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ABSTRACT

Evaluated was grade 6 of a televised arithmetic program, "Patterns in Arithmetic." The major goal was to determine whether or not the technique of item-sampling could be incorporated into a design effective for formative curriculum evaluation. Approximately 60 classes near Madison, Wisconsin, participated in the study. Tests were administered at four times during the year to a random sample of students to provide a profile of performance change. As a result of these tests, changes were made in the television presentations. It was concluded that item-sampling had the following advantages over traditional achievement testing: (1) more detailed information is provided, (2) planning and review are more effective, (3) transient aspects become evident, (4) revision is more effectively accomplished, and (5) it is more economical of time and money. (Author/RS)

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THE FORMATIVE EVALUATION OF PATTERNS IN ARITHMETIC GRADE 6 USING ITEM SAMPLING

Report from the Project on
Individually Guided Elementary Mathematics
Phase 2: Analysis of Mathematics Instruction

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
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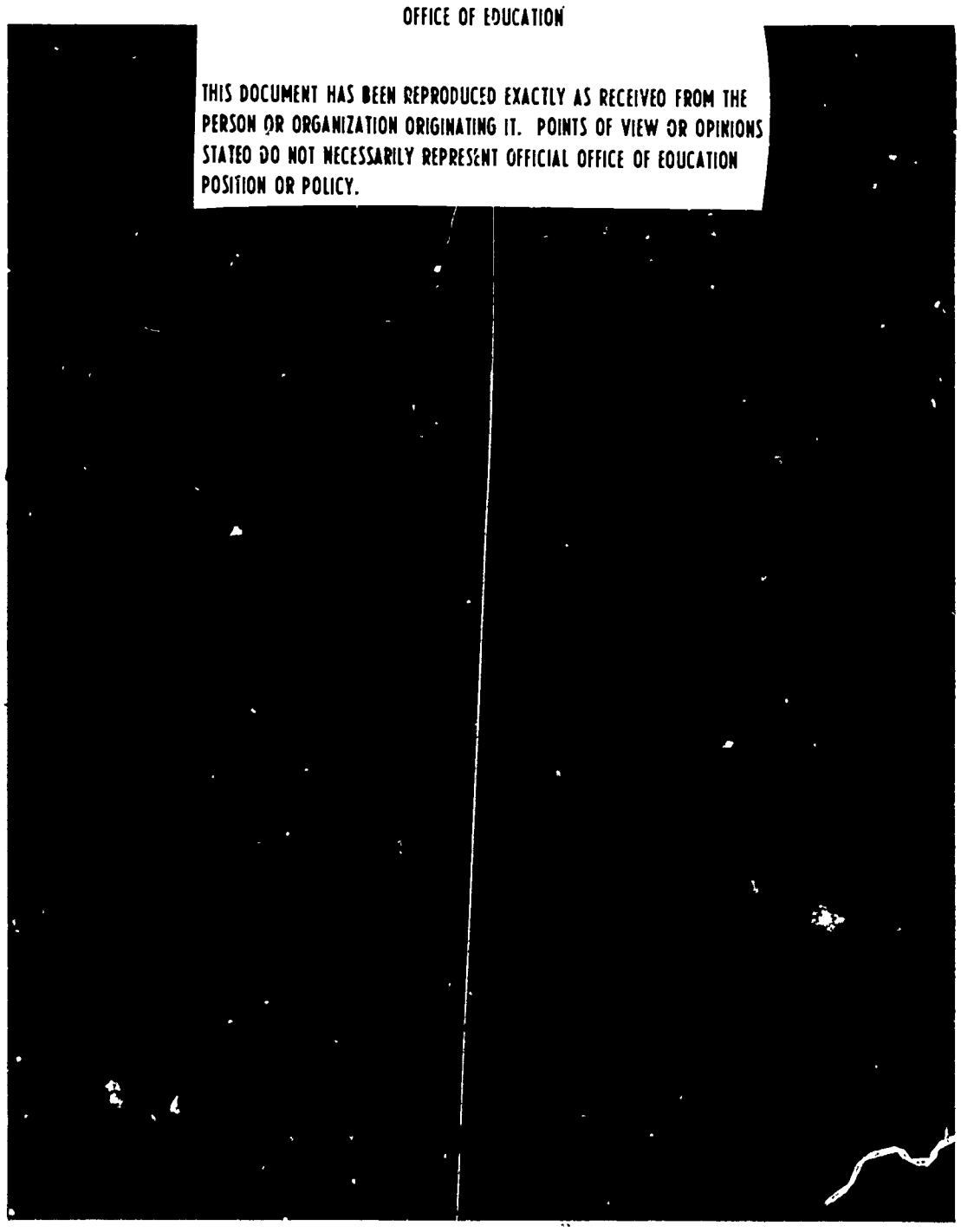

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Individually Guided Elementary Mathematics
Phase 2: Analysis of Mathematics Instruction

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Center for Cognitive Learning
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Madison, Wisconsin

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The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from Phase 2 of the Project on Prototypic Instructional Systems in Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

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ABSTRACT

Purpose

The study had one major goal: To determine whether the technique of "item-sampling" could be incorporated into a design effective for formative curriculum evaluation. The study evaluated only grade 6 of a television arithmetic program, Patterns In Arithmetic (PIA). Other goals of the study were (1) To explore the practical relationship between the population and item parameters (2) To discuss ways formative curriculum evaluation can be used and (3) To suggest advantages item-sampling has over traditional achievement testing.

Procedure

A large pool of items was assembled that represented objectives to be covered by PIA-6. The items were selected while the program was in the planning stage. Approximately 60 classes near Madison, Wisconsin, and using PIA-6 participated in the study. Twelve, 20-item tests were constructed from the item pool. These tests were administered at four times during the year to a random sample of students. At each administration one gets a measure of the difficulty of all 12 x 20 = 240 items. These measures provide a profile of change across the school year. Changes in performance on a given item were evaluated in terms of expected behaviors following instruction. When observed behavior was at a level below that of expected behavior, revisions were suggested. Criterion referencing of items was used to indicate

expected behaviors. Testing done early in the school year was used for planning programs that were written and taped later in the year.

Results

Several important changes were made in PIA-6 as a result of the formative evaluation. It was found that 120-130 random responses to an item were sufficient to yield an estimate of item difficulty that was useful for making developmental year decisions. It became clear that the success of a curriculum study using item-sampling depends largely upon the quality of the hand-picked items that constitute the item pool. Much of the planning should be devoted to selecting items that are sensitive to the behavioral objectives of the curriculum. The role of the evaluator became clearly defined. He must be an active part of the developmental project and have more than a passing knowledge of the curriculum and its objectives. Criterion referencing the items makes the evaluation more objective and may clarify the objectives of the curriculum. Items should be criterion referenced before tests are assembled. If this is done several undesirable items are likely to be discovered.

Conclusions

The item-sampling technique can be applied effectively to formative curriculum evaluation. It has the following advantages over traditional achievement testing:

1. More detailed information is provided.
2. Planning and review are more effective.
3. Transient aspects become evident.
4. Revision is more effectively accomplished.
5. It is more economical of time and money.

CHAPTER I

INTRODUCTION

There is a growing demand for effective curriculum evaluation. Since the introduction of new mathematics curricula over a decade ago, scholars have been critical of their evaluation efforts. Criticism has been in part aimed at the use of standardized achievement tests to evaluate and compare new programs. Standard tests were designed to measure other objectives than those espoused by the new mathematics programs and they are intended to evaluate individuals, not curricula.

In addition, funding agencies for curriculum projects (e.g., the United States Office of Education) are emphatically demanding more quality verification of the products being produced. These agencies want to know what they are getting for their money, and they want something other than a subjective guess.

The primary purpose of this study is to present an evaluation of the Grade 6 Patterns in Arithmetic program (PIA) which has been developed by the University of Wisconsin Research and Development Center under the direction of Professor Henry Van Engen.

The Wisconsin Research and Development Center for Cognitive Learning was established at the University of Wisconsin in 1964. It is at present one of nine federally funded R and D Centers. PIA was the first major project undertaken by the Center. This program is a

complete arithmetic program utilizing television for grades 1-6. As PIA was a Federally funded project, its developmental staff felt the pressure for extensive, objective evaluation. Grades 1-5 of the program were produced during the years 1965-1968. A variety of evaluation activities were carried out during these years.

For Grade 6 a new technique was suggested as a method for obtaining more extensive information about the effectiveness of PIA. This technique is called "item-sampling." In contrast to the traditional type of testing in which every pupil gets the same test, the item-sampling technique allows pupils to take one of several different tests. Conceptually, item-sampling considers a pool of m items distributed over k tests. The k tests are then administered to the population it is desired to measure. For example, suppose there are 100 items about which one desires information. Instead of having each pupil respond to all 100 items, one could construct five, 20-item tests and administer these tests. In general, the maximum number of different tests that can be used is a function of the number of pupils available for testing and the desired accuracy of the sampling estimates.

A pilot run using item-sampling was made at the end of the developmental year of PIA Grade 5. As a result of the success of that trial, a more involved design was conceived for the developmental year evaluation of PIA Grade 6. The design was to test at four times during the year, each time obtaining information on the same set of 240 items by item-sampling. The resulting data would provide a profile of change for <u>each</u> item across the year for the population. These item profiles would provide project personnel with an objective means of assessing

strengths and weaknesses of PIA Grade 6. Materials could be revised accordingly.

Therefore, the subject of this study is the application of item-sampling to developmental curriculum evaluation. Relevant literature will be reviewed in Chapter II.

CHAPTER II

BACKGROUND OF THE PROBLEM TO BE INVESTIGATED

The criticism which has been voiced against the way in which curriculum evaluations have been carried out seems to run the gamut--all the way from the clarity of the program objectives to the final evaluation which reports the merits of the program.

Scriven (1967) identified two broad types of evaluation--formative and summative. Formative evaluation is done during the developmental stages of production in order to identify weaknesses and guide the staff in the revision of materials. The distinguishing feature of a formative evaluation is that the product still remains open to change. Summative evaluation is done on the finished version of the product in order to provide the potential consumer with evidence relative to its effectiveness. In general, results of a summative evaluation are not used to make additional refinements of the product.

Stake notes that "most contemporary evaluation of instruction begins and ends with achievement testing." (Stake, 1967, p. 5) This is not to say that achievement testing is necessarily bad, but he emphasizes that the primary purpose of an achievement test is to provide an instrument which reliably discriminates among individuals for the purpose of instruction and guidance. Standardized achievement tests cover more general skills common to content areas that are likely to be taught in a particular program and are designed to be "fair" to all students.

Stake believes "the standard achievement test is unlikely to encompass the scope or penetrate the depth of a particular curriculum being evaluated." (Stake, 1967, p. 6)

Curriculum evaluation is difficult to accomplish for the unique as well as common features of various curricula must be evaluated both objectively by measuring the outcomes of instruction and by personal judgements as to the quality and appropriateness of goals. There are several points of view currently being advocated as to the relative emphasis which should be placed on formative versus summative evaluation and comparative versus non-comparative evaluation. Cronbach is more enthusiastic about using both formative and summative, non-comparative evaluations. He feels that "in an experiment where treatments differ in a dozen respects, no understanding is gained from the fact that the experiment shows a numerical advantage in favor of a new course." (Cronbach, 1963, p. 5) Scriven on the other hand emphasizes the summative, comparative evaluation. He argues that "if we want to pin down the exact reasons for differences between programs, it is quite true that 'small scale, well-controlled studies can profitably be used to compare alternate versions of the same course' whereas large-scale overall comparison will not be so valuable. But that in no way eliminates the need for comparative studies at some point in our evaluation procedures." (Scriven, 1967, p. 65) Scriven's most convincing argument for comparative evaluation is that one can avoid having to determine an absolute scale for reference. For example, suppose you are comparing arithmetic programs A and B with respect to computing with rational numbers. If $\bar{X}_a$ and $\bar{X}_b$ are pretest means and $\bar{Y}_a$ and $\bar{Y}_b$ are posttest means for groups A and B respectively, then the differences $\bar{Y}_a - \bar{X}_a$

and $\bar{Y}_b - \bar{X}_b$ are more meaningful in measuring growth in computing with rational numbers than $\bar{Y}_a - \bar{X}_a$ would be alone. On the other hand, suppose you are using items for which comprehensive normative information is available. If the item difficulties are known for the pretest period as well as the posttest period, then the difference $Y_a - X_a$, where Y_a represents the post item difficulty and X_a represents the pre item difficulty, would form a useful basis for item comparisons. Also, Cronbach has argued that it is difficult to equate classes taking competing courses and that experiments "putting one course against another, are rarely definitive enough to justify their cost." (Cronbach, 1963, p. 5) Despite his emphasis on experimental, non-comparative evaluation Cronbach, nevertheless, realizes the need to compare courses if intelligent decisions are to be made by administrators, supervisors, etc., but he does not see comparative evaluation as the dominant goal. Professor Cronbach is obviously emphasizing curriculum perfection, while Professor Scriven is more concerned with protecting the interest of the American consumer.

Tyler finds existing achievement tests of limited value for evaluating complex educational objectives. As an example he cites his experience in a project concerned with assessing the process of education.

> The purpose is to appraise the educational progress of large populations in order to provide the public with dependable information to help in the understanding of educational problems and needs and to guide in efforts to develop sound public policy regarding education. This type of evaluation is not focused upon individual students, classrooms, schools, or school systems, but is to furnish over-all information about the educational attainments of large numbers of people. Although the purpose is not identical with that of current achievement testing programs, we thought that available tests and/or test items would serve our purposes, but this turned out not to be the case. (Tyler, 1967, p. 13)

One reason Tyler feels that achievement tests are of limited value is that the items are designed to discriminate among individuals. For assessing the process of education, items are also needed which show what most of the children know as well as what the more advanced few are learning.

How Should Course Evaluation be Accomplished.

Cronbach and others have promoted the concept of "item-sampling" as being a very useful technique for both formative and summative evaluation. The use of item-sampling was first proposed by Lord (1962). The following definition of item-sampling will be used in this study. "In the item-sampling technique, a set of m items is randomly broken up into k subsets of items. The k subsets of items are then randomly assigned to p pupils or subjects. Each subject takes only a portion of the complete set of items." (Cahen, Romberg, Zwirner, 1968, p. 2) For example, suppose there are 100 items about which one desires information. Instead of having each pupil respond to all 100 items, one could construct five, 20-item tests and administer these randomly among pupils. For the results to be useful there must be a sufficient number of responses to each test so that a stable estimate of the difficulty of each item is obtained.

The original purpose of item sampling was to estimate descriptive statistics for a group taking a standardized test so that results could be compared with the norms. Early work by Lord (1962, 1965) revolved around applying the general work in sampling done by Hooke (1956a, 1956b) and Tukey (1950) to testing. To establish empirical evidence to support the technique, Lord (1962) used item sampling to check how close group statistics could be estimated by sampling responses from a test which

had been taken in its entirety. Lord sampled items with replacement (which he admitted was a mistake) and still found that the test mean for the group could be reasonably estimated.

Lord and Novick discuss the use of item-sampling in research design in Chapter 8 of their book Statistical Theories of Mental Test Scores (1969). The advantages cited are these:

> 1. If only a limited amount of time can be demanded of each research subject, the total amount of information obtained from a given number of subjects may be greatly increased by item sampling.
>
> 2. If a test can be administered to only one examinee at a time, the examiner's time may be the limiting factor; more information about a group of examinees may be obtained by giving a few items to each examinee than by giving the entire test to just a few examinees.
>
> 3. With certain tests, scoring costs may be the limiting factor; in this case, it would be better to score a few items from the answer sheets of every examinee than to score every item on the answer sheets of a few examinees.
> (Lord and Novick, 1968, p. 252)

Since the primary purpose of item-sampling is to obtain a stable estimate of the difficulty of each item, one commonly assumes that the item performance is independent of the context in which it occurs, Lord points out that empirical studies have tended to support this assumption. He also mentions that item-sampling is not appropriate on speeded tests. If one does not have a chance to respond to an item, performance on this item is not accurately estimated. Various theoretical results and formulas for estimating descriptive statistics are included in Lord's Chapter 8. For this study there is no pre existing test on which performance must be estimated. Therefore, a discussion of the results and formulas associated with estimating test parameters is not included in this chapter. Since Lord's initial efforts, several individuals and

groups of individuals have pointed out the possibilities of item-sampling.

Husek and Sirotnik have recently advocated the use of item-sampling in growth studies. The example they cite is the following: "If a course has 100 students and the researcher is interested in obtaining some index of growth of the students with respect to a measure containing 100 items, by using item sampling it is possible to obtain data on all 100 items at several times during the term without any student necessarily taking any item twice." (Husek and Sirotnik, 1968, p. 2) At present item-sampling has not been used in any reported growth studies.

Plumlee (1964) illustrates an interesting possible use of item-sampling. She found item-sampling could be used to norm a test which would then be given to prospective employees. The population the norms would be based upon would be current employees. Item samples would be given to the employees and results used to establish a test mean. Each employee would be involved in only a few minutes of testing time and would respond to a small sample of items in the universe. To obtain empirical evidence to support the technique Plumlee obtained data on the actual test performance of 200 clerical applicants on a 30-item test. The total population of 200 applicants was divided randomly into 10 subgroups with 20 in each group. Each of these 10 subgroups could be used to provide an estimate of the entire group mean and standard deviation. As an alternative to using the performance of one subgroup on all 30-items to estimate group performance, Plumlee selected at random 3 questions from each applicant's test for each of the 10 subgroups. Lord's formula was then applied to estimate group performance from the partial data randomly taken from the subgroups. Plumlee found "that the estimated test mean predicted by item-sampling is closer to

the norms population mean than all but 2 of the 10 examinee-sampling predicgions; however, the prediction, by item-sampling, of the total-test standard deviation is closer to the norms population standard deviation than only one examinee-sampling prediction." (Plumlee, 1964, p. 625) One interesting point made by Plumlee has implications when item sampling is applied practically anywhere and particularly in educational evaluation. She said that "supervisors who fear that experimental test results may be used against their employees may be more convincingly assured if each employee takes only a part of a test and different employees take different parts." (Plumlee, 1964, p. 624) Similarly, teachers with classes participating in an experimental program may be more agreeable and comfortable if they are aware that results are not going to be used to evaluate individuals.

Item-sampling has also been used extensively in the National Longitudinal Study of Mathematical Abilities (NLSMA) in order to measure a multitude of dimensions that conventional testing procedures would not allow. NLSMA used item-sampling in a summative-comparative sense and results seem promising. (Cahen, Romberg, Zwirner, 1968) Prior to the NLSMA application of item-sampling almost all studies related to the method attempted to validate the potential utility of the technique by sampling from existing banks of data. (Lord, 1962; Plumlee, 1964) Since "complete" data was available the item-sampling estimates of group mean and variance could be compared with actual results. One of the questions asked by NLSMA was how would data actually obtained under conditions of item-sampling compare with actual performance by the group on a complete test. To answer this question parallel forms of a 50 item Educational Testing Service Cooperative Arithmetic Test (1964) were

administered on consecutive days. The first day Form B of the test was administered under item-sampling conditions (5 items on 10 different tests) while on the second day the groups were given the entire Form A test. The item-sampling estimates of the groups' mean were almost always too high. Two hypotheses were proposed to account for the inflated estimates.

1) Students were allowed slightly more time per item under item-sampling conditions.

2) The student confronted with a short set of test items may vary his test taking behavior and be motivated differently than when confronted with the entire test.

Since the means estimated by item-sampling in the NLSMA study were significantly higher than the observed means on a parallel form, it is natural to question its usefulness. For example, can one risk the possible error in item-sampling estimates. The answer to this question depends upon the use one intends to make of item-sampling. If several groups are to be compared using item sampling, then factors which affect one group presumably equally affect the other. The two hypotheses mentioned above to account for the inflated item-sampling estimates are no longer tenable since all groups are exposed to a more uniform item-sampling setting. In fact, if a test were normed under conditions of item-sampling, it is reasonable to expect that comparisons with norms would be more valid. The following quote from the NLSMA report summarizes some of the uses as well as problems presented by item-sampling.

> The item-sampling technique should have great application for curriculum evaluation where information about the group rather than individual performance has high feedback payoff for the innovator and evaluator.... In one testing session in the study over 10 scales were item sampled, including measures of pupils' attitude toward mathematics....

> Depending upon the desired level of precision, it may be possible to determine how many different sets of items need be sampled and the number of students required on each form or booklet in order to stay within the specified error range.... The data indicated that precision of estimation is monotonically related to the number of students tested.... The study indicated that care must be taken in the future with the problem of establishing accurate time limits for item-sampling administration when the purpose of item sampling is to estimate absolute rather than relative parameters of school performance.
> (Cahen, Romberg, Zwirner, 1968, p. 12-14)

Knapp has suggested the use of item-sampling in obtaining a more realistic set of normative data for standardized achievement tests. Often test developers experience difficulty in getting cooperation from school officials to generate data for establishing norms. Knapp, like Lord, reasons that schools would be more willing to cooperate with testers if pupils involved were required to take only a small sample of items in a fraction of the normal testing time. The unique feature of Knapp's work is his application of the balanced incomplete block design (BIBD) to item-sampling. The parameters he is interested in estimating are the mean, variance, and internal-consistency reliability coefficient of a test. Knapp presents some empirical results to demonstrate the effectiveness of the BIBD technique. In concluding he summarizes: "This method applies the logic of item-sampling and balanced incomplete block designs to parameter estimation and is extremely economical of testing time." (Knapp, 1968, p. 272) One weak point of the study was his failure to give reason why one would expect the BIBD design to be superior to other less restrictive designs. Another implied assumption made by Knapp is that normative data based upon item-sampling would serve as a valid base for another group taking the entire test. Results of the NLSMA study described above tend to cast doubt upon the validity of this assumption.

Stuffelbeam is much opposed to the traditional application of experimental design to evaluation problems. This method, first used in agricultural experiments, involves testing hypotheses about the effect of treatments where treatments (in this case, programs) are randomly assigned to plots (here classes). His four objections to the experimental design approach are: 1) It interferes with continual improvement of the program since significant differences make no sense unless the program remains the same. 2) Although the experimental design may be useful for summative evaluation, it is practically useless for making decisions during the formative stages. 3) It is difficult to control potentially confounding variables by randomization. 4) Although one can be confident of the findings in a well controlled experimental situation, it is not necessarily true that the results will be generalizable to a typical classroom situation where factors are not so nicely controlled. Also, Stufflebeam considers the problem of process evaluation.

> As opposed to experimental design evaluation, process evaluation does not require control over assignment of subjects to treatments, nor that the treatments be held constant. Its purpose is to assist project personnel to make their decisions a bit more rational in their continual efforts to improve the quality of the program.... The process evaluator focuses his attention on theoretically important variates, but he also remains alert to any unanticipated but significant events. Under process evaluation, information is collected daily, organized systematically, analyzed periodically, e.g., weekly, and reported as often as project personnel require such information. (Stufflebeam, 1968, p. 35-36)

The preceding paragraphs have pointed out the need for more effective curriculum evaluation--both formative and summative. A case against using the traditional experimental design technique was made and the technique of item-sampling was explored in several studies. Granted that item-sampling makes it possible to gather more information than,

for example, traditional standardized achievement testing, the problem of what to do with the information still remains. One of the obvious unstated assumptions made by advocates of item-sampling is that the items upon which information is gathered test important objectives. In particular, if item-sampling is applied to formative curriculum evaluation, the chosen items should provide feedback that aid the curriculum developer in improving (revising) his product. For a given curriculum it is fashionable to state goals in terms of "behavioral objectives"--i.e., as a result of exposure to this curriculum what should the individual be able to do. If an item is selected to test one of the objectives of instruction, two important and related questions may be asked.

1. How involved is the item with respect to the objective tested?

   Is it a simple, direct question or one which requires an insightful application? More succinctly, where does the item fit in Bloom's Taxonomy?

2. For a given group of individuals, approximately what per cent should respond correctly to the item if the curriculum is to be considered successful with respect to the tested objective?

For the curriculum developer the answer to question 2 is crucial and obviously dependent upon the answer to question 1.

The technique of "criterion referencing" has been used to answer a slight twist applied to question 2--namely, for a given individual, approximately what per cent of a group of items should be answered correctly if the individual is to be considered successful with respect to the objective(s) tested by the items. Criterion referencing is discussed in the next section and Chapter V proposes a modification to the technique that will apply to formative curriculum evaluation.

Measuring Success

From our earliest experiences in school we are aware that individuals differ in aptitude. In fact, educators are aware that aptitude has many dimensions and that one must be careful to label the kind of learning (e.g., mathematics, history) when speaking of ones aptitude. Many aptitude tests have been written which predict (correlate highly with) an achievement test given at a later date. It is likely that the high aptitude-achievement correlation has tended to set goals for teaching. Carroll (1963) has indicated that if pupils come into a course with normally distributed aptitude scores and if they are all given the same instruction, then the end result will be a normally distributed set of achievement scores. Carroll sees no reason why the focus of instruction can not be changed so that the net result after instruction would be a skewed distribution of achievement scores with a long left tail. Most pupils would be able to demonstrate mastery of the given task and the achievement scores would likely be less correlated with traditional aptitude tests. Use of the phrase "mastery of a learning task" implies that given a task one can objectively specify a criterion (score on a test, etc.) that provides a valid dichotomous classification of individuals-- one group being "masters of the task" and the other group being "non-masters". The score which serves to split the group is referred to as the criterion score and testing which uses such a score is referred to as criterion referenced testing.

Bloom (1968) believes that approximately 95 percent of all individuals can learn a subject up to a high level of mastery. To accomplish this some students require more time and varied approaches to the material to real criterion level, i.e., one must allow for individual differences.

Bloom discusses several preconditions and operating procedures which determine the success of mastery learning.

1. One must be able to determine when a student has demonstrated mastery of a task.

2. Standards of mastery should be relatively independent of inter-student competition. At the same time previous experience with student performance should be useful in establishing a criterion for mastery.

3. Feedback should be frequent and on small fundamental units of instruction.

4. The diagnosis should be accompanied by a prescription so students can overcome difficulties.

To date, most applications of criterion referenced tests have been made with the criterion score set for the purpose of making a decision about the learner. Glaser (1968) has developed a program of individually guided instruction (IPI) for elementary school children. His program uses the concept of a criterion score to determine whether a pupil has mastered a unit of instruction or whether he needs to repeat the unit. A curriculum is viewed as a long branching chain with each of the upper branches depending upon those beneath. Thus, having "mastered" the lower level branches one can proceed to master those branches above. The entire program is individualized in the sense that the pupil may proceed at his own rate through each unit of a subject. Every pupil is exposed to the same units of instruction, but some pupils achieve criterion scores at various rates. Teacher aids take care of the testing and scoring of the criterion referenced test. The setting of criterion scores was done by members of Glaser's staff.

So far, most users of criterion referenced tests have set a seemingly high criterion score. A criterion of 80 - 90% correct responses is not an uncommon level. A closer look at the IPI program shows

clearly why such a high criterion level is not unreasonable. First, the criterion referenced tests are generally composed of items from the lower to middle level of the cognitive scale since most items test routine skills and simple applications of concepts presented. Secondly, the criterion referenced test is administered immediately following the instructional unit it is designed to test. Thirdly, the pupil is conditioned for some items on the test by being exposed to very similar items introduced in the instructional unit. Fourthly, if a pupil does not meet criterion he repeats the same unit again and takes the same test over. Therefore, a criterion level of 80 - 90% is not an unreasonable goal.

The above discussion of criterion scores related to mastery learning seems to imply that they are only useful for making decisions about individuals. However, important decisions must also be made in formative curriculum evaluation. Weaknesses must be identified so materials can be improved. Therefore, it is reasonable to ask if the single score criterion used to dichotomously classify individuals can be modified to be useful in formative curriculum evaluation. This question is attacked in Chapter V.

CHAPTER III

THE STUDY

The Setting of the Study

The Wisconsin Research and Development Center for Cognitive Learning was established at the University of Wisconsin in 1964, and is at present one of nine R and D Centers funded under contract with the United States Office of Education. One of the first major projects undertaken by the Center was to develop, implement, and research a six-year television program in mathematics for the elementary school. Aside from the TV aspect, a distinctive feature of the program is its emphasis on inservice education for teachers while providing a sound program in arithmetic for pupils. The program, hereafter referred to as Patterns in Arithmetic (PIA), was developed over the period 1965-69. Grades 1 and 3 were developed the first year, grades 2 and 4 the second, grade 5 the third, and grade 6 the fourth and final year of the project.

Three distinct aspects of the PIA program can be identified at each level: (1) The individual TV tapes (2) Teacher Notes and (3) Pupil Exercises. In general, decisions relative to the success of individual TV tapes were made on a subjective basis after viewing the program in the classroom and getting the comments from teachers at regularly scheduled meeting. Modifications to the Teacher Notes and Pupil Exercises were based upon classroom visits, meetings with teachers, and regularly

scheduled testing activities.

Several evaluation techniques were used in the field testing of grades 1-5 of the program. The techniques ranged from periodic check-up exercises during the developmental years of the program to extensive standard achievement testing done in grades 1-3 following the developmental year. (Braswell, Romberg, 1969) Also, for grades 1 and 3, Educational Testing Service developed several test instruments specifically designed to fit the terminology and objectives of PIA. These tests were administered after the developmental year when grades 1 and 3 were in final form. In addition to the testing activities, periodic meetings were held with the teachers using the developmental materials, and regular classroom visits by members of the PIA staff were used in evaluating the materials.

Testing done during the development of programs for grades 1-5 consisted of preparing periodic check-up exercises based upon content most recently covered. While useful, this method provided little opportunity to measure what happened to a concept or skill over the entire year. Nor did the testing provide the staff that was responsible for sequencing and planning program content with concrete evidence relative to what pupils knew about a topic before it was presented. It is reasonable to believe that the staff could assemble more efficient materials if reliable data representing program objectives over the entire year were available. A step directed toward this goal was taken at the end of PIA Grade 5.

Near the end of the developmental year of PIA-5 a modest pool of items was developed to assess the degree in which program objectives

had been accomplished and to provide staff with data covering a variety of concepts and skills that would be useful in program planning over the summer for PIA-6. In all 180 items were arranged on nine, 20-item tests. The tests were mailed in quasi-random order to the various classes participating in the developmental year program. Typically, about three members of each class responded to each of the nine tests. The net result was that between 154 and 172 pupils responded to each test, and reasonably stable estimates of performance on 180 items were secured.

To interpret the data nine major content areas (e.g., fractions, geometry, division) were identified for PIA-5. Each item was classified as belonging to one or more of these areas. Of the 180 items, 40 could be identified as relating to fractions. Traditional testing would have provided information on only a total of approximately 40 items with perhaps 6 or 8 related to fractions. Each of the nine content areas had many important dimensions and using 180 items it was possible to evaluate most of them. It should be clear that one is not particularly interested in a test mean or even individual scores. These aspects are only of passing interest. The useful results are the item difficulties and the evaluator merely "uses" individuals to provide stable estimates of item difficulties.

The information provided by this testing procedure was more comprehensive and informative than that provided by the earlier procedure of using a single set of check-up exercises with only 30 or 40 items. With this background there was reason to believe that for PIA-6, a similar but more sophisticated evaluation scheme could be used over the entire

year to provide a profile of change.

The Present Study

This study extends the use of item-sampling to formative curriculum evaluation. The paragraphs below discuss briefly the nature and scope of the study. Other chapters will give a detailed account of the method, results, and conclusions. One anticipated conclusion is that item-sampling can be used effectively as one tool in formative curriculum evaluation. However, considerable preparation must be made if the method is to succeed and the evaluator must be keenly aware of the relationship that exists between the parameters of his population and the conclusions he hopes to draw.

Problems Investigated

Since this study was a formative evaluation of the PIA-6 program, all problems revolved around the technique used for gathering information about pupil performance on a particular set of items relevant to that program. One purpose of gathering the data was to provide periodic information to the staff so that intelligent decisions could be made during development. The study was designed so that information related to particular items as well as more general content areas could be provided. The study also proposes a modification to criterion referencing as applied to individuals that was useful for objectively interpreting the data provided by item-sampling.

The use of item-sampling involves four dependent dimensions: 1) the sample size 2) the accuracy of the sampling estimates 3) the total number of items used and 4) the number of items per test. This study sheds light on the practical relationship among these dimensions as they relate to formative curriculum evaluation.

Design of the Study

During the school year a pupil is exposed to many "treatments" in a variety of subject matter areas. In arithmetic, for example, a pupil may be exposed to five periods of instruction per week. Each treatment is assumed to have some effect in developing concepts and improving skills. For PIA-6 the main instruction occurs twice weekly and is highlighted by a short TV arithmetic program. In all there are 64, 15 minute programs designed to be shown twice weekly. Teachers have definite responsibilities before, during, and after each TV program. Together, the pretelecast, telecast, and follow-up activities provide a concentrated treatment. Ideally, it is desirable to get pre- and post-treatment measures for each program to evaluate its effectiveness. When this is economically unfeasible a compromise must be made and data is collected less frequently. For this study it was decided to collect extensive data four (approximately equally spaced) times during the year. The entire PIA-6 arithmetic program can be viewed as 64 distinct programs or treatments. Periodically the group of sixth graders participating in the study were tested. The results of the testing were used to evaluate the treatments that preceded the testing period. Figure 1 illustrates the general procedure.

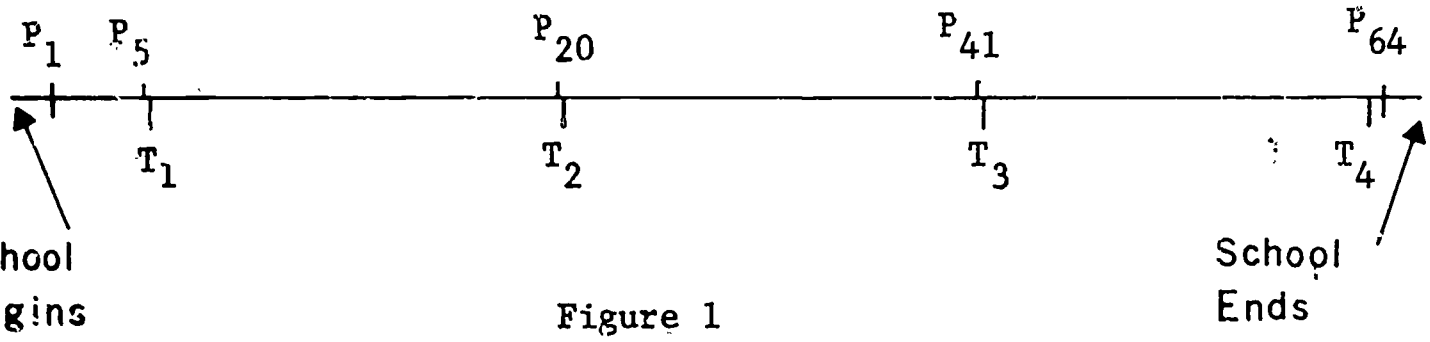

Figure 1

Testing Schema

The horizontal line represents the school year. Above this line P_1-P_{64} represent the 64 programs provided by PIA-6. Below the line T_1, T_2, T_3, and T_4 represent the four testing periods as they occurred during the year. T_1 followed Program 5, T_2 followed Program 20, T_3 followed Program 41 and T_4 followed Program 63.

The design described above is similar to the "Equivalent Time-Samples Design" described by Campbell and Stanley (1963). Observations are made periodically and the experimental variable is introduced repeatedly. If the experimental variable (treatment) is having an effect, one would expect a discontinuity in the measurements made at T_1, T_2, T_3 and T_4. If the same test items are used at each testing period one gets a profile of item change across the year.

In Appendix B a growth profile is plotted for each item. The profile is located beside the test item. Figure 2 illustrates the item profile for item 14 on Test 11. The four testing periods are represented along the horizontal axis and the percent responding correctly (item difficulty) is plotted along the vertical axis. The triangular region(s) along the horizontal axis represents major program coverage at that point during the school year. The location of the triangle is a good approximation of where topic coverage occurred relative to the four testing periods. For this example, the item was very difficult at T_1 and T_2 since there was no attention given to the problem

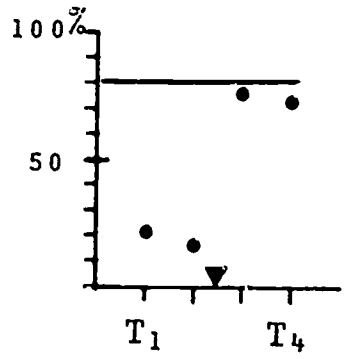

Figure 2

Growth Profile for
Item 14, Test 11

by the program. However, at T_3 there is growth as a result of extensive coverage immediately following T_2. The horizontal line at 80% represents the criterion level associated with this item. The five criterion levels are defined in Chapter V. This item was slightly below criterion at T_3 and T_4.

A profile is obviously useful in evaluating the effectiveness of the program with respect to the item considered. If comparable samples of pupils can be provided at each testing period, then baseline data provided by testing at T_1 can be used to determine the effects of the intervening treatments. Moreover, if the treatment related to a given time occurs between T_2 and T_3, then data from T_1 and T_2 provide baseline data to compare the results from T_3. The method used for obtaining comparable samples of pupils will be discussed in the next chapter. From Figure 1 it is clear that no baseline data was available to measure the success of Programs 1-5 since these programs were presented prior to T_1.

This design is also useful where the treatment may be transient in nature. For example, a concept taught just before T_2 may show up extremely well at T_2, but show considerable decay at T_3. The occurrence of this event would normally suggest revision or the need for review exercises. Item profiles were used during the developmental year to modify the program materials. Examples that show how the data was used are included in Chapter VI.

The sixth grade pupils participating in the study were divided into two groups. One group used PIA-5 the previous year and the other group used some other program. Group membership was determined by noting how a pupil answered the question "Did you use Patterns In Arithmetic in

grade 5?" Separate data analyses were done for each group. The group distinction served as a basis for assessing the cumulative effects of PIA. For example, it was expected that those using PIA-6 who had also participated in PIA-5 would do better on some of the more dependent topics. In particular, geometry should be an area where previous experience in PIA would be beneficial. Any attempt to compare these two groups must be made with reservation since the size of the group that did not use PIA-5 was fairly small and hence the standard error of the estimate is rather large.

The test items were gathered during the summer of 1968 and covered objectives developed during the planning sessions over the spring and summer months of 1968. Some test items were written by the PIA staff and a sizeable number were taken from existing test instruments. The sources of the latter type and general characteristics of the items will be discussed in Chapter IV.

Following the selection of the item pool twelve, 20-item tests were assembled. The twelve different tests served as a basis for all testing done during the year. The testing periods were in September (T_1), November (T_2), February (T_3) and May (T_4). At each testing period <u>all</u> tests were administered to the two groups--the group that used PIA in grade 5 and the group that used some other program in grade 5. The tests were numbered from 1-12 and pupils were <u>assigned</u> a test number at each testing period. It was desirable that a pupil take a <u>different</u> test at each testing period while maintaining a degree of randomness that would not affect the item-sampling model. A class list was kept for all classes and a copy sent to the teacher to indicate which

numbered test a pupil should take. A sample class list is included in Appendix B. The rationale for having a pupil take a different test each time and the method of assigning tests to pupils is discussed in Chapter IV.

A general definition of item-sampling was presented in Chapter I. From the discussion above it is clear that this definition was approximately satisfied at each of the four testing periods. Namely, a set of 240 items (12 tests, 20 items per test) was broken up into twelve tests (subsets) of twenty items each. The break-up was not strictly random as required by item-sampling. To meet conditions (2) and (3) described above it was necessary to control the assignment of items to tests. A strict random assignment of items to tests would not necessarily have provided "balanced" tests. Each test was then assigned randomly at each testing period to a subset of pupils in the study group.

Many, if not most, of the items were difficult at the beginning of the year. However, at various points during the year most of the items showed a pattern of decreasing difficulty, especially on the test administration following the period of topic coverage. In most cases, item growth could be explained directly by the program.

The technique used in the study should be of interest to other persons involved in formative curriculum evaluation. It is certainly reasonable to believe that using two, three, or even four hundred well-chosen items for a formative evaluation should provide more information to project personnel than using a 40 or 50 item standardized test.

When doing any kind of testing it is theoretically nice to ask the

question "How can I get the most information per testing time involved?" Once you have selected a pool of items to cover program objectives then comes the question of how to make the most efficient use of these items. This study sheds light on how many responses to an item are necessary to obtain a stable P^+ (percent responding correctly to an item). Will 100 random responses to an item produce a more useful P^+ than 50 responses, or are 150 responses necessary? For a given pool of items the answer to this question can be used to reduce the testing time involved or decrease the population required for the study.

CHAPTER IV

THE INSTRUMENTS AND THE POPULATION

The Instruments

The final product consisted of twelve, 20-item tests (see Appendix B). Since each class would be taking a variety of tests at each of the testing periods, it was desirable that they be similar in length. In addition to the length factor the following other similarities existed between the tests.

1) The first 13 problems were multiple choice and the last 7 were completion (work out the answer and record it).
2) Every long division problem with a remainder was to be worked on the back of the answer sheet.
3) All answers were recorded on a one page answer sheet which was stapled to the question sheets, but removed at the beginning of the testing session.
4) No matter which test was taken, all pupils received the same set of basic instructions from the teacher.

Previous testing done in PI.. .. ..emonstrated the general format adopted was convenient and that instructions were easy to follow. All multiple choice items included 3 or 4 distractors. Usually both types appeared on a test, but there was no evidence that this caused any difficulty.

Constructing the instruments was the most difficult part of the

study. They had to be constructed well in advance of the beginning of the 1968-69 school year since the first administration was in September, 1968. The first TV program was taped on July 9, 1968. By this date the entire pool of items had already been constructed and the tests were on the assembly line. It is obviously very difficult to extrapolate program content over an entire year. A crude outline of program content was available and it was from this outline that Table 1A was constructed. Table 2A specifies approximately the percent of coverage each major topic and sub-topic should receive relative to the entire set of twelve tests. These tables are located in Appendix A.

Some topics were not completely planned prior to construction of the tests. Non-metric geometry was one such topic. Rather than risk including irrelevant items, the less well defined areas were not adequately covered. Quarterly Check-Up exercises designed for inclusion in Pupil Manuals were used to test these less well defined areas. The Check-Up exercises were composed after related programs had been planned in detail.

Basic terminology established in PIA grades 1-5 was used in the writing of items for PIA-6. The items chosen to satisfy Tables 1A and 2A came from three sources: 1) Tests supplied by Educational Testing Service (ETS) 2) Tests used in the National Longitudinal Study of Mathematical Abilities 3) Items written by the PIA staff. The following tests furnished by ETS were reviewed and one or more items were used from each of these tests:

The Cooperative Mathematics Test: Arithmetic; Forms A, B, and C

The Ohio Survey Tests, Grades 6 and 8, 1967 edition

The Cooperative School and College Ability Tests (CSCAT), Forms 4A, 4B, and 5B

It should be mentioned that some items used from existing tests were modified slightly to be consistent with the terminology and objectives of PIA. This is a reasonable thing to do because no attempt is being made to compare PIA pupils to any previously established norms.

The task of assembling the pool of items occupied the majority of the month of June, 1968. Items were collected for each major heading and subheading listed in Table 1A. They were assembled by topic area (e.g., all items on fractions were placed together) and reviewed by at least two other staff members. Items were collected in excess of the number needed to allow for culling. Various staff members made comments on the appropriateness, terminology, etc. of each item. Following this stage a reduced pool of "acceptable" items was formed. The final reduced pool contained about 350 items from which 240 were to be selected to fit the twelve, 20-item tests. There is nothing magic about the number 12. In fact, the present study would have been better if more item types could have been used. However, as for any study, its scope depends upon several important factors: the availability of staff, the funds to support the project, and size of the population.

In constructing instruments for a formative evaluation one ideally seeks maximum information at minimun inconvenience to teachers and administrators. Class time for testing is certainly a factor. In the present study it was felt that 30-35 minutes of testing time at four times during the year was not unreasonable. By using tests of 20 items

each, it was possible to take up a short amount of class time and yet get information on 240 items each time.

Once the acceptable pool of items had been established there remained the task of assembling each of the tests. To accomplish this all items were cut apart from a listing by content area and placed on several large tables. Homogeneous (by content area) items were placed together. The reason for this was to make it easier to get broad coverage on each test. In fact, every effort was made to include a variety of topic coverage and item difficulties on each test. There were two main reasons for having broad coverage on each test. First, if one test contained all items representing one content area (e.g., decimals) then performance at T_1 (the first testing period) would have been almost nil since fifth graders have had little experience with decimals. What makes this bad is not that performance would be almost nil, but that other tests which had more familiar types of items (e.g., measurement) would have required more time to complete. It is not desirable to have one test that requires 5 minutes for completion (because the pupil knows nothing about the questions) and another that requires 15-20 minutes for completion (because the content is more familiar). Second, by having broad coverage on each test one gets a more realistic picture of group performance on a given topic simply because more pupils respond to items which cover the topic. This tends to make the results of the testing more generalizable.

Although the above are the two main reasons for having broad coverage the pupils probably prefer variety to homogeneity in coverage. Thus pupils who are weak in one area may find problems they can work in other areas and not get discouraged. In addition to a variety of cover-

age on each test there were also a variety of item difficulties. Each test started with a very easy item (even at T_1), but the items did not get progressively harder. Easy and hard items were intermixed throughout the test to avoid having the pupil become discouraged early. The fact that these tests were not speed tests makes the intermixing of easy and difficult items more reasonable. In fact, although a time limit of 35 minutes was specified for the testing session, rarely did a pupil fail to respond to the final item. Thus, each test was constructed by selecting items from a variety of content areas and representing a variety of levels of difficulty. After Test 9 was constructed a bit of accounting was done to see how the overall coverage compared with Table 2A. Tests 10-12 were then constructed keeping in mind those areas which were neglected in Tests 1-9. Also, a few test items were interchanged to provide for better balance. When all tests had been assembled via cutting and pasting, each item was proofed. Each test was then typed and proofed again before being duplicated. A few errors were made in the final version, but none serious enough to cause problems. Some test items were modified slightly from the first testing period. These items and the modified form are given in Appendix B. No items were changed which represented content covered in PIA-6 before T_1 (the initial testing period). Significant changes in an item would interfere with one's ability to interpret results. If one is going to make statements about pupil performance on items or groups of items that are administered at several times during the year, it is absolutely necessary to hold the item constant and not manipulate instructions, terminology, distractors, or even the location of the item

on the test. The effect of such manipulation is not an objective of a formative evaluation.

At each of the four testing periods (T_1, T_2, T_3, T_4) one gets a measure of item difficulty for each of the 240 items. The extent to which these measures (estimates) are useful depends upon how accurately the estimates approximated the true item difficulty for the population participating in PIA-6. For a population (number of classes) of fixed size this accuracy depends upon how well the sample represents the population, i.e., is the sample random. If one is confident that the sample is random then the response pattern follows a binomial distribution and a confidence interval can be placed about the difficulty for each item. In general, very easy or very hard items require fewer random responses to establish stable estimates of item difficulty than middle difficulty items (those which are answered correctly by 40-60% of the respondents).

TABLE 1
STANDARD ERROR OF MEASUREMENT FOR RANDOM SAMPLE SIZES 130 AND 30 WHEN P IS THE TRUE PROPORTION KNOWING THE CORRECT ANSWER

	STANDARD ERROR	
P(q)	N = 130	N = 30
0.1 (0.9)	0.024	0.055
0.2 (0.8)	0.035	0.073
0.3 (0.7)	0.040	0.084
0.4 (0.6)	0.042	0.089
0.5 (0.5)	0.044	0.091

Table 1 shows the standard error of measurement for random samples of size 130 and 30. This table assumes a binomial model with p representing the true proportion who know the correct answer. It is clear from

this table that the error of measurement increases as p increases from 0.1 to 0.5. The standard error function for the binomial distribution is, of course, symmetric. The numbers 130 and 30 were chosen because they approximate the average number of responses to each test for groups which will be discussed later. From this table it is obvious that for a random sample of size 30 one does not get a very stable estimate of item difficulty, particularly for p in the range 0.2 - 0.8.

To get a measure of how stable the estimates of item difficulty actually were, four items appeared on two different tests. For example: the problems at Test 3, item 7 and Test 10, item 3 were the same. Three other problems appeared on two different tests. Both the group that used PIA-5 and the group which did not use PIA-5 responded to every test. Since these groups differed in size it was possible to observe how stable the sampling estimates were for three cases: (1) The group not using PIA-5 (N-Group) (2) The group that used PIA-5 (Y-Group) and (3) The N-Group and Y-Group combined (C-Group). On the average about 30 students responded to each test from the N-Group, 90 students from the Y-Group and 120 students from the C-Group. In general there was a tendency for the larger group size to yield more stable (less variable) estimates (see Table 2). A few cases occurred in which the estimates for the smaller sample sizes were closer than those based upon a larger sample size. However, this is not unusual.

Table 2 clearly shows that about 120 random responses will produce a fairly useful estimate of the item difficulty--useful in the sense that a given item shows stability when located on two different tests. It is important to know that the response pattern of the sample is

TABLE 2

ITEM STABILITY REPORTED AS PERCENT RESPONDING CORRECTLY AT T_1-T_4 FOR FOUR TEST ITEMS THAT APPEARED ON TWO DIFFERENT TESTS. RESULTS ARE REPORTED FOR THREE GROUPS: N-GROUP, APPROXIMATE SIZE 30; Y-GROUP, APPROXIMATE SIZE 90; C-GROUP, APPROXIMATE SIZE 120

ITEM	ITEM LOCATION	T_1 PERCENT N	Y	C	T_2 PERCENT N	Y	C	T_3 PERCENT N	Y	C	T_4 PERCENT N	Y	C
A	(3,7)*	63	84	78	68	82	76	83	82	83	89	86	87
	(10,3)	65	77	75	83	80	81	73	88	83	87	93	92
B	(6,16)	6	23	18	46	64	56	61	49	52	64	64	64
	(8,14)	6	18	15	41	65	59	53	68	63	66	70	69
C	(9,13)	7	10	10	27	22	23	19	33	29	43	41	41
	(12,6)	18	15	17	28	23	30	30	26	28	16	39	33
D	(1,19)	Modified			68	55	58	67	65	66	61	62	62
	(7,19)	at T_1			48	56	55	56	61	59	49	54	53

* (3,7) Refers to Test 3, item 7

typical of the group because the inferences one makes from the growth profiles are based upon this assumption. At each testing period a different random sample of pupils respond to any given item. If a significant change in performance on an item occurs one must be confident that the change is a result of instruction and not a function of the particular sample responding to the item. Chapter VIII will discuss the error of the sampling estimates in detail.

Characteristics of the Population

Letters inviting participation in the developmental year of PIA-6 were sent to administrators representing areas which had participated in the earlier developmental years of PIA. The tapes were aired during the developmental year over WHA-TV, the University of Wisconsin TV station. Since reception range of WHA is limited to within approximately 50 miles of Madison, Wisconsin, participants were restricted to this area. Based upon response to the invitation, approximately 100 classes expressed interest in participating in PIA during the developmental year. Test materials for the first testing period were sent to all schools who "signed-up" during the summer of 1968.

A letter informed prospective participants of the planned testing procedure. Of the 96 sets of test materials sent out, 62 sets were administered and returned as directed. The classes represented by these 62 sets of materials formed the population for the formative evaluation. Most of the classes that participated in the study represented medium to small size rural Wisconsin communities. Six classes from Monona Grove, an upper, middle-class suburb of Madison, Wisconsin, and three classes from Janesville, Wisconsin, a city of about 50,000

also participated in the study. None of the classes participating in the study represented large industrial communities. Also, it is likely that few participants represented minority groups.

Since many areas involved in the study had participated in PIA-5, it was expected that the majority of participants would not be new to the Patterns program. In fact, 1085 of the 1581 pupils participating in T_1 participated in PIA-5 during the 1967-68 developmental year. Table 3 shows how characteristics of the population changed over the four testing periods. New pupils are those who participated in the testing at a given period, but did not participate previously. No new pupils are included in the analysis following T_2. Pupils lost are those absent or transferred based on results from T_1 and T_2.

TABLE 3

CHARACTERISTICS OF THE POPULATION

	Testing Period			
	T_1	T_2	T_3	T_4
Number classes	62	58	58	57
Number Pupils	1579	1484	1465	1432
New Pupils	-	52	0	0
Pupils Lost	-	87	120	105
New Classes	-	0	0	0

For the initial testing period materials (see Appendix B) were sent to school personnel since the names and addresses of most teachers were not available. Depending upon the school district the following types of individuals acted as coordinator for the project: mathematics supervisor, school principal, district administrator, elementary super-

visor, or a staff member under the superintendent of schools. These individuals performed an invaluable service, not only in the testing program, but in providing overall support for and implementation of the project.

A form was sent out with each package of materials (see Appendix B) to provide the staff with an accurate record of names and addresses of teachers participating in the testing program. As answer sheets were returned for each class, a detailed class list was made for each of the 62 classes who returned materials. The class list contained a place to record the following information:

1) The number of pupils in the class
2) The complete name and address of the teacher
3) Approximate time required for the testing for each of the four testing periods
4) The number of pupils absent
5) The name of each pupil participating in the testing
6) The number of the test taken by each pupil at each testing period
7) A place to indicate participation status in PIA-5.

A sample class list is included in Appendix B.

Assigning Pupils to Tests

Following the initial testing period tests were assigned to each pupil for the remaining three periods. Since the method for accomplishing this was one of the important decisions in the evaluation, a detailed discussion of the procedure will be given.

First, it was desirable that each pupil take a different test at

each of the four testing periods. The reasons for this are:

1) That some learning specifically related to the test might result. It is not unreasonable to expect that the effect of learning on taking the same test would be different from its effect on taking a different test. Learning, as used here, includes not only becoming familiar with the test format, instructions, etc., but also being affected by the test content. The latter effect would be more likely in this testing situation than in more traditional testing because the test content involves topics that range over the year. Being asked a question on a topic that one is not prepared to answer at the beginning of the year could provide a cue that would influence performance once the content related to that topic is reached on the program.

2) It would be likely that unequal numbers of pupils would duplicate responding to a given test each time if tests were assigned on a random basis. For example, 10 pupils might take Test 1 at both the first and second periods, while 20 pupils might duplicate taking Test 8. It would not be easy to assess the effect of this on the results.

3) The way a pupil approaches or answers a problem initially could affect his response if he received the same test again.

4) Although not likely, without restricted random assignment a few pupils could get the same test all four times. It is more likely that several pupils would take a given test three times.

At the initial testing period the 12 tests were sent out in a repeating pattern of Tests 1-12 (the pattern not necessarily beginning with Test 1) and the teacher was instructed to distribute the tests just like they arrived. Following the instructions led to two desirable outcomes. First, approximately equal numbers of pupils responded to each test (see Table 7, Chapter VI). Second, a class size of between 11 and 24 pupils took each test at least once; one between 23 and 36 took each test at least twice, etc. The second aspect provided balance in the distribution of tests across classes. To insure that this balance continued for the remaining testing periods a restricted random assignment of tests to pupils was made. The restrictions were that no pupil get the same test twice and that the balance established during the initial testing period continue. For example, if 28 pupils participated in the initial testing period then on each subsequent period each of the twelve tests would be assigned to at least two pupils and four (28-24) randomly chosen tests would be taken three times. Hence, for a given class size, n, responding to the initial testing period, $n \div 12$ provides a quotient and remainder--the quotient determined the minumum number (m) of responses to each test and the remainder (r) determined how many tests are taken by one extra person.

To accomplish the restricted random assignment, the following procedure was used:

1. Determine m and r.
2. Use $(m + 1) \times 12$ playing cards numbered 1-12 repeatedly as a model for assigning tests to pupils.
3. Shuffle the cards thoroughly, cut the deck and note the number

on the top card. Compare this with the test number taken by the first person at T_1. If the number is the same, put the card at the bottom of the deck and look at the number on the next card, repeating the above process. If the number is different, then record this number as his assigned test for T_2. Record a tally for this test.

4. After the first person is assigned a test continue, without shuffling to determine the test the second person should take. Each time a test is assigned record a tally. When r tests have been assigned to one extra person then allow only the minimum number of responses from this point on. Any number that can not be used when it comes up always goes to the bottom of the deck.

5. After assigning tests for T_2 shuffle the entire deck thoroughly and repeat the above process for T_3 and finally, for T_4. When assigning tests for T_3 care must be taken to avoid duplicating a test taken at T_1 or T_2. T_3 must also be considered when tests are assigned to T_4.

The only conflict that can arise in this procedure is that near the bottom of the class list one might not have any possible "legal" assignment. For example, to satisfy the restriction that each test gets taken the minimum number of times it might be necessary to assign Test 3 to the last person. However, suppose Test 3 was taken by this person at T_1! To escape this dilemma a random exchange was made with a test assigned earlier on the list.

From the above discussion it is clear that once the original class

list was made and the test taken at T_1 recorded, assignment of tests to pupils could be made for the remaining three testing periods. In November of 1968 all pupils were assigned tests for the remaining three periods. Thus, the future testing sequence for each pupil was known shortly after the first set of testing materials had been returned. In some cases this sequence was never used as three classes dropped out of the study and some pupils transferred out of a given class. New pupils appearing in classes at T_2 were included in the data analysis, but following T_2 no new pupils were included in the analysis that did not participate at T_1 or T_2.

Chapter V discusses criterion referencing as it was used in this study.

CHAPTER V

CRITERION REFERENCING THE ITEMS

Criterion referencing as discussed in Chapter II applied mainly to individuals. In order to make decisions about individuals it is reasonable to set a specific mastery level score (criterion score). However, to use criterion referencing in a formative evaluation it is reasonable to modify the single score criterion established for a set of items. The formative evaluator is interested in item profiles and performance within the various content areas rather than the total test score. The total test score, although an indicator of growth, does not pinpoint where the growth has occurred. While it would be difficult to set a criterion level for each item, it is reasonable to classify items and to set a lower-bound criterion for each classification. Items used in the formative evaluation of PIA-6 were classified in one of five types. (Romberg, 1969)

(1) Mastery Level 1 (A): Items in this category are expected to be very easy by the end of the year. Most every pupil should have mastered the content of the item.

An example of an A-Level item is given in Figure 3.

Write "four and twenty-seven hundredths" as a decimal.

Figure 3. Sample A-Level Item

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(2) Mastery Level 2 (B): These items represent topics which receive major emphasis during the year. While items in this category test important objectives, a very high level of mastery is not expected. The majority of PIA-6 items fall in this category.

An example of a B-Level item is given in Figure 4.

Which number below is nearest to zero on the number line?

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) $\frac{1}{16}$

Figure 4. Sample B-Level Item

---

(3) Mastery Level 3 (C): Items in this category represent more complicated aspects of content covered in PIA-6. C-Level items do not test extensions of concepts presented in PIA-6. Story problems which lead to involved computations as well as problems which are conceptually difficult for the average pupil are C-Level items.

An example of a C-Level item is given in Figure 5.

Imagine 3 sacks that contain marbles as above.

Sack A contains 1 white and 1 red marble

Sack B contains 3 white and 3 red marbles

Sack C contains 1 white and 2 red marbles

If you reach in a sack and pick a marble without looking, which sack would the probability of a red marble be the greatest?

a) Sack A b) Sack B c) Sack C d) It makes no difference

Figure 5. Sample C-Level Item

(4) Transfer Level 1 (X): Transfer Level 1 items involve a minor extension of concepts presented in PIA-6. The introduction of new notation or a problem which requires some insight belongs in the X category.

An example of a X-Level item is given in Figure 6.

Which number below is between $\frac{5}{6}$ and 1?

a) $\frac{1}{3} + \frac{1}{3}$ b) $\frac{5}{6} + 1$ c) $\frac{5}{6} + \frac{1}{7}$ d) $0.30 + 0.20$

Figure 6. Sample X-Level Item

(5) Transfer Level 2 (Y): These items are the more difficult ones used in the testing. Such items are usually conceptually difficult, represent an extension of program content, and contain difficult computations.

An example of a Y-Level item is given in Figure 7.

A triangle n inches on a side has a perimeter of 60.3 inches. What is the perimeter of a triangle n + 2 inches on a side?

Figure 7. Sample Y-Level Item

Before these classifications can be used to interpret results they must be quantified. Quantification of each classification consisted of placing a lower-bound criterion on the item difficulty (percent responding correctly). This lower-bound criterion reflected the aspirations of the PIA staff for performance at the end of Grade 6. Table 4 indicates the lower-bound criterion for item difficulties for each classification level. Multiple-choice and free response items are considered separately since the former type involves an element of chance.

TABLE 4

LOWER-BOUND CRITERION FOR EACH CLASSIFICATION LEVEL

Level	Lower-Bound	
	Multiple Choice	Free Response
A	85	80
B	65	60
C	40	30
X	60	50
Y	35	10

For example, a multiple-choice item which is classified level B has a lower-bound criterion of 65. If at the end of PIA-6, 73% of the pupils respond correctly to this item, then criterion is met. On the other hand, if only 32% respond correctly, a serious problem may exist and revision might be in order. Classification of items in this way makes it possible to objectively interpret the test results in terms of expected behaviors and use these results to suggest modifications. Another objective met by this system is that the goals of the curriculum developer are conveyed to the reader.

Table 5 shows how the items used in this study were classified (Items 1-8 on Test 4 were not classified). Two staff members worked together to classify each item. There was disagreement. It was often difficult to decide whether an item should be level B or level X. Level B is the intermediate mastery level and level X represents a

minor extension. Figure 2, Chapter III, illustrates how the classification was used for item 14 on Test 11 (an A-Level, free response item). The horizontal line at 80% indicates criterion for this level item. A graph similar to that in Figure 2 is located beside each classified item in Appendix B.

Although the criterion referencing of items is useful and informative, it is nevertheless arbitrary. An item may not reach criterion for several reasons:

1. The item was measuring a skill other than the one intended.
2. Poor instruction.
3. Coverage related to the item was not as originally planned. If items are chosen before detailed planning, it is possible that some topics will not get the intended treatment. Items related to such topics will naturally be more difficult than anticipated.
4. Criterion was set too high.
5. The estimate of item difficulty is subject to sampling error.

Some of the same reasons may also explain why an item reaches criterion. In particular, (1) and (5) could be responsible for some items reaching criterion.

In summary, a criterion level is an arbitrary level assigned by the developmental staff that reflects their aspirations--it should not be considered as absolute.

TABLE 5

CLASSIFICATION OF ITEMS FOR TESTS 1-12

Item	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7	Test 8	Test 9	Test 10	Test 11	Test 12
1	A	A	A		A	A	A	A	A	A	A	B
2	A	B	B		B	X	X	A	B	B	B	X
3	B	Y	B		B	X	B	X	C	A	C	Y
4	A	A	X		B	B	A	B	B	B	Y	X
5	B	B	X		B	B	B	A	B	B	B	B
6	X	X	X		X	B	C	X	B	B	C	B
7	X	B	A		A	B	A	C	X	C	Y	X
8	A	X	A		B	B	B	X	B	X	B	A
9	B	B	X	A	B	B	B	X	B	X	C	X
10	A	A	B	A	X	B	B	A	Y	Y	Y	Y
11	A	B	Y	X	B	X	X	Y	X	C	C	A
12	A	X	B	B	Y	B	B	B	X	B	B	C
13	X	C	A	A	A	X	B	B	B	B	X	X
14	A	A	A	B	A	B	A	B	A	A	A	A
15	B	B	A	B	B	B	B	B	B	B	B	B
16	B	B	B	B	B	B	A	A	A	B	A	A
17	B	B	B	A	B	A	B	B	A	B	B	B
18	A	B	B	B	C	B	B	A	C	B	C	B
19	B	C	B	A	B	B	B	B	B	B	B	C
20	A	X	B	A	A	Y	C	C	B	C	Y	Y

CHAPTER VI

THE DATA

Item Data and Test Characteristics

A copy of each test is located in Appendix B. Typically, every test was taken by two or three pupils in each class at each testing period. The common set of instructions for accomplishing each administration is also located in Appendix B.

For item-sampling to function properly pupils must have a chance to respond to each item. Therefore, no rigid time limit was imposed. Since it is well known that there are pupils who will work "forever" on a test if given the opportunity, a conservative, maximum time limit of 35 minutes was suggested. The teacher could, of course, take the tests up before 35 minutes. Table 6 summarizes the modal administration date (in terms of programs 1-64) and the average time required for a class to complete the tests (exclusive of distribution of materials and the reading of directions). From this table it is clear that 35 minutes was a conservative time limit.

TABLE 6

LOCATION AND TIME REQUIRED FOR TESTING FOR T_1-T_4

Testing Period	Program Location of Testing	Average Classtime Required
T_1	Between Programs 5 and 6	23.8 minutes
T_2	Between Programs 20 and 21	24.8 minutes
T_3	Between Programs 41 and 42	23.9 minutes
T_4	Between Programs 63 and 64	23.8 minutes

The twelve, 20-item tests were administered four times during the 1968-69 school year. They were administered in September, November, February, and May. Patterns In Arithmetic-6 has 64 distinct programs. The testing periods occurred at Program 5 (T_1), Program 20 (T_2), Program 41 (T_3), and Program 63 (T_4). In order to explain what happens at a given testing period, it is necessary to know the specific content covered prior to that testing period. For example, to interpret test results at T_2 it is necessary to know that between T_1 and T_2 Programs 6-20 were viewed. The effect of the TV programs and related pupil exercises can then be measured by observing pupil performance on related items. Also, the effectiveness of review exercises in maintaining and improving skills and understanding can likewise be measured.

This chapter will present the item data and test characteristics at each of the four testing periods. The information reported for a given testing period is as follows:

1) Item data (reported in percent responding correctly) will be reported for three groups: a group which participated in PIA-5 (Y-Group); a group which did not participate in PIA-5 (N-Group); and a combined group (C-Group) consisting of those pupils in each of the first two groups and any pupils that could not be classified as Y-Group or N-Group. The group distinction made it possible to measure the cumulative effect of PIA. However, the small size of the N-Group makes it impossible to make strong generalizations.

2) The number (N) of pupils taking each test for each of the three groups.

3) The test mean (MEAN), standard deviation (STDEV), and reliability (R) for each test and for each of the three groups.

4) The item correlations between the Y-Group and N-Group, r_{YN}, for each of the twelve tests. For a given test a high correlation between item performance could imply

> that the Y-Group and N-Group are operating from a fairly common base with respect to the concepts and skills being measured by the test. A low correlation between item performance could imply that one group was operating differently with respect to some content measured by the test. For example, if a test item involved terminology or concepts common to PIA but foreign to those pupils using other arithmetic programs, then performance by the PIA group would likely be considerably above that of the other group. This would tend to lower the correlation between item performance.

Table 7 presents the item data and test characteristics for T_1, the September testing period. The data for T_2-T_4 is presented in Appendix C.

Beside each test item in Appendix B is a small graph. This graph represents a growth profile for the item across the year. Figure 2, Chapter III, illustrates the appearance of a sample profile. The four testing periods are plotted along the horizontal axis and the corresponding percents responding correctly are plotted along the vertical axis. These graphs are based upon data reported in this chapter and Appendix C for the C-Group described above.

Data Presented by Content Area

The last section presented an overall picture of how the tests functioned at each testing period. Since each test represents quite varied coverage, it was necessary to assemble the items by content area in order to interpret results. This chapter considers each major topic covered in PIA-6. After reading the concluding chapters it should be clear that the data reported herein represents performance on materials written during the developmental year. Modifications on the "first generation" PIA-6 materials based upon the

TABLE 7: TEST RESULTS FOR T_1

Item No.	Test 1			Test 2			Test 3			Test 4		
	Y	N	C	Y	N	C	Y	N	C	Y	N	C
1	75	65	73	90	85	88	81	80	80	74	48	67
2	71	43	63	82	85	82	73	61	70	76	74	77
3	69	43	62	37	29	35	75	39	64	78	87	80
4	78	73	76	39	26	35	10	21	14	73	71	73
5	26	19	24	25	18	23	38	37	37	78	61	73
6	51	43	47	65	59	65	32	18	28	39	39	36
7	76	65	71	14	12	13	84	63	78	29	29	31
8	97	95	95	34	38	34	51	50	52	38	32	37
9	30	24	29	52	41	49	9	5	8	30	13	27
10	71	70	71	75	65	71	25	21	23	62	55	60
11	69	81	72	38	35	35	22	21	21	37	19	32
12	93	92	93	33	29	31	63	37	55	54	35	50
13	26	22	25	34	18	29	25	37	27	54	23	46
14	64	59	62	85	44	71	65	61	64	34	35	35
15	33	11	27	9	6	8	80	63	75	6	3	6
16	5	0	4	12	15	13	9	8	9	30	35	31
17	34	14	29	50	65	53	25	29	26	36	45	40
18	45	19	38	47	47	46	49	39	46	6	3	5
19	36	32	36	0	0	0	33	37	35	84	94	85
20	18	27	23	47	29	42	49	53	49	65	48	61
N	89	37	133	92	34	133	88	38	132	89	31	127
MEAN	10.65	8.97	10.20	8.66	7.47	8.21	8.95	7.79	8.63	9.83	8.42	9.51
STDEV	3.35	2.81	3.40	3.15	3.24	3.25	3.00	3.18	3.15	3.53	3.95	3.70
R	0.71	0.59	0.71	0.66	0.69	0.69	0.61	0.63	0.64	.72	0.80	0.75
r_{YN}	0.92			0.90			0.88			0.89		

TABLE 7: TEST RESULTS FOR T_1 (CONTINUED)

Item No.	Test 5			Test 6			Test 7			Test 8		
	Y	N	C	Y	N	C	Y	N	C	Y	N	C
1	98	79	93	90	88	88	91	79	89	95	94	95
2	64	33	55	18	22	19	15	17	15	89	82	87
3	70	64	68	25	28	24	74	66	73	11	9	10
4	53	24	46	6	6	6	31	31	31	56	42	53
5	4	9	5	77	72	77	43	28	42	51	39	47
6	11	18	13	36	16	29	43	48	44	53	52	53
7	94	91	93	51	41	49	99	90	97	19	39	24
8	64	64	63	54	50	55	58	52	56	71	55	67
9	28	36	30	37	25	33	42	41	40	68	55	64
10	65	52	59	52	28	48	73	62	70	58	67	60
11	34	27	32	33	28	33	23	24	23	11	9	12
12	8	12	9	23	47	30	65	59	64	49	33	44
13	55	36	49	27	31	29	69	48	64	28	24	27
14	55	33	40	52	53	54	72	48	67	18	6	15
15	55	24	47	5	6	5	24	10	20	1	0	1
16	26	24	26	23	6	18	70	66	69	93	79	88
17	5	6	5	0	0	0	3	7	4	12	12	12
18	10	6	9	5	0	3	2	3	3	67	52	61
19	23	6	18	23	31	26	53	41	51	46	48	47
20	65	76	69	4	0	2	0	0	0	6	9	7
N	88	33	128	84	32	125	88	29	124	95	33	135
MEAN	8.75	7.21	7.90	6.40	5.78	6.27	9.5	8.21	9.23	9.02	8.06	8.74
STDEV	2.80	2.45	2.72	2.82	2.97	2.79	2.76	3.29	2.90	2.78	2.97	2.88
R	0.60	0.45	0.57	0.63	0.69	0.62	0.59	0.69	0.63	0.61	0.64	0.63
r_{YN}	0.89			0.91			0.97			0.95		

TABLE 7: TEST RESULTS FOR T_1 (CONTINUED)

Item No.	Test 9			Test 10			Test 11			Test 12		
	Y	N	C	Y	N	C	Y	N	C	Y	N	C
1	52	66	55	79	73	76	97	97	97	70	65	69
2	68	49	62	48	46	49	35	25	33	43	44	43
3	46	56	50	77	65	75	38	34	37	2	3	2
4	5	0	4	86	68	81	22	38	26	23	9	19
5	88	83	87	59	35	53	34	22	31	5	3	5
6	30	29	31	63	57	60	35	19	30	15	18	17
7	28	17	25	64	49	59	29	34	29	83	91	84
8	52	41	48	34	32	33	56	53	54	83	88	83
9	13	12	13	42	38	41	32	28	31	33	24	29
10	39	34	38	35	27	31	13	19	14	31	38	32
11	70	66	68	7	2	6	84	78	82	79	50	72
12	63	44	57	41	41	41	74	69	73	43	38	43
13	10	7	10	74	27	61	75	69	75	38	32	35
14	23	20	21	80	62	75	21	28	23	88	65	81
15	14	10	13	11	14	11	38	28	35	47	26	43
16	84	61	76	27	27	27	16	3	13	84	74	82
17	87	59	79	6	8	7	55	53	55	70	59	64
18	17	22	18	20	3	14	16	19	18	77	65	73
19	72	59	67	9	3	7	9	6	9	14	6	13
20	40	44	40	2	0	1	0	3	1	2	0	2
N	93	41	141	95	37	138	98	32	136	86	34	129
MEAN	9.00	7.78	8.61	8.64	6.76	8.09	7.80	7.25	7.65	9.30	7.97	8.91
STDEV	3.08	3.50	3.24	3.01	3.03	3.13	2.40	2.91	2.49	2.91	3.23	3.00
R	0.66	0.73	0.68	0.65	0.65	0.68	0.42	0.62	0.47	0.65	0.72	0.66
r_{YN}	0.92			0.96			0.91			0.95		

formative evaluation should strengthen those areas which appeared weak.

For each topic all related items were retrieved from the twelve tests. For example, all items related to fractions were grouped together. Frequently an additional breakdown was made, especially for the more significant topics like fractions, decimals, and geometry. The data for each topic is presented according to the following format:

1. The major topic (content area) is identified (e.g., Fractions).

2. Subtopics within the major topic area are identified where necessary (e.g., Addition and Subtraction of Fractions).

3. The location of the item and a summary of its content is indicated. The item location is indicated by a number pair such as (3, 16) which indicates Test 3, Item 16. The content summary is often very abbreviated and it may be necessary to refer to the specific item in Appendix B.

4. Results are reported at each testing period for the Y-Group (the group using PIA-5) and the N-Group (the group not using PIA-5).

5. A code is used between testing periods to indicate how PIA-6 coverage related to the item:

    a) A "x" indicates major attention was given to the content tested by the item.

    b) An "r" indicates significant review related to the item appeared.

    c) A "c" indicates that coverage related to the item appeared. However, items having this code often require several skills, some of which may not have been covered.

How to Interpret Topic Data

The first major topic presented is that of fractions (see Table 8). The first item involves addition of fractions with unequal denominators. The item can be found on Test 1, Item 15. For the Y-Group, 33 per cent responded correctly at T_1 and 51 per cent responded correctly at T_2. The * between T_1 and T_2 indicates that this skill received major attention between T_1 and T_2 (Programs 6-20). If one would like to know how many pupils responded to this item, Table 7 indicates that 89 pupils in the Y-Group responded at T_1. The 89 responses represent random responses from approximately 45 different classes. This is true in general for the Y-Group: the number responding to a given test (item) represent a random sample of pupils from approximately 45 classes that had previously used PIA-5. The number responding in the N-Group represent a random sample of pupils from approximately 15 classes that had previously used an arithmetic program in grade 5 other than PIA-5. A word of caution -- since only about 30 pupils belong to the N-Group, the item data for this group must be interpreted with care.

The data for the topic "Fractions" is presented in this chapter. The following topics are presented in a similar fashion in Appendix C: Decimals, Ratio, Geometry-Measurement, Geometry--Non-Metric, Integers, Long Division, Sentence, Number Line, Relation and Functions, Charts and Graphs, and Probability. Some items were appropriately classified in more than one category.

TABLE 8: RESULTS BY CONTENT AREA: FRACTIONS

		Results											
		T_1			T_2			T_3			T_4		
		Y	N		Y	N		Y	N		Y	N	
A. Addition and Subtraction													
Location	**Content**												
(1,15)	Add: $\frac{1}{8} + \frac{5}{6}$	33	11	*	56	64	r	61	56	r	62	68	
(2,14)	Add: $\frac{2}{3} + \frac{7}{3}$	85	44	r	88	63	r	73	54	r	80	63	
(3,17)	Subtract: $5 - 2\frac{3}{7}$	25	29	*	42	43		61	51	r	85	63	
(4,14)	Subtract: $\frac{3}{5} - \frac{1}{4}$	34	35	*	61	55		66	58	r	75	69	
(5,14)	Add mixed numbers: $1\frac{1}{4} + 2\frac{3}{8}$	55	33	*	82	58		61	58	r	6[illegible]	58	
(6,16)	Add: $0.3 + \frac{1}{2}$	23	6	*	64	46	c	49	61	c	64	64	
(7,15)	Subtract: $1\frac{3}{8} - \frac{1}{2}$	24	10	*	59	35		51	61	r	61	54	
(8,16)	Add: $\frac{1}{8} + \frac{5}{8}$	93	79	r	97	83	r	96	92	r	90	100	
(9,17)	Add: $\frac{35}{36} + \frac{1}{36}$	87	59	r	88	83	r	85	74	r	88	87	
(10,14)	Subtract: $\frac{3}{15} - \frac{1}{15}$	80	62	r	94	79		91	81	r	97	94	
(12,16)	Subtract: $3\frac{1}{2} - 3\frac{1}{2}$	84	74	r	92	86		91	94		92	97	

TABLE 8: FRACTIONS (CONTINUED)

B. Multiplication and Division		Results													
		T_1			T_2			T_3			T_4				
Location	Content	Y	N		Y	N		Y	N		Y	N			
(1,20)	Multiply: $\frac{4}{5} \times \frac{9}{8}$	18	27	*	75	75	r	80	69		69	86			
(2,9)	If $2 \times N = \frac{1}{3}$, N = ?	52	41		46	54	*	60	46	r	70	63			
(2,15)	Divide: $8 \div \frac{1}{10}$	9	6		5	4		28	11	*	47	48			
(2,16)	$\frac{1}{3} \times \square \times 5 = 5$	12	15		12	0	*	46	46	r	47	33			
(2,18)	Story problem involving $6 \times 3\frac{1}{2}$	c 47	47		45	54	*	59	60	*	66	70			
(3,16)	What is $\frac{1}{3}$ of $\frac{1}{5}$?	9	8	*	37	14	r	47	51		55	37			
(4,16)	What is $\frac{2}{3}$ of 27?	30	35	*	37	32	r	58	56		69	52			
(5,9)	Write "four fifths of twenty"	28	36	*	40	36	r	70	61		74	55			
(5,16)	Divide: $\frac{9}{2} \div 3$	26	24		22	36		39	42	*	60	61			
(6,4)	Multiply, cancel: $\frac{4}{7} \times \frac{3}{4} \times \frac{7}{12}$	6	6	*	36	26	*	63	61		71	61			
(6,15)	Divide: $\frac{3}{5} \div \frac{9}{10}$	5	6		3	9		16	33	*	52	42			
(7,17)	Multiply: $\frac{2}{5} \times (\frac{5}{2} \times 11)$	3	7	c	11	10	*	62	53		53	69			
(8,15)	Multiply: $3\frac{1}{7} \times 2\frac{1}{9}$	1	0		9	3	*	35	26		30	49			

TABLE 8: FRACTIONS (CONTINUED)

		Results											
		T_1			T_2			T_3			T_4		
		Y	N		Y	N		Y	N		Y	N	
B. Multiplication and Division (ctd.)													
Location	**Content**												
(9,15)	Divide: $7\frac{1}{2} \div 5$	14	10		10	3		33	23	*	57	63	
(11,14)	Multiply: $4 \times 3\frac{1}{2}$	21	28		12	14	*	80	71		69	82	
(11,19)	Story problem involving $10 \times 35\frac{9}{10}$ ...	9	6		6	5	*	42	40		49	50	
C. Number Line													
Location	**Content**												
(1,4)	Name $1\frac{3}{4}$ on number line	78	73	r	81	86	r	86	81		89	93	
(5,2)	Betweenness on number line	64	33	*	75	53		76	53	c	70	61	
(7,5)	Name point B? $\frac{11}{4}$	43	28	*	68	45	r	83	75		63	51	
(11,12)	Which number nearest zero? $\frac{1}{16}$	74	69	r	78	78	c	89	79		88	89	
(12,4)	Name a pt. on the number line: $\frac{5}{8}$	23	9	r	23	33	c	26	33		34	19	

TABLE 8: FRACTIONS (CONTINUED)

		Results										
		T_1			T_2			T_3			T_4	
		Y	N		Y	N		Y	N		Y	N
D. Terminology												
Location	**Content**											
(3,16)	What is $\frac{1}{3}$ of $\frac{1}{5}$?	9	8	*	23	11	r	47	51	55	37	
(3,20)	Basic fraction for $\frac{36}{42}$	49	53	r	58	50	r	74	66	81	71	
(4,9)	Which corresponds to a counting number?.	30	13	c	48	46	*	66	53	58	59	
(4,16)	What is $\frac{2}{3}$ of 27?	30	35	*	37	32	r	58	56	69	52	
(5,9)	Write "four fifths of twenty"	28	36	*	40	36	r	70	69	74	55	
(8,5)	Change $3\frac{1}{2}$ to fraction	51	39	*	77	66	r	78	63	r 87	97	
(9,2)	Change $\frac{15}{8}$ to mixed number	68	49	*	82	60	r	77	71	r 89	80	
(9,6)	LCD for $\frac{1}{6}$ and $\frac{1}{8}$	30	29	*	76	60		53	55			
E. Sentences												
Location	**Content**											
(2,5)	$F = (\frac{9}{5} \times c) + 32$; $C = 0$. $F = ?$	25	18	c	41	38	*	51	34	r 62	56	
(2,9)	$2 \times n = \frac{1}{3}$; $n = ?$	52	41		46	54	*	60	46	r 70	63	

TABLE 8: FRACTIONS (CONTINUED)

		Results											
		T_1			T_2			T_3			T_4		
		Y	N		Y	N		Y	N		Y	N	
E. Sentences (ctd.)													
Location	**Content**												
(2,16)	$\frac{1}{3}$ x ? x 5 = 5	12	15	c	12	0	*	46	46	c	47	33	
(10,20)	F = ($\frac{9}{5}$ x c) + 32; c = 30. F = ?	2	0		11	3	*	40	27	r	45	42	
(11,2)	2 x n = 9; n = ?	35	25	c	49	35	*	72	67		72	61	
F. Other Topics													
Location	**Content**												
(2,4)	Convert 2 $\frac{1}{2}$ to decimal	39	26	*	68	67	r	75	63		82	78	
(2,7)	Find S for 0.083 = 83 x $\frac{1}{S}$	14	12	*	44	33		52	29	c	43	30	
(2,8)	Scale Drawing: Read 2 $\frac{1}{2}$ on ruler	c 34	38	c	46	29	c	62	31	*	60	59	
(3,8)	Find area shaded part of rect. ($\frac{1}{3}$)	c 51	50	r	54	39	r	62	54	r	64	70	
(4,9)	Which fraction is counting number? $\frac{6}{3}$ ..	30	13	r	48	46		66	53		58	59	
(5,5)	m = 4; $\frac{(3 \times m) + 8}{m}$ = ?	4	9		22	19	*	45	22	r	41	19	
(5,12)	Relate $\frac{1}{n}$ to $\frac{1}{n+1}$ (n = counting #)	8	12	c	14	14	c	10	6		10	16	

TABLE 8: FRACTIONS (CONTINUED)

		Results											
		T_1			T_2			T_3			T_4		
		Y		N	Y		N	Y		N	Y		N
F. Other Topics (ctd.)													
Location	Content												
(5,18)	Story problem involving $6 \div \frac{2}{3}$	c	10	6	6		8	c	23	28	*	30	23
(6,3)	Five boys share 3 melons equally		25	28	c 41		37	c	30	33	c	31	45
(6,9)	Area $\frac{1}{2}$ of sq. (2" on a side)	c	37	25	c 32		51	*	35	36	c	46	67
(6,18)	4 BLACK, 7 RED: Prob. of RED		5	0	5		3		6	15	*	47	61
(7,10)	Which sentence is true? $\frac{1}{12} = \frac{6}{72}$	r	73	62	r 81		58		70	56	c	73	71
(7,12)	Which regions shaded show $\frac{1}{3}$?		65	59	c 63		58		63	71	c	68	60
(8,7)	$1\frac{1}{2}$ qts. is what fraction of gal.	c	19	39	c 29		41	c	36	39		40	40
(8,9)	Comma notation for fractions		68	55	c 75		66		80	79		90	80
(9,3)	$4\frac{1}{2}$" represents 72 mi.; 7" represents ? mi.	c	46	56	41		43	c	48	48	*	62	60
(9,11)	Distance between $1\frac{1}{2}$ and $3\frac{1}{2}$ on # line		70	66	c 68		60	c	75	77		78	77
(9,13)	Area of sq. $\frac{1}{2}$" on a side	c	10	7	c 22		27	*	33	19	c	41	43
(10,2)	$\frac{1}{3}$ of what number = 9?		48	46	c 47		34	*	50	43	r	49	48

TABLE 8: FRACTIONS (CONTINUED)

		Results													
		T_1			T_2			T_3			T_4				
		Y	N		Y	N		Y	N		Y	N			
F. Other Topics (ctd.)															
Location	**Content**														
(10,6)	Which fraction is ≠ other four?	63	57	r	67	66		68	68		87	71			
(11,4)	Ratio of lengths $\frac{1}{2}$" : $\frac{3}{4}$"	c 22	38		23	14		14	10	c	34	36			
(11,10)	Formula $L = \frac{1}{4} \times p$	13	19		20	14	*	35	29	c	22	25			
(12,1)	Which fraction is least? $\frac{1}{820}$	70	65	*	91	83		86	94		83	84			
(12,5)	Area of 3, 4, 5 right triangle	5	3		6	8	*	16	12	r	29	19			
(12,13)	Which number is between $\frac{5}{6}$ and 1?	38	32	r	42	50		41	48		38	28			

CHAPTER VII

USE OF THE RESULTS

For Revision and Review

The data presented in Chapter VI and Appendix C was for each testing period and several content areas. This data was used to plot a graph which shows a growth profile across the school year for every test item. These graphs are located in Appendix B adjacent to the corresponding test item. The data by content area and the growth graphs present a comprehensive picture of what happened to important concepts presented in PIA-6.

Although the graph plots were not available until the final testing (T_4) was completed, the data by content area reported in Chapter VI was updated at each testing period and made available to PIA staff. Typically, a meeting was held a few days after a testing period and the results of the testing were reviewed and interpreted. Items related to topics covered by intervening programs were the focus of discussion. Skills and concepts which had been covered earlier in the year were also examined to see how well they were being retained. The problem writer for the Pupil Manual was present at these meetings and received suggestions for topics which should be reviewed and exercises that could be revised. All members of the PIA staff proposed or submitted problems for the Pupil Manual. In particular, the evaluator and the problem writer discussed the pupil exercises for most programs. A

rather persistent question asked by the problem writer was "What kind of review exercises do you think are appropriate here?"

Significant results of the testing undoubtedly functioned in subtle ways that were not always observable. It is, however, appropriate to discuss observable ways the test data was used to develop and modify the materials.

Major Revision

The single most important result of the testing involved Programs 15 and 54. These two programs reviewed and extended decimal numeration and density. Both programs, particularly Program 54, provided instruction and exercises on ordering decimals. One of the primary objectives of these programs was to teach pupils how to order decimals such as .238, .24, .3, and .05 (e.g., .05 < .238 < .24 <.3). Test 9, item 4, was designed to test a skill necessary before decimals can be ordered effectively (see Figure 8).

Which number is the greatest?

a) 0.03

b) 0.29

c) 0.293

d) 0.2093

Test 9, Item 4

Figure 8

The response pattern to this problem was as follows -- T_1: 4% correct; T_2: 8% correct; T_3: 9% correct; T_4: 46% correct. A look at the response pattern at T_1, T_2, and T_3 revealed that the 90%+ pupils who

responded incorrectly were about equally divided among the three distractors - 0.03, 0.29, and 0.2093. Three hypotheses relating to misconceptions pupils have about decimals were offered to account for these results.

1) The group choosing 0.03 focused on its non-zero digit, 3 being larger than the digit 2 which is the first non-zero digit in the other choices (the fact that the 3 is in the hundredths place is overlooked).
2) The group choosing 0.29 did so because they knew that one-thousandth < one-hundredth and then generalize incorrectly that thousandths are less than hundredths--hence, the incorrect conclusion: 0.293 < 0.29.
3) The group choosing 0.2093 focused on the number of digits rather than appropriate place value. Since 0.2093 is a "longer number" than the other choices, then it must be the greatest.

Program 54 was completely reworked as a result of the testing. While it was too late to make changes in the Program 54 tape for developmental year use, the Teacher Notes were modified considerably to alert teachers to the three misconceptions pupils have about decimals. These misconceptions along with recommendations for aleviating them were detailed in the Teacher Notes. That these recommendations were of value can be seen by comparing the results at T_4 with those from T_1 - T_3. The TV tape for Program 54 was redone with particular emphasis on ordering.

The effect of this redo could not, of course, be measured during

the developmental year. Program 15 emphasized a verbal approach to ordering--it was thought that if pupils could read decimals, then they could compare them. While this program was effective in teaching pupils how to read and write decimals (see Test 4, items 13 and 20; Test 11, item 16), it was not effective in teaching order. Therefore, the redo of Program 54 approached order using a scanning process--often a digit by digit comparison. For example, to compare 63.816 and 63.82 one observes that corresponding digits are the same until you reach the hundredths digits. Then, since two hundredths is greater than one hundredth, it follows that $63.82 > 63.816$. Counter examples to the misconceptions pupils have about decimals were used on the program.

Other Revision Suggested by the Testing

Another weak area uncovered by the testing was measurement. In particular, perimeter, area (of a rectangular region) and volume (of a box) were all covered prior to T_1. However, results on the testing clearly indicated that perimeter and area were often confused and what was done with the specified dimensions on a volume problem bordered on randomness. Specific review exercises were designed to distinguish perimeter and area. Also, volume was explored in greater detail on a later program and review exercises were included. From the test results (see Table 9) it can be seen that these aspects of measurement did show improvement following T_1.

The problem of finding the area of a triangle was not easy at any time during the year (see Table 9, item 7). The formulas for the area of a parallelogram and area of a triangle were introduced in a single program - Program 30. It is possible that one program

TABLE 9

TEST RESULTS FOR 3 PERIMETER ITEMS, 4 AREA ITEMS AND 1 VOLUME ITEM

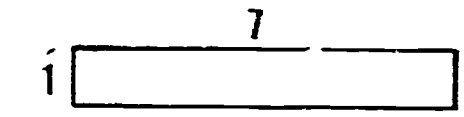
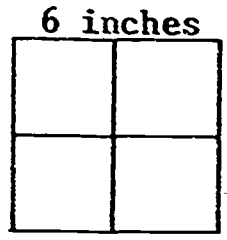
Item	C-Group Percent Correct T_1	T_2	T_3	T_4	Mastery Level
1. What is the distance around the rectangle below?  a) 7 b) 8 c) 14 d) 16	73	71	68	80	A
 2. The large square above has been divided into four small squares. What is the perimeter of a small square? a) 3 inches b) 6 inches c) 12 inches d) 24 inches	28	32	43	51	X
3. A square is nine inches on a side. What is its perimeter? a) 9 x 9 b) 9 + 9 c) 4 x 9 d) 2 x 9	60	56	70	64	A

TABLE 9 (Continued)

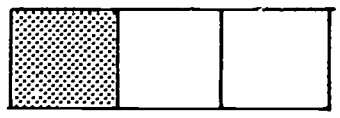
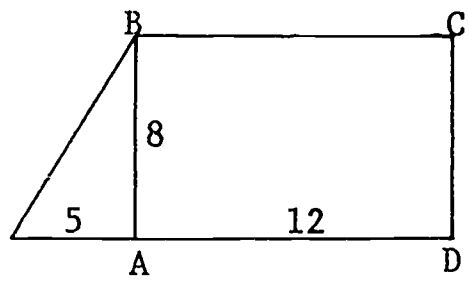
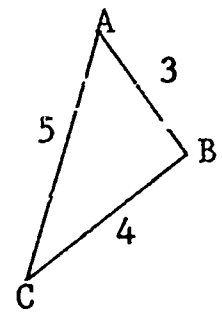
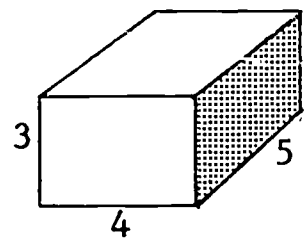
Item	C-Group Percent Correct T_1	T_2	T_3	T_4	Mastery Level
4. If the area of the rectangle above is 18, what is the area of the shaded part? a) $18 \div \frac{1}{3}$ b) $\frac{1}{3}$  c) 3 d) 6	52	52	60	65	A
5. What is the area of a rectangle 8 feet long and 3 feet wide? a) 11 sq. ft. b) 22 sq. ft. c) 12 sq. ft. d) 24 sq. ft.	60	72	85	79	A
 6. If ABCD is a rectangle, what is the area of the entire figure (the rectangle and the triangle)? a) 116 b) 480 c) 136 d) 25	30	20	31	47	C

TABLE 9 (Continued)

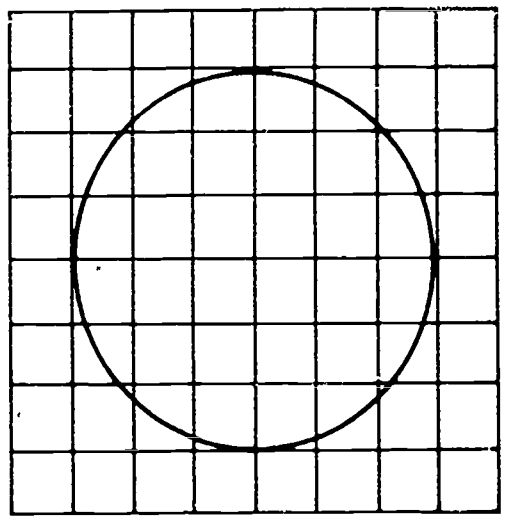
Item	C-Group Percent Correct T_1	T_2	T_3	T_4	Mastery Level
7. Triangle ABC is a right triangle. What is its area? a) 12 b) 10 c) $\frac{1}{2}$ x 15 d) 6 	5	6	15	27	B
8. What is the volume of the box? a) 12 b) 24 c) 48 d) 60 	31	68	80	79	A

was not sufficient for the concepts presented.

The basic approach to area in PIA was in terms of "inner" and "outer" area of a closed curve. It was realized in PIA-5 that the term "outer area" was very confusing and was in fact a misnomer for many pupils. The outer area was naturally associated with the grid squares located "outside" the given closed curve. An attempt was made in PIA-6 to review outer area (Program 3) and make teachers aware of the problem pupils have with this concept. Test 10, item 5, pictured a grid over a circle and asked for the outer area of the circle. Performance on this item at T_1 showed a sizeable gain over results on a similar problem given at the end of PIA-5. However,

in the absence of significant review the concept showed considerable decay over the year (see Figure 9).

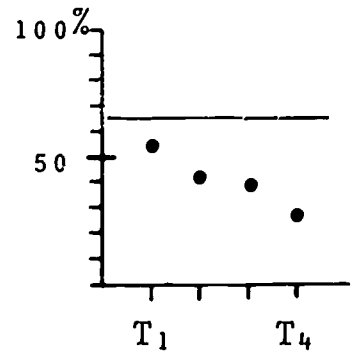
Test 10, Item 5



In the figure above, use the grid to determine the outer area of the circle. What is the outer area of the circle?

a) 16
b) 28
c) 36
d) 64

Growth Profile for
Test 10, Item 5



Test 10, Item 5 and Growth Profile
Figure 9

While PIA staff remain convinced that their basic approach to area was a sound one, it is clear that the term "outer area" was confusing for many 5th and 6th graders.

Problem solving is a skill which one likes to think of as evolving, yet capable of being effected by the teacher and the curriculum. It is safe to say that most pupils in which problem solving skill is manifest have developed this skill independently of both text-books and teachers. During the evaluation of PIA-6 it was

observed that two test problems were much alike; however, one was much easier than the other (see Figure 10).

Test 5, Item 6

If (E,F) represents the number of edges (E) and number of faces (F), what is (E,F) for a cube?

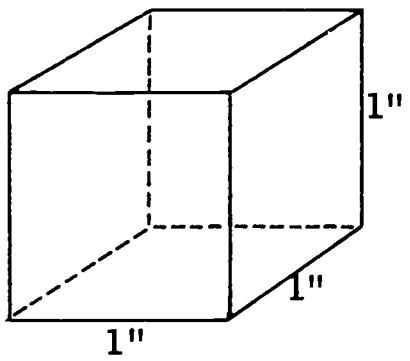
a) (4,4)

b) (6,12)

c) (12,4)

d) (12,6)

Test 9, Item 12



If E = number of EDGES
F = number of FACES
V = VOLUME

What is (E,F,V) for the cube above?

a) (9,4,3)

b) (12,6,1)

c) (9,6,1)

d) (12,4,3)

Test 5, Item 6 and Test 9, Item 12
Figure 10

The results at T_1 showed 13% responded correctly to item 6 (33% took choice b) while 57% responded correctly to item 12. These results suggested that the appearance of the visual for item 12 made it much easier than it would have been otherwise. With the visual they could count edges and faces.

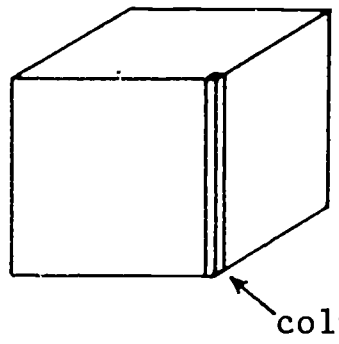
The question naturally arises "Why don't pupils construct a sketch if a sketch makes the problem so much easier?" Here, it seems, is a place where a problem solving skill, sketching, can be

taught--a skill which will be useful not only in arithmetic, but throughout one's mathematical career. At least two sections of exercises in PIA-6 were modified (Programs 31 and 60) to encourage pupils to sketch a figure for the problem. Also, a section (Program 8) involving cuts through solids was modified to have the pupils draw cuts to satisfy certain conditions rather than have all cuts predrawn.

Many test items (e.g., Test 1, item 19; Test 8, item 17) involved long division. The PIA approach to long division was to introduce it in grade 4 in terms of repeated subtraction after having established that $a \div b = n$ and $a = n \times b$ are equivalent sentences. The concept of repeated subtraction was used to solve sentences like $n \times 24 = 96$ before the division process was introduced. Having introduced long division by repeated subtraction in PIA-4, a series of refinements were made in PIA-5, but the traditional long division algorithm was not introduced until Program 17 of PIA-6. The basic reason behind the lengthy PIA approach to long division was to promote understanding of the algorithm. While the repeated subtraction process is easy to understand it is often time-consuming. Moreover, when one encounters a problem like $236.2 \div 0.65$, it is desirable that the algorithm be used. Every long division problem involving counting numbers with a remainder was worked on the back of the pupil's answer sheet. It was of interest to see what fraction of the pupils were using the algorithm introduced in Program 17. At T_2 a check showed that approximately $\frac{1}{3}$ of the pupils were still using the "old" repeated subtraction process. When this was discovered the Teacher Notes for Program 17 were revised to encourage a more complete shift

to the algorithm. The fact that many pupils were evidently more comfortable using the old method was not taken as a bad sign. On the contrary, it suggested that a certain group of pupils understood the repeated subtraction approach and found it more meaningful than the algorithm. By T_4 almost all pupils were using the standard algorithm.

The use of ratios in problem solving was one of the key mathematical ideas that permeated much of grade 6. Ratios were reviewed early in the year and used to convert from one standard unit of measure to another. Later in the year ratios were used in similar figures and scale drawings. The final application of ratio was to percent. A ratio approach to percent avoided the necessity of treating percent problems as three distinct cases. In general, ratio was a strong area for PIA. From the items listed under ratio (see Appendix C) it is clear that there were many difficult problems in this area. Most items showed nice growth over the year and skills were maintained through repeated use of ratio to solve many types of problems. The power and flexibility of the ratio approach in problem solving was evidenced by one pupil's approach to a problem that was <u>not</u> classified as a ratio problem. The problem was located at Test 12, item 20 (see Figure 11).



One cubic foot of water weighs 62.4 pounds. How much does a column of water one inch square and one foot high weigh? Write a sentence to show what must be done to get the answer. You do not have to find the answer.

Test 12, Item 20
Figure 11

It was expected that the correct answer would be written in the form $62.4 \div 144 = n$. However one 6th grade girl answered the problem with the sentence $\frac{144}{62.4} = \frac{1}{n}$, a valid and rather amazing application of the idea of ratio by a 6th grader.

5 One objective of the testing was to determine how computation skills changed over the year. It was necessary to review many skills covered in PIA-5 and several computation problems were devoted to these previously introduced skills. For example, problems like $\frac{1}{8} + \frac{5}{8}$ and 571 x 68 were introduced prior to PIA-6 but reviewed at various points in PIA-6. Most old skills were maintained at a high level. New computation skills that were introduced often showed dramatic growth on the testing period following coverage of the topic.

The most obvious trouble spots in computation involved the multiplication of two mixed numbers (e.g., $3\frac{1}{7} \times 2\frac{1}{9}$) and problems of

mixed types (e.g., $0.3 + \frac{1}{2}$, $7\frac{1}{2} \div 5$). The latter problem was given particular attention in the Pupil Exercises. Review exercises required pupils to add, subtract, multiply, and divide numbers written in a form unsuitable to operate upon directly. For example, $0.3 + \frac{1}{2}$ would be changed to $\frac{3}{10} + \frac{5}{10} = \frac{8}{10}$ or $0.3 + 0.5 = 0.8$. Early in the year many pupils were answering $7\frac{1}{2} \div 5$ as 1 with a remainder of $2\frac{1}{2}$. Although this is not an incorrect answer (if one views division as repeated subtraction), it is not a desirable one. Later in the year, after division of fractions was covered, the tendency to answer the problem this way greatly decreased. The response became $7\frac{1}{2} \div 5 = \frac{15}{2} \div 5 = \frac{3}{2}$ or $1\frac{1}{2}$.

Program 62 reviewed problem solving and more specialized sentence forms. The reason for reviewing these sentences was that pupils had developed additional computation skills and a program was needed to tie together the newly acquired skills and familiar sentence forms. One sentence introduced on the TV program was

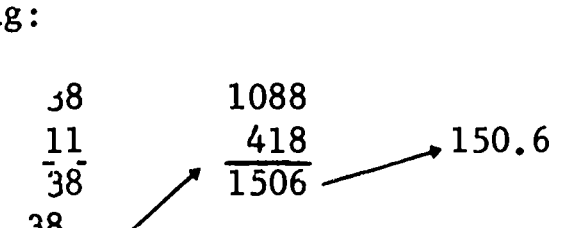
$$F = 500 - (6.2 \times 35).$$

Here it is necessary to find the product first and then subtract this from 500 to obtain F. Problem 18 on Test 7 was similar to this problem. The pupil had to compute

$$1088 + (38 \times 1.1) = ?$$

The main difference between this problem and the one used on the TV program is the operation--subtraction on the TV problem and addition on the test problem. PIA staff expected to see a nice gain on this problem at T_4 since Program 62 occurred near the fourth testing period. At T_3 15% of the group responded correctly while at T_4 20%

responded correctly. This result was surprising and disappointing. To get some insight into the difficulty students had with this problem, the answer sheets were carefully analyzed. The correct answer to the problem 1088 + (38 x 1.1) = ? is 1129.8. However, 33 of 92 responses in the Y-Group were 150.6! The students were doing the following:



They did not locate the decimal point on the intermediate product and the digits for the sum were misaligned.

For Program Planning

Administration of the entire pool of 240 items at four times not only made it possible to evaluate the effectiveness of the curriculum, but it also indicated how students attack certain problems and what correct and incorrect "hunches" they have before instruction is given. For example, it was found that before instruction the problem $\frac{4}{5} \times \frac{9}{8}$ was answered correctly by 23% of the pupils.

Another problem of interest was 0.2 x 0.6 = ? Students had no specific instruction in multiplying by decimals until Program 35. However, Program 15 covered decimal numeration and Program 20 covered multiplication of fractions. By T_2 the problem could have been worked as $0.2 \times 0.3 = \frac{2}{10} \times \frac{3}{10} = \frac{6}{100} = 0.06$. At T_1 6% responded correctly while at T_2 18% responded correctly. This indicated that a few students were able to synthesize the instruction from Programs 15 and 20. Another interesting aspect of this problem was the number of

students who responded 0.2 x 0.3 = 0.6! At T_1 88 students out of 127 made the free response 0.6. At T_3, after instruction in how to multiply decimals, 62 out of 128 still responded 0.6.

Probability is not a traditional topic in the elementary school. PIA-6 introduced probability late in the school year (Programs 56-57). Eleven problems in the item pool were related to probability. Since T_3 occurred at Program 41 three measures on the difficulty of these items were available to assist in the planning of the programs on probability. It was found that the terms "most likely", "least likely", and "equally likely" were understood by most pupils. A spinner problem was quite easy and suggested that spinners could be used as an aid to teach probability. Although pupils were aware of certain informal aspects of probability, they did not know how to assign a number to represent the probability of an event. For example, Test 6, item 18, referred to a sack that contained 4 BLACK, and 7 RED balls. The pupil was required to state the probability of drawing a RED ball. The response pattern was as follows: T_1: 3% correct; T_2: 5% correct; T_3: 9% correct and T_4: 61% correct. T_1, T_2, and T_3 represent pre-instruction knowledge. Clearly, one had to assume the pupils knew nothing about assigning probabilities. Another skill that pupils must master before more involved probability problems can be solved is that of the "product rule" for determining the number of outcomes to an experiment. Test 20, item 9 showed several pupils could already apply this rule to a simple case. However, a significant number of pupils were adding rather than multiplying.

Another topic about which pupils knew a surprising amount before instruction was reading graphs and charts. In fact, the pupils knew so much about this topic that little gain was evidenced across the year on most of the items. Rather than devote the pupil exercises to retrieving data from graphs and charts, pupils were required to graph data (given a chart) and to make a chart and graph from raw data.

Weaknesses in the Item Pool

Evaluating a curriculum using item sampling with a pool of items conceived before the curriculum is finalized has two main disadvantages.

1. Irrelevant items may be included in the item pool, and
2. Not all important aspects of the curriculum may be covered by the item pool.

For the evaluation of PIA-6 the latter event was more common than the former. The following are important aspects of PIA-6 which were not adequately tested by the item pool:

1. Angle Measure (Programs 18-19): Use of the protractor; angles between parallel lines when intersected by a diagonal; terminology associated with angles; the proper way to name an angle.
2. Coordinate Geometry (Programs 38, 40): Locating integers on the number line; representing "trips" on the number line using vectors; locating ordered pairs of integers in a coordinate system.
3. Inequalities (Programs 37, 39): Graphing inequality sentences; equivalent inequality sentences.

4. Similar figures (Programs 51-53): Meaning of similar figures; projections and special cuts of solids which yield similar figures; dilations.
5. Fractions: The equivalence of $\frac{a}{b}$ and $a \div b$; use of the term reciprocal; converting a fraction to a mixed number using long division.
6. Reflections (Programs 24-25): Finding the image of a given figure; locating the line of reflection when the figure and image are given; properties of reflected figures.

The formative evaluation would have been lacking without some evaluation on the aspects mentioned above. Therefore, quarterly Check-Up Tests were sent to teachers to serve the dual purpose of classroom evaluation and formative evaluation of topics not adequately covered by the item pool. The teachers using PIA-6 had expressed the need for a test that they could use. The twelve, 20-item tests could not be used effectively for classroom evaluation because each class took all tests. Moreover, prior to T_4, some of the test items represented topics that were yet to be presented.

Each Check-Up Test contained about 30 items and covered areas of interest to teachers and particularly the neglected areas listed above. Teachers were not requested to return these tests and administration was optional--cooperation over four testing periods was all that could be expected. Notes relating to the Check-Up Tests expressed the desire on our part to get feedback on these tests. Typically, about 10 classes returned the tests at each quarter and

about half of these were rescored by PIA staff. These tests seemed to be popular and useful for the teacher.

The four Check-Up Tests were modified for inclusion in the Pupil Manual for PIA-6. Check-Up 2, Check-Up 3, and Check-Up 4 were simply retyped using a slightly different format and more effective art. Check-Up 1 was not very satisfactory for classroom evaluation and was modified. The Check-Up Tests and summary statistics are included in Appendix D. Problems not included on the original Check-Up 1 are indicated. No summary statistics are available for Check-Up 1 and Check-Up 4.

CHAPTER VIII

CONCLUSIONS

The purpose of this study was to explore the use of item-sampling in formative curriculum evaluation. A pool of items was conceived that represented content to be covered and skills to be mastered by a sixth grade TV arithmetic program, Patterns In Arithmetic. Items were selected from the pool to fit twelve, 20-item tests. These tests were administered four times during the 1968-69 school year to the population participating in the study. Results from the testing periods were used for program planning, to suggest appropriate review, and to indicate major revision. The previous chapter gives examples of these aspects.

It is clear that the use of item-sampling in formative curriculum development makes planning and revision more systematic than traditional testing procedures. It is possible to monitor simultaneously many dimensions of the curriculum. Moreover, the item profile provides a "history" of the item across the year. If effective revisions are made, the developmental year "history" is but a temporary state. The revised version should prove superior to the developmental year product.

It is appropriate to discuss some of the practical decisions one faces when item-sampling is used in curriculum development.

Item Stability and the Population

In Chapter III the problem of obtaining a stable estimate of item difficulty was discussed. Data was presented for four items (see Table 2) that appeared on two different tests. It was found that 120-130 responses generated under item-sampling conditions produced a fairly stable estimate of item difficulty. However, 30-40 responses generated under similar conditions produced less reliable (more variable) estimates.

Although desirable, it is not necessary to have extremely precise estimates of the "true" difficulty of the items for purposes of formative curriculum evaluation. The study group itself is but an "estimate" of the larger target population of potential users. Moreover, one is normally not trying to compare groups using a statistical test of significance. The formative evaluator monitors the developmental year product and signals its successes and failures. Hence, the estimates of item difficulties provided by item-sampling must only be accurate enough to expose weaknesses within the curriculum. It is the joint duty of the evaluator and the curriculum developer to decide upon how accurate the estimates must be. Table 10 indicates the standard error (S.E.) of the estimate for several sample sizes, N, and true item difficulties, p. This table assumes the binomial model and uses the formula:

$$S.E. = \sqrt{\frac{pq}{N}} \qquad (q = 1 - p)$$

TABLE 10

STANDARD ERROR OF THE ESTIMATE

q	N / p	30	60	90	120	150	180	210
.9	.1	.055	.039	.032	.027	.024	.022	.021
.8	.2	.073	.052	.042	.037	.033	.030	.028
.7	.3	.084	.059	.048	.042	.037	.034	.032
.6	.4	.089	.063	.052	.045	.040	.037	.033
.5	.5	.091	.065	.053	.046	.041	.037	.034

The binomial model is appropriate under conditions of item-sampling. This can be seen by considering the probability of a given individual responding correctly to an item. For a fixed population and a given item there exists a number p which corresponds to the fraction who "know" the correct answer to the problem. If an individual is selected at random from the population then the a priori probability that he will respond correctly is p. If a sample of N individuals is selected from the population to respond to the item then it is clear that the probability of k correct responses is:

$P(k) = \binom{N}{k} p^k q^{N-k}$, the $(k+1)$st term in the expansion of the binomial $(q + p)^N$.

It is well known that as N gets larger, the binomial distribution approaches the normal distribution. The parameters μ and σ^2 for the normal are p and Npq respectively for the binomial. While Npq represents the variance of the number of successes (correct responses), the standard error is more frequently reported. For a given N and p the standard error function tells how likely it is that a fixed interval centered at

p will contain the observed value of p, namely $\hat{p}$. The fact that the distribution of $\hat{p}$ rapidly approaches the normal distribution with mean p and variance $\frac{pq}{N}$ makes it possible to use a table for the normal distribution to express what fraction of the time a given interval will contain $\hat{p}$.

Using the normal approximation, the interval $p \pm$ S.E. will cover $\hat{p}$ about 68% of the time while $p \pm 2$ S.E. will cover $\hat{p}$ about 95% of the time. Table 10 shows that for 90 random responses to an item with true difficulty p = .4, the standard error of the estimates is .051. This means that if a random sample of size 90 responded to the item and an estimate, $\hat{p}$, of the true item difficulty was calculated, then the interval $.4 \pm 0.051$ (.349 to .451) would contain $\hat{p}$ about 68% of the time.

In practice one does not know p and is trying to obtain a useful estimate of p. Therefore, while in theory one can speak of the likelihood that $p \pm$ S.E. contains $\hat{p}$, in practice one does not know p. What is generally done is to treat $\hat{p}$ like p and use a standard error that is only approximately correct. For example, suppose $\hat{p} = .30$ and true (but unknown) p = .35. If $\hat{p}$ is based upon 120 responses then the table value .041 is used as an approximation to the standard error.

An interesting observation can be made from Table 10 by comparing the columns for N = 30, 120, and 210. The standard error of the N = 120 column is about one half that of corresponding entries in the N = 30 column. The increase of 90 responses doubles the accuracy of the estimate. However, suppose 90 more responses are obtained making a total of 210 responses. It is clear that while the standard error

decreases, the accuracy is only slightly improved. Thus, even though 210 responses clearly provides a more stable estimate of item difficulty than 120 responses, it does not provide the formative evaluator with substantially more decision-making capability. Since the standard error is calculated using the formula $\sqrt{\frac{pq}{N}}$ it is clear that for fixed p the standard error varies as $\sqrt{\frac{1}{N}}$. Hence, one must multiply his sample size by 4 to cut the standard error in half. For the above example 480 random responses would be needed to cut in half the error for 120 responses.

The evaluator must not take every slight drop in item performance as a definite sign that review is needed--he must be aware that chance variation will occur. One hundred twenty responses are certainly enough to alert the evaluator to a serious problem. However, to determine if a topic needs review it may be necessary to group several items related to the concept or skill. A slight variation in a single item might be a chance result.

The following factors should be considered in determining N:

1) The size of the population that is to be sampled

2) The number of items about which one desires to get information

3) The desired accuracy of the estimates

4) The amount of time that is available for testing.

The following example illustrates how these factors operate together.

Suppose a study involving item-sampling is being planned and it is estimated that 1,300 pupils will be participating in the study. These 1,300 pupils probably represent a larger target population. The evaluator

and the developer agree that for middle-difficulty items a standard error of .04 is sufficiently small for making developmental decisions. From Table 10, 150 random responses to an item are necessary to provide this accuracy. The integer part of the quotient 1300 ÷ 150 is 8. Therefore, the evaluator can use 8 different tests to obtain information. It was indicated earlier that for administrative efficiency the tests should be approximately the same length and suitable for administration under uniform instructions. Once the number of tests is decided upon the next decision is how many items per test are reasonable. If there are 400 items one would like information about, then 400 ÷ 8 = 50 items per test may be used. However, 50 items per test may be unreasonable in terms of the time required. This would be particularly true if the tests were to be administered several times. Therefore, a compromise must be met. The evaluator could increase the size of the population, decrease the number of items, or do both. For a given study it is likely that many workable combinations will exist. The various combinations should be given careful consideration well in advance of the study.

Selection of the Item Pool

The success of any evaluation depends upon the quality of the items about which one gathers information. There is no conceivable way for poorly chosen items to provide useful results. One of the primary advantages of item-sampling over traditional standardized achievement testing is that the evaluator can hand-pick the items. Therefore, much thought and effort should go into building an item pool that contains

only informative items relative to the objectives of the curriculum being evaluated. If one of the objectives of the curriculum is to promote a "positive attitude" toward a subject, then items designed to measure attitudes are certainly "informative" items. The present study would have been more effective if the evaluator had looked at each item in the pool and asked the question "Is this item worth being responded to?" The result would have been to toss out about 20 items that were included in the final item pool.

Multiple-choice items should be chosen that have distractors which give insight into common misconceptions. If revision is going to correct errors, it is important to know what errors need correcting. Free response items are also useful in formative evaluation. This is particularly true in mathematics where it is informative to observe the approach one takes or notice how the pattern of response changes following instruction. Frequently given incorrect responses often indicates trouble spots.

A few items that are easy before instruction may be desirable. Most items that are initially easy should be items which test concepts or skills that the developer hopes to maintain at a high level during the year. Items which are much easier or harder than expected before instruction are indeed useful. These items may suggest to the developer that he modify his planned approach to a topic because pupils know more about the topic than he thought. However, too many initially easy items weaken the usefulness of the item pool for evaluating the curriculum.

Characteristics of Test Instruments

For item-sampling to function properly it is necessary that every pupil have the opportunity to respond to each test item. It is just as important that the last item on a test be attempted as the first if a true measure of the difficulty of the item is to be obtained. While it is reasonable to place the more difficult items at the end of a speeded or rigorously timed test, item-sampling instruments are likely to be more effective if easy and hard items are intermixed. If a pupil senses that all the hard items are at the end of the test, then upon reading two or three problems that he can not work he may become discouraged and give up.

It is likely that questions other than multiple-choice will be used during a formative evaluation. Since pupils rarely become discouraged when responding to a multiple-choice item, these should be first on the tests. This study always started a test with an easy item and then varied the difficulty of the items over the remainder of the test.

The use of item-sampling in a growth study implies that improvement in skills and concepts is expected during the period of the study. Since the item pool is administered repeatedly over the year it is natural for the evaluator and the curriculum developer to be concerned about the effect of the initial testing period--the period where many of the concepts and skills required by the test items have not been covered. Some pupils expect to do extremely well on every test while others do the best they can and forget the test upon finishing it. The former group

is likely to feel more comfortable (less anxious) if the instructions for the tests make it clear that they will be hard. Also, teachers should be informed about the purpose of the testing and that while initially the tests will be hard, they will become progressively easier. A preliminary letter to teachers describing the testing procedure and a copy of the instructions for the first administration are included in Appendix B.

Referencing the Items

A test item is included in a formative evaluation because it evaluates a goal of instruction. The curriculum developer has at least a rough idea about the level of performance he expects on the item following instruction. He also knows if the item represents a simple recall of a fact, a complex extension of instruction, or something between these two extremes. Therefore, it is reasonable for the curriculum development staff (including the evaluator) to decide upon a classification system for items used in the study. There is a tendency to use too many categories for classifying items. Chapter V discusses the categories used to classify items for PIA-6. Five categories were used: three mastery levels and two transfer levels. A lower bound criterion was specified for each category. The lower bound reflected the aspirations of the PIA staff for the item following instruction.

Items should be classified before instruction begins for several important reasons:

1) As the items are classified the project becomes more cognizant of its goals.

2) A few useless and poorly worded items are likely to be discovered.

3) The classification can be done objectively.

The curriculum developer may feel uneasy about classifying items and then having to report that certain items failed to meet criterion. However, formative evaluation does not represent evaluation of the revised product, but rather an evaluation of the developmental year effort. The advantages of classifying each item far outweigh any fear of failing to meet criterion.

Use of Results During the Formative Year

If the evaluator keeps test results a "secret" they are of no value. For PIA-6 the test items were grouped into homogeneous categories. These categories were updated at each testing period and copies were given to each staff member. The "*", "r", and "c" coding system used between testing periods (see Chapter VI) was useful. It is recommended that only two symbols be used- one for extensive coverage and one for significant review. The "c" symbol was used between testing periods to indicate "related coverage." It was extremely difficult to decide if coverage related to a particular item had occurred between testing periods. The "*" category was used to signal major coverage and could almost be placed by looking at the program titles. The symbol "r" was used to indicate that significant review related to the item had occurred. Whether review had occurred could be objectively determined. The use of a code between testing periods is desirable. It indicates at a glance how the item was affected by instruction over

the year.

Another aid to the curriculum project would be a brief summary of the highlights of the testing period. This summary could include interesting results on items related to intervening programs, topics which need review, and results which have implications for future program planning. The present study would have been more effective if greater attention had been given to this summary. The evaluator must realize that one of his jobs is to help interpret results--the curriculum developer often does not have time to become absorbed in test data.

Summary

Item-sampling proved to be useful in formative curriculum evaluation. It was shown that item-sampling could be successfully used in a growth study where it is desirable to obtain a profile of change over a period of time. It is clear that item-sampling provides more detailed information to staff than traditional testing that uses a single test with items not necessarily designed for the curriculum. The fact that the evaluator can hand-pick all test items is a major advantage in item-sampling. Since the "items" are the focus of the evaluation, considerable care must be taken in their selection. Although the design of an item-sampling study is simple, the parameters involved in such a study (e.g., population size, sample size responding to each test, number of items per test, etc.) must be decided upon well in advance of the study. An example was given earlier in this chapter which illustrates how these parameters operate together. It was suggested that

once an estimate of the population size is known, and the accuracy of the sampling estimates is specified by staff, the number of tests that may be used is determined. To administer several different tests to a given class, uniform instructions and tests of similar length are required. Hence, once the number of tests that can be used is known, the number of items about which one can gather information is a simple function of time available for testing.

Criterion referencing of items was used in this study and is recommended, provided that items are classified early in the study--preferably before any testing is done. By referencing the items before the initial testing period it is likely that undesirable items will be uncovered. This would strengthen the item pool. Together, item-sampling and the criterion referencing (classifying) of items provide staff with an efficient, objective and useful evaluation techinque.

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Technical Report No. 113 (Part II)

THE FORMATIVE EVALUATION OF PATTERNS IN ARITHMETIC GRADE 6 USING ITEM SAMPLING

Report from the Project on
Individually Guided Elementary Mathematics
Phase 2: Analysis of Mathematics Instruction

T. A. Romberg, and John G. Harvey
Principal Investigators

By James Braswell

Thomas A. Romberg, Assistant Professor
of Curriculum and Instruction and Chairman of the Examining Committee

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Center for Cognitive Learning
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Madison, Wisconsin

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APPENDIX A

CONTENT SPECIFICATIONS FOR PIA-6 TESTS

APPENDIX A

Table 1A represents proposed topic coverage for Patterns In Arithmetic-6. Although it was developed during the summer preceeding the actual production of the program, this outline represents a fairly accurate outline for the final product. This table together with Table 2A was used as a guide in developing and collating the test items.

TABLE 1A

PROPOSED TOPIC COVERAGE FOR PIA-6

I. Geometry (30%)

A. Measurement

1. Linear: Review standard units of linear measure; introduce metric system; review perimeter; conversion of units by ratio

2. Area: Review standard units of area measure; inner and outer area; formula for area of rectangle; use of ratios to convert units; ability to compute area or [illegible]gle, parallelogram

3. Angle: Introduce terms such as ray, acute, obtuse, straight and vertical angles; use of protractor to measure a given angle or angle of rotation;

point out congruence is equivalent to equal measure.

4. Volume: The concept of volume and if possible extend the idea of inner and outer area measure to inner and outer volume; standard units; volume of box (rectangular solid); perspective drawings

B. Non-metric aspects of geometry

1. Sections: Notion of a plane section of a solid; use of terms plane, region, polygon, parallel; 2-D drawings of 3-D figures; use sections of solid figures to get rectangle, square, circle, ellipse

2. Reflections: Review construction of a reflection pointing out orientation shift; congruence after reflection

3. Dilations: Construction and properties; similarity (review); (Similarity) show two similar figures can be related by combinations of rigid motions and/or dilations; notion that corresponding sides of similar figures are proportional; scale drawings as application of similarity; make simple scale drawings

4. Symmetries: Lines of symmetry of figures and relation to dilations and reflections

II. Fractions (30%)

A. Interpretation: The meaning of a fraction (shaded figures, number line)

B. Equal Fractions: Fractions associated with counting numbers,

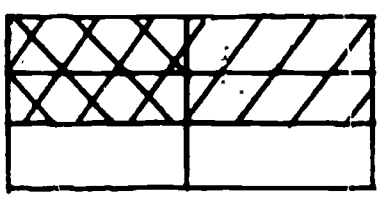
i.e., $\frac{6}{3}$; equivalent fractions; cross product test

C. Operations: (+, -, x) Review addition and subtraction of fractions, least common denominator; meaning of $\frac{2}{3}$ of 6 as an introduction to multiplication of fractions; number line argument for $6 \times \frac{1}{2}$...then $6 \times \frac{1}{2} = \frac{1}{2} \times 6$ - but $6 \times \frac{1}{2} = \frac{12}{2} \times \frac{1}{2} = ?$

We know the answer must be 3 from the number line argument--but $\frac{12}{4}$ is another name for 3. Hence, $\frac{12}{2} \times \frac{1}{2} = \frac{(12 \times 1)}{(2 \times 2)}$. Equivalence of $\frac{2}{3}$ of 6 and $\frac{2}{3} \times 6$; use of regions to motivate multiplication of rationals; use of a rectangular model to illustrate commutativity, i.e.,

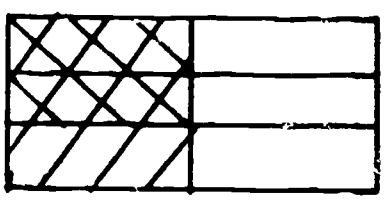
$\frac{1}{2} \times \frac{2}{3} = \frac{2}{3} \times \frac{1}{2}$;

$\frac{1}{2} \times \frac{2}{3}$



$\frac{2}{6}$

$\frac{2}{3} \times \frac{1}{2}$



$\frac{2}{6}$

Eventually get to general rule for multiplication of fractions; give attention to $\frac{1}{2} \times \frac{2}{3}$ and $\frac{3}{6} \times \frac{9}{12}$ are equivalent (uniqueness aspect); mixed numbers via distributive property, e.g., $4 \times 3\frac{1}{2} = 4 \times (3 + \frac{1}{2}) = (4 \times 3) + (4 \times \frac{1}{2})$ other combinations such as $\frac{2}{3} \times 3\frac{1}{2}$, $2\frac{3}{4} \times 6\frac{2}{3}$, etc.

D. Decimals: Review notation for decimals; interpretation of decimals; writing a rational number as a decimal when (a) denominator is 10, 100, 1000, etc. (b) denominator is not so nice; addition and subtracrion of decimals; multiplication of decimals; division of decimals, e.g., $10.6 \div 0.3$, $0.6 \div 3$, etc.

E. Operations ($\div$): Division of rational numbers including (a) a review of equivalence of the statements $n = 12 \div 4$ and $4 \times n = 12$. Use this to conclude $12 \div 4$ and $\frac{12}{4}$ are the same since $4 \times n = 12$ implies $\frac{1}{4} \times 4 \times n = \frac{1}{4} \times 12 = \frac{12}{4}$ (b) the solution of problems like $n = 5 \div \frac{1}{2}$, $\frac{1}{2} \times n = 5$ (c) problems like $n = \frac{2}{3} \div \frac{1}{4}$ (d) division of mixed numbers (e) simplifying fractions to a basic fraction, cancellation to simplify computation (f) tabulate functions using rational numbers without using the word function

III. Counting Numbers (15%)

A. Operations - Properties of 0 and 1

1. Review multiplication, expecially problems involving 3 digits such as 263×491

2. Division algorithm, writing the sentence to correspond to division with (without) a remainder

B. Factors

1. Prime numbers, composite numbers

2. Greatest common divisor, least common multiple

3. Exponents ($16 = 2^4$), represent a composite number as a product of primes using exponents

C. Equivalence

1. Equivalent sentences involving integers (fractions);

$2 \times n = 16 \longleftrightarrow \frac{1}{2} \times 2 \times n = \frac{1}{2} \times 16$

Story problem applications

2. Equivalent conditions involving integers (fractions)- e.g.,

$4 \times n < 8 \longleftrightarrow n < 2$

D. Functions - tabulate

1. Graph a relation on the number line

2. Introduce coordinates and plot ordered pairs

IV. Ratio (5%)

A. Cross product

1. Solution of problems like $\frac{2}{3} = \frac{a}{27}$ using cross product

2. Converting from one standard unit to another (e.g., cm. to inches)

B. Percent

1. $65\% \longleftrightarrow \frac{65}{100}$

2. $\frac{a}{b} \longrightarrow$ decimal $\longrightarrow$ % $(\frac{3}{4} \longrightarrow .75 \longrightarrow 75\%)$

V. Miscellaneous (20%)

A. Probability

1. Sample space

2. Events as subset of the sample space of events

3. Assign probability to an event by ratio of number of ways favorable to the total number of outcomes for equally likely events

4. Complement of an event - given the probability of an event, what is the probability of its complement

B. Statistics

1. Nature of statistics
2. Use of graphs to represent statistics visually
3. Measures of central tendency - the average, mode (?)

C. Problem Solving - extensions of program content

1. Involving fractions
2. Equivalent sentences
3. Reading graphs, probability
4. Synthesis of various aspects above

TABLE 2A

ITEM ALLOCATION BY TOPIC

The percentages indicated below were used as a guide in collating the tests. When considered together the twelve tests represent closely content and percent coverage specified in this table.

I. Measurement (8%)

A. Linear (2%)
B. Area (3%)
C. Volume (2%)
D. Angle (1%)

II. Ratio-Cross Product (3-5%)

III. Scale Drawings (2%)

IV. Fractions (25%)

A. Meaning of fractions (3%)
B. Equal fractions (3%)
C. Ordering fractions (3%)
D. Addition and Subtraction (7%)

E. Multiplication and Division (8%)
F. Cancelling common factors (1%)

V. Decimals (13%)

A. Interpretation of decimals (3%)
B. Converting $\frac{a}{b}$ to decimal (2%)
C. Addition and Subtraction (3%)
D. Multiplication and Division (5%)

VI. Percent (7%)

VII. Division (Long) (6%)

A. Understanding the algorithm (2%)
B. Operating with the algorithm (4%)

VIII. Integers (4%)

A. Multiplication (2%)
B. Negatives (2%)

IX. Reading Charts and Graphs (2%)

X. Number Line (2%)

A. Related to fractions (1%)
B. Related to decimals (1%)

XI. Equivalent Sentences (2%)

XII. Probability (3%)

A. Counting processes (1%)
B. The certain and impossible event
C. Assigning probabilities, recognizing the most (least) probable event (1-2%)

XIII. Functions (3%)

XIV. Statistics (3%)

XV. Geometry - Non Metric (10%)

A. Visualize 3-D objects (2%)
B. Reflections (4%)
C. Rotations* (2%)
D. Symmetry (1%)
E. Dilations* (1%)

XVI. Applications - Extensions: involving notation, more complicated story problems, science problems (7%)

*This topic was not tested.

APPENDIX B

MATERIALS USED IN THE TESTING

This appendix contains copies of the various materials that were involved in the testing. In order, the following materials are included:

B1) A preliminary letter explaining the testing procedure

B2) A copy of the instructions used for T_1

B3) A copy of the instructions used for T_4

B4) The form which was completed by the teacher and returned at each testing period

B5) A sample class list

B6) A copy of each test as it appeared following the first testing period

B7) A description of changes made in a few items following T_1.

◀ B1 ▶

TESTING PROCEDURE FOR PIA-6

Two types of testing will be done during the developmental year of PIA-6. One type will be the traditional type of classroom evaluation of pupils. This will involve a mid-year and end-of-year test of pupil achievement. This testing will provide information about class performance and pupil achievement.

The second type of testing will represent a somewhat new approach to evaluation. While a program is being developed it is important for project personnel to have periodic feedback to see if program objectives are being met. It would be nice to have feedback after each program, but this is not practical. To compromise we have decided to use a large pool of items administered 4 times during the year to supplement and complement the periodic meetings with teachers participating in PIA-6. The item pool will reflect program content and most items will be designed to measure specific objectives of instruction. However, a few items will be designed to measure extensions of ideas developed in the programs.

Ten or twelve, 20-item tests are planned for this second type of testing. Each test will contain a different set of items and all tests will be administered at each testing period. Each test will be divided into two sections - multiple choice and computation (problem solving).

The tests will be sent directly to you; we request that you administer the tests and return the materials to us for grading. Since your class will be working on 10 or 12 different tests, the problem of timing will be important. Testing done in PIA-5 demonstrated that 30-40 minutes was sufficient time for a class taking 9 different 20-item tests. It is likely that a similar time limit will serve for PIA-6. Considerable effort will be made to make the tests approximately the same length.

It may seem odd that all items will be given at each of the four testing periods. However, information gathered on each item at each period can be useful for developmental purposes. For example, the initial testing period will provide a base for each item so that growth during the year can be determined. It is expected that maximum growth on an item will occur on the testing period immediately following PIA-6 coverage of the concept involved in the item. Since some topics are covered more than once during the year, it is expected that some items will show gain more than once. In addition to providing a base for each item, the initial testing period will provide the staff with valuable information to aid in the planning of programs.

The active cooperation of every teacher participating in PIA-6 is important if this second type of evaluation is to be effective. You should understand what this type of evaluation will not do as well as what it will do. It will not provide information about individuals or classes, but about the entire group of pupils participating in PIA-6. These test results will reflect group performance on the entire pool of items. The first type of testing will provide information useful for individual and classroom purposes.

Since for the second type of evaluation the same tests will be used at each of the testing periods, two aspects are worth mentioning.

(1) These tests will probably be very difficult at the beginning of the year, but should become progressively easier. Pupils should be aware of this and encouraged to do their best even though there will be items beyond their comprehension.

(2) Some pupils may take the same test more than once. This is undesirable and can be minimized by the way we send the tests to you and the manner in which you distribute the test materials. More detailed information about this point will be included in the directions for administering the tests.

There will be two groups of pupils participating in PIA-6. Those who participated in PIA-5 and those who did not. We would like to make a distinction between these two groups of pupils. Therefore, we will ask you to send a class list to us about mid-September indicating whether each pupil did or did not participate in PIA-5. The class list will serve another function. It will be used as a check to see how many pupils take the same test more than once. You will not have to keep track of this nor will you be expected to correct any tests of the second type.

◀ B2 ▶

PATTERNS IN ARITHMETIC
TESTING PROGRAM

In an earlier letter we mentioned that two types of testing will be done during the developmental year of Patterns In Arithmetic (PIA-6).

one type will be the traditional type of classroom evaluation of pupils and will consist of a mid-year and end-of-year test of pupil achievement. This testing will provide information about class performance and pupil achievement.

The second type of testing will represent a new approach to evaluation. While the program is being developed it is important for project personnel to have periodic feedback to see if program objectives are being met. To accomplish this we have decided to use a large pool of items places in 12 distinct tests. These tests will be administered 4 times during the year to supplement the periodic meetings with teachers and administrators. Copies of these tests are included with this letter. Administer the tests before the September 27 meeting. At that meeting aspects of the testing activities will be discussed.

Before giving the tests please read the following General Comments and Directions for the Testing.

GENERAL COMMENTS

1. There are twelve different 20-item "Question Sheets" (tests) with many questions relating to topics that will not be covered until later this year. The "Question Sheets" are numbered from 1-12 in the upper left-hand corner of page 1.
2. The same set of twelve, 20-item tests will be used for the 4 testing periods: September, December, February and May. Since we do not want a pupil to take the same test twice, a procedure will be explained below for accomplishing this.
3. The tests will be difficult at the beginning of grade 6, but should

become progressively easier during the year.

4. This phase of the testing will not provide you with any useful information about individual pupils or about your class as a whole. Information gathered will be analyzed on a group basis and not a per class or per individual basis. For example, we expect about 90-100 classes to participate in the developmental program and testing activities. This means that any given test (out of the 12 different ones) will be taken by only 2 or 3 pupils in your class. We will take the 2 or 3 pupils that respond to test 1 in your class and put their papers together with all other pupils responding to test 1 and analyze these as a group. The same procedure will be followed for each of the 12 tests.

5. You will not be expected to grade any of these tests. The answer sheets should be returned to us immediately after the testing and the test questions destroyed. A new set of tests will be sent each time the tests are to be given.

6. IMPORTANT: Directions for Passing Out the Question Sheets.

As mentioned above, the same set of tests will be used 4 times. Since it is undesirable for a pupil to take the same test twice, the tests must be passed out in a planned order. If you will look through the "Question Sheets" you will see that they are arranged in a repeating pattern of 12. For example, the pattern could be 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4,---Do Not Disturb this pattern.

The pattern arrangement will insure that each test gets taken about the same number of times. Pass the "Question Sheets" out,

in sequence, until every pupil has one. The pupil will put his name in the upper right-hand corner of the answer sheet which is attached to the "Question Sheets." We will make a class list for your class from the names on the answer sheets. The class list will contain the number of the "Question Sheets" taken by each pupil. This will be returned to you at the second testing period so that you may make certain that no pupil gets the same test again. Hence, it is essential that you enclose your name and school address with the answer sheets. A form for this information is enclosed.

SPECIFIC DIRECTIONS FOR THE TESTING-SEPTEMBER PERIOD

READ OVER CAREFULLY BEFORE THE TESTING

1. Reserve about 45 minutes for the testing.
2. Make sure each pupil has two pencils and an eraser. The back of the test cover and the question sheets can be used for scratch paper.

Follow the steps below and read the material in quotes to your class. Before passing out the tests say slowly -

> "I have some quesions about arithmetic that I want you to answer. Some of the questions will be difficult and you may not know the answers. Now I am going to pass the questions out, but do not open them until I say to."

Now pass out the "Question Sheets" one-by-one in the order you receive them. After everyone has a question sheet have each pupil tear off carefully the answer sheet that is stapled with the questions and is the last page. Then say -

"Has everyone removed the answer sheet? (pause) Now write your name clearly in the upper right-hand corner." (Allow time for this)

Now say -

"Did you use TV arithmetic in grade 5? If your answer is <u>yes</u> write yes under your name. If your answer is <u>no</u> write no under your name." (Repeat if necessary)

Then say -

"There are twenty questions in all. Some questions are multiple choice. You should guess at the answer <u>only</u> if you can rule out some of the choices. <u>All</u> answers go on the answer sheet. Now open your questions to the first page and lets read the instructions. (pause briefly) . . . You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and <u>circle</u> your choice on the <u>answer sheet</u> provided. You should have enough time to work on every question. Do not spend too much time on any problem."

Now ask informally if there are any questions. If there are no questions say -

"You will have about 30 minutes to answer all the questions . . . now begin."

Feel free to answer any questions during the testing that are <u>not</u> related to the solution of a problem. After about 18-20 minutes wander about the class. If some pupils appear to be moving too slowly, suggest that they start on problems 14-20.

When you feel the majority of pupils are finished, collect the "Answer Sheets" and "Question Sheets." If about everyone has not finished after 35 minutes, collect the materials anyway.

(SEE NEXT PAGE FOR RETURNING MATERIALS)

RETURNING MATERIALS (September, 1968)

Return only each pupils answer sheet and this sheet.* Destroy all "Question Sheets" soon after the testing is complete.

Please complete the information below and return it with the answer sheets in the enclosed envelope.

Teacher Name _____

School _____

School Address _____

_____ Zip _____

How many pupils are in your class? _____

About how long did the average pupil spend on the test? _____

Number of pupils absent? _____

*In an earlier letter a class list was requested. This will not be necessary since we will construct the list from the answer sheets.

PATTERNS IN ARITHMETIC - 6
TESTING PROGRAM

Enclosed find test materials and directions necessary for completing part 4 of our 4 part testing program. This is perhaps the most important of the four testing periods. May I suggest that upon completion of the testing program you retain one copy of each of the twelve tests.

A report will be sent to each teacher and administrator involved in the study. The tests will be useful for interpreting the results and can serve as a source of questions for class evaluation next year.

WHEN TO ADMINISTER THE TESTS: On May 15, Program 63 on exponents will be shown. Administer the tests a day or two after you have completed the exercises associated with Program 63. Depending upon your class, May 16, 19, 20, or 21 should provide an adequate choice of days.

Before giving the test please read over the following general comments and Specific Directions for Testing.

GENERAL COMMENTS

1. The same twelve, 20-item tests are again used. The number of each test is in the square in the upper, right-hand corner of the cover.

2. IMPORTANT:

A class list for your class is enclosed. It was prepared from answer sheets returned at T_1 and modified following T_2 and T_3. The test number that each pupil should receive is indicated beside his name under the column headed T_4 (to correspond to the fourth testing period). This column represents a random assignment of pupils to tests and it is very important that this assignment remain undisturbed. A few extra tests are included for pupils whose names are not on your class list. It makes no difference which test these pupils take. The important thing is to make sure that each pupil named on the class list gets the test number specified under T_4.

Considerable effort has been made to send the tests to you in the same pattern that they appear under T_4 on the class list. It is

advisable that you put the name of each pupil on the front cover of the test he is to receive. For example, if it is indicated under T_4 that the first person on your class list is to receive test number 7, then you should place his name on the front cover of test 7 booklet. Your booklets are hopefully arranged so that the first booklet should correspond to the first person on your class list, etc.

3. Columns T_1, T_2 and T_3 on your class list have been marked out to lessen the chance for error.

4. Be sure and have each pupil circle yes or no on his answer sheet to indicate whether he participated in PIA-5. This information is still needed to group tests.

5. You will not be expected to grade any of these tests. The answer sheets, attached to the back of each test should be returned to us immediately after the testing. Copies of each test may be saved for your own use next year and for interpreting data which you will receive in the form of a "report on the project."

SPECIFIC DIRECTIONS FOR THE TESTING - MAY PERIOD

READ OVER CAREFULLY BEFORE THE TESTING

1. Reserve about 45 minutes for the testing.
2. Make sure each pupil has two pencils and an eraser. The back of the test cover and the question sheets can be used for scratch paper.

Follow the steps below and read the material in quotes to your

class. Before passing out the tests say slowly -

> "Today I have some more questions about arithmetic that I want you to answer. Now I am going to pass the questions out, but do not open them until I say to."

Now pass out the tests one-by-one making sure each pupil gets the test number indicated in column T_4 on your class list. The number of each test is on the front cover in the square in the upper, right-hand corner.

After everyone has a test, have each pupil tear off carefully the answer sheet that is stapled with the questions and is the last page of each test. Then say -

> "Has everyone removed the answer sheet? (pause) Now write your name clearly in the upper right-hand corner."
> (Allow time for this)

Now say -

> "Did you use TV arithmetic in grade 5? If your answer is yes circle "yes" under your name. If your answer is no circle "no" under your name." (Repeat if necessary)

Then say -

> "There are twenty questions in all. Some questions are multiple choice. You should guess at the answer only if you can rule out some of the choices. All answers go on the answer sheet. Now open your questions to the first page and lets read the instructions. (pause briefly) . . . You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and circle your choice on the answer sheet provided. You should have enough time to work on every question. Do not spend too much time on any problem."

Now ask informally if there are any questions. If there are no questions say -

"You will have about 30 minutes to answer all the questions . . . now begin."

Feel free to answer any questions during the testing that are not related to the solution of a problem. After about 18-20 minutes wander about the class. If some pupils appear to be moving too slowly, suggest that they start on problems 14-20.

When you feel the majority of pupils are finished, collect the "Answer Sheets" and "Question Sheets." If about everyone has not finished after 35 minutes, collect the materials anyway.

(SEE NEXT PAGE FOR RETURNING MATERIALS)

B4

RETURNING MATERIALS (MAY, 1969)

Return only each pupils answer sheet and this sheet.

Please complete the information below and return it with the answer sheets in the enclosed envelope.

Teacher Name _____

School _____

School Address _____

_____ Zip _____

Date Tests were Administered _____

About how long did the average pupil spend on the test? _____

ENCLOSE THIS FORM WITH THE ANSWER SHEETS

◀ B5 ▶

Class Size _____

Name _____

School _____

Address _____

	T_1	T_2	T_3	T_4
Time	20	25	25	30
Absent	0	2	3	2

	Pupil Name	PIA-5	T_1	T_2	T_3	T_4
1.	Janet M.	+	9	11	5	2
2.	Paul R.	+	5	6	9	8
3.	Betty S.	+	11	9	5	1
4.	Tama B.	+	12	2	1	5
5.	Carl F.	+	8	9	4	5
6.	William C.	+	7	5	9	8
7.	John B.	-	2	3	8	10
8.	Richard M.	+	6	12	10	11
9.	Fred K.	+	9	12	1	7
10.	Jerry J.	+	4	1	11	8
11.	Karen S.	+	8	1	2	6
12.	Vera B.	+	12	4	3	6
13.	Herb K.	+	3	1	6	12
14.	Dan W.	-	3	6	1	9
15.	Tom R.	+	5	3	4	12
16.	Evelyn H.	+	10	7	12	3
17.	Larry S.	+	9	10	7	4
18.	Kay S.	+	11	10	8	2
19.	Bobby P.	+	2	5	12	4
20.	Rochelle M.	+	7	2	11	10
21.	Joyce B.	+	1	7	6	11
22.	Fred W.	+	10	4	3	1
23.	Irv K.	+	8	11	10	7
24.	June V.	-	4	8	7	9
25.	Henry V.	+	6	8	2	3

PART I

TWELVE TESTS AND GROWTH GRAPHS

PATTERNS IN ARITHMETIC
Grade 6
1

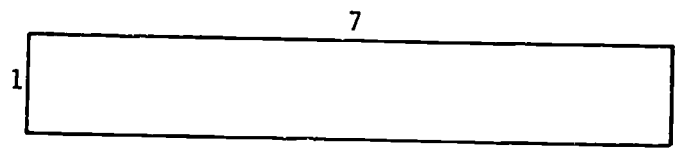
TEST 1

QUESTION SHEETS

Instructions: You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and circle your choice on the answer sheet provided.

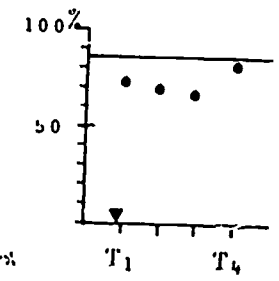
You should have enough time to work on every question. Do not spend too much time on any problem.

1. What is the distance around the rectangle below?



a) 7
b) 8
c) 14
d) 16

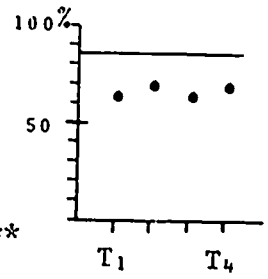
****



2. Which of the following sentences is equivalent to n x 6 = 48?

a) n = 6 x 48
b) 6 ÷ 48 = n
c) n ÷ 6 = 48
d) 48 ÷ 6 = n

****



3. What is the greatest number that divides both 28 and 42?

a) 2
b) 7
c) 14
d) 42

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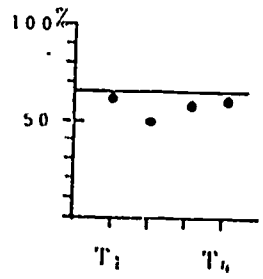
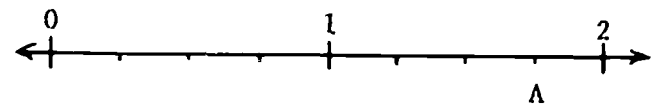
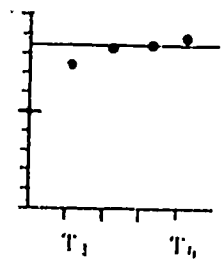




4. On the number line above, point A can be named by

a) $\frac{7}{8}$
b) $1\frac{3}{4}$
c) $\frac{8}{7}$
d) $\frac{3}{4}$



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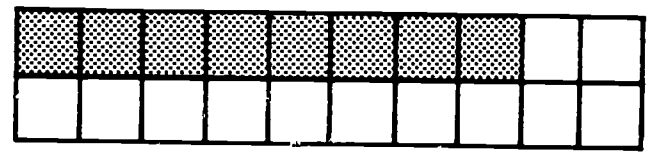
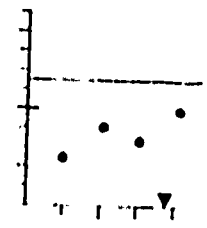


5. What percent of the above figure is shaded?

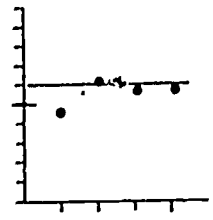
a) 40
b) 50
c) 25
d) 8

****



6. Light travels about 186,000 miles per second. About how long will it take light to travel 186 miles?

a) 0.001 second
b) 0.01 second
c) 1000 seconds
d) 3 hours



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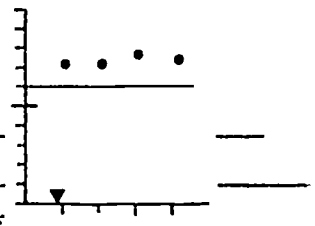
7. We have many standard units for linear measure. Four of these are yard, inch, foot and meter. Relative lengths of these four standard units are represented by the line segments below. Which line segment represents one foot?

a) _
b) ———
c) ———
d) ———



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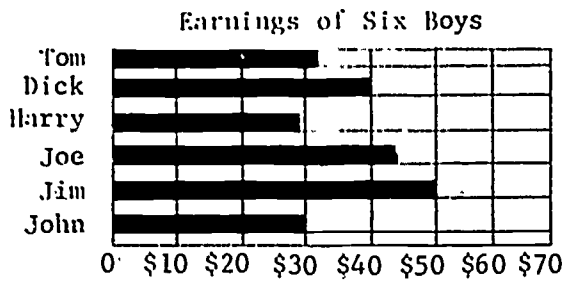
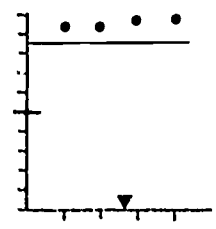
Questions 8 and 9 refer to the Graph below



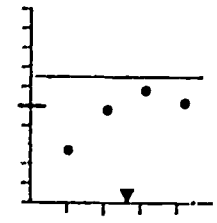
8. Which three boys earned about the same amount of money?

a) Tom, Harry and John
b) Dick, Joe and John
c) Tom, Joe and Jim
d) Tom, Harry and Joe



9. Estimate the difference between the greatest and least amounts of money earned.

a) about $17
b) about $19
c) about $20
d) about $21
e) about $25



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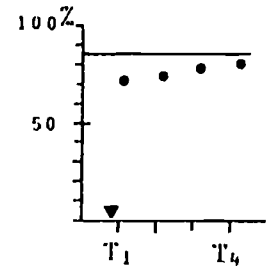
10. The relationship $\frac{\text{inches}}{\text{yards}}$ is expressed by

a) $\frac{1}{1}$
b) $\frac{3}{1}$
c) $\frac{12}{1}$
d) $\frac{24}{1}$
e) $\frac{36}{1}$



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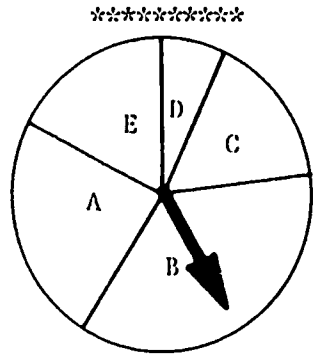
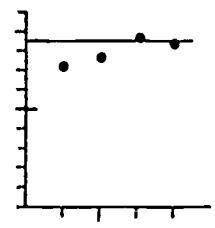


11. The arrow above can spin freely about the center of the circle. Which section of the circle is the arrow least likely to stop on?

a) A
b) B
c) C
d) D
e) E



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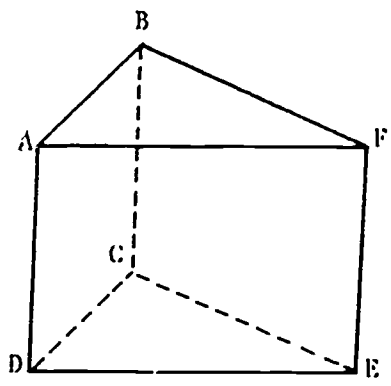


12. In the above triangular prism, the bottom face is named by

    a) ABF

    b) CDE

    c) BCEF

    d) ABCD

****

13. A triangle has a perimeter of 60.3 inches. If each side is 2 inches longer what would be the perimeter?

    a) 62.3

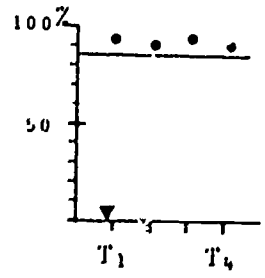
    b) 64.3

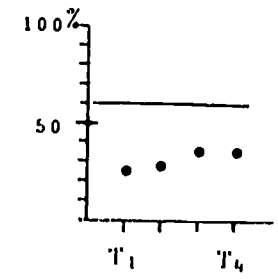
    c) 66.3

    d) 68.3

****

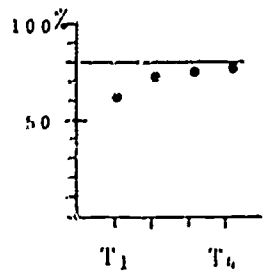
GO TO NEXT PAGE



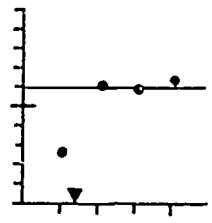


Work the remaining problems on scratch paper or beside the problem and place your answer on the answer sheet in the space provided.

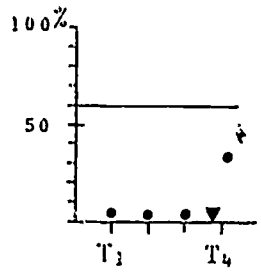
14. 571 x 68 = ?



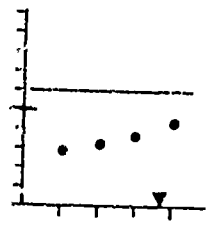
15. $\frac{1}{8} + \frac{5}{6} = ?$



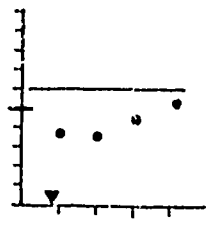
16. What is 15% of 80?



17. Three pieces of paper weigh 0.03 pounds. How much will 12 pieces of paper weigh?

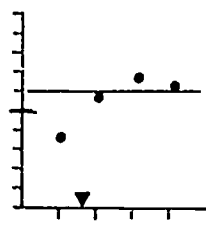


18. $\frac{?}{40} = \frac{160}{200}$

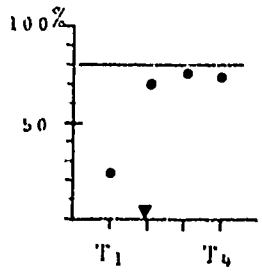


Work problem 19 on the back of your answer sheet. Leave your answer there.

19. 3879 ÷ 27 = ?



20. $\frac{4}{5} \times \frac{9}{8} = ?$



PATTERNS IN ARITHMETIC
Grade 6
1

Name _____

Did you use TV arithmetic in grade 5? Yes No

ANSWER SHEET

Circle your choice

1.	a	b	c	d		8.	a	b	c	d	
2.	a	b	c	d		9.	a	b	c	d	e
3.	a	b	c	d		10.	a	b	c	d	e
4.	a	b	c	d		11.	a	b	c	d	e
5.	a	b	c	d		12.	a	b	c	d	
6.	a	b	c	d		13.	a	b	c	d	
7.	a	b	c	d							

Place your answers for problems 14-20 in the blanks below.

14. _____

15. _____

16. _____

17. _____

18. _____

19. Answer on back of this sheet.

20. _____

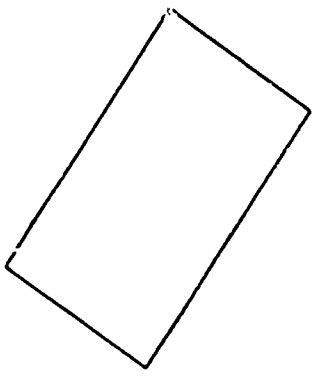
PATTERNS IN ARITHMETIC
Grade 6
?

TEST 2

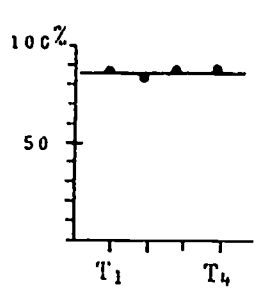
QUESTION SHEETS

Instructions: You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and circle your choice on the answer sheet provided.

You should have enough time to work on every question. Do not spend too much time on any problem.



1. The above figure is a
   a) circle
   b) pentagon
   c) rectangle
   d) square
   e) triangle



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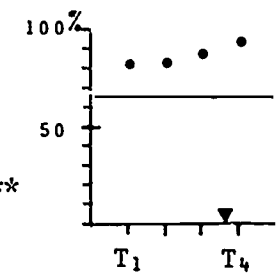
2. Mr. Jackson drove Monday, Tuesday and Wednesday and traveled 1260 miles. How many miles did he average per day?
   a) 420
   b) 500
   c) 630
   d) 1260



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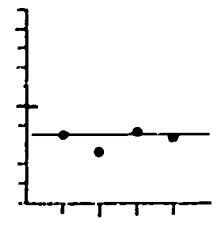
3. The ratio of $\frac{\text{liters}}{\text{gallons}}$ is $\frac{3.79}{1}$. From this ratio, 1 liter must be about
   a) 1 cup
   b) 1 pint
   c) 1 quart
   d) $\frac{1}{2}$ gallon



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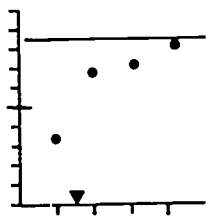
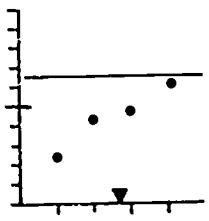
4. Another way to write $2\frac{1}{2}$ is:
   a) $0.2\frac{1}{2}$
   b) 2.5
   c) 0.25
   d) 2.05



****

5. If $F = (\frac{9}{5} \times C) + 32$, what is F when $C = 0$?
   a) $\frac{9}{5} + 32$
   b) 32
   c) $\frac{90}{5} + 32$
   d) 0



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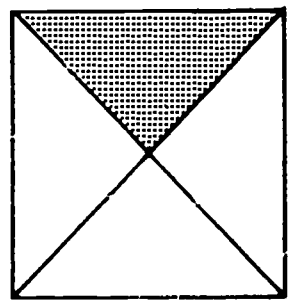
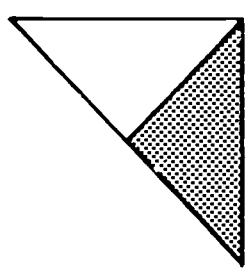
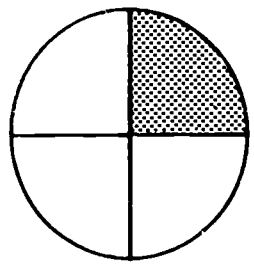


6. Which figure below has the same percent shaded as the figure above.

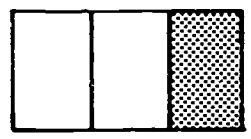
a)



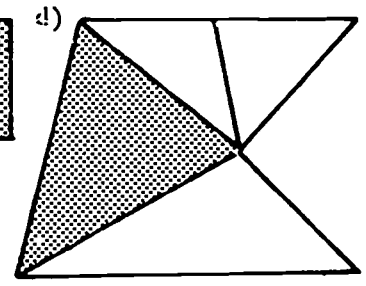
b)

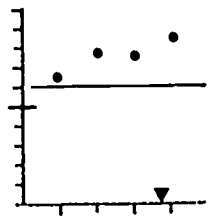


c)



d)





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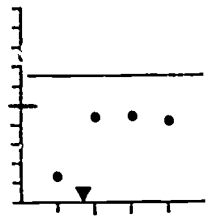
7. The sentence $0.083 = 83 \times \frac{1}{S}$ is true when S = ?

a) 0.001

b) 10

c) 100

d) 1000



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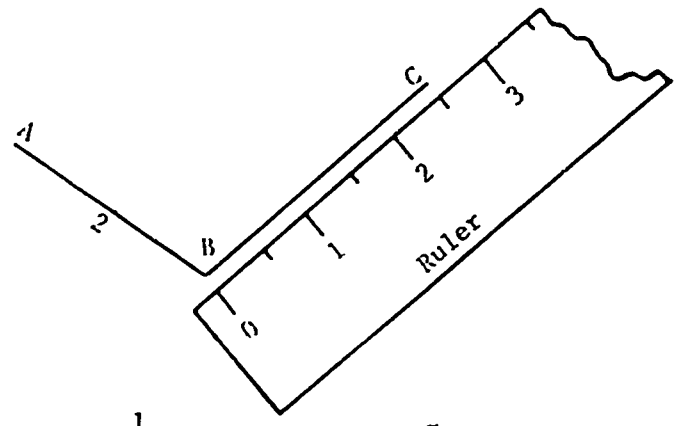
8. If 1 inch represents 150 miles. How many miles is it from A to C if you go through B?



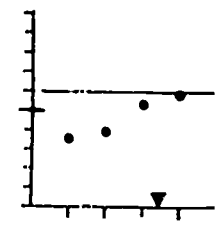
a) $4\frac{1}{2}$

b) 150

c) 30

d) 600

e) 675



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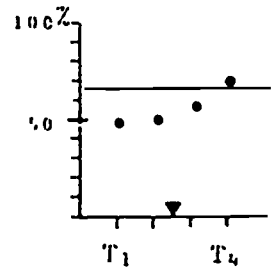
9. If $2 \times N = \frac{1}{3}$, what is N?

a) $\frac{1}{6}$

b) $\frac{2}{3}$

c) $\frac{3}{2}$

d) 6



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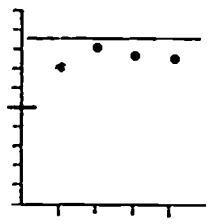
Day	Probability of rain
Monday	0
Tuesday	0.20
Wednesday	0.40
Thursday	1.00
Friday	0.30

10. From the chart above, which day is it most likely to rain.

a) Monday
b) Tuesday
c) Wednesday
d) Thursday
e) Friday



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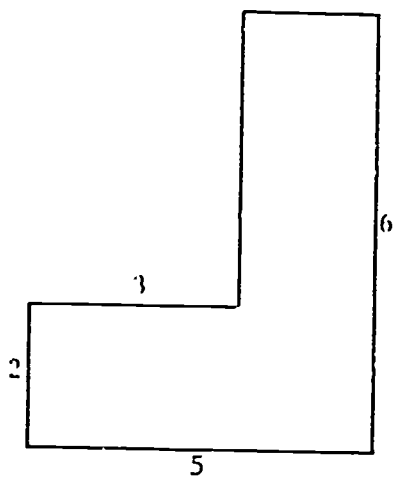
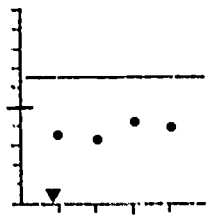


11. What is the area of the figure above?

a) 15
b) 16
c) 18
d) 22
e) 30



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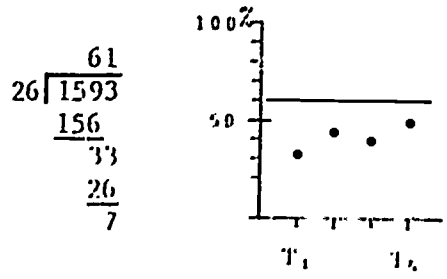

12. In the above long division problem the 156 really stands for

a) 159 - 3
b) 26 x 61
c) 60 x 26
d) 7 x 26

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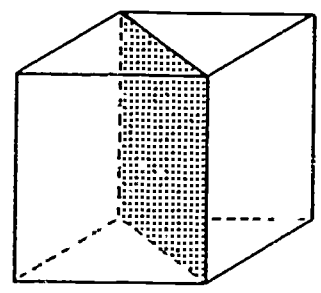


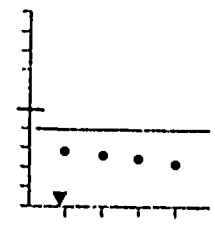
13. A cut is made through the above cube to give the shaded face. What is the shape of this shaded face?

a) a rectangle but not a square
b) a square
c) a parallelogram but not a rectangle
d) a parallelogram but not a square

****

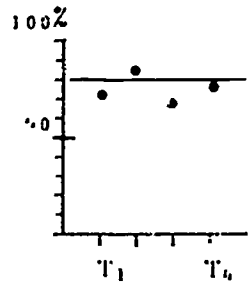
BAD ITEM

GO TO NEXT PAGE

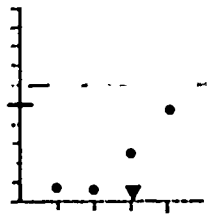


Work the remaining problems on scratch paper or beside the problem and place your answer on the answer sheet in the space provided.

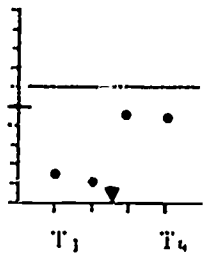
14. $\frac{2}{3} + \frac{7}{3} = ?$



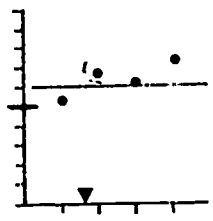
15. $8 \div \frac{1}{10} = ?$



16. $\frac{1}{3} \times \boxed{?} \times 5 = 5$

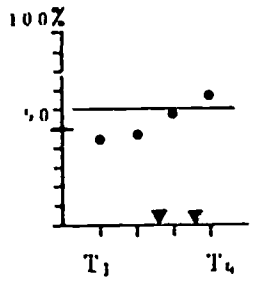


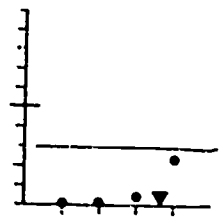
Work problem 17 on the back of your answer sheet. Leave your answer there.



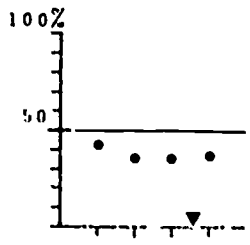
17. $8708 \div 9 = ?$

18. One inch on a map represents 6 miles. How many miles does $3\frac{1}{2}$ inches on the map represent?





19. John has a stack of 500 pieces of notebook paper. How much does <u>each piece</u> <u>weigh</u> if all 500 pieces weigh 3 pounds?



20. Jim saw a sign in the store window which said "20% OFF ON ALL SKATES." Jim bought a pair of skates for $8. How much were the skates originally?

PATTERNS IN ARITHMETIC
Grade 6
2

Name _____

Did you use TV arithmetic in grade 5? Yes No

ANSWER SHEET

Circle your choice

1.	a	b	c	d	e	8.	a	b	c	d	e
2.	a	b	c	d		9.	a	b	c	d	
3.	a	b	c	d		10.	a	b	c	d	e
4.	a	b	c	d		11.	a	b	c	d	e
5.	a	b	c	d		12.	a	b	c	d	
6.	a	b	c	d		13.	a	b	c	d	
7.	a	b	c	d							

Place your answers for problems 14-20 in the blanks below.

14. _____

15. _____

16. _____

17. Answer on back of this sheet.

18. _____

19. _____

20. _____

PATTERNS IN ARITHMETIC
Grade 6
3

TEST 3

QUESTION SHEETS

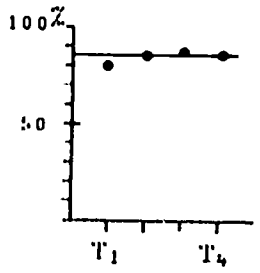
Instructions: You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and circle your choice on the answer sheet provided.

You should have enough time to work on every question. Do not spend too much time on any problem.

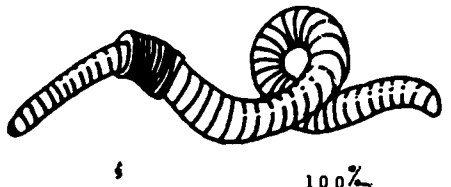
1. If N - 6 = 8, then N is

   a) 68
   b) 48
   c) 14
   d) 2

****

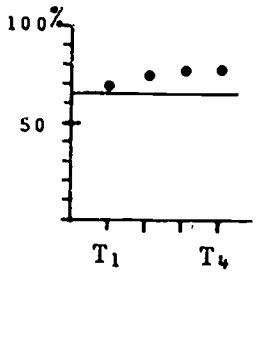


2. What unit of measure would be most appropriate to express the length of the earth worm below?



   a) square inches
   b) centimeters
   c) yards
   d) feet

****

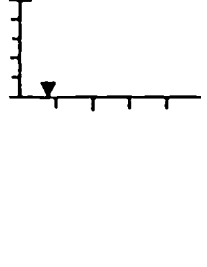


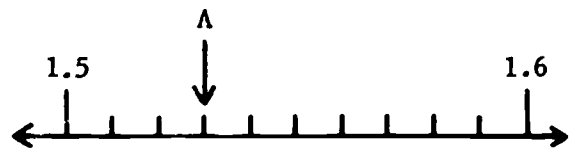
3. What number in place of ☐ will make the following true?

$$\frac{4}{10} = \frac{\square}{25}$$

   a) 5
   b) 10
   c) 19
   d) 31
   e) $62\frac{1}{2}$

****

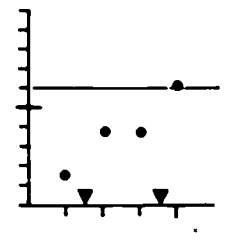


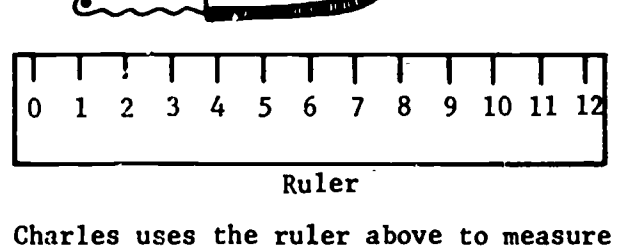


4. In the figure above, the part of the number line from 1.5 to 1.6 has been divided into ten equal spaces. What number names point A?

   a) 4.5
   b) 3.0
   c) 1.8
   d) 1.53

****



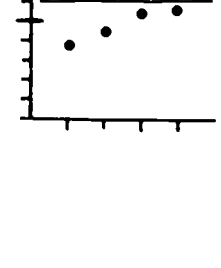


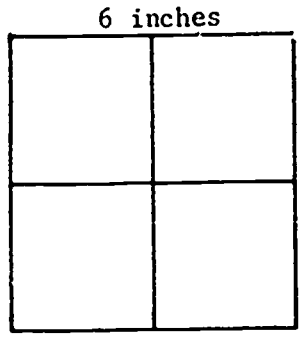
Ruler

5. Charles uses the ruler above to measure the length of a knife. How long is the knife?

   a) 7 inches
   b) 8 inches
   c) $\frac{1}{2}$ foot
   d) 9 inches

****

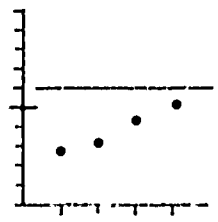




6. The large square above has been divided into four small squares. What is the perimeter of a small square?

a) 3 inches
b) 6 inches
c) 12 inches
d) 24 inches

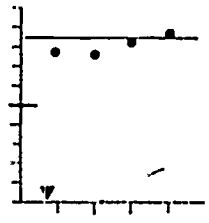
****

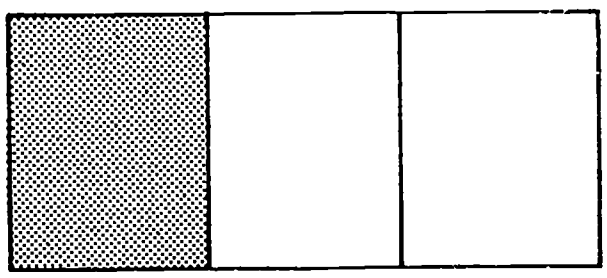


7. Candy bars are 3 for 20¢. How many could you buy for $1.00?

a) 6
b) 15
c) 40
d) 60

****

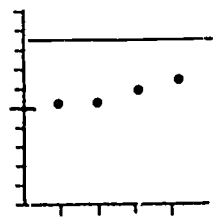




8. If the area of the rectangle above is 18, what is the area of the shaded part?

a) $18 \div \frac{1}{3}$
b) $\frac{1}{3}$
c) 3
d) 6

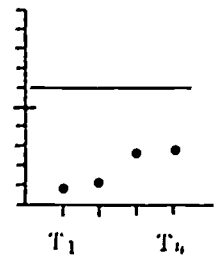
****



9. Which sentence below is not equivalent to $0.9 \times C = 27$?

a) $9 \times C = 270$
b) $C = 3.0$
c) $C = 30$
d) $C = 27 \div 0.9$

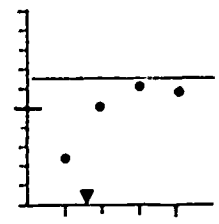
****



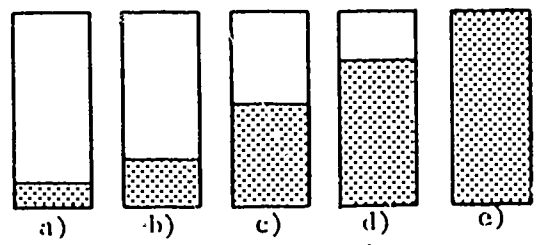
10. $300 + 8 + \frac{2}{5} =$

a) 384.0
b) 382.5
c) 308.4
d) 308.2

****



11. Which of the following 10-gallon cans most nearly contains 10 quarts of water?



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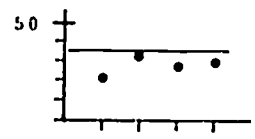
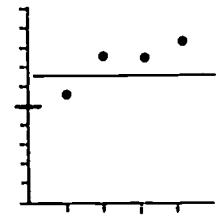


12. Which of the following expresses 24 as a product of prime numbers?

a) 3×8
b) $2 \times 3 \times 4$
c) 4×6
d) $2 \times 2 \times 2 \times 3$

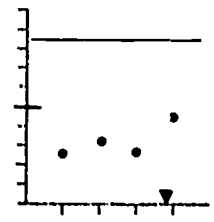
****



13. Out of 100 cars tested, 4 were found to have bad tires. What percent had bad tires?

a) 4
b) 0.4
c) 0.04
d) 40

****



Work the remaining problems on scratch paper or beside the problem and place your answer on the answer sheet in the space provided.

14. 80 x 407 = ?

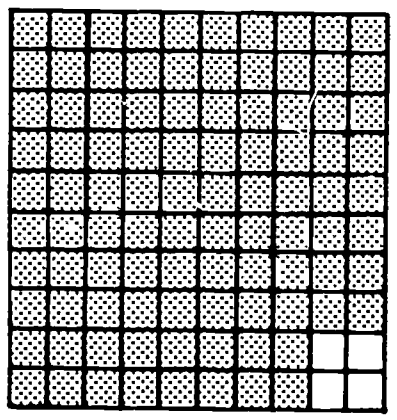
15. 483 ÷ 21 = ?

16. What is $\frac{1}{3}$ of $\frac{1}{5}$?

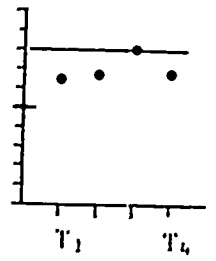
17. $5 - 2\frac{3}{7} - ?$

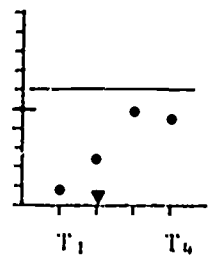
18. If 1 inch = 2.54 centimeters, how many centimeters are in a foot?

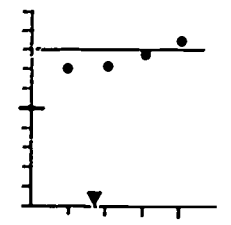
19. What percent of the large square is shaded?

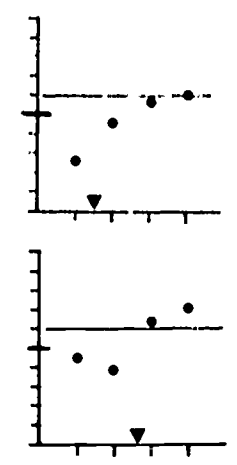


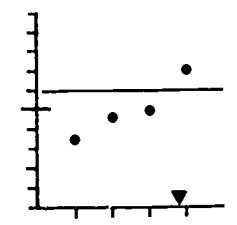
20. Find the basic fraction for $\frac{36}{42}$.

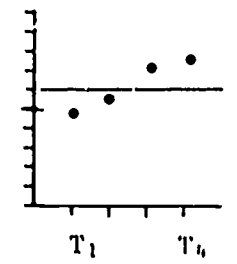












PATTERNS IN ARITHMETIC
Grade 6
3

Name _____

Did you use TV arithmetic in grade 5? Yes No

ANSWER SHEET

Circle your choice

1. a b c d
2. a b c d
3. a b c d e
4. a b c d
5. a b c d
6. a b c d
7. a b c d
8. a b c d
9. a b c d
10. a b c d
11. a b c d e
12. a b c d
13. a b c d

Place your answers for problems 14-20 in the blanks below.

14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____

ERIC

PATTERNS IN ARITHMETIC
Grade 6
4

TEST 4

QUESTION SHEETS

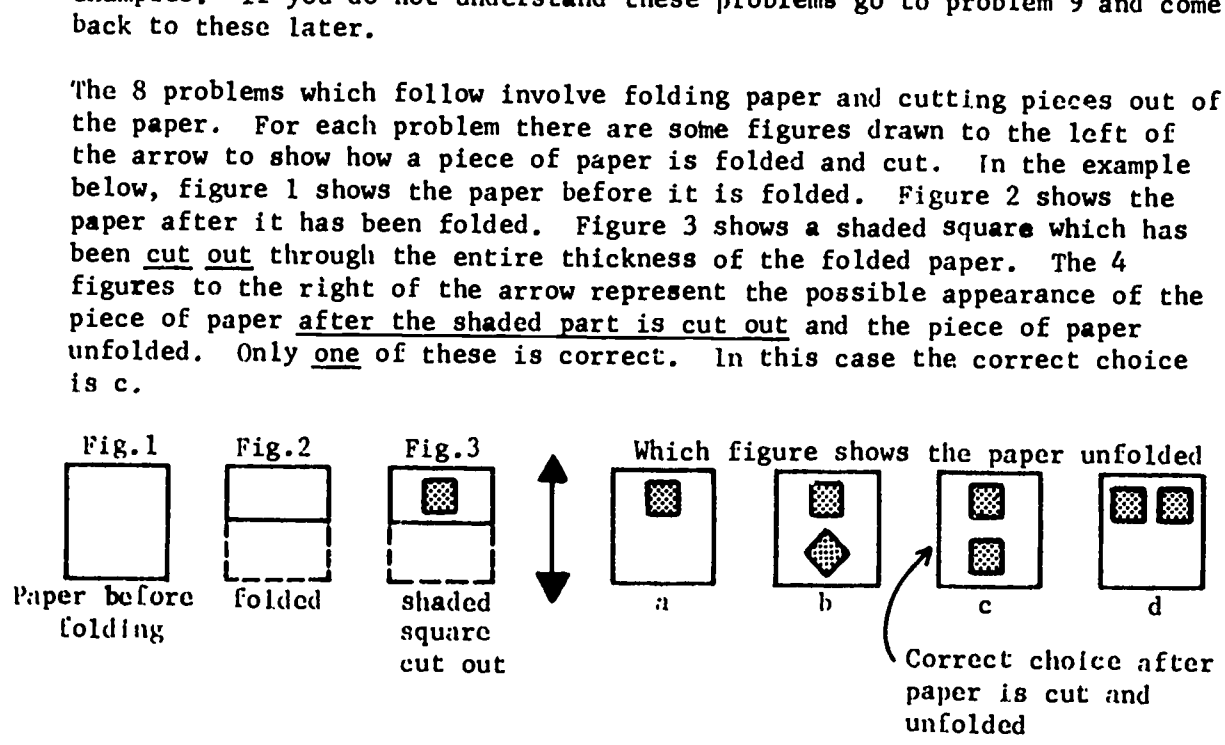
Instructions: You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and circle your choice on the answer sheet provided.

You should have enough time to work on every question. Do not spend too much time on any problem.

Read the paragraph below very carefully. Take your time to understand the examples. If you do not understand these problems go to problem 9 and come back to these later.

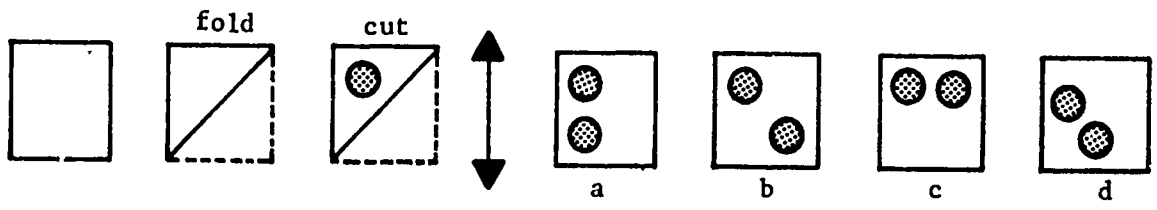
The 8 problems which follow involve folding paper and cutting pieces out of the paper. For each problem there are some figures drawn to the left of the arrow to show how a piece of paper is folded and cut. In the example below, figure 1 shows the paper before it is folded. Figure 2 shows the paper after it has been folded. Figure 3 shows a shaded square which has been cut out through the entire thickness of the folded paper. The 4 figures to the right of the arrow represent the possible appearance of the piece of paper after the shaded part is cut out and the piece of paper unfolded. Only one of these is correct. In this case the correct choice is c.

Example 1



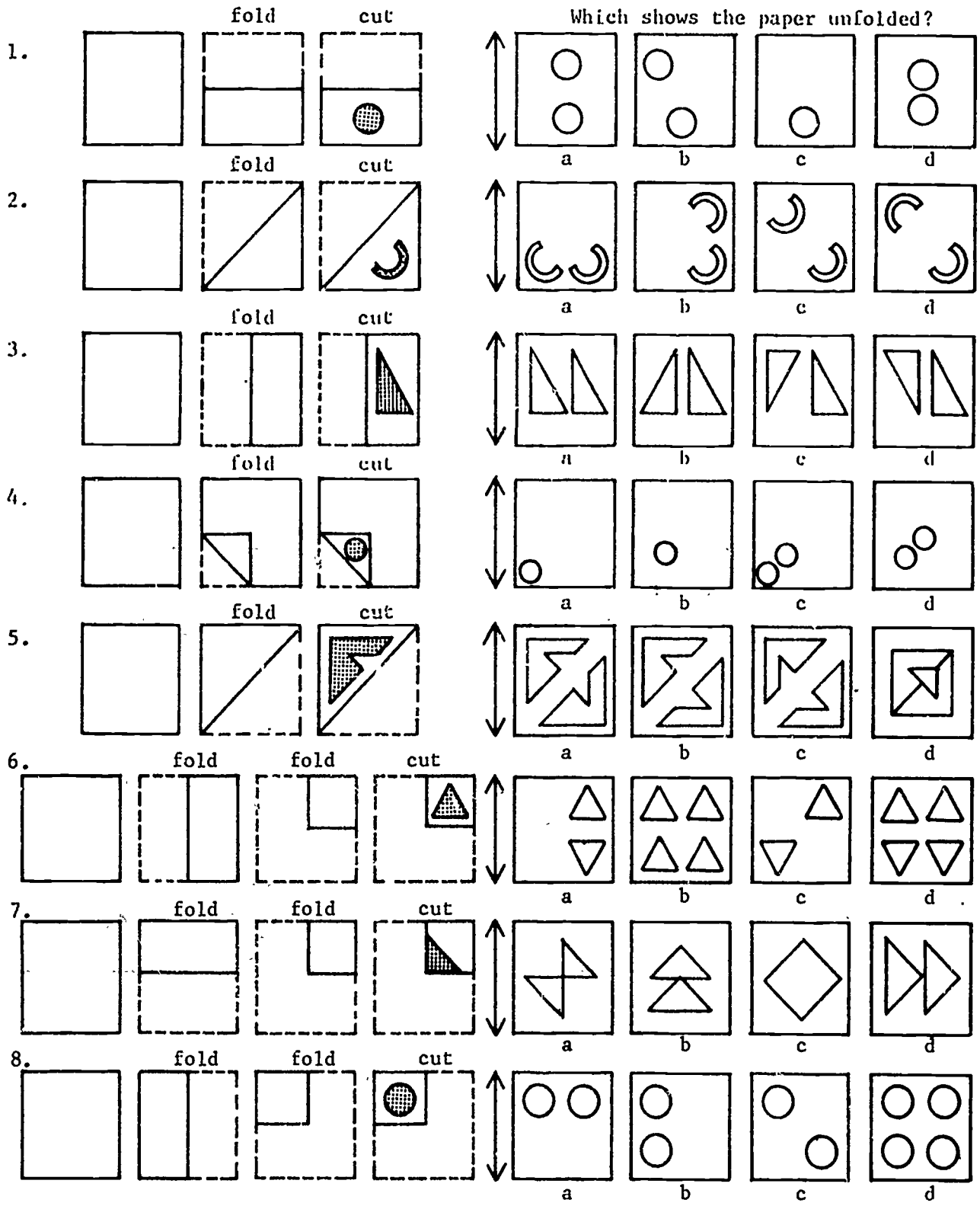
Now look closely at example 2 and observe that b is the correct choice.

Example 2



For each problem below look how the paper is folded and cut. Then choose the figure that shows how the paper looks after it is unfolded.

Circle your choice on the answer sheet.



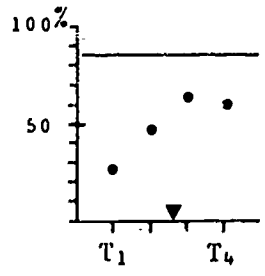
9. Which of the following corresponds to a counting number?

a) $\frac{6}{3}$

b) $\frac{4}{8}$

c) $\frac{1}{2}$

d) $1\frac{1}{2}$



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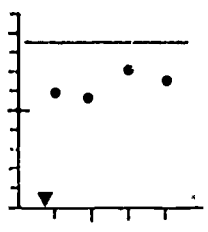
10. A square is nine inches on a side. What is its perimeter?

a) 9 x 9

b) 9 + 9

c) 4 x 9

d) 2 x 9



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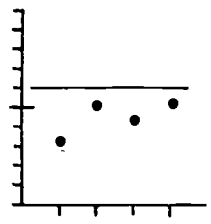
11. John measured the top of his desk and found its area. Which number could be the area of John's desk?

a) 2 ft.

b) 90 sq. yds.

c) 750 sq. in.

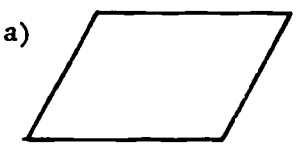
d) 1 meter



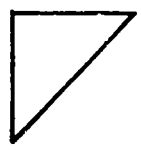
****

12. In which figure are all angles congruent?

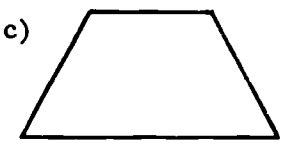
a)



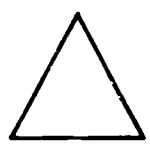
b)



c)



d)



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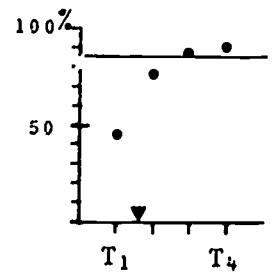
13. Another way to write 0.25 is:

a) $\frac{25}{1}$

b) $\frac{25}{10}$

c) $\frac{25}{100}$

d) $\frac{25}{1000}$



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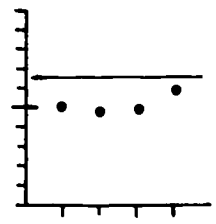
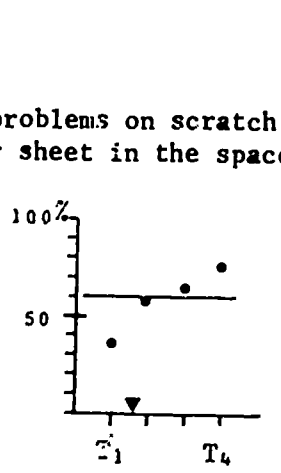
GO TO NEXT PAGE

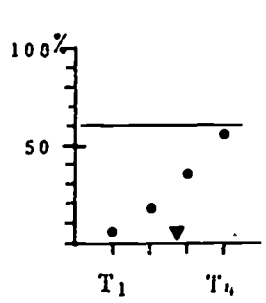


Work the remaining problems on scratch paper or beside the problem and place your answer on the answer sheet in the space provided.

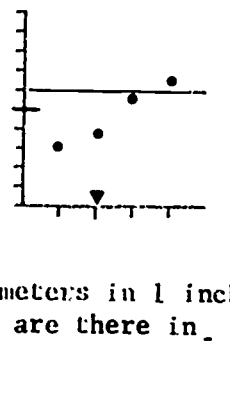
14. $\frac{3}{5} - \frac{1}{4} = ?$



15. 0.2 x 0.3 = ?

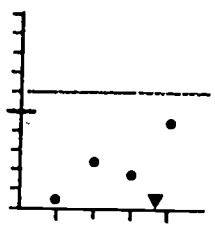


16. What is $\frac{2}{3}$ of 27?

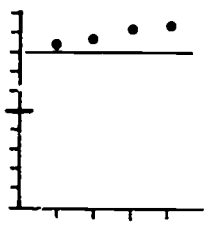


17. There are 2.54 centimeters in 1 inch. How many centimeters are there in 10 inches?

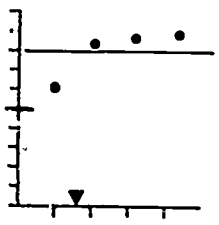
18. 16 is what percent of 20?



19. A box contains 121 red marbles 87 green marbles 148 yellow marbles and 18 black marbles. If you close your eyes and pick a marble which color are you most likely to pick?



20. Write "four and twenty-seven hundredths" as a decimal.



PATTERNS IN ARITHMETIC
Grade 6
4

Name _____

Did you use TV arithmetic in grade 5? Yes No

ANSWER SHEET

Circle your choice

1.	a	b	c	d		8.	a	b	c	d
2.	a	b	c	d		9.	a	b	c	d
3.	a	b	c	d		10.	a	b	c	d
4.	a	b	c	d		11.	a	b	c	d
5.	a	b	c	d		12.	a	b	c	d
6.	a	b	c	d		13.	a	b	c	d
7.	a	b	c	d						

Place your answers for problems 14-20 in the blanks below.

14. _____

15. _____

16. _____

17. _____

18. _____

19. _____

20. _____

PATTERNS IN ARITHMETIC
Grade 6
5

TEST 5

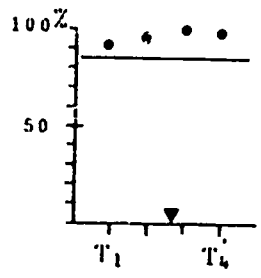
QUESTION SHEETS

Instructions: You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and circle your choice on the answer sheet provided.

You should have enough time to work on every question. Do not spend too much time on any problem.

1. The sentence 3 x N = 12 is true when N = ?

   a) 2
   b) 4
   c) 9
   d) 36



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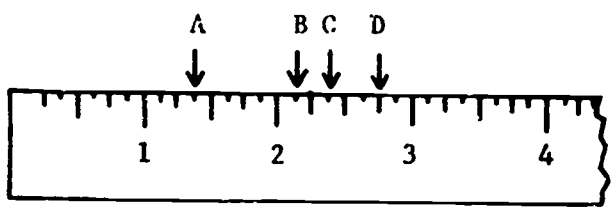
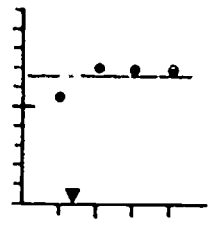


2. Which arrow above is pointing to a number between $2\frac{5}{8}$ and $2\frac{7}{8}$?

   a) arrow A
   b) arrow B
   c) arrow C
   d) arrow D



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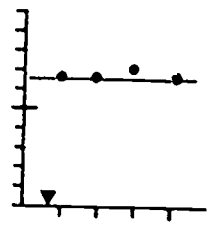
3. Which object below is about 1 meter long?

   a) Book
   b) Yardstick
   c) Car
   d) Bus



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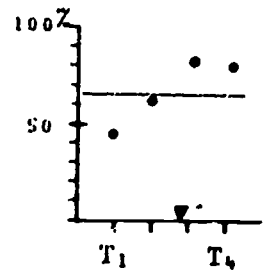
4. -2 + (-7) = ?

   a) 9
   b) 5
   c) -9
   d) -5
   e) 14



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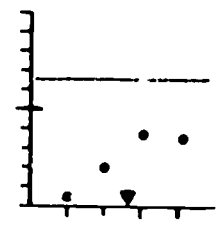
5. If we replace m by 4, then

$$\frac{(3 \times m) + 8}{m}$$ will be?

   a) 12
   b) $\frac{12}{4}$
   c) 5
   d) 20



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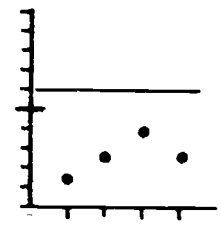
6. If (E,F) represents the number of edges (E) and the number of faces (F), what is (E,F) for a cube?

   a) (4, 4)
   b) (6, 12)
   c) (12, 4)
   d) (12, 6)



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$$9\overline{)284}\quad 31,\ 27,\ 14,\ 9,\ 5$$

7. In the division example above, the remainder is

   a) 31

   b) 9

   c) 284

   d) 5

****

8. Kay made 200 cookies and used 8 cups of sugar. The next day she used the same recipe, but made only 50 cookies How many cups of sugar did she use?

   a) 2

   b) 4

   c) 6

   d) 8

****

9. Another way to write "four fifths of twenty" is

   a) (4 x 5) x 20

   b) $\frac{4}{5} \div 20$

   c) $\frac{4}{5} \times 20$

   d) 0.45 x 20

****

10. If two distances are equal, what is the ratio of the first distance to the second distance?

    a) 0

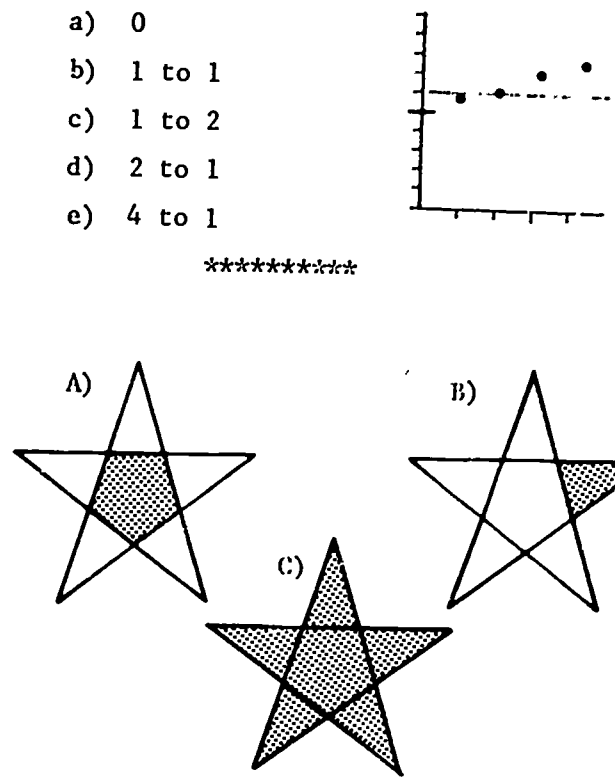
    b) 1 to 1

    c) 1 to 2

    d) 2 to 1

    e) 4 to 1

****



11. The area of the shaded region in figure A is 5 square units. The area of the shaded region in figure B is 2 square units. The area of the shaded region in figure C is

    a) 6 square units

    b) 7 square units

    c) 15 square units

    d) you can not tell

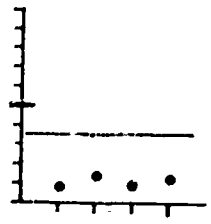
****

12. If n is a counting number, how is $\frac{1}{n}$ related to $\frac{1}{n+1}$?

a) $\frac{1}{n} > \frac{1}{n+1}$

b) $\frac{1}{n+1} < \frac{1}{n}$

c) $\frac{1}{n+1} = \frac{1}{n}$



d) you cannot tell until you know what n is.

****

	Inches of Rain
City A	12.7
City B	100.0
City C	2.35
City D	5.1

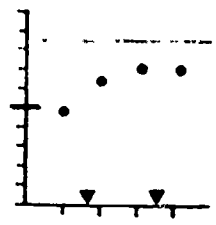
13. The chart above shows the amount of rain for 4 cities. Which city had the least amount of rain?

a) City A

b) City B

c) City C

d) City D



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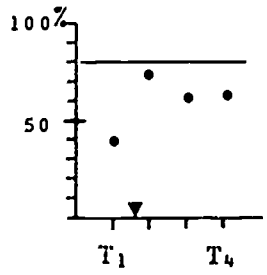
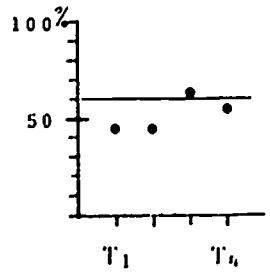
GO TO NEXT PAGE

Work the remaining problems on scratch paper or beside the problem and place your answer on the answer sheet in the space provided.

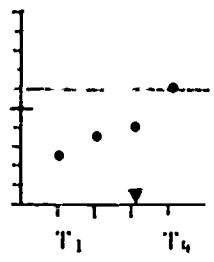
14. $1\frac{1}{4} + 2\frac{3}{8} = ?$



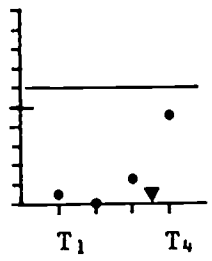
15. 708 x 397 = ?



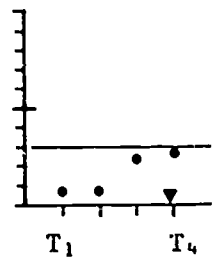
16. $\frac{9}{2} + 3 = ?$



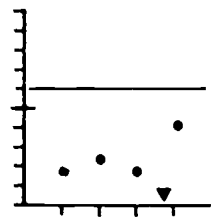
17. 0.483 ÷ 0.21 = ?



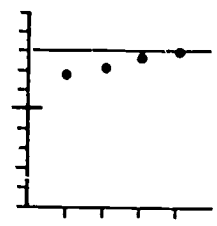
18. Jane made a cake. The recipe called for $\frac{2}{3}$ cup of sugar for each cup of flour. If she used 6 cups of sugar, how many cups of flour should she use?



19. There are 25 children in Mrs. Strangman's class, but on Monday only 20 were present. What percent were present?



20. Judy had ten dollars and she bought a sweater for $4.15. How much money does she have left?



PATTERNS IN ARITHMETIC
Grade 6
5

Name _____

Did you use TV arithmetic in grade 5? Yes No

ANSWER SHEET

Circle your choice

1. a b c d
2. a b c d
3. a b c d
4. a b c d e
5. a b c d
6. a b c d
7. a b c d
8. a b c d
9. a b c d
10. a b c d e
11. a b c d
12. a b c d
13. a b c d

Place your answers for problems 14-20 in the blanks below.

14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____

PATTERNS IN ARITHMETIC
Grade 6
6

TEST 6

QUESTION SHEETS

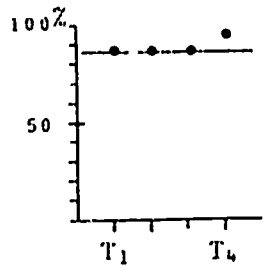
Instructions: You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and circle your choice on the answer sheet provided.

You should have enough time to work on every question. Do not spend too much time on any problem.

1. Which number below is two hundred ten thousand, eighty-six?

   a) 210,086
   b) 21,086
   c) 210,860
   d) 2,001,086

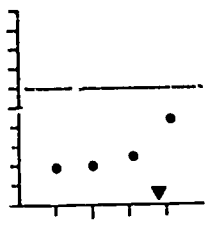
****



2. Which number best represents the probability that a person born 300 years ago is still alive today?

   a) 0
   b) 0.01
   c) $\frac{1}{300}$
   d) 1

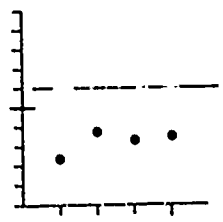
****



3. Five boys share three melons equally. What is each boy's share?

   a) $\frac{1}{5}$ melon
   b) $\frac{5}{3}$ melon
   c) $\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ melon
   d) $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ melon

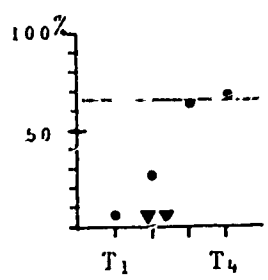
****

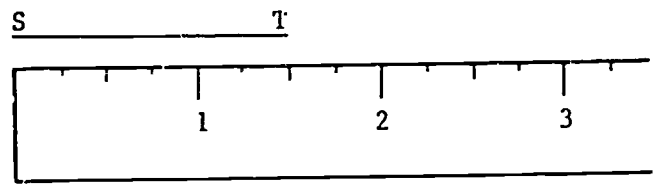


4. $\frac{4}{7} \times \frac{3}{4} \times \frac{7}{12} = ?$

   a) $\frac{1}{12}$
   b) $\frac{3}{8}$
   c) $\frac{1}{4}$
   d) $\frac{1}{3}$

****

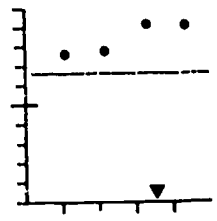




5. In the figure above, the line ST is drawn to scale: 1 inch to 100 feet. What is the distance represented by ST?

   a) 200
   b) 175
   c) 150
   d) 125
   e) 100

****



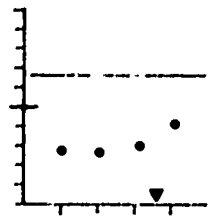
6. For which sentence below is $N \neq 1$.

a) $N = \frac{1}{3} \times 3$

b) $\frac{N}{3} = \frac{2}{6}$

c) $N = \frac{6}{6}$

d) $N = 3 \div \frac{1}{3}$



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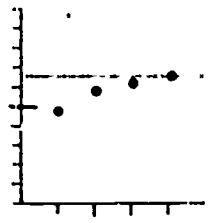
7. The earth's crust is composed of many things. The chart below lists some of them. Which statement below is not true?

Contents of the Earth's Crust

Oxygen	46.6%
Silicon	27.7%
Aluminum	8.1%
Iron	5.0%
Calcium	3.6%
Sodium	2.9%
Others	6.1%



a) There is more oxygen than anything else.

b) There is less sodium than iron.

c) Silicon and Aluminum account for less than 30%.

d) There is about 9 times as much oxygen as iron.

****

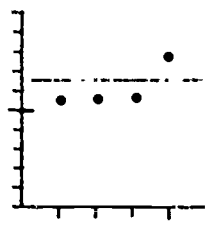
8. Which object below is usually a cylinder?

a) a coke bottle

b) a tin can

c) an egg

d) carton of milk



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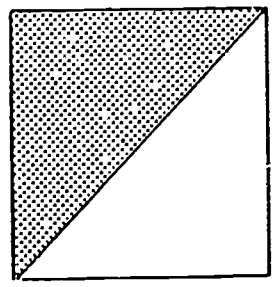
9. If the square below is 2 inches on a side, what is the area of the shaded triangle?

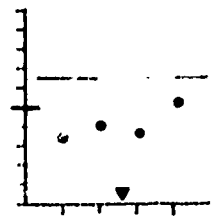


a) 2 square inches

b) 4 square inches

c) 1 square inch

d) $\frac{1}{2}$ square inch



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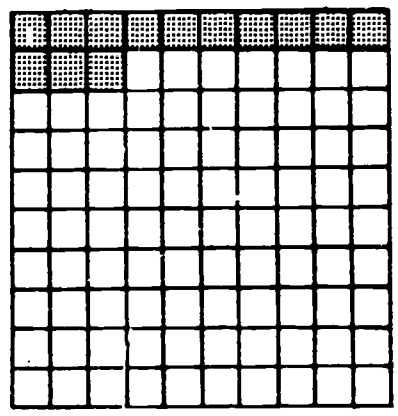


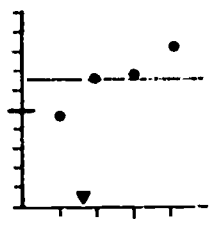
10. What part of the large square above is shaded?

a) 0.13

b) $\frac{1}{10} + \frac{3}{10}$

c) 13

d) 1.3



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11. When one whole number is divided by another whole number, which of the following is always true?

a) The divisor is always smaller than the quotient.

b) The remainder is smaller than the divisor.

c) The quotient is smaller than the divisor.

d) The remainder is smaller than the quotient.

e) The dividend is smaller than the remainder.

****

12. Which decimal below is nearest to one half on the number line?

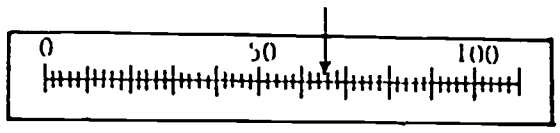
a) 0.152

b) 0.470

c) 0.510

d) 0.501

****



13. On the scale above, the reading indicated by the arrow is between

a) 58 and 60

b) 60 and 62

c) 62 and 64

d) 64 and 66

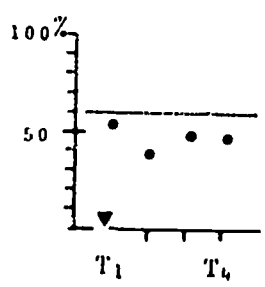
e) 66 and 68

****

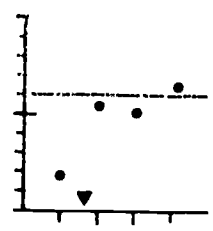
GO TO NEXT PAGE

Work the remaining problems on scratch paper or beside the problem and place your answer on the answer sheet in the space provided.

14. 5 yd. 2 ft.
    2 yd. 2 ft.
    ? yd. 1 ft.

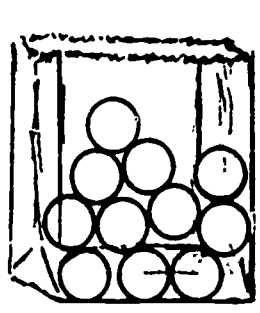


15. $\frac{3}{5} + \frac{9}{10} = ?$



16. $0.3 + \frac{1}{2} = ?$

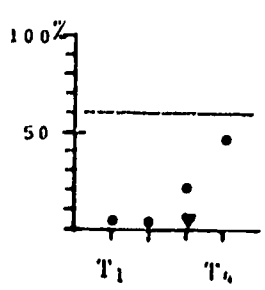
17. $39.2 \div 100 = ?$

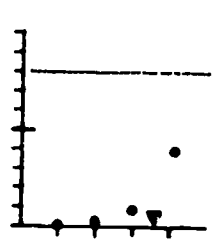


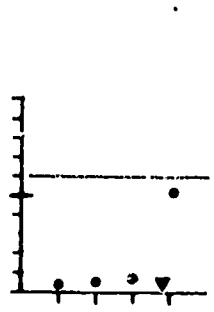
18. The sack above contains 4 BLACK balls and 7 RED ones. If you pick a ball without looking, what is the probability that it will be RED?

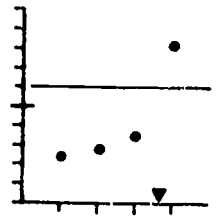
19. Find the <u>average</u> of the numbers 21, 33, 26 and 20.

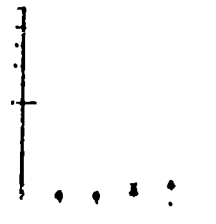
20. A triangle n inches on a side has a perimeter of 60.3 inches. What is the perimeter of a triangle n + 2 inches on a side?











PATTERNS IN ARITHMETIC
Grade 6
6

Name _____

Did you use TV arithmetic in grade 5? Yes No

ANSWER SHEET

Circle your choice

1. a b c d
2. a b c d
3. a b c d
4. a b c d
5. a b c d e
6. a b c d
7. a b c d
8. a b c d
9. a b c d
10. a b c d
11. a b c d e
12. a b c d
13. a b c d e

Place your answers for problems 14-20 in the blanks below.

14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____

ERIC

PATTERNS IN ARITHMETIC
Grade 6
7

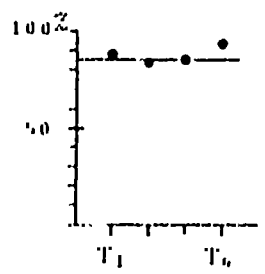
TEST 7
QUESTION SHEETS

Instructions: You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and circle your choice on the answer sheet provided.

You should have enough time to work on every question. Do not spend too much time on any problem.

1. Another name for 2083 is

   a) 20 + 83
   b) 200 + 80 + 3
   c) 2000 + 800 + 3
   d) 2000 + 80 + 3



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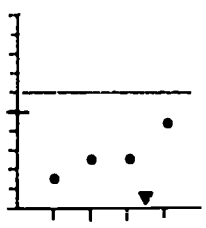
2. Another way to write 30.3 is

   a) 303 ÷ 10
   b) $\frac{303}{100}$
   c) 300 + 0.3
   d) 3 + 0.3



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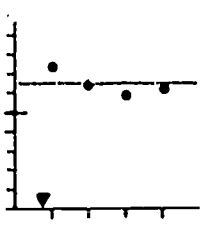
3. Three of our standard units of measure are meter, centimeter and foot. Which units are in the ratio $\frac{100}{1}$.

   a) $\frac{\text{centimeter}}{\text{foot}}$
   b) $\frac{\text{foot}}{\text{centimeter}}$
   c) $\frac{\text{centimeter}}{\text{meter}}$
   d) $\frac{\text{foot}}{\text{meter}}$



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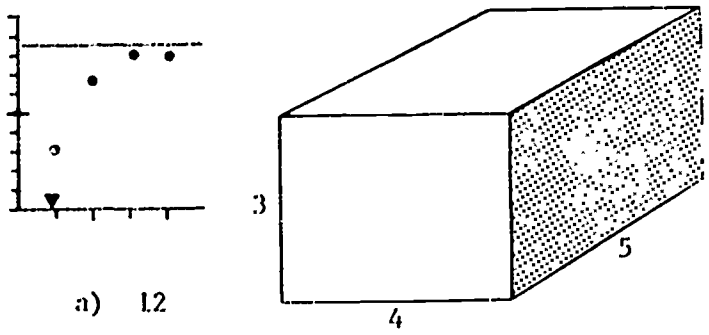
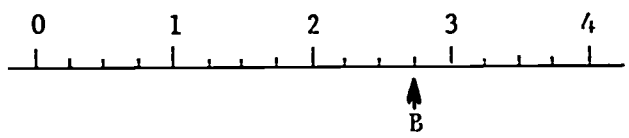
4. What is the volume of the box?



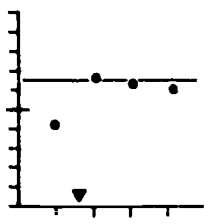
   a) 12
   b) 24
   c) 48
   d) 60

****



5. On the number line above, which number can be used to name point B?

   a) $\frac{11}{4}$
   b) $\frac{11}{16}$
   c) $2\frac{3}{11}$
   d) $\frac{3}{4}$



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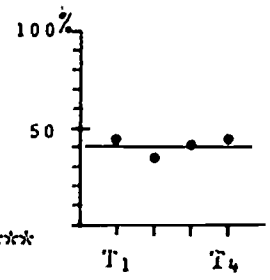
6. Bill's father bought new tires. After he had driven 3,000 miles 25% of the tread was worn off. How many more miles do you think he can drive before all the tread is gone?

a) 3,000
b) 6,000
c) 9,000
d) 12,000



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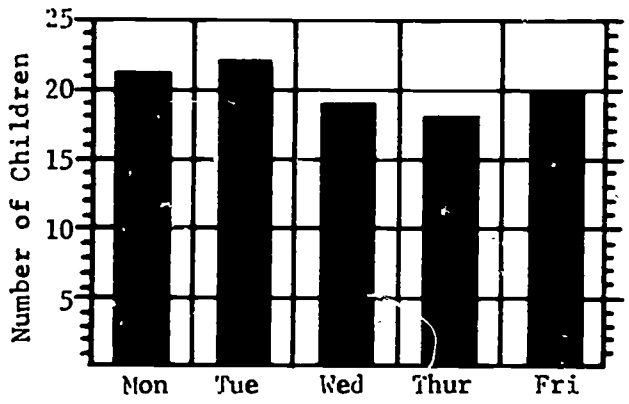
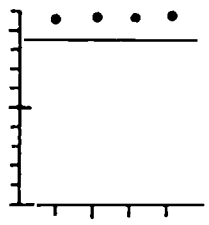
Use the chart below to answer the next two questions.



7. On which day did the greatest number of children in this class attend school?

a) Monday
b) Tuesday
c) Wednesday
d) Friday



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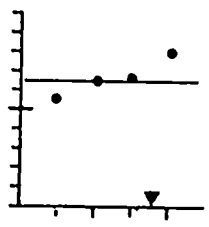
8. If the average attendance for the week was 20 pupils per day, by how many pupils was the attendance for Thursday below the average?

a) 0
b) 1
c) 2
d) 3



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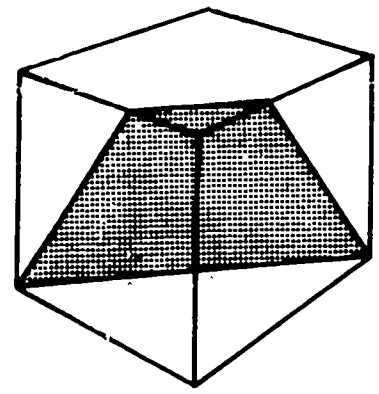
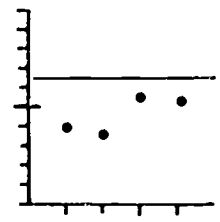


9. A cut is made in the cube above to give the shaded surface. What is the name of this shaded surface?

a) parallelogram
b) triangle
c) rectangle
d) trapezoid



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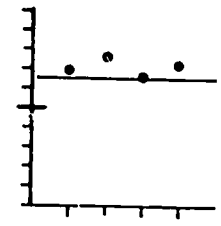
10. Which sentence below is true?

a) $\frac{1}{3} = \frac{7}{20}$
b) $\frac{4}{3} = \frac{9}{7}$
c) $\frac{1}{12} = \frac{6}{72}$
d) $\frac{4}{11} = \frac{11}{4}$



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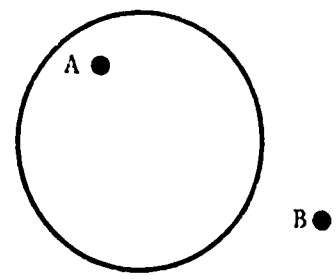
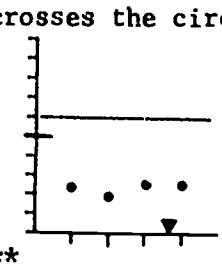


11. In the figure above point A is joined to point B with a curve. What is the probability the curve crosses the circle?

a) 1
b) 100
c) less than one
d) greater than one



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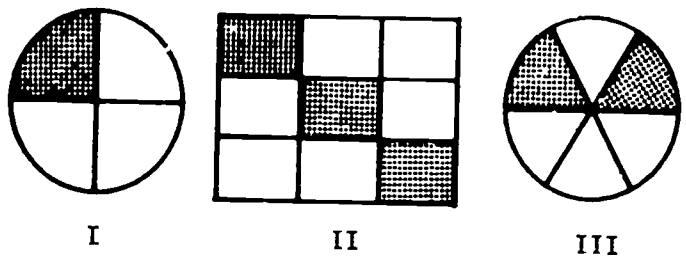
12. Which of the regions shaded show a fraction equal to $\frac{1}{3}$?



I II III

a) I
b) I and II
c) II and III
d) I and III

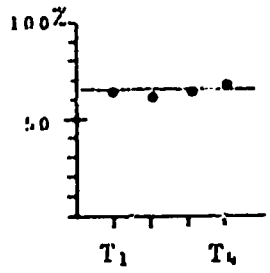
****

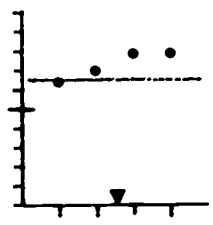
13. If S = (3 x T) + 5, what is S when T = 8?

a) 29
b) 24
c) 5
d) 1

****

GO TO NEXT PAGE

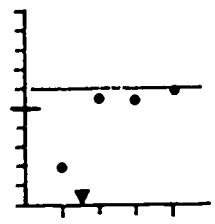




Work the remaining problems on scratch paper or beside the problem and place your answer on the answer sheet in the space provided.

14. 0.23 + 0,85 = ?

15. $1\frac{3}{8} - \frac{1}{2}$ = ?



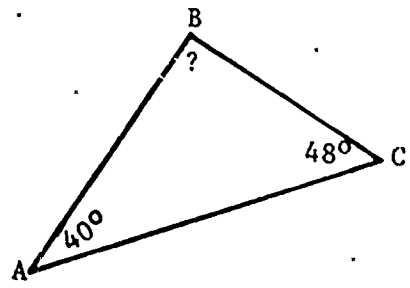
16. 73 x 48 = ?

17. $\frac{2}{5}$ x ($\frac{5}{2}$ x 11) = ?

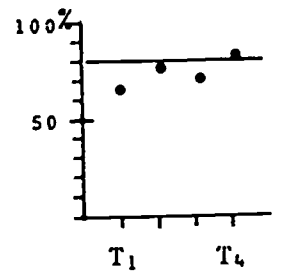
18. 1088 + (38 x 1.1) = ?

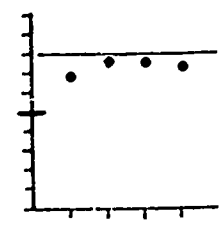
Work <u>problem 19</u> on the back of your answer sheet. Leave your answer there.

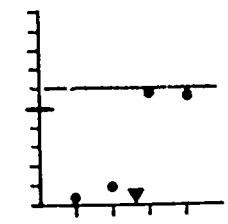
19. 3879 ÷ 27 = ?

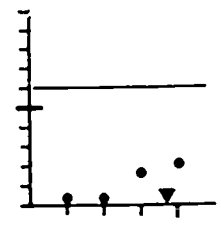


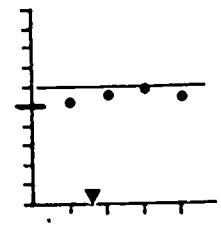
20. For the above triangle, what is the measure of the angle at B.

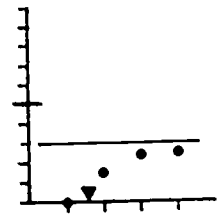












PATTERNS IN ARITHMETIC
Grade 6
7

Name _____

Did you use TV arithmetic in grade 5? Yes No

ANSWER SHEET

Circle your choice

1. a b c d
2. a b c d
3. a b c d
4. a b c d
5. a b c d
6. a b c d
7. a b c d
8. a b c d
9. a b c d
10. a b c d
11. a b c d
12. a b c d
13. a b c d

Place your answers for problems 14-20 in the blanks below.

14. _____
15. _____
16. _____
17. _____
18. _____
19. Answer on back of this sheet.
20. _____

PATTERNS IN ARITHMETIC
Grade 6
8

TEST 8

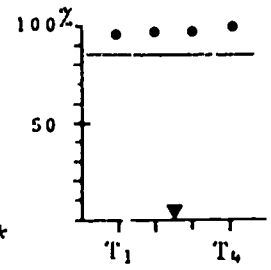
QUESTION SHEETS

Instructions: You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and circle your choice on the answer sheet provided.

You should have enough time to work on every question. Do not spend too much time on any problem.

1. The sentence 3 x N = 12 is true when N = ?

   a) 2
   b) 4
   c) 9
   d) 36



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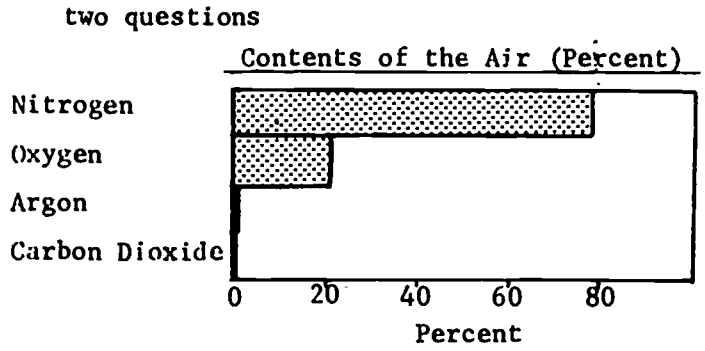
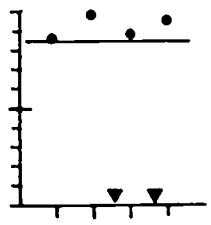
Use the chart below to answer the next two questions



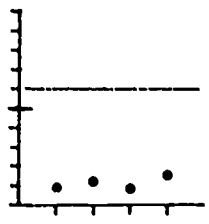
2. About what percent of the air is oxygen?

   a) 2
   b) 20
   c) 80
   d) 19
   e) 40



3. The ratio of nitrogen to oxygen is about

   a) 60 to 20
   b) 1 to 4
   c) 4 to 1
   d) 20 to 80



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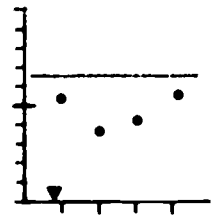
4. 2 out of 5 is how many out of 100?

   a) 2
   b) 40
   c) 20
   d) 25
   e) 30



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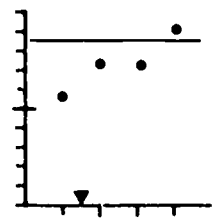
5. $3\frac{1}{2}$ =

   a) $\frac{3}{2}$
   b) $\frac{5}{2}$
   c) $\frac{6}{2}$
   d) $\frac{7}{2}$



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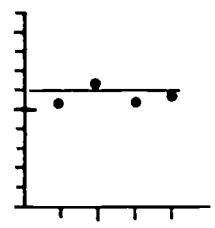
$$9\overline{)284}\quad 31,\ 27,\ 14,\ 9,\ 5$$



6. In the division problem above the quotient is

   a) 31
   b) 9
   c) 284
   d) 5

****

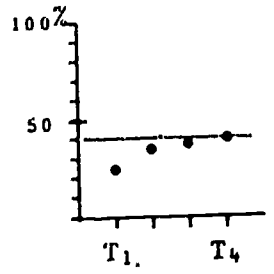
7. $1\frac{1}{2}$ quarts is what fraction of a gallon?

a) $\frac{3}{8}$

b) $\frac{1}{2}$

c) $\frac{3}{2}$

d) $\frac{8}{3}$



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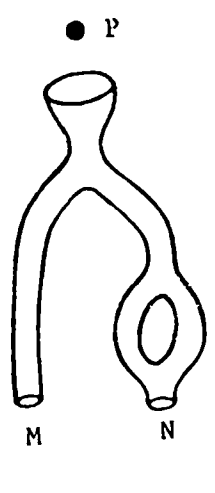


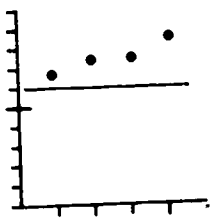
8. A marble is dropped into the tube at P and comes out at M or N. How many different paths can the ball travel?

a) 1

b) 2

c) 3

d) 4



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9. Suppose we decided to write fractions in a different way. For example, instead of $\frac{2}{3}$ we would write (2,3) and instead of $\frac{7}{5}$ we would write (7,5). What would be the sum of (1,5) and (3,5)?

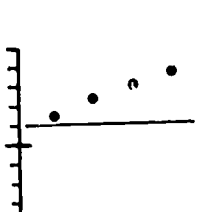
a) 4

b) (4,5)

c) 14

d) (3,10)

e) (3,25)



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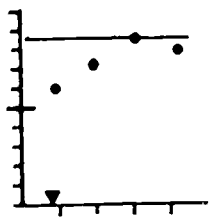
10. What is the area of a rectangle 8 feet long and 3 feet wide?

a) 11 sq. ft.

b) 22 sq. ft.

c) 12 sq. ft.

d) 24 sq. ft.



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11. Water and sugar are mixed in the ratio: 3 parts water to 1 part sugar. What percent of the mixture is water?

a) 1

b) 3

c) 60

d) 75

e) can't tell

****

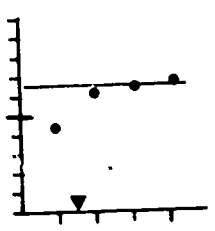
12. Another way to name angle DEC is

a) angle 1

b) angle 2

c) angle 3

d) angle 4



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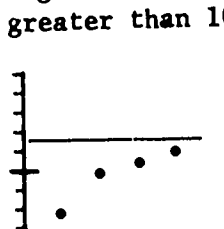
13. All but one number below is greater than 100. Which choice is <u>not</u> greater than 100?

a) 20 x 6

b) 21.63

c) 100 ÷ 0.5

d) 100.1



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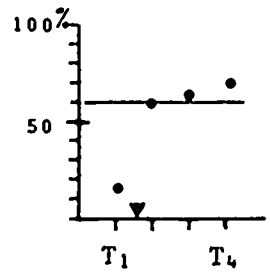
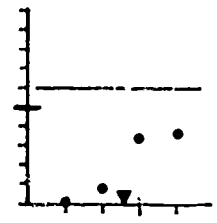
GO TO NEXT PAGE

Work the remaining problems on scratch paper or beside the problem and place your answer on the answer sheet in the space provided.

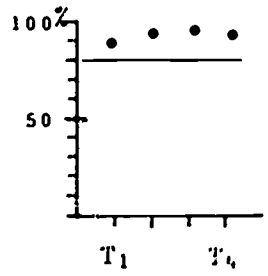
14. $0.3 + \frac{1}{2} = ?$



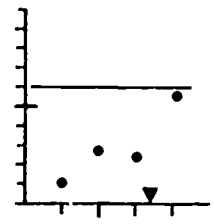
15. $3\frac{1}{7} \times 2\frac{1}{9} = ?$



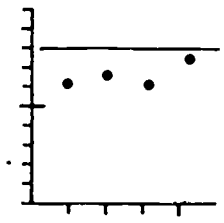
16. $\frac{1}{8} + \frac{5}{8} = \frac{?}{8}$



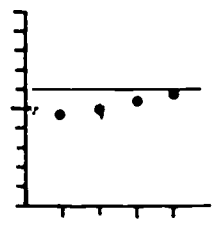
17. $6.624 \div 16 = ?$



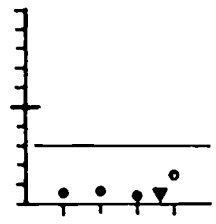
18. $300 \times 800 = ?$



19. Charles wanted to make a dog house of wood. He would need 10 square feet for the bottom, 20 square feet for the sides, and 12 square feet for the roof. Wood cost 25 cents a square foot. How much would the wood for his dog house cost?



20. A bicycle originally cost $47. It is on sale at 20% off the original price. What is the sale price?



PATTERNS IN ARITHMETIC
Grade 6
8

Name _____

Did you use TV arithmetic in grade 5? Yes No

ANSWER SHEET

Circle your choice

1.	a	b	c	d		8.	a	b	c	d	
2.	a	b	c	d	e	9.	a	b	c	d	e
3.	a	b	c	d		10.	a	b	c	d	
4.	a	b	c	d	e	11.	a	b	c	d	e
5.	a	b	c	d		12.	a	b	c	d	
6.	a	b	c	d		13.	a	b	c	d	
7.	a	b	c	d							

Place your answers for problems 14-20 in the blanks below.

14. _____

15. _____

16. _____

17. _____

18. _____

19. _____

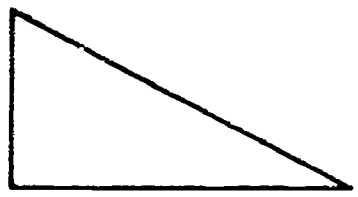
20. _____

PATTERNS IN ARITHMETIC
Grade 6
9

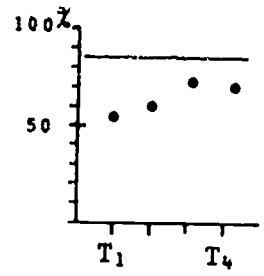
TEST 9
QUESTION SHEETS

Instructions: You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and circle your choice on the answer sheet provided.

You should have enough time to work on every question. Do not spend too much time on any problem.



1. The above figure is a
   a) circle
   b) rectangle
   c) square
   d) triangle
   e) parallelogram



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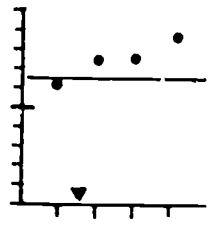
2. Which choice below is equal to $\frac{15}{8}$?
   a) $\frac{1}{8} + \frac{5}{8}$
   b) $1\frac{5}{8}$
   c) $7 + \frac{1}{8}$
   d) $1\frac{7}{8}$



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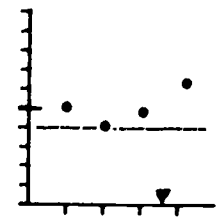
3. If the scale length of $4\frac{1}{2}$ inches represents an actual distance of 72 miles, how many miles does the scale length of 7 inches represent?
   a) 2
   b) 56
   c) $74\frac{1}{2}$
   d) 112
   e) 504



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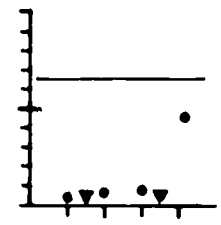
4. Which number is the greatest?
   a) 0.03
   b) 0.29
   c) 0.293
   d) 0.2093



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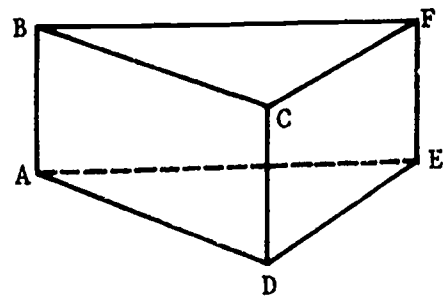
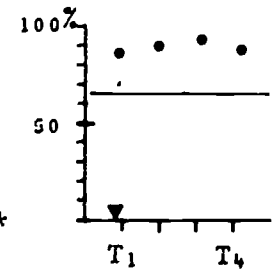


5. In the above prism, the back face is named by

a) BCF
b) ABFE
c) CDEF
d) ADE



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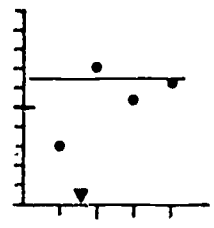
6. Which is the least common denominator for $\frac{1}{6}$ and $\frac{1}{8}$?

a) 6
b) 8
c) 14
d) 24
e) 48



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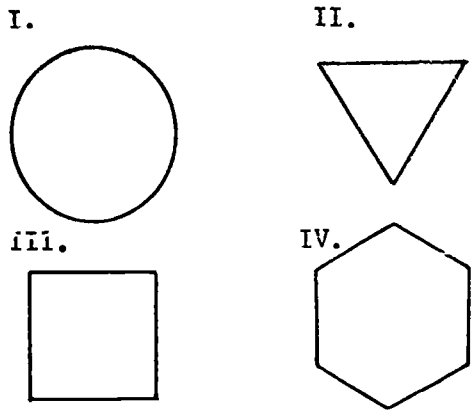
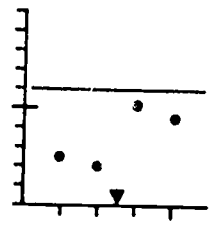


7. Which of the above figures has more than 6 lines of symmetry?

a) I
b) II
c) III
d) IV



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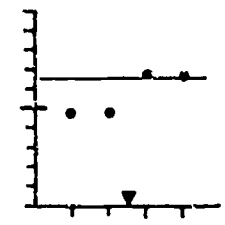
8. In a football game the Red Team was penalized 15 yards and on the next play passed for a gain of 11 yards. Which sentence tells what happened to the Red Team on the two plays?

a) 15 + 11 = n
b) 15 - 11 = n
c) -15 + 11 = n
d) 11 - 15 = n



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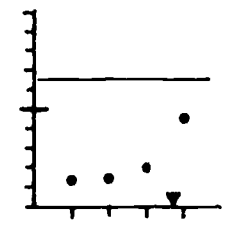
9. 3 is what percent of 6?

a) 0.5
b) 2
c) 3
d) 50
e) 200



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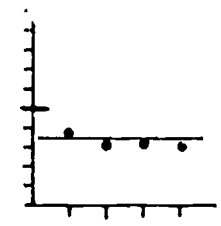
10. Jane is going to make cookies. She rolls out the dough and starts to cut out the cookies. Jane has 3 shapes of cookie cutters - a circle, a square, and a star. If each cutter has the same area, which cookie cutter would probably give the most cookies after the dough is rolled out once?

a) the circle
b) the square
c) the star
d) all the same



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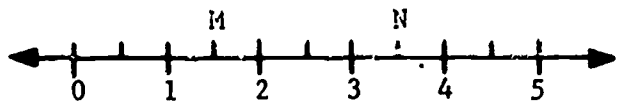


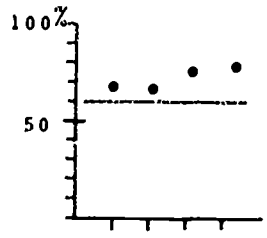
11. What is the distance from M to N on the number line?

a) 2

b) $1\frac{1}{2}$

c) 3

d) $3\frac{1}{2}$



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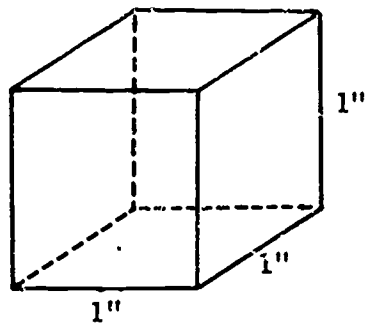


12. If E = number of EDGES
F = number of FACES
V = VOLUME

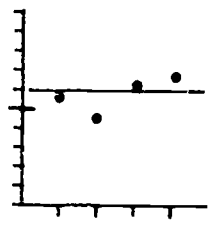
What is (E,F,V) for the cube above?

a) (9,4,3)

b) (12,6,1)

c) (9,6,1)

d) (12,4,3)



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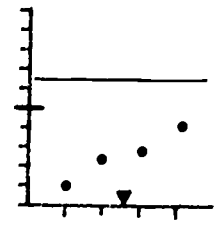
13. What is the area of a square $\frac{1}{2}$ inch on a side?

a) $\frac{1}{2}$ square inch

b) 1 square inch

c) $\frac{1}{4}$ square inch

d) 4 square inches



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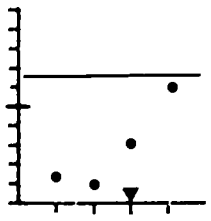
GO TO NEXT PAGE

Work the remaining problems on scratch paper or beside the problem and place your answer on the answer sheet in the space provided.

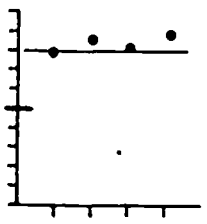
14. 0.6 x 30 = ?

15. $7\frac{1}{2} \div 5 = ?$



16. 19 x 2010 = ?

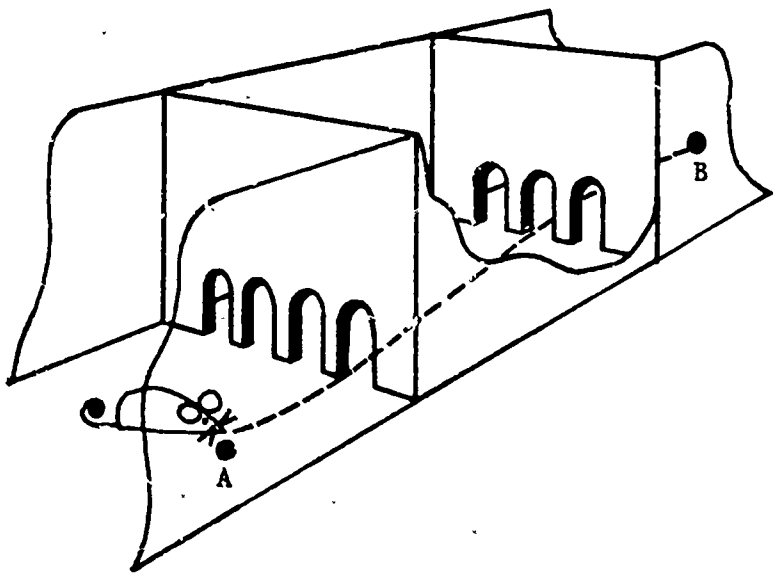
17. $\frac{35}{36} + \frac{1}{36} = ?$

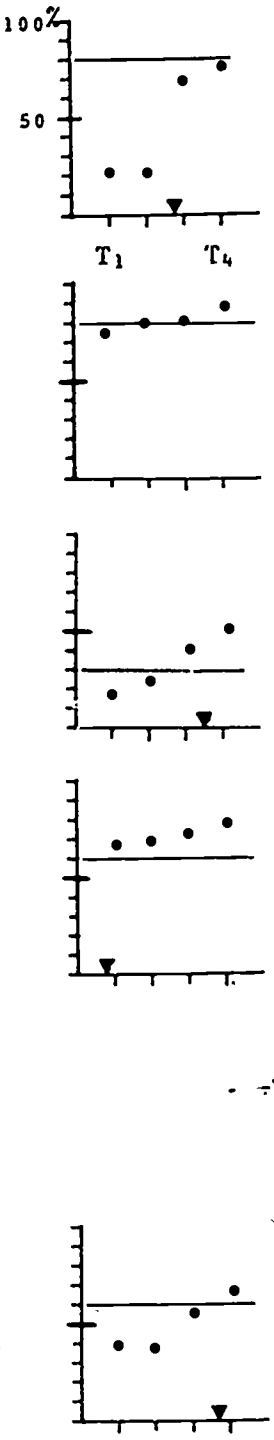


18. Oranges cost 79¢ a dozen. To the nearest penny, how much would one orange cost?

19. If the ratio of $\frac{\text{grams}}{\text{pounds}}$ is $\frac{453}{1}$ then 3 pounds is how many grams?

20. A mouse has two walls to go through and several holes. If he starts at point A, how many routes can he take to get to point B? The dotted line shows one possible way.





PATTERNS IN ARITHMETIC
Grade 6
9

Name _____

Did you use TV arithmetic in grade 5? Yes No

ANSWER SHEET

Circle your choice

1.	a	b	c	d	e	8.	a	b	c	d
2.	a	b	c	d		9.	a	b	c	d	e
3.	a	b	c	d	e	10.	a	b	c	d
4.	a	b	c	d		11.	a	b	c	d
5.	a	b	c	d		12.	a	b	c	d
6.	a	b	c	d	e	13.	a	b	c	d
7.	a	b	c	d						

Place your answers for problems 14-20 in the blanks below.

14. _____

15. _____

16. _____

17. _____

18. _____

19. _____

20. _____

PATTERNS IN ARITHMETIC
Grade 6
10

TEST 10
QUESTION SHEETS

Instructions: You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and circle your choice on the answer sheet provided.

You should have enough time to work on every question. Do not spend too much time on any problem.

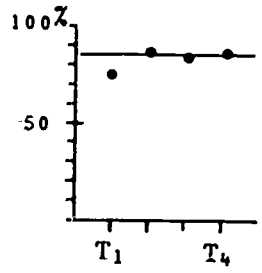
1. The sentence 23 + N = 130 is true when N = ?

   a) 117

   b) 107

   c) 153

   d) 17



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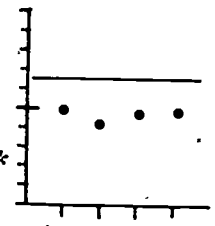
2. One third of what number is 9?

   a) 3

   b) 6

   c) 18

   d) 27



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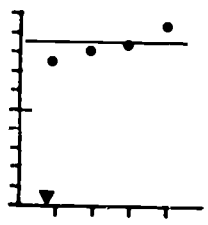
3. Candy bars are 3 for 20¢. How many could you buy for $1.00?

   a) 6

   b) 15

   c) 20

   d) 23



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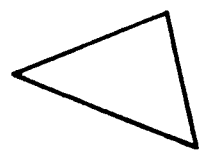
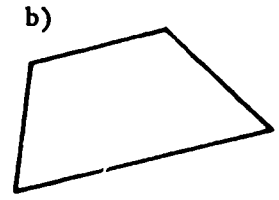
4. When a triangle is reflected through a line, which figure below could be its image?

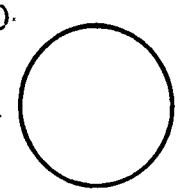
a)



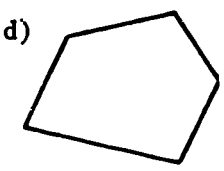
b)



c)



d)



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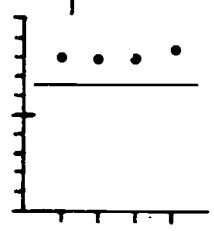
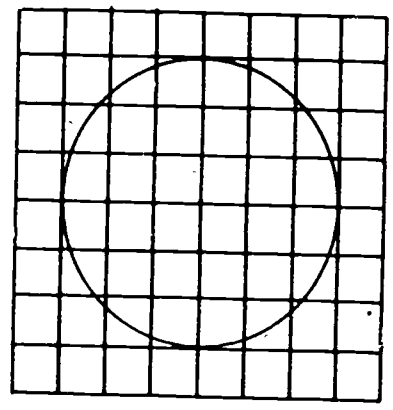




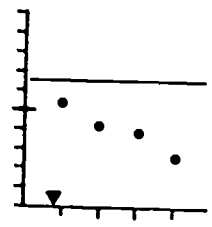
5. In the figure above, use the grid to determine the outer area of the circle. What is the outer area of the circle?

   a) 16

   b) 28

   c) 36

   d) 64



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6. Which fraction is not equal to the other four?

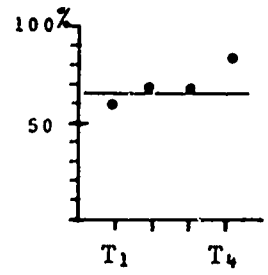
a) $\frac{1}{2}$

b) $\frac{9}{27}$

c) $\frac{1}{3}$

d) $\frac{7}{21}$

e) $\frac{8}{24}$



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7. An odometer registers the distance traveled by an automobile. If a car's odometer registered 7910 miles at the beginning of a trip, what should it register after the car was driven 3 hours at the average speed of 30 miles per hour?

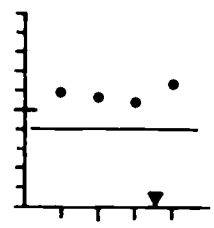
a) 90

b) 7920

c) 7943

d) 7940

e) 8000



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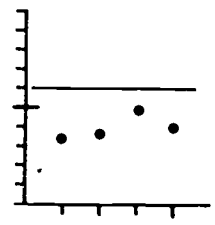
8. Which unit of linear measure is the most precise?

a) centimeter

b) inch

c) foot

d) yard



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9. If a school is composed of 50 freshmen, 40 sophomores, 60 juniors and 30 seniors, then

$$\frac{\text{number of juniors}}{\text{number of students}} = (?)$$

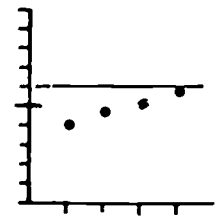
a) $\frac{1}{4}$

b) $\frac{1}{3}$

c) $\frac{3}{5}$

d) $\frac{2}{3}$

e) $\frac{10}{3}$



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10. The picture below represents a lady bug about 4 times actual size. About how long is a lady bug?

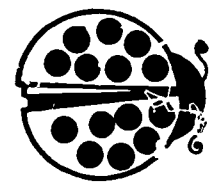
1 inch 1 cm

a) $\frac{1}{4}$ inch

b) 1 centimeter

c) $\frac{1}{2}$ inch

d) 2 centimeters



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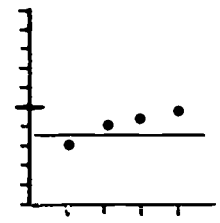
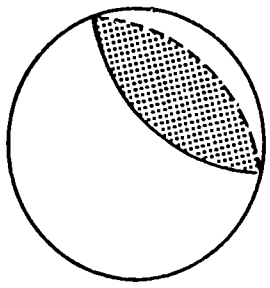




11. The sphere above is cut to give the shaded surface. What is the name of this shaded surface?

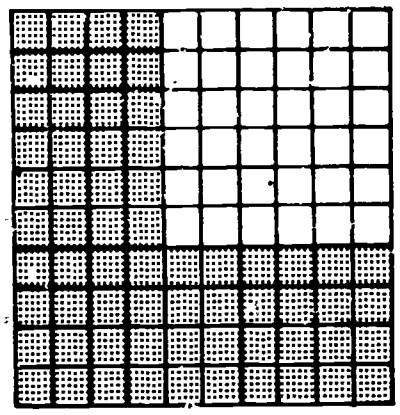
a) ellipse

b) circle

c) polygon

d) egg shaped

****



12. What percent of the above figure is shaded?

a) 36

b) 0.36

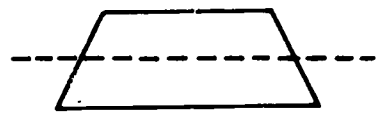
c) 64

d) 0.64

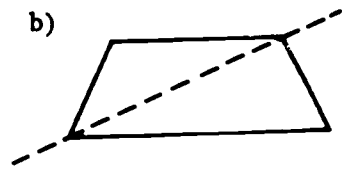
****

13. For which choice below does the dotted line represent a line of symmetry?

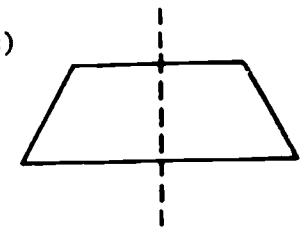
a)



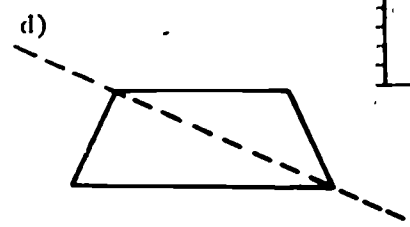
b)



c)



d)



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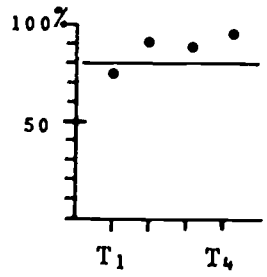
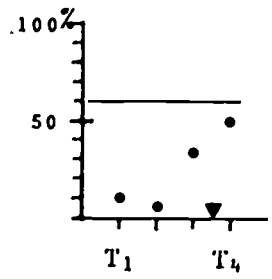
GO TO NEXT PAGE

Work the remaining problems on scratch paper or beside the problem and place your answer on the answer sheet in the space provided.

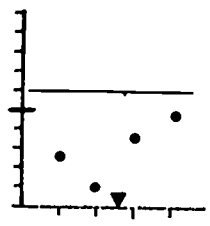
14. $\frac{3}{15} - \frac{1}{15} = ?$



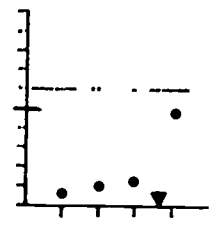
15. $8 \div \frac{1}{10} = ?$



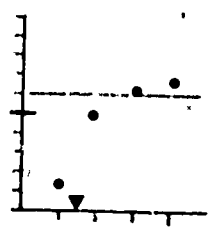
16. $\frac{2.54}{1} = \frac{?}{10}$



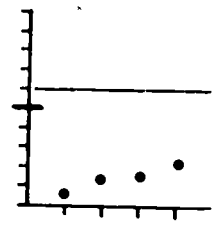
17. $2186.73 \div 801 = ?$



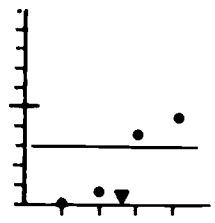
18. Write $\frac{3}{5}$ as a decimal.



19. How many square <u>yards</u> of carpet are needed to fully cover the floor of a room 12 <u>feet</u> by 15 <u>feet</u>?



20. The sentence $F = (\frac{9}{5} \times C) + 32$ shows how F is related to C. If C = 30, what is F?



PATTERNS IN ARITHMETIC
Grade 6
10

Name _____

Did you use TV arithmetic in grade 5? Yes No

ANSWER SHEET

Circle your choice

1. a b c d
2. a b c d
3. a b c d
4. a b c d
5. a b c d
6. a b c d e
7. a b c d e
8. a b c d
9. a b c d e
10. a b c d
11. a b c d
12. a b c d
13. a b c d

Place your answers for problems 14-20 in the blanks below.

14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____

PATTERNS IN ARITHMETIC
Grade 6
11

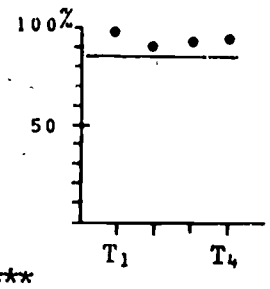
TEST 11
QUESTION SHEETS

Instructions: You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and circle your choice on the answer sheet provided.

You should have enough time to work on every question. Do not spend too much time on any problem.

1. A book has 123 pages and Nancy has read 83. Which sentence shows how many more pages are left to be read?

   a) P = 123 + 83
   b) P = 123 - 83
   c) P = 123 + 83
   d) P = 123 x 83



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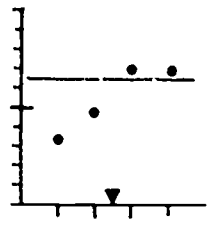
2. If 2 x n = 9, what is n?

   a) $\frac{2}{9}$
   b) $\frac{9}{2}$
   c) 11
   d) 18



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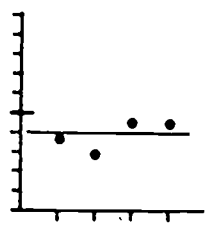
3. The bank received 5% interest on a loan of $300.00 for 1 year. How much interest was received?

   a) $5.00
   b) $6.00
   c) $1.50
   d) $15.00



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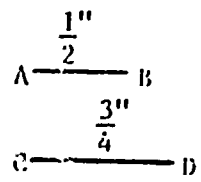
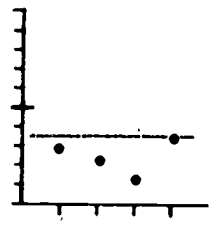
4. 

In the figure above,

$$\frac{\text{length of AB}}{\text{length of CD}} = (?)$$

   a) $\frac{1}{2}$
   b) $\frac{1}{3}$
   c) $\frac{2}{3}$
   d) $\frac{3}{2}$
   e) $\frac{5}{3}$



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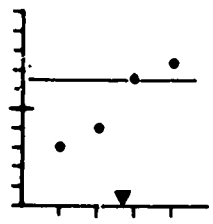
5. One way to reach 2 on the number line is

   a) 2 x 0
   b) -3 + 5
   c) -1 + (-1)
   d) 5 + (-2)



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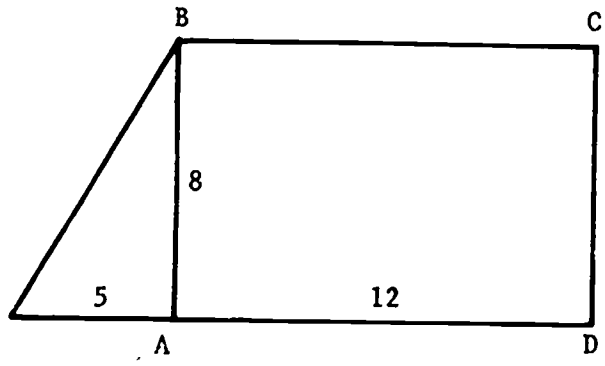
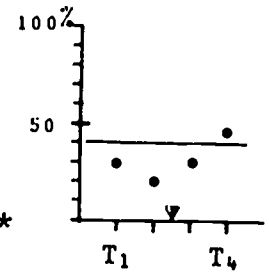


6. If ABCD is a rectangle, what is the area of the entire figure (the rectangle and the triangle)?

a) 116
b) 480
c) 136
d) 25



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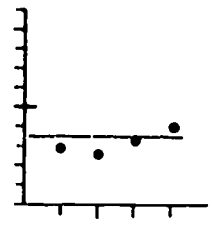
7. When a counting number is divided by 23, what is the greatest possible remainder?

a) 0
b) 5
c) 22
d) 23
e) 99



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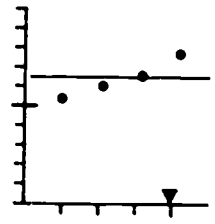
8. $1000 = 10 \times 10 \times 10 = 10^3$

$100 = 10 \times 10 = 10^2$

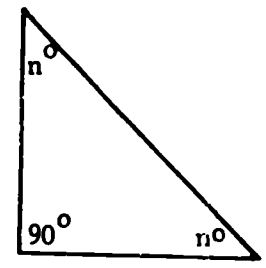
$1000 \times 100 = ?$

a) 5
b) 6
c) 10^5
d) 10^6

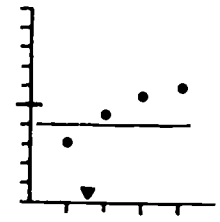


****

9. In the triangle below, n = ?



a) 60
b) 45
c) 30
d) 15



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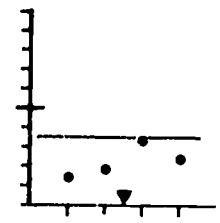
10. Which sentence shows how to find the length (L) of the side of a square when you know the perimeter (P)?

a) $L = \frac{1}{4} \times P$
b) $L = 4 \times P$
c) $L = P \div \frac{1}{4}$
d) $L + 4 = P$



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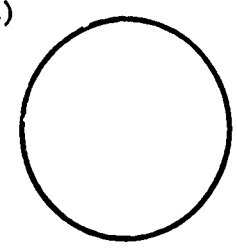
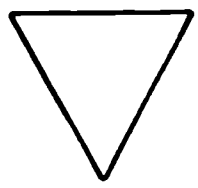
GO TO NEXT PAGE

11. Charles is using a grid with a square centimeter unit to determine the inner and outer area of a figure. He finds the inner and outer area are equal. Which figure below could he have measured?

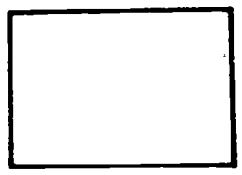
a)



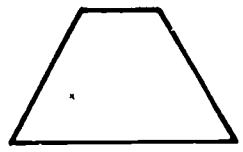
b)



c)



d)



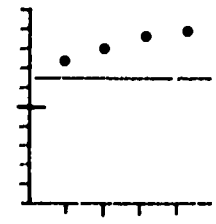
12. Which number below is nearest to zero on the number line?

a) $\frac{1}{2}$

b) $\frac{1}{4}$

c) $\frac{1}{8}$

d) $\frac{1}{16}$



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13. Imagine a bag that contains 5 red marbles and 2 blue marbles. How many blue marbles would you need to put in the bag if you want to draw a blue marble about as often as a red one?

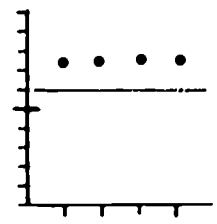
a) 5

b) 3

c) 2

d) 7

e) 10



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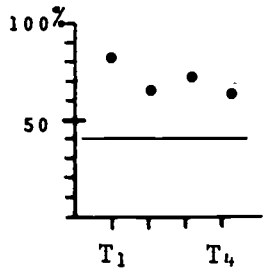
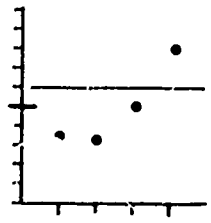


Work the remaining problems on scratch paper or beside the problem and place your answer on the answer sheet in the space provided.

14. $4 \times 3\frac{1}{2} = ?$

15. $27.6 \div 3 = ?$



16. Write "two hundred one and fifty-three thousandths" as a decimal.

Work <u>problem 17</u> on the back of your answer sheet. Leave your answer there.

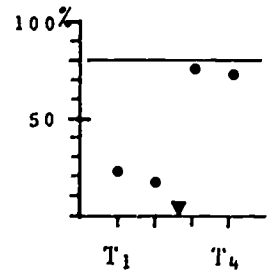
17. $8642 \div 90 = ?$

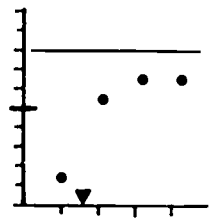
18. It is estimated that by the year 1975, 50% of the people will be less than 28 years old. If this is true, and by 1975 there are 220 million people, how many will be less than 28 years old?

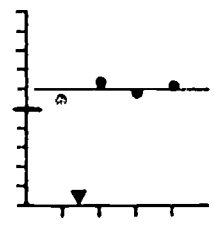
19. Gasoline costs $35\frac{9}{10}$ cents per gallon.

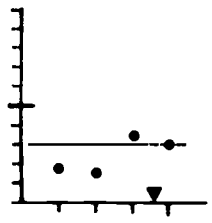
How much will 10 gallons cost?

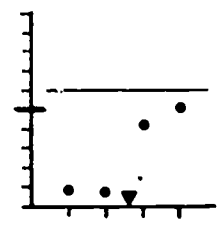
20. When the temperature is <u>$32^{\circ}F$</u> sound travels in air at about <u>1,088</u> feet per second. For each $1^{\circ}F$ increase in temperature sound travels 1.1 foot per second faster. How fast would sound travel at <u>$70^{\circ}F$</u>?

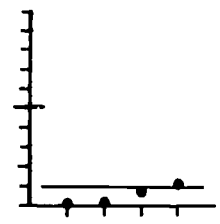












PATTERNS IN ARITHMETIC
Grade 6
11

Name _____

Did you use TV arithmetic in grade 5? Yes No

ANSWER SHEET

Circle your choice

1. a b c d
2. a b c d
3. a b c d
4. a b c d e
5. a b c d
6. a b c d
7. a b c d e
8. a b c d
9. a b c d
10. a b c d
11. a b c d
12. a b c d
13. a b c d e

Place your answers for problems 14-20 in the blanks below.

14. _____
15. _____ _____
16. _____
17. Answer on back of this sheet.
18. _____
19. _____
20. _____

PATTERNS IN ARITHMETIC
Grade 6
12

TEST 12

QUESTION SHEETS

Instructions: You may write anywhere on the question sheets. Questions 1-13 are multiple choice. You should decide which choice is correct and <u>circle</u> your choice on the <u>answer sheet</u> provided.

You should have enough time to work on every question. Do <u>not</u> spend too much time on any problem.

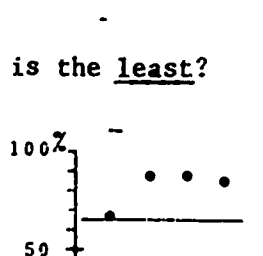
1. Which fraction below is the <u>least</u>?

a) $\frac{1}{2}$

b) $\frac{2}{3}$

c) $\frac{3}{10}$

d) $\frac{1}{820}$



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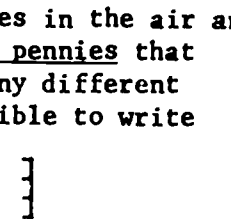
3. Suppose you toss 5 pennies in the air and write down the <u>number of pennies</u> that fall "heads-up." How many different numbers would it be possible to write down?

a) 8

b) 6

c) 5

d) 4



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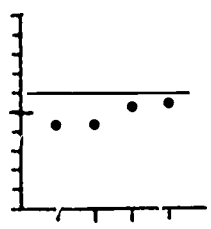
2. The drawing below is a picture of a Euglena. It is drawn about 100 times its actual size. About how long is a Euglena?

a) 1 millimeter

b) 1 centimeter

c) 1 inch

d) 1 yard



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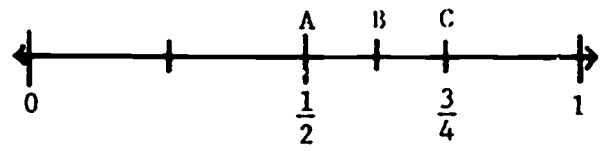


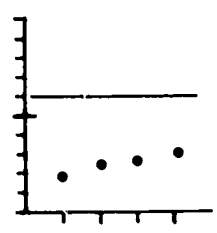
4. On the number line above, point B is halfway between point A and point C. What number names point B?

a) $\frac{1}{2} + \frac{1}{2}$

b) $\frac{1}{3}$

c) $\frac{5}{8}$

d) $\frac{2}{3}$



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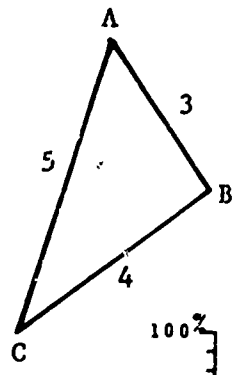
5. Triangle ABC is a right triangle. What is its area?

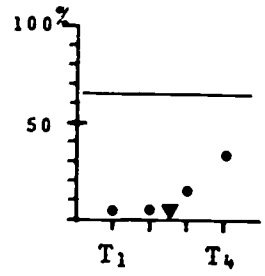


a) 12

b) 10

c) $\frac{1}{2}$ x 15

d) 6



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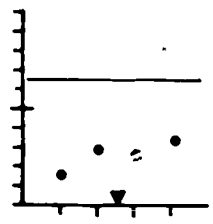
6. What is the area of a square $\frac{1}{2}$ in. on a side?

a) $\frac{1}{2}$ sq. in.

b) 1 sq. in.

c) $\frac{1}{4}$ sq. in.

d) 4 sq. in.



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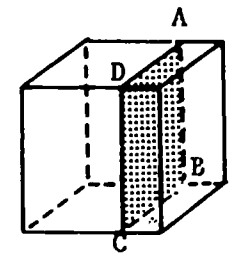
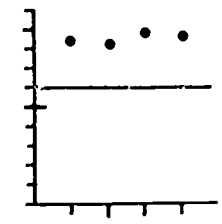
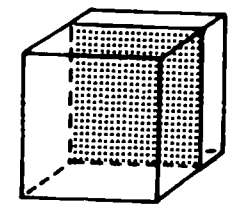


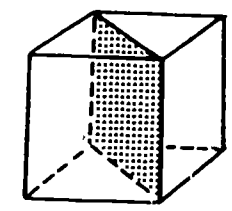


7. A cut is made through the above cube to give the shaded face. Which figure below represents another cut with a shaded face <u>congruent</u> to ABCD?

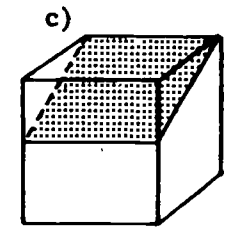
a)



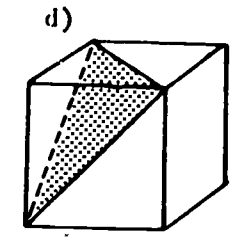
b)



c)



d)



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8. 15 is divisible by 3 with no remainder. What is the next higher counting number divisible by 3 with no remainder?

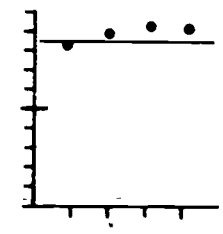
a) 16

b) 17

c) 18

d) 19

e) 20



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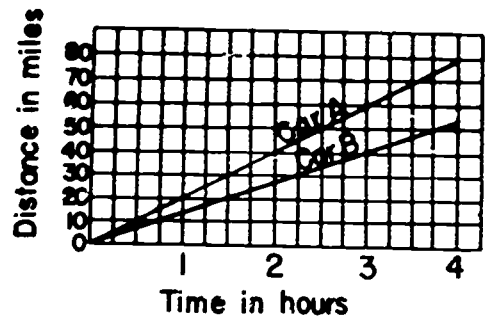
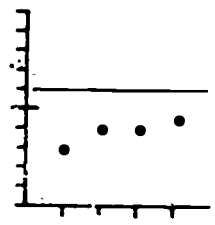
Questions 9 and 10 refer to the graph below.



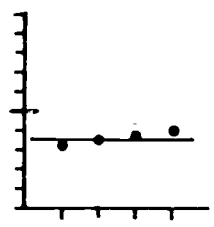
9. Three hours after starting, car A is how many miles ahead of car B?

a) 2
b) 10
c) 15
d) 20
e) 25



10. How much longer does it take car B to go 50 miles than it does for car A to go 50 miles?

a) 1 hour 15 minutes
b) 1 hour 30 minutes
c) 2 hours
d) 2 hours 30 minutes
e) 2 hours 45 minutes



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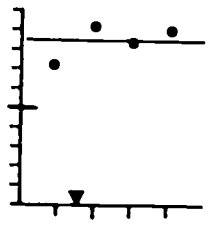
$$17\overline{)215}\quad 12,\ 17,\ 45,\ 34,\ 11$$

11. After the long division above has been done, which sentence below can be written to express the result?

a) 215 = (12 x 17) + 11
b) 215 + 11 = 12 x 17
c) 45 - 11 = 34
d) 215 - 170 = 45



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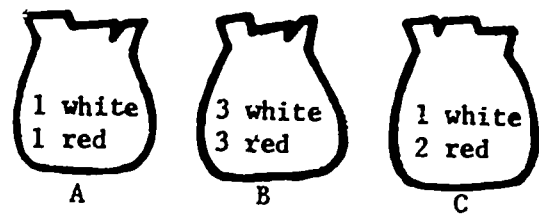


12. Imagine 3 sacks that contain marbles as above.

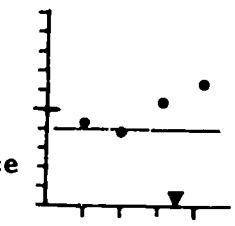
Sack A contains 1 white and 1 red marble

Sack B contains 3 white and 3 red marbles

Sack C contains 1 white and 2 red marbles

If you reach in a sack and pick a marble without looking, which sack would the probability of a red ball be the greatest.

a) Sack A
b) Sack B
c) Sack C
d) It makes no difference



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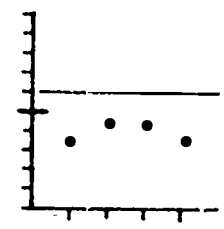
13. Which number below is between $\frac{5}{6}$ and 1?

a) $\frac{1}{3} + \frac{1}{3}$
b) $\frac{5}{6} + 1$
c) $\frac{5}{6} + \frac{1}{7}$
d) 0.30 + 0.20



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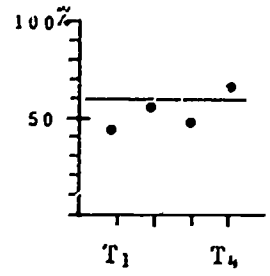
GO TO NEXT PAGE

Work the remaining problems on scratch paper or beside the problem and place your answer on the answer sheet in the space provided.

14. 0.8 - 0.3 = ?

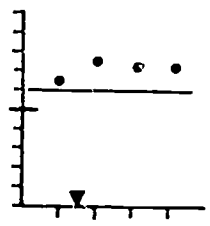
15. 627 x 388 = ?

16. $3\frac{1}{2} - 3\frac{1}{2} = ?$



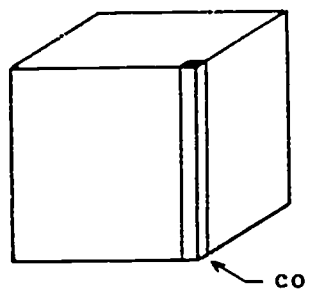
Work problem 17 on the back of sheet. Leave your answer there

17. 3120 ÷ 500 = ?

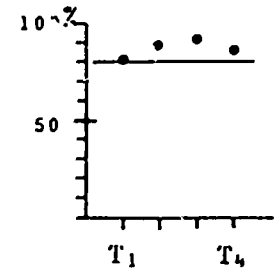


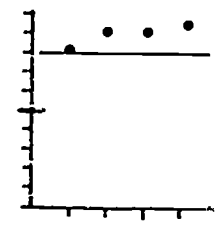
18. A car can travel 60 miles on 5 gallons of gas. At this rate, how many miles will it travel on 1 gallon of gas?

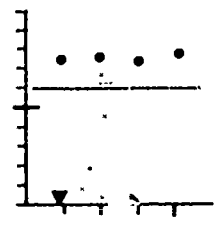
19. A $20 tent is on sale for $19. By what percent has the price been reduced?

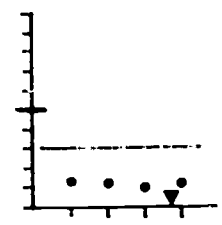


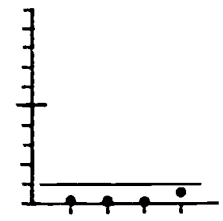
20. One cubic foot of water weighs 62.4 pounds. How much does a column of water one inch square and one foot high weigh? Write a sentence to show what must be done to get the answer. You do not have to find the answer.











PATTERNS IN ARITHMETIC
Grade 6
12

Name _____

Did you use TV arithmetic in grade 5? Yes No

ANSWER SHEET

Circle your choice

1. a b c d
2. a b c d
3. a b c d
4. a b c d
5. a b c d
6. a b c d
7. a b c d
8. a b c d e
9. a b c d e
10. a b c d e
11. a b c d
12. a b c d
13. a b c d

Place your answers for problems 14-20 in the blanks below.

14. _____
15. _____
16. _____
17. Answer on back of this sheet
18. _____
19. _____
20. _____

◀ B7 ▶

CHANGES IN ITEMS FOLLOWING T_1

Problem	Change
(1,19)	At T_1 the problem was 3879 ÷ 27 = ? with no instructions to work the problem on the back of the answer sheet. Following T_1 the instructions stated: "Work problem 19 on the back of your answer sheet. Leave your answer there." The problem was the same as T_1: 3879 ÷ 27 = ?
(1,11)	The words "least likely" were not underlined at T_1.
(2,19)	At T_1 the problem read: "John has a stack of 500 pieces of notebook paper. How much does each piece weigh if all 500 pieces weigh 15 pounds?" After T_1 the 15 pounds was changed to 3 pounds.
(3,17)	The ? mark was added after T_1.
(3,19)	At T_1 a single row and column of the square grid were shaded and the correct answer was 19%. Many pupils put 20% because they counted the row and column intersection twice. The correct answer following T_1 was 96%.
(7,12)	At T_1 figure 1 had two parts shaded and after T_1 only the upper-left quarter circle was shaded. This improved the problem considerably.
(8,4)	At T_1 the problem read: "2 out of 50 is how many out of 100?" The choices were a) 2, b) 4, c) 20, d) 25, e) 30. Following T_1 the problem read: "2 out of 5 is how many out of 100?" The choices were changed to: a) 2, b) 40, c) 20, d) 25, e) 30.

(8,12) The same figure was used at all testing periods. However, at T_1 the question was: "In the figure above, angle AEB is the verticle angle of which angle?"

(10,2) The only change made was to reorder the choices. Originally they were a) 27, b) 18, c) 6, d) 3.

(10,5) The only change made was to reorder the choices. Originally they were a) 16, b) 28, c) 36, d) 54.

Following T_2 no new changes were made in item content, location, or choice of distractors. The instructions were modified slightly at each testing period. Familiarity with the testing prodecure could be assumed by T_3, but yet some formality was necessary for the sake of uniformity.

In summary, every attempt was made to hold both the tests and instructions as constant as possible so that when item profiles over the year are analyzed one could be confident that changes in performance were not affected by these potential variables.

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Technical Report No. 113 (Part III)

THE FORMATIVE EVALUATION OF PATTERNS IN ARITHMETIC GRADE 6 USING ITEM SAMPLING

Report from the Project on
Individually Guided Elementary Mathematics
Phase 2: Analysis of Mathematics Instruction

T. A. Romberg, and John G. Harvey
Principal Investigators

By James Braswell

Thomas A. Romberg, Assistant Professor
of Curriculum and Instruction and Chairman of the Examining Committee

Wisconsin Research and Development
Center for Cognitive Learning
The University of Wisconsin
Madison, Wisconsin

March 1970

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APPENDIX C

RESULTS OF THE TESTING FOR T_2-T_4 AND SUMMARY BY CONTENT AREA*

* The tables included in Appendix C are explained in Chapter VI.

TABLE 1C: TEST RESULTS FOR T_2

Item No.	Test 1			Test 2			Test 3			Test 4		
	Y	N	C	Y	N	C	Y	N	C	Y	N	C
1	67	75	71	84	83	84	92	71	85	87	68	83
2	69	64	68	84	79	83	77	57	74	88	71	83
3	55	46	50	30	17	26	65	21	57	84	68	79
4	81	86	83	68	67	69	37	29	39	73	75	73
5	44	29	40	41	38	41	48	39	44	80	64	78
6	60	68	62	79	79	78	37	14	32	49	39	44
7	71	75	71	44	33	44	82	68	76	34	36	34
8	97	93	95	46	29	40	54	39	52	55	50	52
9	48	50	48	46	54	50	18	0	11	48	46	49
10	72	79	74	83	63	80	48	54	51	52	64	56
11	71	89	76	34	29	31	34	29	33	54	46	51
12	92	93	90	41	33	43	79	61	74	51	46	48
13	27	32	28	26	42	26	32	39	32	82	61	77
14	75	68	71	88	63	83	75	54	68	61	55	59
15	56	64	61	5	4	7	79	64	76	20	14	18
16	4	0	3	12	0	10	23	11	23	37	32	38
17	39	11	32	63	75	66	42	43	45	32	36	36
18	36	25	36	45	54	48	34	36	38	24	11	24
19	55	68	58	1	0	1	51	32	45	88	86	88
20	75	75	70	34	38	36	58	50	56	85	79	84
N	75	28	120	82	24	121	71	28	117	82	28	120
MEAN	11.92	11.89	11.86	9.57	8.79	9.46	10.62	8.11	10.15	11.83	10.46	11.53
STDEV	3.94	3.45	3.73	3.15	2.80	3.05	3.91	4.50	4.21	3.95	4.49	4.07
R	0.79	0.74	0.76	0.66	0.55	0.64	0.77	0.83	0.80	0.79	0.83	0.80
r_{YN}	0.93			0.92			0.84			0.93		

TABLE 1C TEST RESULTS FOR T_2 (CONTINUED)

Item No.	Test 5			Test 6			Test 7			Test 8		
	Y	N	C	Y	N	C	Y	N	C	Y	N	C
1	99	92	97	88	83	87	86	74	84	97	97	97
2	75	53	70	19	17	20	22	32	26	100	93	98
3	69	64	67	41	37	39	67	58	64	9	7	12
4	65	53	62	36	26	28	70	58	68	35	41	37
5	22	19	20	79	69	78	68	45	66	77	66	74
6	26	28	26	30	14	28	34	32	34	65	62	63
7	96	94	96	58	57	59	99	97	98	29	41	34
8	61	56	63	59	51	56	68	65	65	74	76	74
9	40	36	39	32	51	40	38	35	36	75	66	73
10	63	58	61	67	71	66	81	58	76	78	55	72
11	39	39	38	36	57	40	22	13	19	19	21	23
12	14	14	13	36	37	37	63	58	60	62	55	61
13	68	47	63	23	23	26	75	58	70	49	45	49
14	82	58	74	34	43	39	79	65	77	65	41	59
15	51	33	46	3	9	4	59	35	55	9	3	8
16	44	22	36	64	46	56	71	74	75	97	83	93
17	0	0	0	3	0	2	11	10	10	34	17	28
18	6	8	9	5	3	5	3	3	3	66	62	66
19	25	11	23	25	26	29	56	48	55	55	31	50
20	75	64	72	3	3	2	15	13	16	5	7	8
N	72	36	123	73	35	123	73	31	122	77	29	119
MEAN	10.21	8.50	9.75	7.36	7.23	7.37	10.89	9.32	10.57	11.01	9.69	10.78
STDEV	3.03	3.29	3.12	3.20	3.48	3.27	3.49	3.94	3.60	3.06	3.49	3.31
R	0.66	0.70	0.67	0.70	0.74	10.68	0.75	0.79	0.76	0.69	0.74	0.72
r_{YN}	0.95			0.93			0.95			0.94		

TABLE 1C: TEST RESULTS FOR T_2 (CONTINUED)

Item No.	Test 9			Test 10			Test 11			Test 12		
	Y	N	C	Y	N	C	Y	N	C	Y	N	C
1	62	63	61	89	83	87	87	89	90	91	83	88
2	82	60	74	47	34	41	49	35	49	42	50	44
3	41	43	41	80	83	81	32	22	29	0	0	1
4	10	0	8	85	69	79	23	14	21	23	33	26
5	70	90	90	40	34	42	47	22	41	6	8	6
6	76	60	70	67	66	68	16	22	20	33	28	30
7	23	7	19	59	52	58	26	19	26	87	81	81
8	53	53	49	39	28	37	56	68	60	90	86	89
9	13	10	14	46	45	47	44	43	45	41	39	39
10	29	37	31	44	27	40	20	14	18	37	33	35
11	68	60	67	12	14	14	64	68	66	94	83	91
12	55	37	46	36	45	39	78	78	80	35	39	38
13	22	27	23	72	38	64	74	76	76	42	50	44
14	26	13	21	94	79	91	12	14	17	86	92	88
15	10	3	9	7	7	6	31	35	33	54	53	55
16	78	83	80	9	14	10	57	57	56	92	86	91
17	88	83	85	11	10	11	57	68	63	81	69	75
18	27	23	26	53	31	50	15	8	15	77	72	76
19	67	67	68	15	7	13	6	5	8	13	8	11
20	45	30	38	11	3	8	2	3	2	3	0	2
N	78	30	125	85	29	131	81	37	134	78	36	129
MEAN	9.64	8.5	9.2	9.15	7.69	8.87	7.98	7.57	8.14	10.27	9.94	10.09
STDEV	2.67	2.29	2.87	2.76	3.00	2.88	2.59	2.28	2.64	2.70	2.83	2.63
R	0.52	0.35	0.59	0.58	0.64	0.61	0.48	0.38	0.51	0.63	0.63	0.59
r_{YN}	0.93			0.94			0.95			0.98		

TABLE 2C: TEST RESULTS FOR T_3

Item No.	Test 1			Test 2			Test 3			Test 4		
	Y	N	C	Y	N	C	Y	N	C	Y	N	C
1	61	86	68	93	77	88	89	77	86	79	78	79
2	68	53	63	91	80	88	72	89	77	85	86	85
3	63	44	58	42	20	36	59	57	58	80	86	82
4	86	81	84	75	63	72	36	43	38	65	58	63
5	35	25	32	51	34	46	53	57	54	80	78	80
6	57	53	56	76	74	75	44	43	43	37	58	43
7	74	78	75	52	29	45	82	83	83	39	39	39
8	99	97	98	62	31	54	62	54	60	61	53	59
9	60	58	59	60	46	56	29	17	26	66	53	63
10	79	75	78	78	71	76	67	43	60	73	64	70
11	92	78	88	47	26	42	22	34	26	41	50	44
12	94	97	95	40	37	39	76	63	73	45	61	49
13	37	36	37	23	23	23	27	31	28	88	83	87
14	74	72	73	73	54	68	84	71	80	66	58	64
15	61	56	59	28	11	24	84	69	79	40	22	35
16	2	6	3	46	46	46	47	51	48	58	56	57
17	44	19	37	64	51	61	61	51	58	74	61	70
18	48	39	45	59	60	59	62	60	62	21	3	17
19	65	67	66	5	3	5	48	49	48	91	92	91
20	80	69	77	37	29	35	74	66	72	93	75	87
N	84	36	120	95	35	130	85	35	120	92	36	128
MEAN	12.76	11.89	12.50	11.01	8.66	10.38	11.80	11.09	11.59	12.82	12.19	12.64
STDEV	3.27	3.58	3.37	3.67	3.27	3.70	3.64	4.48	3.90	3.92	4.71	4.15
R	.69	.75	.71	.74	.65	.74	.73	.82	.76	.79	.86	.81
r_{YN}	0.91			0.92			0.87			0.88		

TABLE 2C: TEST RESULTS FOR T_3 (CONTINUED)

Item No.	Test 5			Test 6			Test 7			Test 8		
	Y	N	C	Y	N	C	Y	N	C	Y	N	C
1	100	100	100	87	88	87	85	86	85	96	97	97
2	76	53	69	29	15	25	21	39	26	91	87	89
3	74	64	71	30	33	33	64	42	58	9	8	9
4	84	78	82	63	61	63	83	75	80	46	34	42
5	45	22	38	89	91	90	67	56	63	78	63	73
6	46	22	39	35	21	31	37	47	40	49	66	54
7	97	94	96	59	67	61	98	100	98	36	39	37
8	74	47	66	54	70	58	61	83	67	74	79	76
9	70	61	67	35	36	36	51	61	54	80	79	80
10	71	67	70	62	82	68	70	56	66	89	74	85
11	60	42	54	39	36	38	25	25	25	25	32	27
12	10	6	9	44	33	41	63	61	63	68	53	63
13	72	64	70	41	45	43	77	83	79	56	50	54
14	61	58	60	43	61	48	72	64	70	68	53	63
15	64	56	62	16	33	21	51	61	54	35	26	33
16	39	42	40	49	61	52	77	69	75	96	92	95
17	11	14	12	7	15	10	62	53	59	24	29	25
18	23	28	24	6	15	9	17	8	15	65	53	61
19	20	14	18	35	36	36	61	56	59	52	61	54
20	78	75	77	6	6	6	24	22	24	5	5	5
N	87	36	123	82	33	115	87	36	123	85	38	123
MEAN	11.75	10.06	11.25	8.30	9.06	8.52	11.66	11.47	11.60	11.44	10.79	11.24
STDEV	3.15	3.61	3.36	3.38	3.82	3.51	3.76	4.30	3.91	3.38	3.44	3.40
R	.68	.75	.71	.70	.78	.72	.76	.82	.78	.74	.72	.73
r_{YN}	0.94			0.92			0.88			0.94		

TABLE 2C: TEST RESULTS FOR T_3 (CONTINUED)

Item No.	Test 9			Test 10			Test 11			Test 12		
	Y	N	C	Y	N	C	Y	N	C	Y	N	C
1	76	68	74	87	76	83	95	88	93	86	94	88
2	77	71	76	50	43	48	72	67	70	54	52	53
3	48	48	48	88	73	83	43	52	46	6	6	6
4	7	16	9	84	68	79	14	10	13	26	33	28
5	94	94	94	33	51	39	71	57	66	16	12	15
6	53	55	54	68	68	68	34	26	31	26	30	28
7	57	39	52	57	46	54	33	33	33	92	82	89
8	72	58	68	49	46	48	67	62	66	91	94	92
9	24	10	20	54	43	50	57	48	54	38	42	39
10	31	35	32	43	43	43	35	29	33	37	42	38
11	75	77	76	6	14	8	69	79	72	86	76	83
12	65	55	62	54	41	50	89	79	86	55	52	54
13	33	19	29	88	59	79	73	79	75	41	48	43
14	67	74	69	91	81	88	80	71	77	90	94	91
15	33	23	30	35	27	33	57	38	50	47	52	48
16	82	84	82	40	27	36	65	62	64	91	94	92
17	85	74	82	17	8	14	57	62	58	71	73	72
18	43	39	42	68	49	62	34	33	34	74	67	72
19	73	71	72	13	19	15	42	40	42	9	12	10
20	56	52	55	40	27	36	8	7	8	1	0	1
N	88	31	119	82	37	119	83	42	125	87	33	120
MEAN	11.5	10.61	11.27	10.66	9.08	10.17	10.95	10.21	10.70	10.38	10.55	10.43
STDEV	3.56	3.81	3.63	3.57	3.79	3.69	3.44	4.05	3.66	3.06	2.97	3.03
R	.73	.76	.74	.74	.74	.75	0.70	0.79	0.74	0.70	0.66	0.69
r_{YN}	0.95			0.92			0.93			0.99		

TABLE 3C: TEST RESULTS FOR T_4

Item No.	Test 1			Test 2			Test 3			Test 4		
	Y	N	C	Y	N	C	Y	N	C	Y	N	C
1	81	79	80	90	85	89	84	89	85	81	90	84
2	74	61	71	93	89	92	74	85	77	83	90	85
3	61	68	62	35	33	34	78	74	77	80	83	81
4	89	93	90	82	78	81	58	70	61	62	84	67
5	52	46	50	62	56	60	57	48	55	83	79	82
6	54	61	56	82	81	82	52	48	51	54	48	53
7	74	64	72	43	30	40	86	89	87	46	45	45
8	97	96	97	60	59	59	64	70	65	62	66	63
9	54	46	52	70	63	68	28	22	27	58	59	58
10	81	79	80	74	74	74	55	59	56	63	66	64
11	83	79	82	37	41	38	25	37	29	52	45	50
12	90	79	87	49	44	48	82	78	81	59	59	59
13	37	32	36	19	22	20	42	59	46	93	83	90
14	81	75	79	80	63	76	70	56	67	75	69	74
15	62	68	63	47	48	47	82	89	83	57	55	56
16	40	21	36	47	33	44	55	37	45	69	52	65
17	43	43	43	71	74	72	85	63	60	75	79	76
18	58	46	56	66	70	67	80	74	70	46	45	45
19	62	61	62	20	22	21	65	63	70	94	86	92
20	69	86	73	36	30	34	81	74	75	90	86	89
N	89	28	117	89	27	116	88	27	115	81	29	110
MEAN	13.40	12.82	13.27	11.63	10.96	11.47	12.61	12.85	12.67	13.81	13.66	13.77
STDEV	3.72	3.70	3.71	3.90	4.59	4.08	4.48	3.96	4.35	4.40	4.68	4.46
R	0.76	0.75	0.75	0.78	0.84	0.80	0.84	0.78	0.82	0.84	0.86	0.85
r_{YN}	0.87			0.96			0.84			0.87		

TABLE 3C: TEST RESULTS FOR T_4 (CONTINUED)

Item No.	Test 5			Test 6			Test 7			Test 8		
	Y	N	C	Y	N	C	Y	N	C	Y	N	C
1	98	100	98	95	94	95	93	91	93	100	100	100
2	70	61	68	39	55	43	41	49	43	95	97	96
3	69	61	67	31	45	35	62	57	61	20	11	17
4	79	81	79	71	61	68	79	77	79	59	49	56
5	41	19	35	91	88	90	63	51	60	87	97	90
6	27	19	25	40	42	41	40	54	44	57	57	57
7	95	100	96	64	67	65	97	100	98	40	40	40
8	70	58	67	79	73	78	80	80	80	86	83	85
9	74	55	69	46	67	52	46	66	51	90	80	87
10	79	68	76	81	79	81	73	71	72	81	71	79
11	38	48	41	43	48	45	22	34	25	38	37	38
12	10	16	12	52	52	52	68	60	66	69	69	69
13	72	61	69	44	55	47	80	74	79	56	66	59
14	63	58	62	47	45	47	79	89	82	70	66	69
15	54	55	54	52	42	49	61	54	59	30	49	36
16	60	61	61	64	64	64	76	60	72	90	100	93
17	49	41	47	39	42	40	53	69	57	56	57	56
18	30	23	28	47	61	51	23	14	20	73	71	73
19	42	35	40	74	67	72	54	49	53	58	57	58
20	79	77	79	9	9	9	28	17	25	15	14	15
N	81	31	112	97	33	130	92	35	127	86	35	121
MEAN	12.00	11.00	11.72	11.10	11.55	11.22	12.21	12.17	12.20	12.70	12.71	12.70
STDEV	3.71	3.70	3.72	3.83	4.41	3.97	3.98	3.60	3.87	3.43	3.02	3.30
R	0.76	0.75	0.76	0.76	0.82	0.78	0.80	0.73	0.78	0.74	0.65	0.72
r_{YN}	0.94			0.92			0.90			0.96		

TABLE 3C: TEST RESULTS FOR T_4 (CONTINUED)

Item No.	Test 9			Test 10			Test 11			Test 12		
	Y	N	C	Y	N	C	Y	N	C	Y	N	C
1	70	70	70	89	77	86	94	96	95	83	84	83
2	89	80	87	49	48	49	72	61	69	58	41	53
3	62	60	62	93	87	92	46	46	46	9	0	6
4	43	53	46	89	65	83	34	36	34	35	19	31
5	91	83	89	25	32	27	75	71	74	29	19	27
6	65	50	61	87	71	83	46	50	47	39	16	33
7	49	33	46	67	48	63	34	57	39	86	91	87
8	73	50	67	38	42	39	82	61	77	92	88	91
9	46	43	46	58	58	58	54	71	58	43	41	43
10	30	30	30	52	35	48	22	25	23	37	47	40
11	78	77	78	13	10	13	64	61	63	90	84	89
12	67	70	67	64	52	61	88	89	88	68	50	64
13	41	43	41	83	55	76	71	82	74	38	28	35
14	77	73	76	97	94	96	69	82	72	85	88	85
15	57	63	59	53	42	50	76	86	79	63	75	66
16	86	93	88	52	35	48	64	64	64	92	97	94
17	88	87	88	55	32	49	62	61	62	72	72	72
18	54	43	51	72	48	66	28	36	30	82	66	77
19	76	77	76	22	19	22	49	50	50	13	9	12
20	67	67	67	45	42	44	12	7	11	7	3	6
N	93	30	123	89	31	120	89	28	117	92	32	124
MEAN	13.11	12.47	12.95	12.03	9.94	11.49	11.43	11.93	11.55	11.21	10.16	10.94
STDEV	4.16	4.68	4.27	3.70	4.39	3.98	3.86	4.06	3.89	3.47	3.09	3.39
R	0.81	0.85	0.82	0.77	0.82	0.79	0.77	0.80	0.78	0.76	0.72	0.75
r_{YN}	0.89			0.91			0.91			0.96		

TABLE 4C: RESULTS BY CONTENT AREA: DECIMALS

		Results													
		T_1			T_2			T_3			T_4				
		Y	N		Y	N		Y	N		Y	N			
D. Story Applications, Interpreting Decimals, Miscellaneous															
Location	**Content**														
(1, 6)	Light 186,000 m.p.s. How long 186 mi? ...	c 51	43		c 60	68		57	53	r	54	61			
(1, 13)	Perimeter 60.3 in. Each side 2" longer ..	c 26	22		c 27	32		c 37	36		37	32			
(1, 17)	Ratio sentence $\frac{3}{.03} = \frac{12}{n}$	c 34	14		39	11		44	19	*	43	43			
(2, 3)	If $\frac{\text{liters}}{\text{gallons}} : \frac{3.79}{1}$ then 1 liter is about?	c 37	29		30	17		c 42	20		c 35	33			
(2, 4)	Recognize $2\frac{1}{2}$ as a decimal	39	26	*	68	67	r	75	63		82	78			
(2, 10)	Which day has greatest probability of rain	75	65		c 83	63	*	78	71	*	74	74			
(3, 10)	Recognize that $300 \div 8 + \frac{2}{5} = 308.4$	25	21	*	48	54	r	67	43		55	59			
(3, 18)	1" = 2.54 cm, 1' = ?cm.	c 49	39		34	36	*	62	60	r	80	74			
(4, 13)	Another way to write 0.25 is ? $\frac{25}{100}$	54	23	*	82	61	r	88	83	r	93	83			
(4, 17)	1" = 2.54 cm, 10" = ?cm.	c 36	45		32	36	*	74	61	r	75	79			
(4, 20)	Write "four and twenty-seven hundreths" ..	65	48	*	85	79	r	91	75		90	86			
(5, 13)	Which city had least amount of rain?	55	36	*	68	47		72	64	*	72	61			
(5, 20)	Judy had $10 and spent $4.15. Left = ? ..	65	76	r	75	64	c	78	75		79	77			

TABLE 4C: RESULTS BY CONTENT AREA: DECIMALS (CONTINUED)

		Results											
		T_1			T_2			T_3			T_4		
		Y	N		Y	N		Y	N		Y	N	
A. Addition and Subtraction													
Location	**Content**												
(6, 16)	$0.3 + \frac{1}{2} = ?$ (0.8)	23	6	*	64	46	c	49	61	c	64	64	
(7, 14)	0.23 + 0.85 = ? (1.08)	72	48	r	79	65	r	72	64		79	89	
(12, 14)	0.8 - 0.3 = ? (0.5)	88	65	r	86	92	r	90	94		85	88	
B. Multiplication and Division													
Location	**Content**												
(4, 15)	0.2 x 0.3 = ? (0.06)	6	3		20	14	*	40	22	r	57	55	
(5, 17)	0.483 ÷ 0.21 = ? (2.3)	5	6		0	0		11	14	*	49	41	
(6, 17)	39.2 ÷ 100 = ? (0.392)	0	0		3	0		7	15	*	39	42	
(7, 18)	1088 + (38 x 1.1) = ? (1129.8)	2	3		3	3	c	17	8	*	23	14	
(8, 17)	6.624 ÷ 16 = ? (0.414)	12	12		34	17		24	29	*	56	57	
(9, 14)	0.6 x 30 = ? (18)	23	20		26	13	*	67	74	r	77	73	

TABLE 4C: RESULTS BY CONTENT AREA: DECIMALS (CONTINUED)

		Results											
		T_1			T_2			T_3			T_4		
		Y		N	Y		N	Y		N	Y		N
B. Multiplication and Division (ctd.)													
Location	**Content**												
(10, 16)	$\frac{2.54}{1} = \frac{?}{10}$ (25.4)	c	27	27	9		14	*	40	27	r	52	35
(10, 17)	2186.73 ÷ 801 = ? (2.73)		6	8	11		10		17	8	*	55	32
(11, 15)	27.6 ÷ 3 = ? (9.2)		38	28	31		35		57	38	*	76	86
C. Sentences, Number Line													
Location	**Content**												
(2, 7)	$0.083 = 83 \times \frac{1}{S}$ is true when S = ?		14	12	*	44	33		52	29	c	43	30
(3, 4)	Identify 1.53 on number line		10	21	*	37	29	c	36	43	*	58	70
(3, 9)	Sentence not equivalent to .9 x c = 27 ..		9	5		18	0	c	29	17	c	28	22
(6, 12)	Which decimal nearest $\frac{1}{2}$ on number line?..		23	47	c	36	37	c	44	33	*	52	52

TABLE 4C: RESULTS BY CONTENT AREA: DECIMALS (CONTINUED)

		Results											
		T_1			T_2			T_3			T_4		
		Y		N	Y		N	Y		N	Y		N
D. Story Applications, Interpreting Decimals, Miscellaneous (ctd.)													
Location	Content												
(6,7)	Chart "Contents of Earth's Crust"	51		41 c	58		57 c	59		67 c	64		67
(6,10)	What part shaded (0.13)	52		28 *	67		71	62		82 c	81		79
(6,20)	Triangle n" on a side has p = 60.3	c 4		0 c	3		3 c	6		6	9		9
(7,2)	Another way to write 30.3 (303 ÷ 10)	15		17 c	22		32	21		39 *	41		49
(8,13)	Which choice is not greater than 100? ...	28		24 c	49		45 c	56		50 c	56		66
(8,20)	Story involving 20% of $47 and subtract..	6		9	5		7	5		5 *	15		14
(9,4)	Which number is greatest? (0.293)	5		0 *	10		0	7		16 *	43		53
(10,18)	Write $\frac{3}{5}$ as a decimal (0.6)	20		3 *	53		31 r	68		49 c	72		48
(11,3)	5% interest on $300 for 1 year	38		34	32		22 c	43		52 c	46		46
(11,16)	Write two hundred and one and $\frac{53}{1000}$ as dec.	16		3 *	57		57 r	65		62 c	64		64
(11,20)	Temperature, sound story problem	0		3	2		3 c	8		7 c	12		7

TABLE 5C: RESULTS BY CONTENT AREA: RATIO

		Results							
		T_1		T_2		T_3		T_4	
		Y	N	Y	N	Y	N	Y	N
A. Related to Measurement and Standard Units									
Location	**Content**								
(1,6)	Time for light to travel 186 miles	c 51	43	c 60	68	57	53	r 54	61
(1,10)	The ratio $\frac{\text{inches}}{\text{yards}} = ?$	* 71	70	r 72	79	79	75	r 81	79
(1,17)	Ratio sentence from story problem: $\frac{3}{.03} = \frac{12}{n}$	c 34	14	39	11	44	19	* 43	43
(1,18)	$\frac{?}{40} = \frac{160}{200}$	* 45	19	36	25	c 48	39	r 58	46
(2,3)	$\frac{\text{liters}}{\text{gallons}} : \frac{3.79}{1}$. 1 liter is about?	c 37	29	30	17	c 42	20	c 35	33
(2,19)	500 pieces weigh 3 lbs.; 1 piece weighs?....	0	0	1	0	5	3	* 20	22
(3,7)	3 candies for 20¢; how many for $1.00?	* 84	63	82	68	82	83	r 86	89
(3,18)	1" = 2.54 cm.; 1' = ? cm.	c 49	39	34	36	* 74	61	r 80	74
(4,17)	1" = 2.54 cm.; 10" = ? cm.	c 36	45	32	36	* 74	47	r 75	79
(5,8)	200 cookies require 8 cups sugar; 50 cookies require ? cups sugar	* 64	64	61	56	74	47	r 70	58
(5,10)	If two distances are equal, what is their ratio?	c 65	52	63	58	c 71	67	c 79	68

TABLE 5C: RESULTS BY CONTENT AREA: RATIO (CONTINUED)

Location	Content	Results T_1 Y	T_1 N	T_2 Y	T_2 N	T_3 Y	T_3 N	T_4 Y	T_4 N
A. Related to Measurement and Standard Units (ctd.)									
(5,18)	Story: $\frac{2}{3}$ cup sugar for 1 cup flour; 6 cups sugar for ? cups flour	c 10	6	6	8	c 23	28	* 30	23
(7,3)	Which units are in the ratio $\frac{100}{1}$?	* 74	66	c 67	58	64	42	c 62	57
(9,18)	Oranges cost 79¢/doz. Cost of 1 orange to nearest penny?	c 17	22	27	23	43	39	* 54	43
(9,19)	$\frac{\text{grams}}{\text{pounds}}$: $\frac{453}{1}$; 3 lbs. = ? grams	* 72	59	67	67	c 73	71	r 76	77
(11,4)	Ratio of lengths $\frac{1}{2}$": $\frac{3}{4}$" ($\frac{2}{3}$)	c 22	38	23	14	14	10	c 34	36
(12,18)	A car goes 60 miles on 5 gallons of gas. How many miles on 1 gallon?	* 45	19	36	25	c 48	39	r 82	66
B. Related to Percent									
(1,5)	What percent is shaded? (40%)	26	19	44	29	35	25	* 52	46
(1,16)	What is 15% of 80?	5	0	4	0	2	6	* 40	21
(2,6)	Which figure has the same percent shaded as the one above?	65	59	79	79	76	74	* 82	81

TABLE 5C: RESULTS BY CONTENT AREA: RATIO (CONTINUED)

		Results											
		T_1			T_2			T_3			T_4		
		Y	N		Y	N		Y	N		Y	N	
B. Related to Percent (ctd.)													
Location	Content												
(2,20)	20% OFF. Sale price $8; original price = ?	47	29		34	38		37	29	*	36	30	
(3,13)	4 cars out of 100 have bad tires. What percent is this?	25	37		32	39		27	31	*	42	59	
(3,19)	What percent is shaded? (96%)	33	37		51	32		48	49	*	65	63	
(4,18)	16 is what percent of 20?	6	3		24	11		21	8	*	46	45	
(5,19)	20 out of 25 were present. What percent were present?	23	6		25	11		20	14	*	42	35	
(6,7)	Contents of earth's crust	51	41	c	58	57	c	58	67	c	64	67	
(7,6)	3,000 miles uses 25% of tire tread. About how many more miles?	43	48		34	32		37	47	*	40	54	
(8,2)	Reading percent from chart	89	82		100	93	*	91	87	*	95	97	
(8,3)	Ratio of nitrogen to oxygen from chart ...	11	9		9	7		9	8	c	20	11	
(8,11)	3 parts water to 1 part sugar. What percent is water?	11	9		19	21		25	32	c	38	37	
(8,20)	Bike originally $47. 20% Off	6	9		5	7		5	5	*	15	14	

TABLE 5C: RESULTS BY CONTENT AREA: RATIO (CONTINUED)

		Results											
		T_1		T_2		T_3			T_4				
		Y	N	Y	N	Y	N		Y	N			
B. Related to Percent (ctd.)													
Location	Content												
(9,9)	Three is what percent of 6?	13	12	13	10	24	10	*	46	43			
(10,12)	What percent is shaded?	41	41	36	45	54	41	*	64	52			
(11,3)	5% interest on $300 for 1 year	38	34	32	22	c	43	52	c	46	46		
(11,18)	Story problem (50% of 220 million)	16	19	15	8		34	33	*	28	36		
(12,19)	$20 tent on sale $19. % reduced?	14	6	13	8		9	12	*	13	9		

TABLE 5C: RESULTS BY CONTENT AREA: RATIO (CONTINUED)

Location	Content	Results T_1 Y	T_1 N	T_2 Y	T_2 N	T_3 Y	T_3 N	T_4 Y	T_4 N
C. Related to Similar Figures and Scale Drawings									
(2,8)	1" represents 150 miles; 2" + $2\frac{1}{2}$" represents ? miles	c 34	38	c 46	29	c 62	31	* 60	59
(2,18)	1" represents 6 miles; $3\frac{1}{2}$" represents ? miles	c 47	47	45	54	* 59	60	* 66	70
(6,5)	1" = 100'; segment ST = ? feet (150') ..	c 77	72	c 79	69	c 89	91	* 91	88
(9,3)	$4\frac{1}{2}$" represents 72 miles; 7" represent? .	c 56	56	41	43	c 48	48	* 62	60
(10,10)	Scale drawing: How long is ladybug?	c 35	27	44	27	43	43	c 52	35
D. Miscellaneous Ratio Coverage									
(8,4)	2 out of 5 is how many out of 100?	* 56	42	35	41	46	34	r 59	49
(10,9)	A class of 50 freshmen, 40 sophomores, etc. $\frac{\text{number of juniors}}{\text{number of students}}$ = ? ($\frac{1}{3}$)	c 42	37	46	45	54	43	c 58	58

TABLE 6C: RESULTS BY CONTENT AREA: GEOMETRY - MEASUREMENT

A. Standard Units

Location	Content	Results T_1 Y	T_1 N		T_2 Y	T_2 N		T_3 Y	T_3 N		T_4 Y	T_4 N
(1,7)	Which segment represents 1 foot?	* 76	65	c	71	75		74	78	c	74	64
(1,10)	The relationship $\frac{\text{inches}}{\text{yards}}$ is ? $\frac{36}{1}$	* 71	70	r	72	79		79	75	r	81	79
(2,3)	$\frac{\text{liters}}{\text{gallons}} = \frac{3.79}{1}$. One liter is about?	c 37	29		30	17	c	42	20	c	35	33
(3,2)	Appropriate unit to express length of worm? cm.	c 73	61	c	77	87		72	86		74	85
(3,11)	Which 10 gal. can contains 10 qt. water?.	c 22	21	c	34	29		22	34	c	25	37
(3,18)	1 in. = 2.54 cm. How many cm. in a ft.? .	c 49	39		34	36	*	62	60	r	80	74
(5,3)	Which object is about 1 meter long? (yd. stick)	* 70	64		69	64		74	64		79	61
(6,14)	(5 yd.2 ft.) + (2 yd. 2 ft.) = ? yd. 1 ft.	* 52	53		34	43		43	61		47	45
(7,3)	Which units are in ratio $\frac{100}{1}$? $\frac{\text{centimeter}}{\text{meter}}$.	* 74	66	c	67	58		64	42	c	62	57
(8,7)	$1\frac{1}{2}$ qts. is what fraction of a gallon? ...	c 19	39	c	29	41	c	36	39	c	40	40
(10,8)	Which unit is most precise? cm.	c 34	32		39	28		49	46		38	42

TABLE 6C: RESULTS BY CONTENT AREA: GEOMETRY - MEASUREMENT (CONTINUED)

Location	Content	Results											
		T_1			T_2			T_3			T_4		
		Y		N	Y		N	Y		N	Y		N
B. Linear (perimeter, scale)													
(1,1)	Distance "around" rectangle; sides 1 and 7	*	75	65	c	67	75		61	86	r	81	79
(1,13)	Triangle has perimeter 60.3. Each side 2" longer	c	26	22	c	27	32	c	37	36		37	32
(2,8)	1": 150 mi.; 2 + 2 $\frac{1}{2}$ in. : ? miles ...	c	34	38	c	46	29	c	62	31	*	60	59
(3,5)	Length of knife; endpoints 1" and 8"	c	38	37	c	48	39		53	57		57	48
(3,6)	Large sq. divided into 4 small sqs. Perimeter small ?	c	32	18		37	14		44	43		52	48
(4,10)	A sq. is 9" on a side. Perimeter = ?	*	62	55		52	64		73	64	c	63	66
(6,20)	Perimeter of triangle n + 2 inches on a side	c	4	0		3	3	c	6	6		9	9
(10,10)	Scale drawing. How long is lady bug?	c	35	27		44	27		43	43	c	52	35
(12,2)	How long is Euglena (scale, picture)?	c	43	44		42	50		54	52	c	58	41

TABLE 6C: RESULTS BY CONTENT AREA: GEOMETRY - MEASUREMENT (CONTINUED)

			Results											
			T_1			T_2			T_3			T_4		
			Y	N		Y	N		Y	N		Y	N	
C. Area														
Location	Content													
(2,11)	Find area of rectangular L-shaped region?	*	38	35	r	34	29	r	47	26	r	37	41	
(3,8)	Area of rect. is 18. Area of shaded part = ?	c	51	50	r	54	39	r	62	54	r	64	70	
(4,11)	Which number could be area of desk top?	c	37	19	c	54	46	c	41	50	c	52	45	
(5,11)	Area of 5 pointed star	c	34	27		39	39	c	60	42	c	38	48	
(6,9)	A sq. 2" on a side. Area of shaded triangle?	c	37	25	c	32	51	*	35	36	c	46	67	
(8,10)	Area of rectangle 8' x 3'	*	58	67	r	78	55	r	89	74	r	81	71	
(8,19)	Doghouse problem: find area, cost?...	c	46	48	c	55	31	c	52	61	c	58	57	
(9,10)	Cookie cutter problem		39	34		29	37		31	35		30	30	
(9,13)	Area of sq. $\frac{1}{2}$ in. on a side	c	10	7	c	22	27	*	33	19	c	41	43	
(10,5)	Find outer area of circle	*	59	35		40	34		33	51		25	32	
(10,19)	Floor 12' x 15'. How many sq. yds.?	c	9	3	c	15	7	c	13	19		22	19	

TABLE 6C: RESULTS BY CONTENT AREA: GEOMETRY - MEASUREMENT (CONTINUED)

		Results											
		T_1		T_2			T_3			T_4			
		Y	N	Y	N		Y	N		Y	N		
C. Area (ctd.)													
Location	Content												
(11,6)	Area of region (rect. and triangle	35	19	16	22	*	34	26	r	46	50		
(11,11)	Which figure has inner area = outer area?	c 84	78	64	68		69	79		64	61		
(12,5)	Area of 3, 4, 5 right triangle?	5	3	6	8	*	16	12	r	29	19		
D. General Coverage													
Location	Content												
(4,12)	In which figure are all angles congruent?	54	35	r	51	46	c	45	61	r	59	59	
(7,4)	Volume of box: 3 x 4 x 5	* 31	31	r	70	58	r	83	75		79	77	
(7,20)	For a triangle, measure $\angle B = ?$	0	0	*	15	13		24	22	c	28	17	
(9,12)	What is (E,F,V) for a 1" cube? (12,6,1)	r 63	44	c	55	37	c	65	55	c	67	70	
(11,9)	Determine angle measure (45°)	32	28	*	44	43	c	57	48		54	71	
(11,20)	Sound problem, extension	0	3		11	3	c	8	7	c	12	7	
(12,20)	Cubic ft. water prob. (wt. 1" sq. col.)	c 2	0	c	3	0		1	0	c	7	3	

TABLE 7C: RESULTS BY CONTENT AREA: GEOMETRY -- NON-METRIC

		Results								
		T_1		T_2			T_3		T_4	
		Y	N	Y	N		Y	N	Y	N
A. Reflections										
Location	**Content**									
(4,1)	Paper folding, circle cut	74	48	87	68	c	79	78	81	90
(4,2)	Paper folding, C-cut	76	74	88	71	c	85	86	83	90
(4,3)	Paper folding, triangle cut	78	87	84	68	c	80	86	80	83
(4,4)	Paper folding, circle cut	73	71	73	75	c	65	58	62	84
(4,5)	Paper folding, irregular figure	78	61	80	64	c	80	78	83	79
(4,6)	Two folds, triangle	39	39	49	39	c	37	58	54	48
(4,7)	Two folds, triangle -- square	29	29	34	36	c	39	39	46	45
(4,8)	Two folds, circle	38	32	55	50	c	61	53	62	66
(10,4)	Triangle reflected; what is image?	86	68	85	69	c	84	68	58	59
B. Sections, 3-D Figures										
Location	**Content**									
(1,12)	Identify bottom face of prism .....*	93	92	92	93	r	94	97	90	79

TABLE 7C: RESULTS BY CONTENT AREA: GEOMETRY -- NON-METRIC (CONTINUED)

			Results									
			T_1		T_2		T_3		T_4			
			Y	N	Y	N	Y	N	Y	N		
B. Sections, 3-D Figures (ctd.)												
Location	Content											
(2,13)	Identify cut through cube as a rect.		34	18	* 26	42	23	23	19	22		
(6,8)	Which object is usually a cylinder (tin can)		54	50	59	51	54	70	79	73		
(7,4)	Volume of box 3 x 4 x 5 (pictured) ...	*	31	31	r 70	58	83	75	79	77		
(7,9)	Identify cut as trapezoid		42	41	c 38	35	51	61	46	66		
(9,5)	Identify back face of prism by ABFE ..	*	88	83	r 90	90	94	94	91	83		
(9,12)	Find (E,F,V) for a cube (pictured) ...	r	64	44	c 37	46	c 65	54	c 67	70		
(10,11)	Identify cut through sphere as circle		7	6	c 12	14	6	14	c 13	10		
(12,7)	Identify a cut congruent to a given cut		83	91	c 87	81	92	82	c 86	91		
(12,20)	Extension, cubic ft. water with visual	c	2	0	3	0	1	0	c 7	3		
C. Other Coverage												
Location	Content											
(2,1)	Identify figure as rectangle	r	90	85	84	83	93	77	90	85		

TABLE 7C: RESULTS BY CONTENT AREA: GEOMETRY -- NON-METRIC (CONTINUED)

C. Other Coverage (ctd.)			Results							
			T_1		T_2		T_3		T_4	
Location	Content		Y	N	Y	N	Y	N	Y	N
(5,6)	What is (E,F) for a cube?	r	11	18	c 26	28	46	22	27	19
(8,12)	Naming an angle		-	-	* 62	55	68	53	69	69
(9,1)	Identify figure as triangle	c	52	66	62	63	76	68	70	70
(9,7)	Which figure has > 6 lines of symmetry		28	17	23	7 *	57	39	49	33
(10,13)	Recognize a line of symmetry		74	27	72	38 *	88	60	83	55

TABLE 8C: RESULTS BY CONTENT AREA: INTEGERS

		Results							
		T_1		T_2		T_3		T_4	
		Y	N	Y	N	Y	N	Y	N
A. Multiplication									
Location	**Content**								
(1,14)	571 x 68 = ? (38,828)	r 64	59	r 75	68	74	72	81	75
(3,14)	80 x 407 = ? (32,560)	r 65	61	r 75	54	84	71	70	56
(5,15)	708 x 397 = ? (281,076)	r 55	24	r 51	33	64	56	54	55
(7,16)	73 x 48 = ? (3,504)	r 70	66	r 71	74	77	69	76	60
(8,18)	300 x 800 = ? (240,000)	r 67	52	r 66	62	65	53	73	71
(9,16)	19 x 2010 = ? (38,190)	r 84	61	r 78	83	82	84	86	93
(12,15)	627 x 388 = ? (243,276)	r 47	26	r 54	53	47	52	63	75
B. Long Division									
(1,19)	3879 ÷ 27 = ? (143 r 18)	36	32	* 55	68	r 65	67	r 62	61
(2,17)	8709 ÷ 9 = ? (967 r 5)	50	65	* 63	75	r 64	51	r 71	74
(3,15)	483 ÷ 21 = ? (23)	r 80	63	* 79	64	r 84	69	r 82	89
(7,19)	3879 ÷ 27 = ? (143 r 18)	53	41	* 56	48	r 61	56	r 54	49
(11,17)	8642 ÷ 90 = ? (96 r 2)	55	53	* 57	68	r 55	62	r 62	61

TABLE 8C: RESULTS BY CONTENT AREA: INTEGERS (CONTINUED)

		Results											
		T_1			T_2			T_3			T_4		
		Y	N		Y	N		Y	N		Y	N	
B. Long Division (ctd.)													
(12,17)	3120 ÷ 500 = ? (6 r 120)	70	59	*	81	69	r	71	73	r	72	72	
(2,12)	Understanding the algorithm	33	29	c	41	33		40	37		49	44	
(5,7)	Identify the remainder	94	91	r	96	94		97	94		95	100	
(6,11)	Knowing remainder < divisor	33	28	c	36	57		39	36	c	43	48	
(8,6)	Identify the quotient	53	52	c	65	62		49	66		57	57	
(11,7)	Greatest possible remainder for N + 23	29	34	c	26	19		33	33	c	34	57	
(12,8)	Next integer after 15 that is divisible by 3	c 83	88	c	90	86		91	94	c	92	88	
(12,11)	Sentence for long division example ..	c 79	50	*	94	83		86	76	r	90	84	
C. Average, Negative Integers, Misc.													
(1,3)	Greatest divisor of 28 and 42	69	43	r	55	46		63	44		61	68	
(2,2)	Travel 1260 mi. in 3 days. Average = ?	82	85		84	79		91	80	*	93	89	
(3,12)	Express 24 as a product of primes ...	63	37	r	79	61		76	63	r	82	78	

TABLE 8C: RESULTS BY CONTENT AREA: INTEGERS (CONTINUED)

		Results											
		T_1			T_2			T_3			T_4		
		Y	N		Y	N		Y	N		Y	N	
C. Average, Negative Integers, Misc. (ctd.)													
(5,4)	-2 + (-7) = ?	53	24		65	53	*	84	78		79	81	
(6,1)	Write two hundred ten thousand, eighty-six	90	88	c	88	83		87	88	c	95	94	
(6,19)	Average: 21, 33, 26, and 20	23	21		25	26		36	36	*	74	67	
(7,1)	Expand 2083 (2000 + 80 + 3)	91	79		86	74		85	86		93	91	
(9,8)	Sentence for loss of 15 followed by a gain of 11 (-15 + 11)	52	41		53	53	*	72	58		73	50	
(9,18)	Oranges 79¢/doz. Cost of 1 orange to the nearest penny	c 17	22		27	23		43	39	*	54	43	
(10,7)	Odometer reading after 3 hours, 30 mph average (7910 ⟶ 8000)	64	49		59	52		57	46	*	67	48	
(11,5)	To reach 2 on number line--(-3 + 5)	34	22		47	22	*	71	57		75	71	
(11,8)	1000 x 100 = ? (10^5)	56	53		56	68		67	62	*	82	61	

TABLE 9C: RESULTS BY CONTENT AREA: LONG DIVISION

			Results											
			T_1			T_2			T_3			T_4		
			Y	N		Y	N		Y	N		Y	N	
A. Computation														
Location		Content												
(1,19)	3879 ÷ 27 = ?	(143 r 18).....	36	32	*	55	68	r	65	67	r	62	61	
(2,17)	8708 ÷ 9 = ?	(967 r 5)	50	65	*	63	75	r	64	51	r	71	74	
(3,15)	483 ÷ 21 = ?	(23)	r 80	63	*	79	64	r	84	69		82	89	
(5,17)	0.483 ÷ 0.21 = ?	(2.3)	5	6		0	0		11	14	*	49	41	
(6,17)	39.2 ÷ 100 = ?	(0.392)	0	0		3	0		7	15	*	39	42	
(7,19)	3879 ÷ 27 = ?	(143 r 18)	53	41	*	56	48	r	61	56	r	54	49	
(8,17)	6.624 ÷ 16 = ?	(0.414)	12	12		34	17		24	29	*	56	57	
(10,17)	2186.73 ÷ 801 = ?	(2.73)	6	8		11	10		17	8	*	55	32	
(11,15)	27.6 ÷ 3 = ?	(9.2)	38	28		31	35		57	38	*	76	86	
(11,17)	8642 ÷ 90 = ?	(96 r 2)	55	53	*	57	68	r	55	62	r	62	61	
(12,17)	3120 ÷ 500 = ?	(6 r 120)	70	59	*	81	69	r	71	73	r	72	72	
B. Understanding and Terminology														
(2,12)	Understanding the algorithm		33	29	c	41	33		40	37		49	44	
(5,7)	Identify remainder in long ÷ example		94	91	r	96	94		97	94		95	100	

TABLE 9C: RESULTS BY CONTENT AREA: LONG DIVISION (CONTINUED)

		Results											
		T_1			T_2			T_3			T_4		
		Y	N		Y	N		Y	N		Y	N	
B. Understanding and Terminology													
Location	Content												
(6,11)	Knowing remainder < divisor	33	28	c	36	57		39	36	c	43	48	
(7,2)	Another way to write 30.3? (303 ÷ 10)	11	17	c	22	32		21	39	*	41	49	
(8,6)	Identify quotient in long ÷ example ...	53	52	c	65	62		49	66		57	57	
(11,7)	Greatest possible remainder for N ÷ 23?	29	34	c	26	19		33	33	c	34	57	
(12,8)	Next integer after 15 that is divisible by 3	c 83	88	c	90	86		91	94	c	92	98	
(12,11)	Sentence for long division example	c 79	50	*	94	83		86	76	r	90	84	

TABLE 10C: RESULTS BY CONTENT AREA: SENTENCE

Location	Content		Results T_1 Y	T_1 N		T_2 Y	T_2 N		T_3 Y	T_3 N		T_4 Y	T_4 N
(1,2)	Which sentence equivalent to n x 6 = 48?	r	71	43	r	69	64		68	53	r	74	61
(2,7)	0.083 = 83 x $\frac{1}{s}$; s = ?		14	12	*	44	33		52	29	c	43	30
(2,9)	2 x N = $\frac{1}{3}$; N = ?		52	41		46	54	*	60	46	r	70	63
(3,1)	N - 6 = 8 ; N = ?	r	81	80	r	92	71		89	77	c	84	89
(3,9)	Which sentence is not equivalent to 0.9 x c = 27?		9	5		18	0	c	29	17	c	28	22
(5,1)	3 x N = 12 ; N = ?	r	98	79	r	99	92	*	100	100	c	98	100
(6,6)	For which sentence is N ≠ 1? N = 3 ÷ $\frac{1}{3}$		36	16		30	14		35	21	*	40	42
(7,10)	Which sentence is true? $\frac{1}{12} = \frac{6}{72}$	r	73	62	r	81	58		70	56	c	73	71
(9,8)	Choose a sentence to represent story ..		52	41		53	53	*	72	58		73	50
(10,1)	23 + N = 130 ; N = ?		79	73	r	89	83		87	76		89	77
(11,1)	Simple story with sentence p = 123 - 83	r	97	97		87	89		95	88		94	96
(11,2)	2 x n = 9 ; n = ?		35	25	c	49	35	*	72	67		72	61
(12,11)	Sentence for long division example	c	79	50	*	94	83		86	76	r	90	84
(12,20)	Sentence to represent solution	c	2	0		3	0		1	0	c	7	3

TABLE 11C : RESULTS BY CONTENT AREA: NUMBER LINE

Location	Content	T_1 Y	T_1 N		T_2 Y	T_2 N		T_3 Y	T_3 N		T_4 Y	T_4 N	
(1, 4)	Name pt. A on number line: $1\frac{3}{4}$	78	73	r	81	86	r	86	81		89	93	
(2, 8)	Ruler application to scale drawing	34	38	c	46	29	c	62	31	*	60	59	
(3, 4)	Which number names point A? 1.53	10	21	*	37	29	c	36	43	*	58	70	
(3, 5)	Length of knife; endpoints 1" and 8"	c 38	37	c	48	39		53	57		57	48	
(5, 2)	Between $2\frac{5}{8}$ and $2\frac{7}{8}$	64	33	*	75	53		76	53	c	70	61	
(6, 5)	Scale drawing: read $1\frac{1}{2}$" on ruler	77	72	c	79	69	c	89	91		91	88	
(6, 12)	Which decimal nearest $\frac{1}{2}$ on number line? .	23	47	c	36	37	c	44	33	*	52	52	
(6, 13)	Reading location of arrow on scale	27	31	c	23	23	c	41	45		44	55	
(7, 5)	Which number names point B? $\frac{11}{4}$	43	28	*	68	45	r	67	56		63	51	
(9, 11)	Distance from M to N on number line?	70	66	c	68	60	c	75	77		78	77	
(11, 5)	Way to reach 2 on number line? -3 + 5 ..	34	22		47	22	*	71	57		75	71	
(11, 12)	Which number nearest 0 on number line? $\frac{1}{16}$	74	69	r	78	78	c	89	79		88	89	
(12, 4)	B is $\frac{1}{2}$ way between A and C, name B	23	9	r	23	33	c	26	33		35	19	

TABLE 12C: RESULTS BY CONTENT AREA: RELATIONS AND FUNCTIONS

Location	Content	Results T_1 Y	T_1 N		T_2 Y	T_2 N		T_3 Y	T_3 N		T_4 Y	T_4 N
(2,5)	$F = (\frac{9}{5} \times C) + 32$, $C = 0$; $F = ?$	25	18	c	41	38	*	51	34	r	62	56
(5,5)	If $m = 4$, $\frac{(3 \times m) + 8}{m} = ?$	4	9		22	19	*	45	22	r	41	19
(5,6)	What is (E,F) for a cube?	r 11	18	c	26	28	c	46	22	c	27	19
(5,12)	How is $\frac{1}{n}$ related to $\frac{1}{n+1}$?	8	12		14	14	c	10	6		10	16
(7,13)	$S = (3 \times T) + 5$; $T = 8$, $S = ?$	69	48		75	58	*	77	83	r	80	74
(9,12)	(E, F, V) for a cube	r 63	44	c	55	37	c	65	55	c	67	70
(10,20)	$F = (\frac{9}{5} \times C) + 32$, $C = 30$; $F = ?$	2	0		11	3	*	40	27	r	45	42
(11,10)	Sentence to relate the length and perimeter of square ($L = \frac{1}{4} \times P$)	13	19		20	14	*	35	29	c	22	25

TABLE 13C : RESULTS BY CONTENT AREA: CHARTS AND GRAPHS

Location	Content	Results T_1 Y	N		T_2 Y	N		T_3 Y	N		T_4 Y	N
(1,8)	Earnings of 6 boys--which 3 earned about the same money?	97	95		97	93	*	99	97		97	96
(1,9)	Estimate the difference between the greatest and least	30	24		48	50	*	60	58	c	54	46
(2,10)	Determine the day most likely for rain. (Decimal chart)	75	65	c	83	63	*	78	71	*	74	74
(5,13)	Which day had the least rain?	55	36	*	68	47		72	64	*	72	61
(6,7)	Contents of the earth's crust	51	41	c	58	57	c	59	67	c	64	67
(7,7)	On which day did the greatest number of pupils attend school?	99	90		99	97	*	98	100		97	100
(7,8)	By how many pupils was the Thursday attendance below average?	58	52		68	65	c	61	83	*	80	80
(8,2)	What percent of the air is oxygen?	89	82		100	93	*	91	87	*	95	97
(8,3)	Reduced ratio of nitrogen to oxygen	11	9		9	7		9	8	c	20	11
(12,9)	After three hours, how far is car A ahead of car B?	33	24		41	39	c	38	42		43	41
(12,10)	How much longer for car B to go 50 miles than car A?	31	38		37	33	c	37	42		37	47

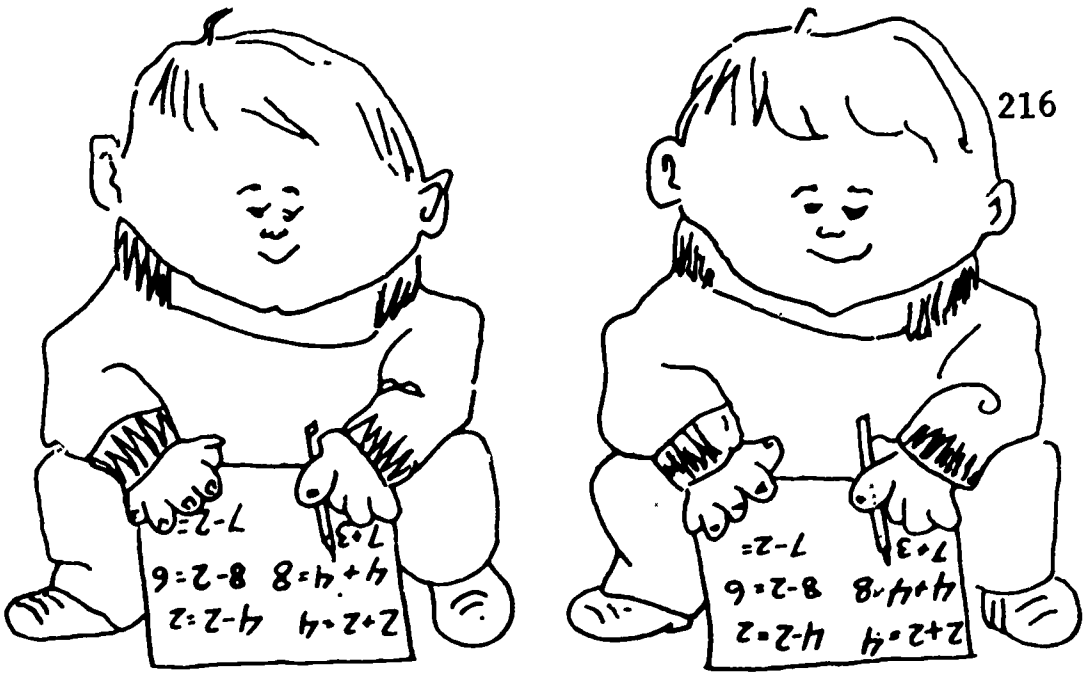
TABLE 14C: RESULTS BY CONTENT AREA: PROBABILITY

Location	Content	Results T_1 Y	T_1 N		T_2 Y	T_2 N	T_3 Y	T_3 N		T_4 Y	T_4 N
(1,11)	Spinner (least likely region to stop on)	69	81		71	89	92	78	c	83	79
(2,10)	Chart (which day is it most likely to rain?)	75	65	c	83	63	78	71	c	74	74
(4,19)	Pick a marble; which color most likely?	84	94		88	86	91	92	c	94	86
(6,2)	Probability a person born 300 years ago is living.	18	22		19	17	29	15	*	39	55
(6,18)	Sack with 4 black, 7 red: P(Red) = ?	5	0		5	3	6	15	*	47	61
(7,11)	Probability of curve crossing circle (1)	23	24		22	13	25	25	*	22	34
(8,8)	Marble dropped in tube. How many paths?	71	55		74	76	74	79	c	86	83
(9,20)	Mouse in maze--How many routes can he take?	40	44		45	30	56	52	*	67	67
(11,13)	Bag with 5 red, 2 blue. How many blue to make chances equal?	75	69		74	76	73	79	c	71	82
(12,3)	Toss 5 pennies. Possible number that fall heads-up.	2	3		0	0	6	6	c	9	0
(12,12)	Which sack is prob. of red ball greatest?	43	38		35	39	55	52	*	68	50

APPENDIX D

CHECK-UP TESTS DESIGNED TO SUPPLEMENT THE FORMATIVE EVALUATION

Patterns in Arithmetic
Check-up 1



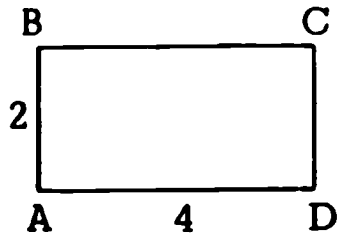
Instructions: Find the answer to each question. Your teacher will tell you where to put your answers.

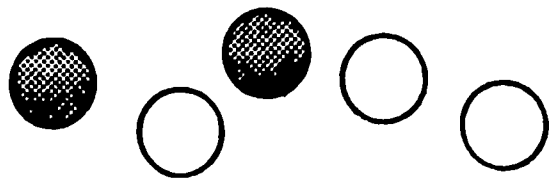
1. Write three hundred twenty-nine thousandths as a decimal.

2. Add the mixed numbers $3\frac{2}{5}$ and $1\frac{1}{6}$. Write your answer as a mixed number.

3. What is the perimeter of rectangle ABCD?

4. What is the area of rectangle ABCD?





5. What fraction of the circles are shaded?

6. Write $\frac{6}{27}$ as a basic fraction.

7. Write the sum $\frac{8}{10} + \frac{7}{1000}$ as a decimal.

8. Find the number that makes the sentence below true.

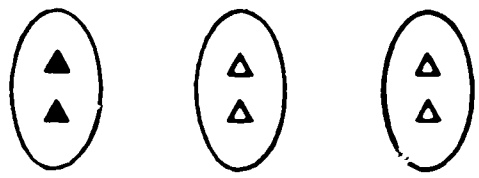
$$\frac{2}{7} + \frac{3}{7} = m$$

9. Which number below is least?

$$23.6 \quad \frac{1}{800} \quad .1 \quad \frac{2}{5} \quad 1$$

10. Solve the sentence $16 \times g = 208$.

11. The picture at the right represents two fractions that are equal. What are these fractions?

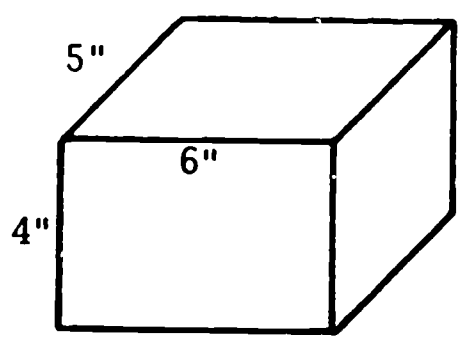


12. Solve the sentence $2\frac{2}{5} - \frac{3}{5} = n$.

13. Write $\frac{123}{1000}$ as a decimal.

14. If 5 pounds of apples cost 60¢, how much will 3 pounds cost?

15. What is the volume of the box?



16. Each side of a square is 1 ft. 3 in. What is the perimeter of the square?

17. Add:
$$\begin{array}{r} 2.61 \\ \underline{3.58} \end{array}$$

18. Mr. Jones collects eggs on his chicken farm each day. One day he collected 6213 eggs and put them into cartons with 12 eggs in each carton. How many cartons did he fill? How many eggs remain?

19. The auditorium at school has 41 rows of seats with 36 seats in each row. One day every seat was taken but 3. Write a sentence to show how many seats were taken. Solve the sentence.

20. One morning the mileage on Bill's car read 12063.8 miles. At the end of the day the mileage read 12317.5 miles. Write a sentence to represent the number of miles traveled during the day. Solve the sentence.

DEVELOPMENTAL YEAR RESULTS FOR CHECK-UP 1

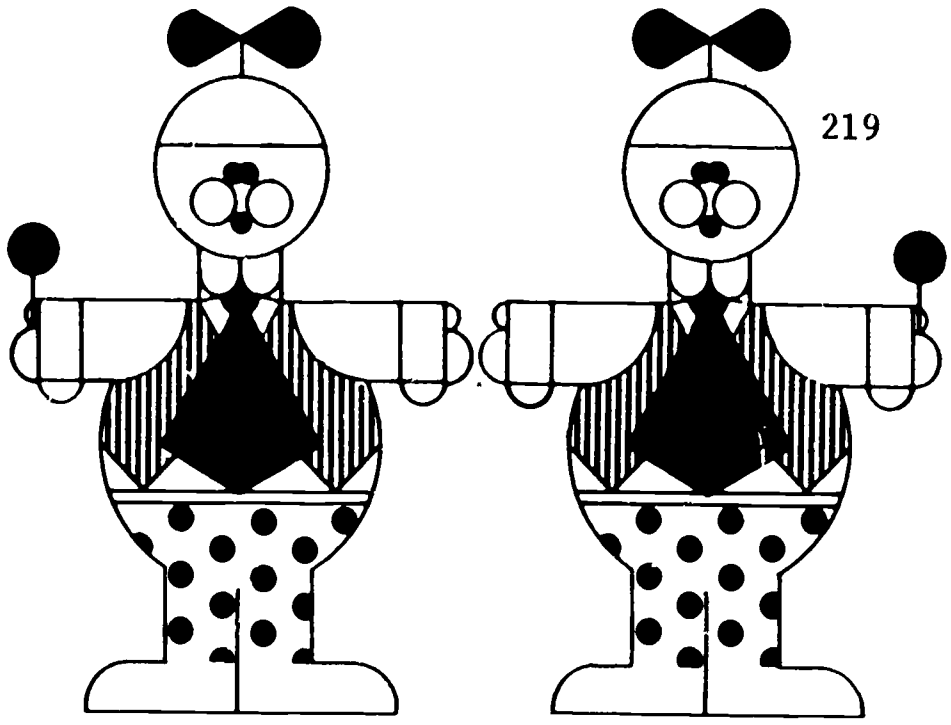
Check-Up 1 was administered following Program 15. It was found to be weak and difficult and was modified for inclusion in the Pupil Exercise Manual. No summary statistics are available for the modified version, however, a few items were the same and the results for these items are summarized in Table D1.

TABLE 1D

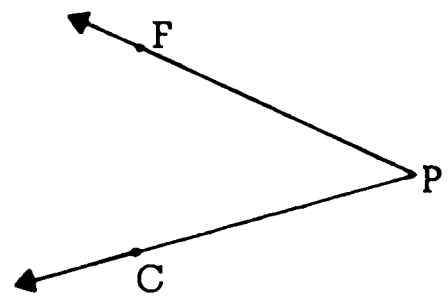
RESULTS FOR SELECTED ITEMS ON CHECK-UP 1.
114 PAPERS WERE SCORED

Item Number	Percent Responding Correctly
1	57
2	67
14	49
18	Cartons 74%, Extra Eggs 80%
19	Sentence 62%, Answer 53%
20	Sentence 72%, Answer 60%

Patterns in Arithmetic
Check-up 2



Instructions: Find the answer to each problem. Your teacher will tell you where to put your answers.



1. Which choice represents the correct way to name the angle above?

   a) ∠ FPC
   b) ∠ FCP
   c) ∠ CFP
   d) ∠ PFC
   e) ∠ PCF

2. $\frac{2}{7} \times \frac{8}{11} = ?$

   a) $\frac{10}{18}$

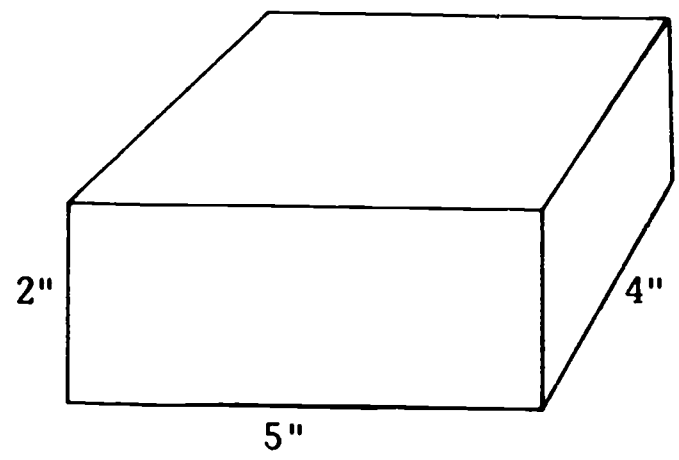
   b) $\frac{10}{77}$

   c) $\frac{16}{77}$

   d) $\frac{16}{18}$

3. There were 126 apples in a basket and $\frac{1}{3}$ of them were green. How many were green?

   a) 34
   b) 42
   c) 84
   d) $125\frac{2}{3}$
   e) 378



4. What is the volume of the rectangular box above?

   a) 10 cu. in.
   b) 11 cu. in.
   c) 11 in.
   d) 40 in.
   e) 40 cu. in.

5. $\frac{2 \times 3 \times 7}{2 \times 7} = ?$

a) 42
b) 21
c) 14
d) 3

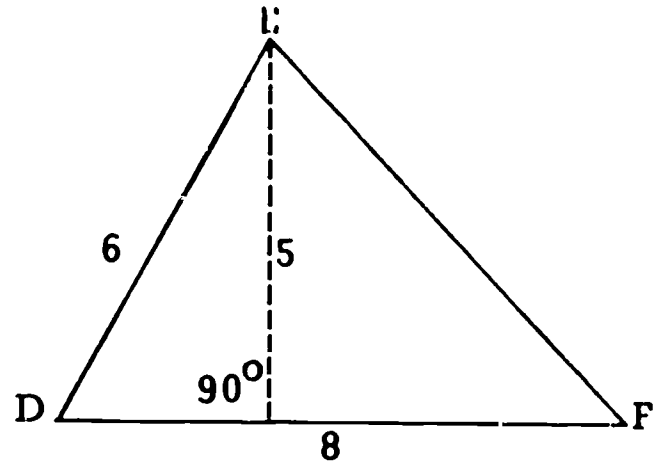
6. Hamburger costs 69¢ per pound. How much would $\frac{2}{3}$ pound of hamburger cost?

a) 99¢
b) 50¢
c) 46¢
d) 40¢
e) 23¢

7. Solve the sentence

$$\frac{3}{5} \times n = 1$$

a) $\frac{5}{3}$
b) $\frac{2}{5}$
c) 2
d) $\frac{3}{5}$
e) 1



8. What is the area of triangle DEF above?

a) 48 sq. units
b) 24 sq. units
c) 20 sq. units
d) 40 sq. units
e) 19 sq. units

9. Which choice below will be greater than 263?

a) $\frac{7}{7} \times 263$
b) $\frac{4}{5} \times 263$
c) $1\frac{1}{3} \times 263$
d) 0.93×263

10. The sum of the angles of a triangle is

a) 90°
b) 180°
c) 270°
d) 300°
e) different for each triangle

For the next 5 problems choose the word which best fits the sentence.

11. The _____ is a standard unit of measure for angles.

a) erg
b) protractor
c) inch
d) centimeter
e) degree

12. The number of square units in the interior of a closed region is called the _____ of the region.

a) perimeter
b) area
c) volume
d) altitude
e) grid

13. One figure that <u>always</u> has four congruent angles is a _____.

a) rectangle
b) triangle
c) parallelogram
d) polygon
e) trapezoid

14. The fractions $\frac{2}{3}$, $\frac{1}{9}$, $\frac{7}{11}$, and $\frac{6}{7}$ are all _____ fractions.

a) stretcher
b) mixed
c) basic
d) prime
e) odd

15. The distance around a square is called the _____ of the square.

a) perimeter
b) area
c) volume
d) edge
e) face

GO TO NEXT PAGE

MATCH

For each problem in List A find the answer, locate it in List B, and place it in the blank. Use scratch paper when necessary. Answers may be used more than once.

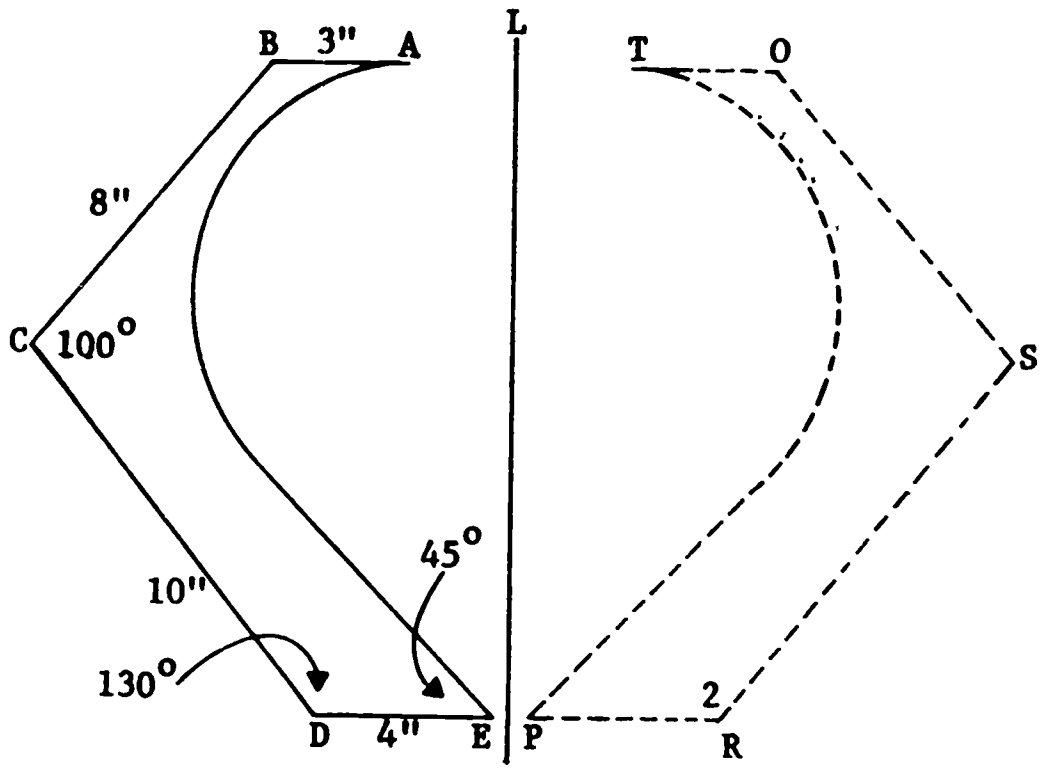
List A	List B
16. _____ 2.0 - 0.4	$6\frac{3}{7}$
17. _____ 1.3 + 0.03	$3\frac{1}{2}$
18. _____ $(\frac{1}{3} \times 21) - 6$	$\frac{6}{9}$
19. _____ $\frac{2}{3} \times \frac{3}{6}$	$\frac{5}{9}$
20. _____ $\frac{4}{5} \times 45$	$\frac{1}{3}$
21. _____ $3\frac{1}{7} \times 21$	33,000
22. _____ $\frac{7}{16} \times (\frac{1}{3} \times \frac{16}{7})$	3
23. _____ thirty-three thousandths	36
24. _____ reciprocal of 3	66
	1
	180
	2.4
	1.6
	1.33
	0.303
	0.033

Work the problems below. Your teacher will tell you where to put your answers.

25. Solve the sentence

$$\frac{21}{15} \times \frac{5}{6} = n$$

and write your answer as a basic fraction.



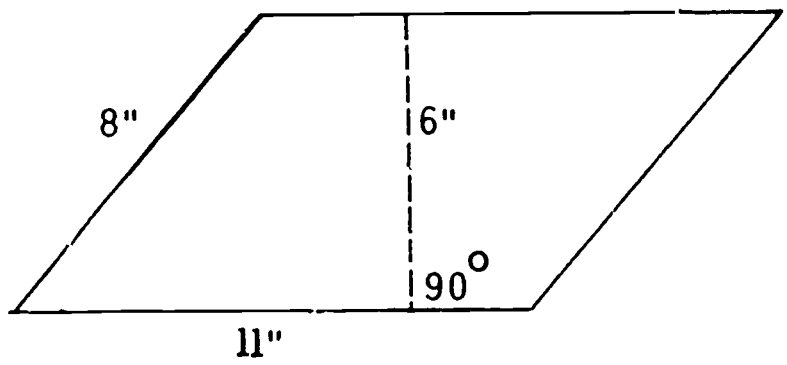
26. When ABCDE is reflected through line L

a. Which letter names the image of point D?

b. How long is the segment OS?

c. How many degrees are in ∠2?

d. If the distance from point C to point S is 21", what is the distance from point C to the line of reflection?



27. Find the area of the above parallelogram. Include units with your answer.

28. Divide: $2342 \div 61$

After you have done the division complete the sentence below.

$2342 = (61 \times _____) + _____$

29. Divide: $72031 \div 9$

After you have done the division write $\frac{72031}{9}$ as a mixed number.

$\frac{72031}{9} = _____$ (mixed number)

DEVELOPMENTAL YEAR RESULTS FOR CHECK-UP 2

Check-Up 2 was administered following Program 30 and 264 papers were scored. The mean score (based upon 100 possible points) was 64.1 and the standard deviation was 21.7. Table D2 shows the percent responding correctly to each item.

TABLE 2D

RESULTS ON CHECK-UP 2 BASED UPON 264 TESTS SCORED

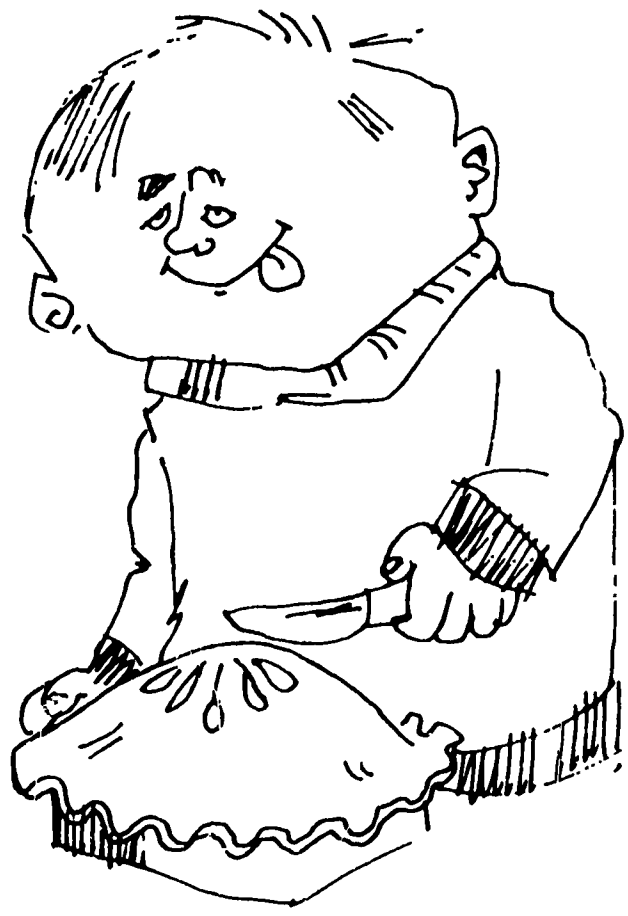
Item	% Correct	Item	% Correct
1	88	17	85
2	98	18	77
3	80	19	74
4	75	20	73
5	74	21	62
6	69	22	66
7	80	23	57
8	33	24	36
9	70	25	22
10	36	26	a) 93 b) 75
11	44		c) 59 d) 63
12	59	27	43
13	56	28	Division 61
14	84		Sentence 64
15	74	29	Division 62
16	78		Sentence 56

Patterns in Arithmetic Check-up 3

$$1 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \cdots + \frac{1}{4}$$

(226 terms)

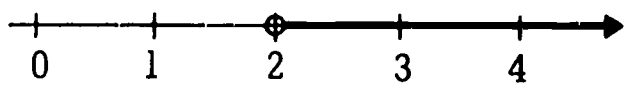
?



Instructions: Find the answer to each problem. Your teacher will tell you where to put your answer.

1. Which choice below shows how to write 973¢ as dollars?

   a) $0.973
   b) $9.73
   c) $97.30
   d) $973.00
   e) $9730.00

2. 

   Which sentence below is represented on the number line graph above?

   a) $n < 2$
   b) $n \geq 2$
   c) $n > 2$
   d) $2 > n$
   e) $2 \leq n$

3. $12 \div \frac{3}{4} = ?$

   a) 36
   b) 48
   c) 9
   d) $11\frac{1}{3}$
   e) 16

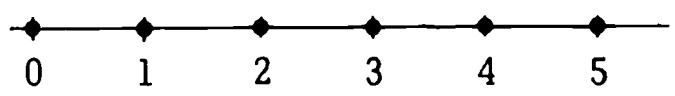
4. To solve the sentence $\frac{2}{3} \times n = 70$

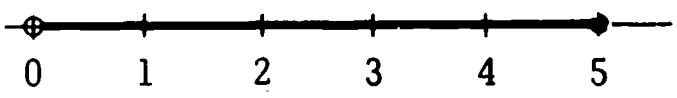
   a) Multiply both sides by $\frac{3}{2}$
   b) Add $\frac{1}{3}$ to both sides
   c) Subtract $\frac{2}{3}$ from both sides
   d) Divide both sides by $\frac{3}{2}$
   e) Add $\frac{1}{3} \times n$ to both sides

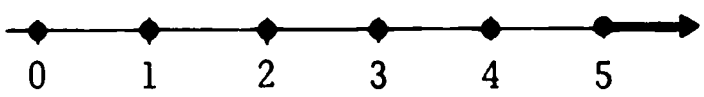
5. Two thirds of all light bulbs tested were bad. If 24 bulbs were bad, how many were tested?

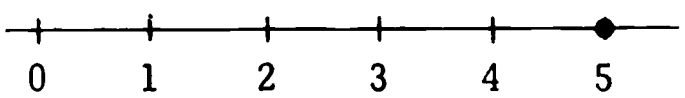
   a) 72
   b) 48
   c) 36
   d) 28
   e) 16

6. Mr. Brown had a leak in a water pipe and placed an empty 5 gallon can under the leak to catch the dripping water. A few hours later he returned to see how much water was in the can. Which number line graph shows how many gallons of water could be in the can?

a)


b)


c)


d)


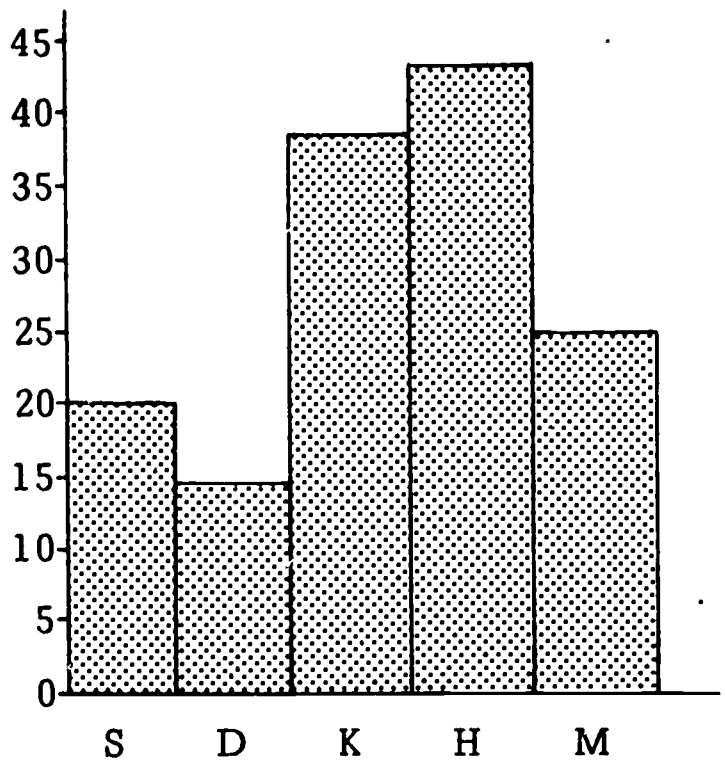
7. 100 x 0.76 = ?

a) 100.76
b) 7.6
c) 76
d) 760
e) 1.76

8. 0.2 x 0.7 = ?

a) 140
b) 14.0
c) 1.4
d) 0.14
e) 0.014

Problems 9 and 10 refer to the graph below.



The graph above shows the average yearly rainfall in inches for San Francisco (S), Denver (D), Key West (K), Hartford (H) and Minneapolis (M).

9. Which city had the greatest amount of rain?

a) San Francisco (S)
b) Denver (D)
c) Key West (K)
d) Hartford (H)
e) Minneapolis (M)

10. What was the average yearly rainfall in Key West (K)?

a) 43 inches
b) 38 inches
c) 33 inches
d) 25 inches
e) 20 inches

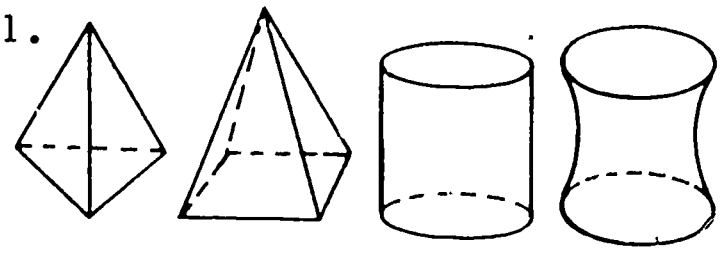
11. 

Fig. a Fig. b Fig. c Fig. d

Each figure above has volume. For which figure could the volume be found by the formula:

Volume = (area of base) x (height)

a) Figure a
b) Figure b
c) Figure c
d) Figure d

---

12. The sentence $n > {}^{-}3$ has many answers. Which choice below is <u>not</u> an answer?

a) $n = 0$
b) $n = {}^{-}1$
c) $n = {}^{+}800$
d) $n = {}^{-}5$
e) $n = {}^{+}3$

---

13. If you divide $\frac{2}{3}$ pie equally among four people, how much would each person get?

a) $\frac{1}{8}$ pie
b) $\frac{1}{7}$ pie
c) $\frac{1}{6}$ pie
d) $\frac{1}{5}$ pie
e) $\frac{1}{4}$ pie

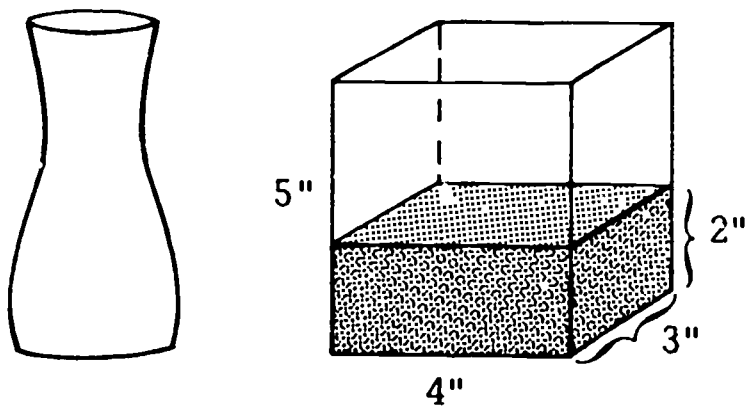
14. $\frac{2}{5} \div \frac{3}{7} = ?$

a) $\frac{14}{15}$
b) $\frac{10}{21}$
c) $\frac{15}{14}$
d) $\frac{5}{12}$
e) $\frac{6}{35}$

---

15. Which sentence below is equivalent to $n - 12 < 3$?

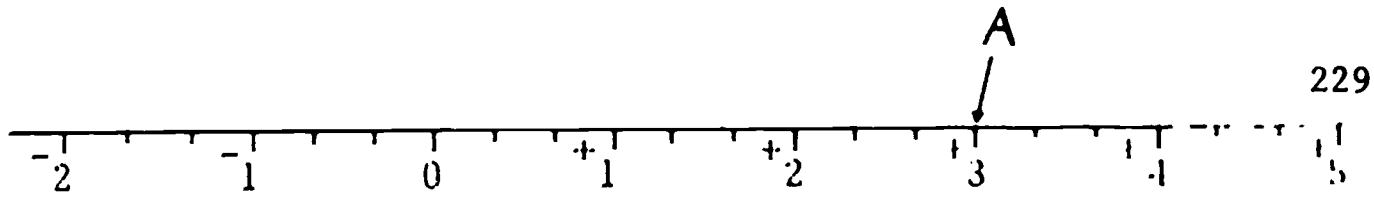
a) $n = 9$
b) $n < 9$
c) $n = 15$
d) $n < 15$
e) $n < 4$



16. To find the volume of the vase above, it was filled with water and the water was then poured into the box. What is the volume of the vase?

a) 12 cu. in.
b) 14 cu. in.
c) 24 cu. in.
d) 36 cu. in.
e) 60 cu. in.

17-21.



Notice how the point A = $^{+}3$ has been indicated on the number line above. In a similar manner indicate <u>carefully</u> and <u>neatly</u> each of the following points:

B = $^{+}5$, C = 0, D = $^{-}1$, E = $^{+}\frac{1}{3}$, F = $^{-}1\frac{1}{3}$

---

22. Which of these numbers is the least: $^{+}5$, 0, $^{-}1$, $^{-}\frac{1}{3}$, $^{-}1\frac{1}{3}$

---

23. Solve the sentence:

$^{+}4 + {}^{-}3 = n$

---

24. Are all positive numbers greater than $^{+}\frac{1}{3}$? Explain your answer briefly.

---

25. A scout troop made money by selling tickets to a circus. They made \$1.05 on each ticket they sold. If they sold 83 tickets, how much did they make?

Sentence _____

Answer _____

---

26. A cake recipe calls for $8\frac{1}{2}$ cups of flour. If Frances wants to make a cake one-third the size of the recipe, how much flour should she use?

Sentence _____

Answer _____

---

27. A board 17 feet long is cut into 3 equal pieces. How long is each piece?

Sentence _____

Answer _____

---

28. Multiply:
$$\begin{array}{r} 4.3 \\ \times \quad 2 \\ \hline \end{array}$$

---

29. Multiply:
$$\begin{array}{r} 2.09 \\ \times \ 1.6 \\ \hline \end{array}$$

---

30. Solve the sentence:

$1\frac{2}{3} \div 6 = n$

DEVELOPMENTAL YEAR RESULTS FOR CHECK-UP 3

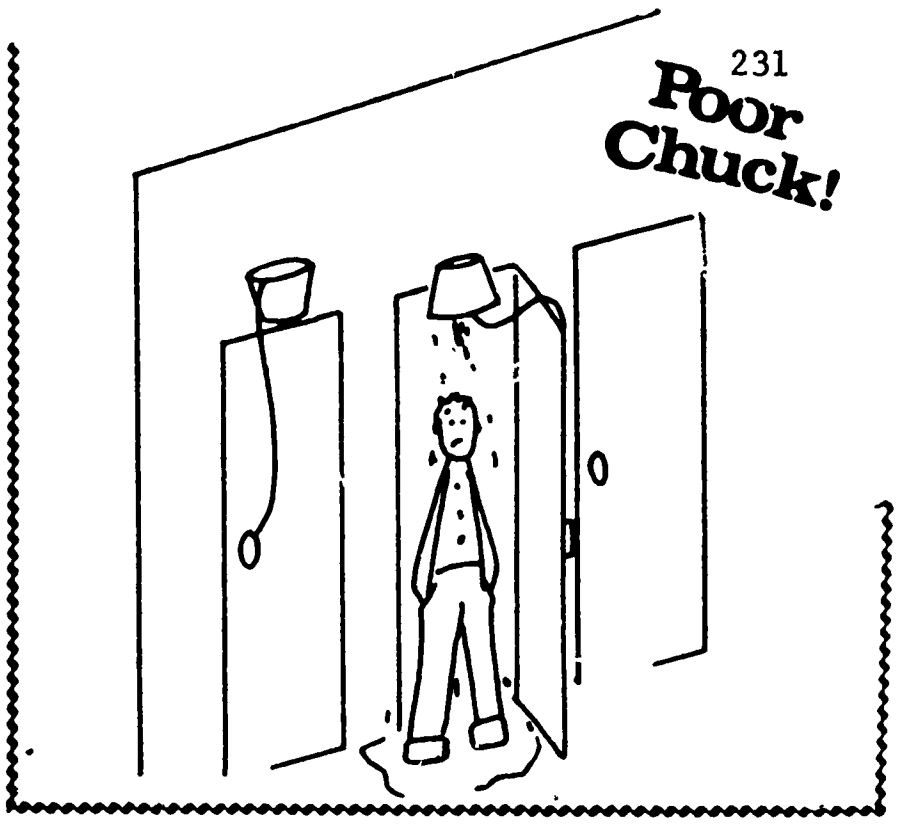
Check-Up 3 was administered following Program 44 and 166 papers were scored. The mean score (based upon 100 possible points) was 68.4 and the standard deviation was 18.4. Table D3 shows the percent responding correctly to each item.

TABLE 3D

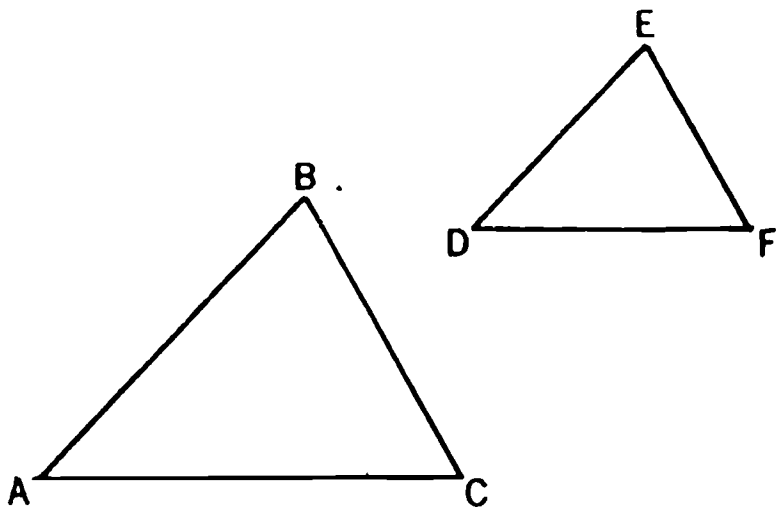
RESULTS ON CHECK-UP 3 BASED UPON 166 TESTS SCORED

Item	% Correct	Item	% Correct
1	83	18	88
2	56	19	92
3	54	20	54
4	64	21	69
5	55	22	82
6	68	23	90
7	49	24	51
8	75	25	Sentence 93
9	99		Answer 69
10	89	26	Sentence 40
11	38		Answer 45
12	75	27	Sentence 88
13	70		Answer 73
14	59	28	92
15	35	29	65
16	63	30	49
17	92		

Patterns in Arithmetic
Check-up 4



Instructions: Find the answer to each problem. Your teacher will tell you where to put your answers.



1. Triangles ABC and DEF are similar. Which angle in triangle ABC corresponds to angle F?

   a) angle A
   b) angle B
   c) angle C
   d) angle D
   e) angle E

2. Which number below is the greatest?

   a) 3.63
   b) 3.08
   c) 3.067
   d) 3.627

3. Which choice gives the same answer as $.3\overline{)1.67}$?

   a) $3\overline{)1.67}$
   b) $3\overline{)16.7}$
   c) $3\overline{).167}$
   d) $3\overline{)167.}$

4. 11.3 x 100 = ?

   a) .1130
   b) 1.130
   c) 113.0
   d) 1130.
   e) 11.300

5. One day an ice cream shop sold 80 ice cream cones. 35% of the cones sold were chocolate. How many chocolate cones were sold?

a) 27

b) 28

c) 35

d) 45

e) 52

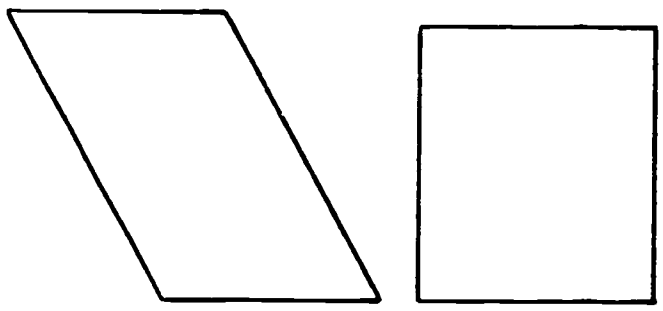


Fig. A Fig. B

6. Is Figure A similar to Figure B?

a) Yes, because both are parallelograms.

b) No, because corresponding angles are not equal.

c) Yes, because each figure has four sides.

d) No, only triangles can be similar.

7. Which number is between 12.6 and 12.7?

a) 12.061

b) 12.073

c) 12.691

d) 12.065

e) 12.600

8. Solve the sentence

k = 5 + (.6 x 7)

a) 9.2

b) 5.42

c) 4.2

d) 10.0

e) 39.2

9. What value of n makes the sentence

2.67 x n = 267 true?

a) .01

b) .10

c) 10

d) 100

e) 1000

10. A bag contains 21 pieces of candy: 13 green and 8 yellow. If you pick a piece from the bag without looking, what is the probability that it will be green?

a) 13

b) $\frac{13}{8}$

c) $\frac{8}{13}$

d) $\frac{8}{21}$

e) $\frac{13}{21}$

11. Which sentence is equivalent to

$142 - n = 30$?

a) $n = 142 + 30$

b) $n = 142 - 30$

c) $142 = 30 - n$

d) $30 = n - 142$

12. During a basketball game a player shot 25 times and made 8 shots. What per cent of the shots did he make?

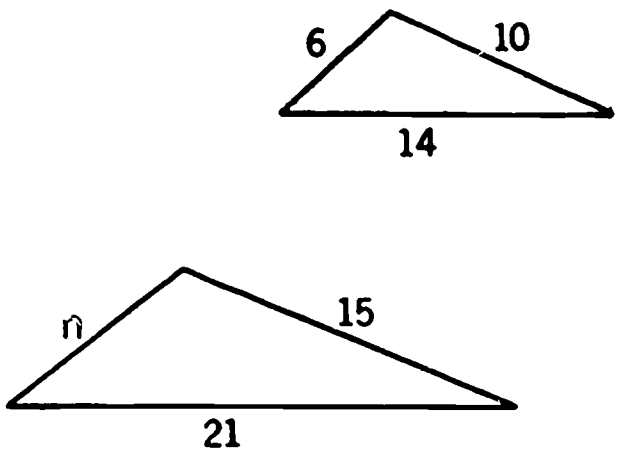
a) 8

b) 25

c) 32

d) 33

e) 68



13. The two triangles above are similar. How long is the side labeled n?

a) 10

b) 9

c) 6

d) 4

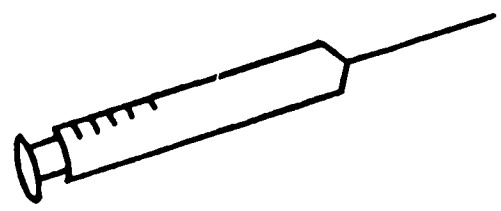
14. How do you write .263 using per cent?

a) .263%

b) 2.63%

c) 26.3%

d) 263%



15. A doctor kept a record of how many shots he gave:

Monday	18
Tuesday	16
Wednesda	26
Thursday	16
Friday	24

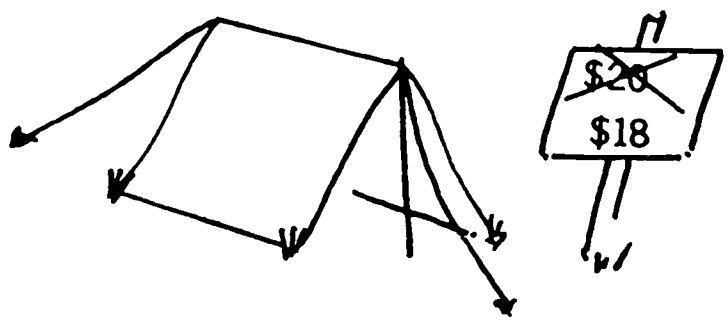
On the average how many shots did he give each day?

a) 16

b) 17

c) 18

d) 20



25. A $20 tent is on sale for $18. By what per cent has the price of the tent been reduced?

Answer _____

26. What is 15% of 70?

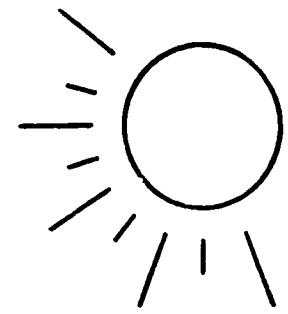
Answer _____

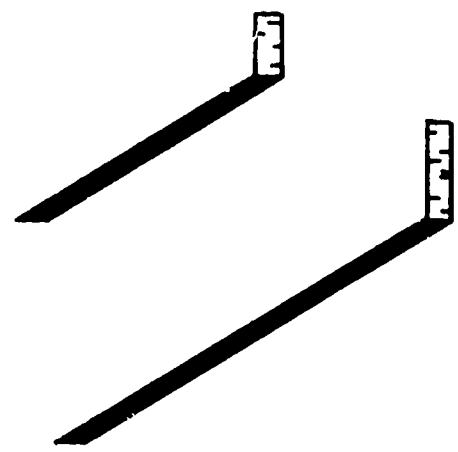
27. Divide: $5.922 \div .47 = ?$

Answer _____

28. If $3^6 = 729$, then $3^7 = ?$

Answer _____





29. A post 2 ft. high casts a shadow 9 ft. long. How long would the shadow of a 3 ft. post be?

Answer _____

Work out the answer for these problems.

16. Divide: $61.5 \div 3 = ?$

Answer _____

17. A box of cereal weighs $4\frac{3}{5}$ oz. How much will 10 boxes weigh?

Sentence _____

Answer _____

18. Write a decimal between 6.23 and 6.24.

Answer _____

19. If the probability of rain is $\frac{7}{10}$, what is the probability that it will not rain?

Answer _____

20. Divide: $0.6 \div 0.3 = ?$

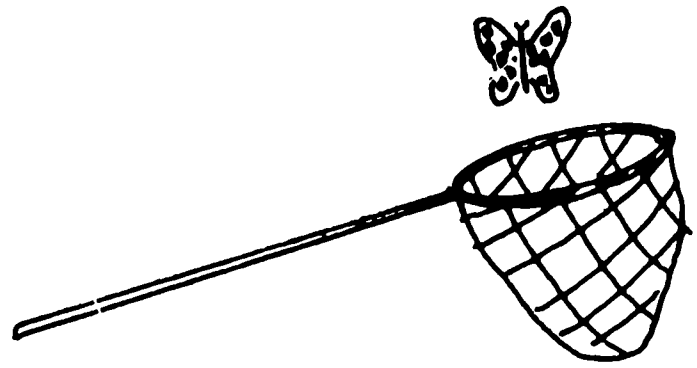
Answer _____

21. Susan took a spelling test of 100 words. She spelled 81 words correctly. What per cent did she spell correctly?

Answer _____

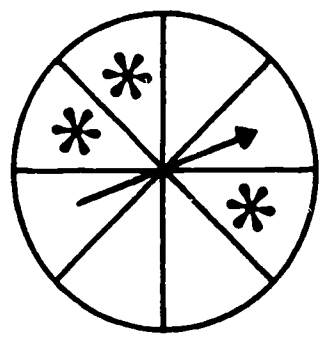
22. Solve the ratio sentence: $\frac{8}{2.4} = \frac{10}{n}$

Answer _____



23. There were 25 sixth graders in Mr. Green's class. They were collecting insects. After 5 weeks 175 insects had been caught. What was the average number of insects caught by each person?

Answer _____



24. Experiment: Flip the spinner. What is the probability that the spinner will stop on a region with a * on it?

Answer _____

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