collisions.

(w means wall)

etc.), since one will usually want the beam particle to collide infrequently with the other particles in the path. Mean free paths on the order of 10^6 cm are common in very good vacuums (10^{-9} mm Hg). Mean free paths for air molecules are on the order of 10^{-6} cms.

The equation for the free path distribution can be derived as follows. We first assume that the probability per unit length k of a particle undergoing a collision between a distance x and x + dx is independent of the distance x the particle has already gone. Then the probability of a collision occurs for a particle in the interval dx is just k dx. If there are N such particles having traveled a distance x without collision, N k dx will undergo collision in the interval dx. This will decrease the number of particles reaching x + dx without collision by an amount dN, the number which underwent collision in the interval dx. One can then write

$$dN = -N\,k\,dx.$$

Upon integration this yields

$$N = N_0 e^{-kx}$$

where N_0 is the number of particles which start at $x = 0$.

Note: The constant $k = 1/\lambda$.

4.2. BROWNIAN MOTION

In 1827, a man named Brown looked into a drop of water in which was suspended many fine particles. He saw the small particles violently agitated in a never-ceasing motion. He correctly conjectured that this motion was due to statistical fluctuations in impacts by the water molecules. Considerable controversy was raised over this explanation, but it survived. All controversy stopped with the work by Einstein in 1905 on this motion. Two of his papers are found in Ann. Phys. **17**, 549 (1905), and **19**, 371 (1906). Here then seems to be the desired connection between the motion of a "large" object and smaller invisible ones which will enable us to determine the mass of the small particles. Presumably we can do this by observing the large particles and invoking the equipartition of energy law. The actual energy measured for these particles fails by a factor of 10^5, see Perrin (*Atoms*, transl. D. L. Hammick [Constable and Co., Ltd., London, 1923]) page 109. The reason for this is that the individual particles undergo so many collisions, on the order of 10^{21}/sec, and the paths are so erratic that the actual time development of the path is not seen; only occasional points on the path are seen. A further discussion of this will be found in Perrin's book.

It would be well to have a look at this chaotic motion anyway. To do this, get a regular 3 1/2 × 4-inch-lantern-slide projector and focus it on a relatively large screen. Then blow some smoke particles into the space where the slide should be. On the screen will be seen some of the smoke particles. Owing to air currents, they will drift in and out of focus and across the field of view. If one looks closely, however, the violent small-scale agitation of Brownian motion is clearly visible.

Perrin has studied such motions in considerable detail. In his book will be found many other ways of observing this interesting phenomenon. One should not overlook the very fine latex spheres used for the calibration of electron microscopes nowadays as an excellent source of particles for viewing Brownian motion.

In Fig. 4.3. one sees a path traced out by a mastic grain (radius equal to .53μ) by Perrin. The particle was followed in the field of view of the microscope for about 140 minutes (thirty seconds between points).

The path is not unlike the photograph of paths of a lucite puck on the air table. However, the photograph shows the actual path whereas the path observed by Perrin is not. In Perrin's words, "As a matter of fact diagrams of this sort... give only a very

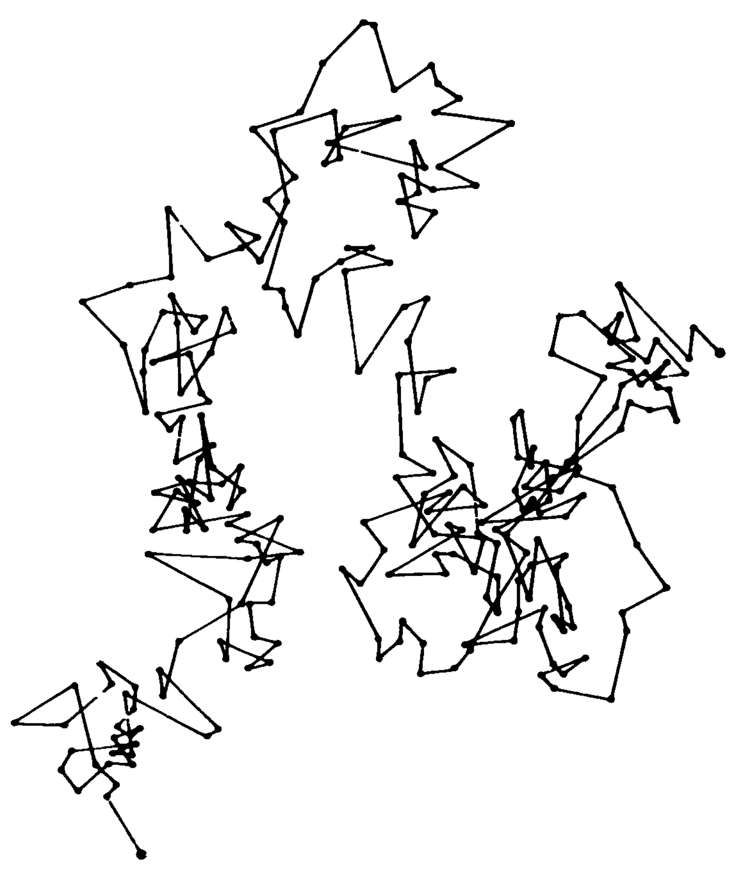


Fig. 4.3. Path of mastic grain, as recorded by Perrin (*Atoms*, p. 116).

meager idea of the extraordinary discontinuity of the actual trajectory. For if the positions were to be marked at intervals of time 100 times shorter, each segment would be replaced by a polygonal contour relatively just as complicated as the whole figure 4.3 and so on."

There is an interesting relation between the distance the particle moves from its initial position after a certain number of steps. The steps can be either the individual steps between collisions or the distance moved every 30 seconds as recorded by Perrin. In both cases the distance from the origin is equal to $\ell\sqrt{N}$, where ℓ is the average length of the step and N is the total number of steps.

Let a particle take steps of average length ℓ in the xy plane. The direction of the path step is assumed to make an angle φ with respect to the x axis. Then the total vector displacement $\vec{r}$ after N steps is

$$\vec{r} = \ell\lfloor \vec{i}(\cos\varphi_1 + \cos\varphi_2 + \ldots + \cos\varphi_n) + \vec{j}(\sin\varphi_1 + \sin\varphi_2 + \ldots + \sin\varphi_n)\rfloor$$

The length of the vector $\vec{r}$ is

$$r = (\vec{r}\cdot\vec{r})^{1/2} = \ell\lfloor \cos^2\varphi_1 + \cos^2\varphi_2 + \ldots + \cos^2\varphi_n + \sin^2\varphi_1 + \sin^2\varphi_2 + \ldots + \sin^2\varphi_n + \sum_{\substack{i \\ i\neq j}}\sum_{j} \cos\varphi_i \cos\varphi_j + \sum_{\substack{i \\ i\neq j}}\sum_{j} \sin\varphi_i \sin\varphi_j\rfloor^{1/2}$$

The last terms will average to zero since the terms are as often plus as minus. The result is then $r = \ell\sqrt{N}$.

You may wish to test this on the pictures of sequences of paths you have made. Remember, however, that the walls confine the motion to a rather small region. By reflecting the motion across the walls, however, you can draw out the path as though no walls were present.

Remember you will have to keep track of single, double, triple reflections and so on. After you have completed the plot, enter the values obtained for the distance moved after every ten collisions. Compare these distances with $\ell\sqrt{N}$, where ℓ is the mean distance and N is the total number of steps.

How did they compare? Were the results better or worse as the number of steps increased? If you cannot make the pictures yourself, use Figs. A.109 to A.126 provided in the Appendix.

EXPERIMENT 5 GRAVITATIONAL SEPARATION

When the table is tilted, the pucks tend to gather at the lower end of the table. The only thing preventing this is the colliding of the pucks with each other and the agitation of the table. The way in which the pucks distribute themselves can be determined in a manner somewhat like an

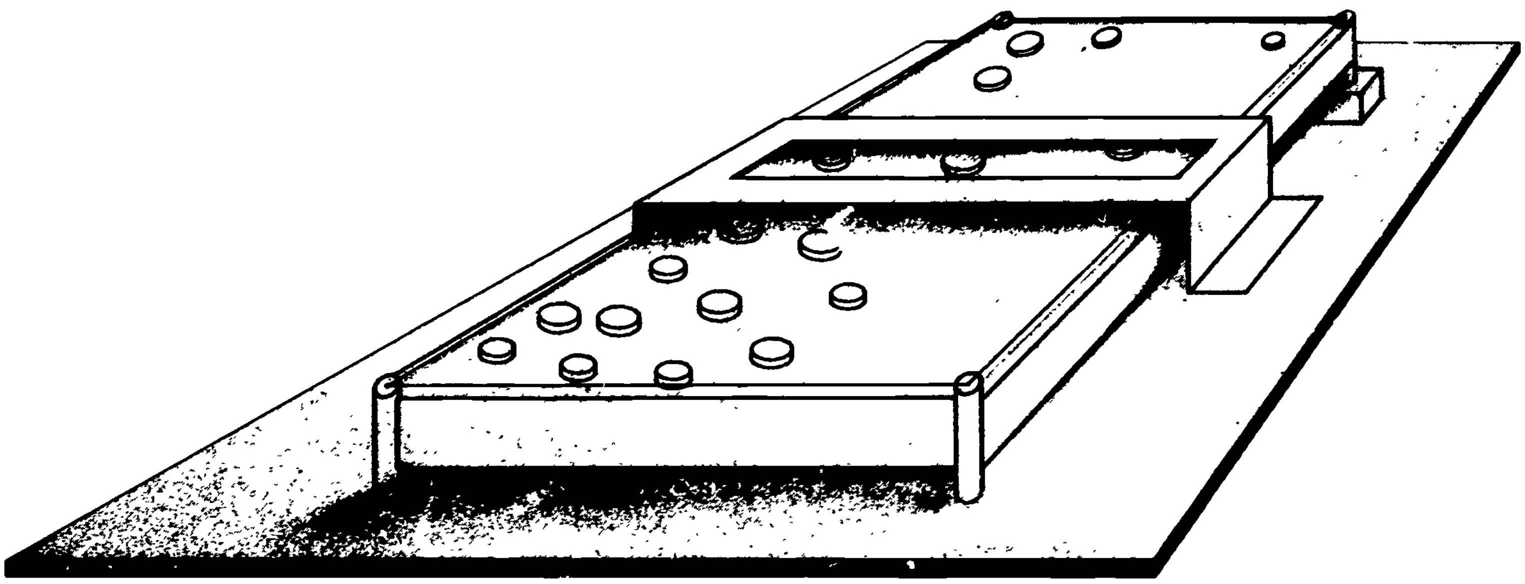


Fig. 5.1. The air table is shown tilted with a cardboard slit in place to enable one to determine the vertical distribution of pucks. (The method for agitating the table walls is not shown).

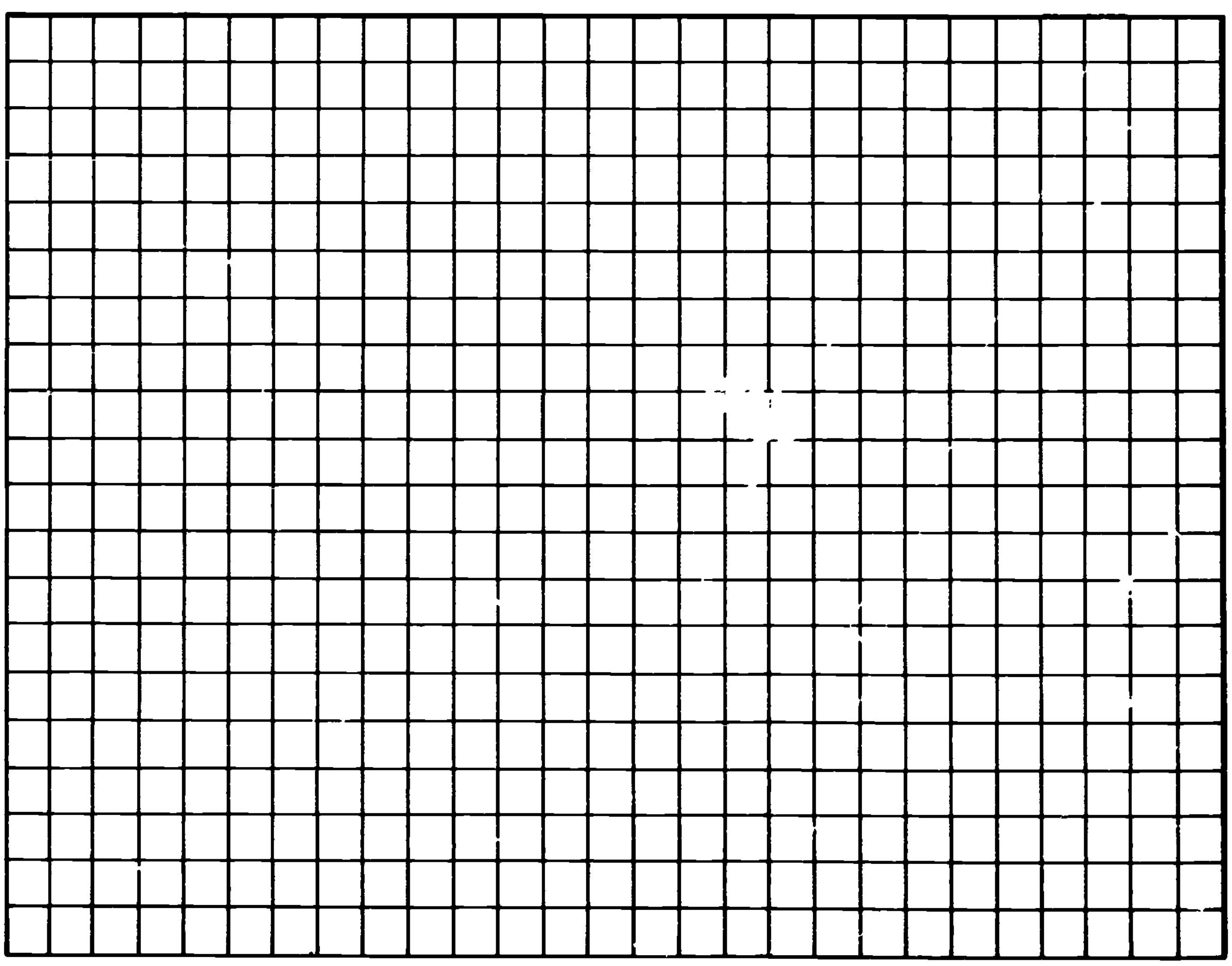


Fig. 5.2. Histogram for vertical distribution.

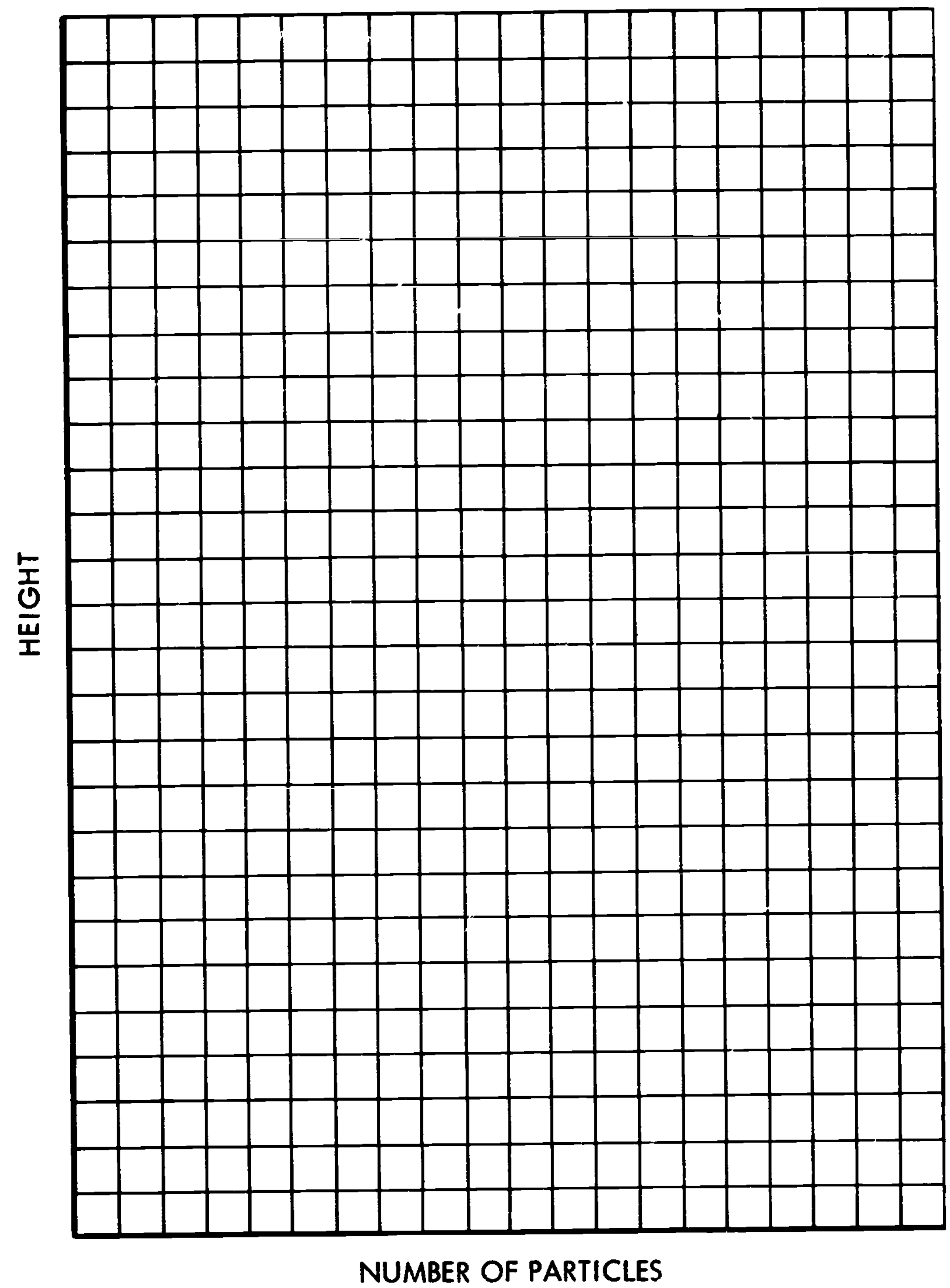


Fig. 5.3. Plot of number of particles versus height.

earlier experiment of Perrin's.

Tilt the table about two degrees. The degree of agitation should be such that very rarely will a puck hit the upper end. Take a piece of cardboard and cut out a rectangle in the center of this sheet as shown in Fig. 5.1. Set this slot across the table with the short width of the slot pointing to the high end of the table. You are going to count pucks in this slot. If you have a stroboscope that can be set for about one flash per 4 seconds or 15 rpm, it will be ideal. Otherwise, you may get a timer which puts out a signal about every 4 seconds. Simply record the number of pucks you see in the slot at each light flash or timer signal. Decide in advance how you will handle fractional pucks and then be consistent. After you have taken a considerable number of readings you will find the average number seen becomes stable. Record this reasonably stable average with the position of the slot on the air table in Table 5.1. Then repeat the experiment for many different positions on the table.

Make a plot of the resulting distribution in Fig. 5.2.

Table 5.1 Vertical distribution.

POSITION ON AIR TABLE																
TOP	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
BOTTOM																

The movie <u>Random Motion</u>* can be used for this experiment. Get a film editor and turn the film to the tilted table portion. Now put a grid on the viewer screen and count the number of pucks touching lines at constant height on the table. This data can be entered into Table 5.1. One then advances a few frames (about 20) and records additional data. Record at least 10 frames worth of data.

We can determine two expressions for the behavior of the puck distribution with height. We can write the ideal gas law for a two-dimensional gas as FA = nkT where F is the force per unit length, A is the area, k is a constant and T is the "temperature." For an isothermal process, FA = constant. If we consider the change in force per unit length at a height between h and h + dh on the table we have.

$dF = \rho g dh \sin \vartheta$, where ρ is mass/area

g = acceleration of gravity and ϑ is the angle of tilt for the table. The ideal gas law can be written $F\rho^{-1}$ = constant from which we can obtain dF in terms of ρ and $d\rho$ to obtain

$$d\rho = -\frac{\rho g \sin \vartheta dh}{C} .$$

This can be integrated to yield $\rho = \rho_0 \exp -\left(\frac{h}{h_0}\right)$. One can replace the mass/area ρ with say the number of pucks per unit area n and write

*See note at the beginning.

$n = n_0 \exp - \left(\frac{h}{h_0}\right)$. Since the pucks occupy considerable area themselves n should be the number of pucks per unit area not covered by pucks.

We can also obtain an expression for an adiabatic gas. An appropriate expression can be obtained in this case from the first law of thermodynamics for a two-dimensional gas.

This law states that $C_A dT = -FdA$ when no heat is being added or subtracted from the system. In this expression C_A is the "specific heat" at constant area. From the ideal gas law we obtain

$$dFA + FdA = nRdT,$$

which together with the above equation yields $(C_A + nR)\, dT = dFA$. The expression $C_A + nR = C_F$ is the specific heat at constant force/length. We can then write

$$AdF = C_F dT \text{ and}$$

$$FdA = - C_A dT$$

or $dF/F = - C_F/C_A\, dA/A$ which integrates to FA^γ = constant where $\gamma = C_F/C_A$.

We can get the last expression in terms of ρ and h.

$$A = \frac{C}{\rho} \text{ and } F = \rho nRT.$$

Hence $FA^\gamma = C^\gamma \rho^{-\gamma+1}\, nRT$ = const, or $\rho^{1-\gamma}\ T$ = const.
The temperature is proportional to the kinetic energy (total energy less potential energy) and hence $\rho^{1-\gamma}(E - mgh \sin\theta)$ = const or of the form $\rho^{1-\gamma} = (A - Bh)$

$$(A - Bh) = \rho^{\gamma-1}.$$

Make a plot of your data and see whether your gas is more nearly isothermal or adiabatic. Now put in the puck with the light and take a picture of its path. Each individual short section of path between collisions has a parabolic shape. This, of course, is due to the fact gravity acts on the particle between collisions. Every individual air molecule will also describe such paths between collisions, even though the paths are much shorter and the molecular speed is much higher. Figure A.108 is such a picture.

EXPERIMENT 6 TIME AND SPACE AVERAGES

In Experiment 1 we took pictures of a set of many different particles. We calculated from these many different particles the velocity distribution. We also arrived at the collision frequency by looking at the number of collisions which occurred for many different particles. Suppose now we look at one particle only and determine its history. For example, as was done in Experiment 4, we follow a particle's path. The irregularities in this path correspond to collisions. How does the number of collisions per second for the one particle compare with the total collision rate? This can be found by counting all the collisions and dividing by the exposure time. Count wall collisions separately from other particle collisions. Enter the numbers in the boxes below and compare them with the numbers you entered in the boxes in Sections 2.1 and 2.2.

Collision/sec one particle	Collision/sec many particles

Can you use the total number of particles to relate these two quantities? This procedure is an example of taking a time average instead of a

Table 6.1 Speed comparison.

space average. The equality of the two types of averages says that what one particle does in time is equivalent to what many different particles are doing at one instant. A more impressive example would be to take a speed distribution on one particle and compare it with the speed distribution of many particles. This can be done by installing a chopper in front of the camera lens, or installing a flashing light on one puck. The chopper or flasher should be operated at a rate of about 10 to 20/sec. This converts all the lines in a photograph like Fig. A.114 into dotted lines. The distance between dots measures the speed. Take such photographs, enter the data in Table 6.1. and construct the corresponding histogram. Compare these results with those obtained in Experiment 1. Be sure to have the same table agitation rate.

6.1. HISTOGRAM

This data for this section can be taken from the movie film since the camera has interrupted the motion about 24 times per second. The data can be removed from the film by using a film reader and plotting the path of the center of one puck through a whole series of free paths and collisions. In fact, the free path data of Section 4.1 can be taken in this fashion as can all of the other data for the experiments described in these pages.

C O N C L U S I O N

The previous pages have outlined a set of experiments using an air table and pucks to demonstrate many of the concepts of kinetic theory. Some of these have been considered in detail, such as the speed distribution of pucks. Other experiments hav been treated somewhat superficially, such as those dealing with time and space averages. Perhaps it would be well in these concluding paragraphs to indicate problem areas (other than kinetic theory) that might be treated experimentally using this same technique.

All of macroscopic physics deals with materials made of atoms whether they be crystals, fluids, or amorphous solids. For many physically important processes the atomic detail can be suppressed, such as is done in thermodynamics with pressure temperature and volume; as in fluid flow with density, stream velocity and pressure; and in electricity with current density, potential and resistivity. The two-dimensional puck array makes it possible to study not only these macroscopic properties of materials but also to study the transition from continuum to particle physics. One experiment related to the contents of this monograph which makes the process clear is the propagation of sound waves. The carriers of the sound waves are clearly the individual atoms acting in concert with each other. However, there exist interesting relations between the mean free path and the wavelengths of sound which can be propagated. These can be studied by fixing a piston to one side of the table which can be driven to generate the "sound waves." It would seem that one would need many more pucks than have been used in the experiments outlined here, but nevertheless it can be studied with this technique. If one puts on magnetic pucks to create repelling forces the properties of crystal structure can be studied as can the transition between solid, liquid and gas as the agitation rate, "effective temperature," is raised. It is interesting to contemplate the crystal structures which would arise as one first used single repelling magnetic discs, then pairs of magnetic discs rigidly coupled, and then triplets of magnetic discs rigidly coupled, and so forth.

The experiments dealing with diffusion and transport phenomena would be most illuminating. Self-diffusion rates can be studied, as can the diffusion of unlike particles. The model may be useful in studying neutronlike diffusion and electric charge motion. The numbers of pucks required and relative puck sizes have not been investigated to cover adequately the transition region from particle to continuum behavior. If the entire region cannot be covered, certainly the region in the vicinity of particle behavior can be studied.

The continuum concepts of viscosity and turbulence can with some difficulty be related to particle dynamics. The statement that the big whirls feed on the little whirls and so on to viscosity breaks down on the particle scale. Nevertheless it should be possible to study net momentum transport across boundaries in a puck gas. An investigation of the pertinent numerical factors would clarify the numbers of pucks needed to adequately represent a vortex, for example.

Experiments on the randomizing of motion should be possible for the student. If such experiments could clearly elucidate the Boltzmann H theorem and the probabilistic nature of entropy, they would be valuable pedagodic tools.

The pucks used in the experiments of this book were made of lucite. The collisions are not energy conserving owing principally to slipping about the vertical axis on collision. The more sophisticated experiments require more particles and hence more collisions with a resultant energy loss. Some study is needed to determine the best material to reduce this frictional loss. Small magnetic discs may be the answer. The resulting long-range force, however, will make it difficult to determine speed distribution and free paths since the speed then changes continuously and the path curves continuously. However, experimental determination of the relations between kinetic energy and potential energy may be possible.

Finally let me add my hope that the student who does the experiments in this monograph finds them as interesting and rewarding as did the author.

A P P E N D I X

These figures are 30% smaller than the original

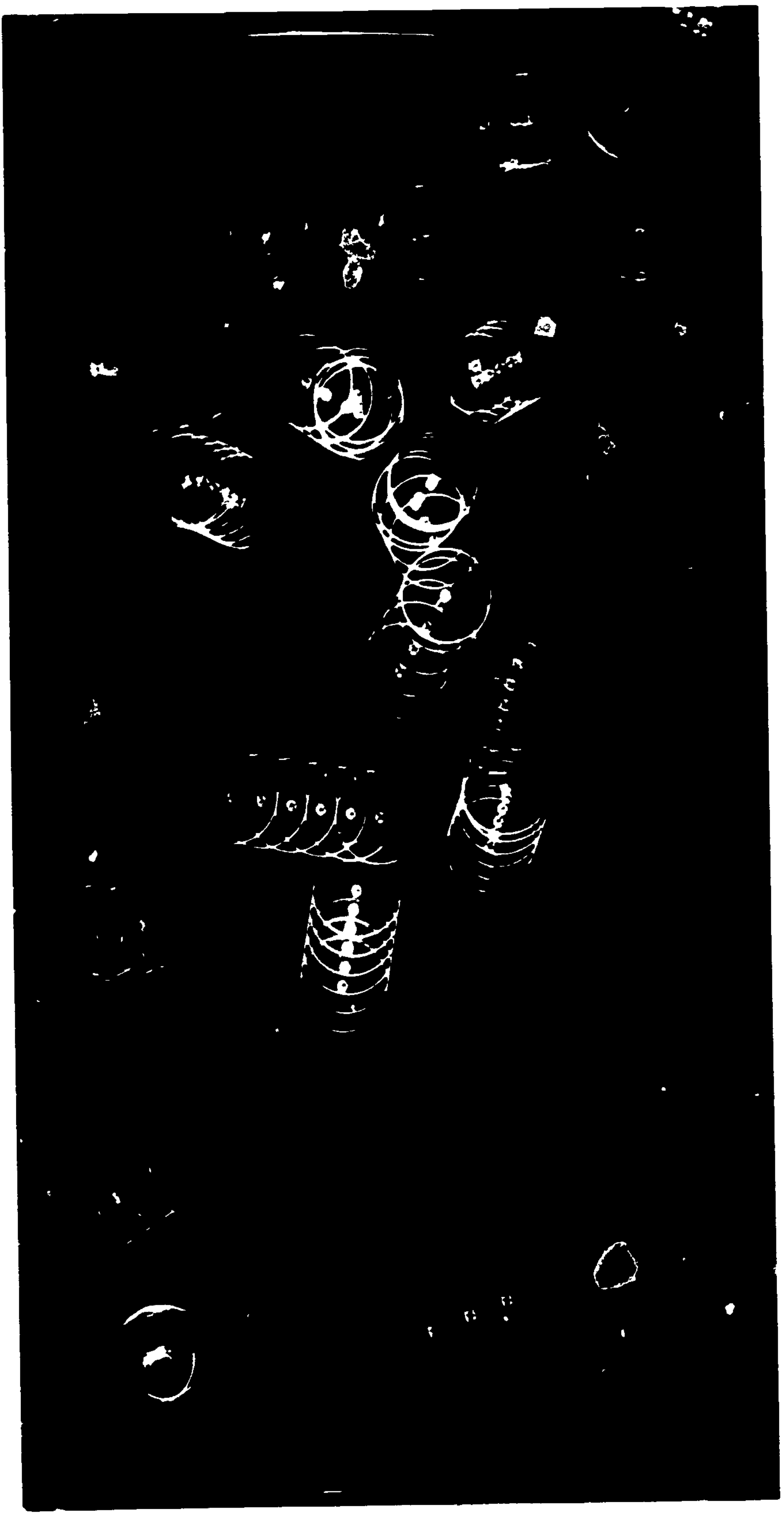


Fig. A.1.

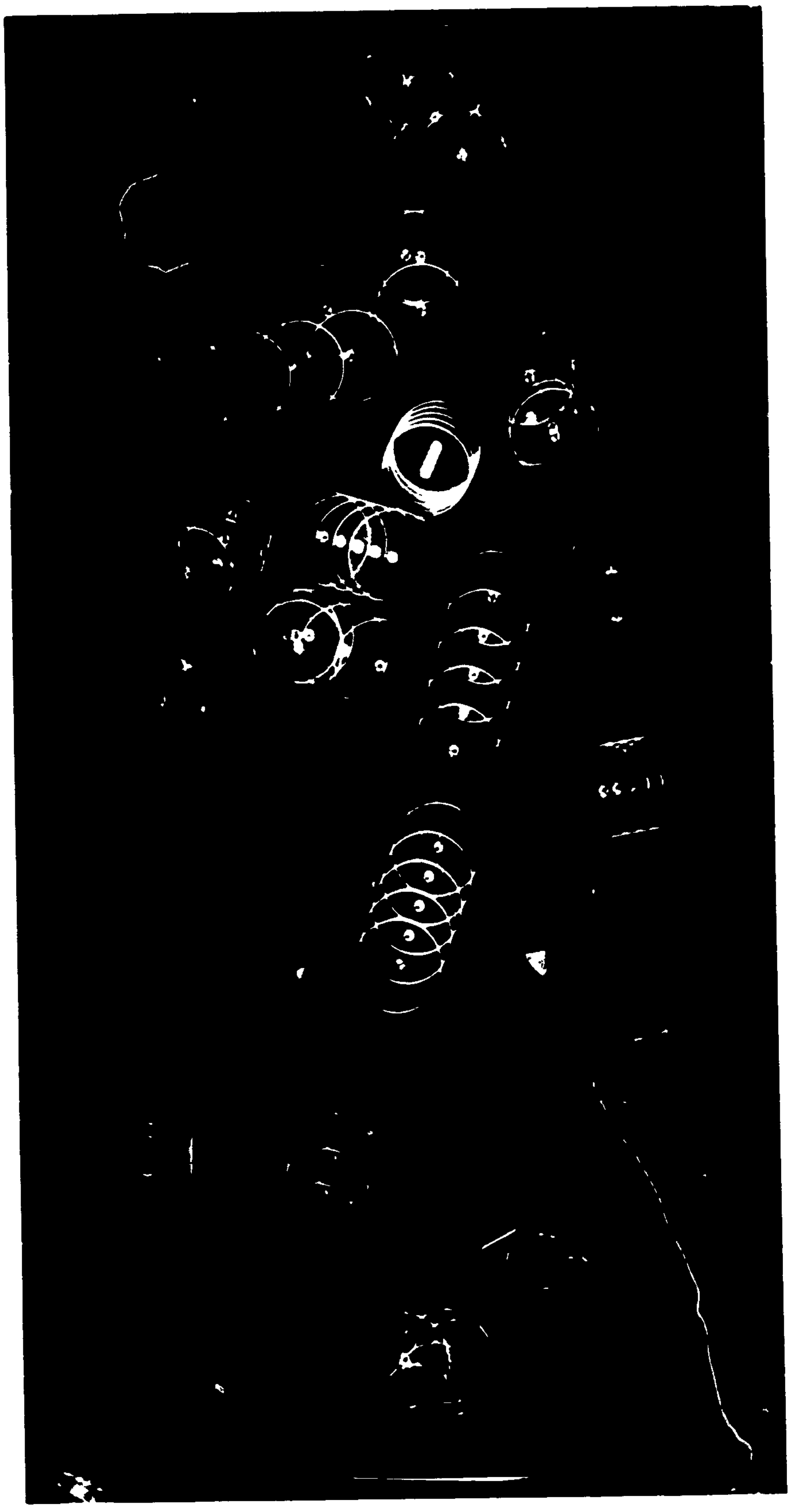


Fig. A.2.

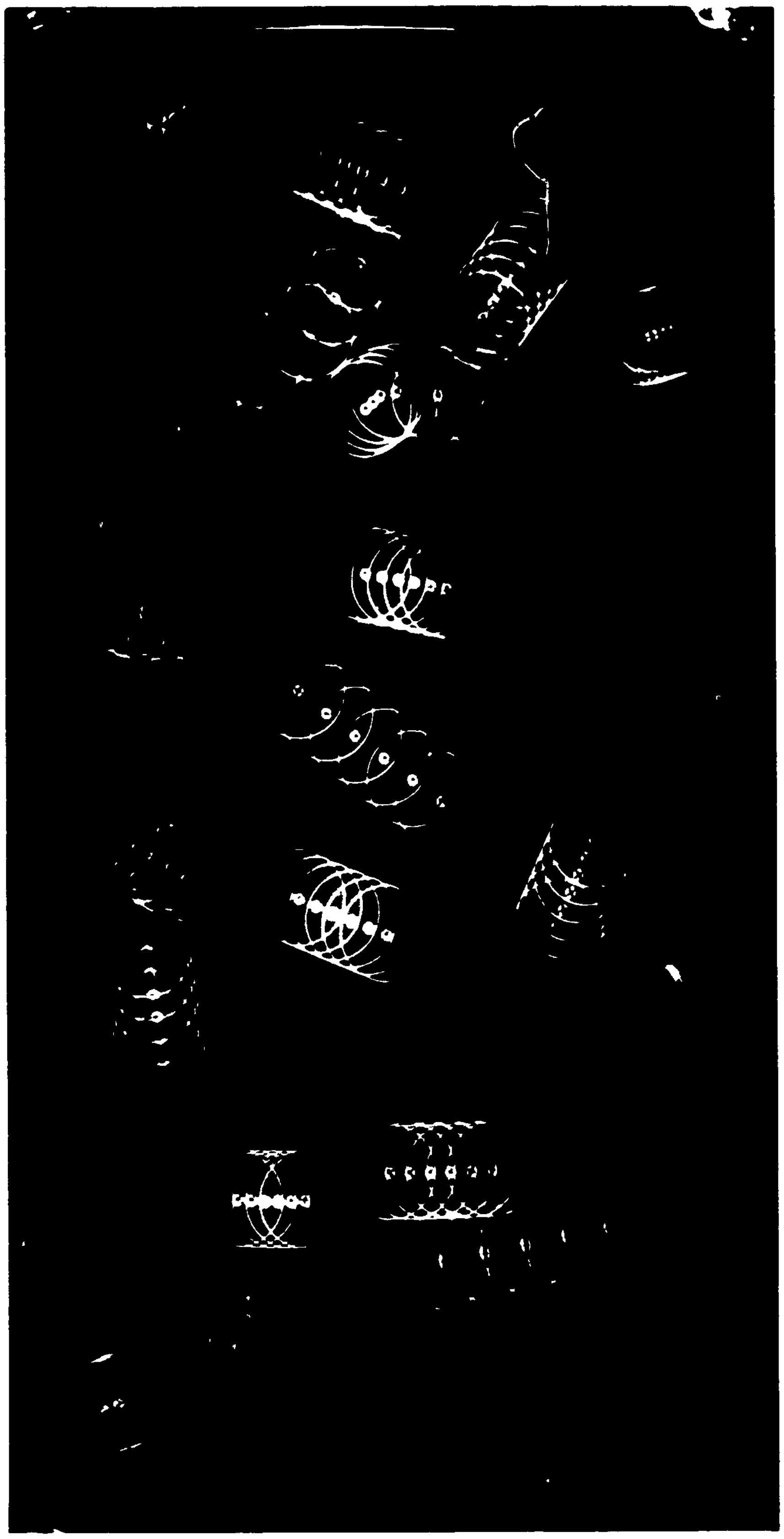


Fig. A.3.

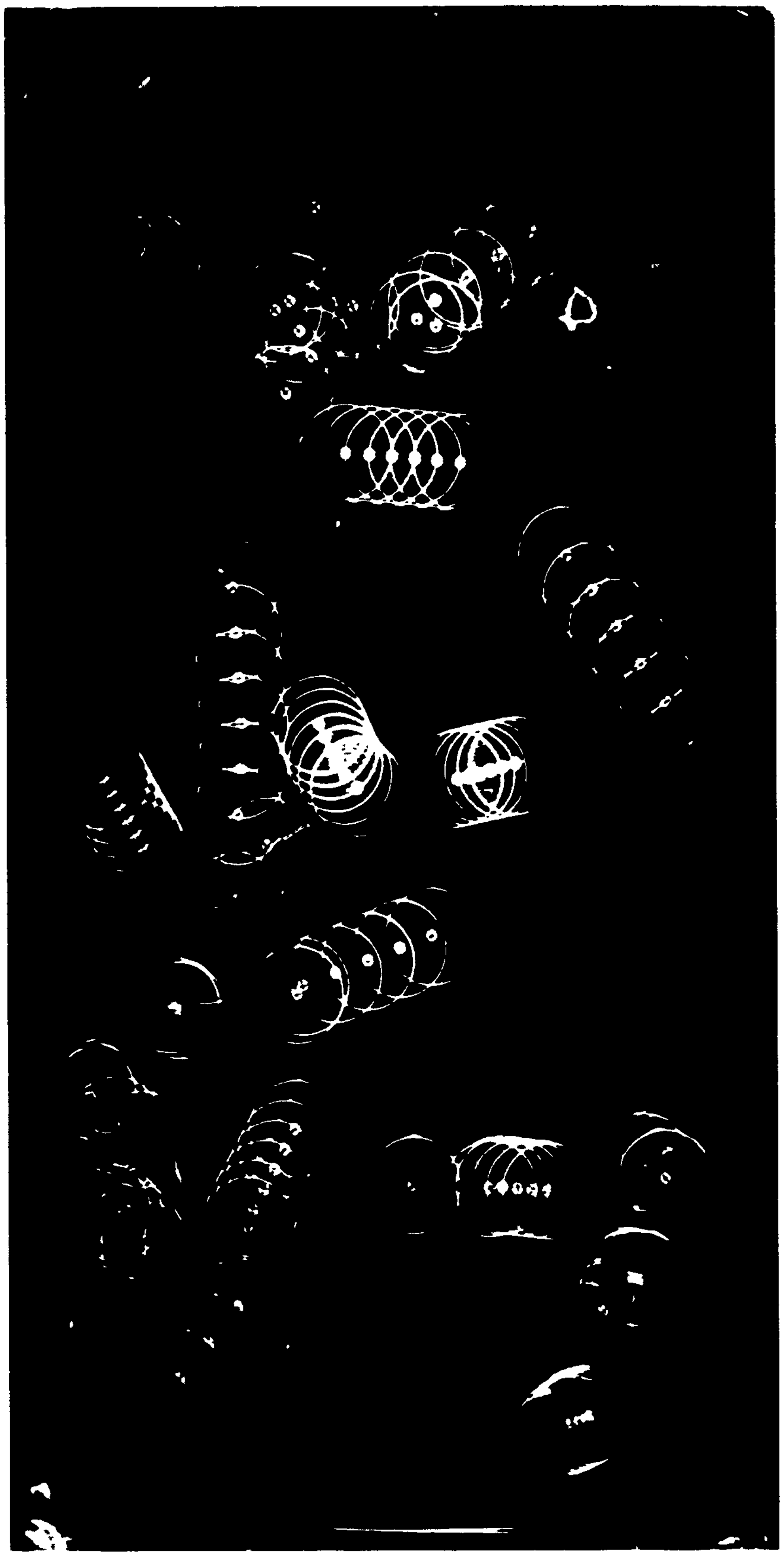


Fig. A.4.

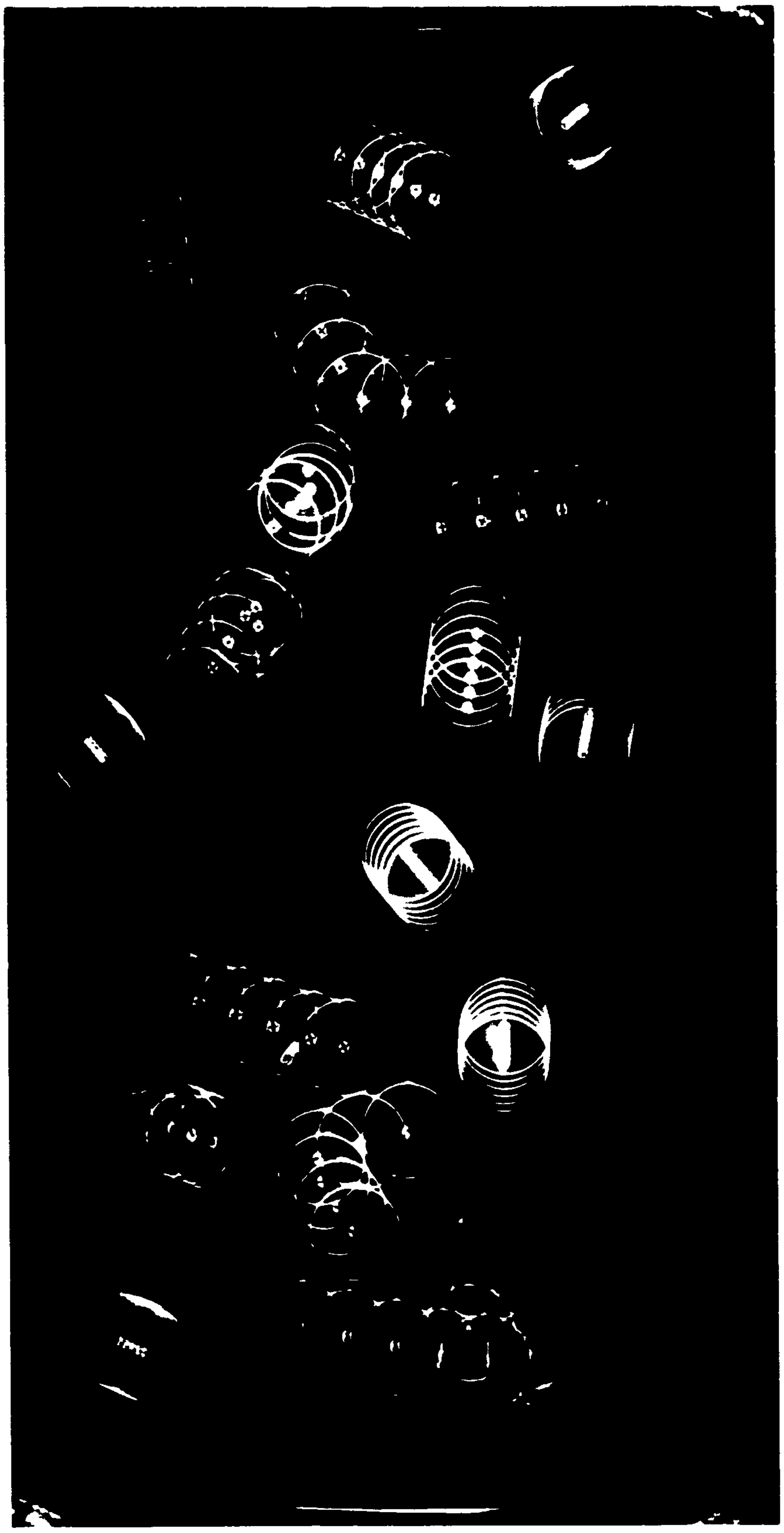


Fig. A.5.

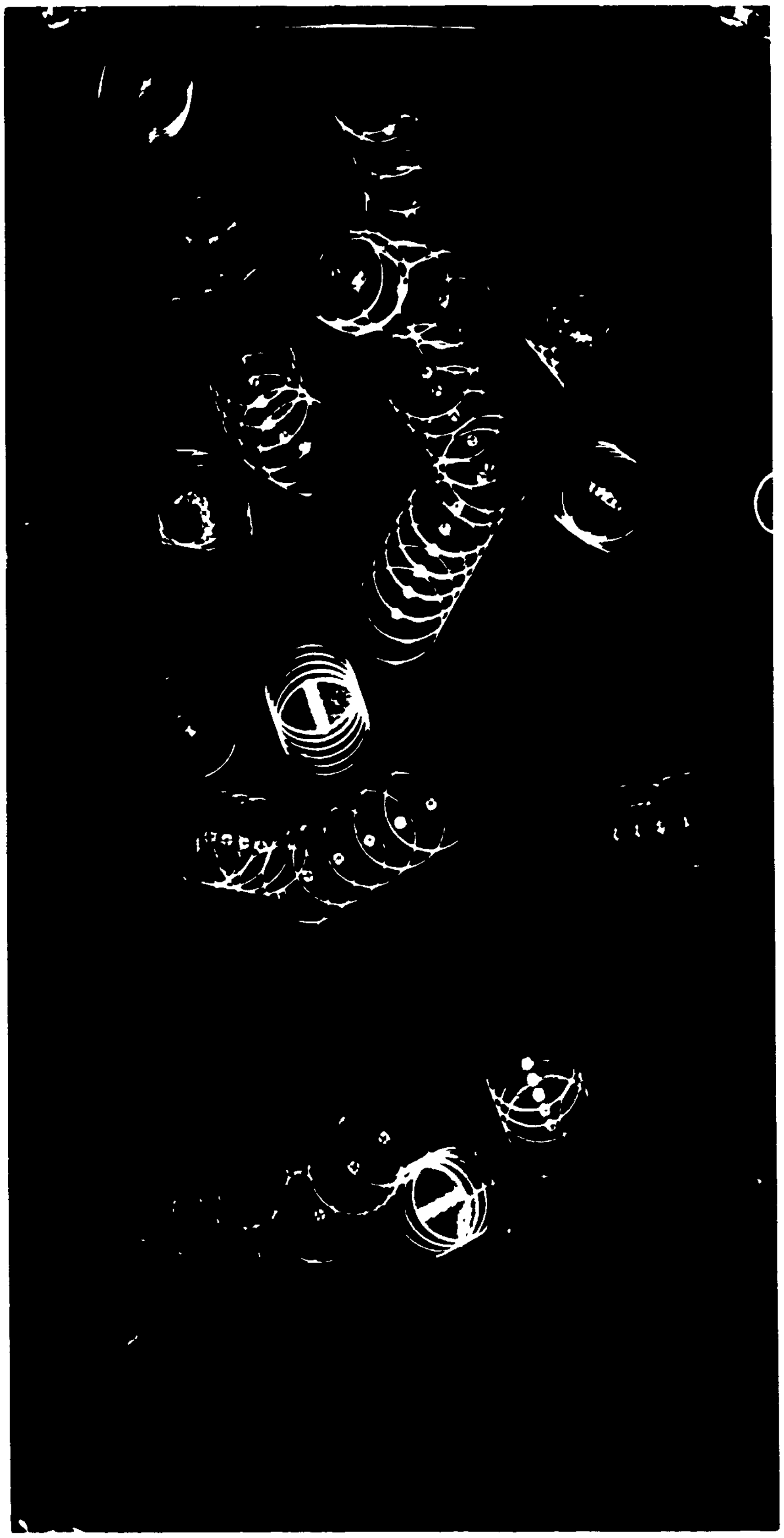


Fig. A.6.

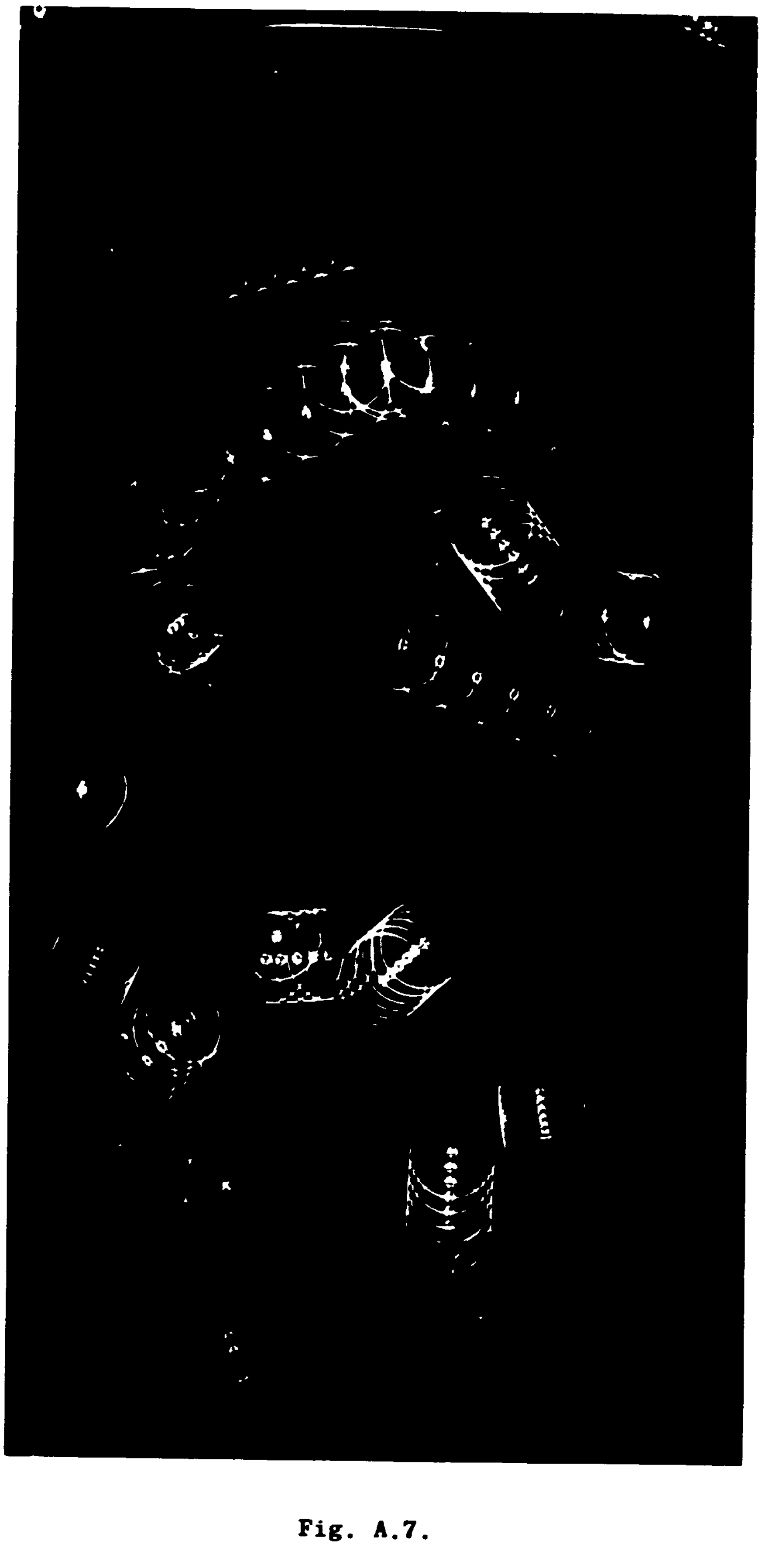


Fig. A.7.

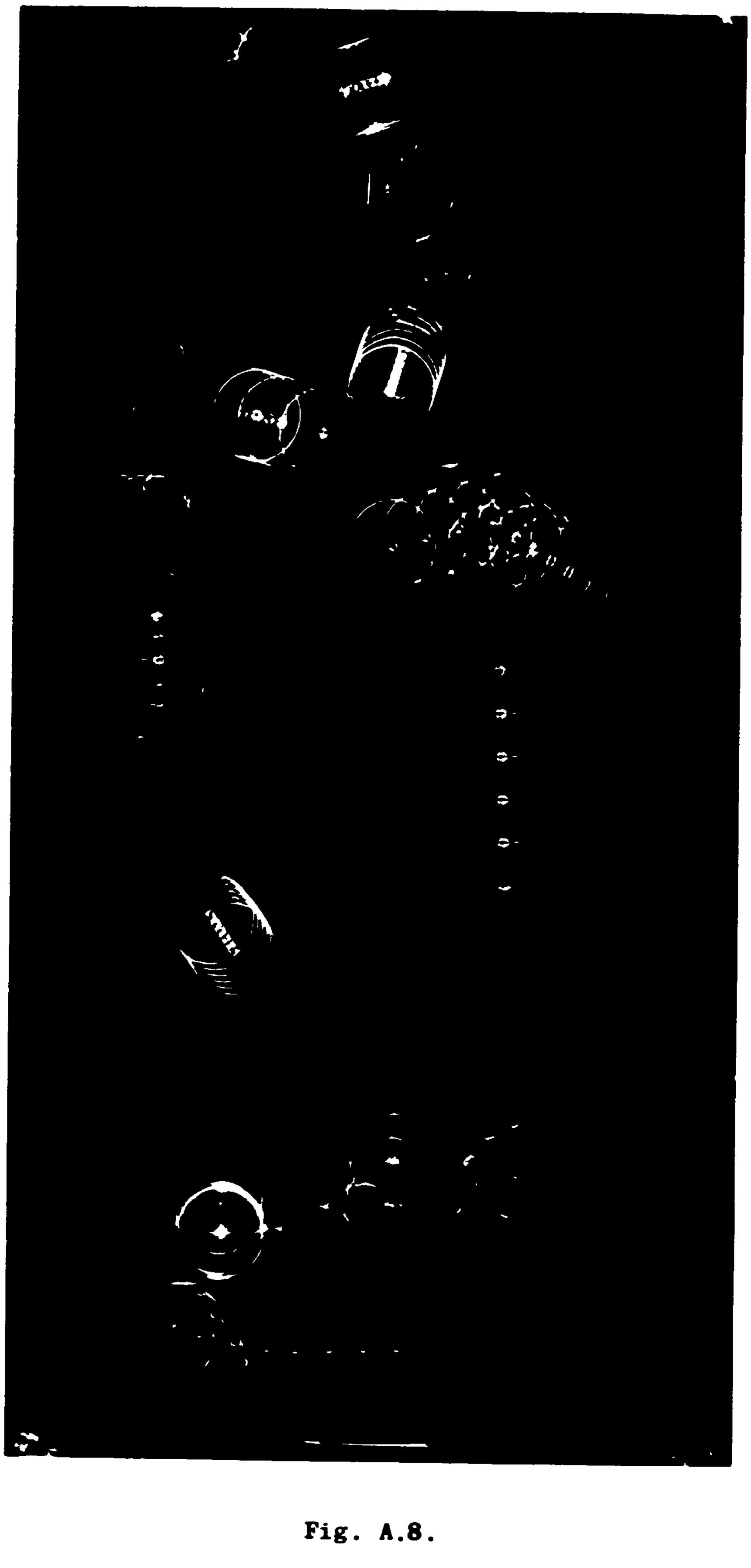


Fig. A.8.

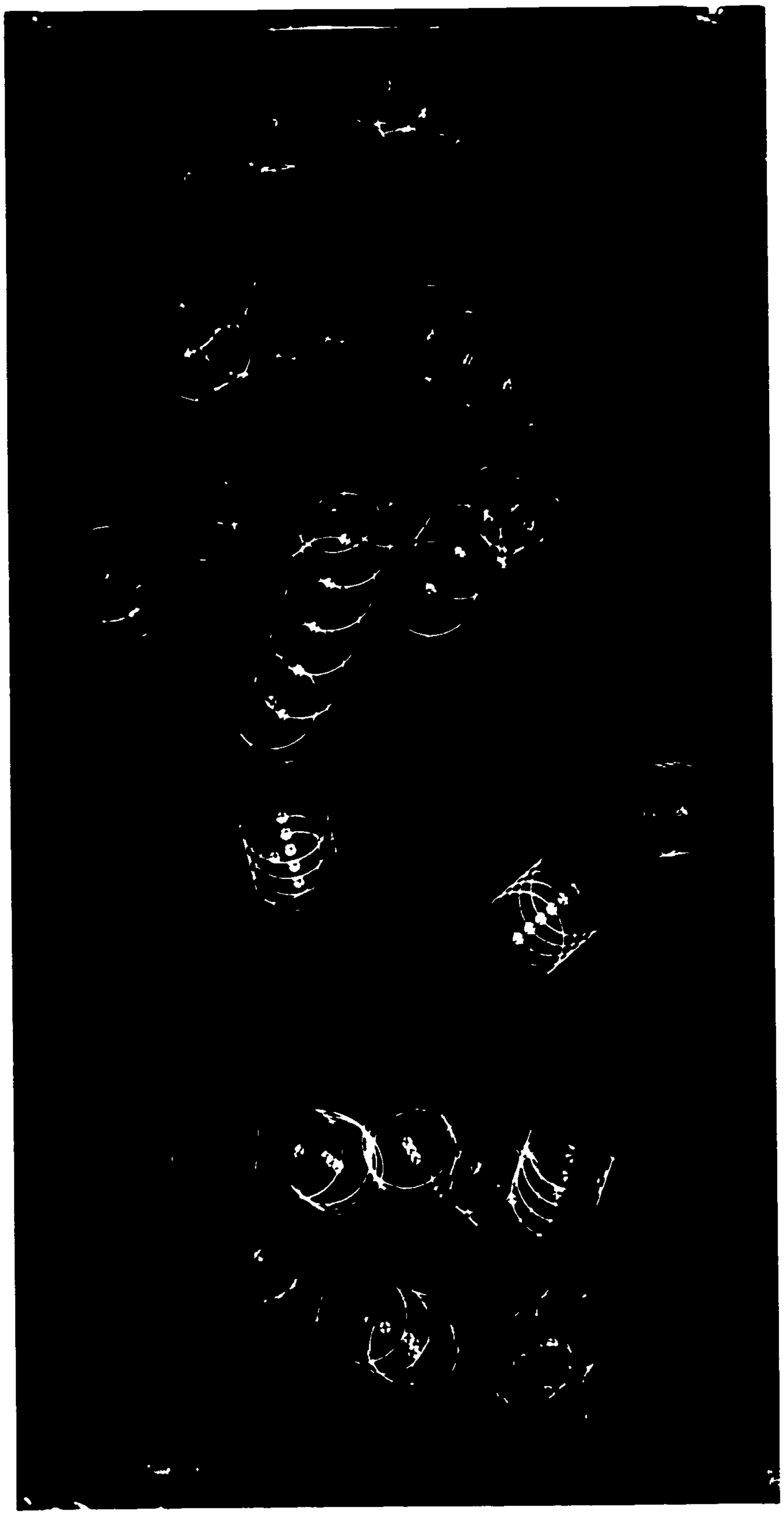


Fig. A.9.

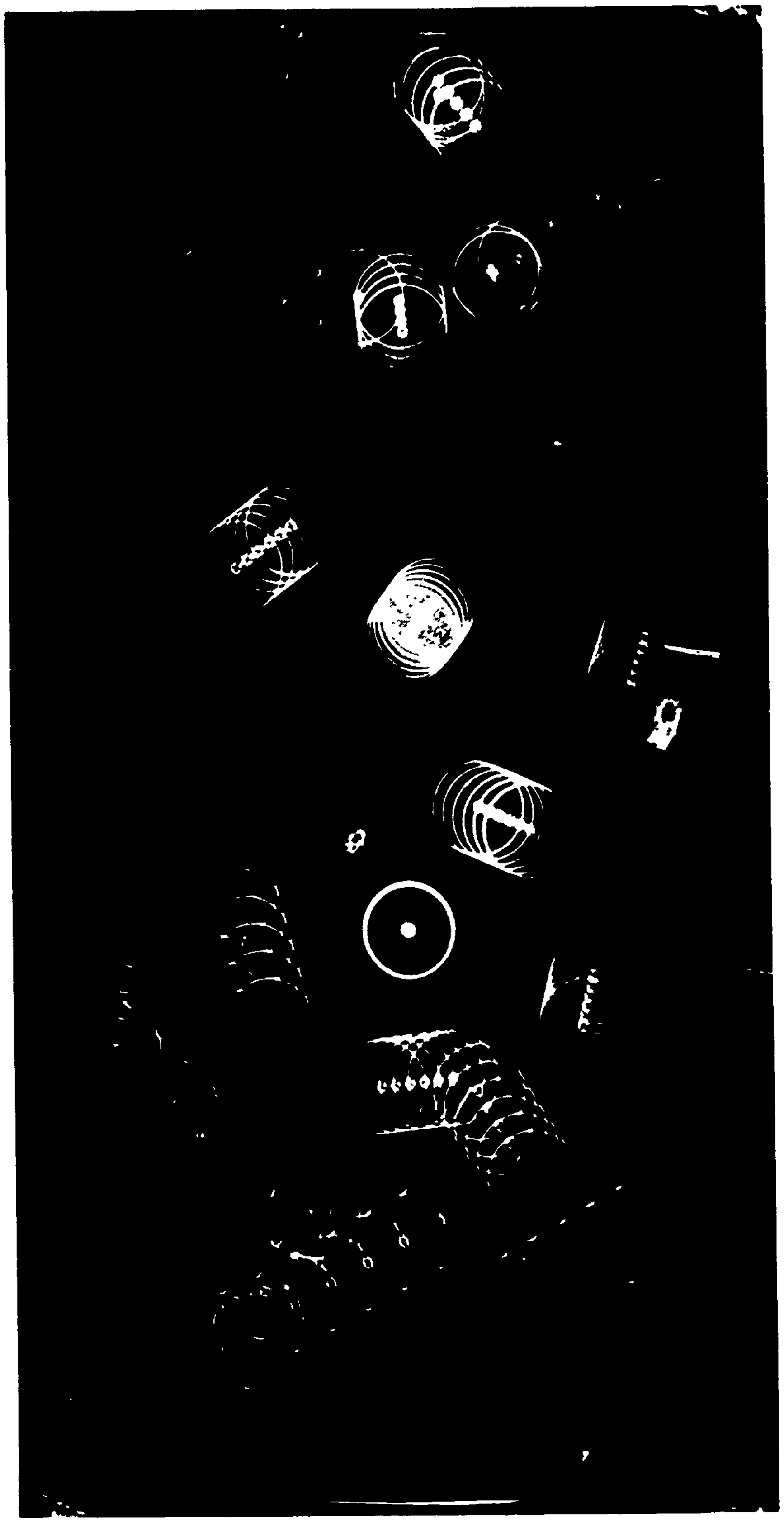


Fig. A.10.

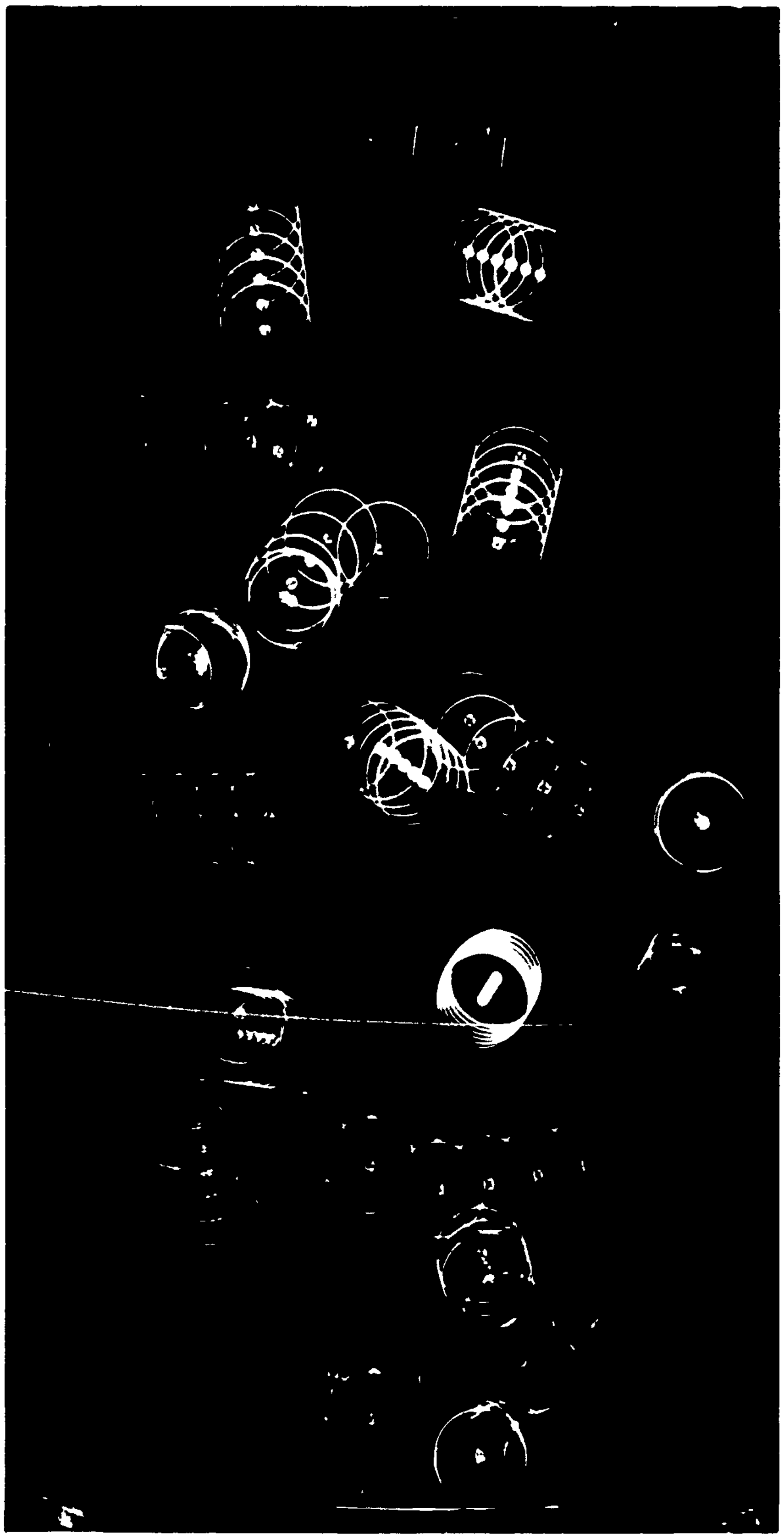


Fig. A.11.

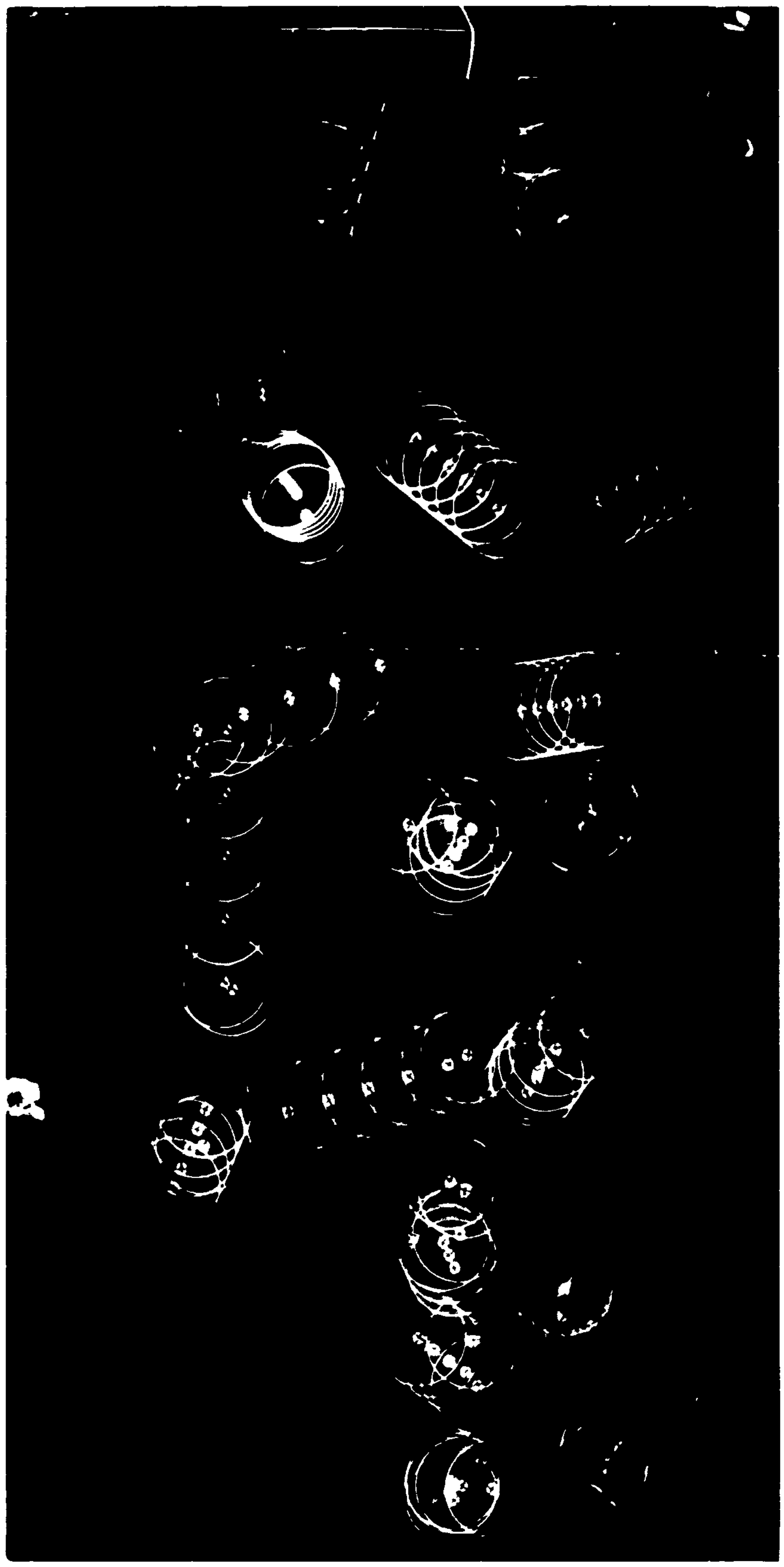


Fig. A.12.

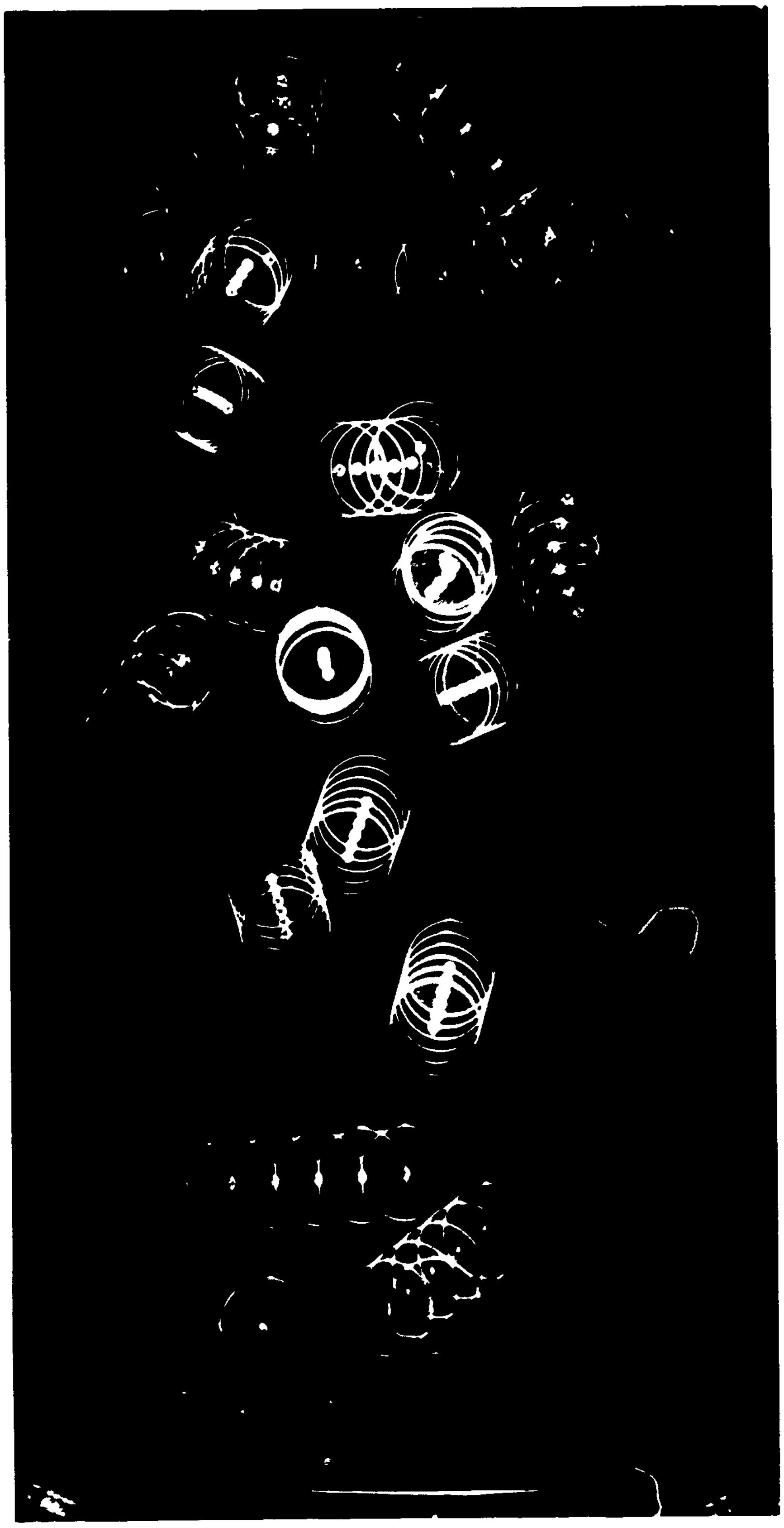


Fig. A.13.

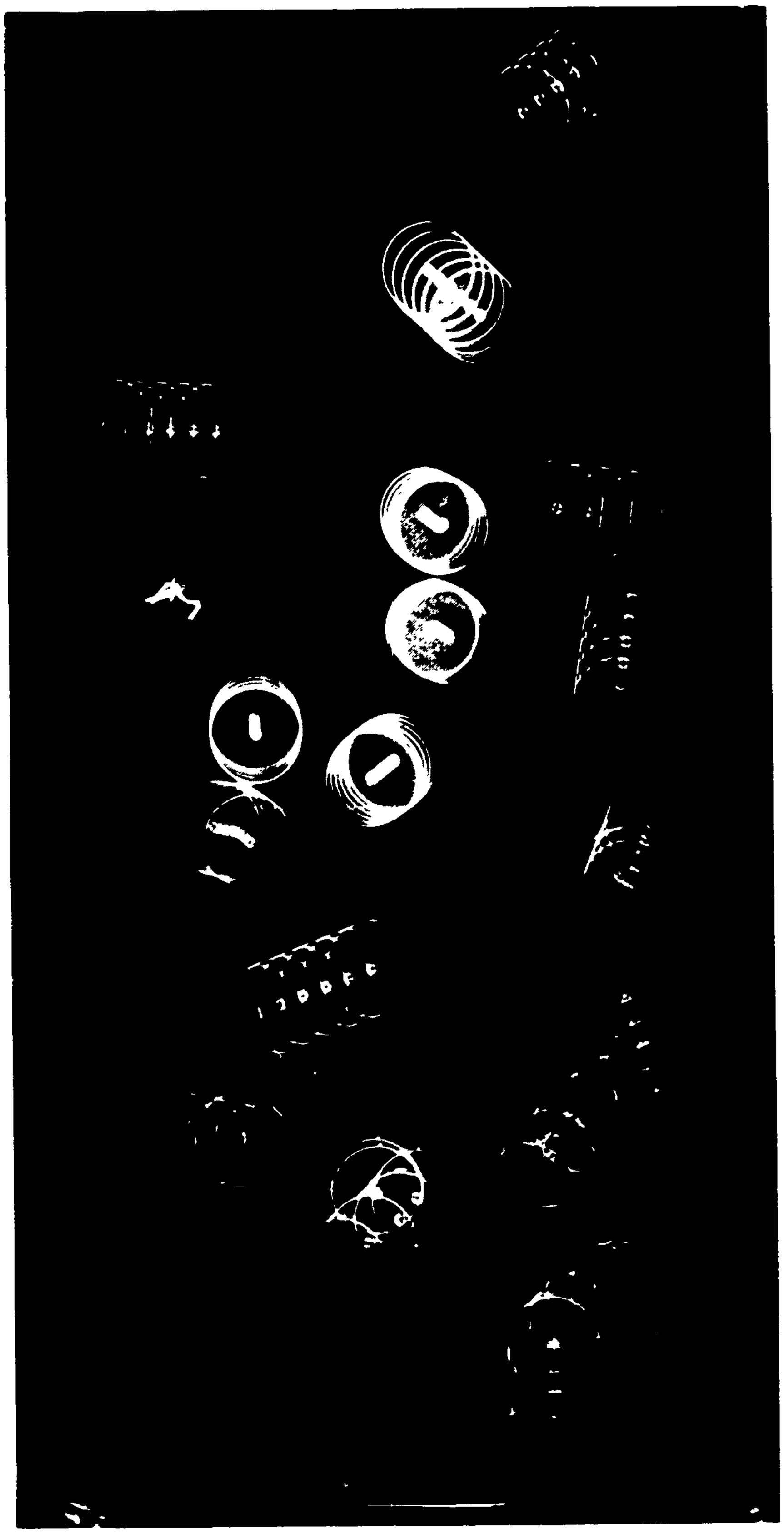


Fig. A.14.

Fig. A.15

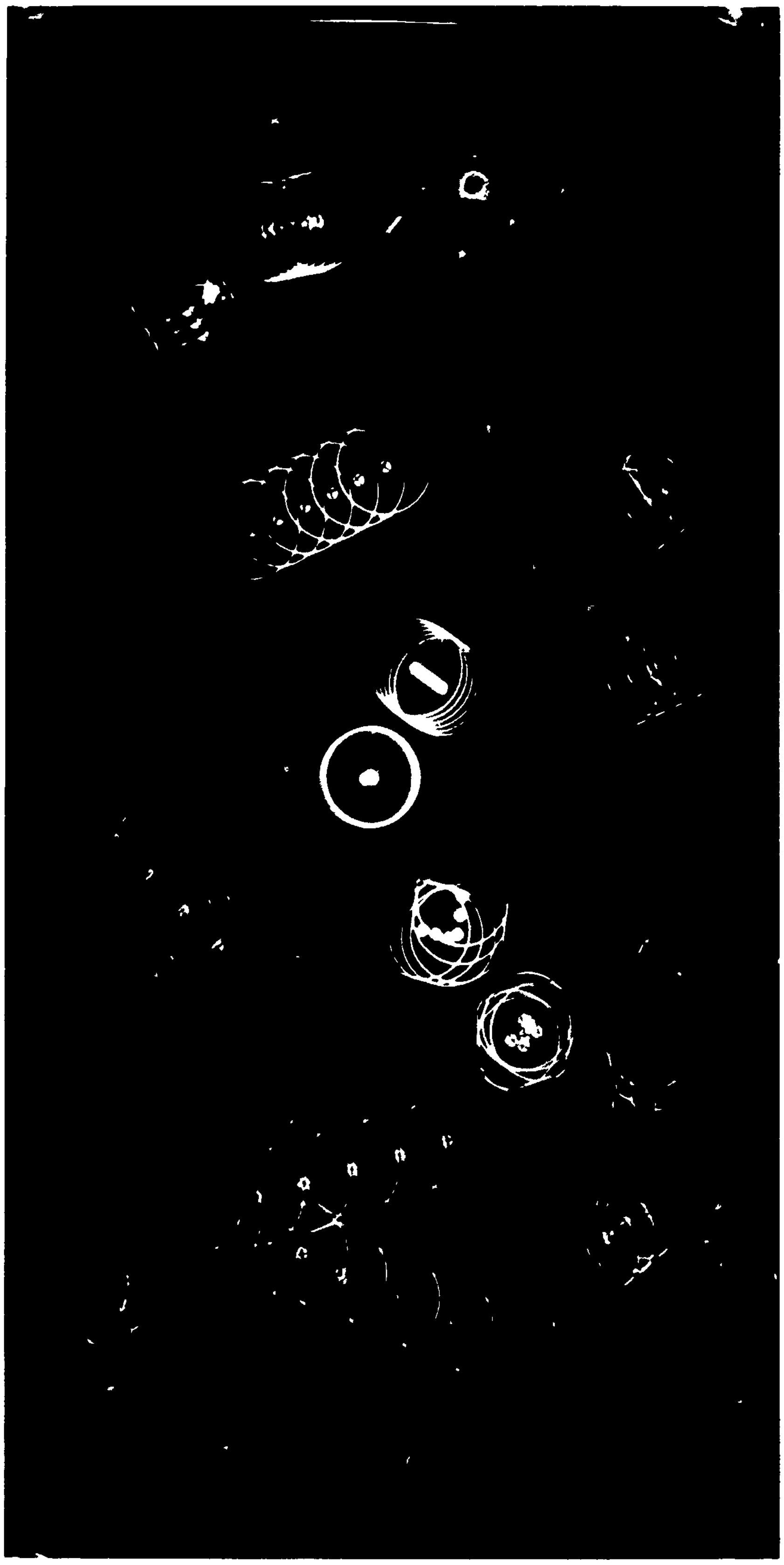


Fig. A.16.

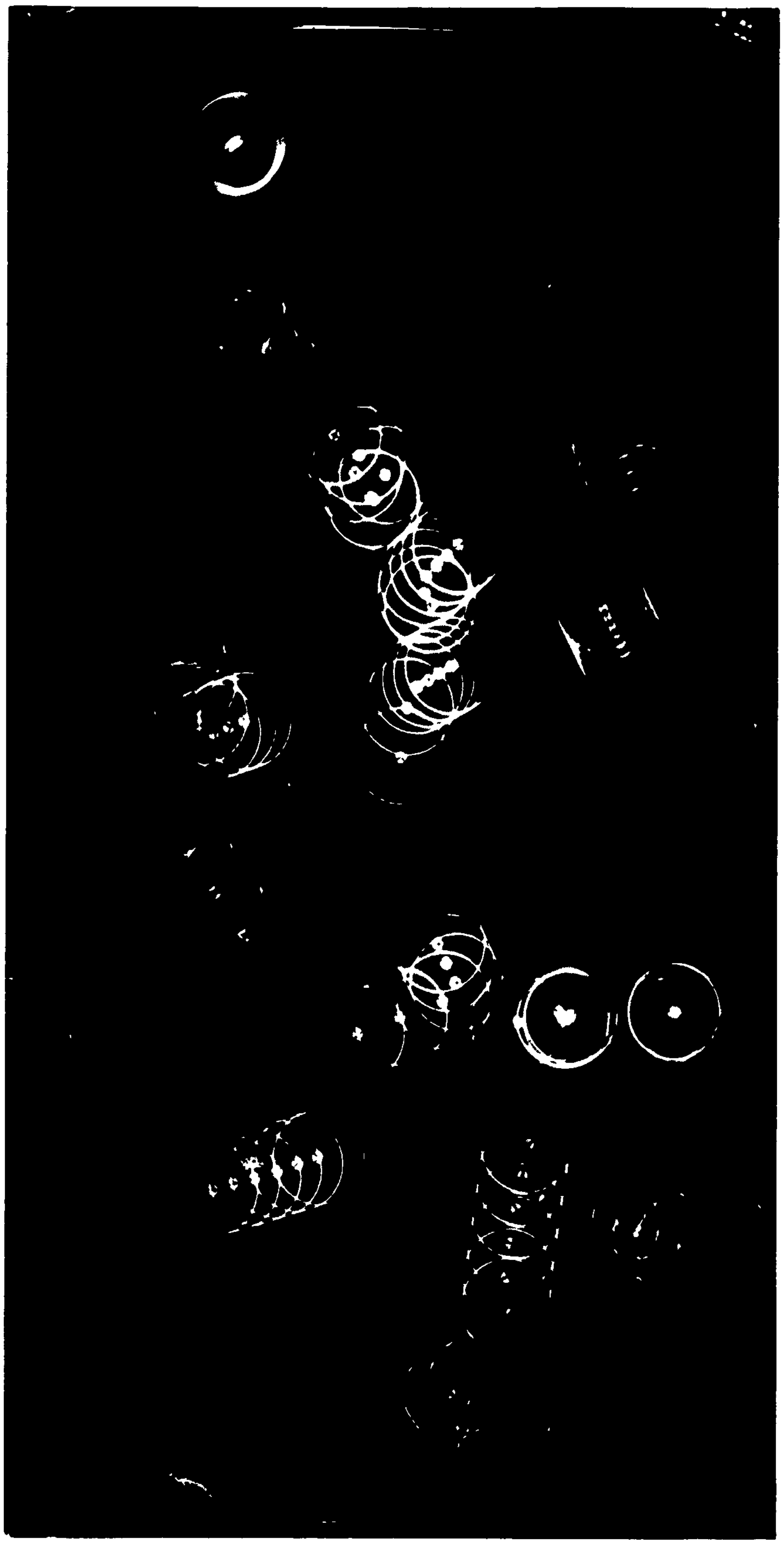


Fig. A.17.

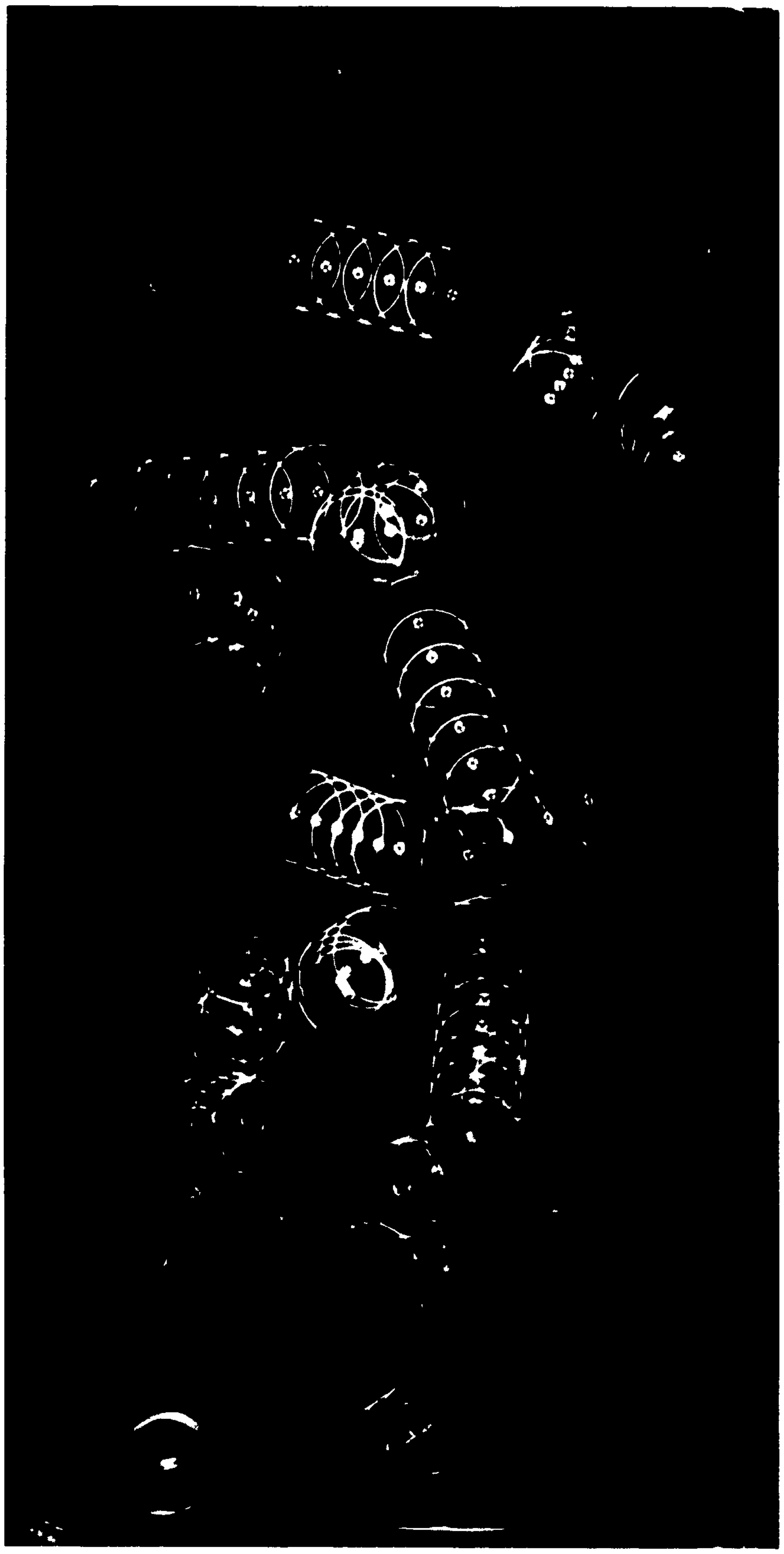


Fig. A.18.

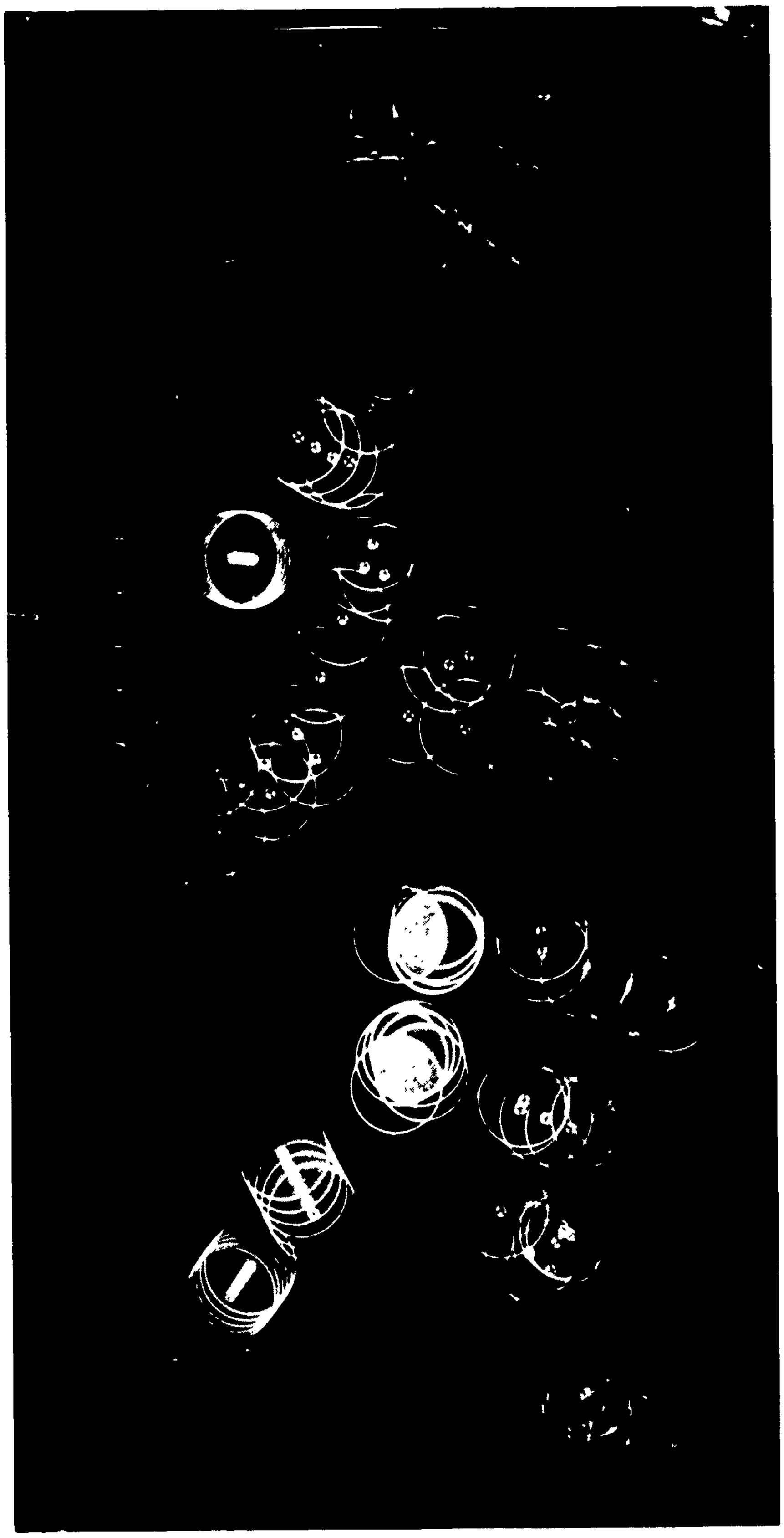


Fig. A.19.

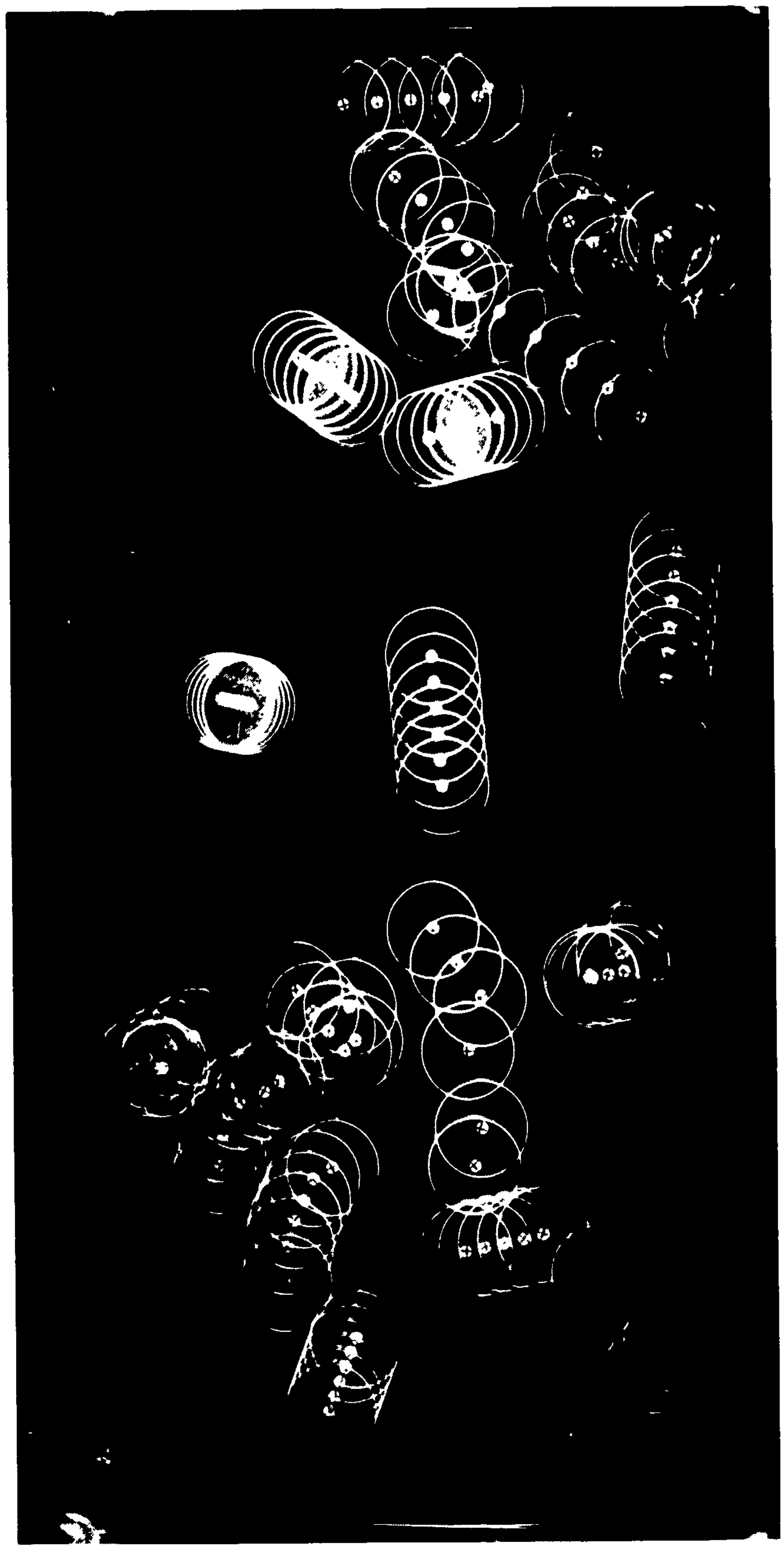


Fig. A.20.

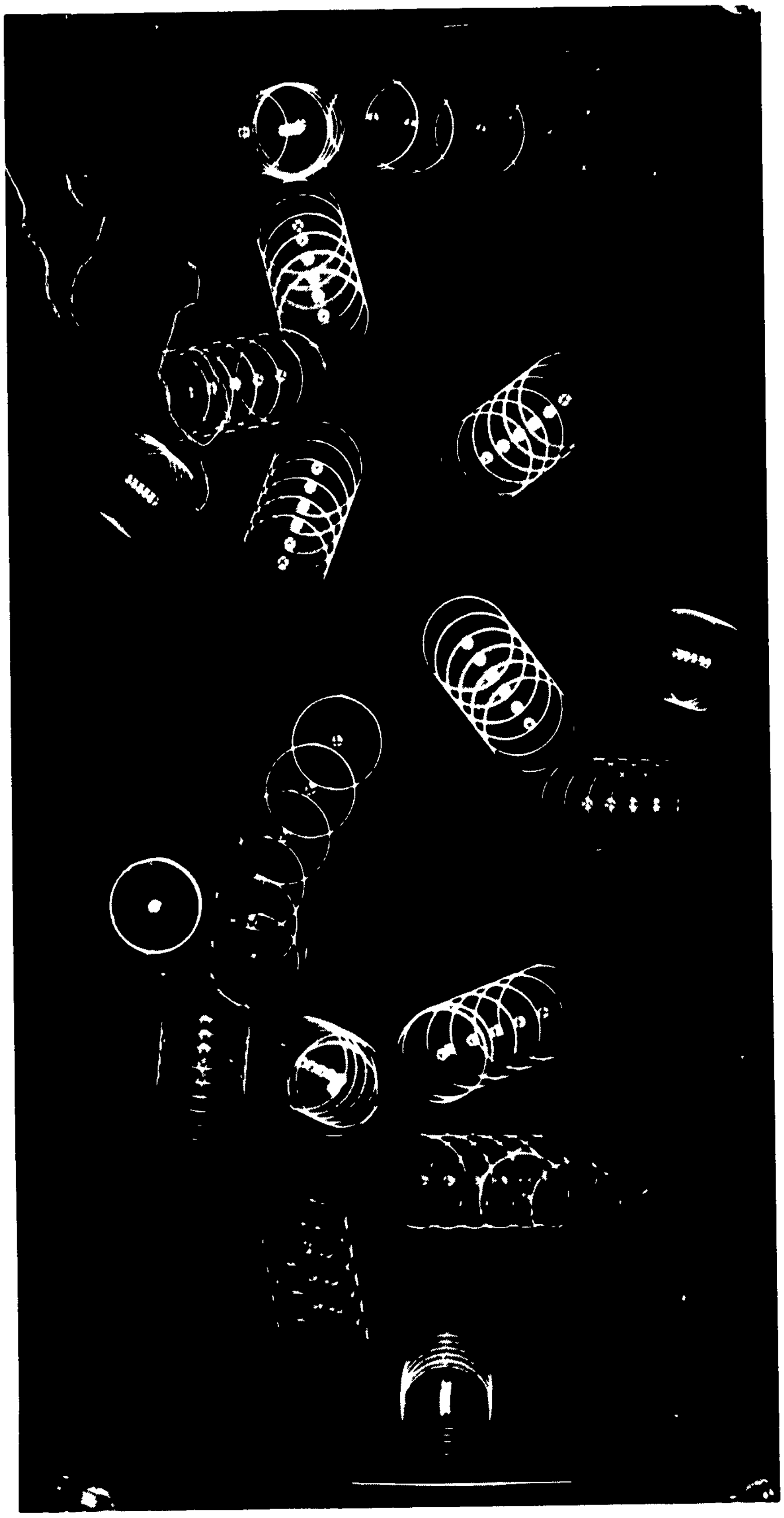


Fig. A.21.

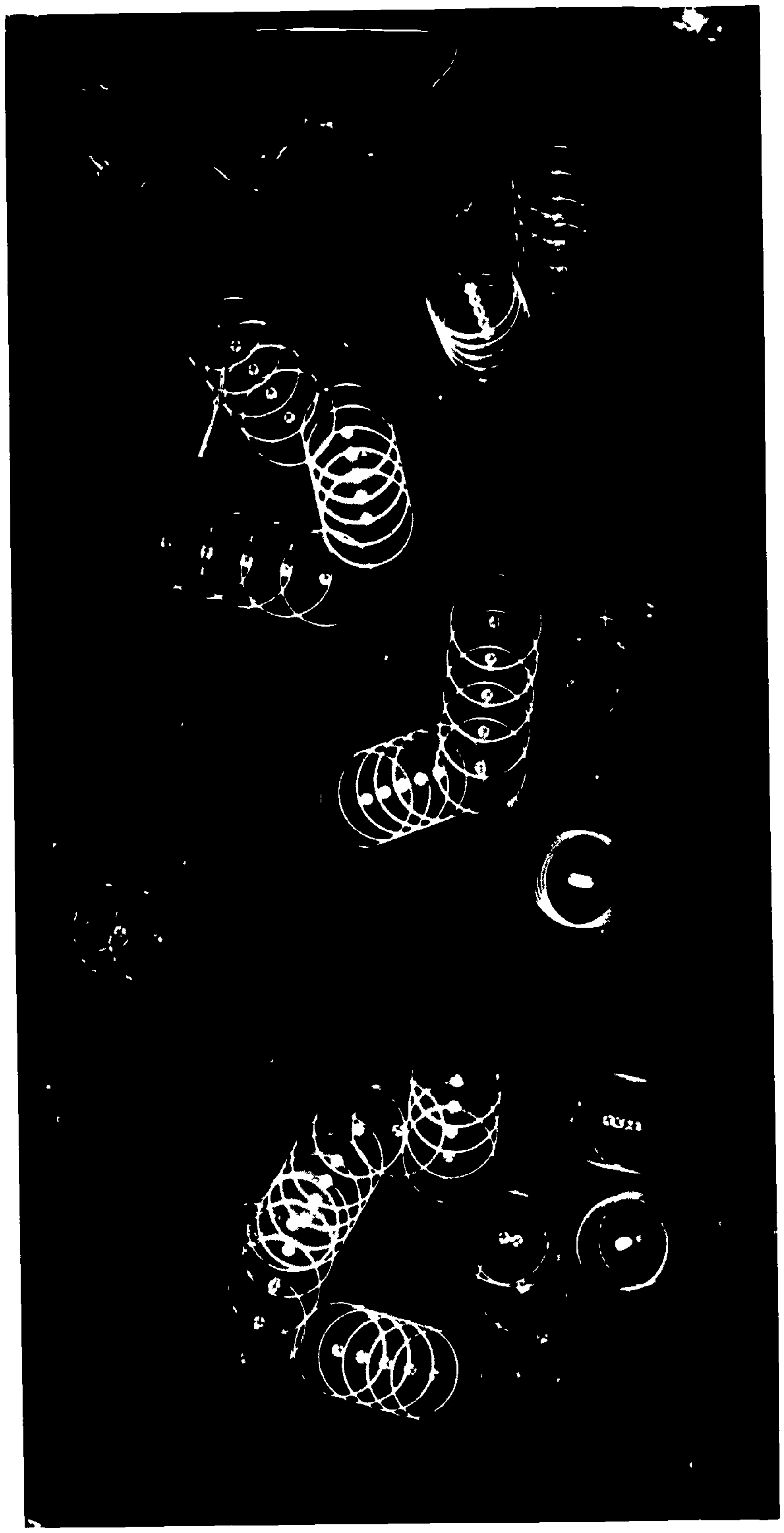


Fig. A.22.

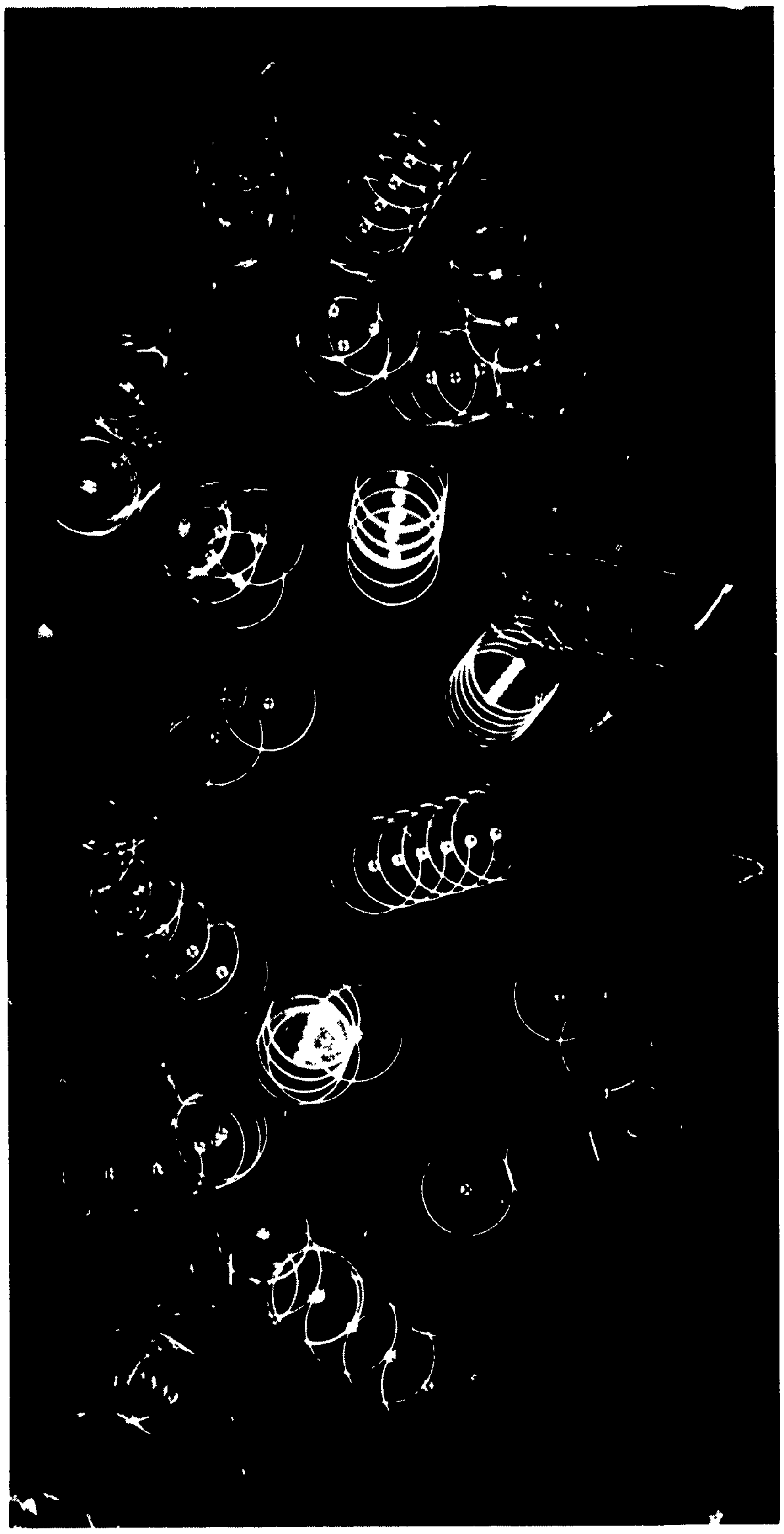


Fig. A.23.

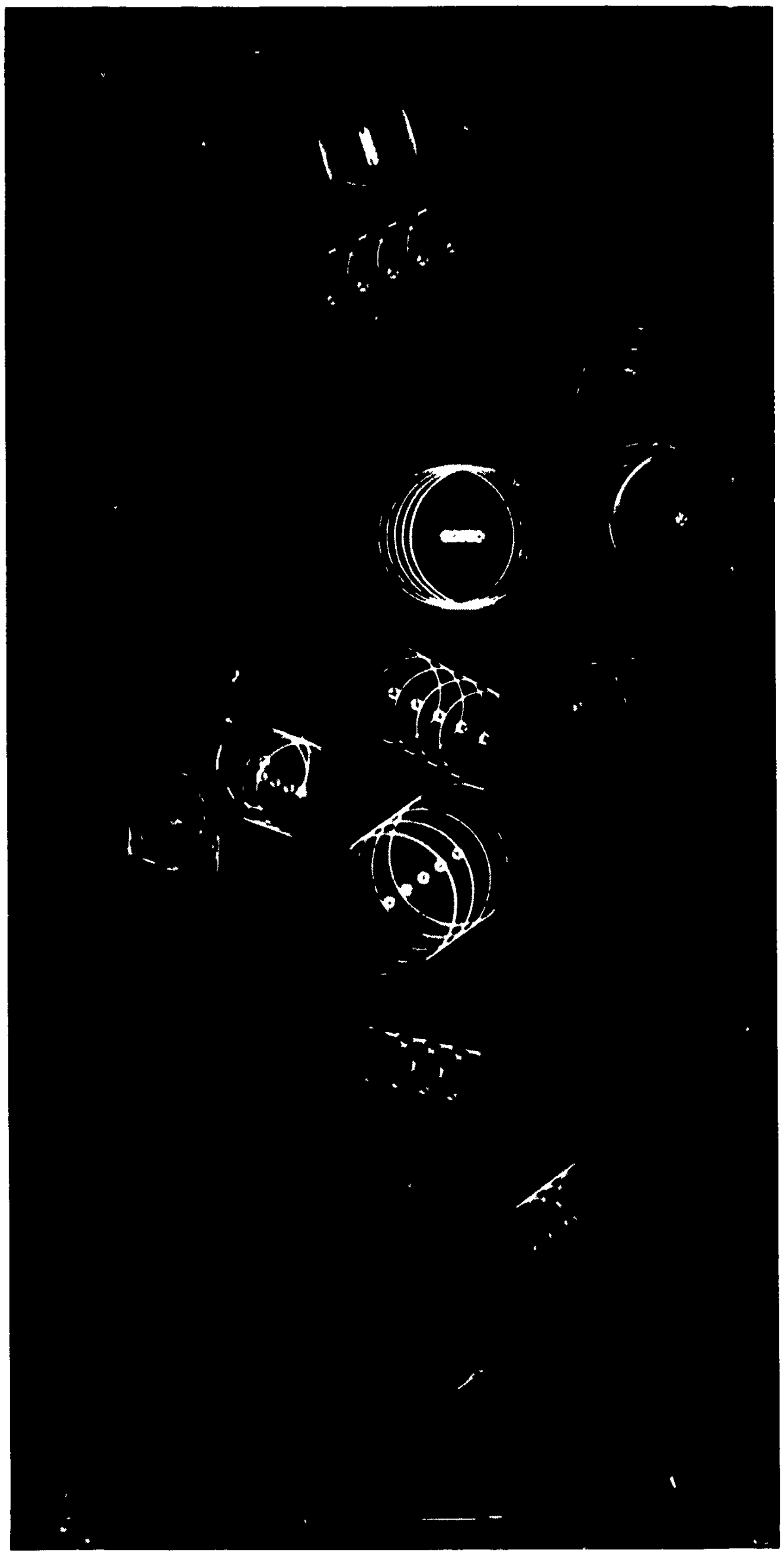


Fig. A.24.

Fig. A.25.

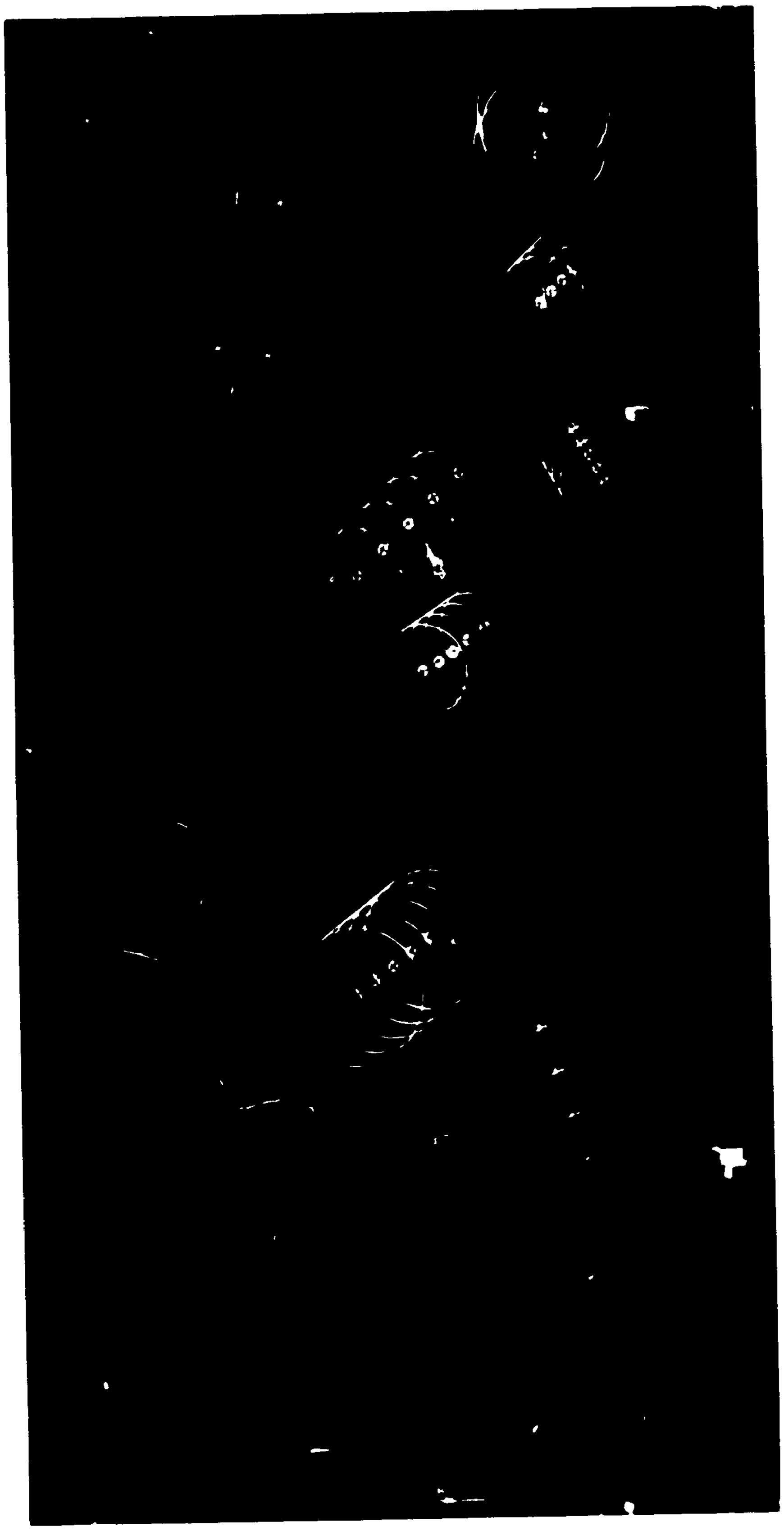


Fig. A.26.

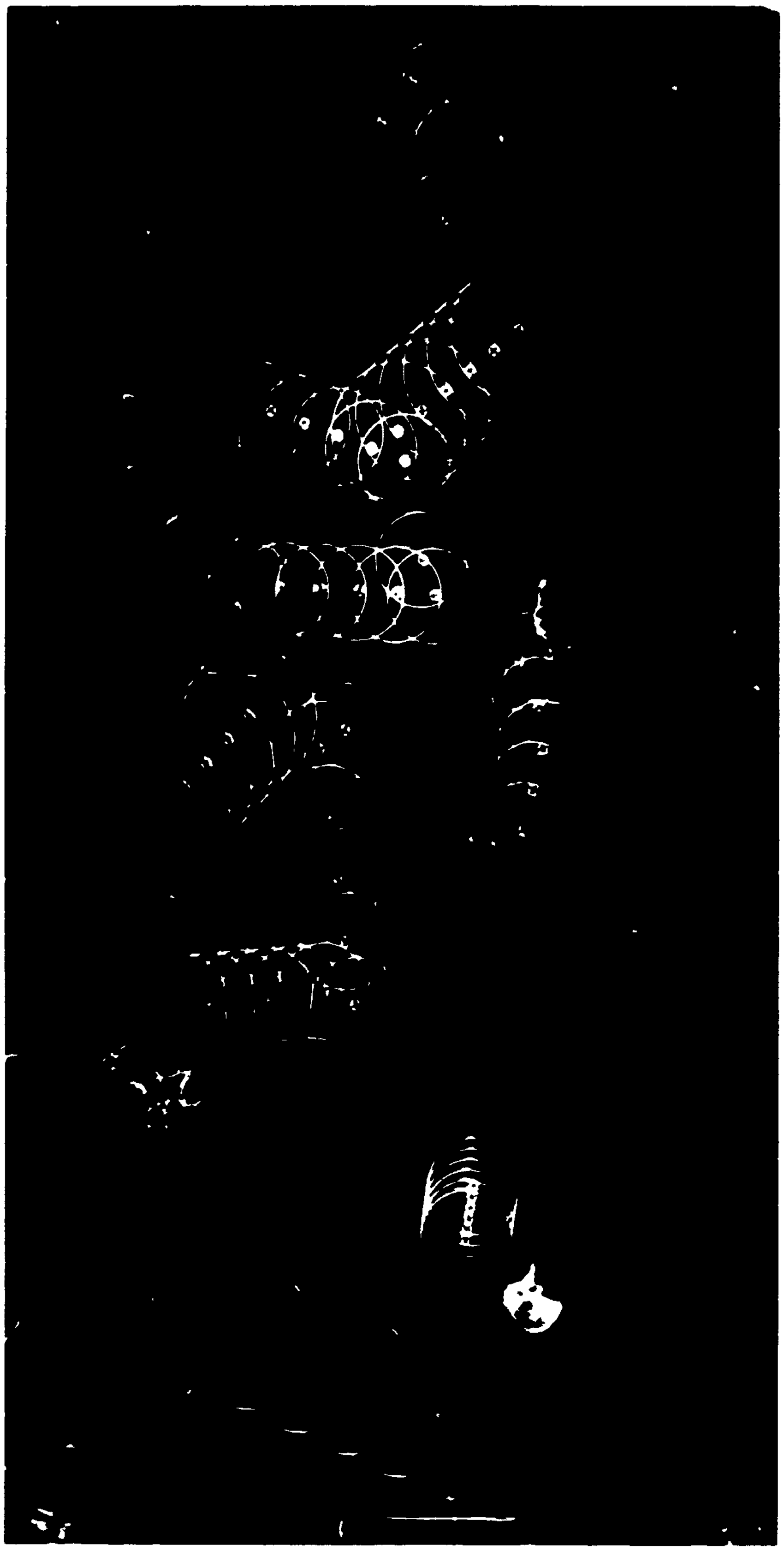


Fig. A.27.

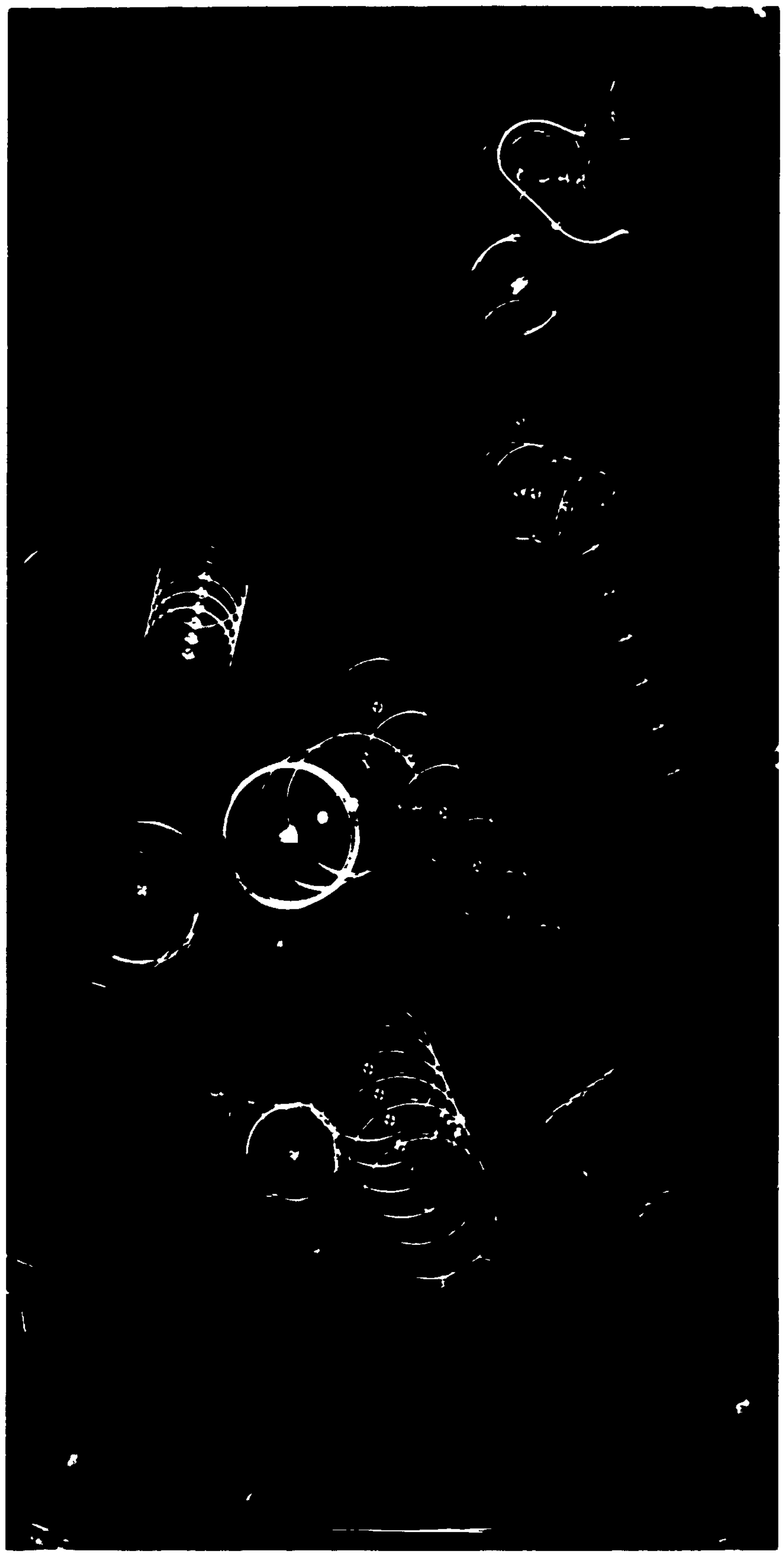


Fig. A.28.

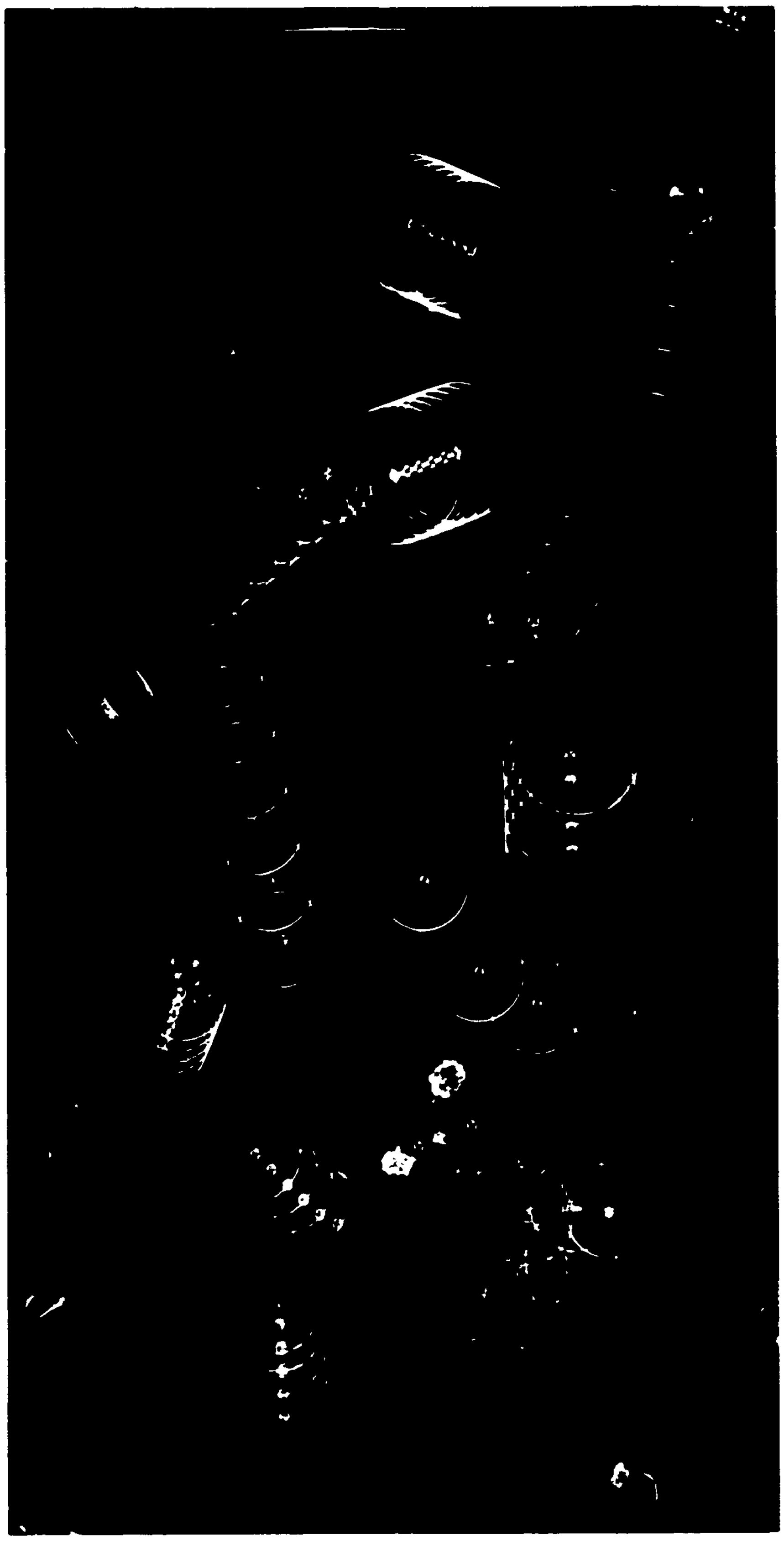


Fig. A.29.

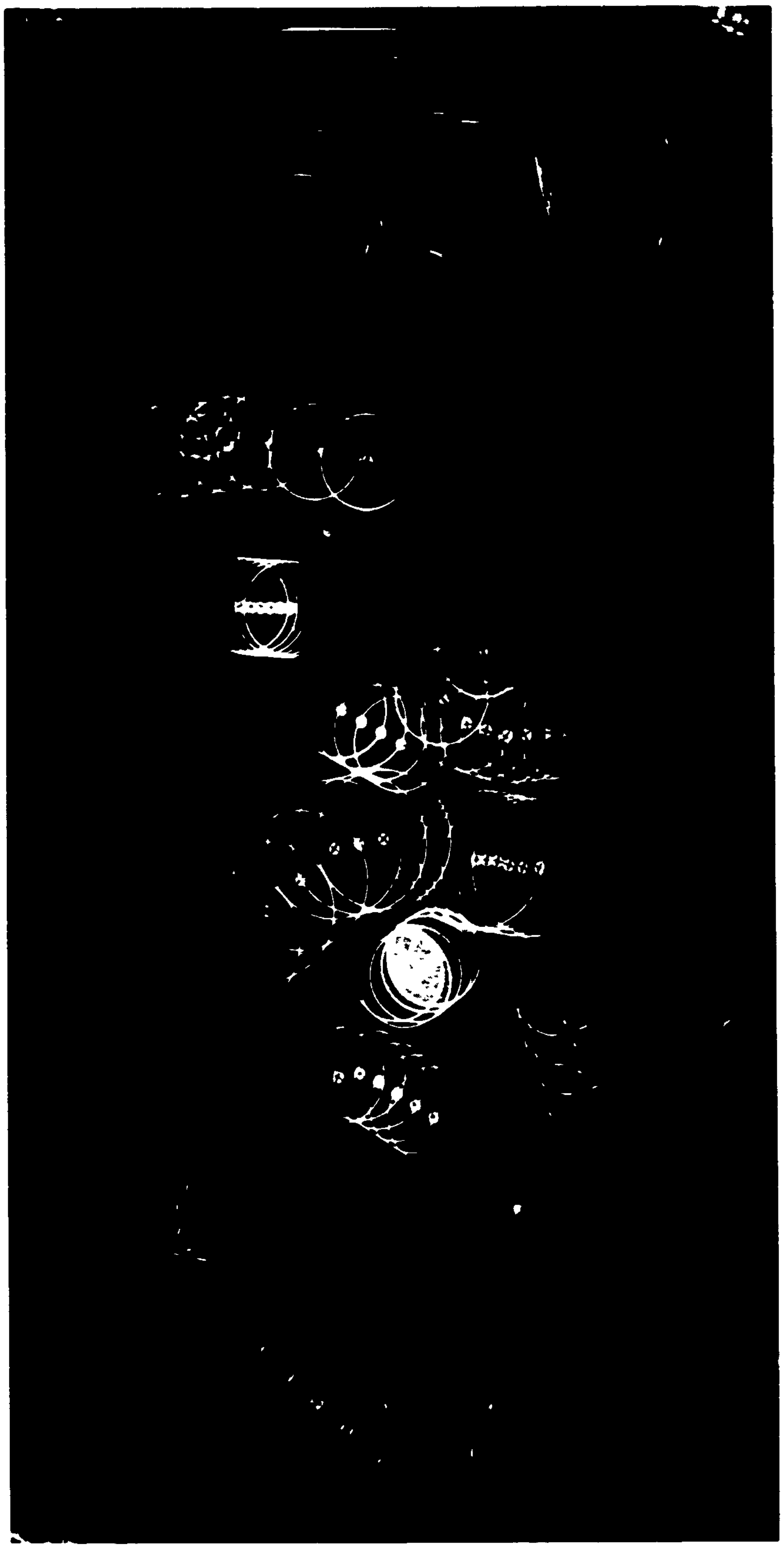


Fig. A.30.

Fig. A.31.

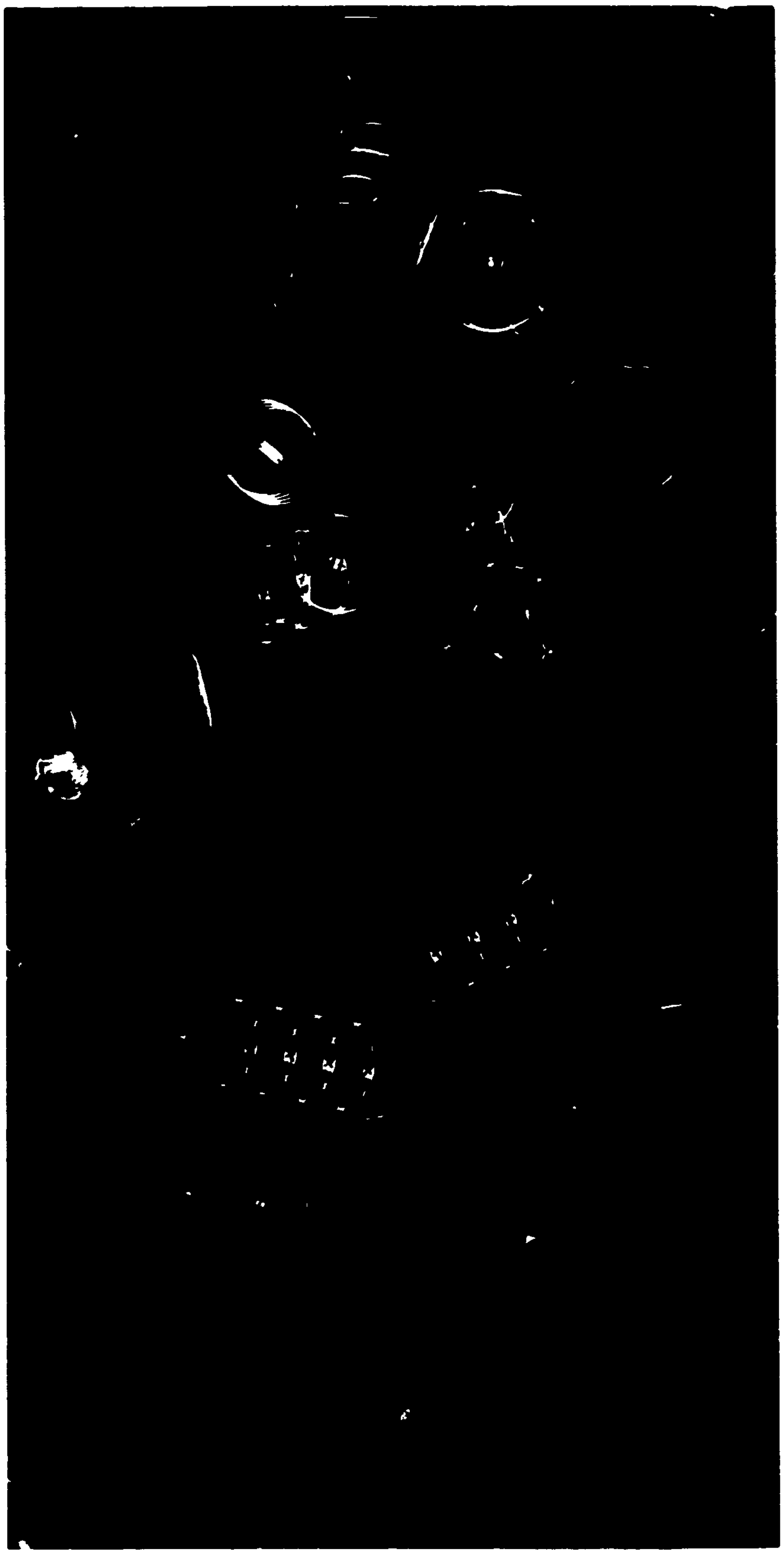


Fig. A.32.

Fig. A.33.

Fig. A.34.

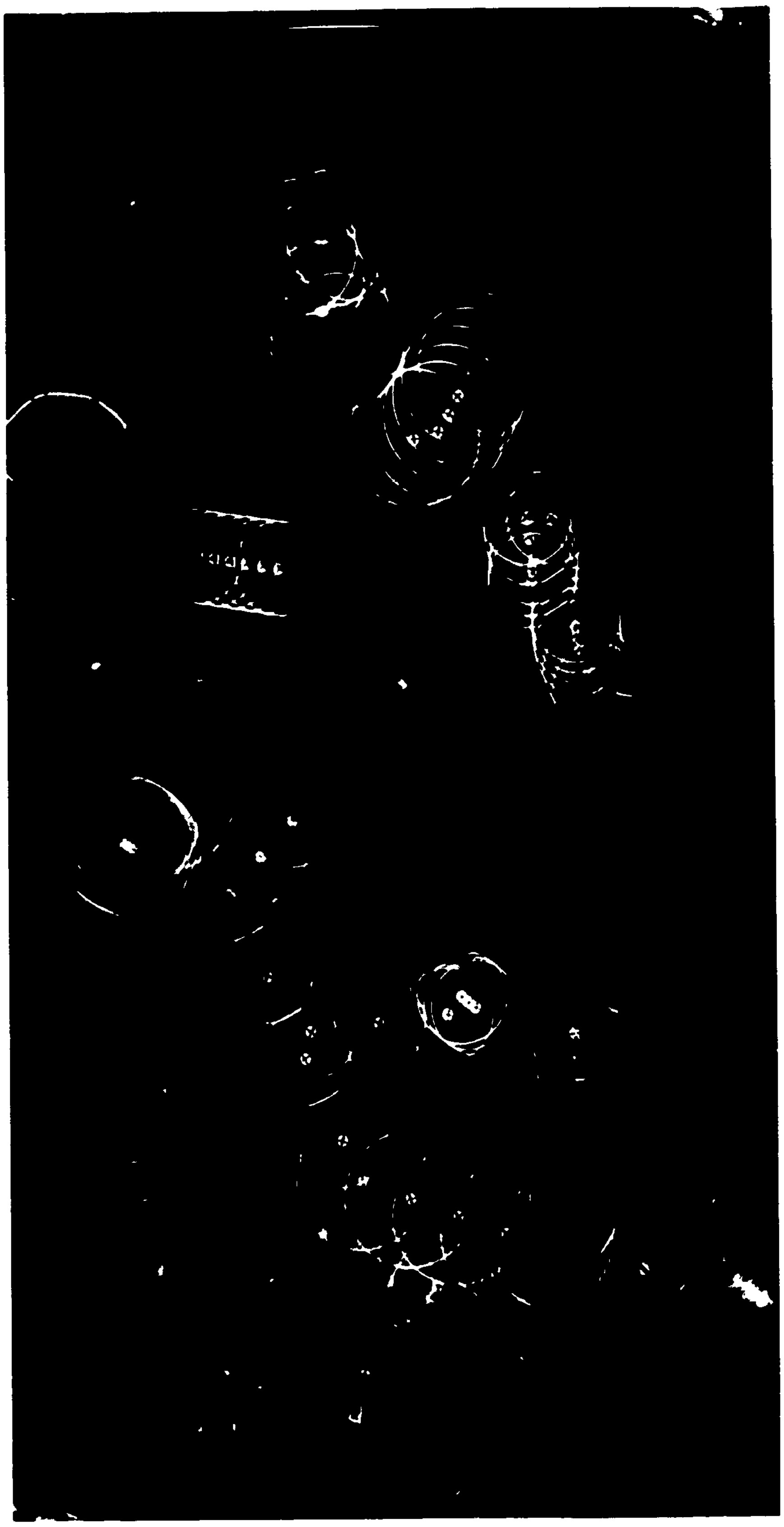


Fig. A.35.

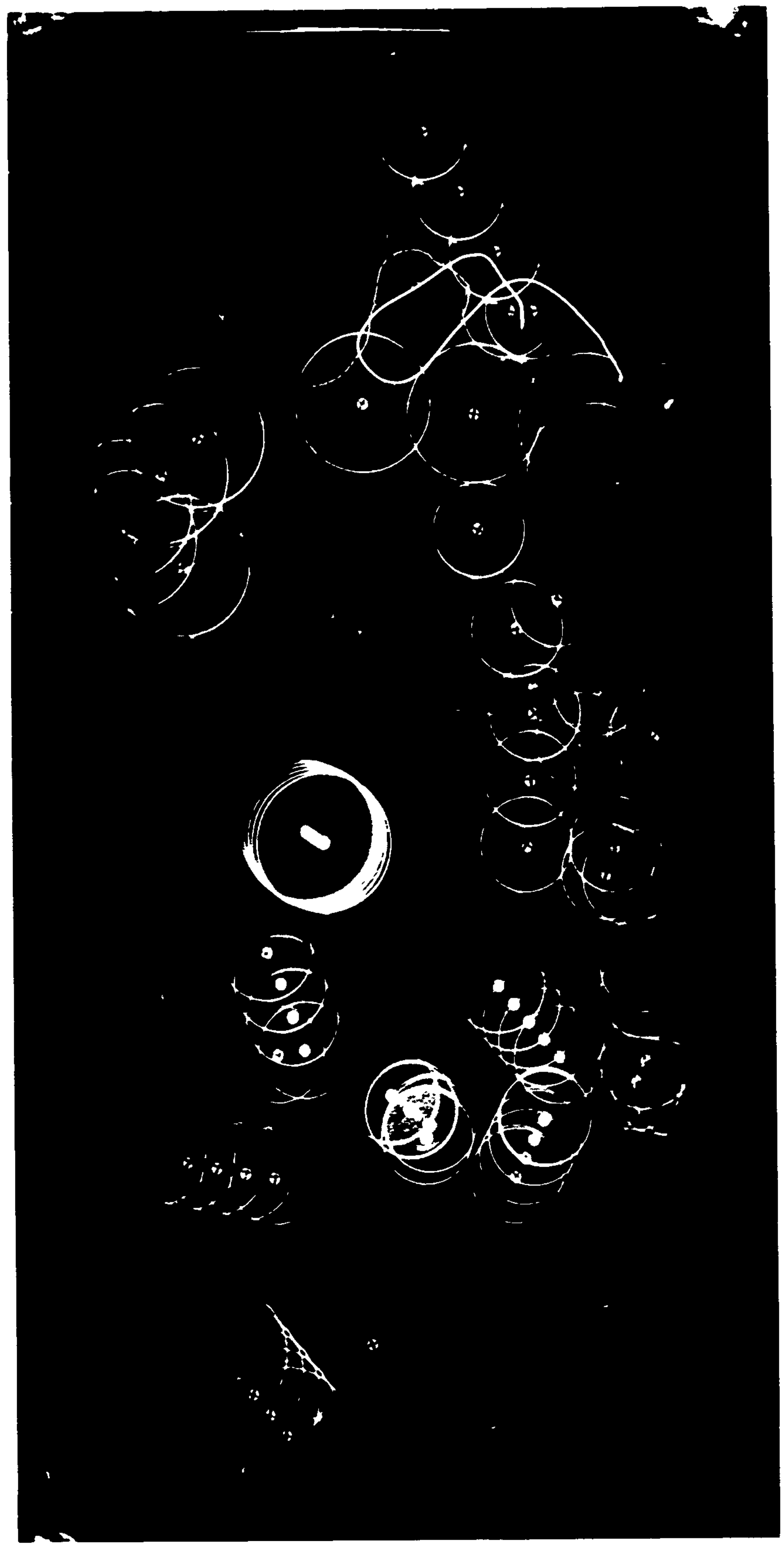


Fig. A.36.

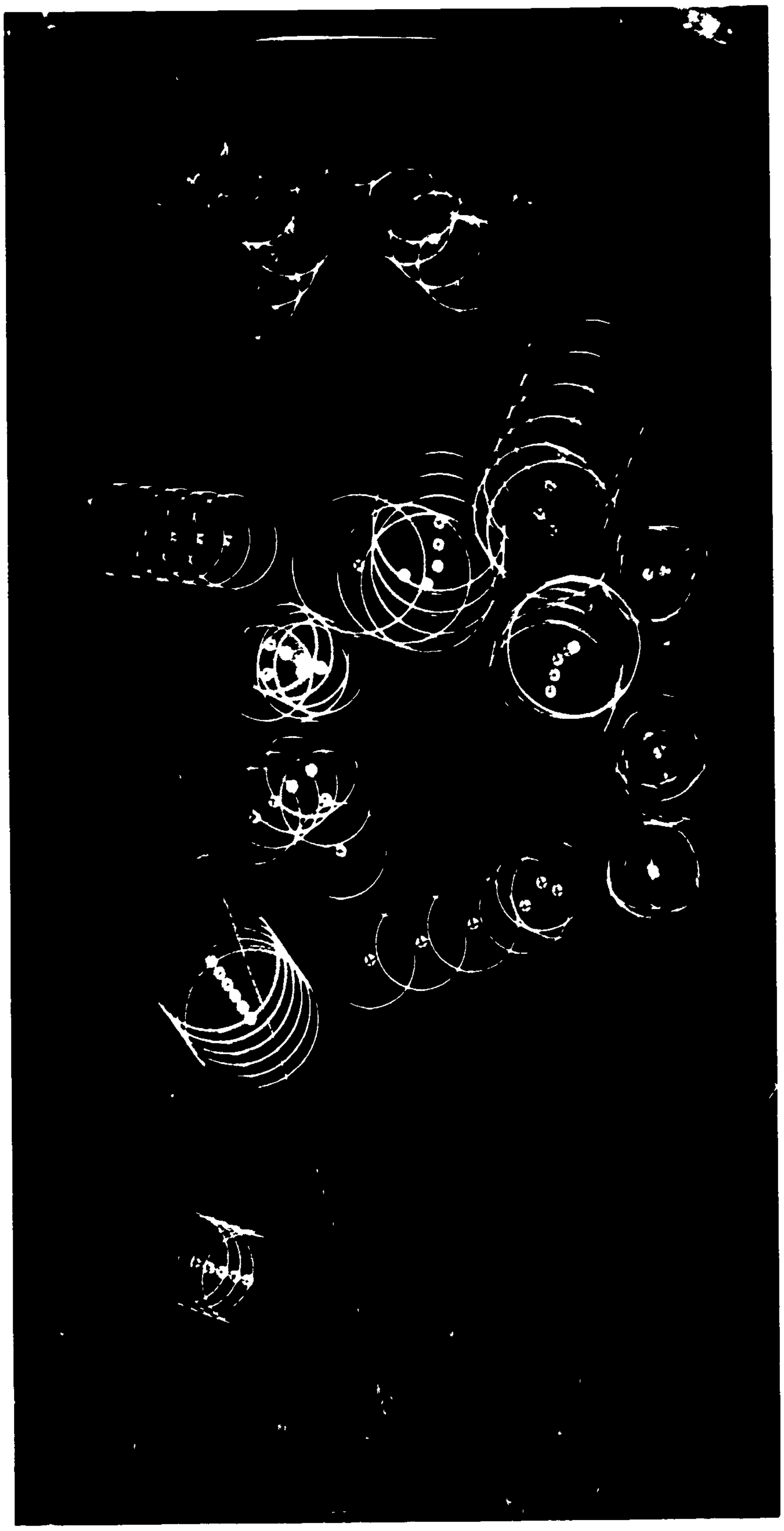


Fig. A.37.

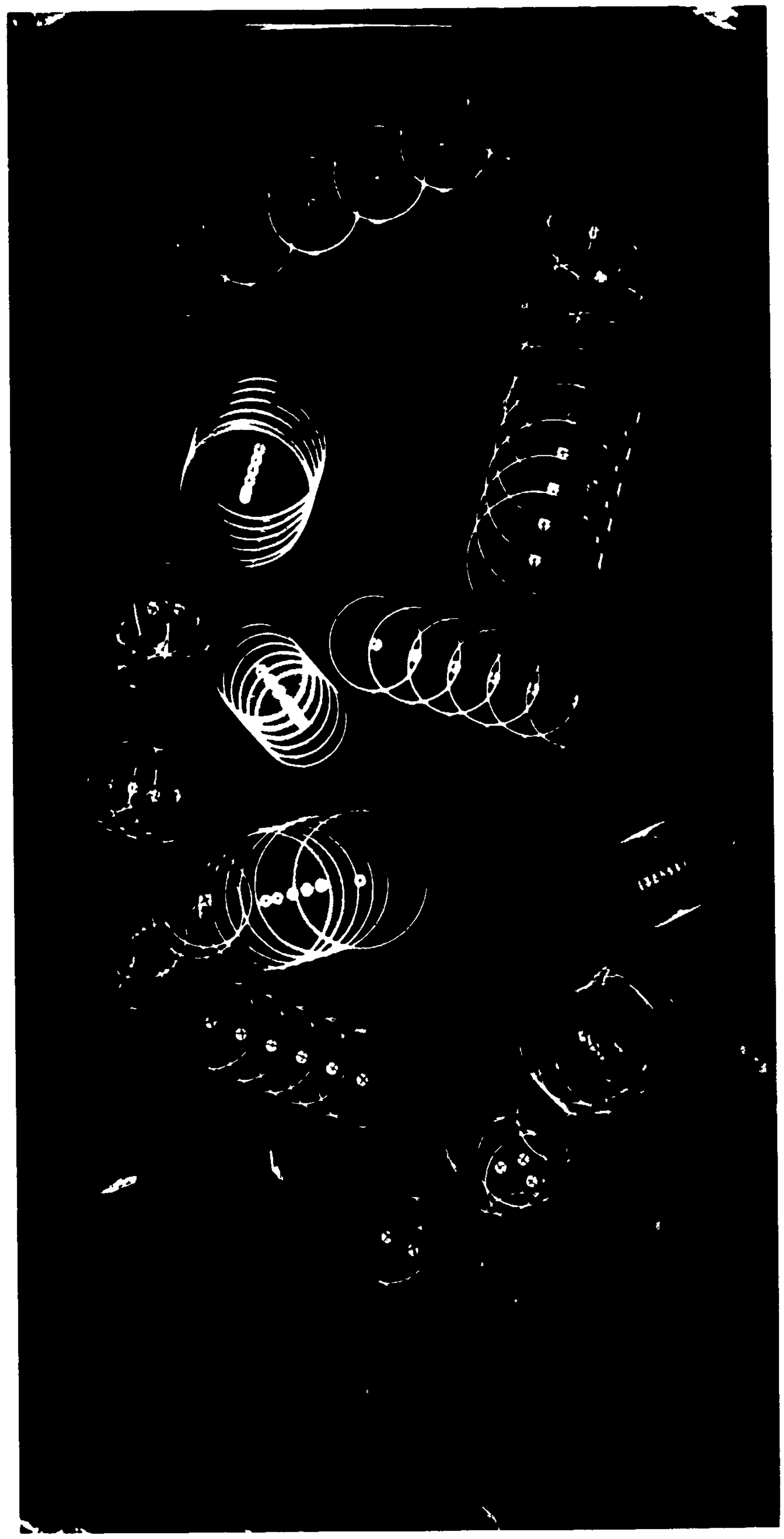


Fig. A.38.

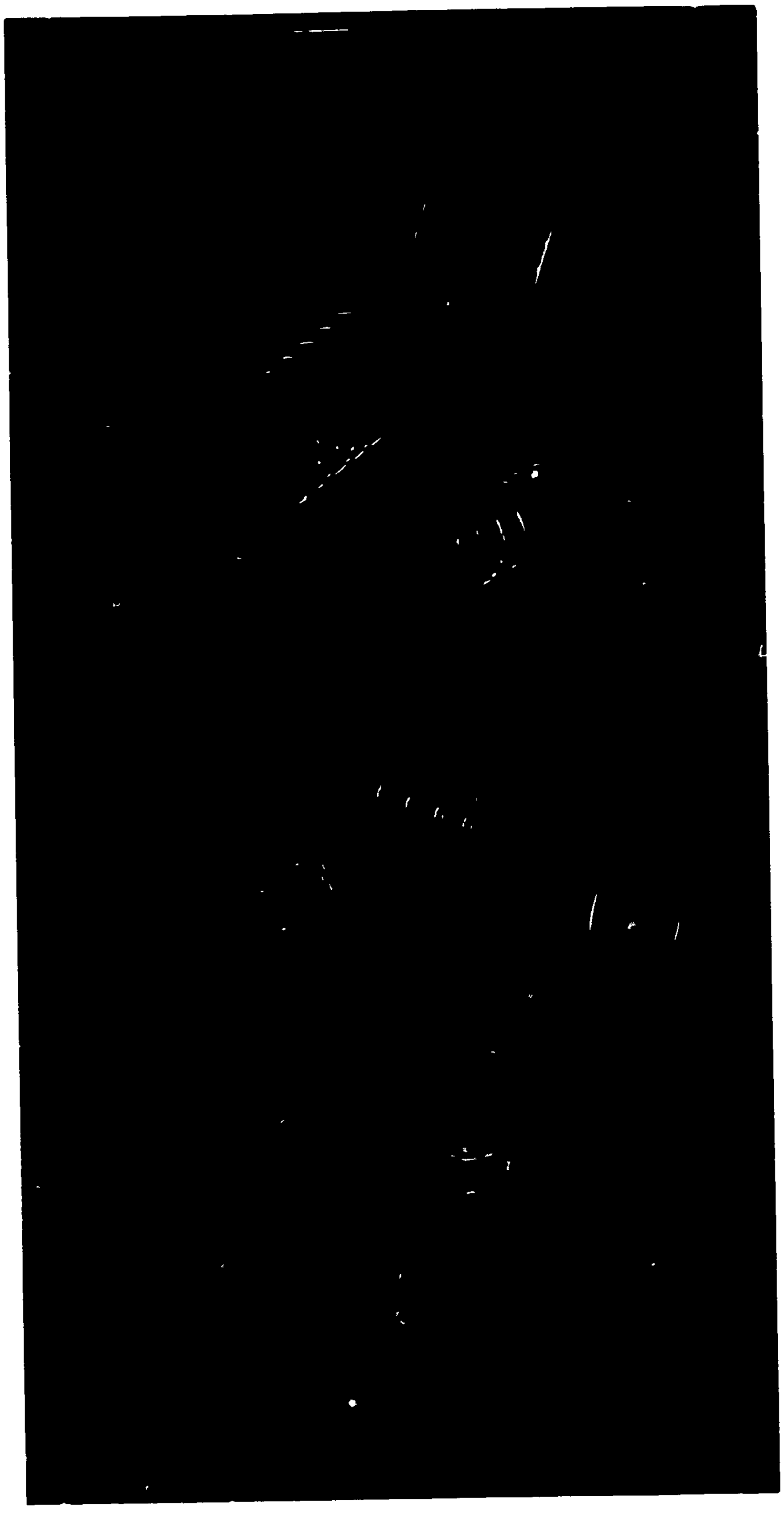


Fig. A.39.

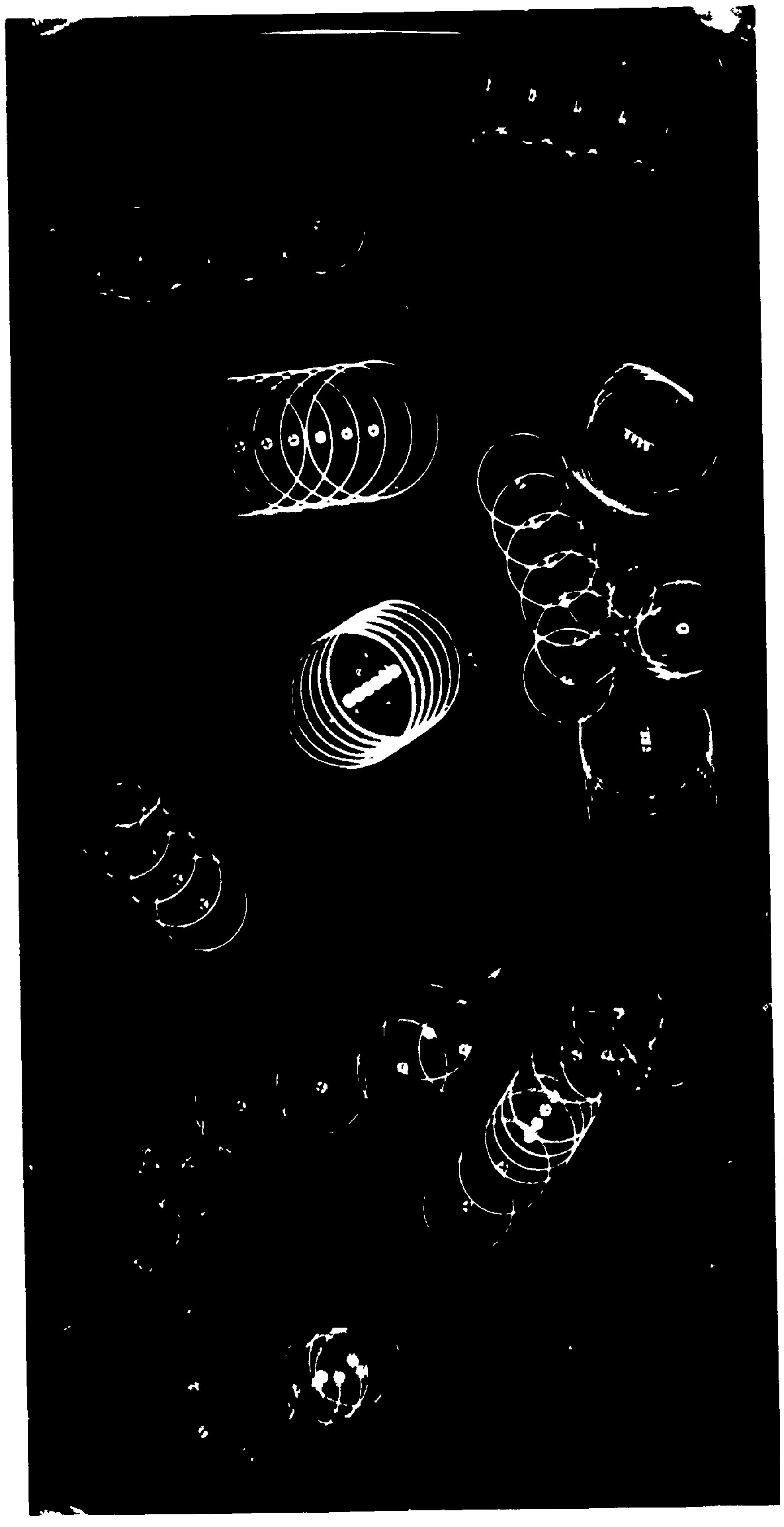


Fig. A.40.

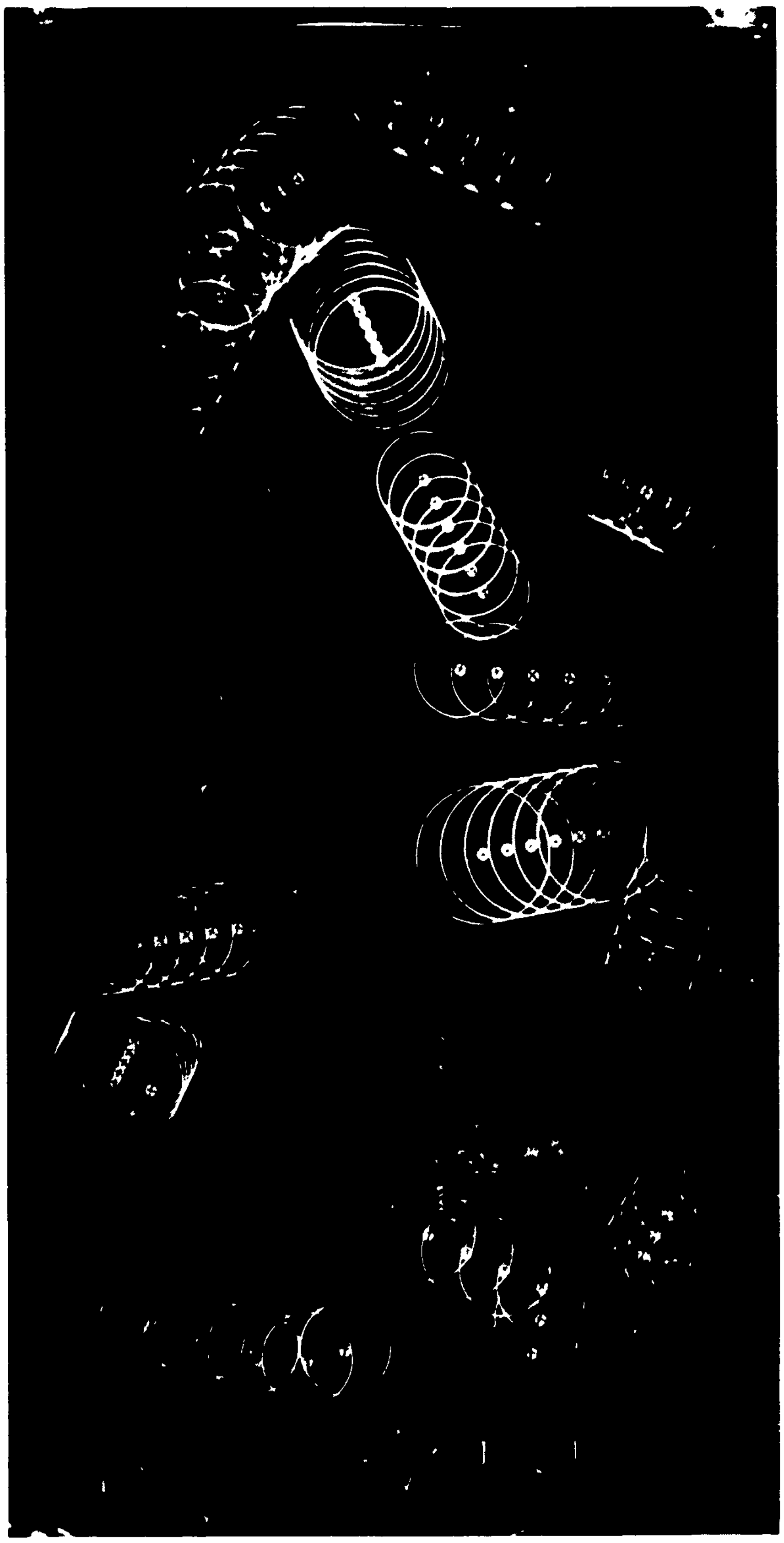


Fig. A.41.

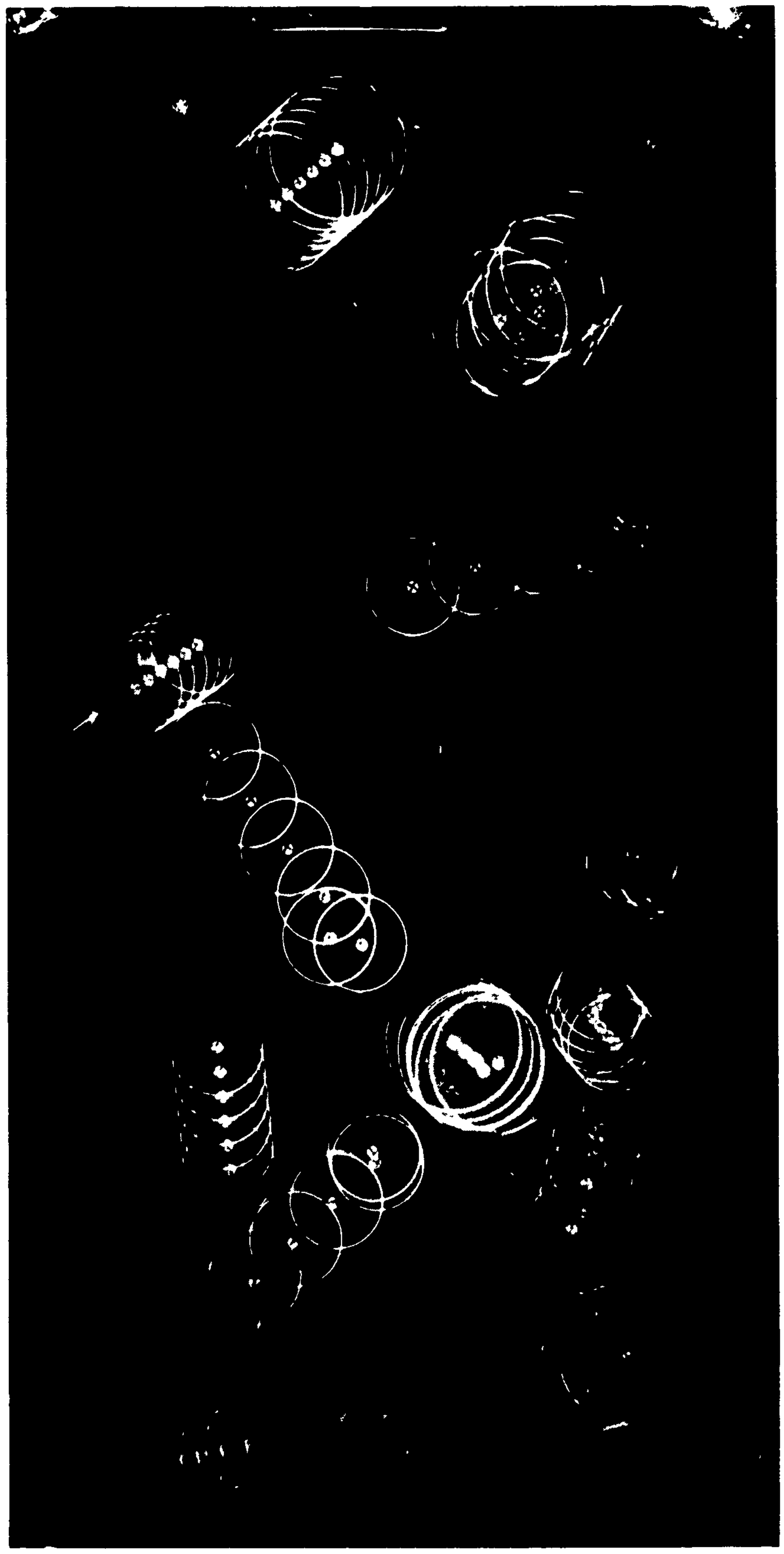


Fig. A.42.

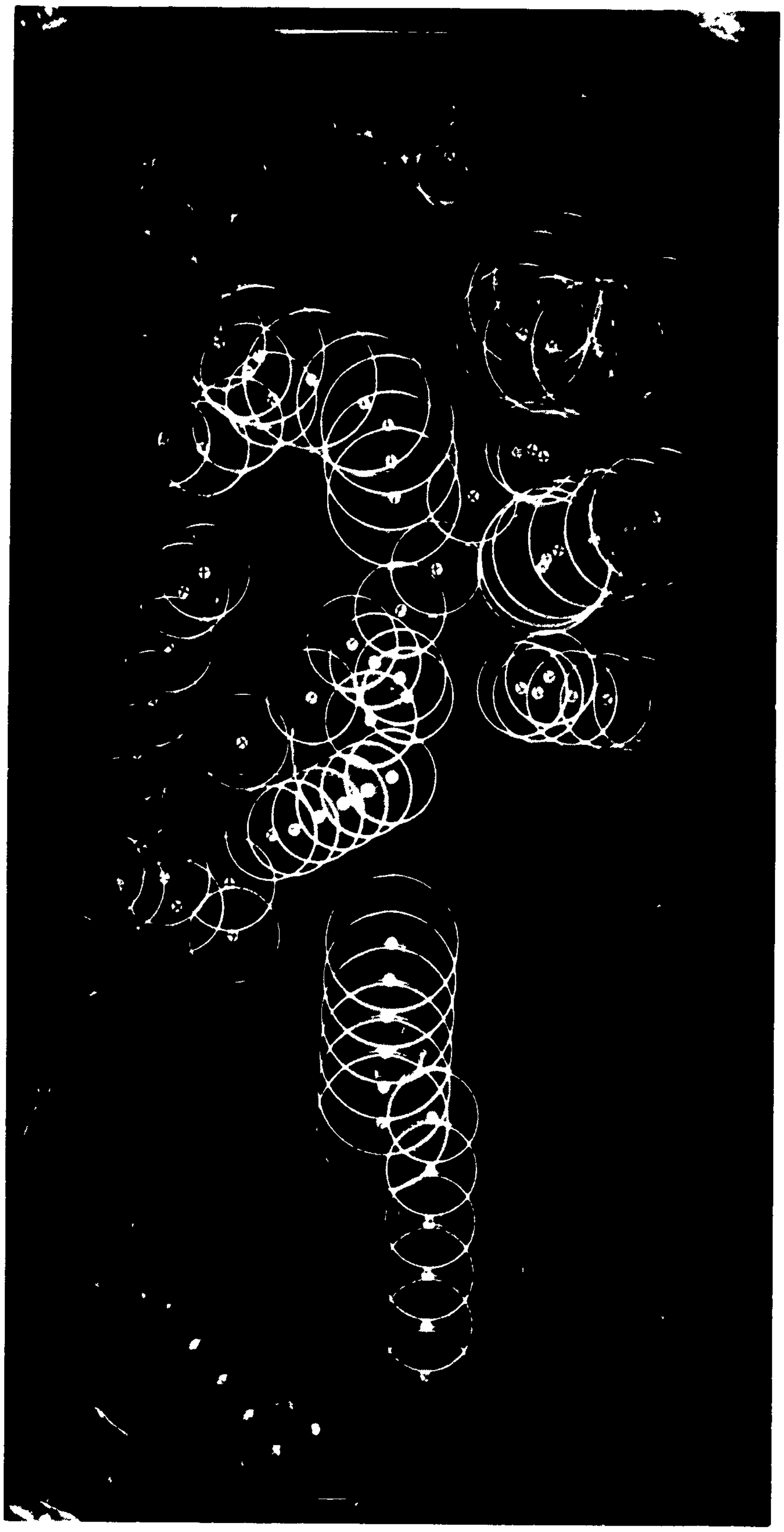


Fig. A.43.

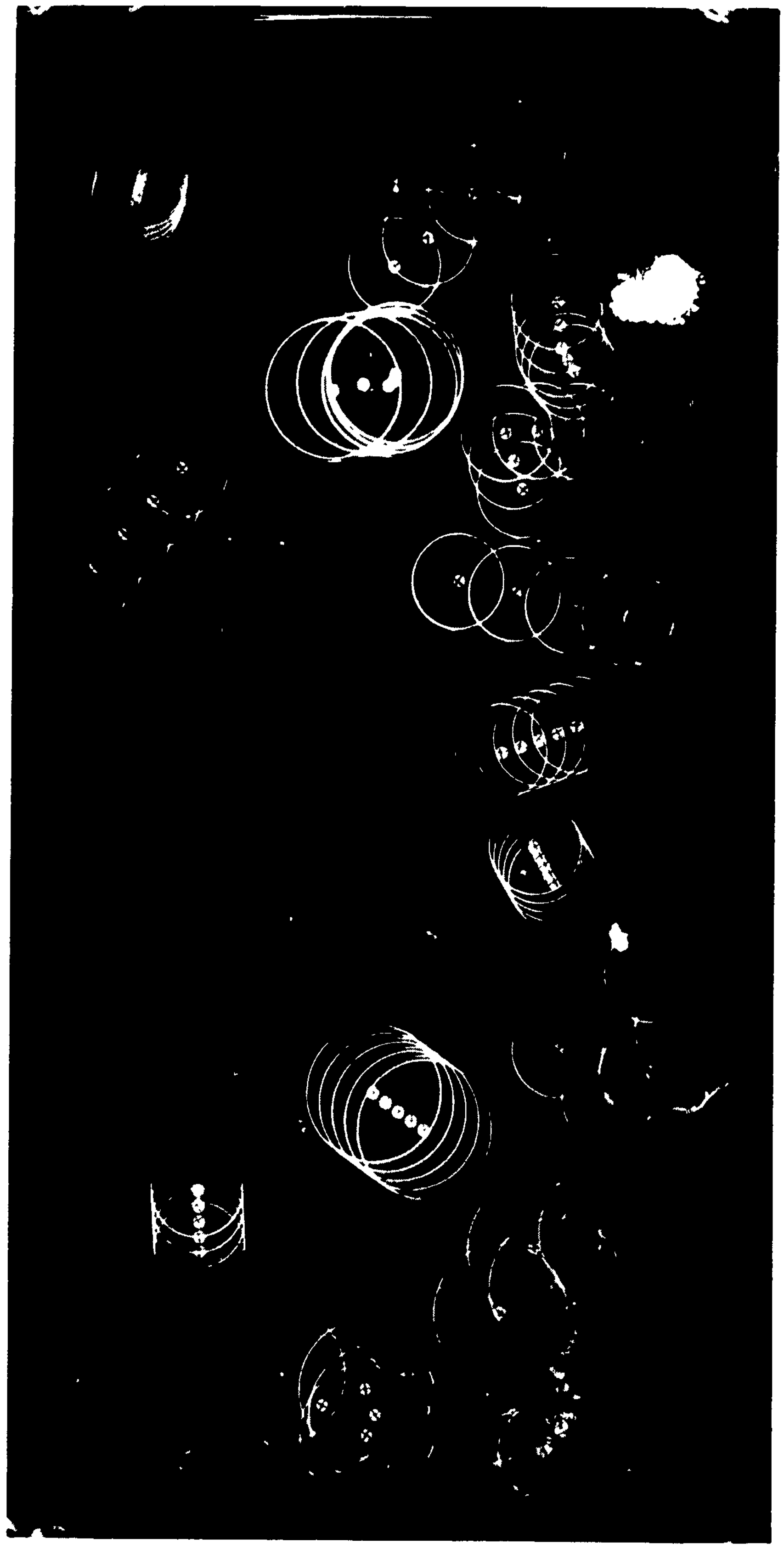


Fig. A.44.

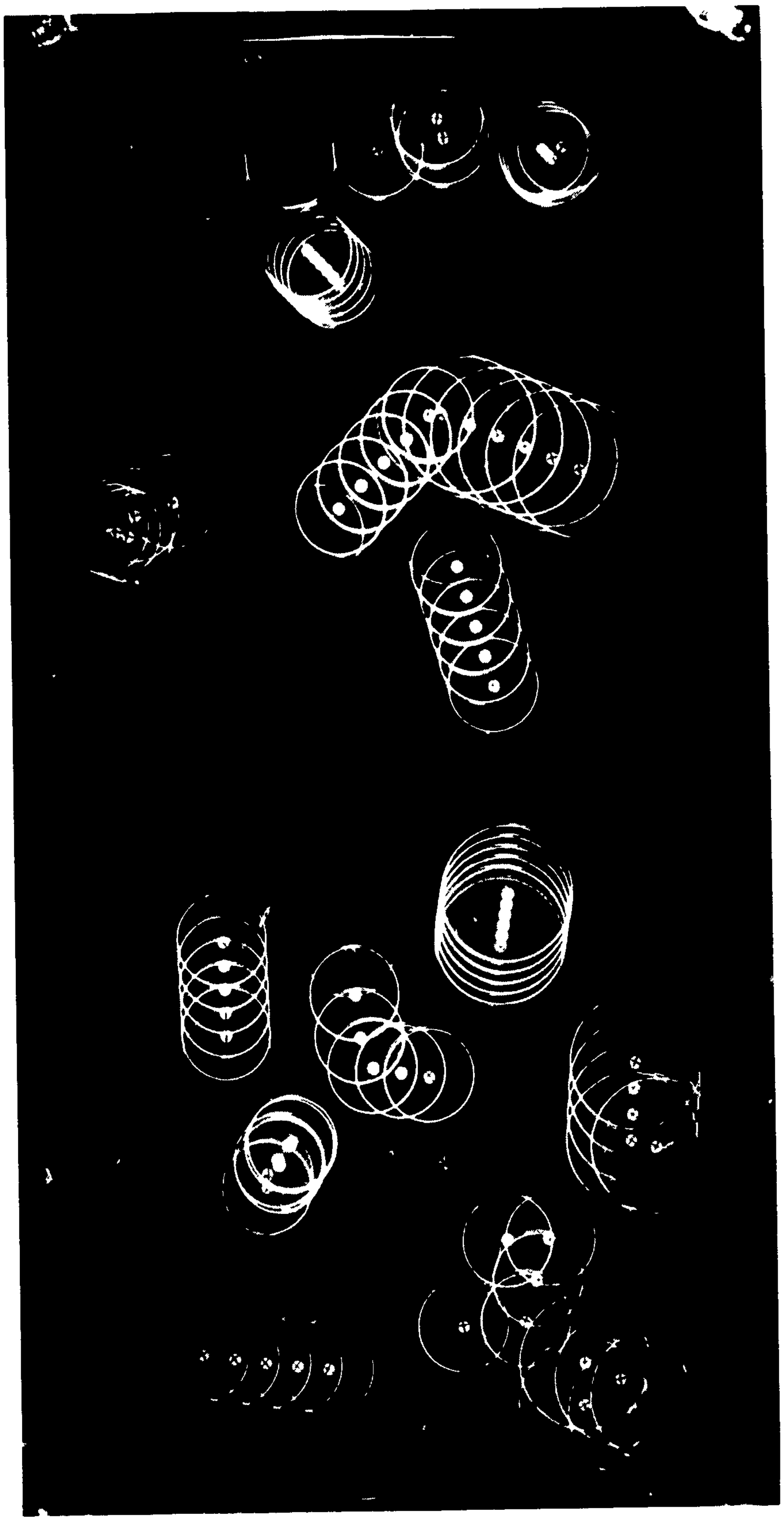


Fig. A.45.

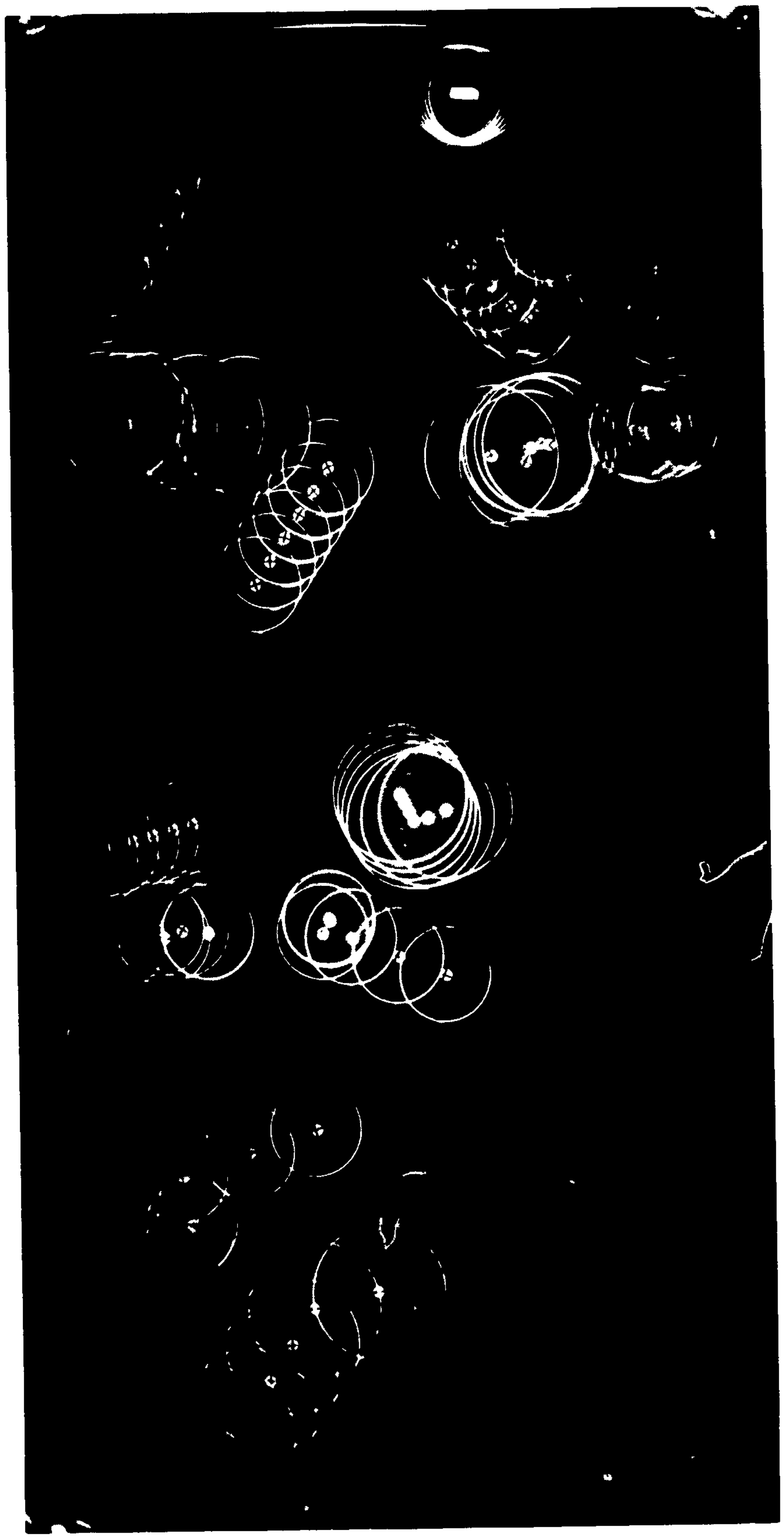


Fig. A.46.

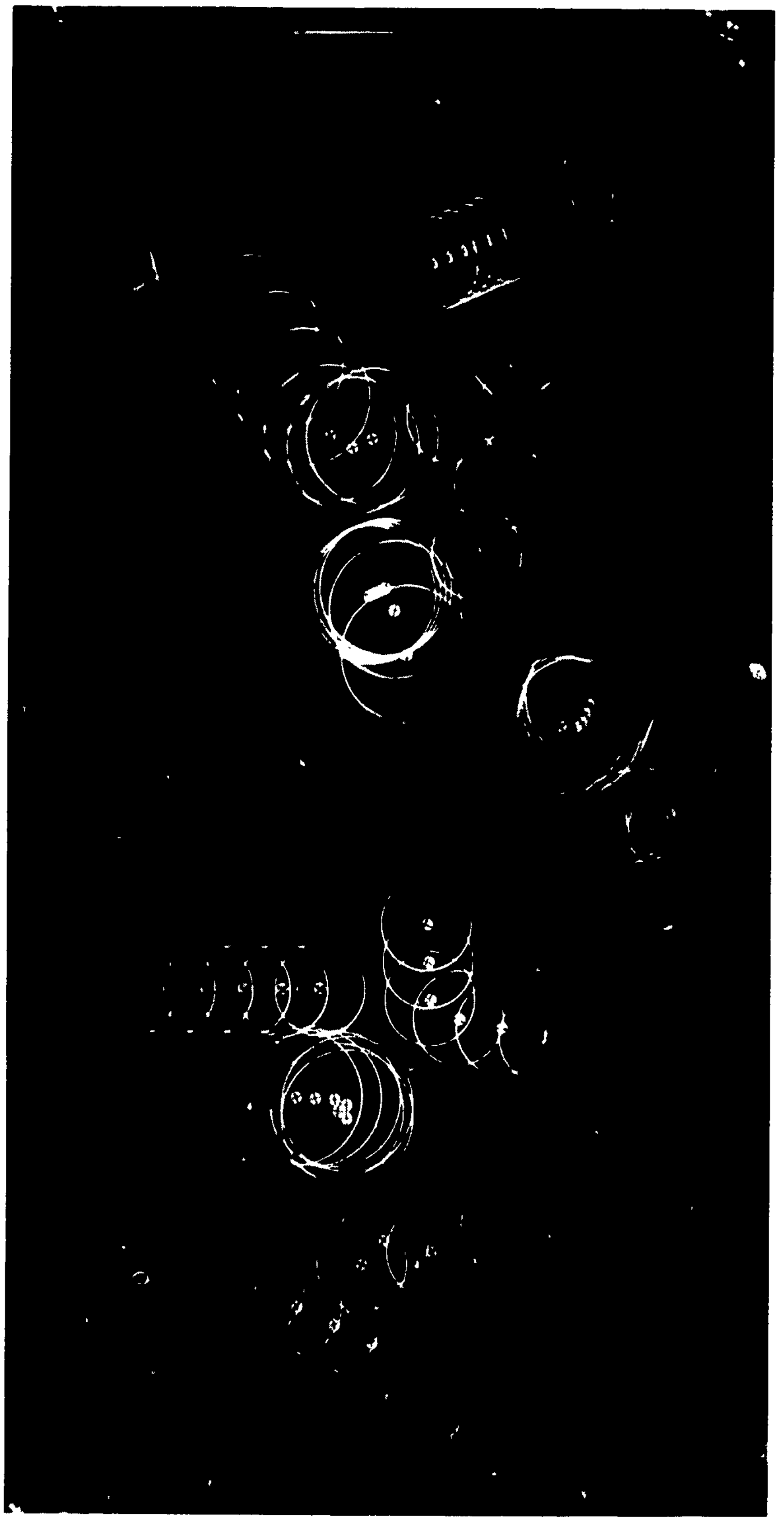


Fig. A.47.

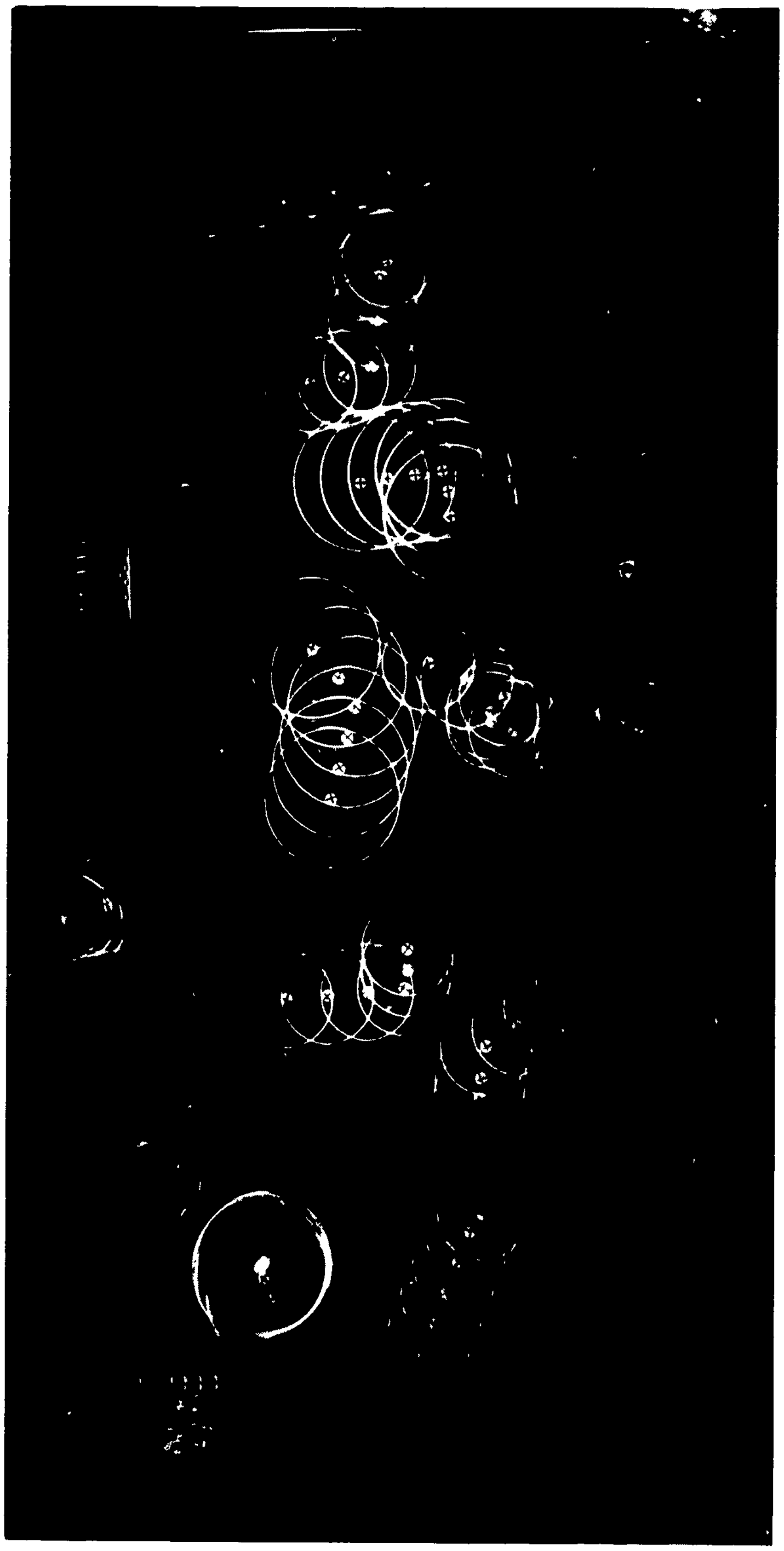


Fig. A.48.

Fig. A.49.

Fig. A.50.

Fig. A.51.

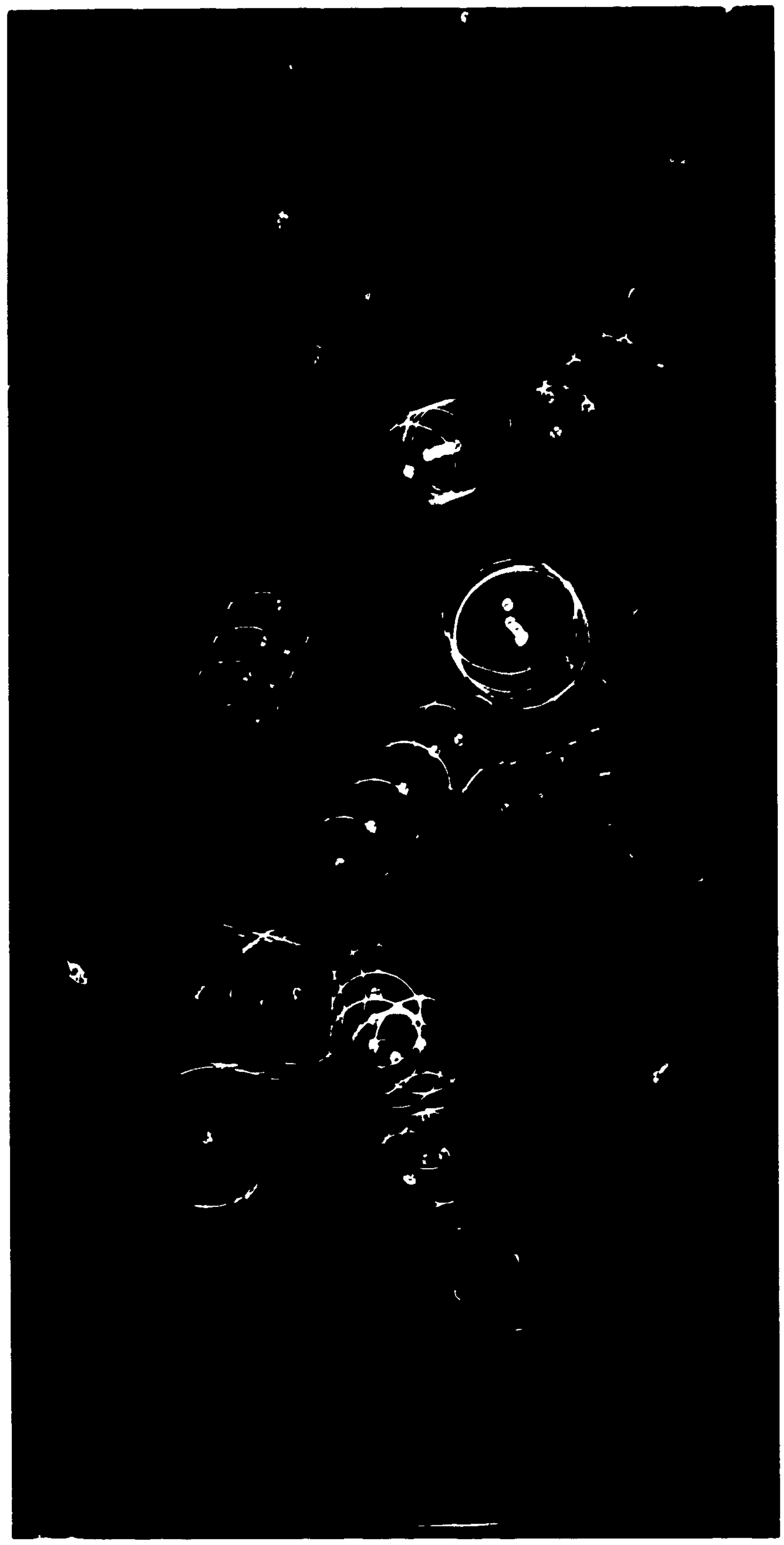


Fig. A.52.

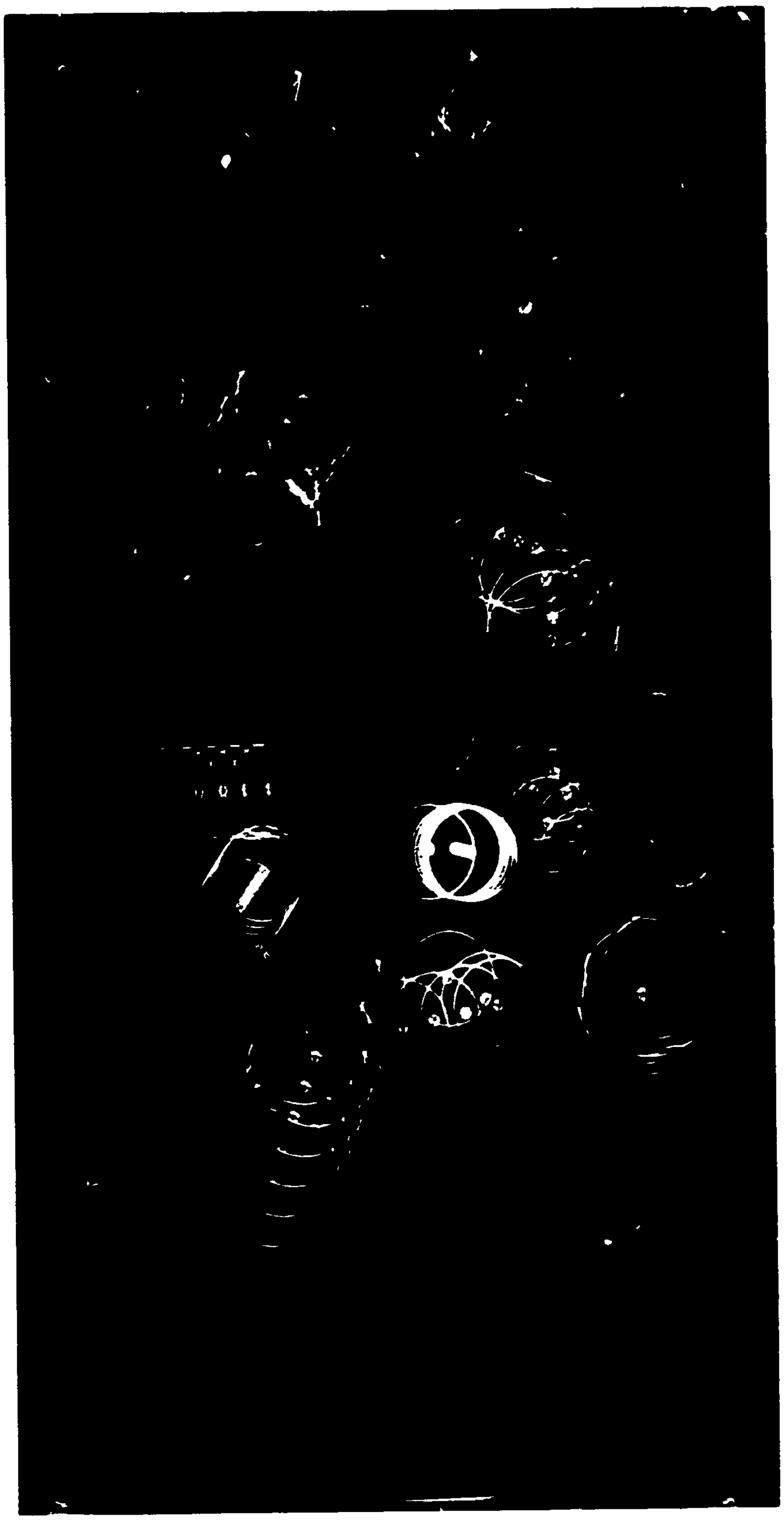


Fig. A.53.

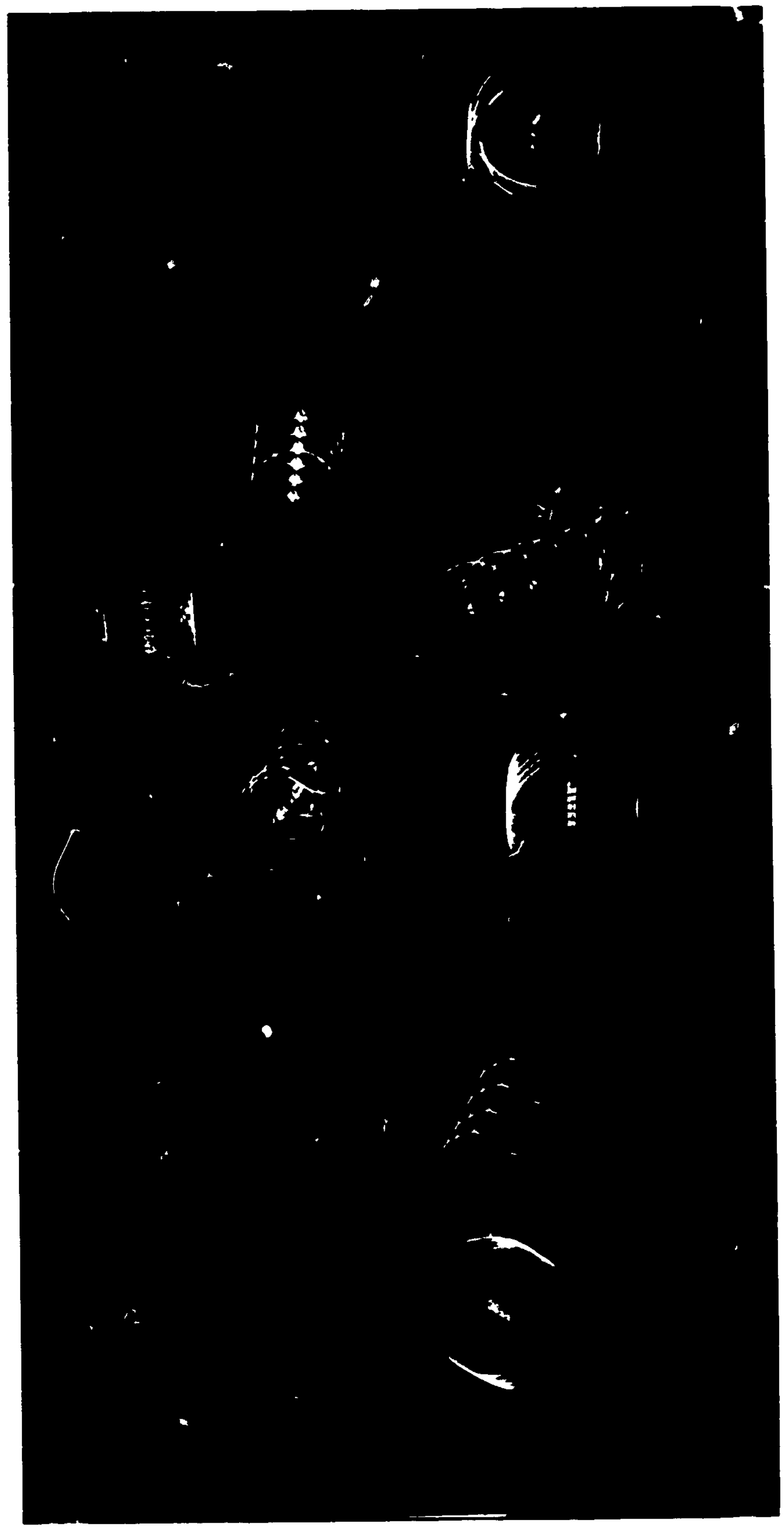


Fig. A.54.

Fig. A.55.

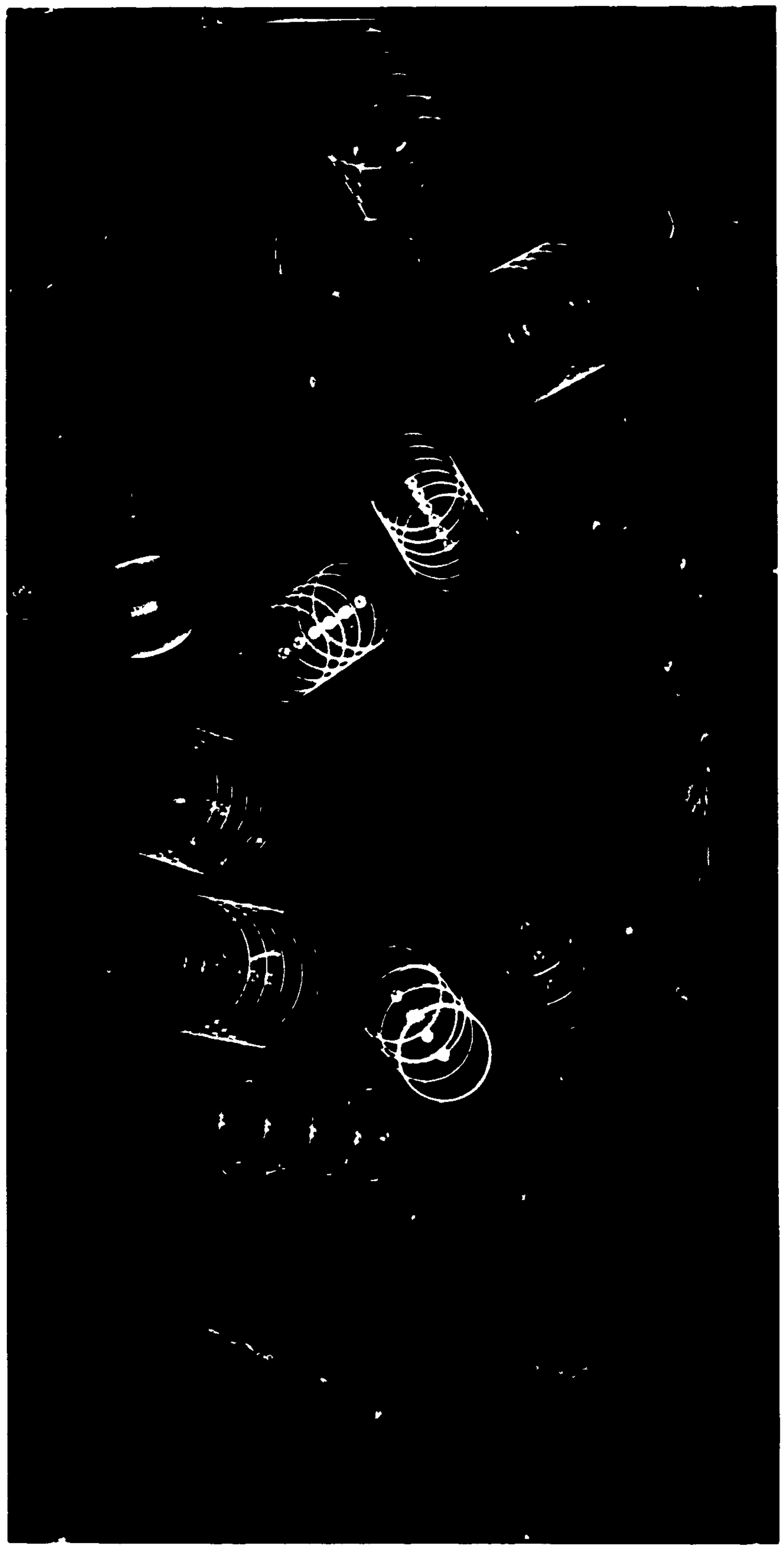


Fig. A.56.

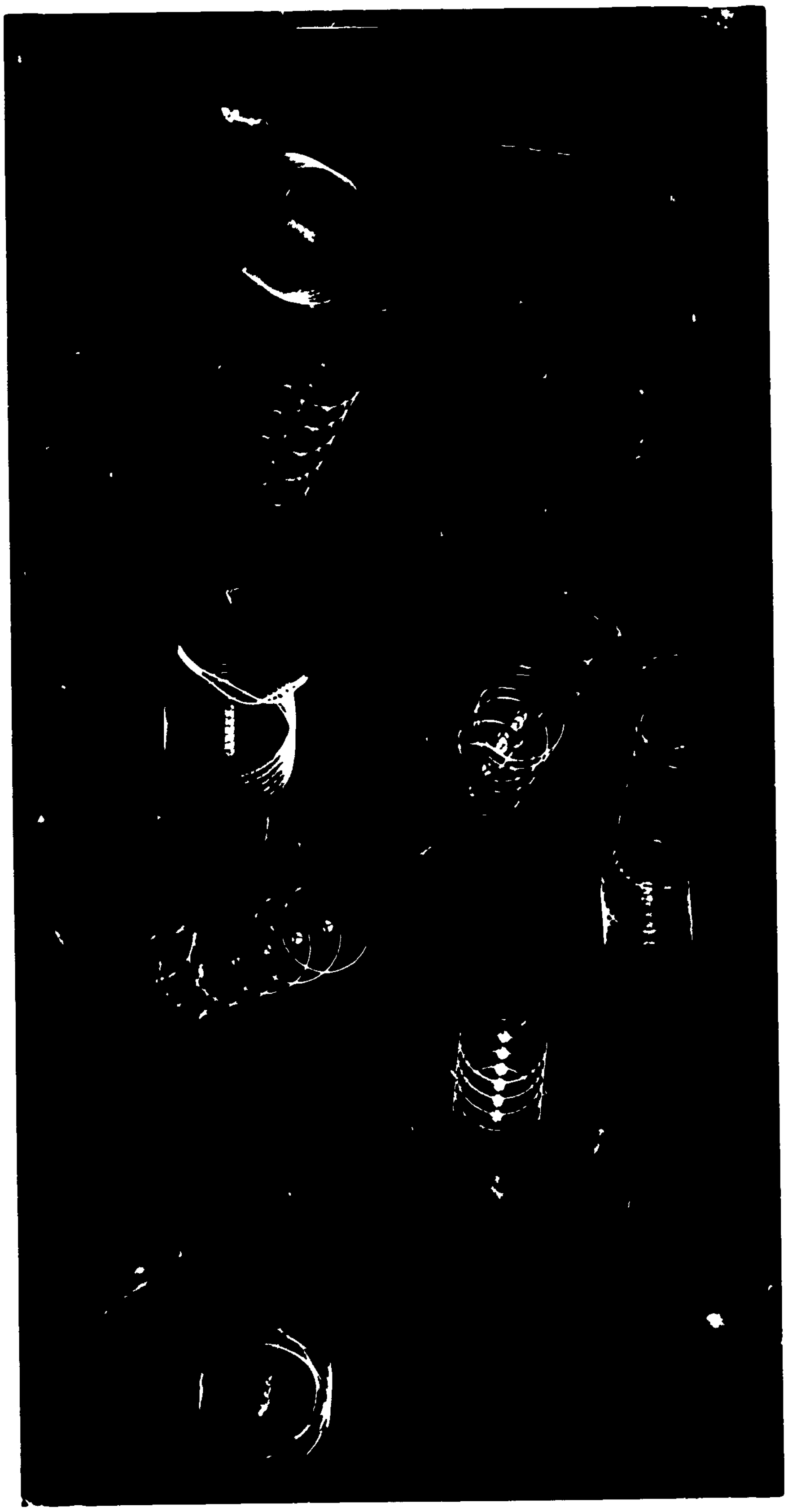


Fig. A.57.

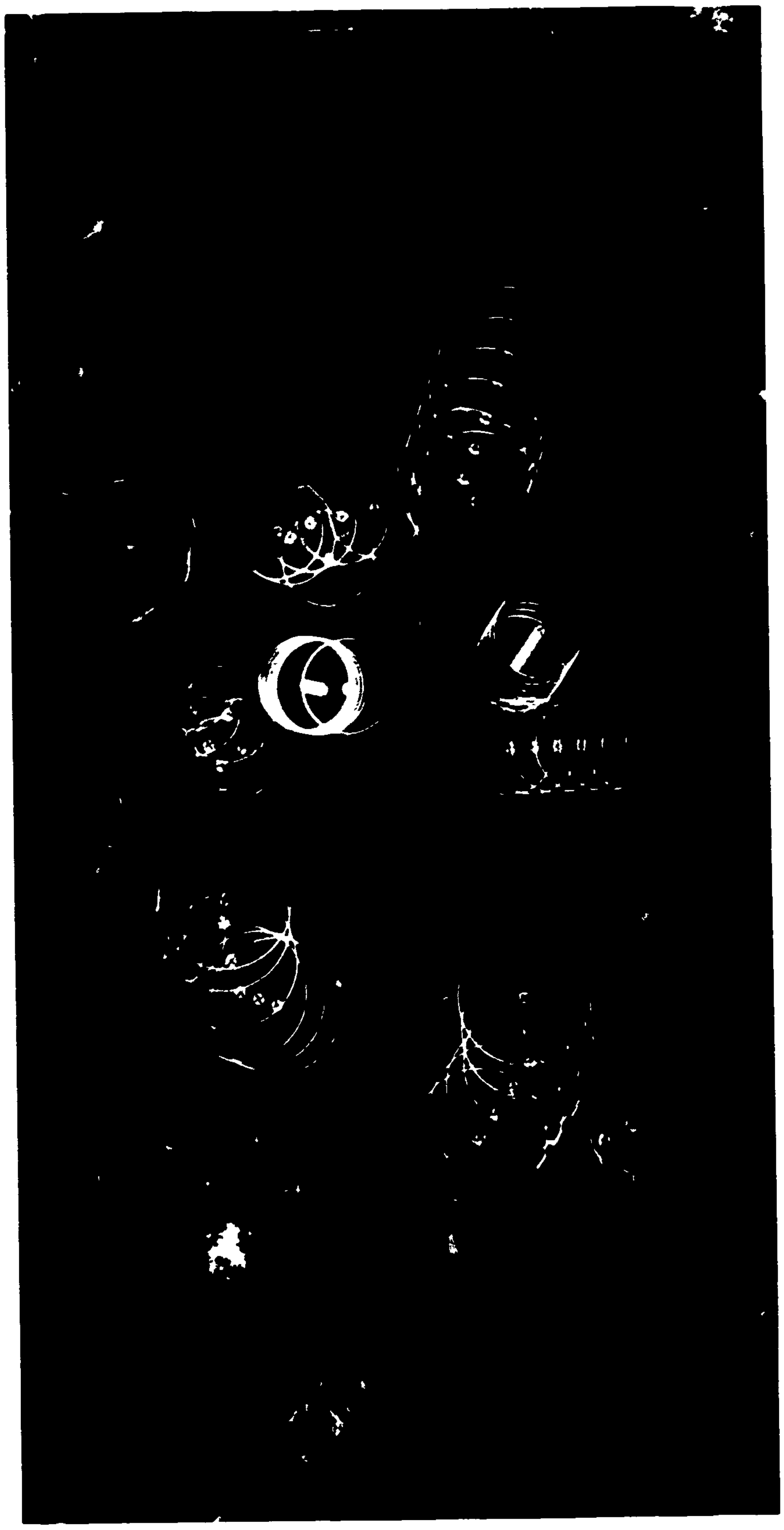


Fig. A.58.

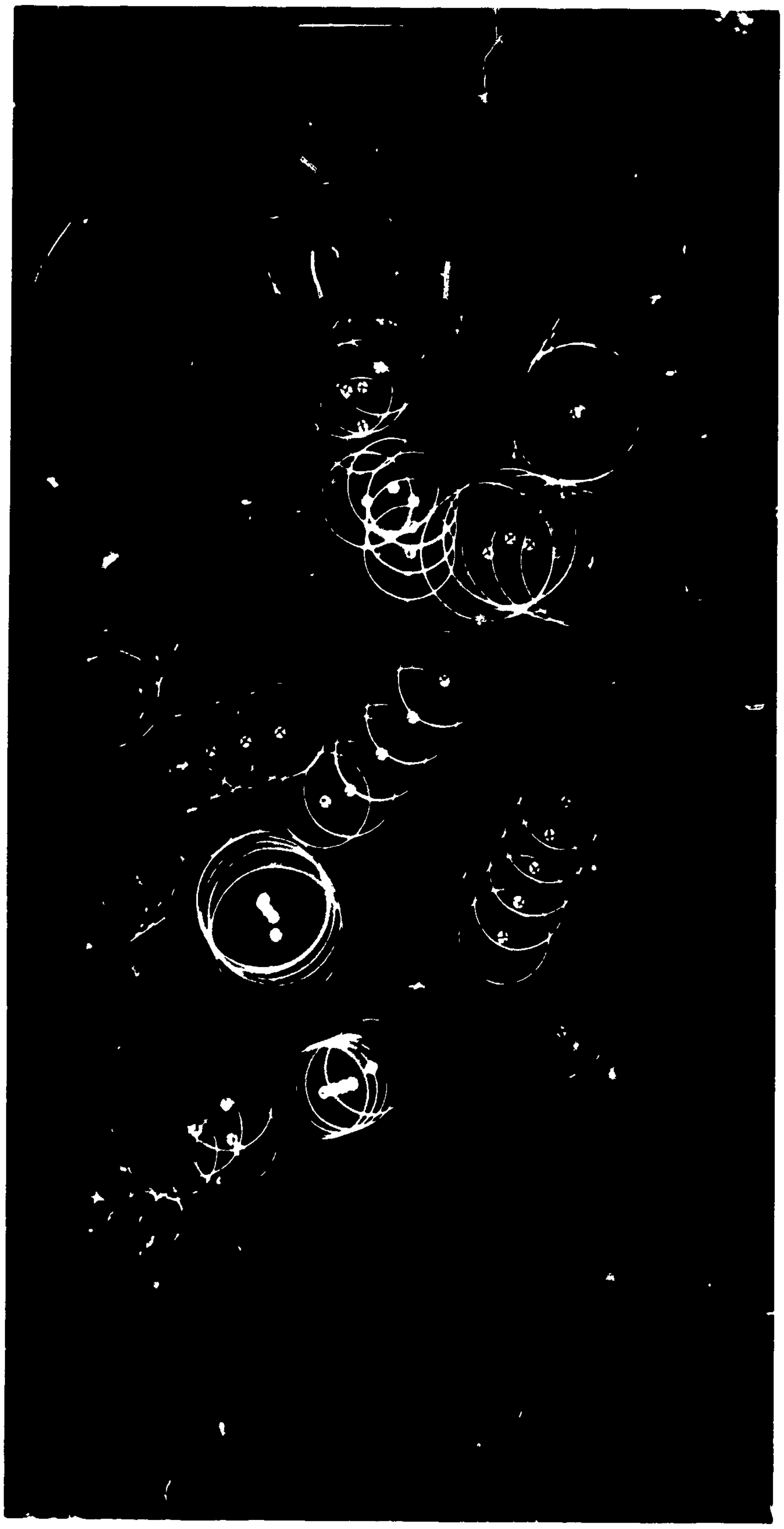


Fig. A.59.

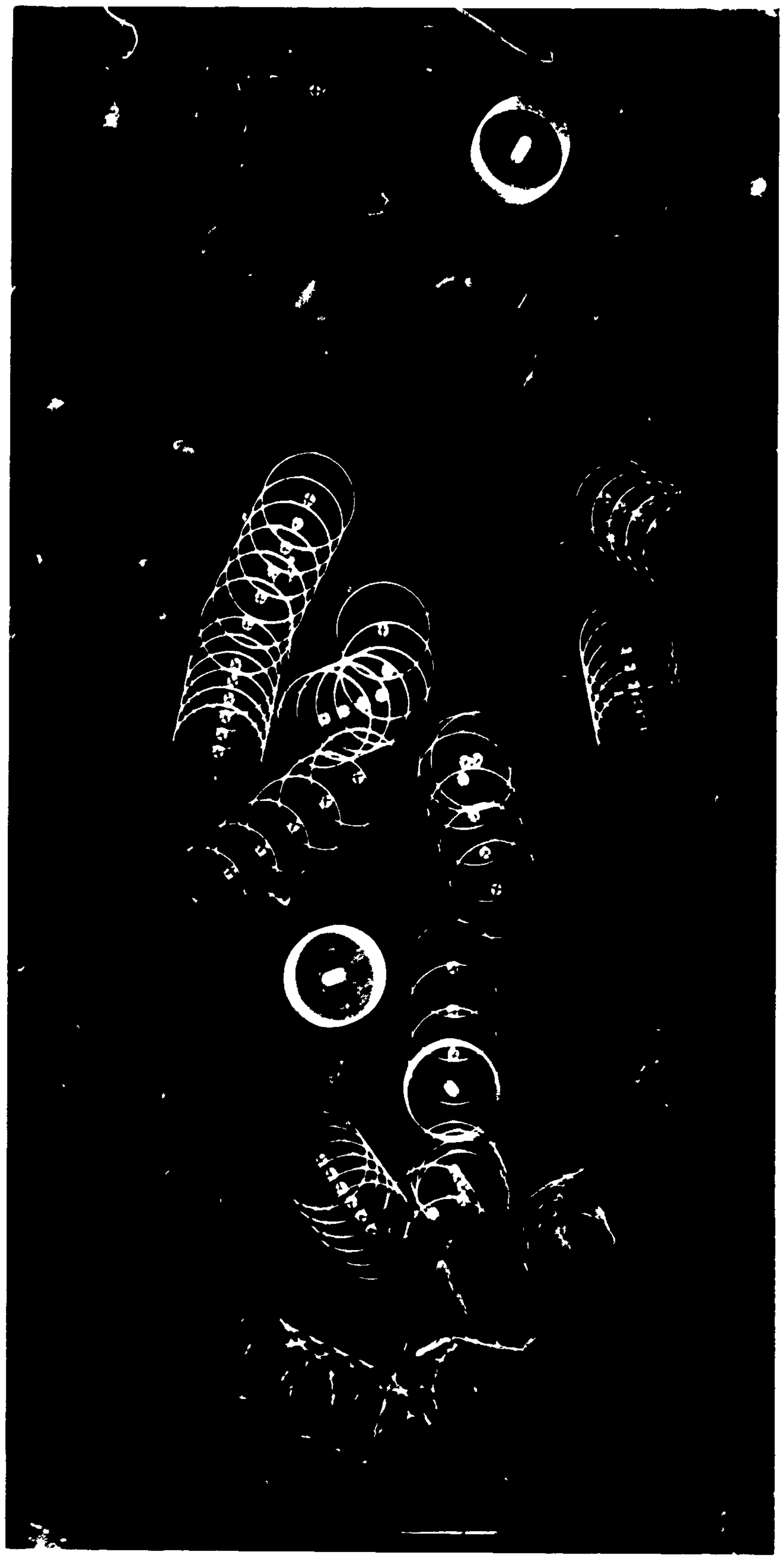


Fig. A.60.

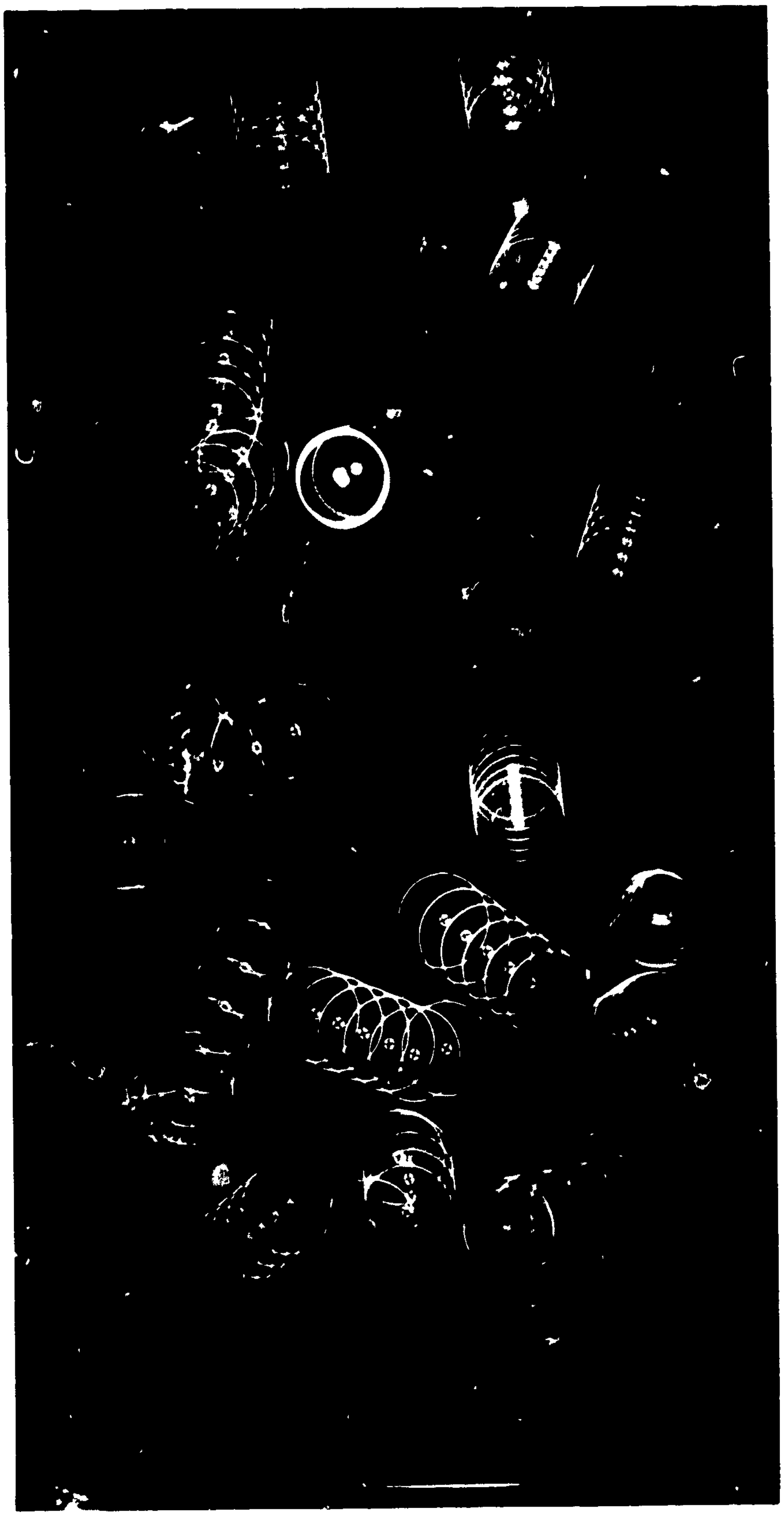


Fig. A.61.

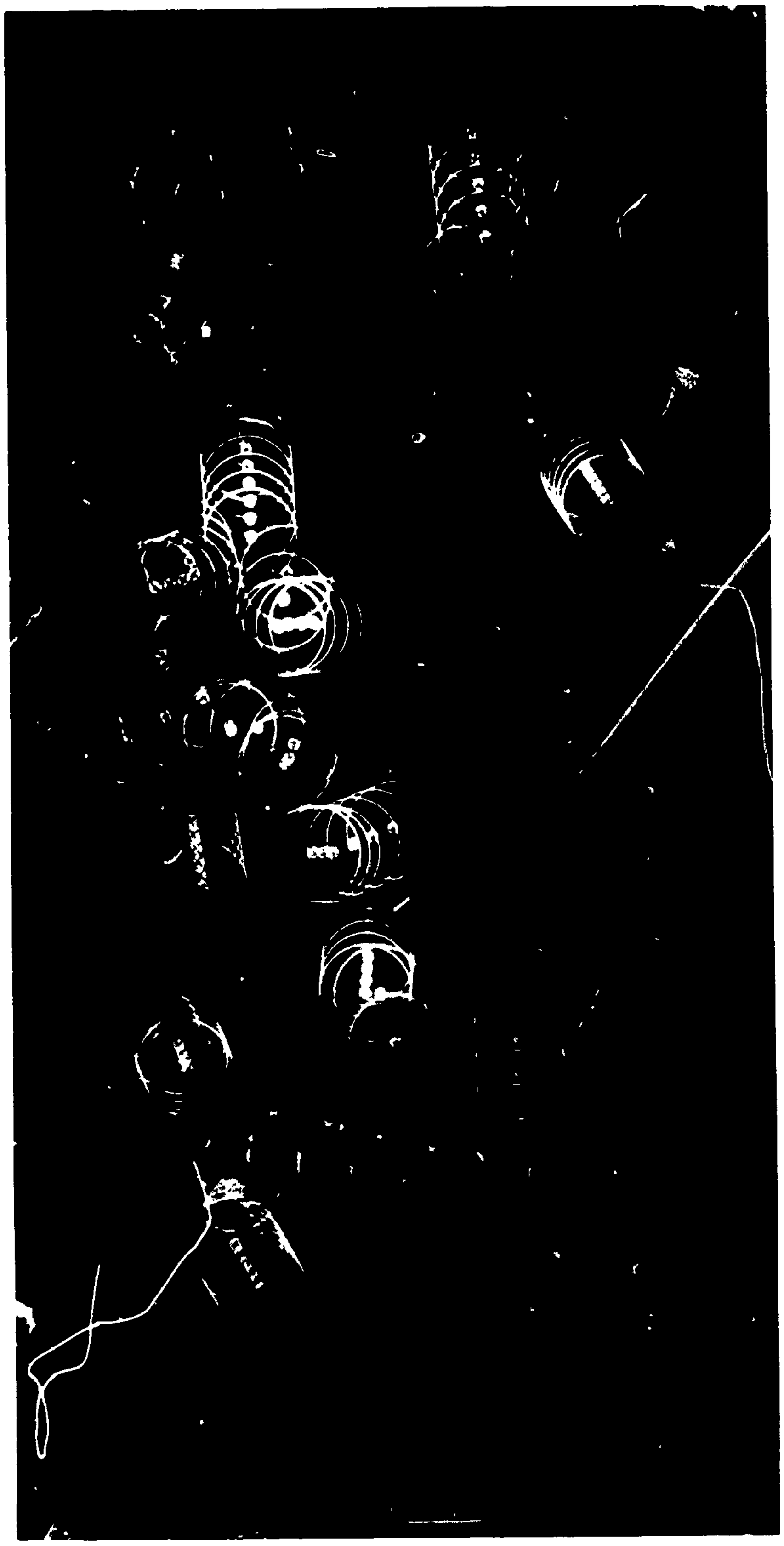


Fig. A.62.

Fig. A.63.

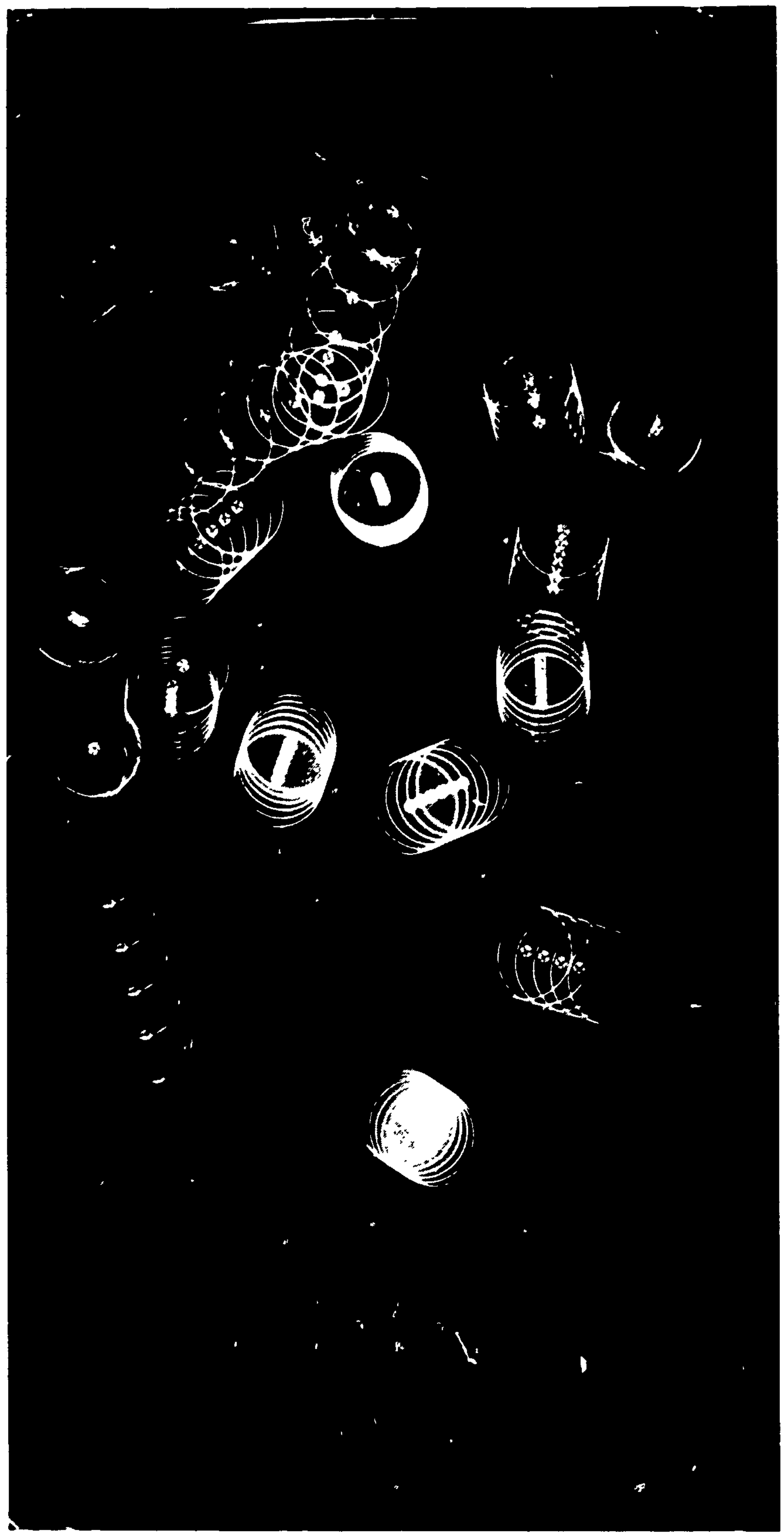


Fig. A.64.

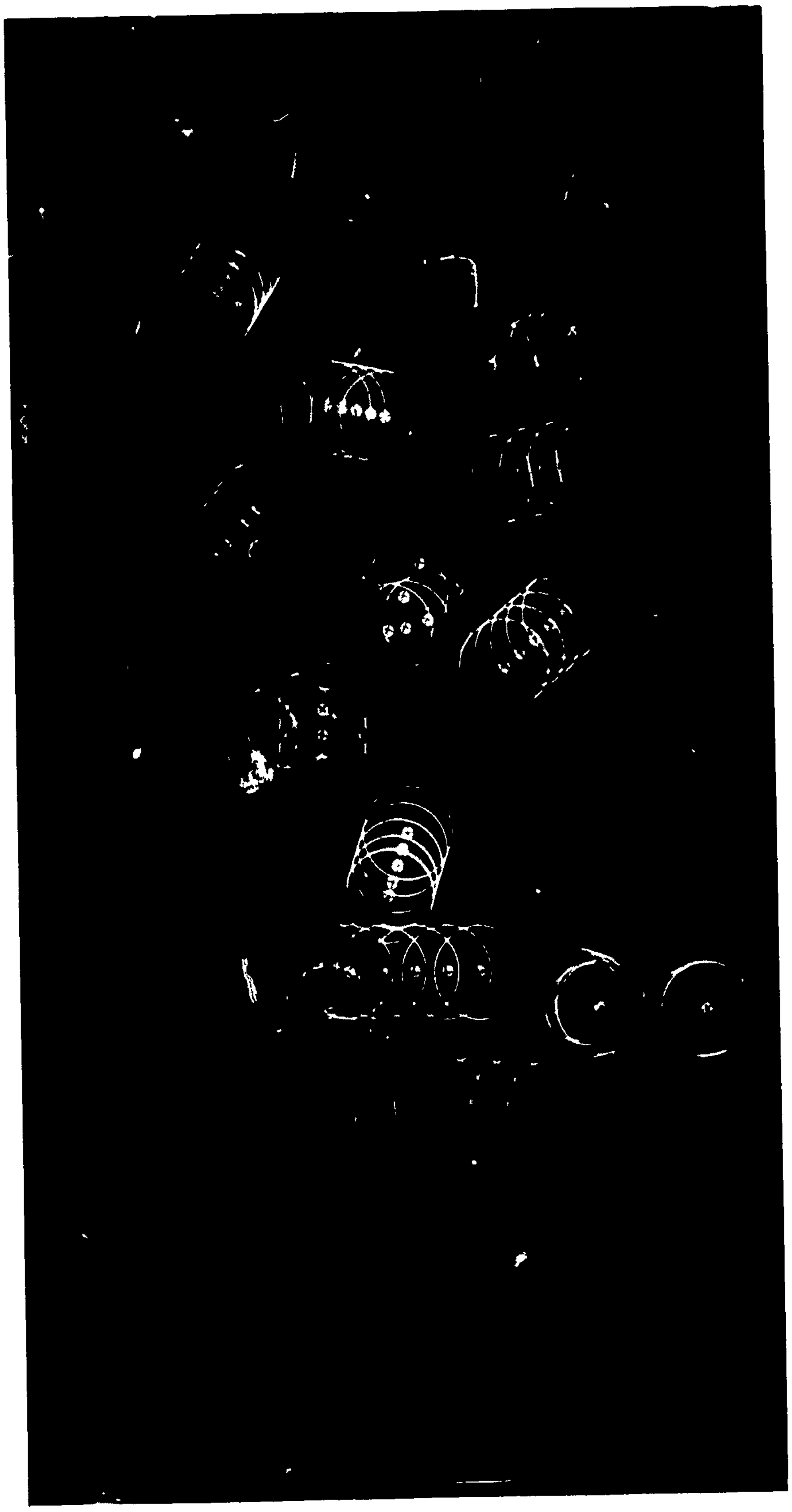


Fig. A.65.

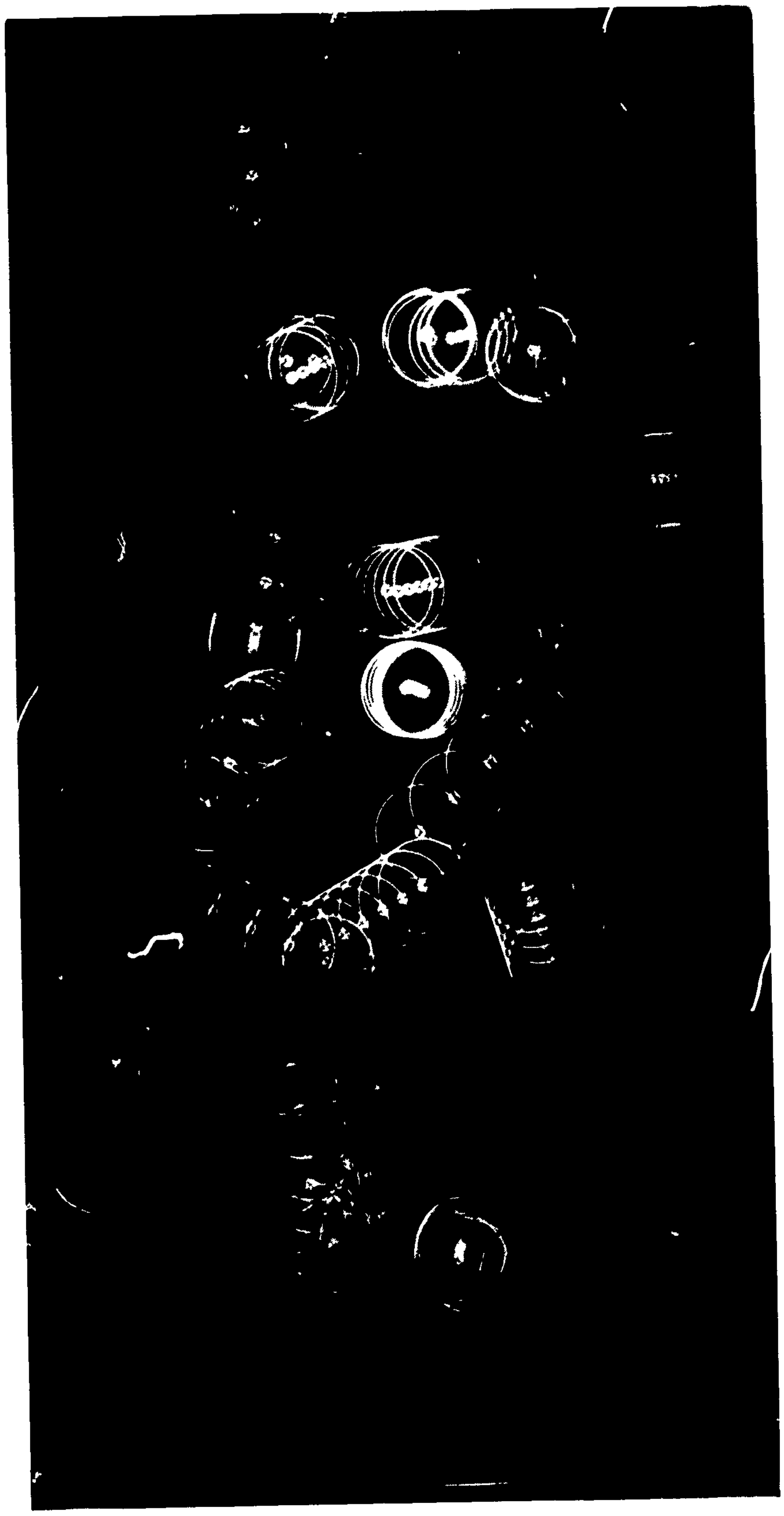


Fig. A.66.

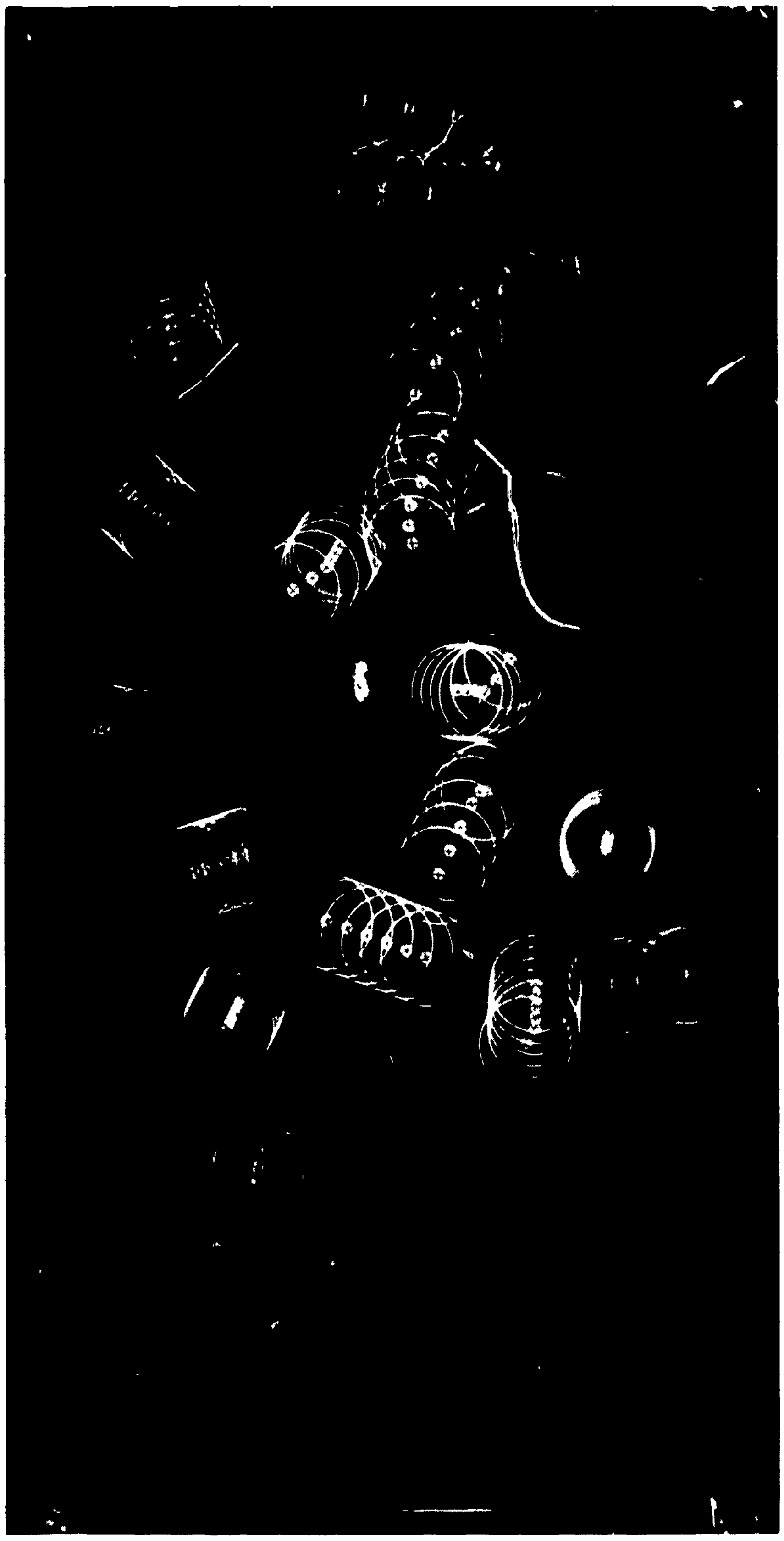


Fig. A.67.

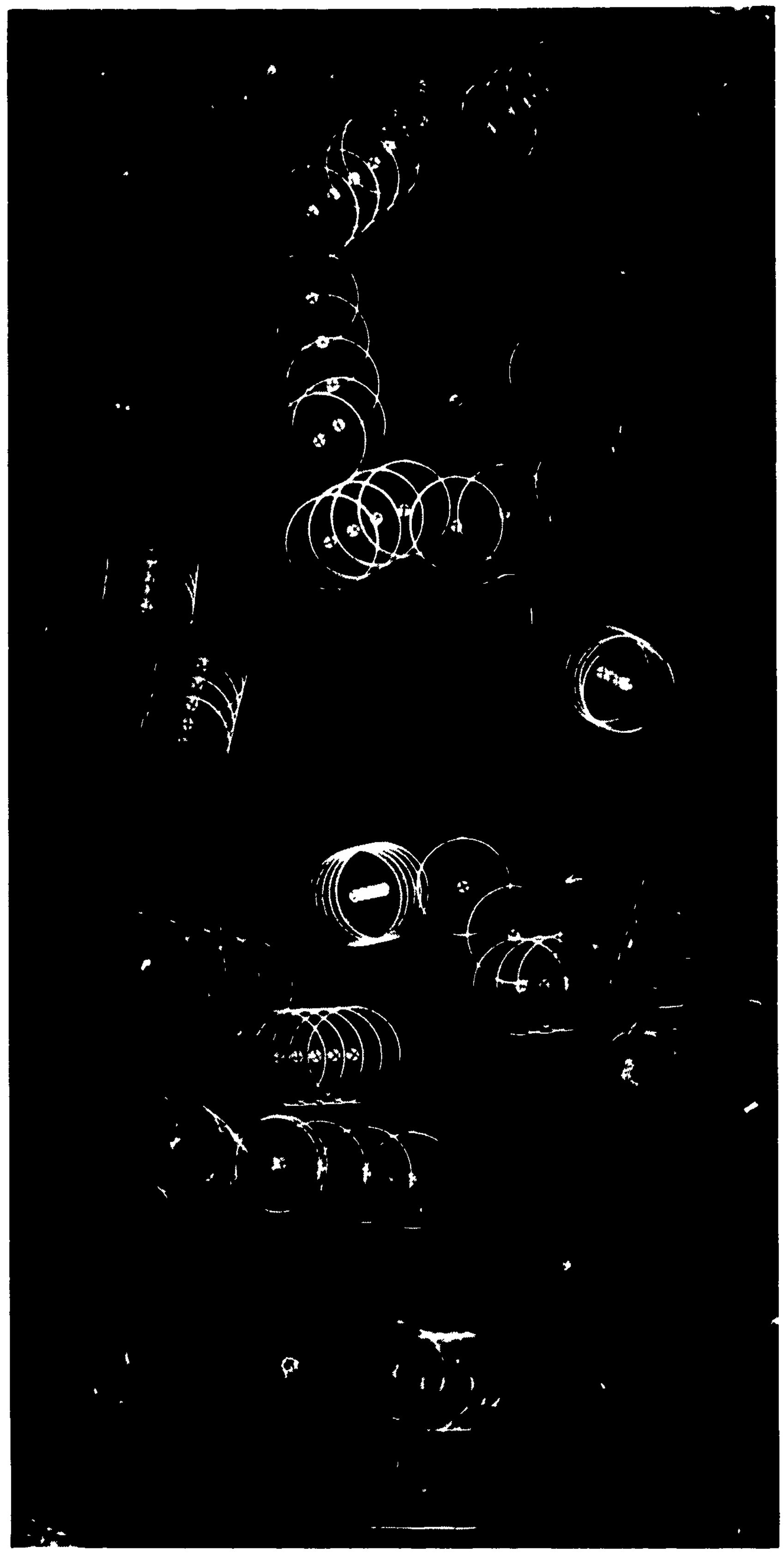


Fig. A.68.

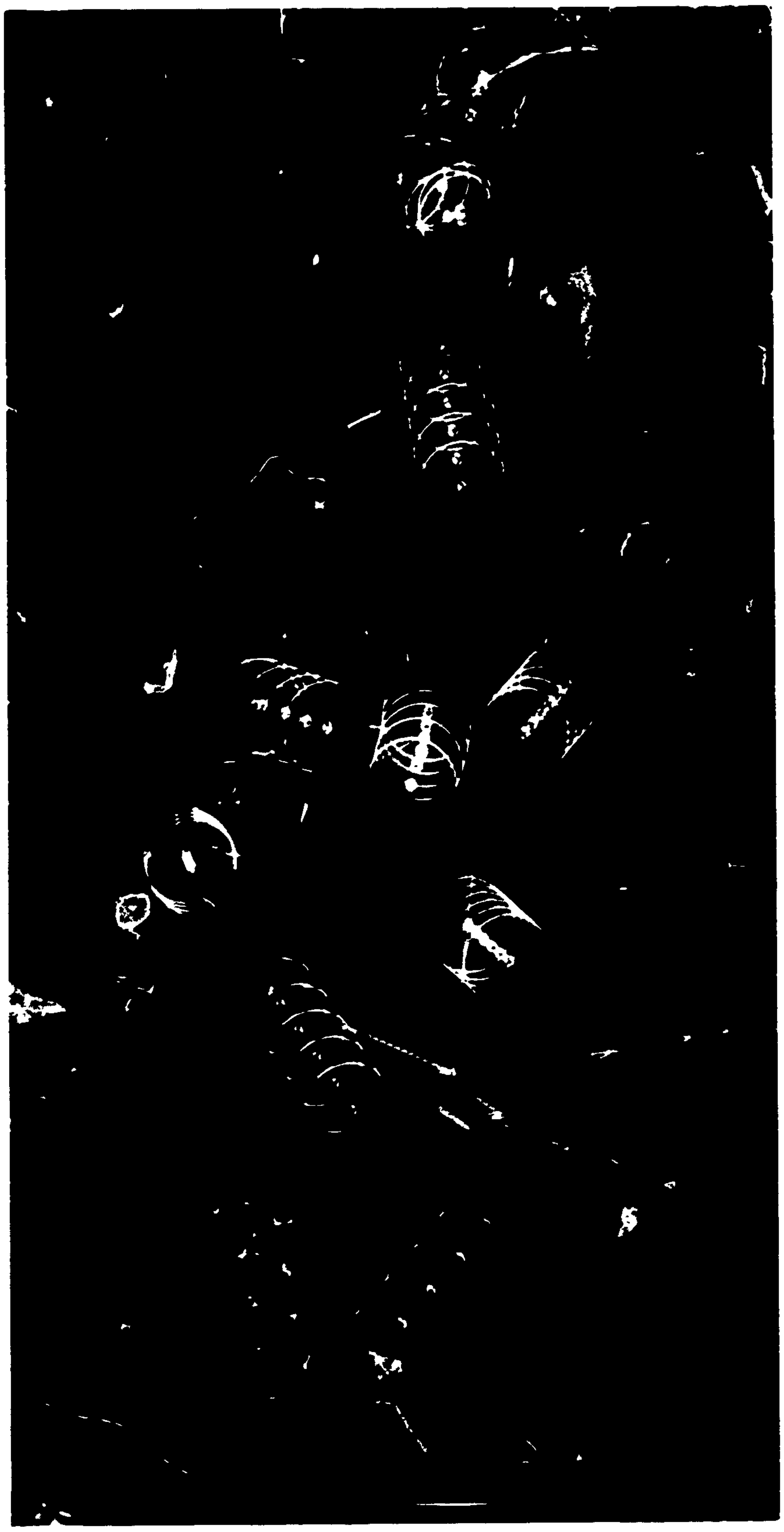


Fig. A.69.

Fig. A.70.

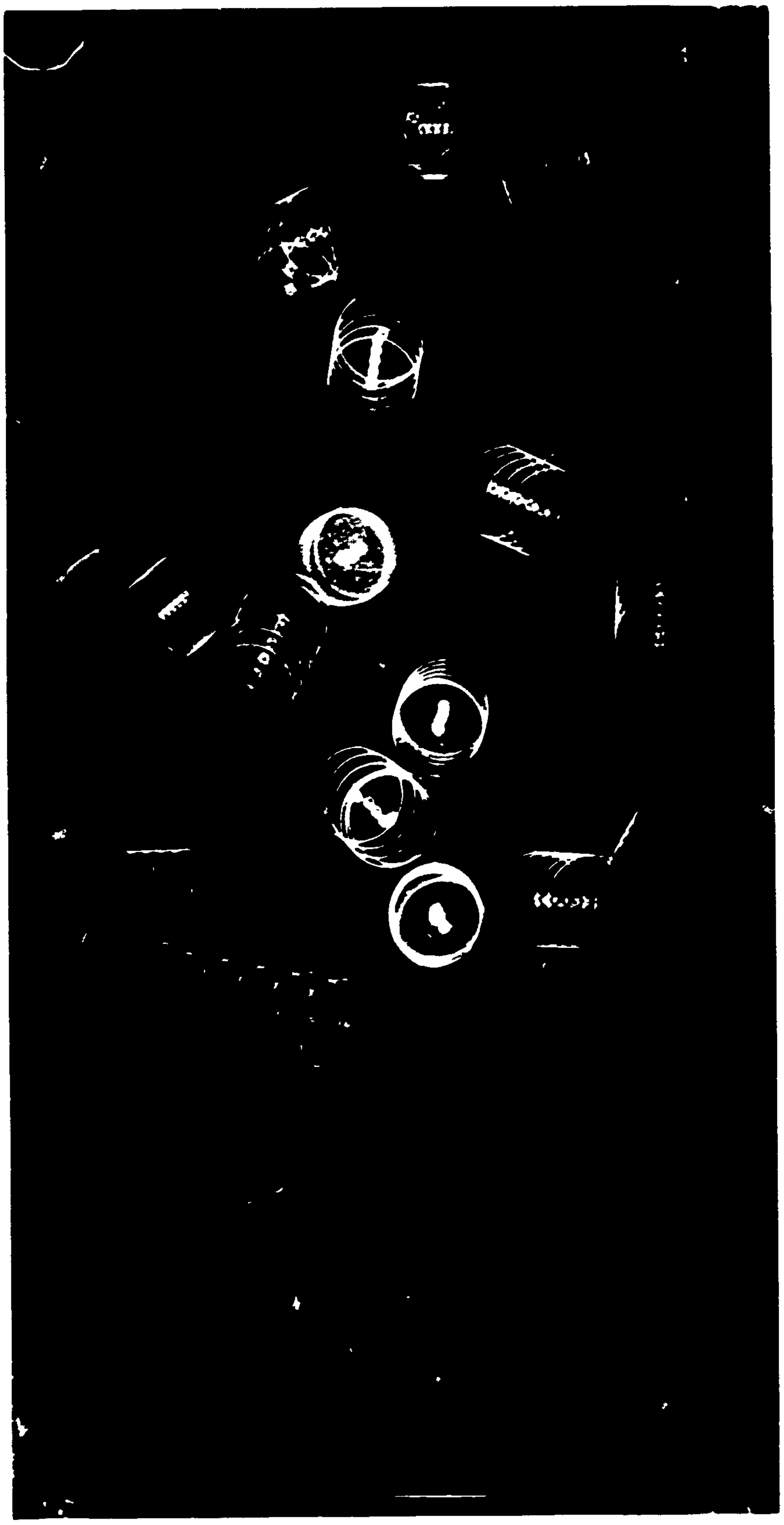


Fig. A.71.

Fig. A.72.

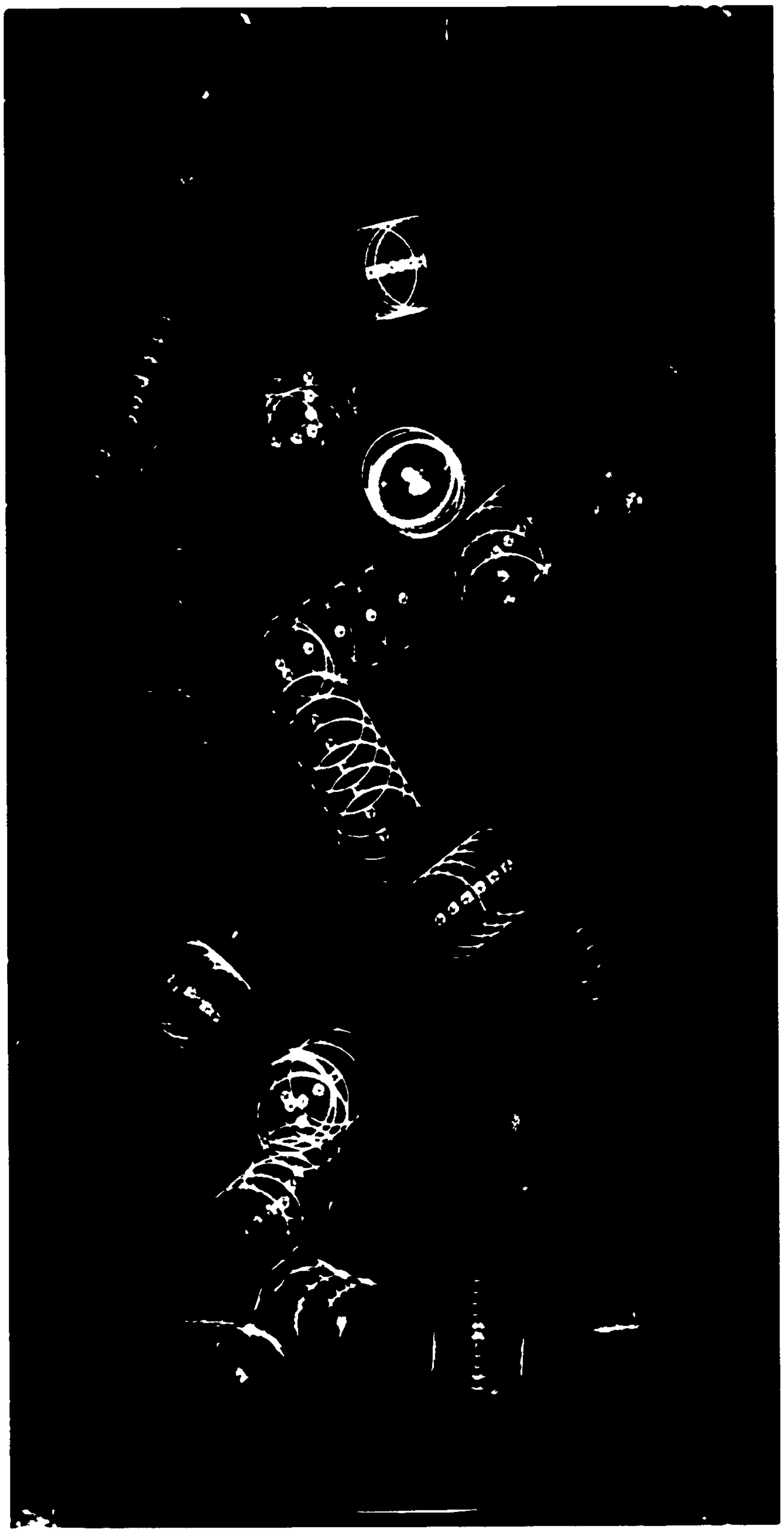


Fig. A.73.

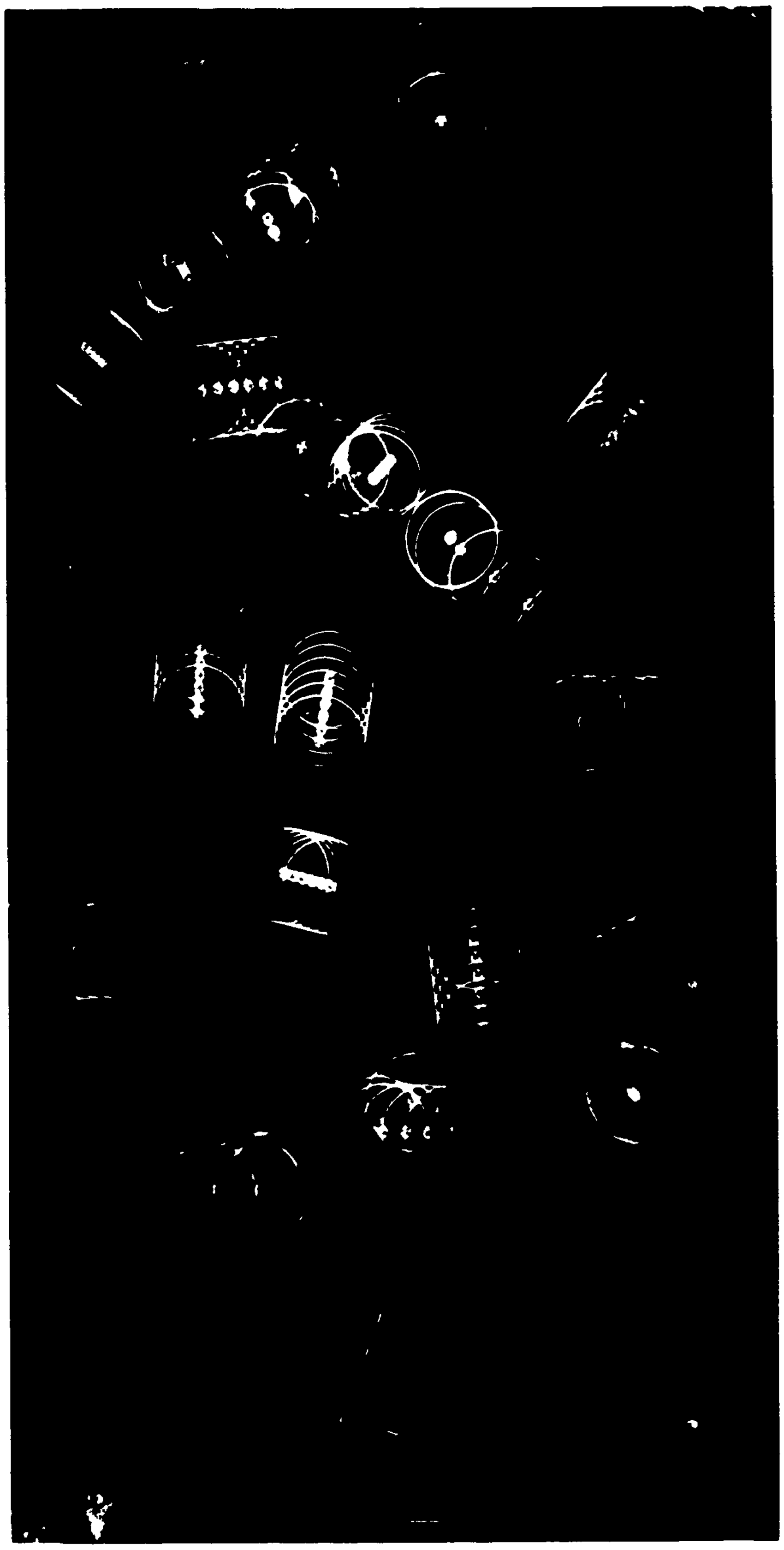


Fig. A.74.

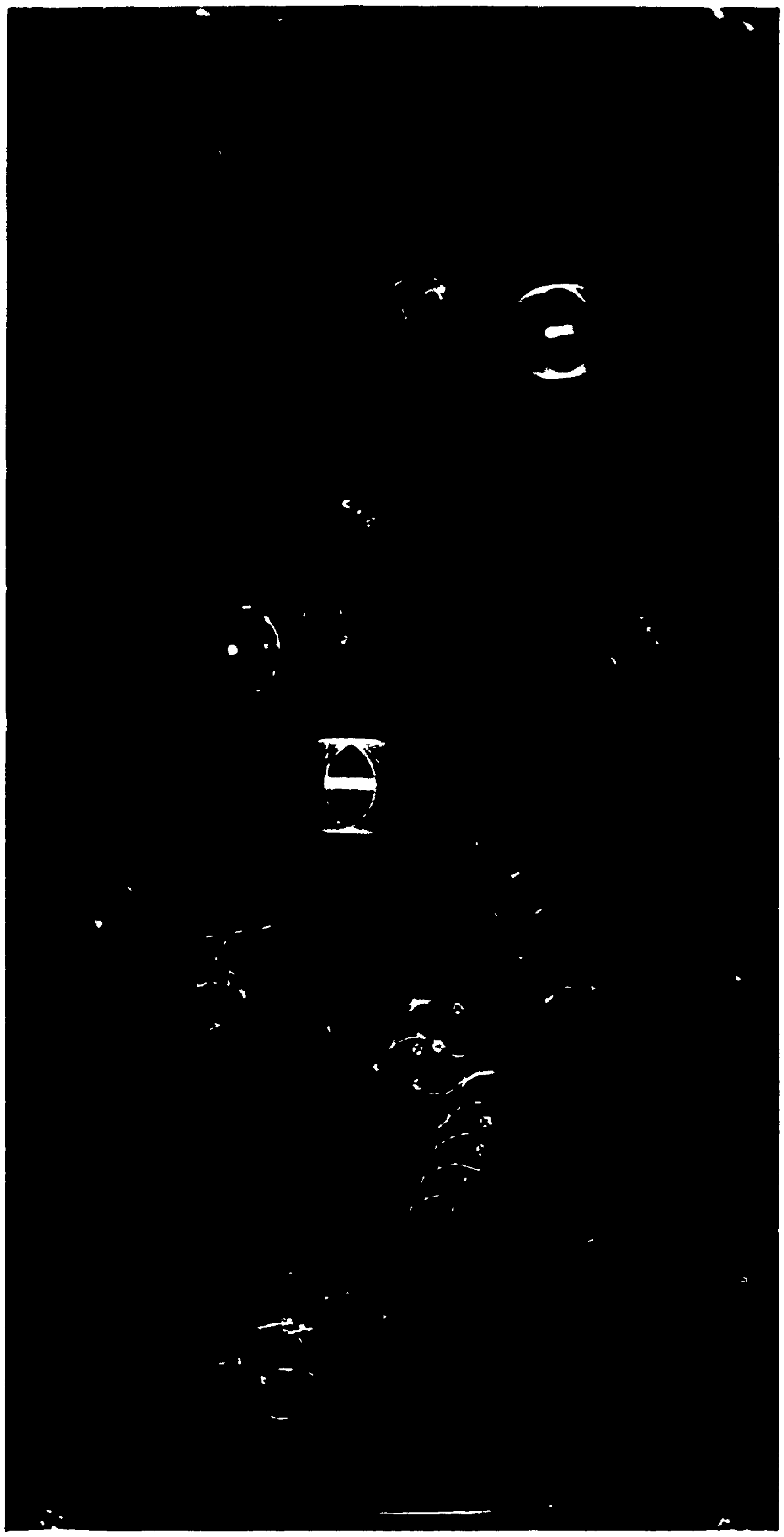


Fig. A.75.

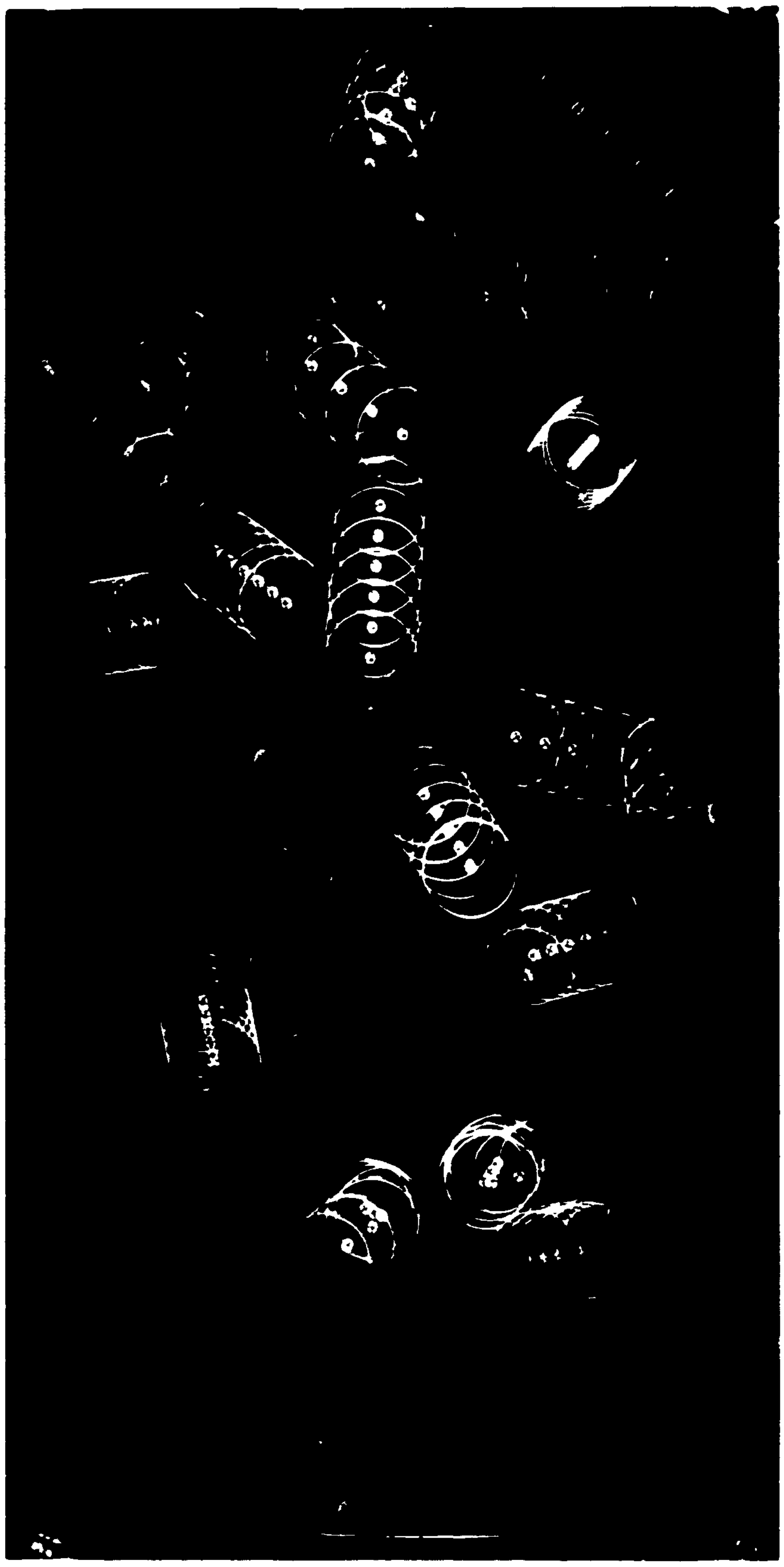


Fig. A.76

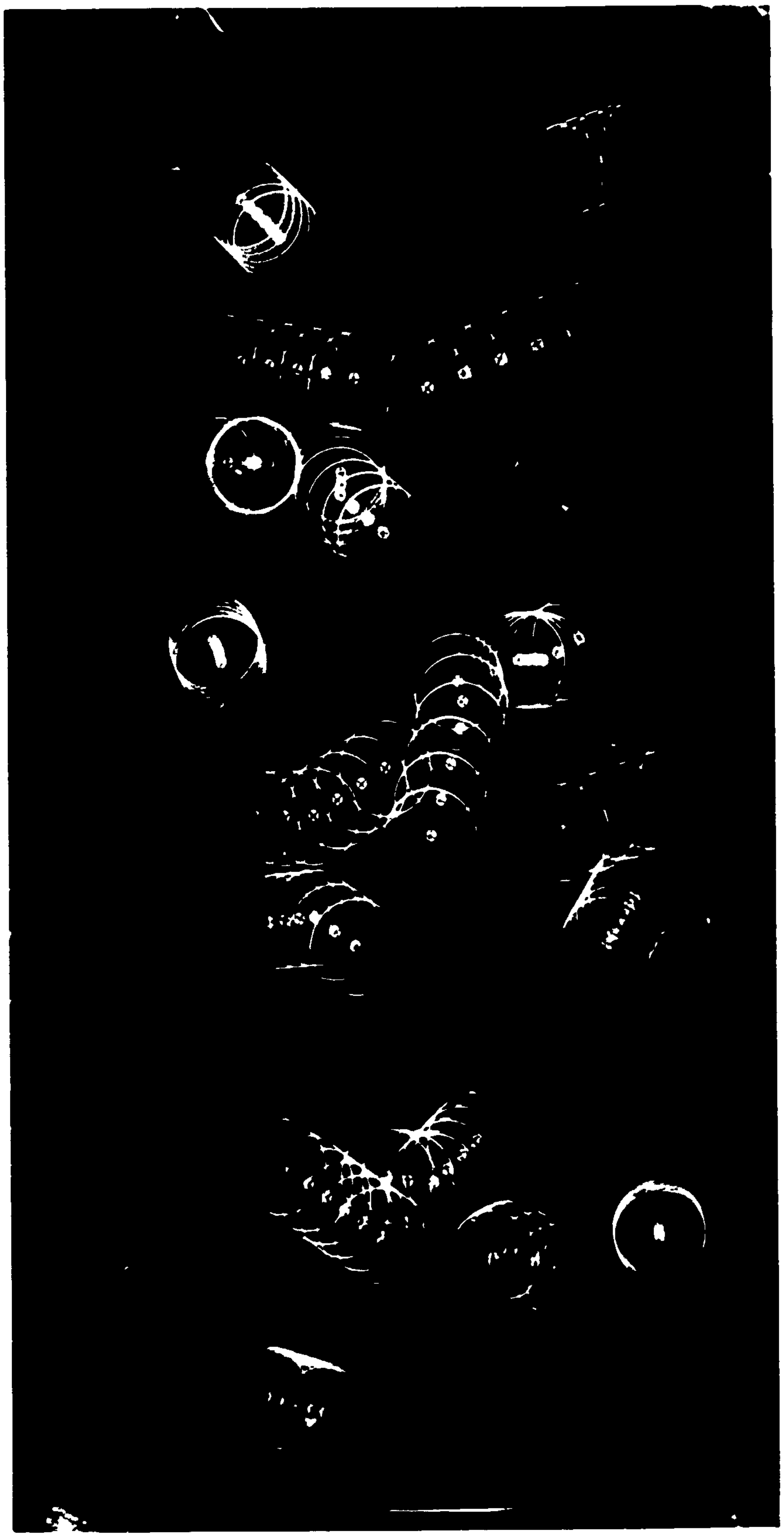


Fig. A.77

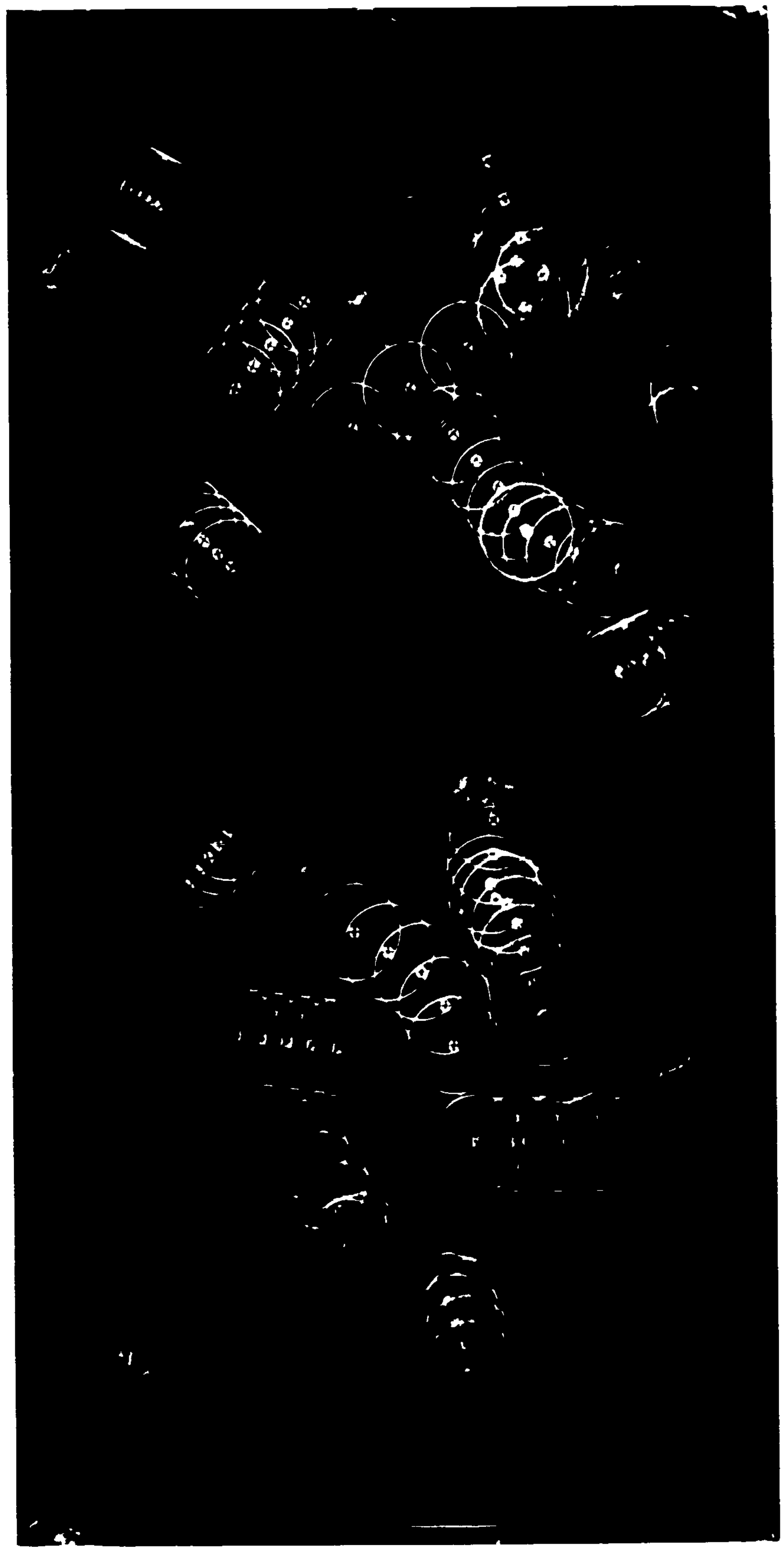


Fig. A.78

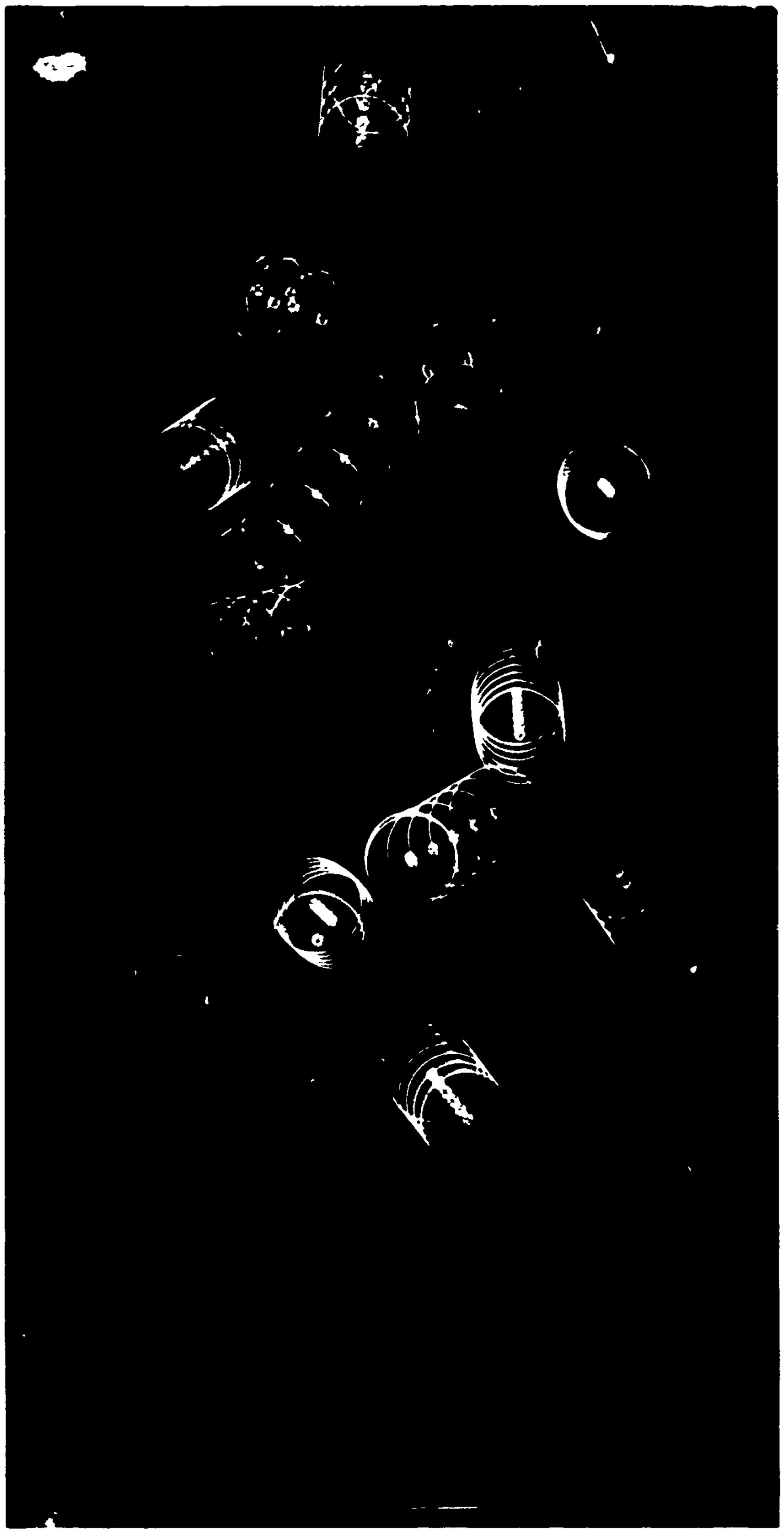


Fig. A.79

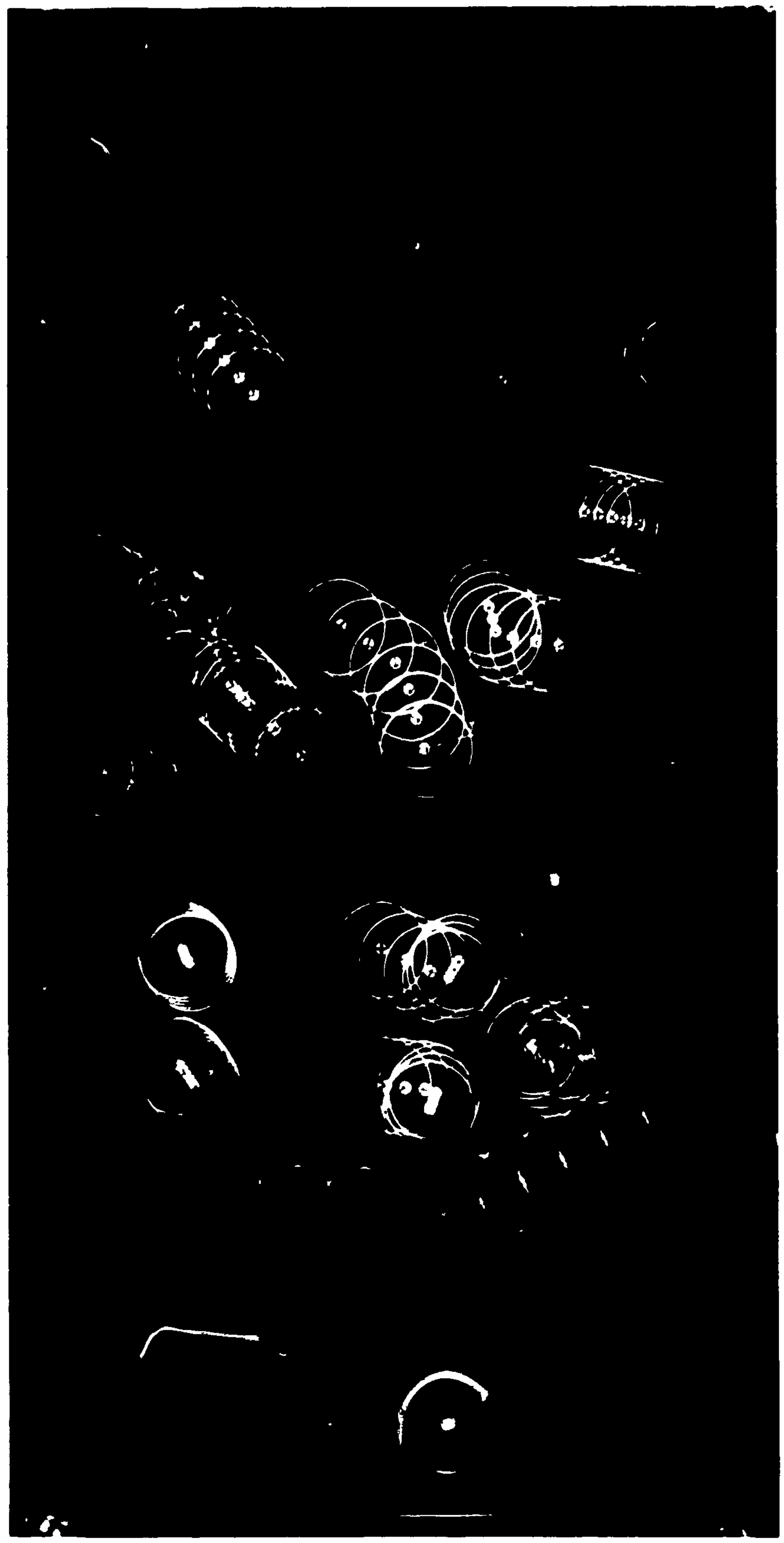


Fig. A.80.

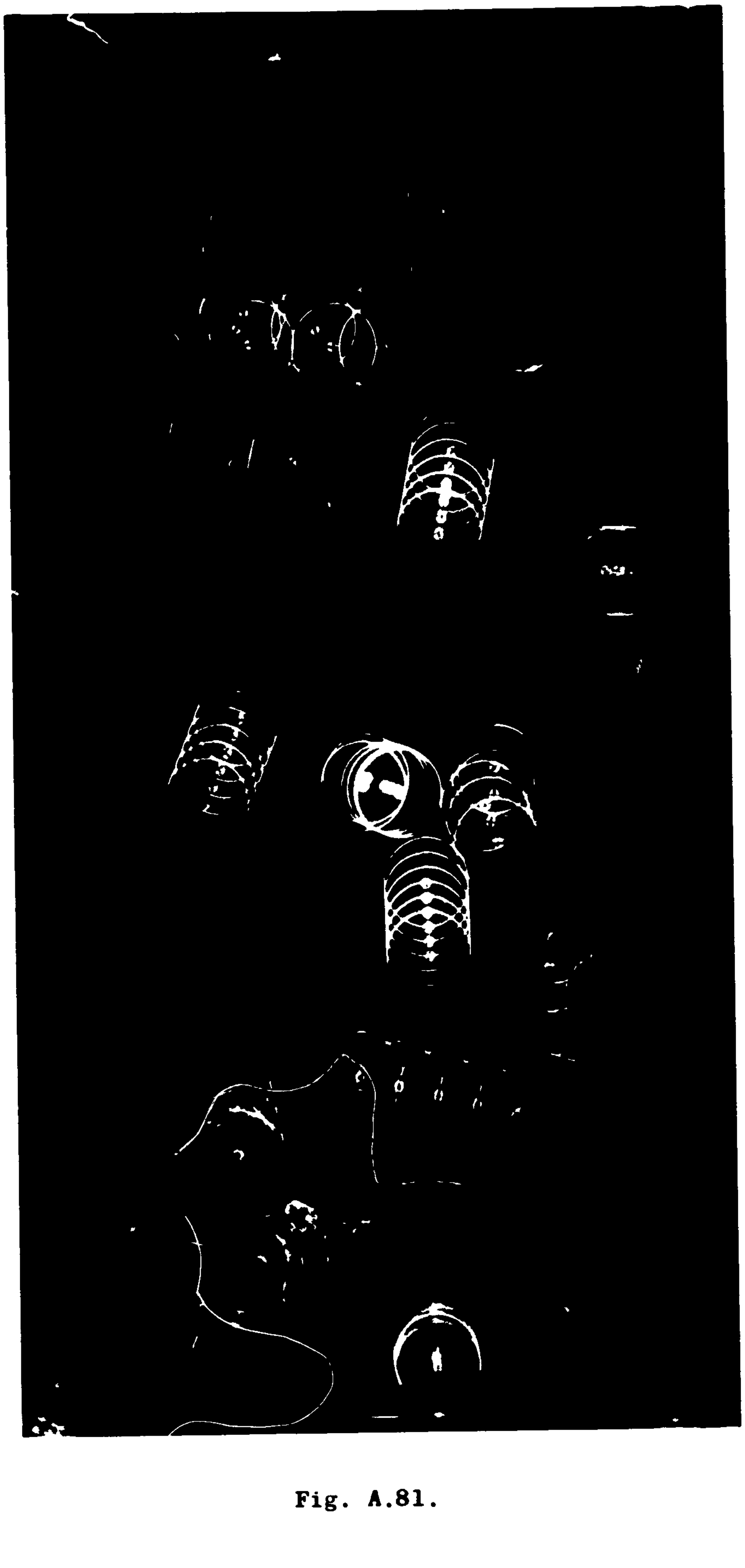


Fig. A.81.

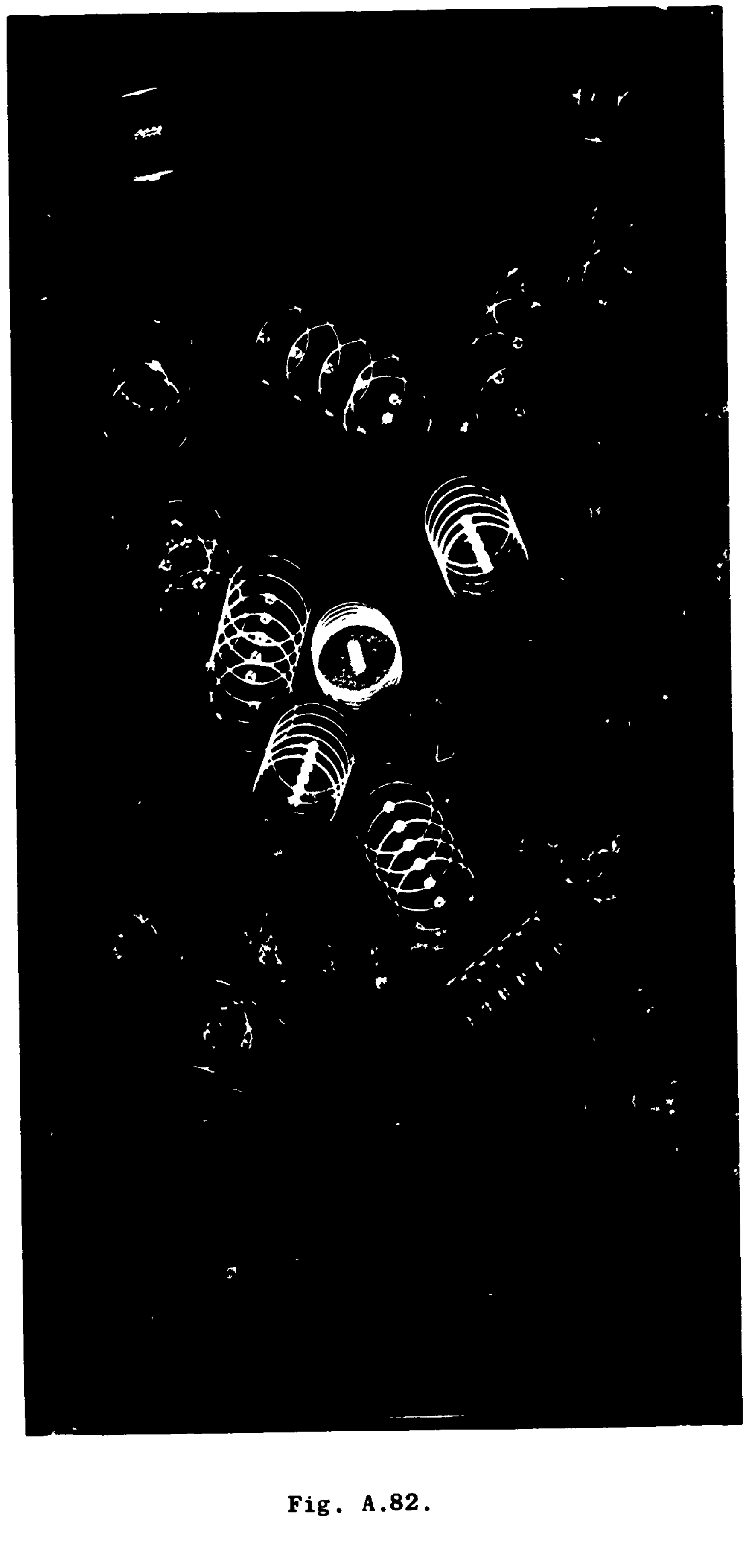


Fig. A.82.

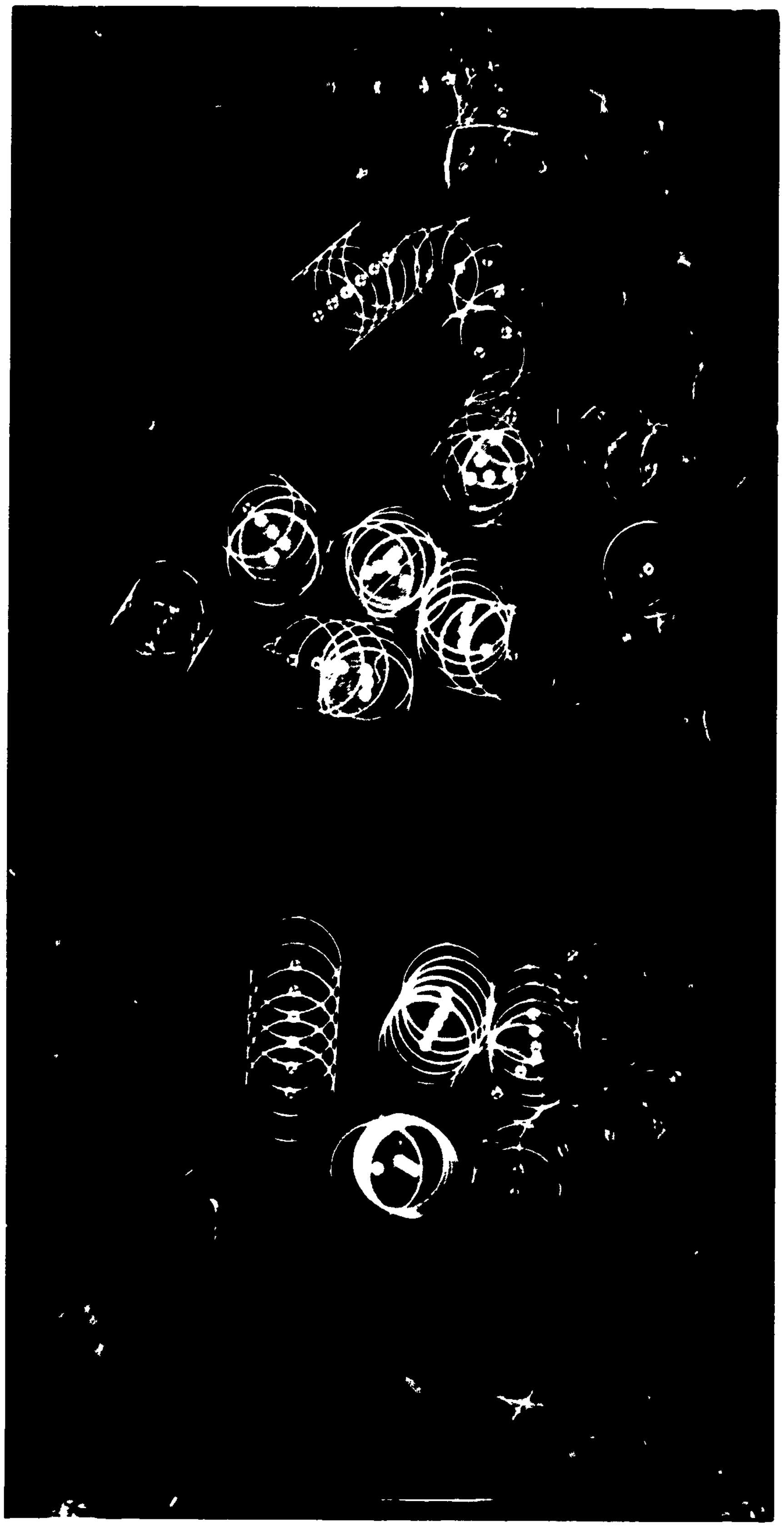


Fig. A.83.

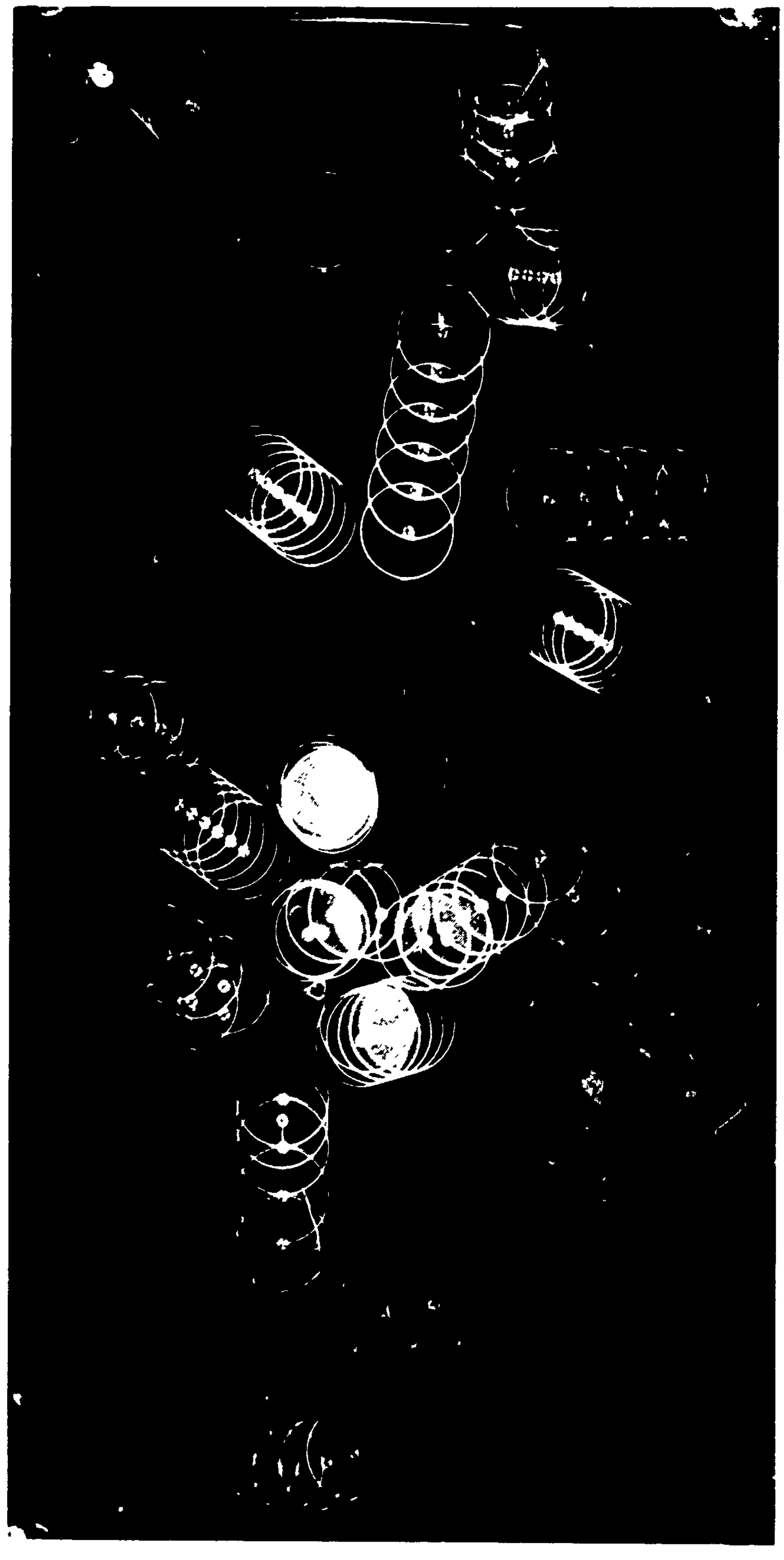


Fig. A.84.

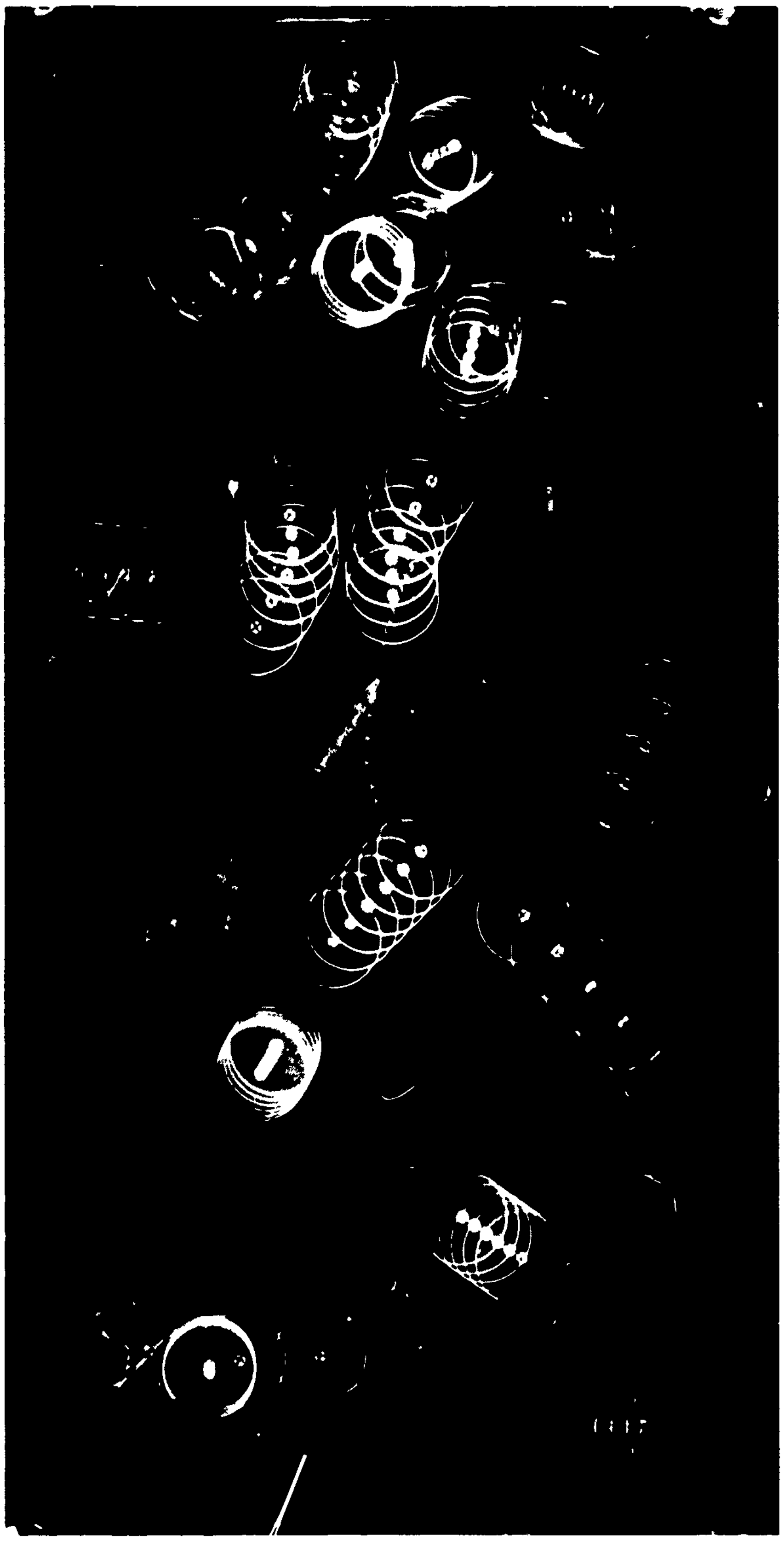


Fig. A.85.

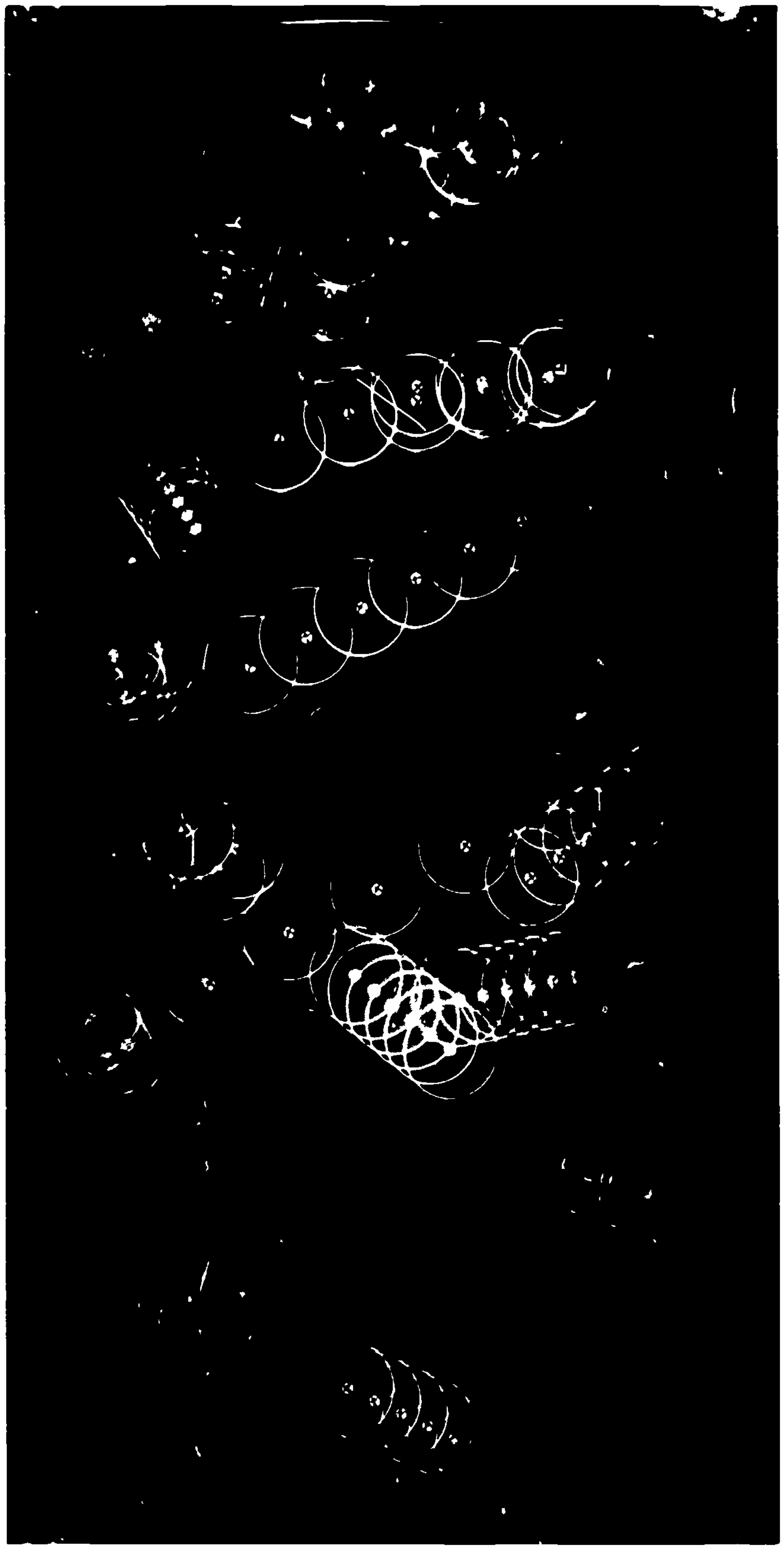


Fig. A.86.

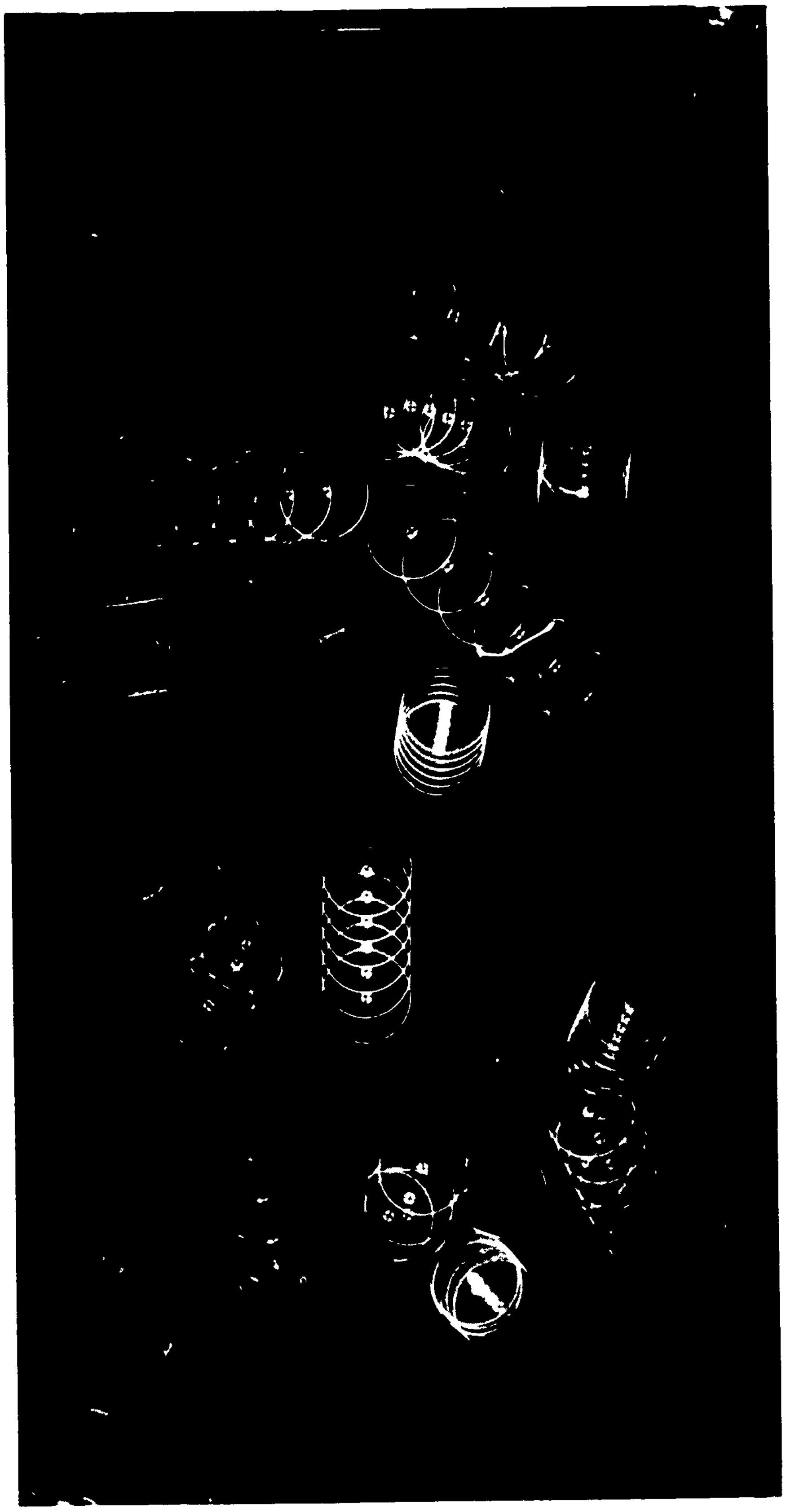


Fig. A.87.

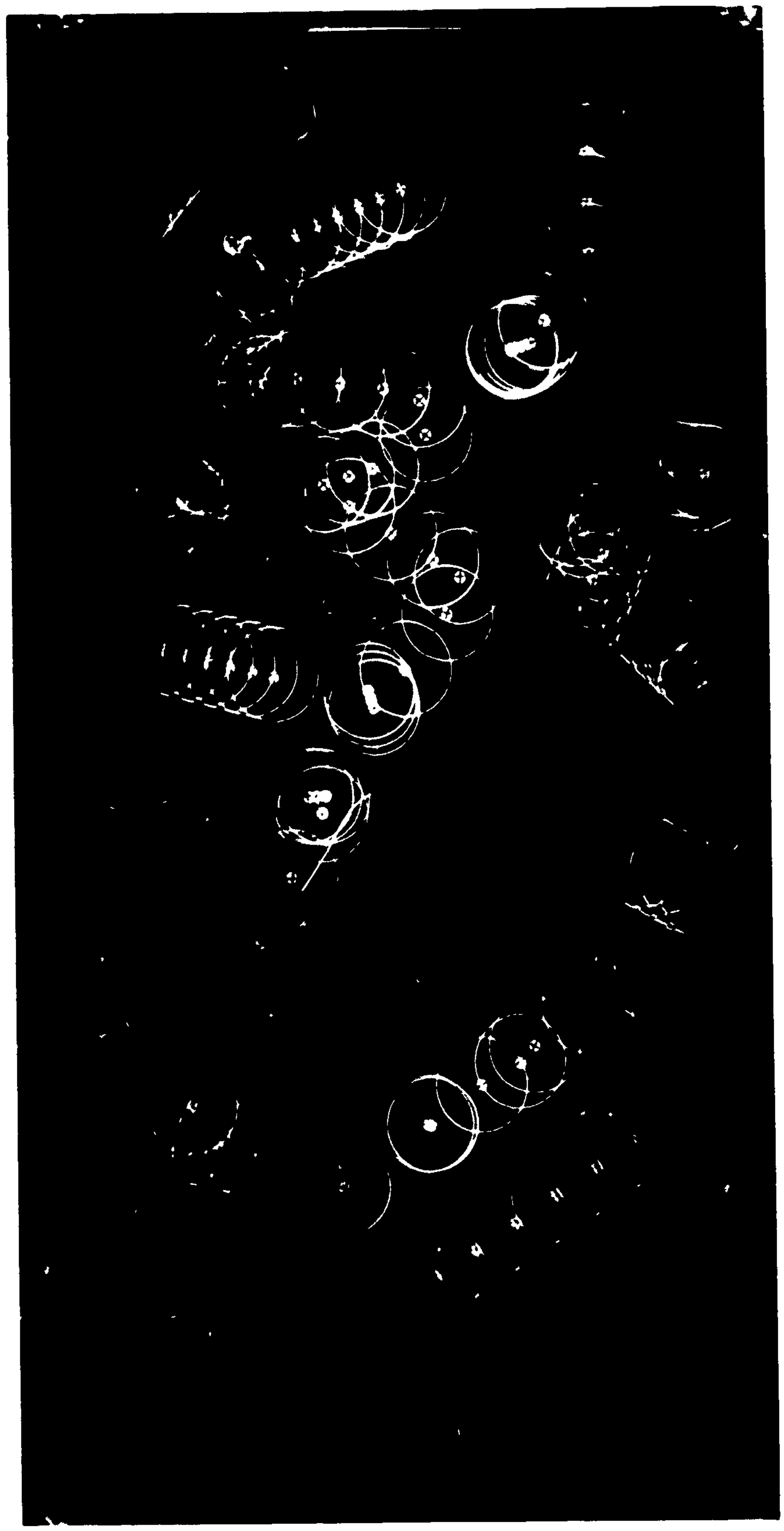


Fig. A.88.

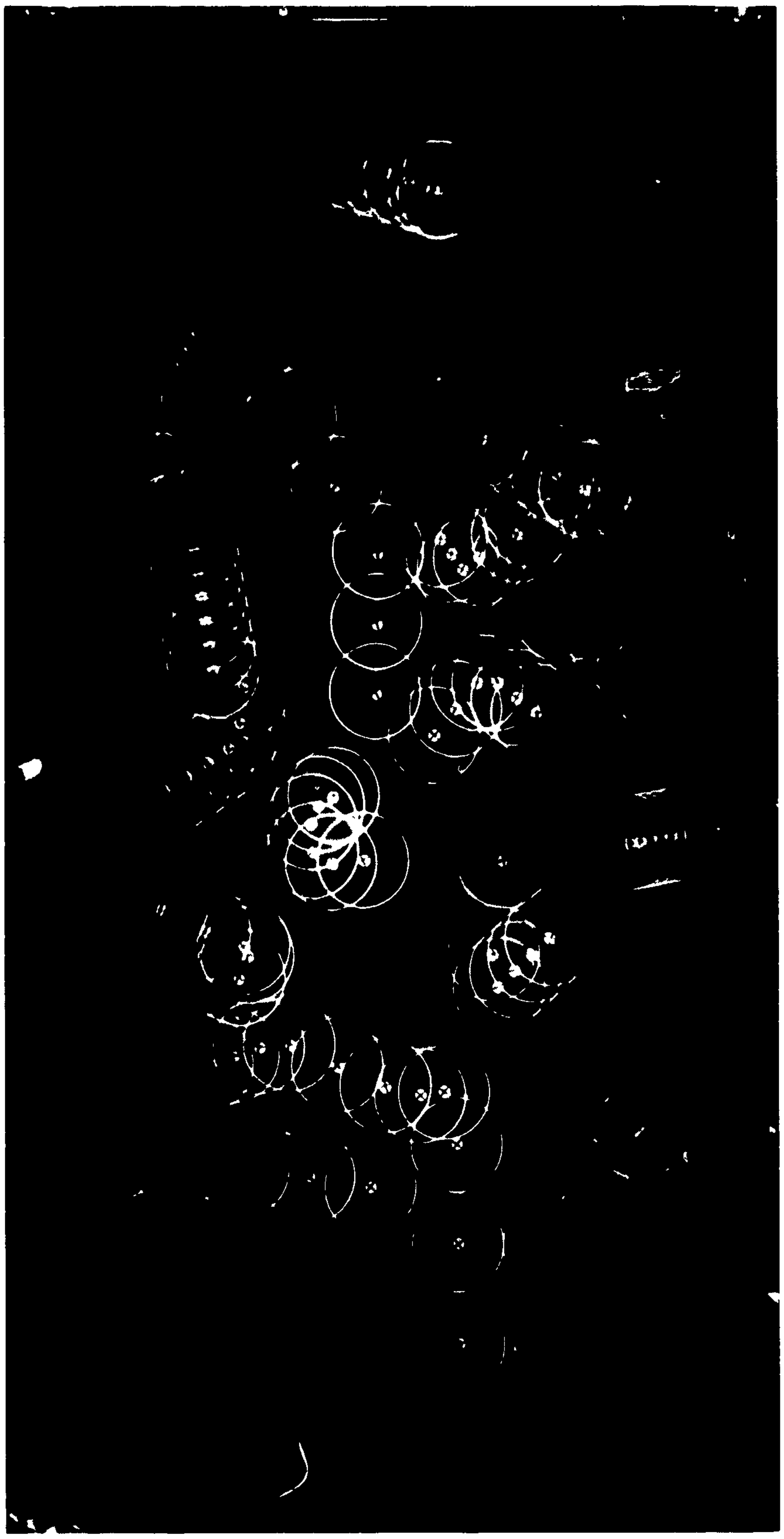


Fig. A.89.

Fig. A.90.

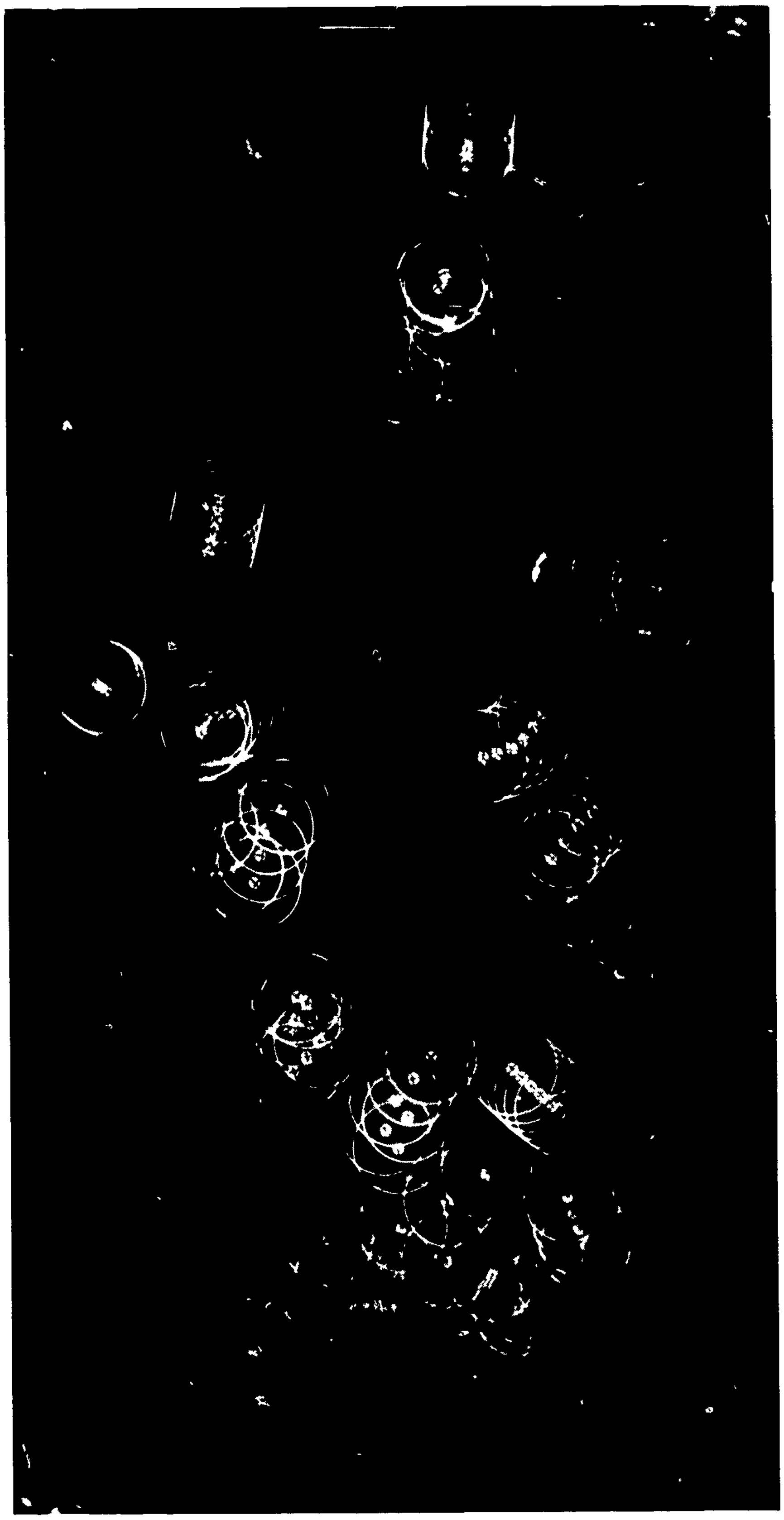


Fig. A.91.

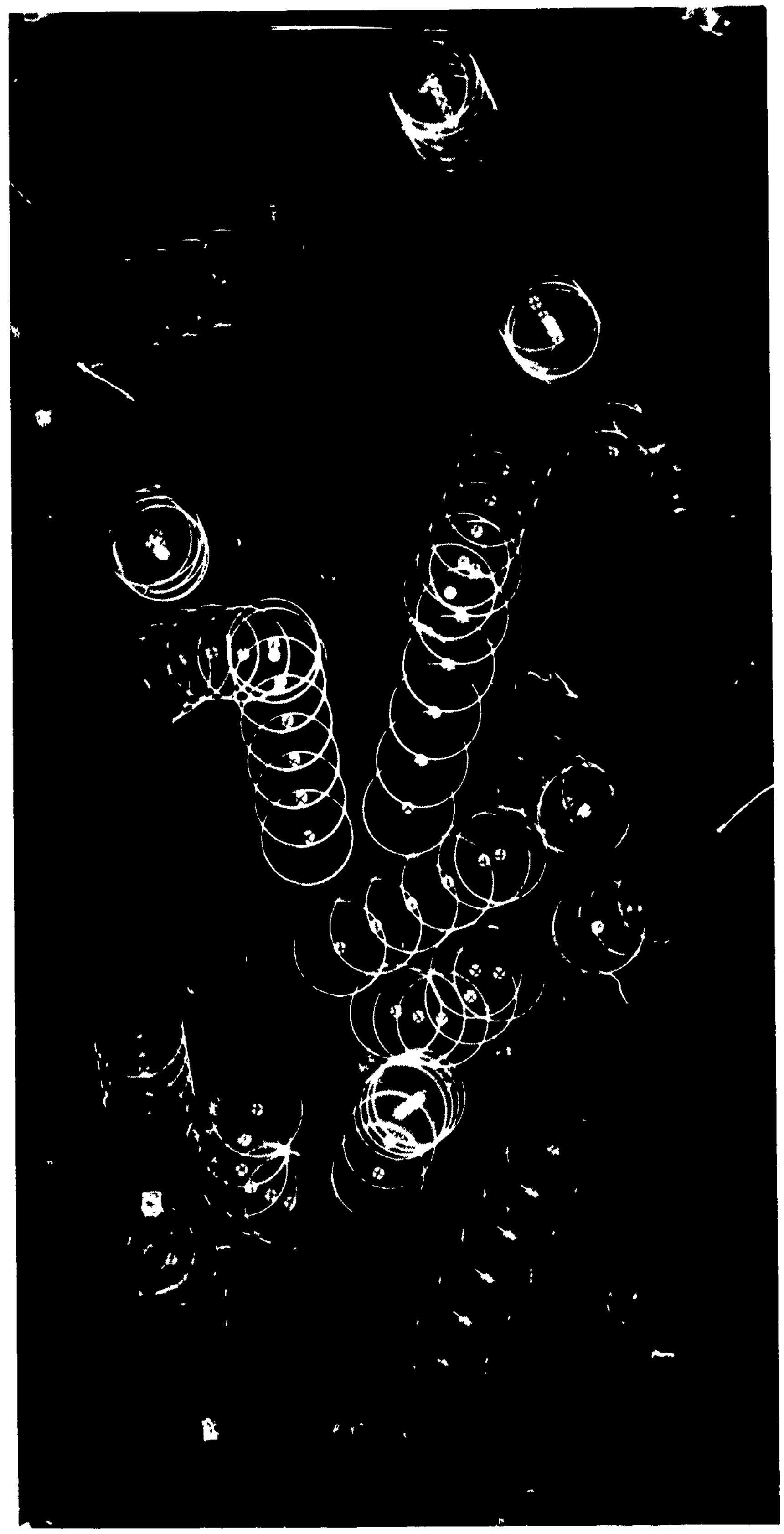


Fig. A.92.

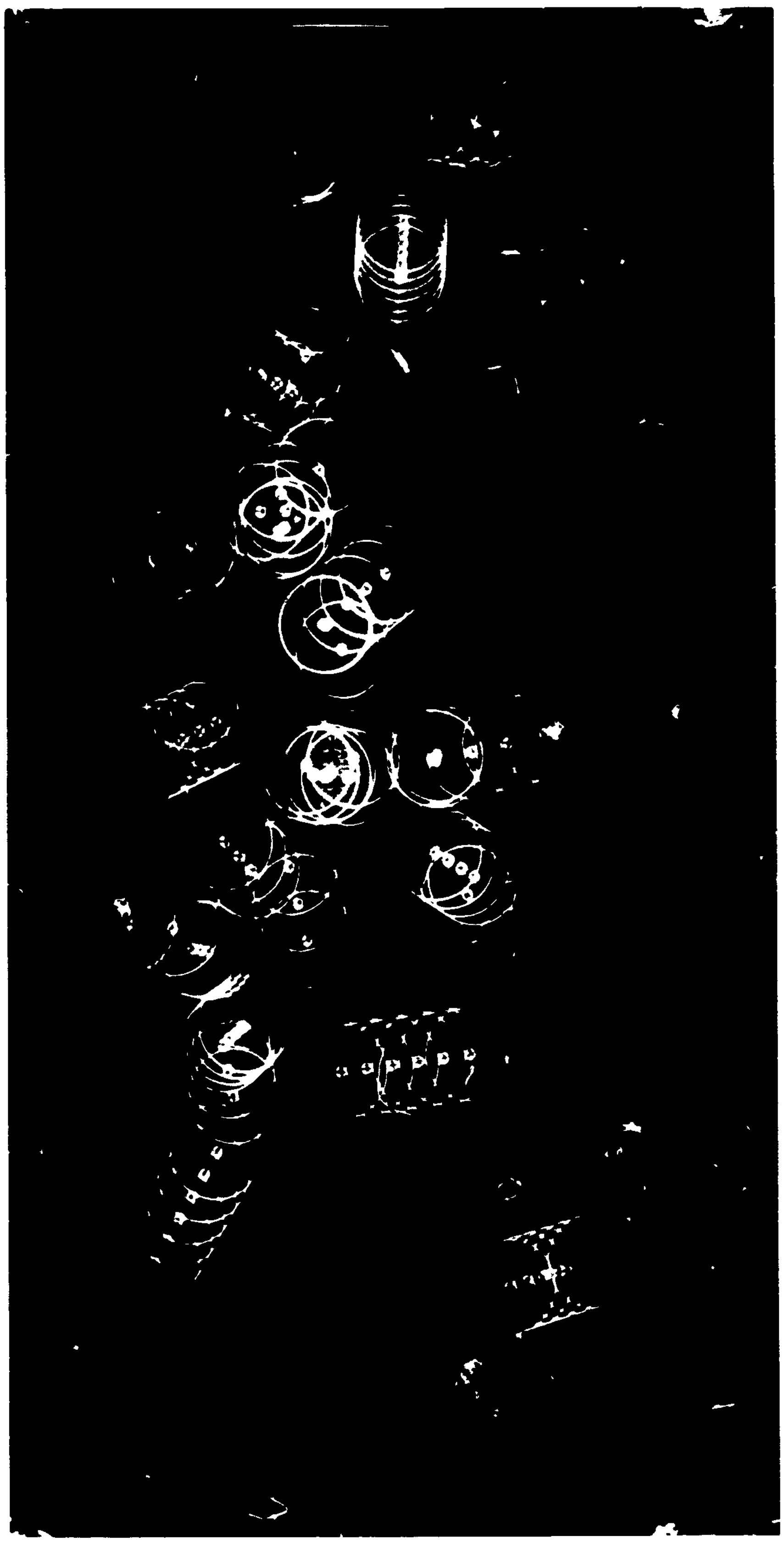


Fig. A.93.

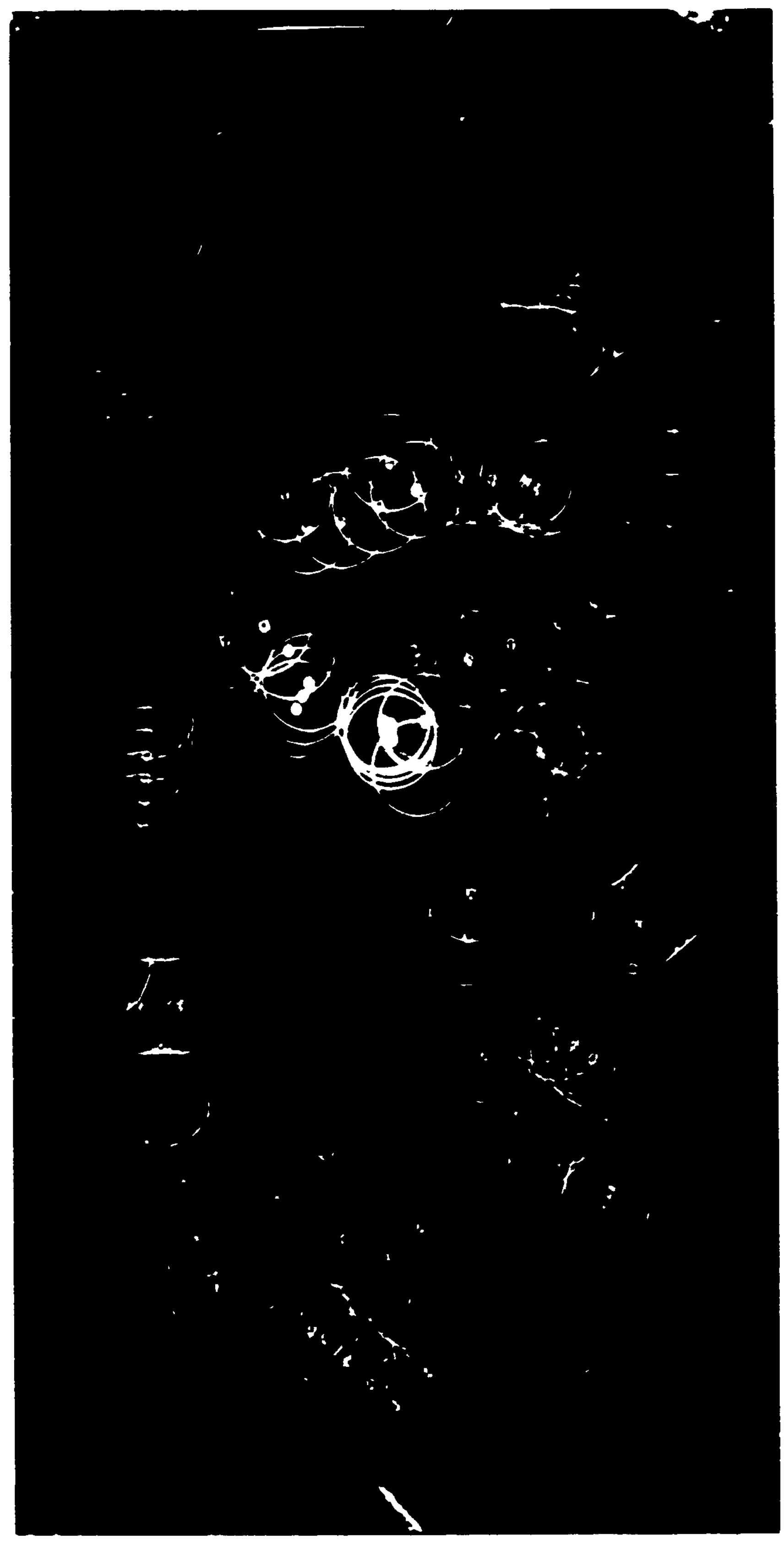


Fig. A.94.

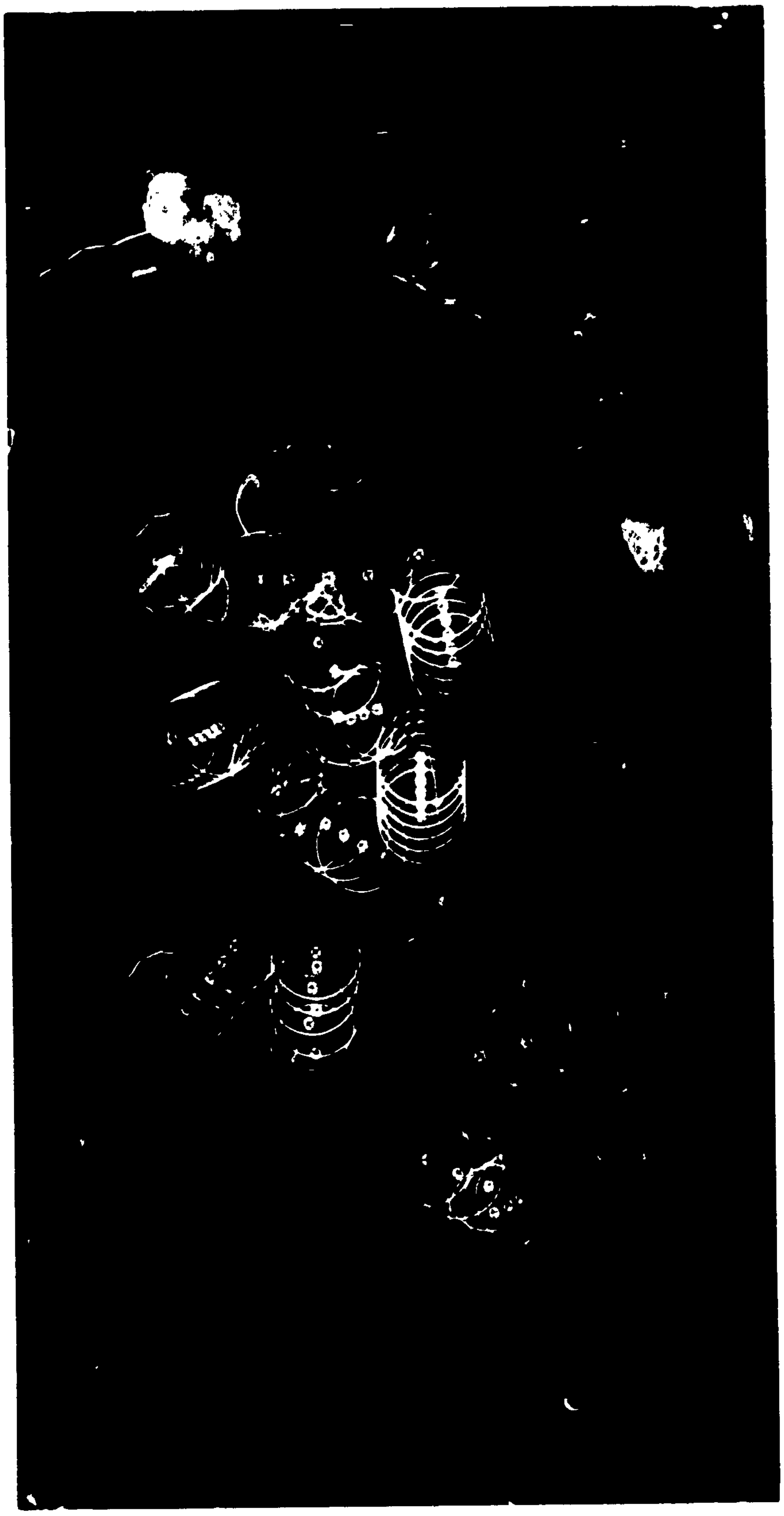


Fig. A.95.

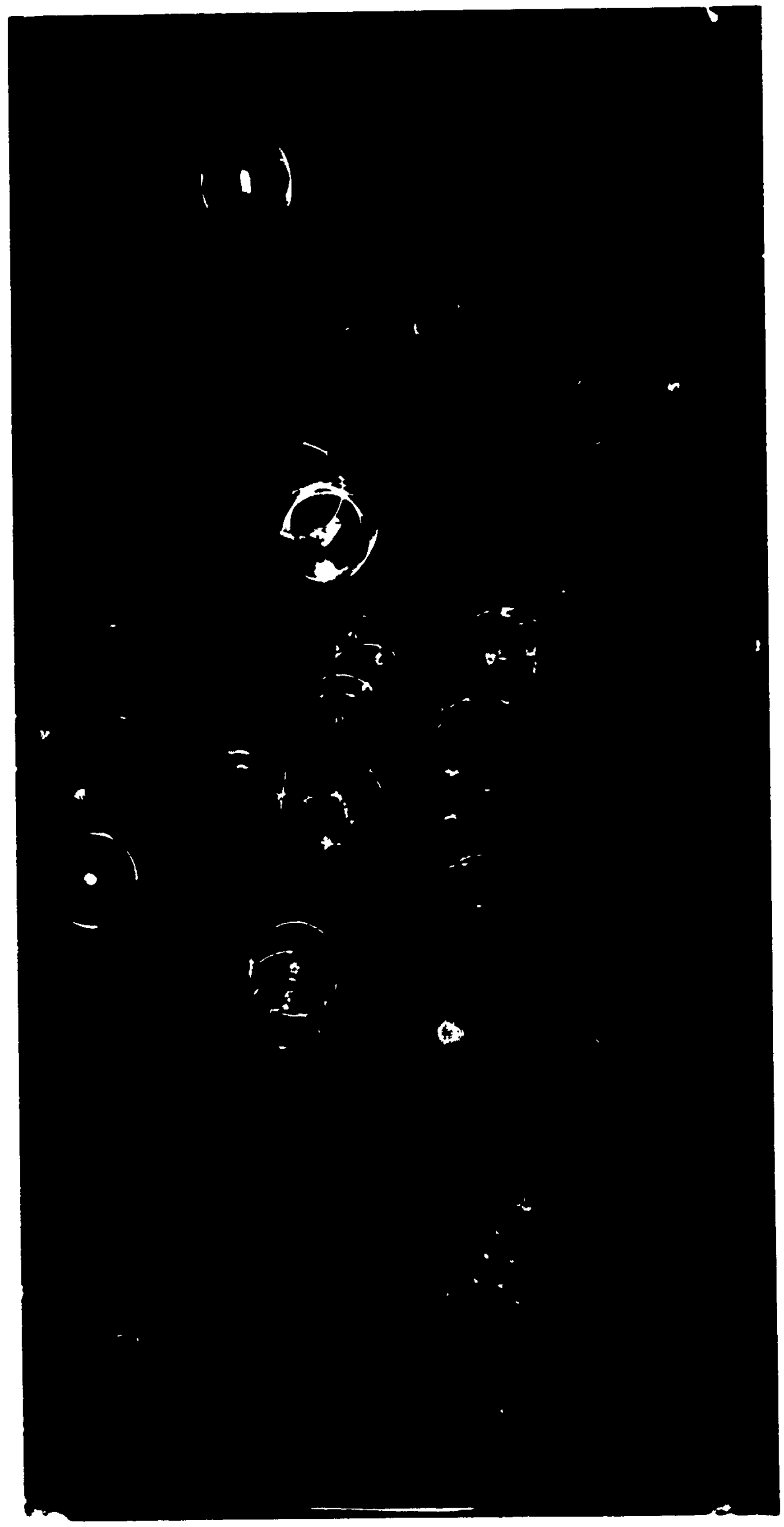


Fig. A.96.

Fig. A.97.

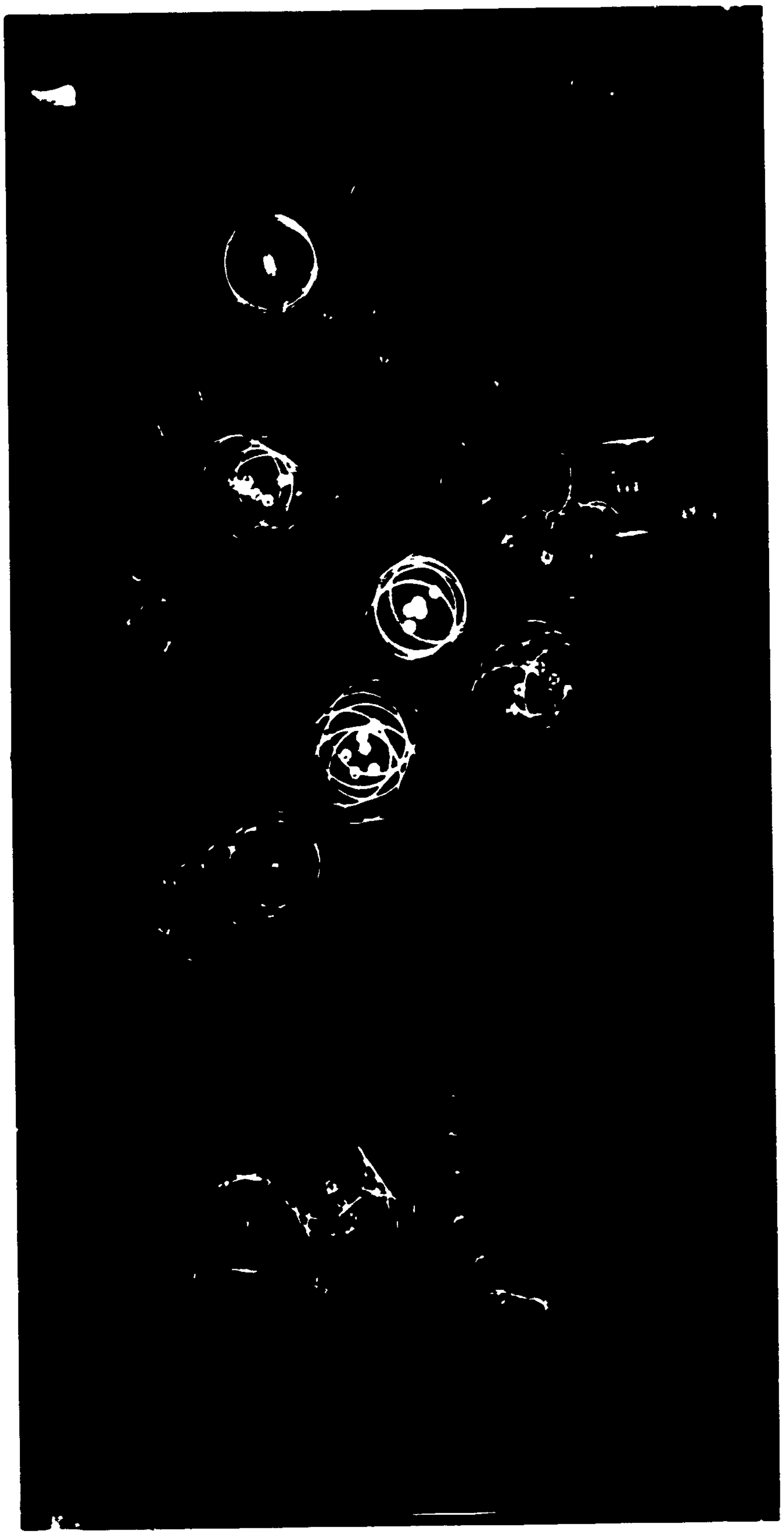


Fig. A.98.

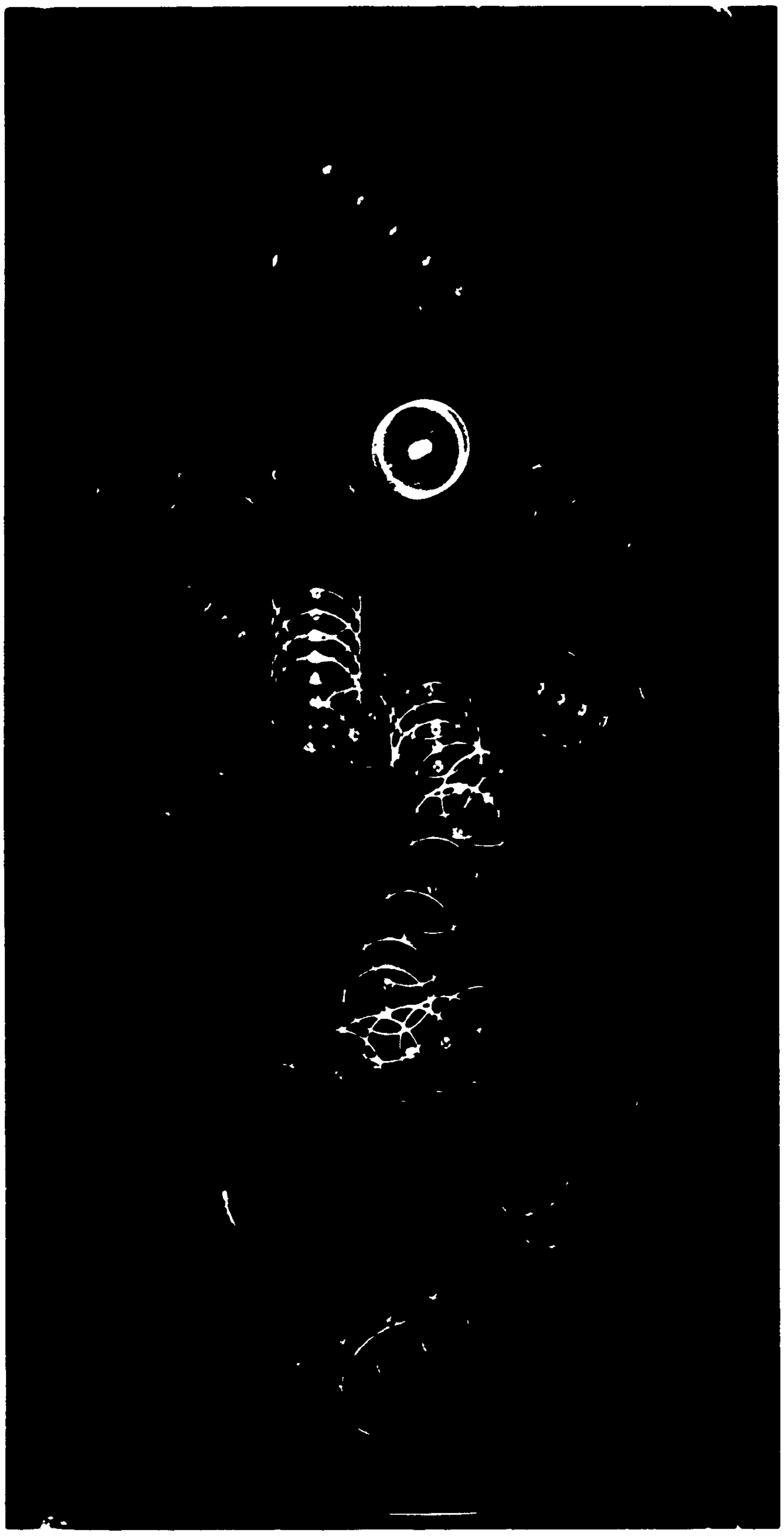


Fig. A.99.

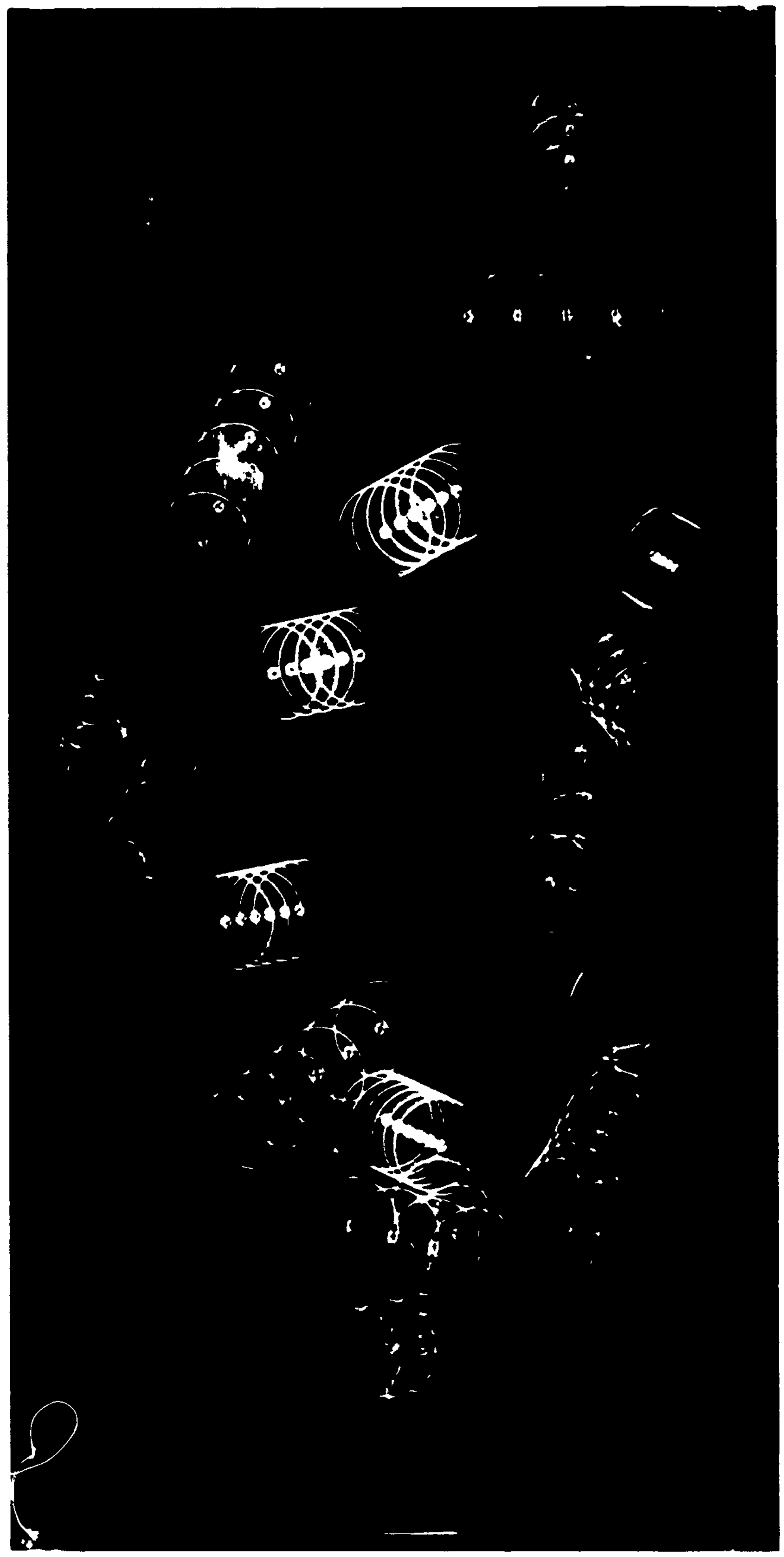


Fig. A.100.

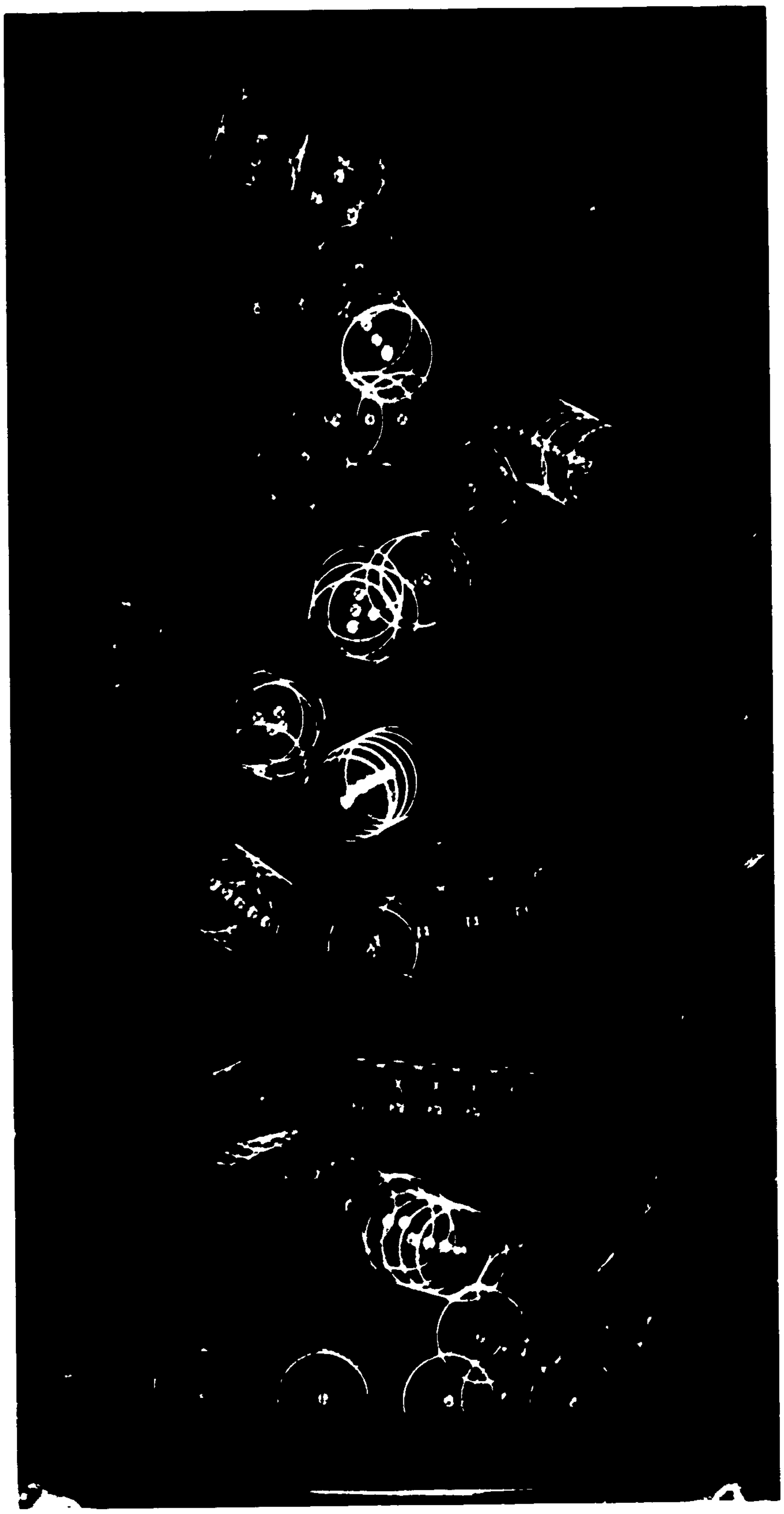


Fig. A.101.

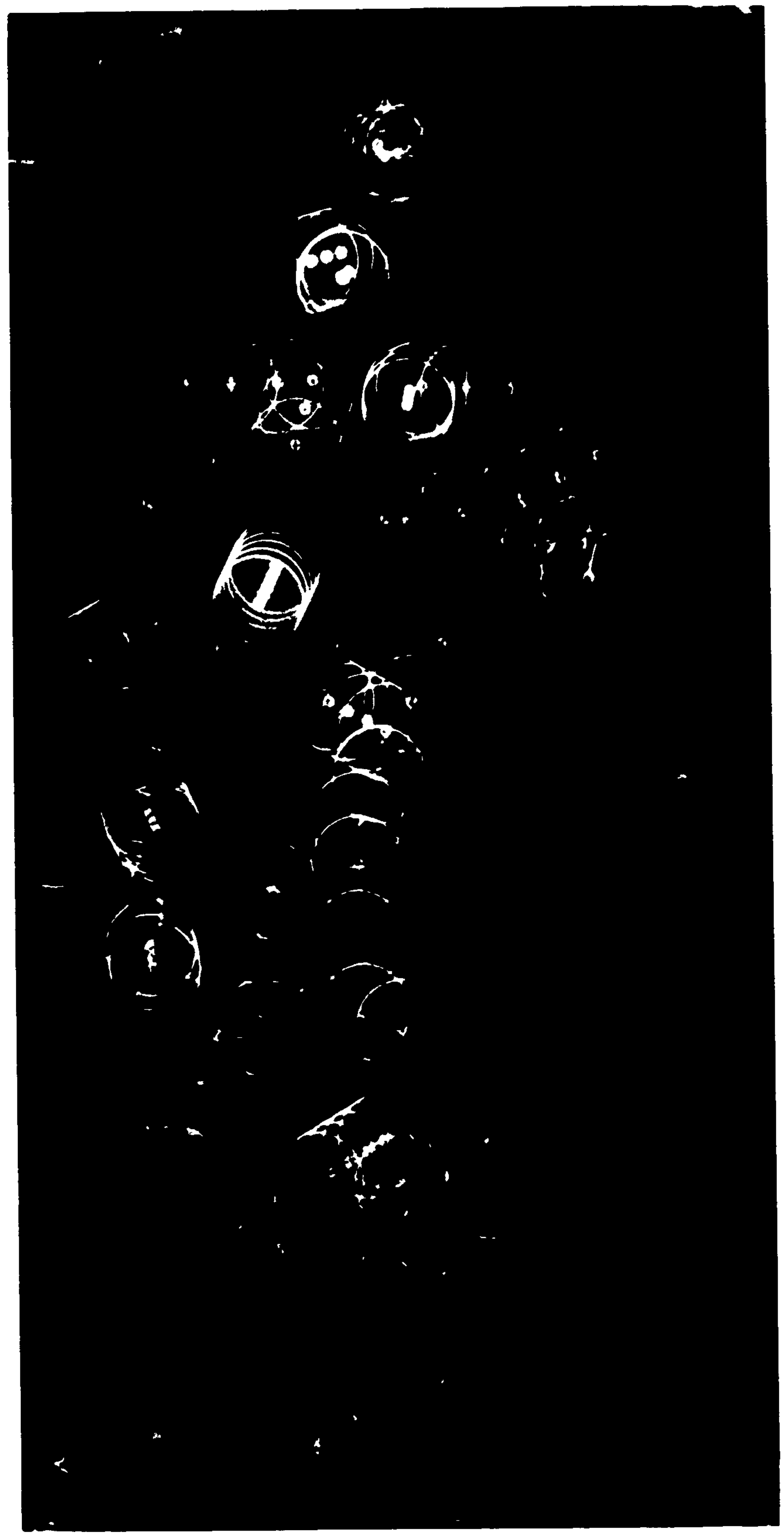


Fig. A.102.

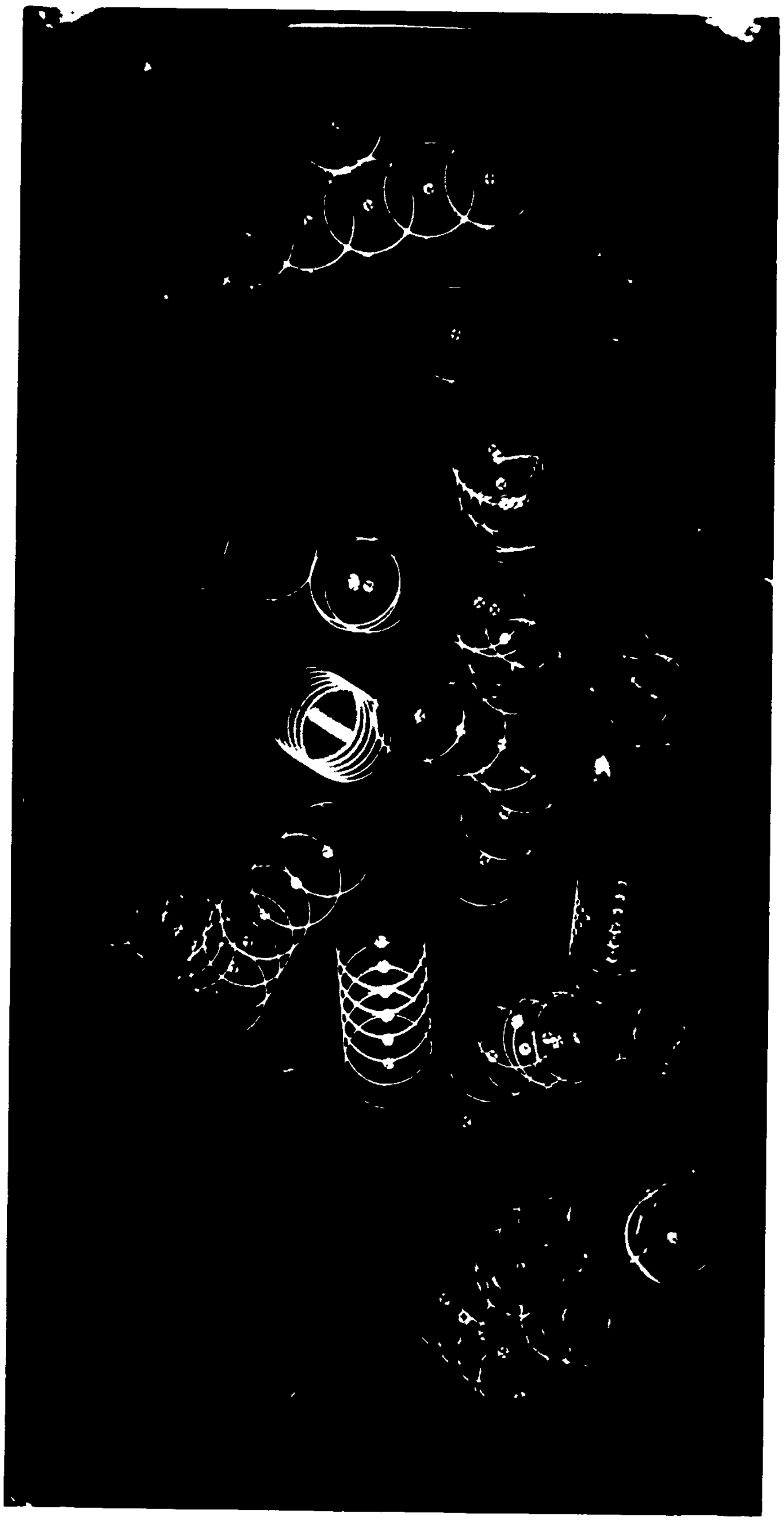


Fig. A.103.

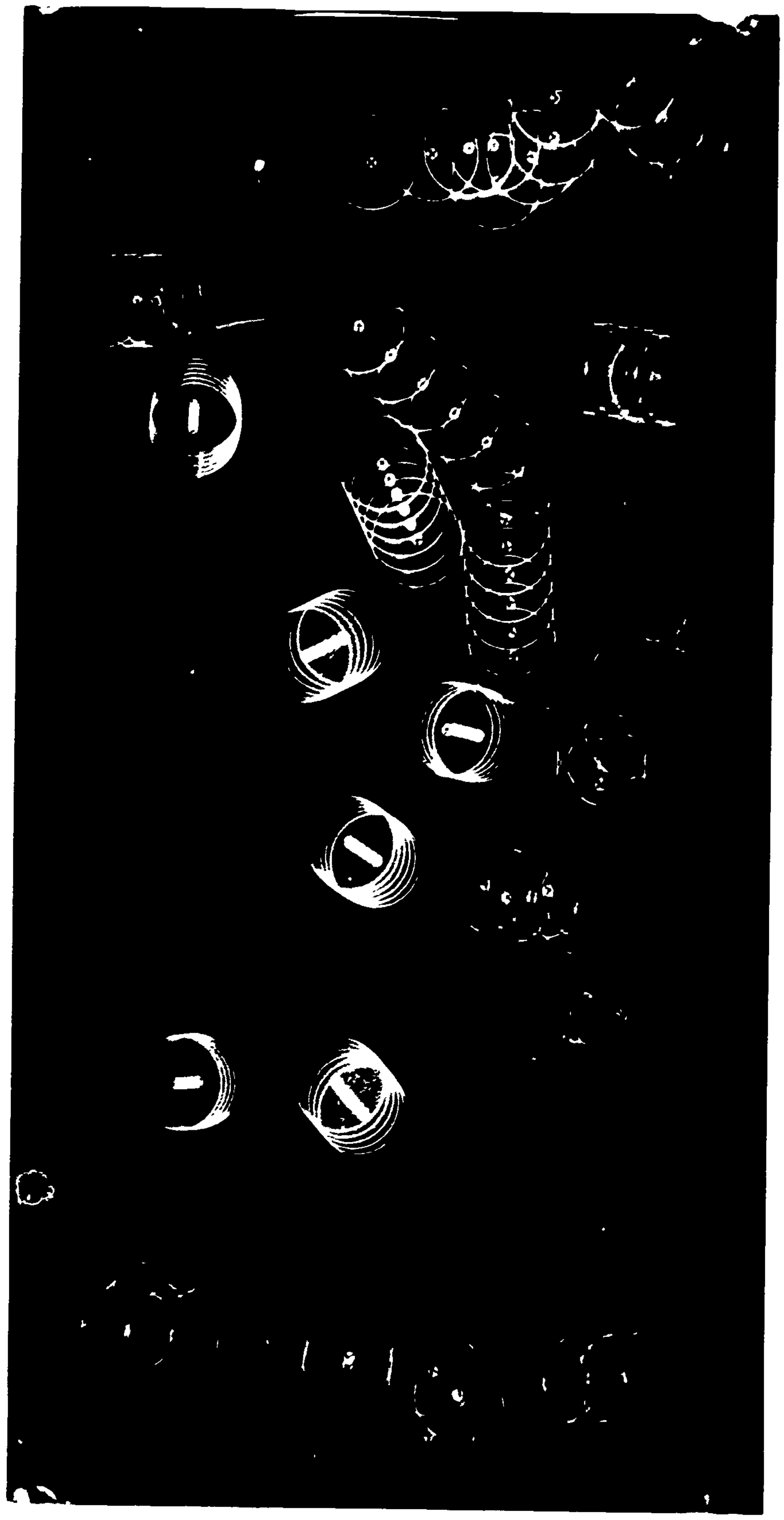


Fig. A.104.

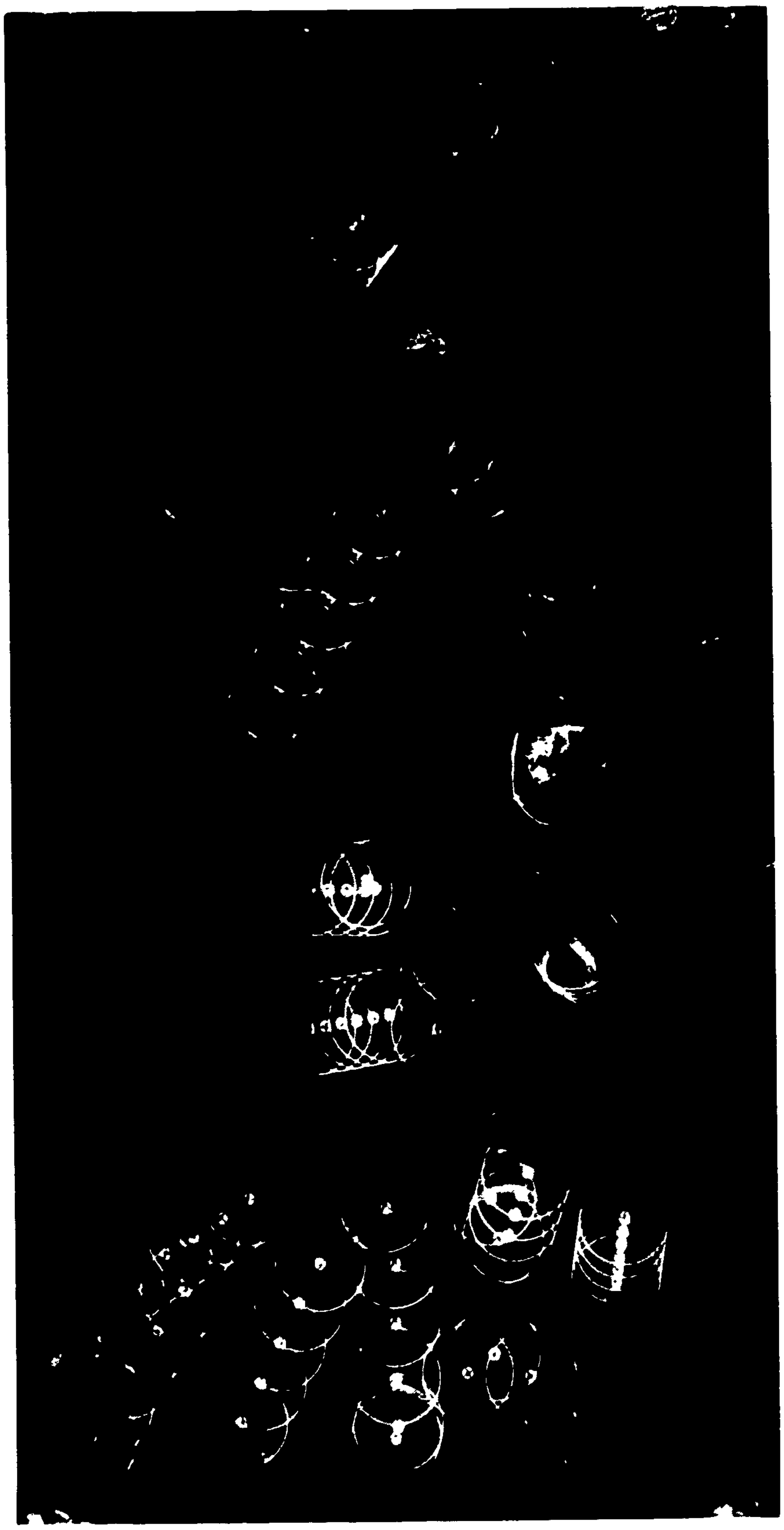


Fig. A.105.

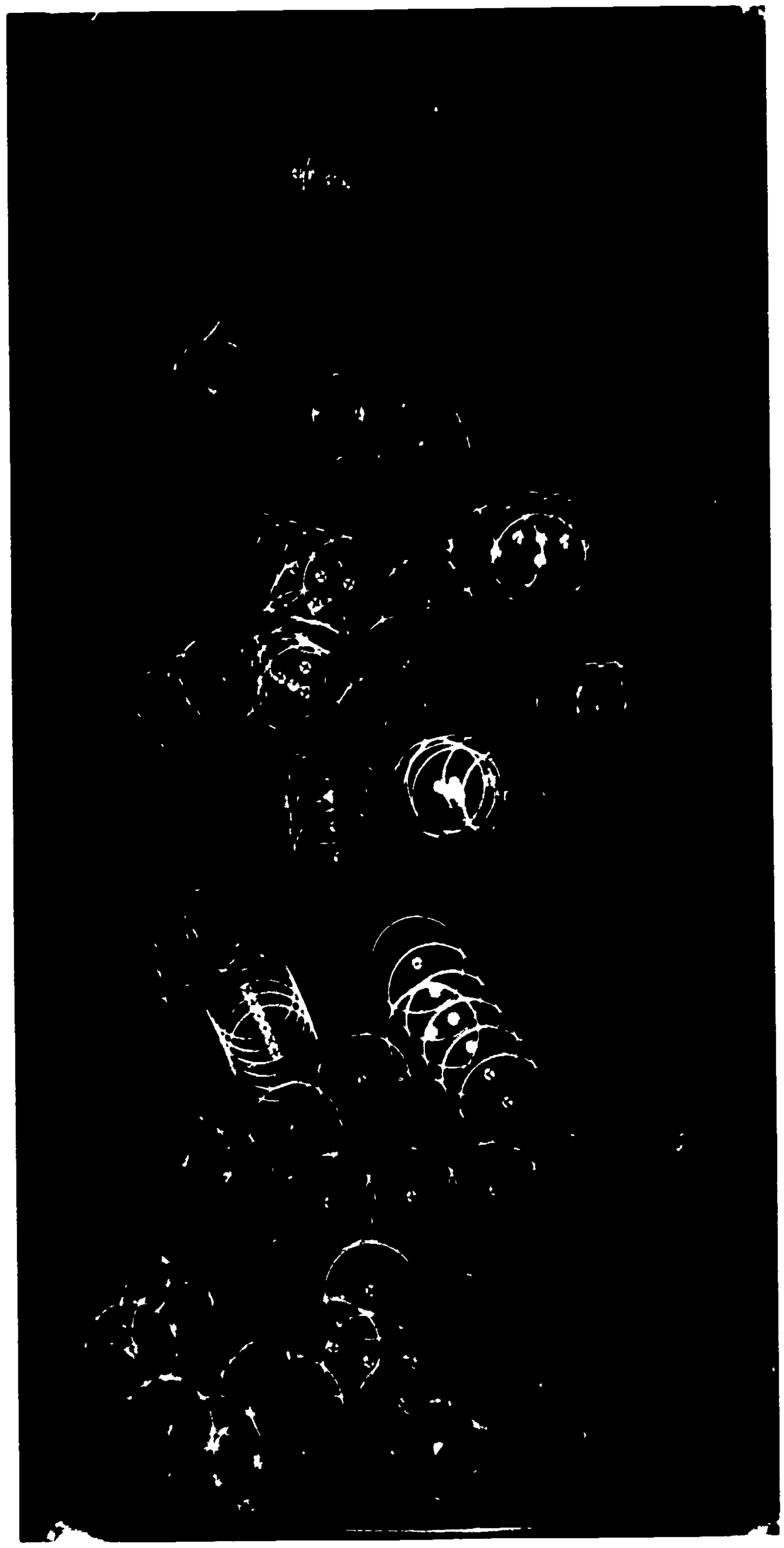


Fig. A.106.

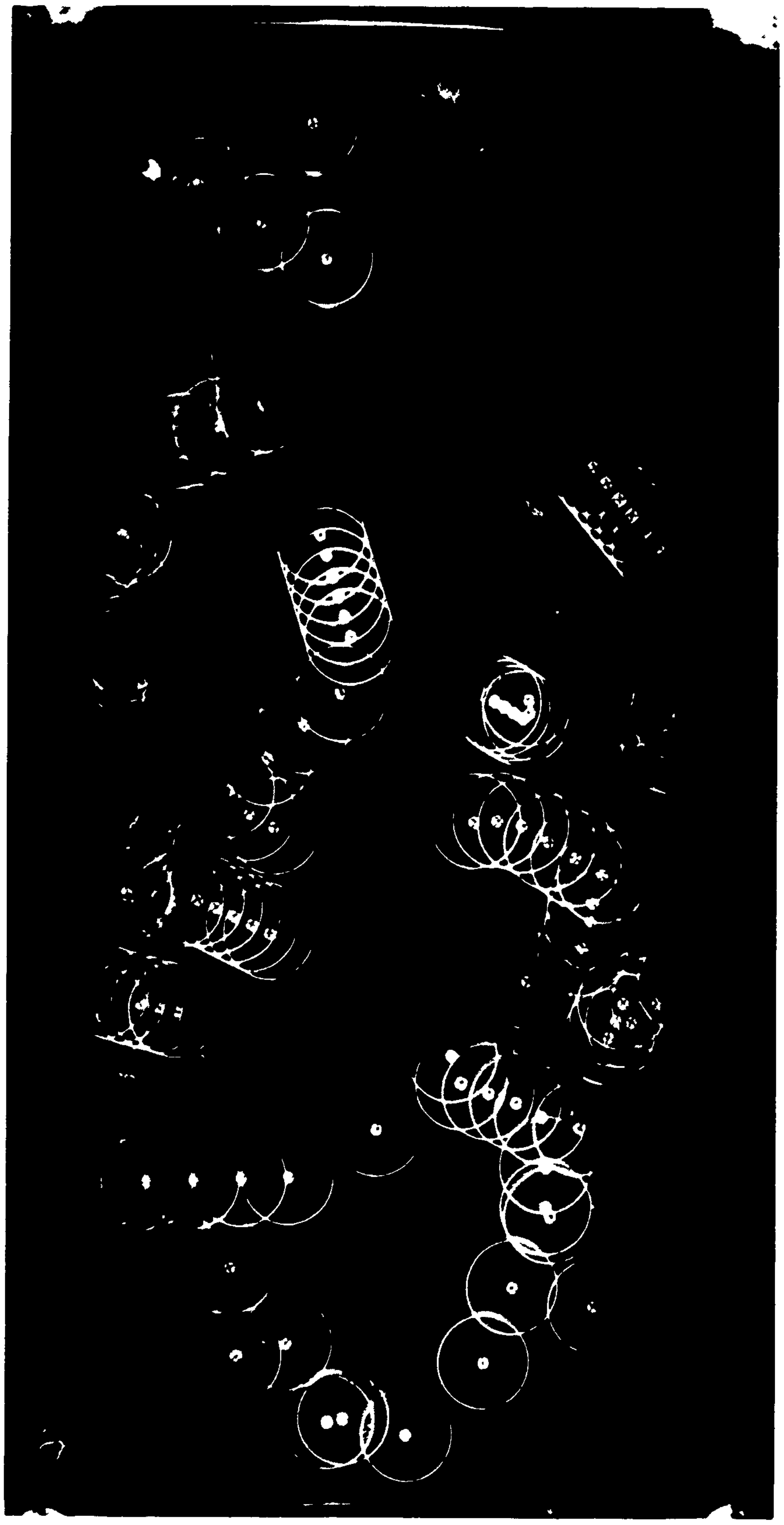


Fig. A.107.

Fig. A.108.

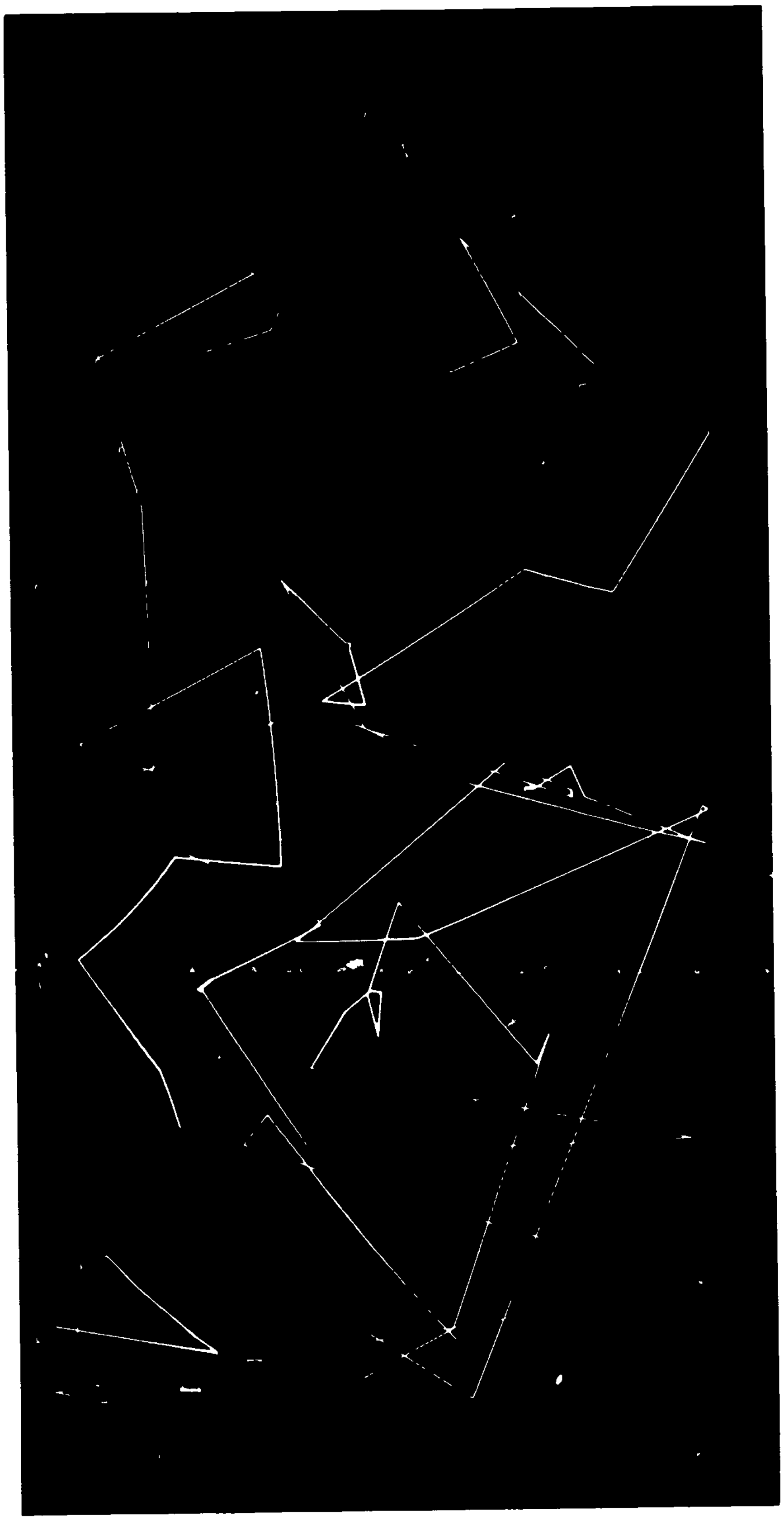


Fig. A.109.

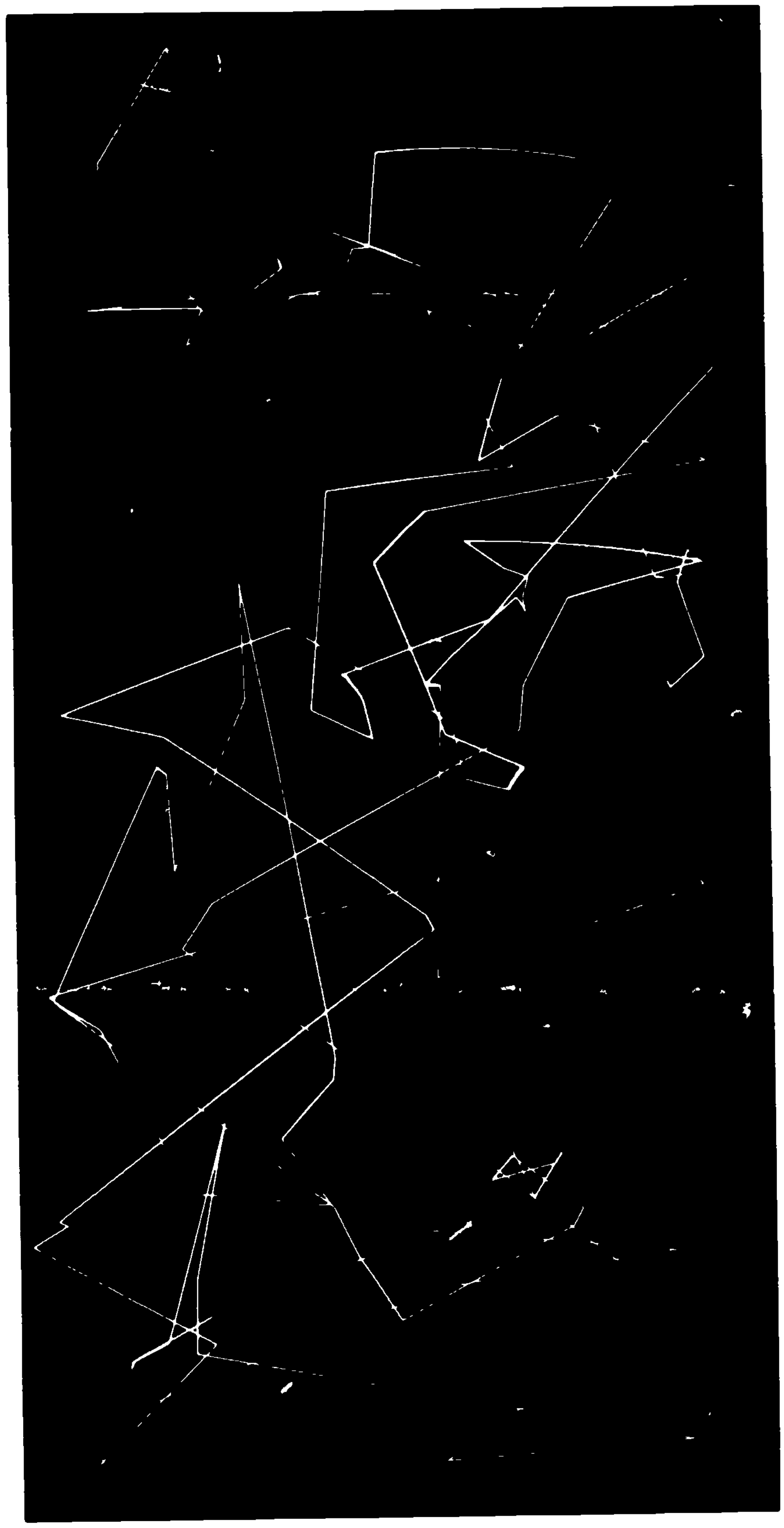


Fig. A.110.

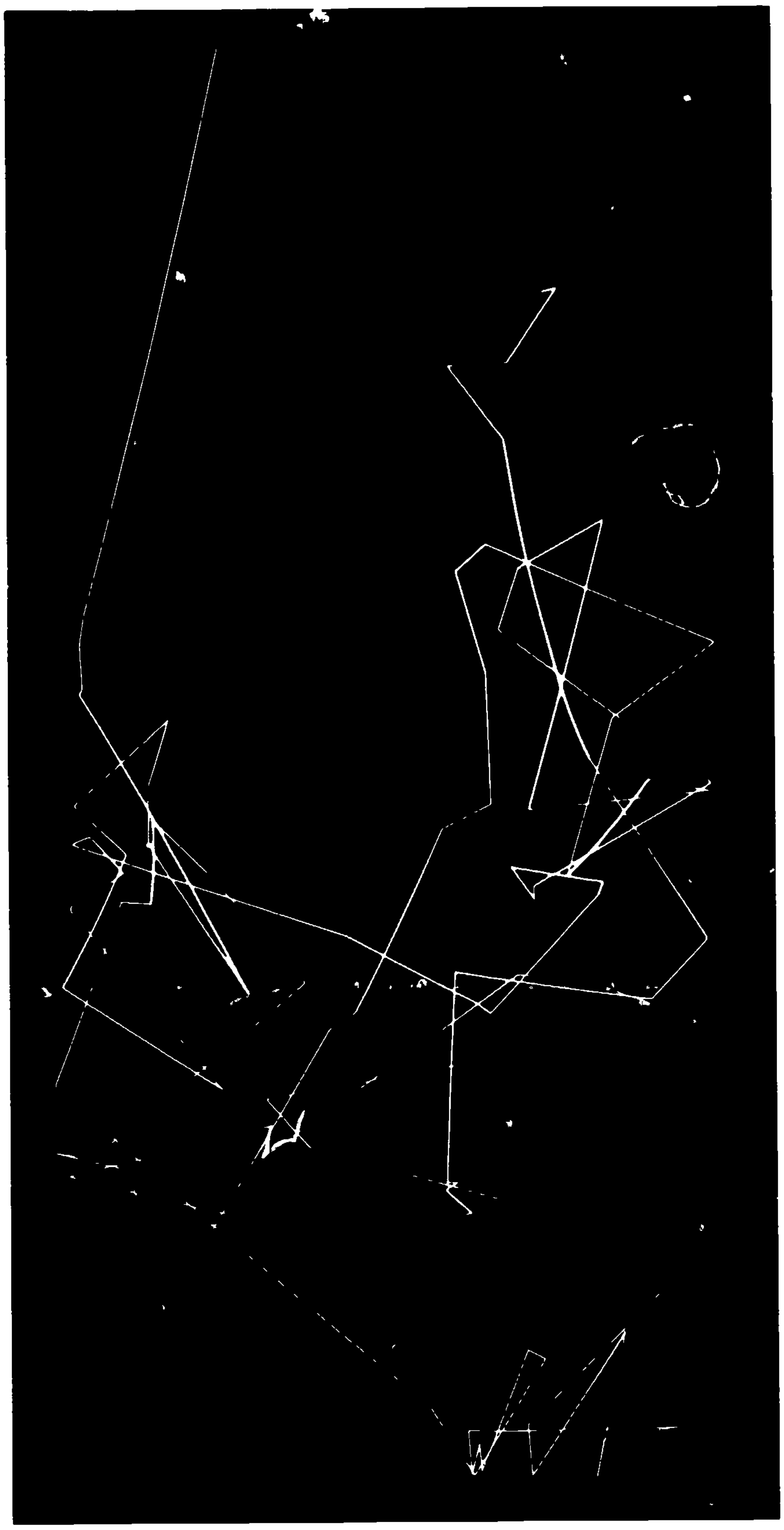


Fig. A.111.

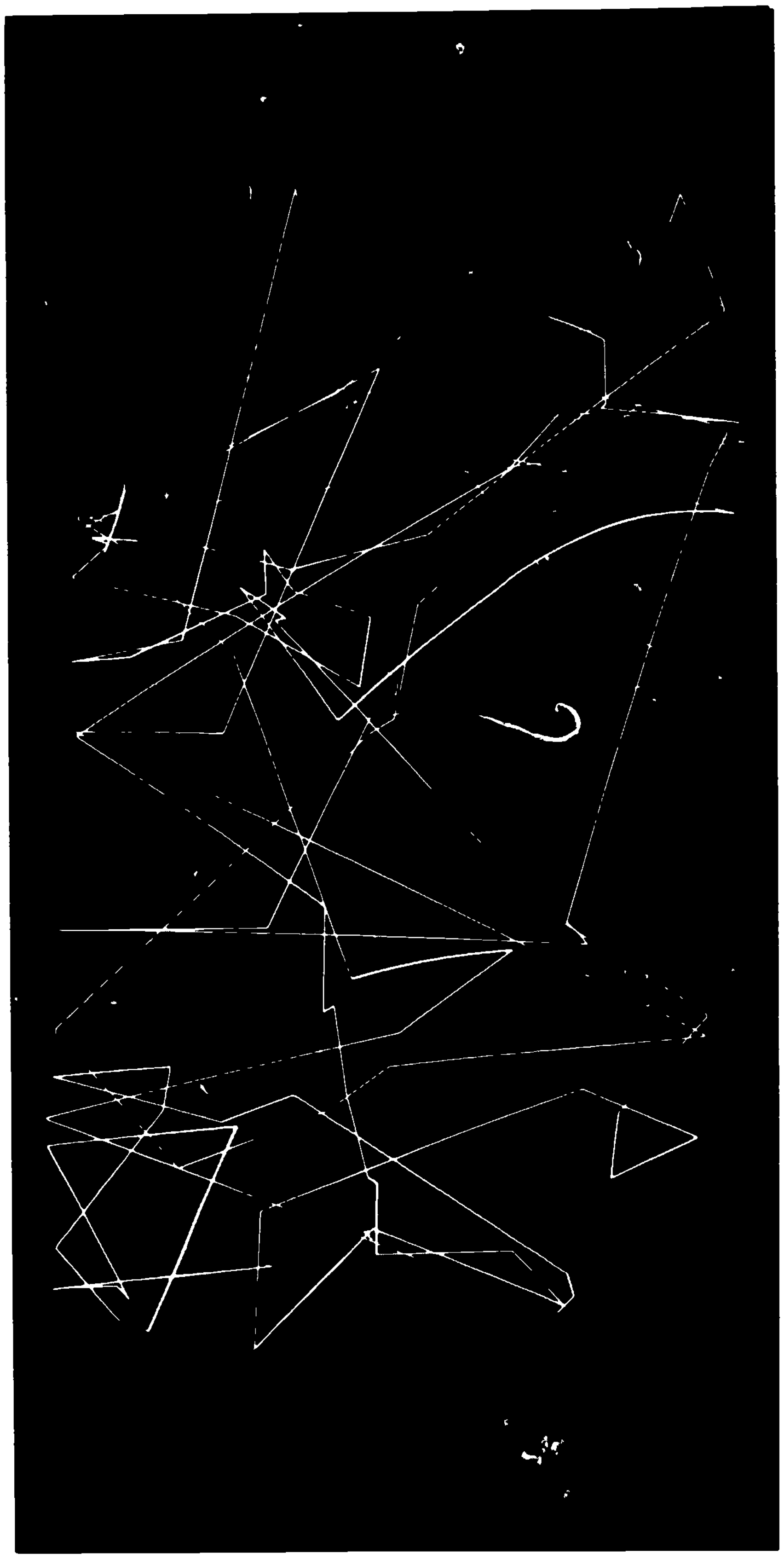


Fig. A.112.

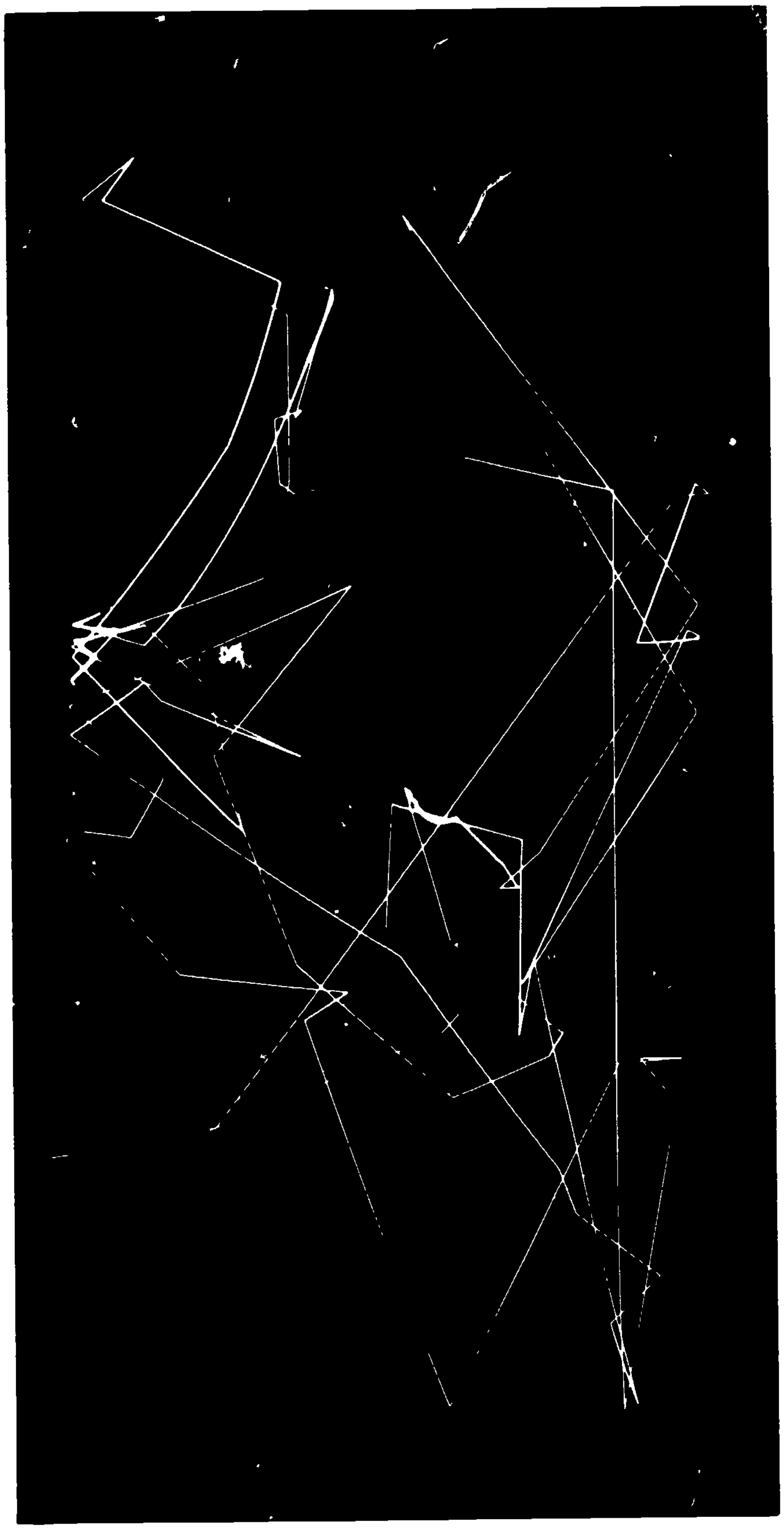


Fig. A.113.

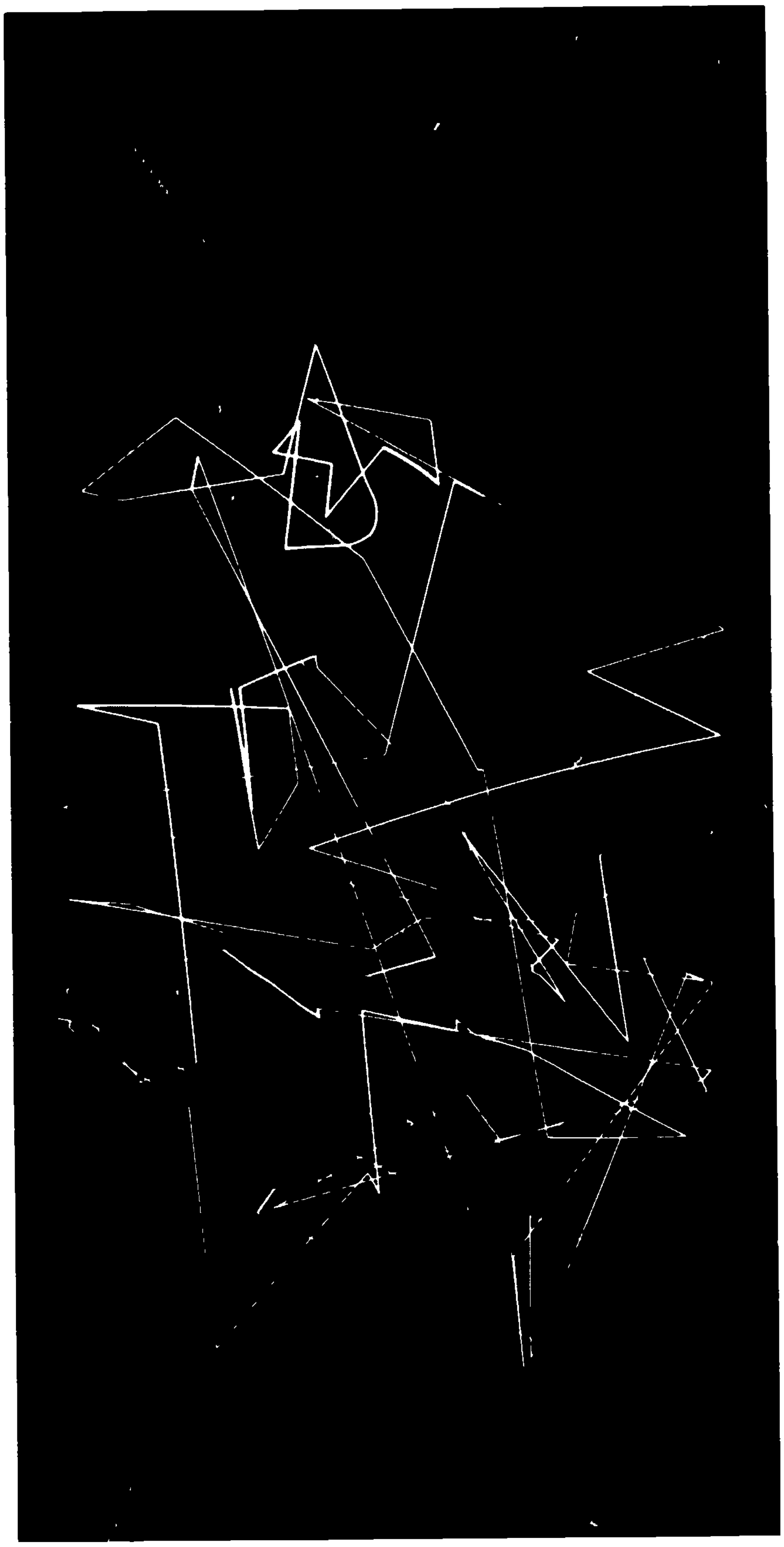


Fig. A.114.

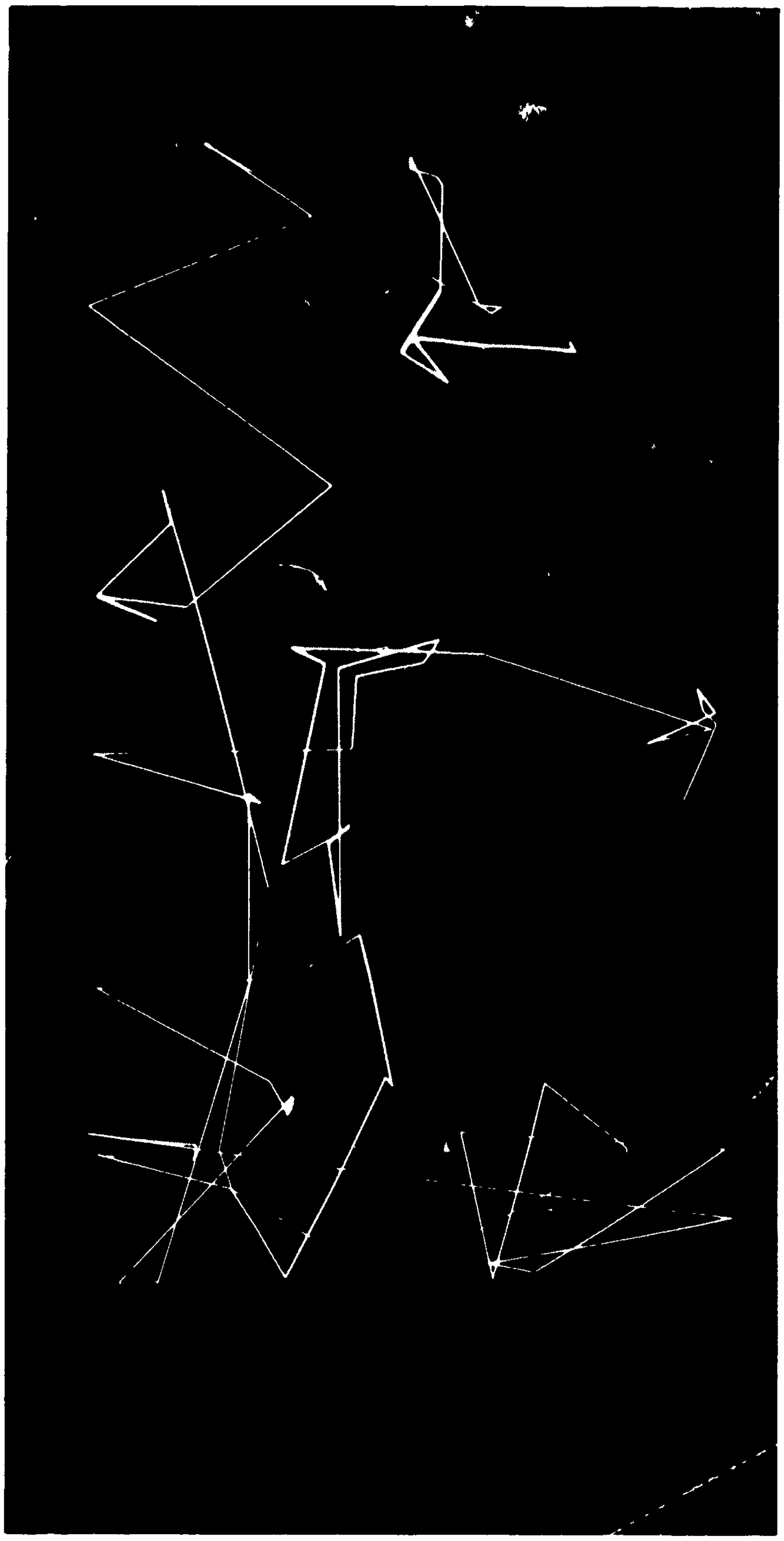


Fig. A.115.

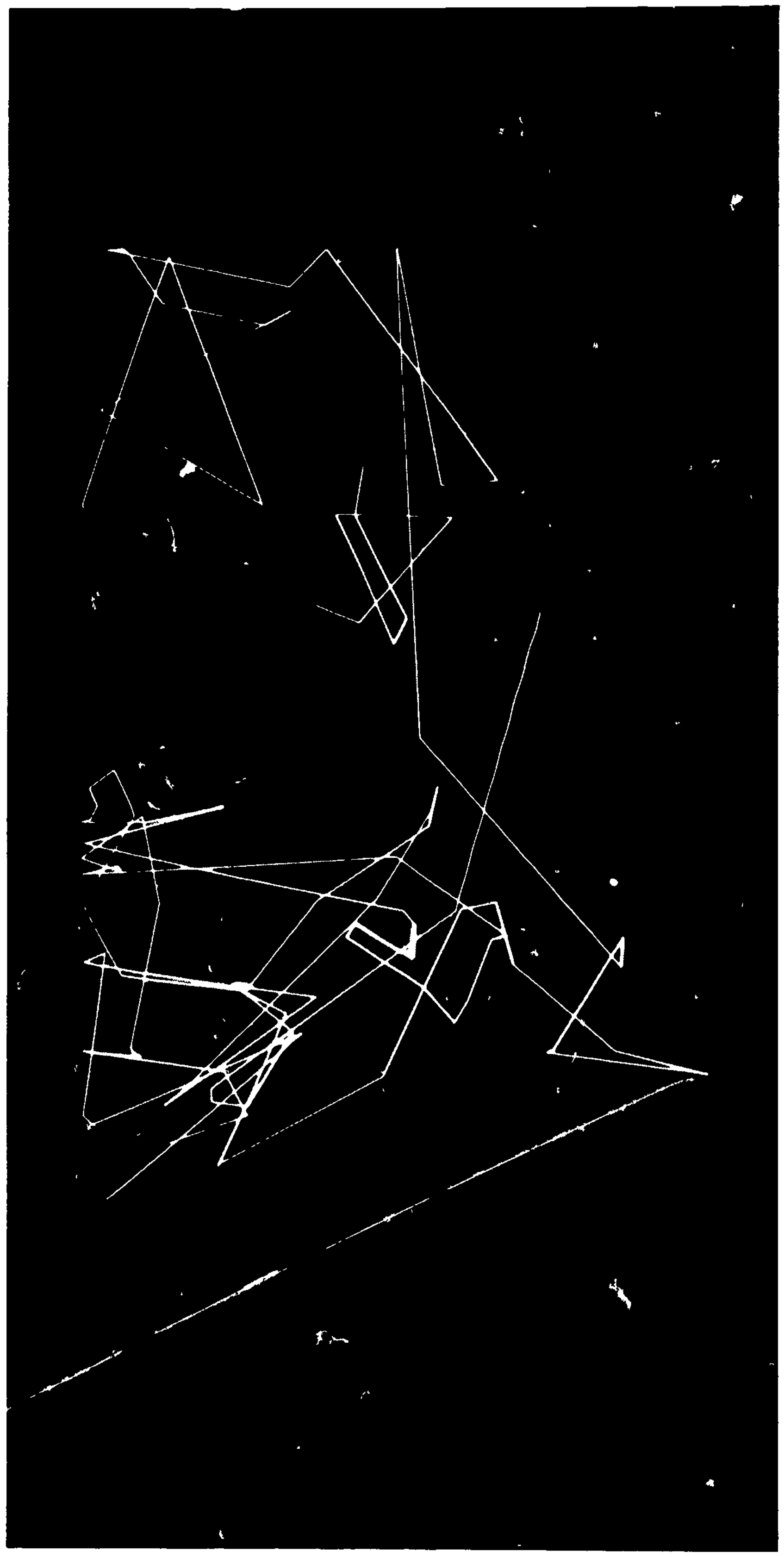


Fig. A.116.

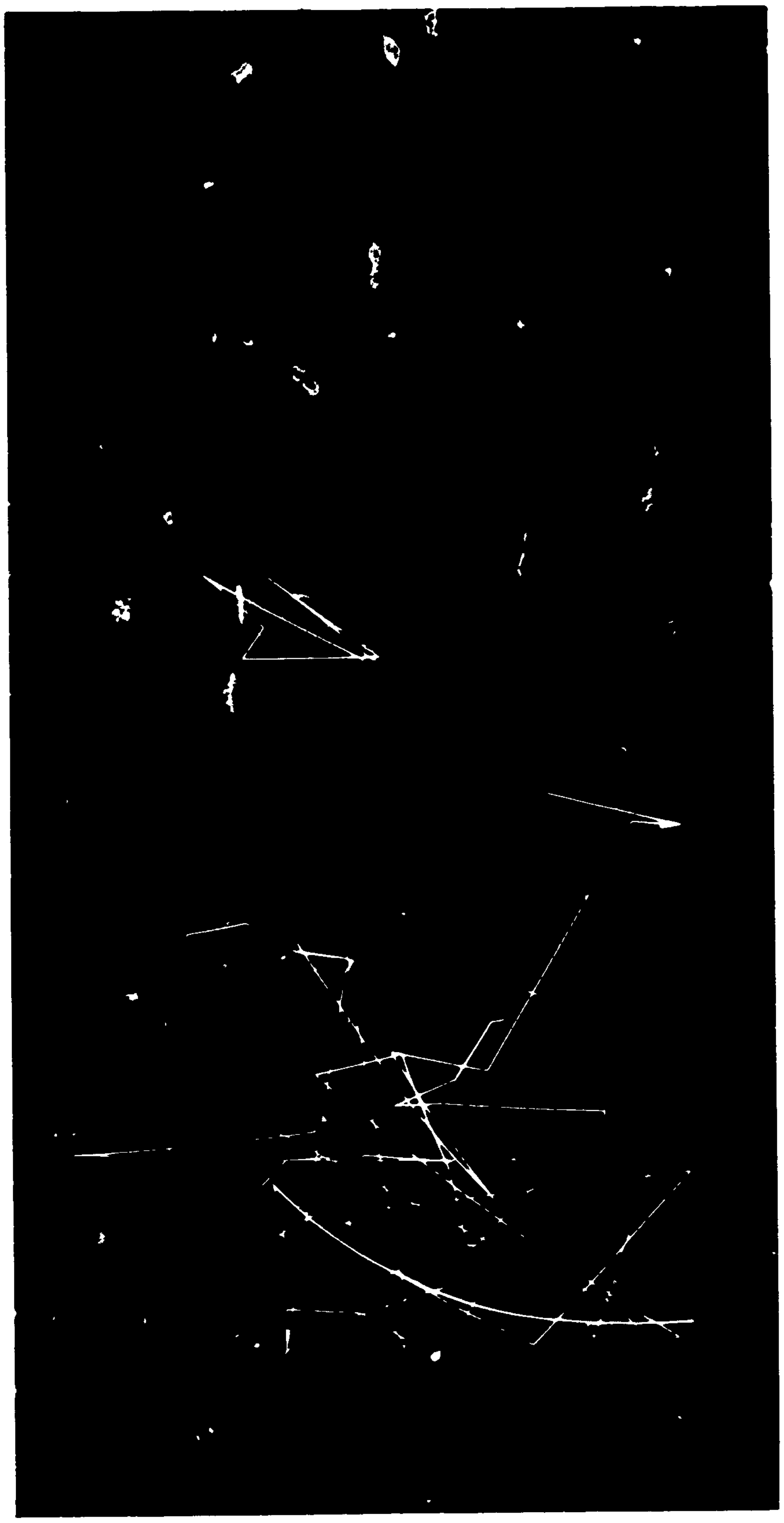


Fig. A.117.

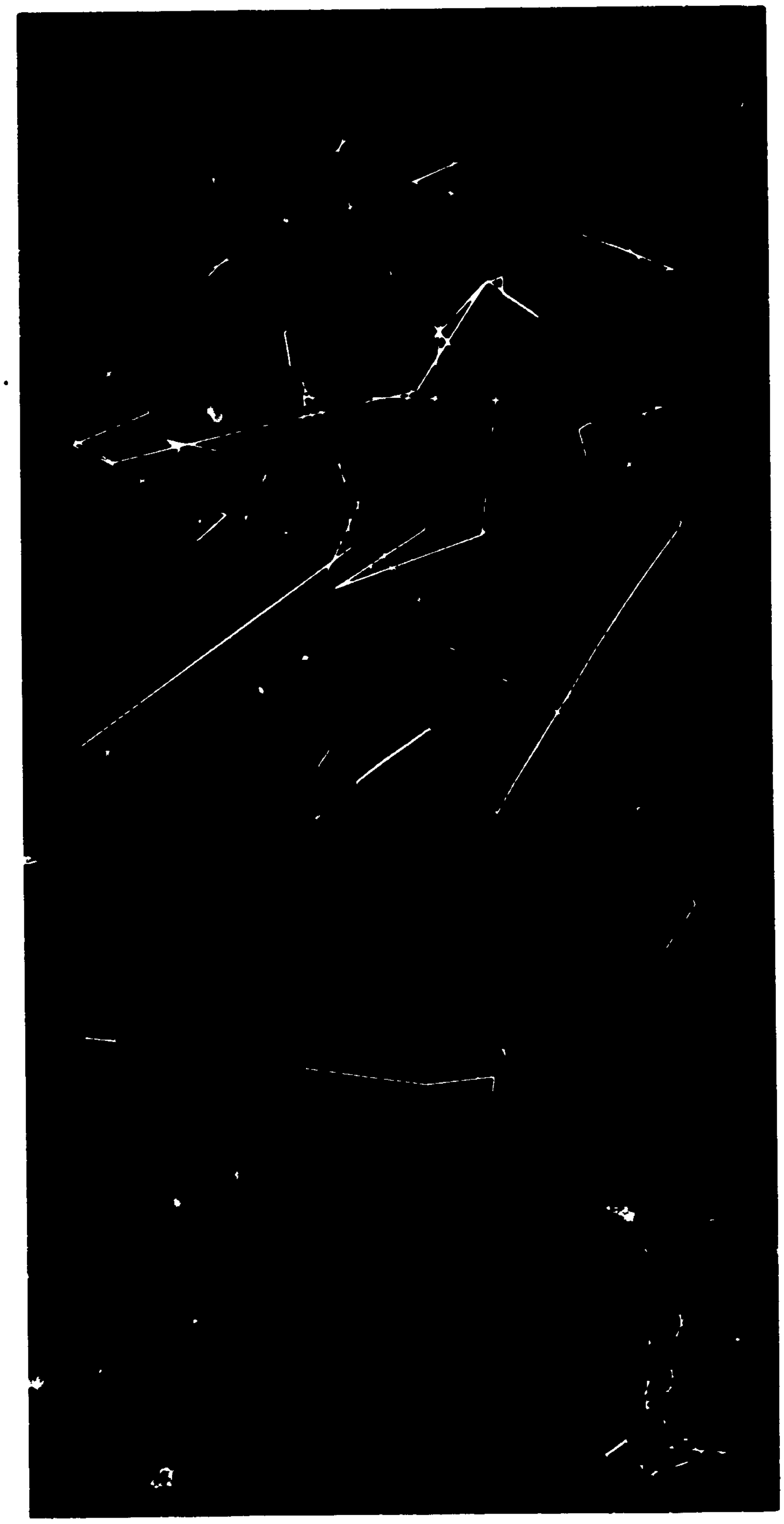


Fig. A.118.

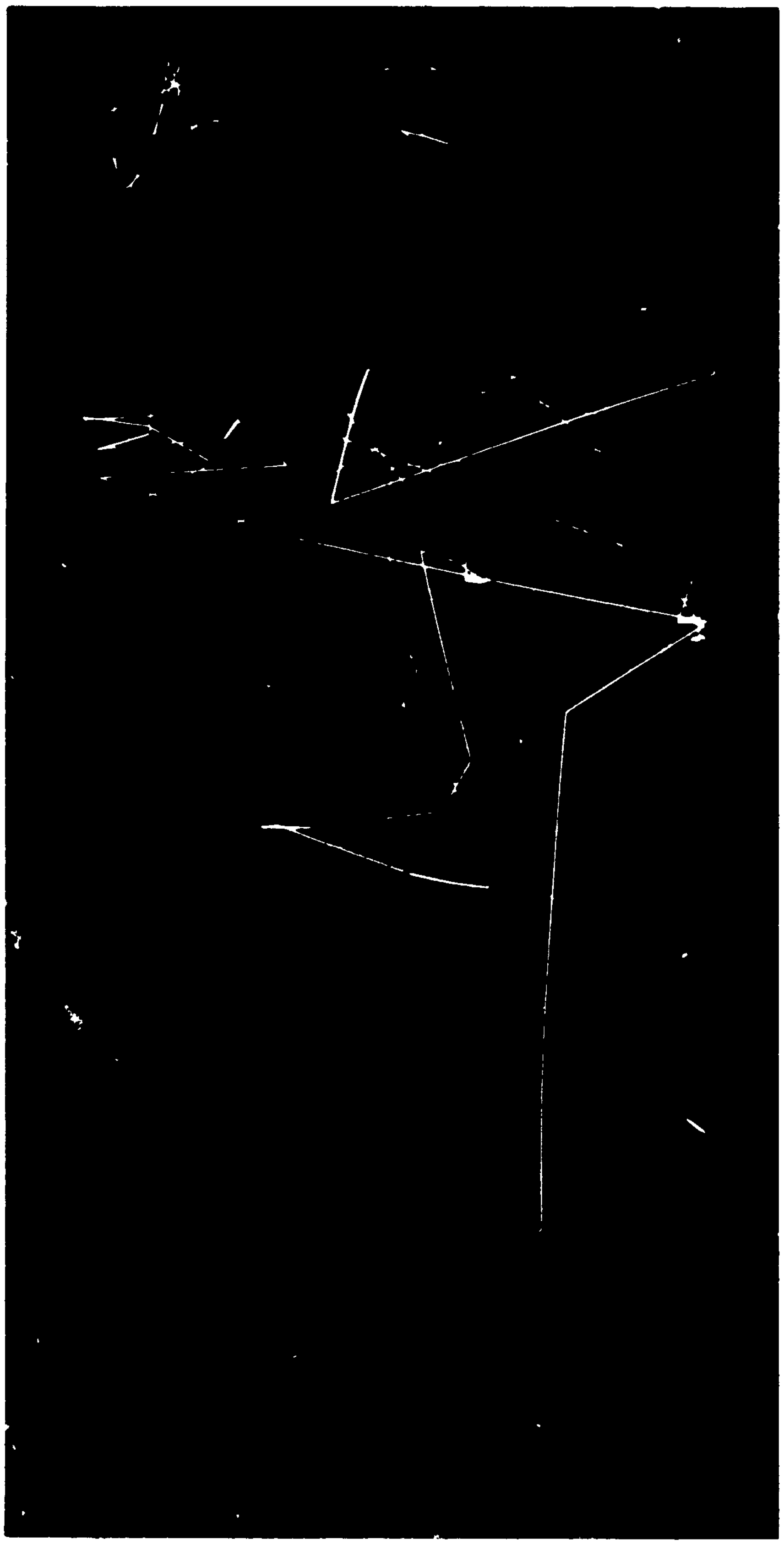


Fig. A.119.

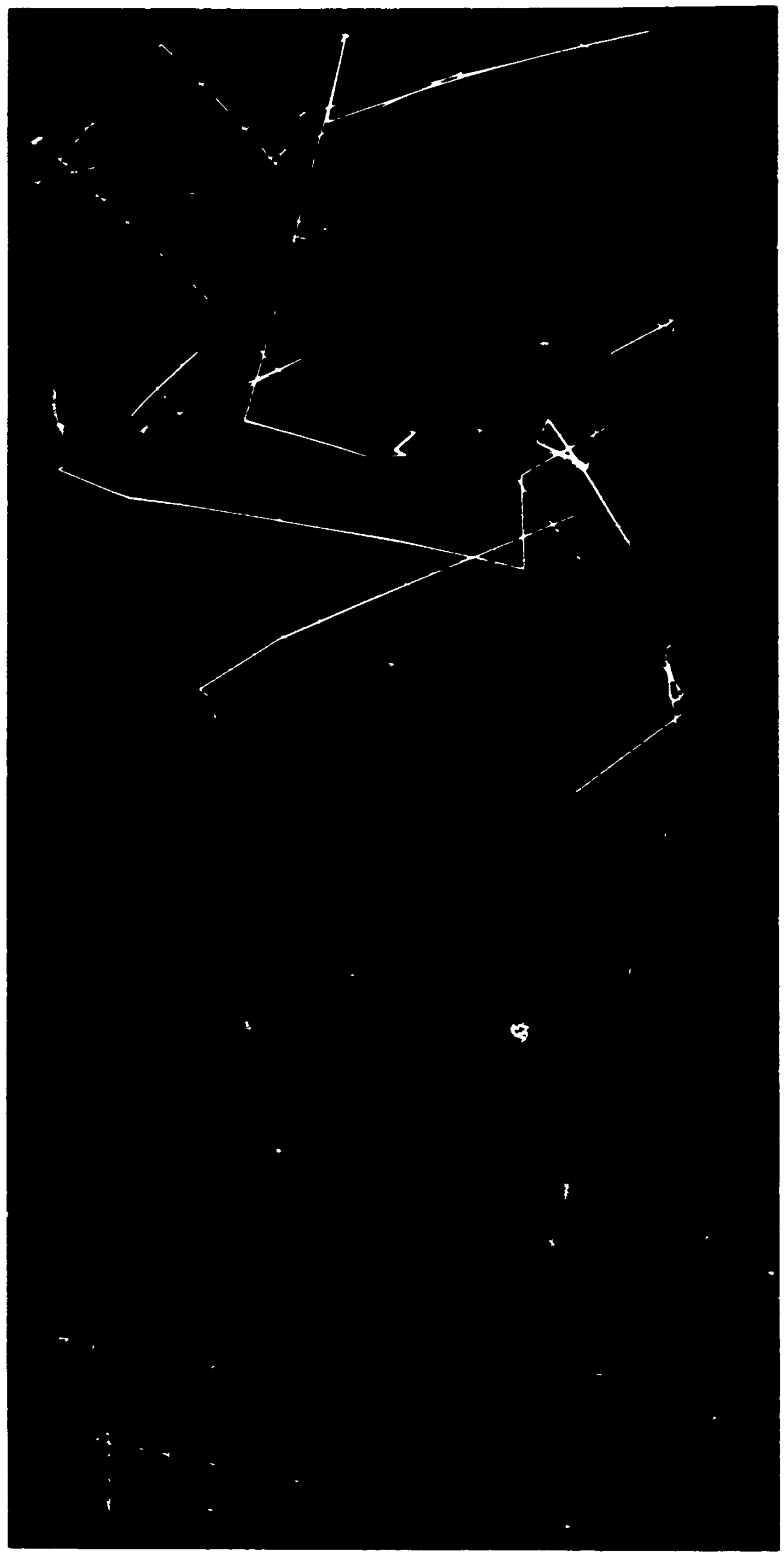


Fig. A.120.

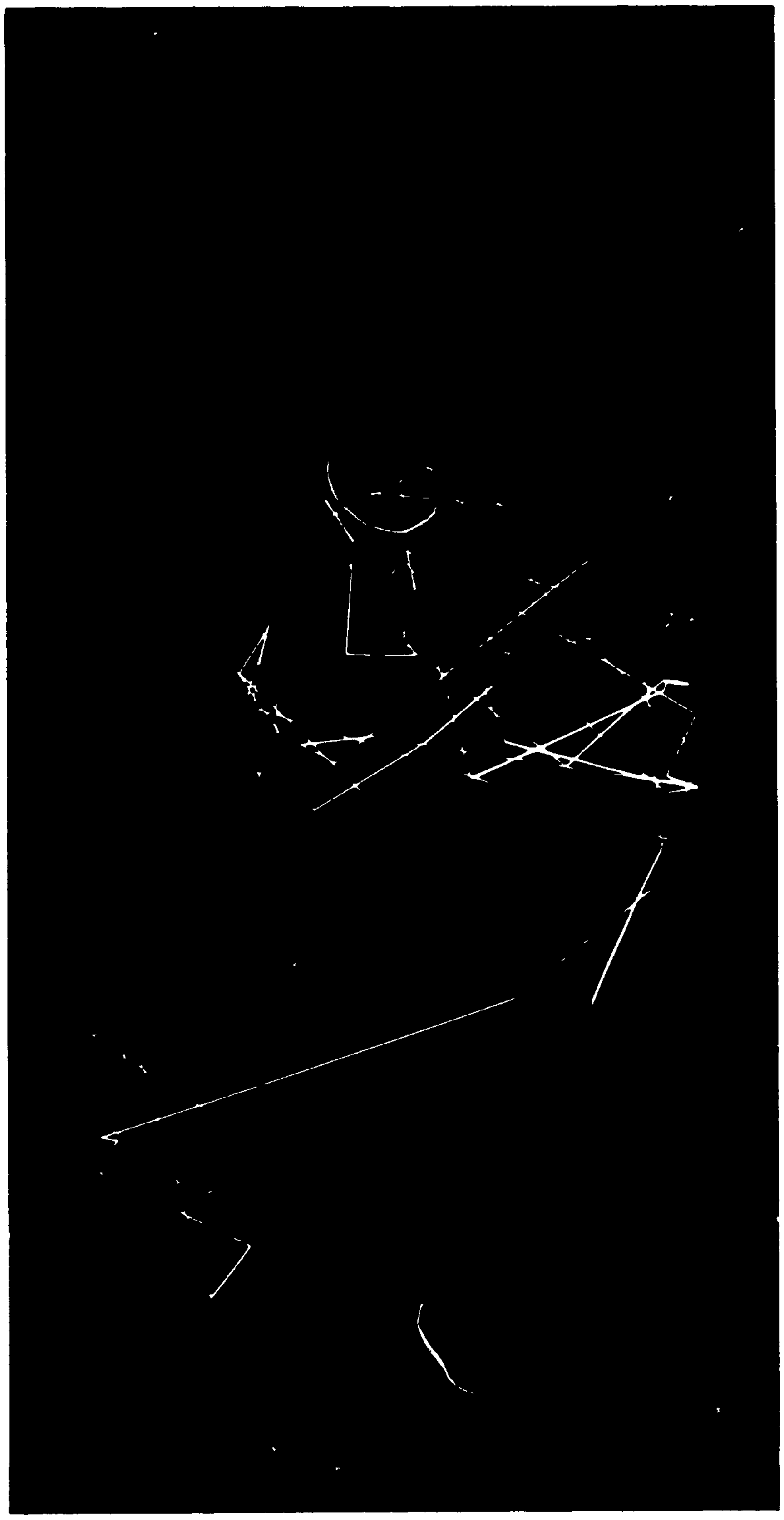


Fig. A.121.

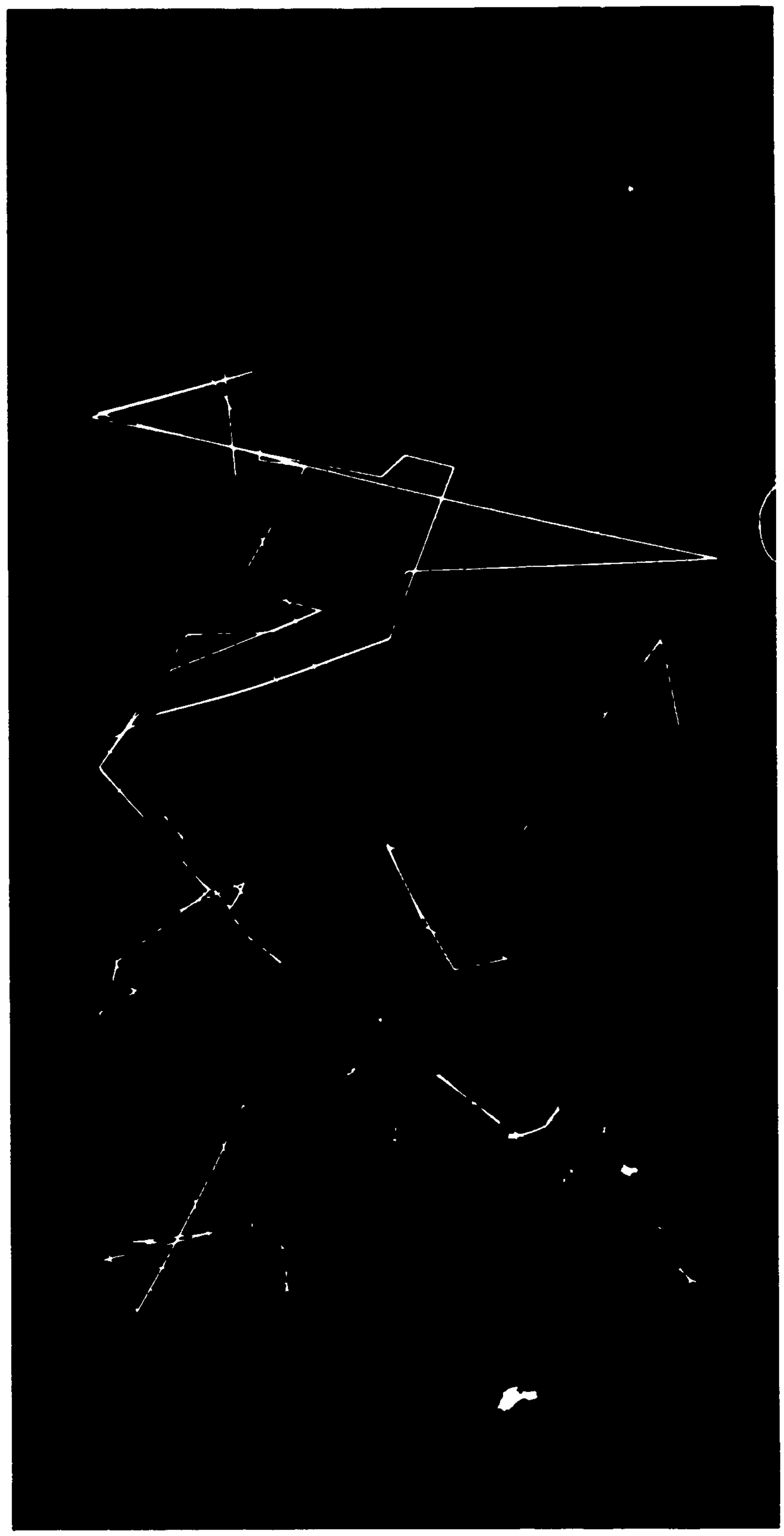


Fig. A.122.

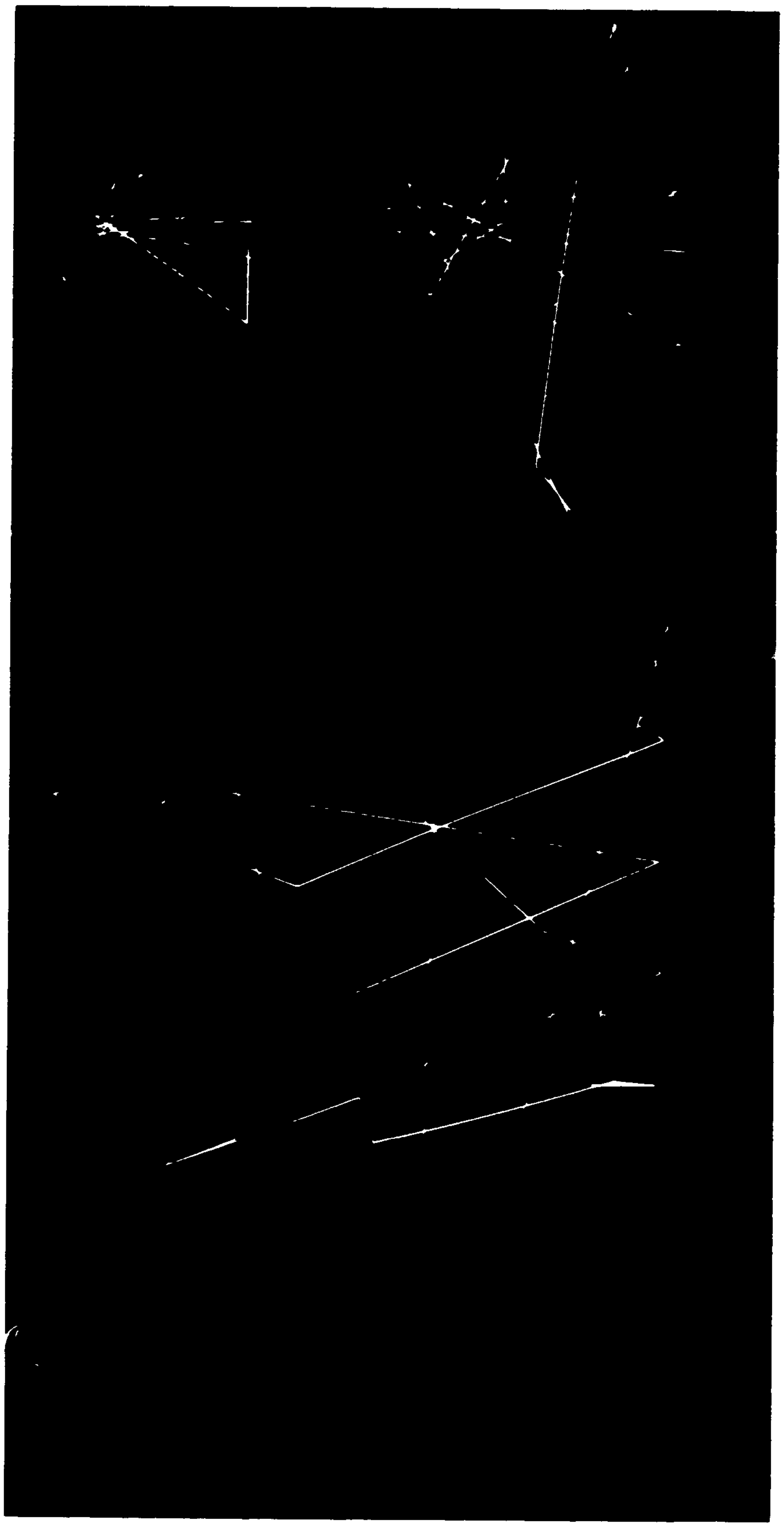


Fig. A.123.

Fig. A.124.

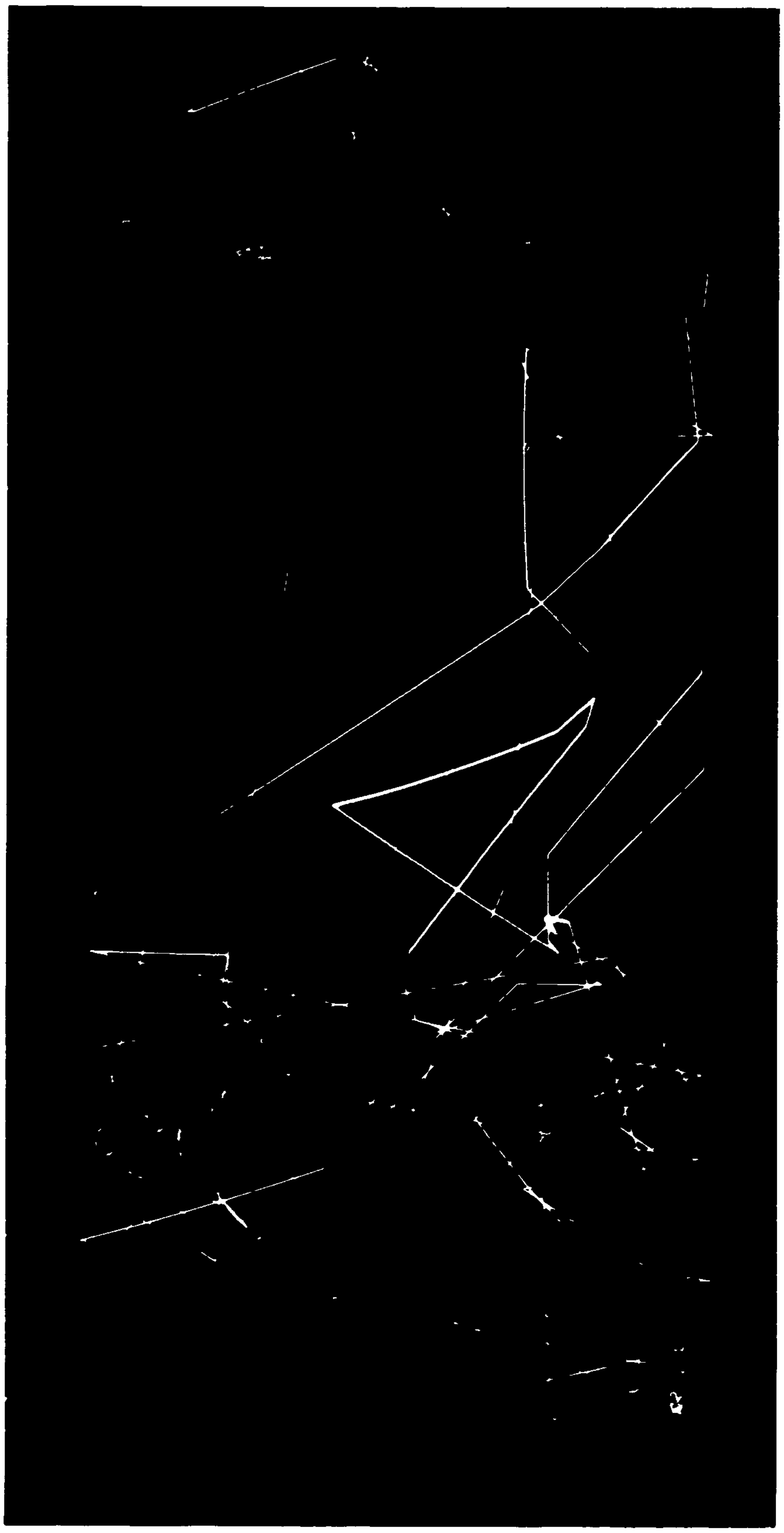


Fig. A.125.

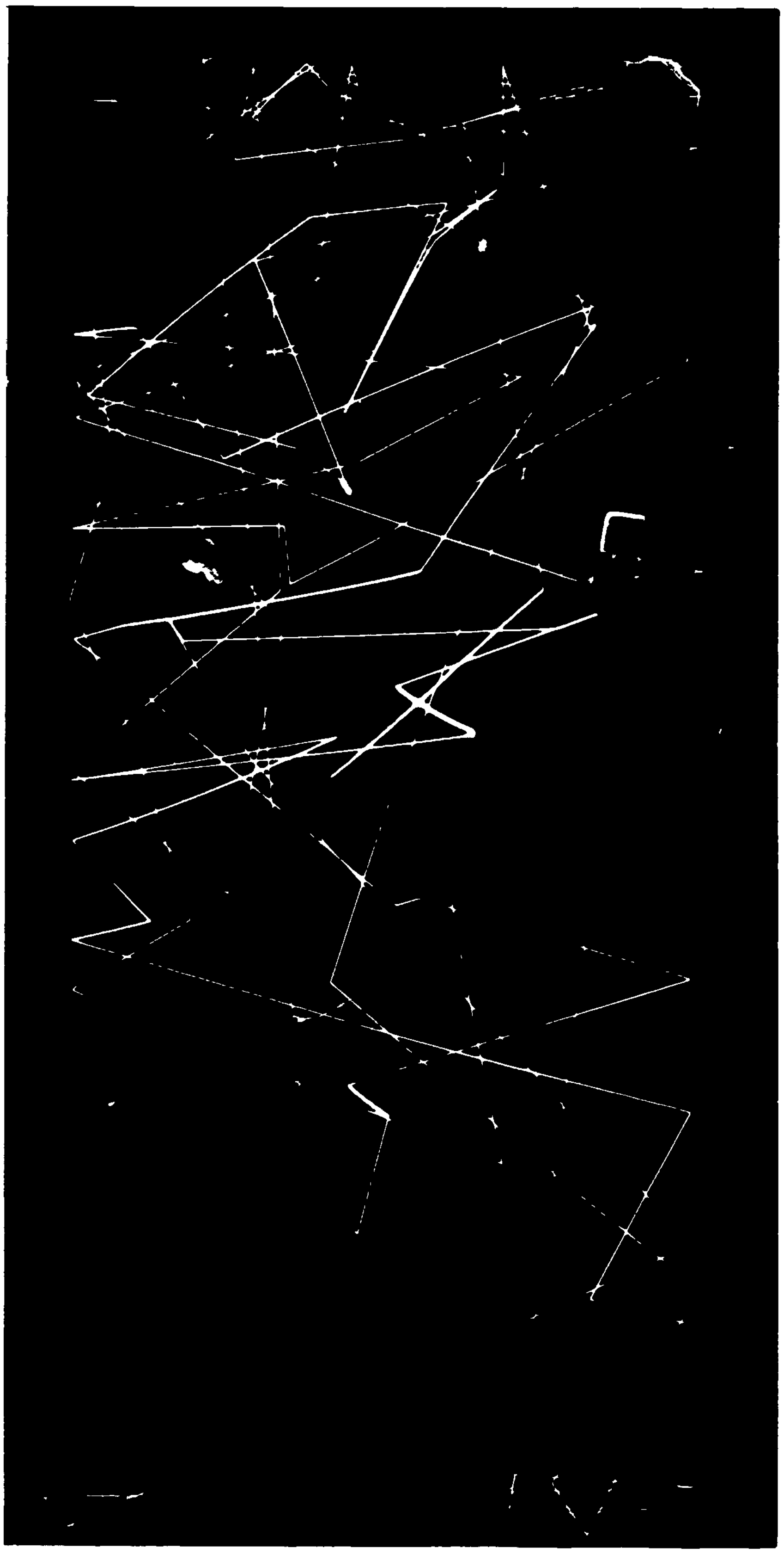


Fig. A.126.