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ABSTRACT

The repeated measures design is of importance to those interested in doing learning studies concerned with repeated trials on a single type of task, repeated trials on different tasks, or both together crossed with and following different treatments. In doing analysis of variance with such data it is assumed that the data fits an additive model. Since the existence of an unrecognized independent variable may, in some cases, imply a violation of the assumption of additivity, a test for non-additivity is needed. Such a test is illustrated. (DG)

A Test for a Neglected Source of Variation: The Individual Difference by Repeated Measures Interaction

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The procedure presented in this paper has been developed to determine in a post hoc manner, if it is likely that the data from a repeated measures experiment, have met the assumption of additivity for a univariate analysis of variance model. Of particular concern was the type of non-additivity which could arise, if a component of the measures taken was some unknown function of one or more unknown individual difference variables and one or more of the independent variables taken into consideration in the experimental design on which the analysis was based. An individual difference variable being some variable with a finite number of levels to which subjects or individuals can be assigned. The procedure will be developed and applied to a set of data.

The importance of determining if it is likely that there is an individual difference variable of which the treatments are a function is demonstrated in an example referred to by Jensen (1967) of a study by Hovland (1939) who performed an experiment in which no statistically significant differences were found between massed practice and distributed practice on paired associate learning tasks. Upon subsequent examination of the data, however, it was determined that 44% of the subjects improved their performance more rapidly on distributed practice and 38% learned faster with massed practice. Thus it is likely that had an additional factor which reflected the individual differences suggested by the previous percentages been included in the design of the experiment, a significant individual difference factor by massed vs. distributed practice interaction would have been found.

Figure 1 is an example of a repeated measures design. Table 1 is its concomitant summary table for a univariate analysis of variance, specifing the appropriate sources of variation, degrees of freedom, expected mean squares and

F ratios under the assumptions of normality, independence, homoscedasticity for treatment groups, additivity of treatment effects as well as equal variances and equal covariances in the residual variance - covariance matrix for repeated measures.

The repeated measures design is of particular importance to those who are interested in doing experiments concerned with learning. For learning studies repeated trials on a single type of task, repeated trials on different tasks, or both together crossed with and following different treatments are not at all uncommon. The design provides for not only efficient use of hard to obtain subjects but as well for a means of investigating important theoretical questions involving the interaction of treatments with tasks over time.

Table 2 is an example of univariate analysis of variance summary data obtained from a learning experiment which fits the design in Figure 1, where A and B are treatment factors with two levels each and R represents three kinds of tasks. The statistics calculated indicate significant B, R, RA, and RB effects at the $\alpha=.05$ level. The critical values of F were based on degrees of freedom adjusted by an estimate of ϵ, where ϵ is a function of Σ the variance-covariance matrix for repeated measures, $\epsilon=F(\Sigma)$. The estimate of ϵ was obtained by substituting the sample variance-covariance matrix in the argument in place of Σ, $\hat{\epsilon}=F(\hat{\Sigma})$. (See Theorem 6.1, Box (1949, 1950)). Because there is a possibility that the data do not fit the additive model assumed in doing the analysis of variance and because of the consequences of such a violation it is suggested the assumption be tested. One approach to testing the hypothesis of additivity would be to employ the Tukey 1 degree of freedom test for non-additivity (Scheffé, 1959) which poses as an alternative to the additive model, a multiplicative model. It is suggested, however, that a non-multiplicative alternative may be present.

Further no extension of the Tukey test to a repeated measures situation could be found. Multiplicative non-additivity can be represented by an ordinal interaction as in Figure 2. This type of non-additivity can be eliminated by means of a non-linear monotonic transformation upon the data which gave rise to it. The type of non-additivity which is indicated by a disordinal interaction e.g. Figure 3, however, cannot be eliminated in this manner. The possible interaction in the Hovland data suggests disordinality.

It is often difficult for an investigator to include all relevant independent variables in the design of his experiment. That is they may be of unique importance to the type of experiment being undertaken and unknown prior to conducting the experiment. Thus it is possible in a given experiment for some unknown variable, which has not been included in the design on which the analysis is based, to exist such that it interacts with one or more of the independent variables with which it is crossed. If such an unknown variable, say U, were to exist it would be one that would classify subjects in some manner and since subjects are crossed with repeated measures, a U x R interaction would be expressed in a repeated measures by subjects nested within A and B (RxS:AB) interaction, the existence of which implies a violation of the assumption of additivity.

Note that if U had as many levels as the experiment had subjects a subsequent experiment could not be designed which took U into account so as to eliminate the RxS interaction, for the U would be perfectly confounded with subjects. It will be assumed that this is not the case.

It should be apparant by now that if an appropriate denominator for an F ratio with mean square RxS:AB in the numerator can be found, we shall have a test for the types of non-additivity with which we are concerned. Inspection of the expected value of mean square RxS:AB indicates that the desired denominator should be a mean square which provides an unbiased estimate of within cell variation. The problem of finding that denominator took two different approaches which fortunately converged upon the same solution.

In the first approach it was noted that because the design which represents subjects crossed with repeated measures (Figure 4) has only one observation per cell, it has no degrees of freedom for an estimate of within cell variation. However, it was also noted that the within cell variance corresponds to what is referred to in classical measurement theory as the variance error of measurement. Then if the classical measurement theoretic approach is taken which assumes that an observation X is composed of a "true" score T and an independent "error" score e, $X = T + e$ from which it follows that σ_X^2 the observed score variance is equal to the sum of the true score variance σ_T^2 and the error variance σ_e^2,

$$\sigma_X^2 = \sigma_T^2 + \sigma_e^2$$

then, if we analogously consider an observed value, X_{RSAB}, as the sum of "true" and "error" parts we have,

$$X_{RSAB} = T_{RSAB} + e_{RSAB} .$$

Therefore, the variance for the source RxS:AB ($\sigma^2_{rs:ab}$)[1] which is estimated by the mean square RxS:AB ($MS_{RS:AB}$) can be partitioned into a "true" interaction part, denoted $\sigma^2_{rs:ab_{(T)}}$, and an uncorrelated "error" part, denoted σ^2_e, we would have the following equality,

$$\sigma^2_{rs:ab} = \sigma^2_{rs:ab_{(T)}} + \sigma^2_e .$$

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[1] The x will be left out of the subscripts for interaction terms. Thus for the source RxS:AB, $MS_{RS:AB}$ is the corresponding mean square.

Again from classical measurement theory

$$\rho_{xx} = \frac{\sigma^2_{rs:ab(T)}}{\sigma^2_{rs:ab}} ,$$

and

$$\sigma^2_e = \sigma^2_{rs:ab} (1 - \rho_{xx}),$$

then the estimate of $\sigma^2_{rs:ab(T)} + \sigma^2_e$ is $MS_{RS:AB}$ and the estimate of σ^2_e is $MS_{RS:AB}(1 - r_{xx})$. If the two estimates are distributed as independent χ^2_s, then

$$\frac{MS_{RS:AB}}{MS_{RS:AB}(1 - r_{xx})} = \frac{1}{1 - r_{xx}}$$

given that $\sigma^2_{rs:ab(T)} = 0$, will be distributed as central F and there is a test for the source of variance RxS:AB.

The second approach involved considering the values assigned to each subject as a composite of constituate parts of items either crossed with or nested within the repeated measures. The items were considered as a random sample from some population of items and thus as a random independent variable. Adding items as nested to the design we arrived at what is represented in Figure 5. Table 3 is it's concomitant summary table. By inspection of expected mean squares the source for the items by subjects nested within A and B and R interaction (IxS:ABR) provides the denominator for the F ratio to test the RxS:AB interaction.

It can be shown most clearly how the two approaches converge if the Hoyt analysis of variance method of estimating r_{xx} is employed. In this case

$$r_{xx} = \frac{MS_{subjects} - MS_{sub\ x\ items}}{MS_{subjects}}$$

or in our case to remove treatment effects

$$r_{xx} = \frac{MS_{RS:AB} - MS_{IRS:AB}}{MS_{RS:AB}}$$

with items considered as crossed,

or $$\frac{MS_{RS:AB} - MS_{IS:ABR}}{MS_{RS:AB}}$$

with items considered as nested as in Table 3.

Then consider only the nested case

$$F = \frac{1}{1 - r_{xx}} = \frac{1}{1 - \frac{MS_{RS:AB} - MS_{IS:ABR}}{MS_{RS:AB}}}$$

$$= \frac{1}{1 - \left[1 - \frac{MS_{IS:ABR}}{MS_{RS:AB}}\right]} = \frac{MS_{RS:AB}}{MS_{IS:ABR}}$$

which is identical to the F test statistic obtained from the design in Figure 2. Similarly for the crossed case $F = \frac{MS_{RS:AB}}{MS_{ISR:AB}}$.

Thus if there is no RxS:AB effect the two ratios would be distributed as central F's[2] with respectively $\varepsilon(r-1)(s-1)g$, $\varepsilon(i-1)(r-1)(s-1)g$ and $\varepsilon(r-1)(s-1)g$, $\varepsilon(i-1)(s-1)rg$ degrees of freedom for the crossed and nested cases. Where ε is a function of the variance-covariance matrix as previously noted.

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[2] An article by Hsu and Feldt (1970) and two dissertations (Hsu, 1968) and (Lunney, 1968) indicate that an analysis of variance can be used on binary data with 20 or more degrees of freedom for error if P the probability of a "one" is between .2 and .8 or with more than 40 d.f. error where $.1<P<.9$.

Returning to the demonstration data the initial 2 x 2 x 3 design was augmented by the inclusion of the random independent variable items to a 2 x 2 x 3 x 4 design. The summary data in Table 4 indicate that for the 2 x 2 x 3 x 4 design there is a significant RxS:AB effect. Now it can be told that another independent variable C(sex) had been included in the original design but ignored for purposes of demonstrating our technique. Table 5 is the summary data for the analysis on the 2 x 2 x 2 x 3 design which includes C. Note the significant BxCxR interaction which would have been predicted from the RxS:AB interaction if U=C. But such a convenient conclusion would be unrealistic and Table 6 shows that if items as a random source is again included there is still a significant RxS:ABC effect. Having no further "put up" situations to draw candidates for a U from, the RxS interactions were plotted and five main types of response curves were detected by ocular inspection. The response curves are graphed in Figure 6.

In conclusion a strategy is suggested for those who plan to do experiments of the type discussed and who are not confidant that they have all relevant independent variables in mind. The suggestion is to run a pilot study and look for non-additivity. If it is found, try to categorize the subjects and then interview them and take other measures in an attempt to determine what occured. Then design the next experiment taking into consideration what was discovered in the pilot.

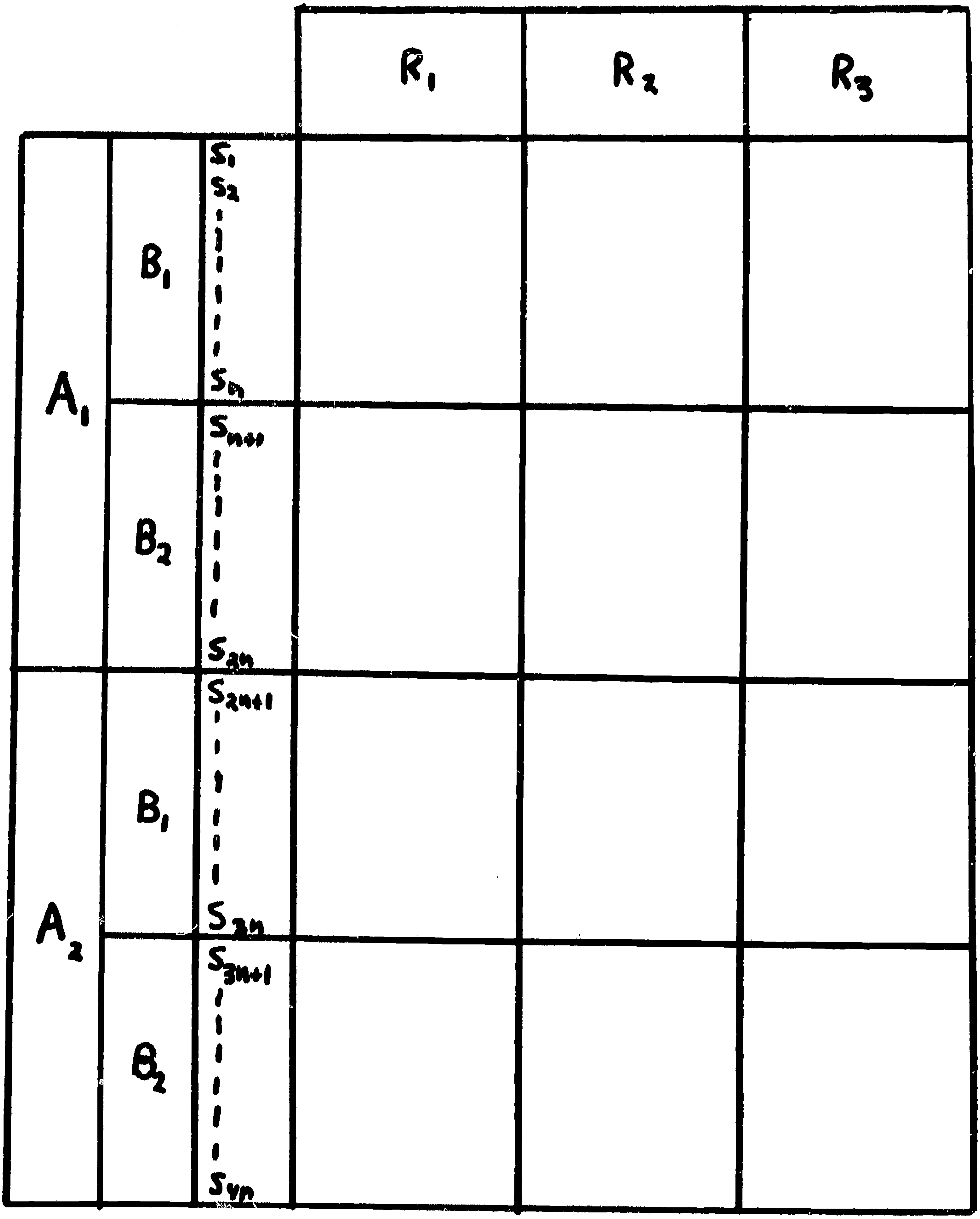

Fig. 1 Experimental design for two independent variables and three repeated measures.

TABLE 1

Analysis of Variance Summary Statistics and Parameters
for a 2 x 2 x 3 Repeated Measures Design

Source	Degrees of Freedom	Expected Mean Squares	F Ratios
A	(a-1)	$\sigma_e^2 + \sigma_{s:ab}^2 + \sigma_a^2$	$\frac{MS_A}{MS_{S:AB}}$
B	(b-1)	$\sigma_e^2 + \sigma_{s:ab}^2 + \sigma_b^2$	$\frac{MS_B}{MS_{S:AB}}$
AB	(a-1)(b-1)	$\sigma_e^2 + \sigma_{s:ab}^2 + \sigma_{ab}^2$	$\frac{MS_{AB}}{MS_{S:AB}}$
S:AB	(s-1)ab	$\sigma_e^2 + \sigma_{s:ab}^2$	
R	(r-1)	$\sigma_e^2 + \sigma_{rs:ab}^2 + \sigma_r^2$	$\frac{MS_R}{MS_{RS:AB}}$
RA	(r-1)(a-1)	$\sigma_e^2 + \sigma_{rs:ab}^2 + \sigma_{ra}^2$	$\frac{MS_{RA}}{MS_{RS:AB}}$
RB	(r-1)(b-1)	$\sigma_e^2 + \sigma_{rs:ab}^2 + \sigma_{rb}^2$	$\frac{MS_{RB}}{MS_{RS:AB}}$
RAB	(r-1)(a-1)(b-1)	$\sigma_e^2 + \sigma_{rs:ab}^2 + \sigma_{rab}^2$	$\frac{MS_{RAB}}{MS_{RS:AB}}$
RS:AB	(r-1)(s-1)ab	$\sigma_e^2 + \sigma_{rs:ab}^2$	

(Note: The coefficients of the variance components have been omitted.)

TABLE 2

Analysis of Variance Summary Data for a 2 x 2 x 3 Repeated Measures Design

Source	Degrees of Freedom	Mean Squares	F Statistics	Critical Values of F[1]
A	1	5.01	1.93	4.00
B	1	57.42	22.17*	4.00
AB	1	0.13	.05	4.00
S:AB	60	2.59		
R	2	40.72	57.35*	3.09
RA	2	5.51	7.76*	3.09
RB	2	5.82	8.20*	3.09
RAB	2	0.26	.37	3.09
RS:AB	120	.71		

[1]Value at $\alpha = .05$, degrees of freedom adjusted by the Box function of the variance-covariance matrix.

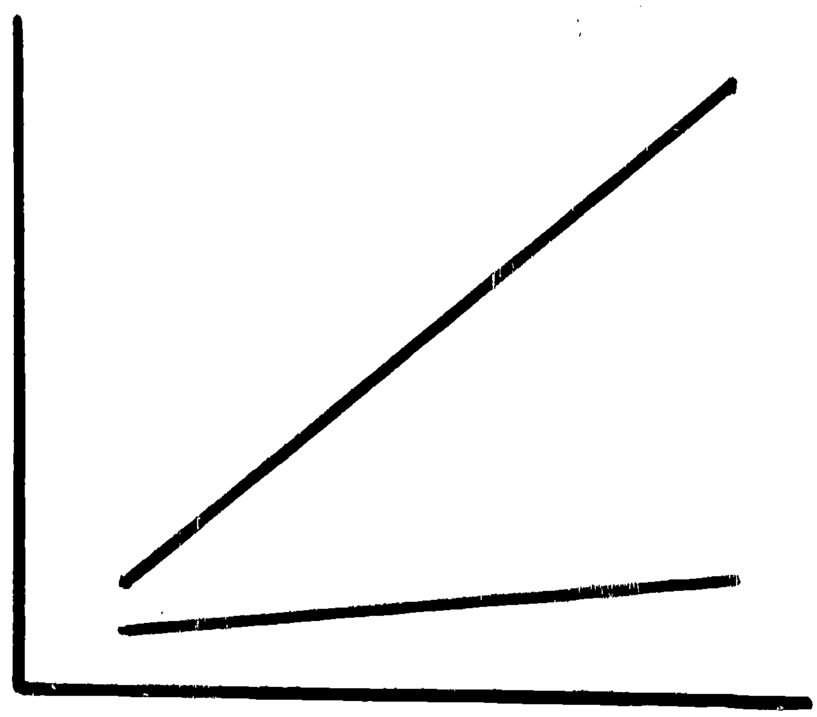

Fig. 2 An example of an ordinal interaction.

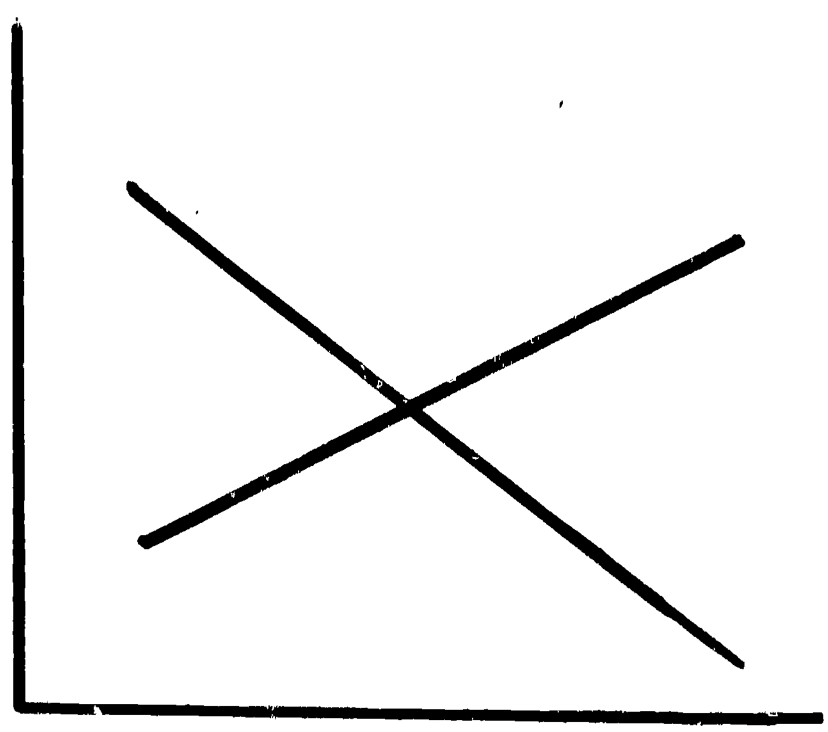

Fig. 3 An example of a disordinal interaction.

	R_1	R_2	R_3
S_1	X_{11}	X_{12}	X_{13}
S_2	X_{21}	X_{22}	X_{23}
S_3	X_{31}	X_{32}	X_{33}
S_4	X_{41}	X_{42}	X_{43}
S_5	X_{51}	X_{52}	X_{53}
$\vdots$			
S_N	X_{n1}	X_{n2}	X_{n3}

Fig. 4 A subjects by repeated measures design with one observation per cell.

			R_1				R_2				R_3			
			I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}	I_{12}
A_1	B_1	S_1, S_2, ⋮ S_n												
	B_2	S_{n+1} ⋮ S_{2n}												
A_2	B_1	S_{2n+1} ⋮ S_{3n}												
	B_2	S_{3n+1} ⋮ S_{4n}												

Fig. 5 Experimental design for two independent variables and four items nested within three repeated measures.

TABLE 3

Analysis of Variance Summary Statistics and Parameters
for a 2 x 2 x 3 x 4 Repeated Measures Design

Source	Degrees of Freedom	Expected Mean Squares	F Ratios
A	(a-1)	$\sigma^2_{is:abr} + \sigma^2_{s:ab} + \sigma^2_{ai:r} + \sigma^2_{a}$	
B	(b-1)	$\sigma^2_{is:abr} + \sigma^2_{s:ab} + \sigma^2_{bi:r} + \sigma^2_{b}$	
AB	(a-1)(b-1)	$\sigma^2_{is:abr} + \sigma^2_{s:ab} + \sigma^2_{abi:r} + \sigma^2_{ab}$	
S:AB	(s-1)(b-1)	$\sigma^2_{is:abr} + \sigma^2_{s:ab}$	
R	(r-1)	$\sigma^2_{is:abr} + \sigma^2_{rs:ab} + \sigma^2_{i:r} + \sigma^2_{r}$	
I:R	(i-1)r	$\sigma^2_{is:abr} + \sigma^2_{i:r}$	$\frac{MS_{I:R}}{MS_{IS:ABR}}$
AR	(a-1)(r-1)	$\sigma^2_{is:abr} + \sigma^2_{rs:ab} + \sigma^2_{ai:r} + \sigma^2_{ar}$	
AI:R	(a-1)(i-1)r	$\sigma^2_{is:abr} + \sigma^2_{ai:r}$	$\frac{MS_{AI:R}}{MS_{IS:ABR}}$
BR	(b-1)(r-1)	$\sigma^2_{is:abr} + \sigma^2_{rs:ab} + \sigma^2_{bi:r} + \sigma^2_{br}$	
BI:R	(b-1)(i-1)r	$\sigma^2_{is:abr} + \sigma^2_{bi:r}$	$\frac{MS_{BI:R}}{MS_{IS:ABR}}$
ABR	(a-1)(b-1)(r-1)	$\sigma^2_{is:abr} + \sigma^2_{rs:ab} + \sigma^2_{abi:r} + \sigma^2_{abr}$	
ABI:R	(a-1)(b-1)(i-1)R	$\sigma^2_{is:abr} + \sigma^2_{abi:r}$	$\frac{MS_{ABI:R}}{MS_{IS:ABR}}$
RS:AB	(r-1)(s-1)ab	$\sigma^2_{is:abr} + \sigma^2_{rs:ab}$	$\frac{MS_{RS:AB}}{MS_{IS:ABR}}$
IS:ABR	(i-1)(s-1)abr	$\sigma^2_{is:abr}$	

(Note: The coefficients of the variance components have been omitted.)

TABLE 4

Analysis of Variance Summary Data for a 2 x 2 x 3 x 4 Repeated Measures Design

Source	Degrees of Freedom	Mean Squares	F Statistics	Critical Values of F[1]
A	1	1.25		
B	1	14.35		
AB	1	.03		
S:AB	60	.64		
R	2	10.18		
I:R	9	1.16	6.56*	2.10
RA	2	1.38		
AI:R	9	0.18	1.06	2.10
RB	2	1.46		
BI:R	9	0.28	1.64	2.10
RAB	2	0.06		
ABI:R	9	0.16	.94	2.10
RS:AB	120	0.17	1.54*	1.28
IS:ABR	540	0.11		

[1]Value at $\alpha = .05$, degrees of freedom adjusted by the Box function of the variance-covariance matrix.

TABLE 5

Analysis of Variance Summary Data for a 2 x 2 x 2 x 3 Repeated Measures Design

Source	Degrees of Freedom	Mean Squares	F Statistics	Critical Values of F[1]
A	1	5.01	1.88	4.00
B	1	57.42	21.51*	4.00
C	1	.88	.33	4.00
AB	1	.13	.05	4.00
AC	1	1.17	.44	4.00
BC	1	2.29	.86	4.00
ABC	1	1.17	.44	4.00
S:ABC	56	2.67		
R	2	40.72	58.88*	3.10
RA	2	5.51	8.10*	3.10
RB	2	5.83	8.57*	3.10
RC	2	1.04	1.53	3.10
RAB	2	.26	.38	3.10
RAC	2	.30	.44	3.10
RBC	2	2.12	3.12*	3.10
RHBC	2	.48	.73	3.10
RS:ABC	112	.68		

[1] Value at $\alpha = .05$, degrees of freedom adjusted by the Box function of the variance-covariance matrix.

TABLE 6

Analysis of Variance Summary Data for a 2 x 2 x 2 x 3 x 4 Repeated Measures Design

Source	Degress of Freedom	Mean Squares	F Statistics	Critical Values of F[1]
A	1	1.25		
B	1	14.36		
C	1	.22		
AB	1	.03		
AC	1	.29		
BC	1	.57		
ABC	1	.29		
S:ABC	56	.67		
R	2	10.18		
I:R	9	1.16	6.82*	2.11
AR	2	1.38		
AI:R	9	.18	1.05	2.11
BR	2	1.46		
BI:R	9	.28	1.64	2.11
CR	2	.26		
CI:R	9	.10	.58	2.11
ABR	2	.06		
ABI:R	9	.16	.94	2.11
ACR	2	.07		
ACI:R	9	.21	1.23	2.11
BCR	2	.53		
BCI:R	9	.11	.65	2.11
ABCR	2	.12		
ABCI:R	9	.13	.76	2.11
RS:ABC	112	.17	1.42*	1.29
IS:ABCR	504	.12		

[1]Value at α=.05, degrees of freedom adjusted by the Box function of the variance-covariance matrix.

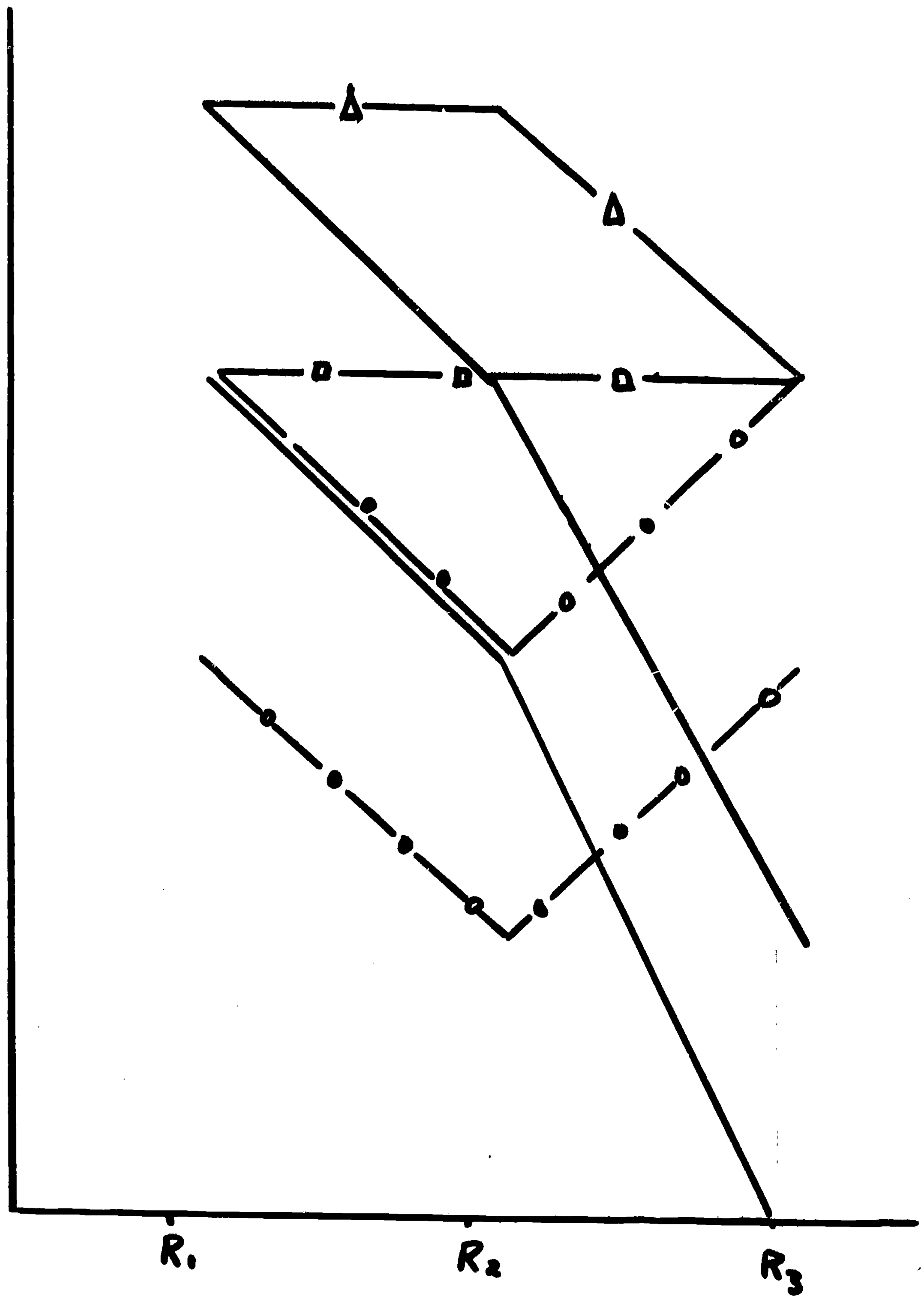

Fig. 6 Some response curves from the demonstration data.

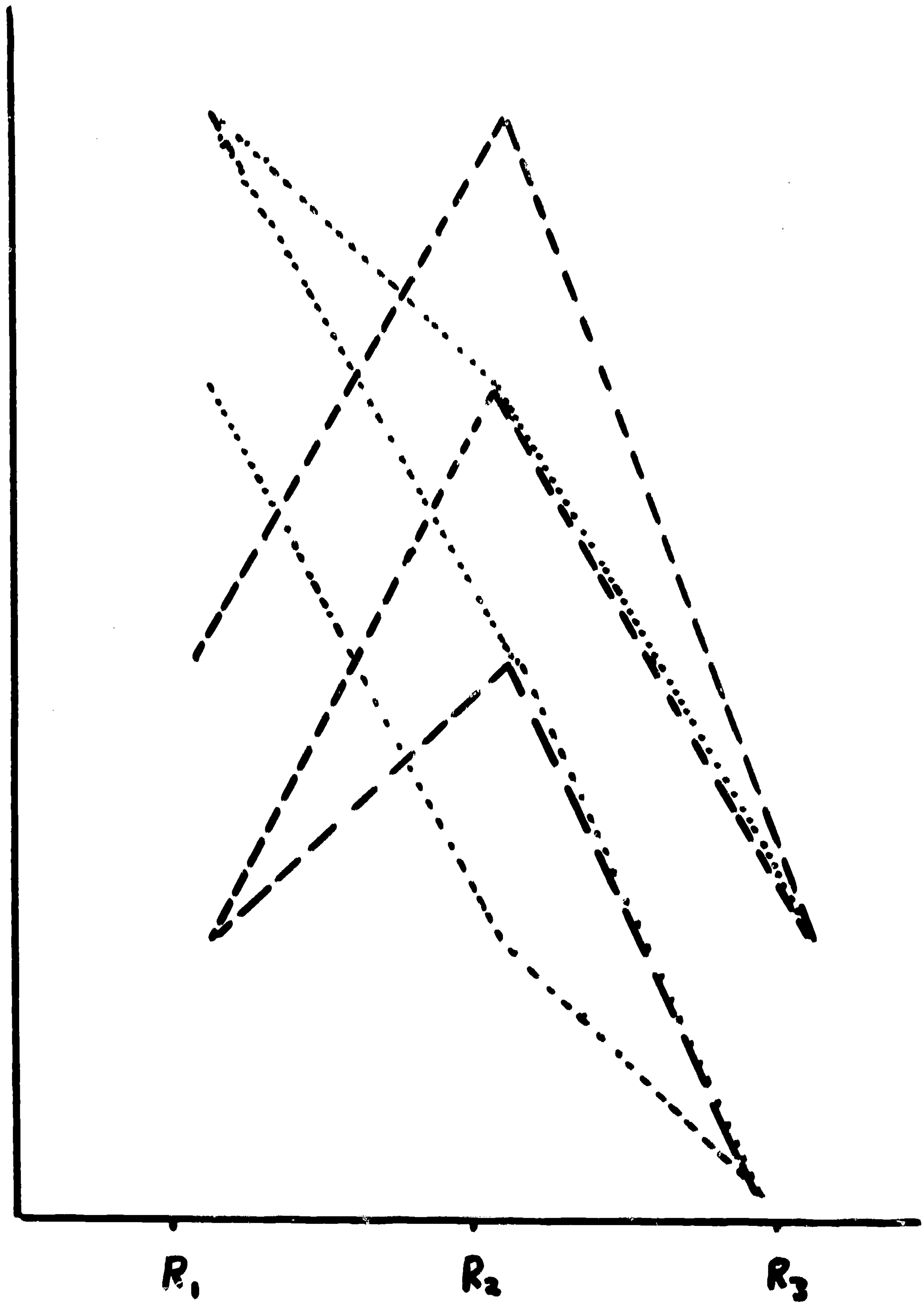

Fig. 6 continued.

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