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AUTHOR SCOTT, JOSEPH A.
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ABSTRACT

THE CHILD'S ABILITY TO IDENTIFY NUMEROUSNESS AS A PROPERTY OF A SET, DISTINCT FROM ALL THE OTHER PROPERTIES OF THAT SET, MAY BE A PREREQUISITE TO CONSERVATION OF NUMEROUSNESS OF SETS. TO TEST THIS THEORY, 11 LESSONS DESIGNED TO DEVELOP THE ABILITY TO IDENTIFY PROPERTIES OF OBJECTS AND OF SETS OF OBJECTS AND TO REPRESENT LENGTH AND NUMEROUSNESS WERE PRESENTED TO KINDERGARTEN AND PRESCHOOL CHILDREN. IT WAS HYPOTHESIZED THAT THIS TRAINING WOULD HELP THE SUBJECTS SCORE HIGHER ON A TEST OF CONSERVATION OF NUMEROUSNESS. THE DIFFERENCE, HOWEVER, BETWEEN EXPERIMENTAL GROUPS, WHO RECEIVED THE TRAINING LESSONS, AND CONTROL GROUPS, WHO RECEIVED NO SPECIAL TRAINING, WAS NOT STATISTICALLY SIGNIFICANT. NEVERTHELESS, SINCE THERE WERE SOME NOTICEABLE DIFFERENCES BETWEEN THE AMOUNT OF IMPROVEMENT IN THE GROUPS, IT WAS CONCLUDED THAT KINDERGARTEN AND PRESCHOOL CHILDREN CAN BE TAUGHT WITH LITTLE DIFFICULTY TO RECOGNIZE, DISCRIMINATE AND LABEL PROPERTIES OF OBJECTS AND SETS, AND THAT THIS TRAINING CAN OFTEN BE ENOUGH TO INCREASE THE CHILDREN'S SCORES ON A TEST OF CONSERVATION OF NUMEROUSNESS. (AUTHOR/MH)

Technical Report No. 92

# THE EFFECT OF SELECTED TRAINING EXPERIENCES ON PERFORMANCE ON A TEST OF CONSERVATION OF NUMEROUSNESS

Report from Phase 2 of the Prototypic Instructional Systems in Elementary Mathematics Project

By Joseph A. Scott

Herbert J. Klausmeier, Professor of Educational Psychology
Chairman of the Examining Committee

Thomas A. Romberg and John G. Harvey, Principal Investigators

Wisconsin Research and Development
Center for Cognitive Learning
The University of Wisconsin
Madison, Wisconsin

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## STATEMENT OF FOCUS

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the imporvement of educational practice.

This Master's Thesis is from Phase 2 of the Project on Prototypic Instructional Systems in Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities, materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

# Chapter I

## INTRODUCTION

Numerousness of sets is among the first basic mathematical concepts to which the elementary school child is introduced. While "numerousness" remains mathematically undefined, analysis of the concept makes it clear that the concept is attained only after an uncounted (at this point at least) number of its attributes and the relationships between them is first attained. Identifying some of these attributes and relationships has been one of the tasks of a group of investigators engaged in the analysis of mathematical concepts at the Wisconsin Research and Development Center for Cognitive Learning (Romberg, Fletcher, and Scott, 1968).

Among the properties, or attributes, of numerousness identified are the following:

1. Numerousness is a unique property of sets.
2. Numerousness identifies the cardinality of a set.
3. Numerousness is a property on which two sets may be compared.
4. Numerousness remains invariant under rearrangement of the elements of a set.

This last property has been called "conservation of numerousness." Originally described and identified by Piaget and Inhelder (1941), it has since been the subject of a number of experimental studies

(e.g. Churchill, 1958; Harper and Steffe, 1968) and has been of some concern to educators in the field of mathematics. To attain the concept of numerousness of sets even in a rudimentary fashion, the elementary school child must have attained to some degree all of the listed attributes, but especially the last. However, Wohlwill (1960) lists conservation as one of the most difficult aspects of the development of "number" and one of the last stages to be reached.

A number of attempts have been made by experimenters to facilitate the attainment of this concept. Churchill (1958), Ito and Hatano (1963), Wallach and Sprott (1964), and Harper and Steffe (1968) were able to show significant differences between experimental and control groups after a period of training. Wohlwill and Lowe (1962) were unable to produce changes between experimental and control groups. In each of the three studies listed, which showed positive results, it is significant to note that a different training technique was used. Churchill trained the children to perform operations such as seriation, matching, ordering, comparing, etc. Wallach and Sprott employed training in the concept of reversals. Harper and Steffe's treatment consisted of training in the concepts of one-to-one correspondence and perceptual rearrangements. Ito and Hatano used repeated counting, and addition and subtraction. The phenomenon therefore appears as little understood and as poorly accounted for as ever.

Piaget states that one of the problems which causes difficulties for the child in the attainment of conservation is his inability to

"decenter" (Flavell, 1963, p. 232) and he lists four steps or stages through which the child must pass before he can conserve.

1. The child centers on a single property within the stimulus situation.
2. The child centers on one property at a time but may shift from one property to another.
3. The child becomes aware that more than one property may be involved and sometimes gives conservation responses. This is the transition period.
4. The child becomes aware of the role of the relevant properties and now gives conservation responses (Flavell, 1963, p. 246).

While the model appears more appropriate to conservation of mass and volume than conservation of numerousness, its potential, especially the centering-decentering aspect, does not appear to have been empirically tested.

By an independent analysis of the concept of "numerousness," through examination of prerequisite concepts and attributes, investigators engaged in the analysis of mathematical concepts at the Wisconsin Research and Development Center for Cognitive Learning arrived at a somewhat different hypothesis, with however, some of the same implications (Romberg, Fletcher, and Scott, 1968).

It is postulated that before the child can attain this concept, at the basic level which will enable him to begin mathematical manipulating he must be able to identify numerousness as a property of a set, and to discriminate this property from all other properties

of that set. Conservation of numerousness is postulated to be the comparing of two sets on the property "numerousness" which is identified before comparison begins, and noting that they are equal on that property, or alternately comparing a set with itself, on the property "numerousness" after rearrangement of the elements and noting the equality. This postulation is not far removed from Piaget's decentering notion. Training the child to be aware of the various properties of an object or set, and training him to be able to focus on any arbitrarily chosen property, could be called decentering.

One of the criticisms which could be aimed at the various studies of conservation is that, although relying almost entirely on verbal communication, the experimenters make no attempt to ensure that they and the subjects are attaching the various labels to the same referents. To assume that a child and an adult respond in the same way to, and have the same referents for "more than," "less than," "same as," etc., is tenuous at best. It is postulated then that the child must learn the labels being used by the teacher or experimenter, and must have the same referent as the teacher or experimenter for these labels, and that this should enhance performance on tests of conservation.

## Statement of Hypotheses

This study is proposed as a test of the following postulation:

a. A group of children (ages 4 - 6) after exposure to a training program consisting of lessons on properties of objects and sets, comparison of objects and sets, and representing objects and sets will attain a higher score on a test of conservation

of numerousness than a similar group not having such training.

b. Children (ages 4-6) receiving this training will attain a higher score on a test which requires them to classify objects, compare objects and sets, and represent objects and sets, than a similar group not receiving such training.

## Subjects

The initial proposal called for the testing and training of 90 kindergarten children attending Sauk Trail Elementary School, Middleton, Wisconsin. Subsequently 48 kindergarten children from Elm Lawn Elementary School in Middleton, and 29 children from Mrs. Cook's Playskool in Maple Bluff, Wisconsin, were used.

## Experimental Materials

### Test Materials

1. A test of conservation of numerousness developed by Harper and Steffe and described in detail in Technical Report No. 38 of the Wisconsin R & D Center for Cognitive Learning (See Appendix A). Instructions were modified in order to test in larger groups.

2. A diagnostic test developed for this study (See Appendix A).

### Training Materials

These consisted of a set of lesson plans together with selected stimulus materials which were presented to the children. The lesson plans and descriptions of the stimulus materials are in Appendix B.

## Experimental Procedure

The initial proposal envisioned the use of 90 subjects in six groups in a modified Solomon Four-Group Design (Campbell and Stanley, 1963, p. 194) as follows:

| | | |
|---|---|---|
| E1 | C1 | C3 |
| Pretest<br>Training<br>Posttest 1 & 2 | Pretest<br>Math Program<br>Posttest 1 & 2 | Pretest<br>-<br>Posttest 1 & 2 |
| E2 | C2 | C4 |
| Training<br>Posttest 1 & 2 | Math Program<br>Posttest 1 & 2 | -<br>Posttest 1 & 2 |

After the Test of Conservation of Numerousness was administered to this group, using the revised instructions, unexpectedly high scores forced the abandonment of this design and the execution of four separate studies.

Exploratory Study 1 was performed to determine if the change in instructions was responsible for the high mean scores. This was necessary before the hypotheses could be tested. The procedure consisted in administering the Test of Conservation of Numerousness ($Test_1$) to 48 students at Elm Lawn Elementary School in Middleton, Wisconsin, using the original instructions for half of the group and the altered instructions to the other half.

Exploratory Study 2a was performed to determine if the students who scored high on $Test_1$ also scored on the Diagnostic Test ($Test_2$). Since ninety subjects at Sauk Trail had already received $Test_1$ and the mean score was too high to make any meaningful test of the hypotheses

it was decided to use this population as the source of this additional information. A stratified random sample of 36 students was chosen from the 90 such that high, medium, and low scorers were equally represented. $Test_2$ was administered to the 36 subjects and a Product-Moment correlation coefficient was obtained from the two sets of scores. A high correlation coefficient here taken jointly with other data would provide supporting evidence for the main hypothesis.

Exploratory Study 2b examined the effects of the training program on the performance of the students with respect to their acquiring specified concepts ($Test_2$). The subjects were thirteen children who had attained the lowest scores on $Test_2$ when it was administered to the 36 students in Study 2a. After the training program, these thirteen were again given $Test_2$.

Exploratory Study 3 examined the effects of the training program on the scores of low scorers on $Test_1$. Sixteen students who had received a score of ten or below when $Test_1$ was intially administered at Sauk Trail received the training program and were again administered $Test_1$. Six students at Elm Lawn School who had attained a score of eleven or below under the same set of instructions (in Exploratory Study 1) were used as a control group. This was a test of the main hypothesis. However, because of the biased sample the results could not be accepted unreservedly.

Experiment 4 then tested the hypotheses. Nursery school children were used. A pretest ($Test_1$) was administered and the class randomly assigned to either the experimental or control group. The experimental group was given the training program. The experimental and control group then received both $Test_1$ and $Test_2$.

## Chapter II

## RELATED RESEARCH

With the renewed interest in Piaget's work and in conservation during the last decade has come a number of studies examining the feasibility of training subjects to conserve. Many of these have been attempts to train Ss to conserve mass or volume. A few have attempted to train Ss to conserve numerousness (often called number). Various techniques have been employed, none with any singular success.

One of the best-known series of studies of conservation is that conducted by Smedslund. Training was attempted in several of the studies. In one study (Smedslund, 1961a) three groups of Ss were used. The first was given "direct reinforcement of conservation of weight over deformation." This involved having Ss weigh balls of plasticene, having the balls deformed, and then weighing them again, to see if the weight was still the same. Treatment for the second group consisted of Ss weighing the plasticene shapes, watching E remove or add some clay and then having S reweigh the balls. The third group, the control group, received no treatment. The results showed that none of the experimental conditions were sufficient for the acquisition of conservation of substance and weight.

In another study Smedslund (1961b) tried to make the child distrustful of immediate sensory clues, so that he might search for other,

more reliable criteria. Eleven six-year-old Ss were given pretest for conservation of substance and weight and for transitivity of weight. The training task which Ss were given involved their weighing pairs of balls of clay. The balls were so arranged that the larger ball usually weighed less than the smaller balls, or when the balls were the same size there was a large difference in weight. There was one session per day for three consecutive days. A posttest was given on the fourth day. No S changed from non-conservation responses to stable conservation responses as a result of the training. Three Ss changed to one correct response on the posttest on conservation of weight.

Smedslund (1961c) described a study in which he tried to produce a number of conflict situations for the child. This was done by asking the child which weighed more or had more substance, the ball or the sausage (deformed ball). A piece of clay was removed from or added to one of the balls. Both balls were then weighed so that S could see that the one with the piece added was heavier or that the one from which a piece had been removed was lighter. This was to reinforce the utilization of addition/subtraction concepts. Thirteen children participated. All had scored zero on a pretest for conservation. Thirty-six trials were run, twelve per day for three consecutive days. Five Ss began giving conservation responses. The other eight did not.

Somewhat greater success was experienced by experimenters attempting to induce conservation of number (numerousness). Churchill (1958) trained an experimental group of children to perform operations such as seriation, matching, ordering, comparing, etc. The group received two

sessions per week of guided play. The results of a posttest showed the experimental group was able to perform significantly better at the "operational level" than the control group. A second retest three months later showed the experimental group still significantly better than the control.

Wohlwill and Lowe (1962) compared the performance of four groups. The first group participated in counting acitvities involving equivalent sets. The second group received a similar treatment and in addition performed operations of addition and subtraction of elements of two sets. The third group was given much practice in noticing that the cardinal value of a set is unchanged regardless of the length of rows into which the objects were placed. The fourth group, the control group, received no treatment. No significant differences were found between the experimental groups or between the experimental groups and the control group.

Ito and Hatano (1963) used three training methods in an attempt to induce conservation. The first involved the repeated confirmation of the invariance of a set by having Ss repeatedly count the set following rearrangement. The second method involved inference based on the change of quantity by addition or subtraction. The third treatment was called "inference based on comprehension of number relations."

This treatment varied from subject to subject as E asked S questions. The three groups showed "remarkable progress" between pre- and posttests. Most gain scores from pre- to posttest were significant. The groups were not, however, compared with each other. Three posttests were administered, the first test on the day after training; the second, a week after training; and the third, two months

after training. There was a slight decline in the scores on the third posttest.

Feigenbaum and Sulkin (1964) examined methods of teaching the principles of correspondence and conservation. Two treatments were used:

a. Reduction of irrelevant stimuli in which Ss were blindfolded, given beads in pairs, and asked to drop one into each of two jars.

b. Reinforcement by addition and subtraction in which Es added or subtracted a bead to/from a pile.

Fourteen out of thirty achieved conservation under Treatment A, while only three out of seventeen achieved conservation under Treatment B. On a retest one week later, ten of the fourteen and two of the three still conserved.

Wallach and Sprott (1964) report an experiment in which a group of subjects were given extended practice with reversibility. Stimulus material consisted of dolls in beds. The dolls were taken out and arranged in different configurations. Ss were then asked if there were as many dolls as beds, more of one than the other, etc. Dolls were then returned to the beds. The situations were repeated until Ss made the correct prediction. The result showed fourteen out of fifteen experimental subjects giving conservation responses on a posttest. A second posttest several weeks later showed the experimental group still scoring significantly better than the control group.

Wallach, Wall, and Anderson (1967) found that six- and seven-year-old children who regarded the number of objects as changing when the set

was rearranged were induced to conserve number by a procedure involving experience with reversibility. Another experience involving addition and subtraction had no effect. The authors argued that recognizing reversibility and not using misleading perceptual cues are both necessary for conservation.

Harper and Steffe (1968) used kindergarten and first grade students. In a six-week training program, twelve half-hour presentations were made. The following concepts were presented: one-to-one correspondence, perceptual rearrangement, as many as, fewer than, addition and subtraction. These concepts were presented to groups in classroom situations by a teacher. The students participated in games and other appropriate activities. The experimental kindergarten group performed significantly better than the kindergarten control group on a posttest. No significant difference was found between experimental and control groups at the first grade level.

## Comments on the Literature Reviewed

The sample of studies reviewed in the previous section is obviously not exhaustive, but is, it is hoped, adequate to make several problems evident. First, it was noted that there has been somewhat more success in attempts to train children in the conservation of numerousness, than in attempts to have children conserve volume, mass, etc. The fact that there seems to be more difficulty in conservation of mass than in conservation of numerousness would suggest that the problem lies not in the subject but in the stimulus material. Second, and more pertinent to this study, it was observed that in the studies where conservation

was successfully induced, experimenters used different techniques. In addition, it was noted that experimenters reporting no success in inducing conservation were using techniques which had been found successful by other experimenters. Table 1 may help to illustrate some of the conflict.

Apparently then, the phenomenon is neither clearly understood nor explained. Several possibilities exist and could be postulated as explanations of the results.

a. The success or failure is due to the experimenter only and not to any of the training techniques.

b. The successes occurred in situations where the child was forced or induced to concentrate on the problem and the training technique was incidental to the success.

c. No single training technique is sufficient. Combinations of techniques or interactions between the technique and the experimenter are necessary.

d. Conservation is a phenomenon that can be induced by many different techniques.

e. All of the successful techniques or experiments include an unidentified common denominator which induces conservation.

f. Nonstandardized tests make the results suspect.

It is believed that all of these and other postulations have some merit and perhaps could be tested. Postulation E appears to be one particularly worthy of examination. It can be said without reference to any of the studies that language, including instructions, is a

Table 1

Training Techniques Used in Successful and Unsuccessful Attempts to Induce Conservation in Children

| | Training Technique Used (and Reference) | | | | |
|---|---|---|---|---|---|
| Conservation Induced | Counting (Ito & Hatano, 1963) | Addition and Subtraction (Ito & Hatano, 1963; Harper & Steffe, 1968) | One-to-One Correspondence (Harper & Steffe, 1968) | Reversibility (Wallach, Wall, & Anderson, 1967) | Removal of Visual Clues (Feigenbaum & Sulkin, 1964) |
| Conservation not Induced | Counting (Wohlwill & Lowe, 1962) | Addition and Subtraction (Wohlwill & Lowe, 1962; Feigenbaum & Sulkin, 1964; Wallach, Wall, & Anderson, 1967; Harper & Steffe, 1968) | | | |

common factor overlapping all studies, and one which allows the greatest possibilities for variation both between and within studies. Another source of variation from which some common factors could emerge is the background and prior training of the children. Still other sources of variation which could produce common factors to elicit sporadic successes are amount of time spent in training the children, age of children, and length of time spent in school. Several of these factors would be extremely hard to control. Language used in the instructions for the various tests could, however, be more standardized and this alone might reduce some of the variation being reported.

The present study seeks to reduce some of the problems suggested by these postulations. The training seeks to establish common referents for labels in both the experimenter and the subjects. Secondly, it attempts to provide a common set of experiences for the children and to provide a common perspective for them in their approach to the type of problem involved in producing conservation responses.

## Chapter III

## EXPERIMENTAL MATERIALS

### Tests

Two tests were used in the series of experiments: 1) a test of conservation of numerousness and 2) a diagnostic test to determine if the children could perform the operations of classifying, comparing objects and sets, and representing objects and sets.

1. The test of conservation of numerousness used was that developed by Harper and Steffe at the Wisconsin Research and Development Center for Cognitive Learning. It consists of twenty problems in which the child is asked to compare two sets of objects and state which has more. The objects are squares and dots. Sets of squares are compared with other sets of squares in five problems. A set of dots is compared with a set of dots in five problems. In the remaining ten problems a disc is placed on each square (or dot), then the discs are moved to cover a set of dots (or squares). The child is then asked to indicate by pointing whether there are more discs or squares, or if there is the same number of both (Harper and Steffe, 1968). The first four problems in this test were used as practice problems. Only the last sixteen, therefore, were scored. In order to test a larger group of

children at one time the instructions were changed in three ways.

a. Children were asked to write an X beside the row or the page which had more objects. If the rows or pages had the same number of objects, the child was asked to put an X on both pages.

b. Redundancy was introduced at many points, e.g. instead of asking "which has more?" the question, "which row has more dots?" etc., was used.

c. Because of what this experimenter felt was ambiguity in the instructions in the problems using the discs, the question on these was changed, and the children were asked if there were more dots than squares rather than if there were more discs than squares.

Children were tested in groups of up to twelve.

The complete set of both the original and revised instructions are included in Appendix A. Throughout the remainder of this report, this test will be referred to as $Test_1$.

2. The diagnostic test consisted of six problems posed to the child. The first asked the child to state a property on which a group of objects had been classified. The second asked him to reclassify the group on the basis of another common property. The third asked him for a decision on which of two objects were longer. The fourth asked him to represent two lengths and to compare the representation. The fifth required him to compare sets and the sixth to represent one set of objects and compare the representation with another set of objects.

The complete set of instructions and description of materials used for each question is included in Appendix A. Throughout the remainder of this report, this will be referred to as $Test_2$.

Training Materials

The training material consisted of eleven presentations, each emphasizing a different concept or process as follows:

Presentation 1. An introduction to describing objects by listing their properties.

Presentation 2. Examining objects and noting their properties.

Presentation 3. Further practice on examining objects and describing properties.

Presentation 4. Classifying and grouping objects according to properties.

Presentation 5. Use of classifying rule.

Presentation 6. Comparing objects on property length.

Presentation 7. Representing length.

Presentation 8. Number is a property of sets.

Presentation 9. Comparing (matching) sets on the property number.

Presentation 10. Representing sets and comparing representations.

Presentation 11. Comparing sets on two or more properties.

Two kinds of materials were used throughout the program.

a. Material specifically prepared and designed to emphasize or focus attention on specific processes, properties, or concepts.

b. Incidental materials in the environment, e.g., the children themselves, their clothes, and objects and furniture in the room.

The prepared materials were the following: for the sections on properties and classifying, the children first described a child's stuffed animal, then a ball and a cube, and finally described and classified pieces of construction paper varying on four dimensions, length, width, color, and dotted or not dotted. The section on comparing used a set of plain wooden blocks varying only on length and produced under the commercial name Number Relation Blocks. Representing length was done with plastic link chains produced commercially under the name of Lots-of-Links. Comparison of sets used construction paper cut-outs which varied on shape, color, and size, the number being different for each dimension. Representation of numerousness was accomplished with felt cut-outs and plastic discs.

The incidental materials used included classifying children on the basis of color of hair, apparel; comparing heights of children; representing waist sizes; etc.

Complete text of the lesson plans and description of the materials is shown in Appendix B. Throughout the remainder of the report, it will be referred to as the Training Program.

## Chapter IV

## EXPLORATORY STUDY 1

### Subjects

Subjects were 48 kindergarten students attending Elm Lawn Elementary School in Middleton, Wisconsin, 23 of whom were in a morning class and 25 in an afternoon class. This was the entire kindergarten class. All children had been at least 5 yrs. old and not yet 6 yrs. old on the previous September 30th.

### Procedure

The children in the morning and in the afternoon groups were randomly assigned to one of two groups. Thus, 11 morning and 13 afternoon children were assigned to Group 1 and 12 morning and 12 afternoon children were assigned to Group 2.

$Test_1$ was administered to Group 1 using the method reported in Technical Report No. 38 of the Wisconsin Research and Development Center for Cognitive Learning and using the instructions indicated in that report. $Test_1$ was administered to Group 2 using revised instructions. For Group 2 the test was administered to a group of 12 students at one time. The test under both conditions were administered in the school library. The children were seated at tables or on the floor in various parts of the room with barricades between them. The instructions were read by the same person to both groups. For Group 1 the experimenter noted and recorded

responses on a score sheet. Group 2 recorded the responses in their booklets.

Results

There were two blocks within each treatment (morning and afternoon groups). A two-way ANOVA, computed by the Finn Multivariate Program (1967) was used to analyze the data. The mean for each group was as follows: Group 1 morning, 10.00; Group 1 afternoon, 9.64; Group 2 morning, 12.25; and Group 2 afternoon, 13.58. The $F$ ratio for Factor A (morning vs. afternoon groups) was 0.241, ($p < .63$). The $F$ ratio for Factor B (Group 1 vs. Group 2) was 5.96, ($p < .02$). The $F$ ratio for Factor AB was .46, ($p < .50$).

Figure 1 shows the distribution of scores for the group receiving the revised and that received the original instructions. Table 2 is the ANOVA Table.

Table 2

Analysis of Variance of Scores for Morning and Afternoon Groups under Original and Revised Instructions

| Source of Variation | SS | df | MS | F | p < |
|---|---|---|---|---|---|
| A | 4.540 | 1 | 4.54 | 0.241 | .626 |
| B | 112.388 | 1 | 112.388 | 5.960 | .091* |
| AB | 8.609 | 1 | 8.609 | 0.457 | .503 |
| ERROR | 829.840 | 44 | 18.86 | | |
| TOTAL | 955.377 | 47 | | | |

* $p < .05$

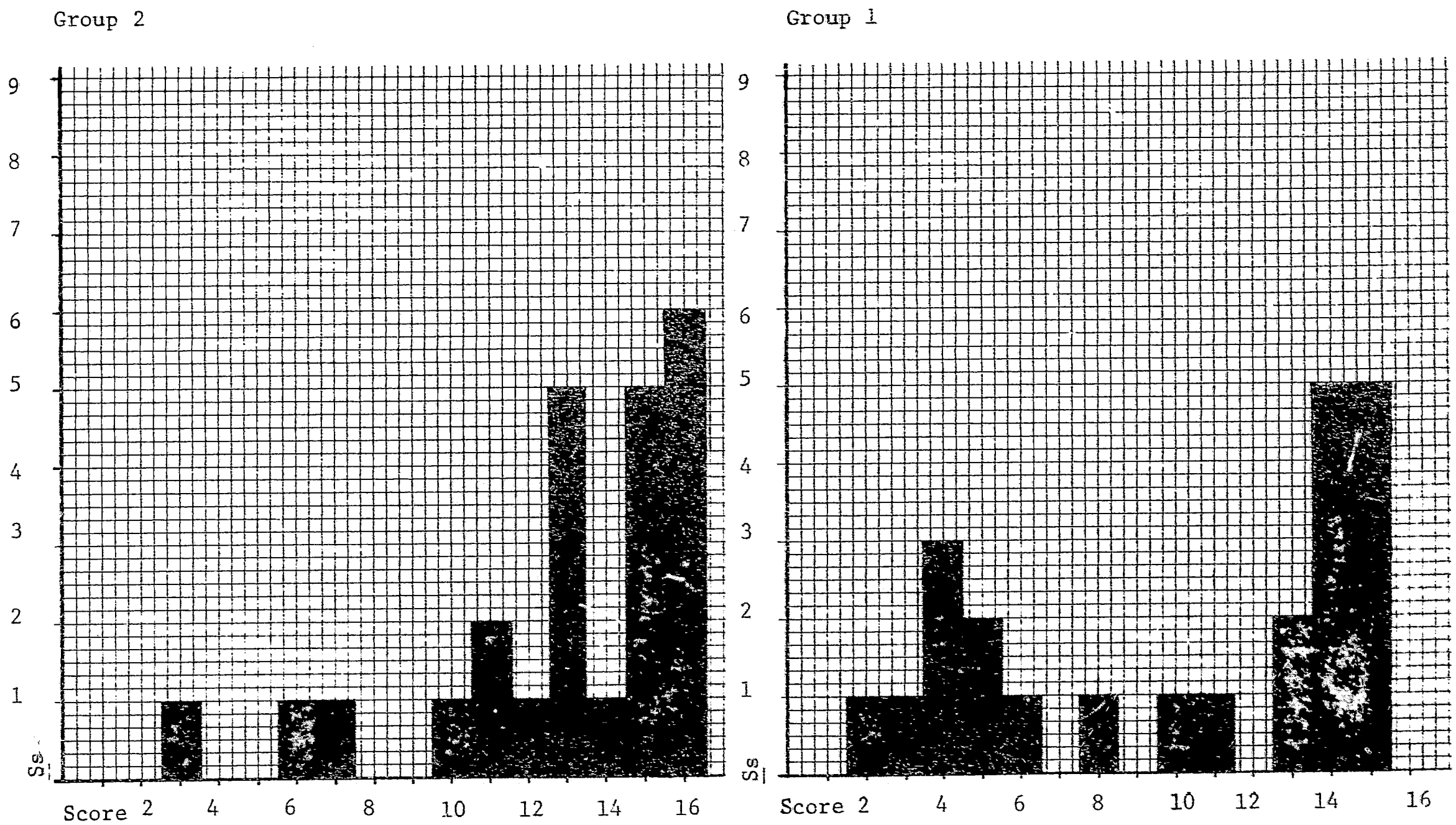


Fig. 1 Distribution of Scores under Revised (Gp 2) and Original (Gp 1) Instructions.

Chapter V

# EXPLORATORY STUDY 2

## Study 2a

### Subjects

Thirty-six students were chosen from among the 90 who had been administered $Test_1$ in Sauk Trail School. The 36 represented both high, medium, and low scorers. The group of 90 was divided in 3 groups of 30 each. The high scorers being in the first group, the medium scorers in the second, and the low scorers in the third. Twelve Ss were then randomly chosen from each group. All children were over age five but not yet age six, on the previous September 30.

### Procedure

The children were individually administered $Test_2$ as described in Appendix B in order to determine their ability to classify, compare objects and sets, and represent objects and sets. A product moment correlation coefficient was then obtained from the two sets of scores ($Test_1$ and $Test_2$) of these 36 children.

### Results

The correlation between the $Test_1$ and $Test_2$ scores for the sample of 36 Ss chosen was .33. This is significantly different from zero correlation ($p < .05$) according to a Fisher Z transformation. Figure 2 shows the correlation of the individual scores.

## Study 2b

### Subjects

Thirteen students from the 36 who were administered $Test_2$ in Study 2a received a score of 6 or below (out of eight). This group of thirteen was used as subjects in the present study.

### Procedure

The children having received $Test_2$ in Study 2a (pretest) were immediately given the Training Program. In completion of the training program these students were again given $Test_2$. (posttest)

### Results

The mean pretest score for the group was 4.38. The mean posttest score was 6.85. I matched pairs $t$ test (Hays 1963, p.333) was used to test the null hypotheses that the $\mu$ of the pretest and posttest were not different. The t score was 3.37 with 12 degrees of freedom. The null hypotheses was rejected ($p < .005$).

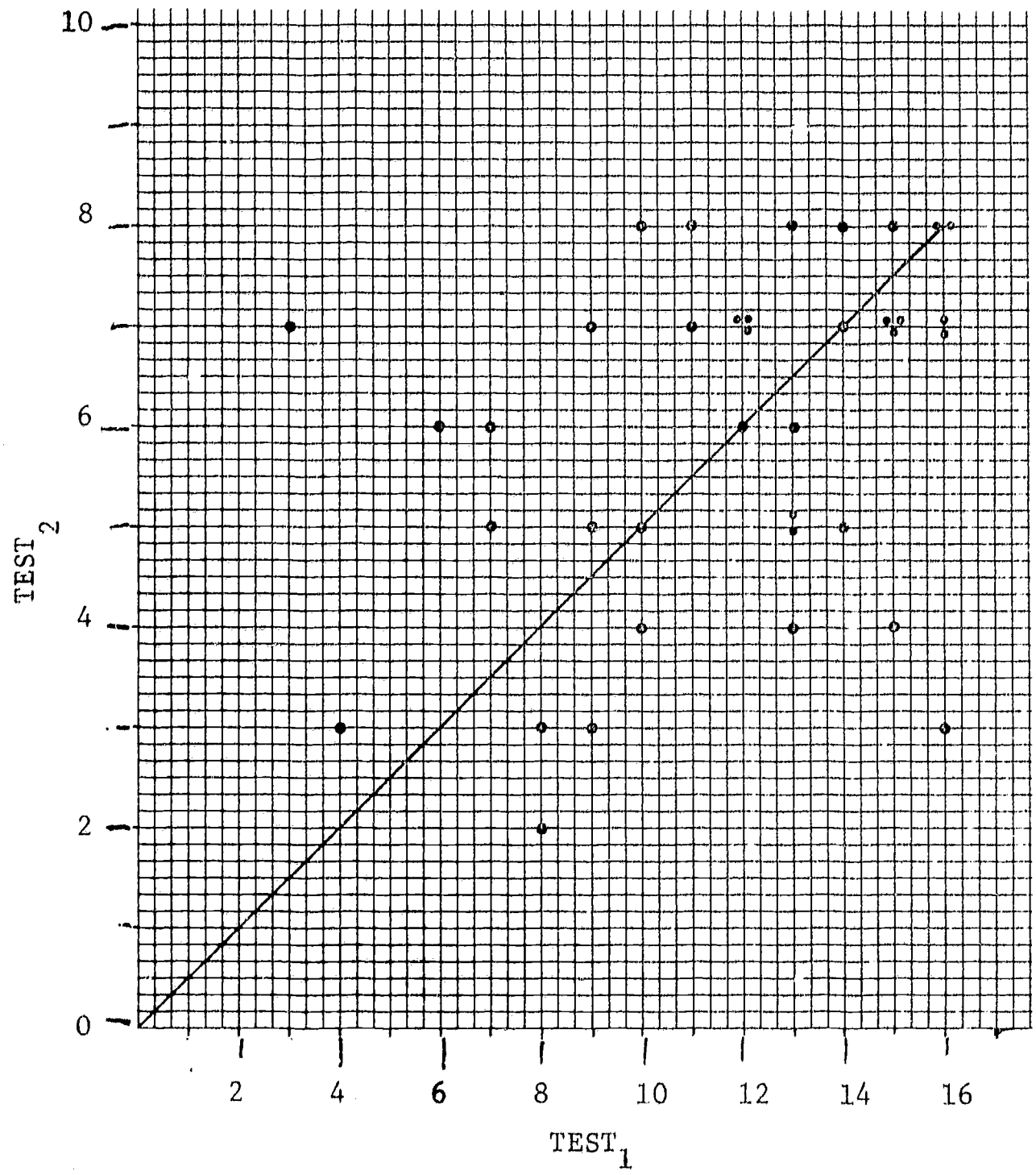


Fig. 2 Distribution of scores for each S on $Test_1$ and on $Test_2$. (Exploratory Study 2a)

## Chapter VI

## EXPLORATORY STUDY 3

### Subjects

Subjects were children enrolled in the kindergarten of Sauk Trail Elementary School, Middleton, Wisconsin. All of the children chosen had obtained a score of 10 or below, out of a possible 16 on $Test_1$, when it was initially administered to the 90 children in the kindergarten class.

### Procedure

The pretest ($Test_1$) was given to all children in the two morning and two afternoon kindergartens. The Training Program (Appendix B) was administered to all students in the two morning and the two afternoon kindergartens. A posttest ($Test_1$) was then administered to the group of students who had scored at or below 10.

In order to provide a control group for this experiment, those students who were tested during Exploratory Study 1 under the revised instructions condition, and who had scored at or below 11 were used. These students were given the posttest ($Test_1$) after a time interval of approximately one month.

### Results

The mean pretest score of the experimental group was 8.06. The mean pretest score of the control group was 8.00. The groups were

considered sufficiently similar to permit comparison of the posttest scores. The posttest score for the control group was 8.5. A *t* test for unmatched pairs was used to test the null hypothesis that the μ of the experimental and control groups are not different. The t-score was 1.51 with 20 degrees of freedom. This was insufficient ($p < .10$) to reject the null hypothesis at the chosen alpha level, $p < .05$. Table 3 presents the summary data for this experiment.

Table 3

Summary Statistics for Exploratory Study 3

| | N | Pretest $\bar{X}$ | Posttest $\bar{X}$ | Gain | Pretest s | Posttest s |
|---|---|---|---|---|---|---|
| Group 1 | 16 | 8.06 | 11.38 | 3.32 | 1.96 | 3.69 |
| Group 2 | 6 | 8.00 | 8.50 | .50 | 2.94 | 4.11 |

| | No. of *Ss* whose scores were Increased | Decreased | Unchanged |
|---|---|---|---|
| Group 1 | 14 *Ss* | 2 *Ss* | 0 *Ss* |
| Group 2 | 2 *Ss* | 2 *Ss* | 2 *Ss* |

## Chapter VII

## EXPERIMENT 4

### Subjects

Subjects in this experiment were 29 preschool children attending Mrs. Cook's Playskool at Maple Bluff, Wisconsin. Children in this experiment were aged 4-5 1/2 years old. 19 were male, 10 were female.

### Procedure

A pretest ($Test_1$) was administered to the 41 children attending the school. Twelve were unable or unwilling to respond. Nine of these, including children from all age levels were either shy or afraid. Three children under four years of age appeared unable to follow the instructions. The remaining 29 were divided into experimental and control groups. The experimental group received the Training Program. The control group received no treatment. At the end of the training period both groups were given two posttests: $Test_1$ and $Test_2$.

### Results

The data for this experiment was analyzed by the Finn (1967) Multivariate Program at UWCC; hence, $F$ ratios rather than $t$ are reported. The pretest score on $Test_1$ for the experimental group was 7.08, for the control group 7.00. These were not significantly different ($p < .90$). The posttest score on $Test_1$ for the experimental group was 8.83, for the control group 7.50. A univariate analysis of the difference scores was used to test the null hypothesis. The $F$

ratio was 3.4769, with 1 and 20 degrees of freedom. This was insufficient ($p$ <.07) to reject the null hypothesis at the chosen alpha level, ($p$ <.05).

The posttest score on $Test_2$ for the experimental group was 4.9, for the control group 3.2. A univariate ANOVA was used to test the null hypothesis that the $\mu$ of the experimental and control groups are not different. The $F$ ratio was 9.3139 with 1 and 20 degrees of freedom. The null hypothesis was rejected ($p$ < .0063).

The correlation coefficient for posttests, $Test_1$ and $Test_2$, was .62 for the entire group. Using Fisher's Z transformation, the coefficient is significantly different from zero ($p$ < .01). For the experimental group the correlation coefficient was .53, significantly different from zero correlation ($p$ <.05), and for the control group the correlation was .83, significantly different from zero correlation ($p$ <.001), using the Fisher Z transformation. The scores for individuals on $Test_1$ and $Test_2$ are shown in Figure 3 for the experimental group and in Figure 4 for the control group. Summary statistics for the experiment are shown in Table 4.

Table 4

Summary Statistics for Experiment 4

| | N | $Test_1$ Pretest $\bar{X}$ | $Test_1$ Posttest $\bar{X}$ | $Test_2$ $\bar{X}$ |
|---|---|---|---|---|
| Experiment Group | 12 | 7.08 | 8.83 | 4.9 |
| Control Group | 10 | 7.00 | 7.50 | 3.2 |

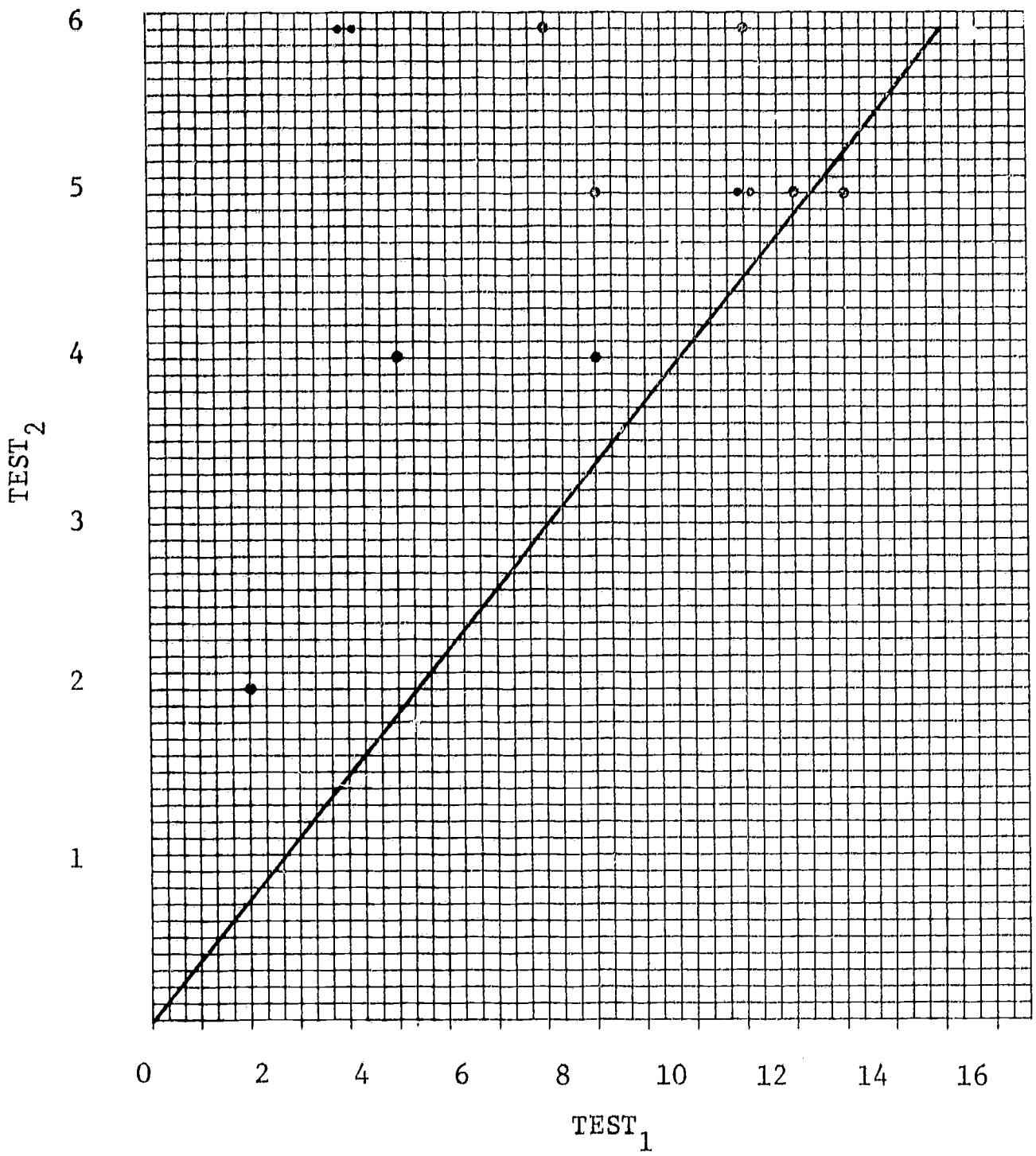


Fig. 3 Distribution of scores for each S on $Test_1$ and $Test_2$. (Experimental Group, Exp.4.)

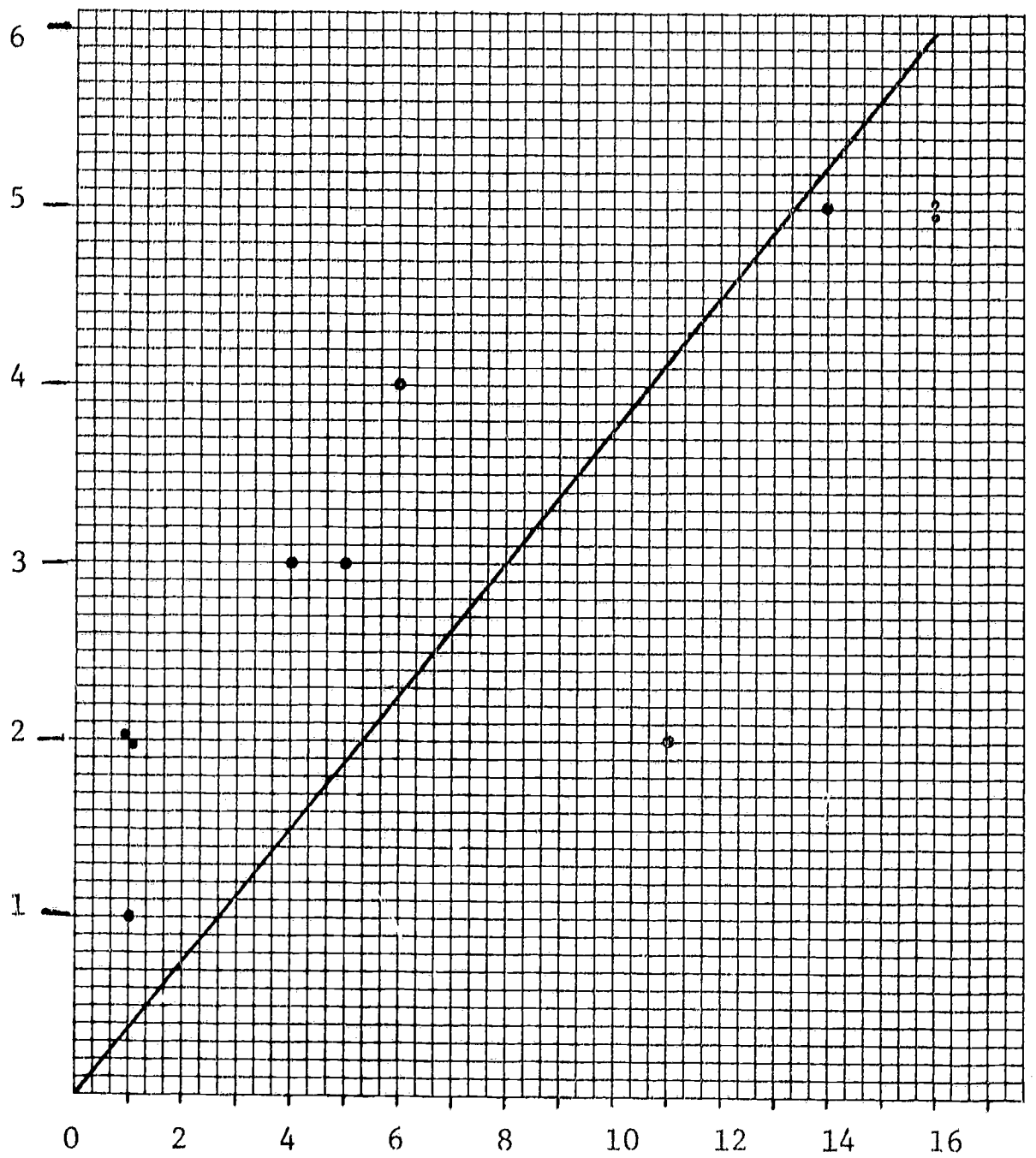


Fig. 4 Distribution of scores for each S on $Test_1$ and $Test_2$. (Control Group, Exp. 4.)

## Chapter VIII

## DISCUSSION AND CONCLUSION

### Discussion

The results cited in the previous chapters appear to support the hypotheses tested. The postulation that children receiving a Training Program consisting of lessons on properties of objects and sets, comparing objects and sets, and representing objects and sets, would score higher than a control group on a test of conservation of numerousness appears to be supported by Exploratory Study 3 and Experiment 4, the results of which though not significant at the chosen alpha level of .05 were, nevertheless, in the anticipated direction. The second postulation, i.e., that children receiving this program would score higher on a test which required them to classify objects, compare objects and sets, and represent objects and sets, than a similar group not receiving the training, is supported by the results in Experiment 4.

Exploratory Study 1 demonstrates the effect of language and instruction on performance on this test of conservation of numerousness. The results of the correlation tests in Exploratory Study 2a and Experiment 4 were somewhat equivocal. There does not, however, seem to be a necessary relationship between conservation scores and the concepts tested by $Test_2$.

Questions arise, however, as to why the results are not more clearcut. There were obviously many uncontrolled variables and we may speculate as to their roles. In addition, there are other fac-

tors not apparent in the design or data which undoubtedly affected performance. The most important of these factors is probably the length of the training program. A six-week program was initially proposed, consisting of approximately the same set of lessons, but with one hour, (three twenty-minute sessions) devoted to each, instead of one twenty-minute session. The condensed version of the program, necessitated by administrative and scheduling problems, contained in the experimenter's opinion, too much expository material and not enough activities. This was most painfully apparent in the nursery school. Another problem resulted from the arrival of summer weather. Again, it was most apparent in the nursery school where the experimental group was not happy at being inside, while their playmates romped on the swings and in the sandboxes. A third factor which undoubtedly had its affects was the following. In order to compare any two sets, in addition to choosing the property numerousness on which to compare them, one of two techniques must be used. The child must either count the objects in each set and make his decision on that basis or he must use one-to-one correspondence of the elements in the set. Few of the children in the nursery school could count. Those with low scores in the kindergarten also appeared to have difficulty in counting. An attempt to teach them one-to-one correspondence, consisted of only one twenty-minute lesson. This was enough for children who had had no prior experience with the technique. Given these limitations, it is felt that were the full six-week program administered to the children, the differences between the experimental

and control group would be more obvious. Even with the limitations it is interesting to note that out of a total of 28 children in the experimental groups, 22 of these increased their score, while 1 remained the same and 5 decreased. In the control group on the other hand 8 increased their score, 2 remained the same, and the score of 8 decreased.

In examining the correlation data an interesting if unexplainable pattern emerged. (See Figure 5.) The correlation coefficient decreases as the group receive more training. The reason for this is not clear, but may be an artifact of the greater within group variation.

One more comment is in order about the conservation phenomena in general. It was noted in an earlier section that many different techniques have been used successfully to induce conservation. This study suggests that there is still another. Beyond this it must be noted that many subjects in control groups receiving no training increase their conservation scores. And, finally, for many years children in schools throughout the world have attained conservation of numerousness, mass and volume without any of the training programs listed.

In the face of these circumstances it seems that one reasonable conclusion is that conservation as a phenomenon is a mere artifact of the particular sequence of concepts to which the child has been exposed. Or perhaps more accurately stated, it is a consequence of the series of concepts to which the child has been exposed and of the parallel of series of concepts which were omitted from his training. Training programs such as the present one, then, only serve to expose the child

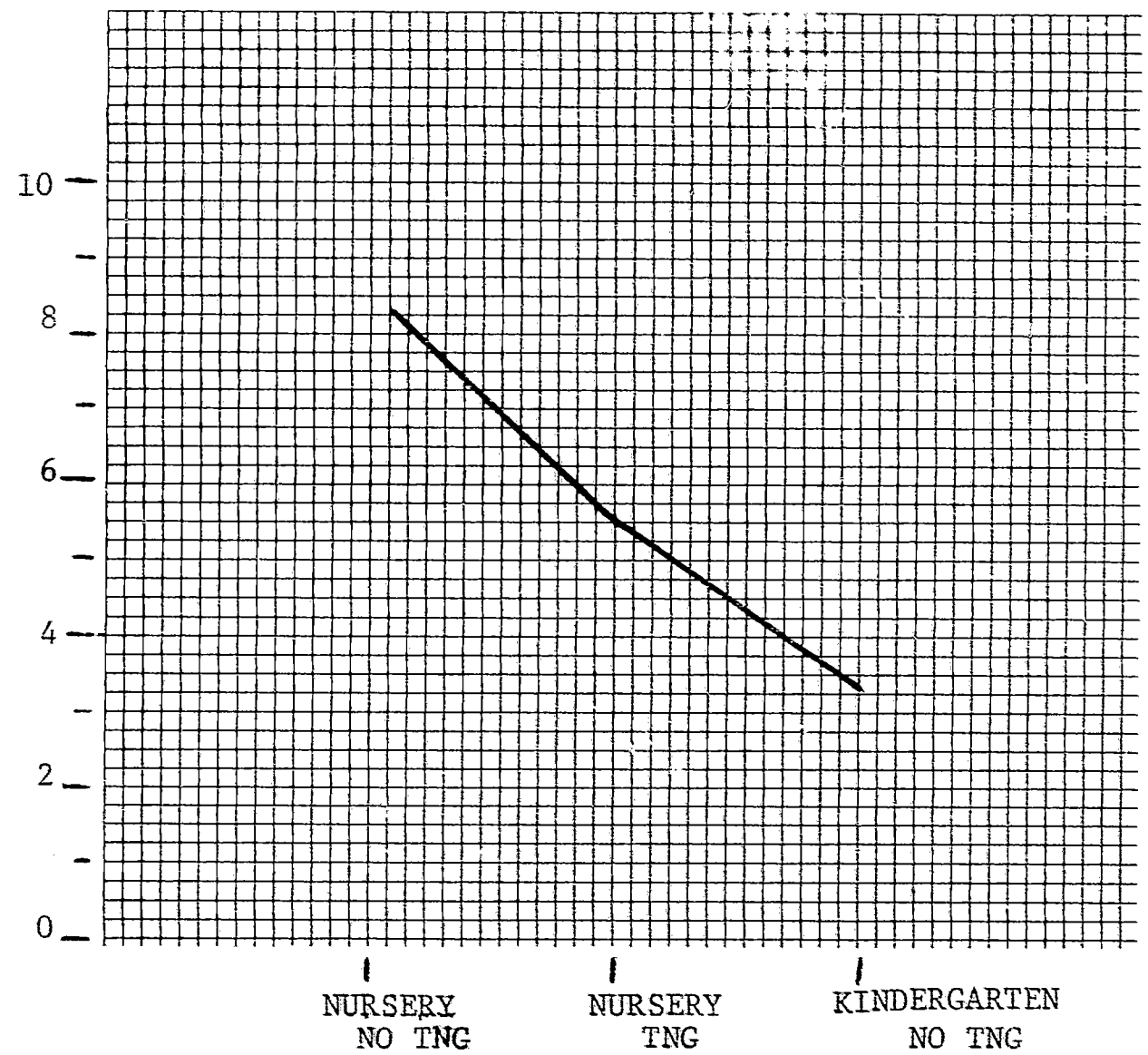


Fig. 5 Correlation coefficients for the three groups given both $Test_1$ and $Test_2$.

to the omitted concepts and are successful only to the extent that they overlap the omissions from the child's training. One obvious conclusion from this is that development of the sequences of logical concepts within any given curricular area may have more long-term value than probing such phenomena as conservation which appear to be mere artifacts of the absence of this development.

## Conclusion

It is concluded that children in kindergarten and many in pre-kindergarten nursery school can be exposed to a program in which they are taught to describe objects in terms of their properties; learn and associate correct labels with properties; observe and indicate properties common to different objects and sets of objects, and this training is sufficient in many cases to increase children's score on a test of conservation of numerousness. It is suggested that a more extensive program, in which children could participate more actively could produce a more marked increase.

Beyond this, however, it is suggested that a more profitable approach to the conservation problem is the more careful analysis of curricular areas, so that attributes and prerequisite concepts in each area are identified, and can be utilized in the child's training.

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# APPENDICES

## APPENDIX A

## ORIGINAL INSTRUCTIONS

### $TEST_1$

W-1 Look at the squares on both pages. Are there the same number of squares on both pages? Or does one page have more than the other? Show me by pointing. Don't talk out loud. If you think both pages have the same number of squares, put a finger on both pages. (Make sure the children are using both hands.) If you think one page has more squares on it, put your finger on that page. Don't take it away until I tell you. Turn your book to the pages with the bee at the top.

W-2 Look at the squares on both pages. Remember what you are supposed to do with your hands. Listen carefully. Are there the same number of squares on both pages? Or does one page have more than the other? Show me by pointing. Turn to the page with the car at the top.

W-3 Three discs. Put the discs on the squares. Notice there are the same number of discs as squares. Now move the discs to cover the dots. Are there the same number of discs as squares? Or are there more of one than the other? Show me by pointing. (Make sure they point with both hands or one depending on whether they think they are the same, etc.) Turn to the page with the tricycle at the top.

W-4 One disc. Place the disc on the dot. Are there the same number of squares as discs? Or are there more of one than the other? Show me. Turn to the page with the butterfly at the top.

1. Look at the squares on both pages. Are there the same number of squares on both pages? Or does one page have more than the other? Show me. Turn to the page with the teddy bear at the top.

2. Six discs. Put the discs on the squares. Now cover the dots with the discs. Are there the same number of discs as squares? Or are there more of one than the other? Show me. Turn to the page with the fish at the top.

3. Five discs. Put the discs on the squares. Now cover the dots with the discs. Are there the same number of squares as discs? Or are there more of one than the other? Show me. Turn to the page with the duck at the top.

4. Five discs. Put the discs on the squares. Move the discs to cover the dots. Are there the same number of discs as squares? Or are there more of one than the other? Show me. Turn to the page with the horse at the top.

5. Look at the squares on both pages. Are there the same number of squares on both pages? Or does one page have more than the other? Show me. Turn to the page with the sheep at the top.

6. Five discs. Put the discs on the squares. Move the discs to cover the dots. Are there the same number of discs as squares? Or are there more of one than the other? Show me. Turn to the page with the bear at the top.

7. Look at the dots in both rows. Are there the same number of dots in both rows? Or does one row have more than

the other? Show me. Turn to the page with the turtle at the top.

8. Look at the squares on both pages. Are there the same number on both pages? Or does one page have more than the other? Show me. Turn to the page with the chicken at the top.

9. Seven discs. Put the discs on the squares. Move some of the discs to cover the dots. Are there the same number of discs as squares? Or are there more of one than the other? Show me. Turn to the page with the tractor at the top.

10. Six discs. Cover each dot with a disc. Are there the same number of squares as discs? Or are there more of one than the other? Show me. Turn to the page with the dog at the top.

11. Eight discs. Put the discs on the squares. Move the discs to cover the dots. Are there the same number of discs as squares? Or are there more of one than the other? Show me. Turn to the page with the penguin at the top.

12. Look at the dots on both pages. Are there the same number of dots on both pages? Or does one page have more than the other? Show me. Turn to the page with the chicken at the top.

13. Look at the dots on both rows. Are there the same number of dots in both rows? Or does one row have more than the other? Show me. Turn to the page with the sheep at the top.

14. Look at the dots on both pages. Are there the same number of dots on both pages? Or does one page have more than the other? Show me. Turn to the page with the clown at the top.

15. Look at the dots in both rows. Are there the same number of dots in both rows? Or does one row have more than the other? Show me. Turn to the page with the owl at the top.

16. Six discs. Put the discs on the squares. Move the discs to cover the dots. Are there the same number of squares as discs? Or are there more of one than the other? Show me.

REVISED INSTRUCTIONS

$TEST_1$

1. Open your books at the first page. (SHOW THEM THE CORRECT ONE. CHECK TO MAKE SURE THEY ARE LOOKING AT THE CORRECT ONE.) Look at the squares on both pages. Are there the same number of squares on both pages? Or does one page have more squares than the other page? If you think that one page has more squares than the other page, put an X on the page which has more squares. If you think that both pages have the same number of squares put an X on both pages. (REPEAT LAST TWO SENTENCES.) Don't talk out loud. (WAIT UNTIL THEY HAVE MARKED THE PAGE(S).) Now turn to the pages with the bee at the top.

2. Look at the squares on both pages--listen carefully. Are there the same number of squares on both pages? Or does one page have more squares than the other page? If you think that one page has more squares than the other page, then put an X on the page which has more squares. If you think both pages have the same number of squares put an X on both pages. (REPEAT THE LAST TWO SENTENCES. WAIT UNTIL THEY HAVE MARKED THE PAGE(S).) Now turn to the page with the car at the top.

3. Take some of the discs, in front of you, put one on each square--one disc on each square. (CHECK THAT THEY DO SO.) Notice that there are the same number of discs as squares. Now move a disc

to cover each dot. Are there the same number of dots as squares? Or is there more of one than the other? If there are more dots put an X beside the dots. If there are more squares put an X beside the squares. If you think that there is just as many squares as dots, put an X beside both the squares and the dots. (CHECK TO MAKE SURE THEY DO SO.) Turn to the page with the tricycle at the top.

4. Put one disc on each dot. (CHECK.) Are there the same number of squares as dots? Or is there more of one than the other? If you think that there are more dots, put an X beside the dots. If you think there are more squares put an X beside the squares. If you think that there is the same number of dots as squares, put an X beside both the squares and the dots. (CHECK.) Turn to the page with the butterfly on the top.

5. Look at the squares on both pages--listen carefully. Are there the same number of squares on both pages? Or does one page have more squares than the other page? If you think that one page has more squares than the other page, then put an X on the page which has more squares. If you think that both pages have the same number of squares put an X on both pages. (REPEAT THE LAST TWO SENTENCES. WAIT UNTIL THEY HAVE MARKED THE PAGE(S).) Now turn to the page with the teddy bear at the top.

6. Take some of the discs, in front of you, put one on each square--one disc on each square. (CHECK THAT THEY DO SO.) Notice that there are the same number of discs as squares. Now move a disc to cover each dot. Are there the same number of dots as squares? Or is

there more of one than the other? If there are more dots put an X beside the dots. If there are more squares put an X beside the squares. If you think that there is just as many squares as dots, put an X beside both the squares and the dots. (CHECK TO MAKE SURE THEY DO SO.) Turn to the page with the fish at the top.

7. Take some of the discs, in front of you, put one on each square--one disc on each square. (CHECK THAT THEY DO SO.) Notice that there are the same number of discs as squares. Now move a disc to cover each dot. Are there the same number of dots as squares? Or is there more of one than the other? If there are more dots put an X beside the dots. If there are more squares put an X beside the squares. If you think that there is just as many squares as dots, put an X beside both the squares and the dots. (CHECK TO MAKE SURE THEY DO SO.) Turn to the page with the duck at the top.

8. Take some of the discs, in front of you, put one on each square--one disc on each square. (CHECK THAT THEY DO SO.) Notice that there are the same number of discs as squares. Now move a disc to cover each dot. Are there the same number of dots as squares? Or is there more of one than the other? If there are more dots put an X beside the dots. If there are more squares put an X beside the squares. If you think that there is just as many squares as dots, put an X beside both the squares and the dots. (CHECK TO MAKE SURE THEY DO SO.) Turn to the page with the horse at the top.

9. Look at the squares on both pages--listen carefully. Are there the same number of squares on both pages? Or does one page have more squares than the other page? If you think that one page has more

squares than the other page, then put an X on the page which has more squares. If you think that both pages have the same number of squares put an X on both pages. (REPEAT THE LAST TWO SENTENCES. WAIT UNTIL THEY HAVE MARKED THE PAGE(S).) Turn to the page with the sheep at the top.

10. Take some of the discs, in front of you, put one on each square--one disc on each square. (CHECK THAT THEY DO SO.) Notice that there are the same number of discs as squares. Now move a disc to cover each dot. Are there the same number of dots as squares? Or is there more of one than the other? If there are more dots put an X beside the dots. If there are more squares put an X beside the squares. If you think that there is just as many squares as dots, put an X beside both the squares and the dots. (CHECK TO MAKE SURE THEY DO SO.) Turn to the page with the bear at the top.

11. Look at the dots in both rows. Are there the same number of dots in both rows? Or is there more dots in one than there is in the other? If you think there is more dots in one row than there is in the other, put an X beside the row which has more dots. If you think that there is the same number of dots in each row, put an X beside each row. (CHECK.) Turn to the page with the turtle on the top.

12. Look at the squares on both pages--listen carefully. Are there the same number of squares on both pages? Or does one page have more squares than the other page? If you think that one page has more squares than the other page, then put an X on the page which has more squares. If you think that both pages have the same number of

squares put an X on both pages. (REPEAT THE LAST TWO SENTENCES. WAIT UNTIL THEY HAVE MARKED THE PAGE(S).) Turn to the page with the chicken on the top.

13. Take some of the discs, in front of you, put one on each square--one disc on each square. (CHECK THAT THEY DO SO.) Notice that there are the same number of discs as squares. Now move a disc to cover each dot. Are there the same number of dots as squares? Or is there more of one than the other? If there are more dots put an X beside the dots. If there are more squares put an X beside the squares. If you think that there is just as many squares as dots, put an X beside both the squares and the dots. (CHECK TO MAKE SURE THEY DO SO.) Turn to the page with the tractor at the top.

14. Put one disc on each dot. (CHECK.) Are there the same number of squares as dots? Or is there more of one than the other? If you think that there are more dots, put an X beside the dots. If you think there are more squares put an X beside the squares. If you think that there is the same number of dots as squares, put an X beside both the squares and the dots. (CHECK.) Turn to the page with the dog at the top.

15. Take some of the discs, in front of you, put one on each square--one disc on each square. (CHECK THAT THEY DO SO.) Notice that there are the same number of discs as squares. Now move a disc to cover each dot. Are there the same number of dots as squares? Or is there more of one than the other? If there are more dots put an X beside the dots. If there are more squares put an X beside the squares. If you think that there is just as many squares as dots,

put an X beside both the squares and the dots. (CHECK TO MAKE SURE THEY DO SO.) Turn to the page with the penguin at the top.

16. Look at the dots on both pages. Are there the same number of dots on each page, or is there more dots on one page than on the other? If you think that there are more dots on one page than the other, then put an X on the page which has more dots. If you think that there are the same number of dots on both pages, put an X on both pages. (CHECK.) Turn to the page with the chicken at the top.

17. Look at the dots in both rows. Are there the same number of dots in both rows? Or is there more dots in one than there is in the other? If you think there is more dots in one row than there is in the other, put an X beside the row which has more dots. If you think that there is the same number of dots in each row, put an X beside each row. (CHECK.) Turn to the page with the sheep at the top.

18. Look at the dots on both pages. Are there the same number of dots on each page, or is there more dots on one page than on the other? If you think that there are more dots on one page than the other, then put an X on the page which has more dots. If you think that there are the same number of dots on both pages, put an X on both pages. (CHECK.) Turn to the page with the clown at the top.

19. Look at the dots on both rows. Are there the same number of dots in both rows? Or is there more dots in one than there is in the other? If you think there is more dots in one row than there is in the other, put an X beside the row which has more dots. If you think that there is the same number of dots in each row, put

an X beside each row. (CHECK.) Turn to the page with the owl at the top.

20. Take some of the discs, in front of you, put one on each square--one disc on each square. (CHECK THAT THEY DO SO.) Notice that there are the same number of discs as squares. Now move a disc to cover each dot. Are there the same number of dots as squares? Or is there more of one than the other? If there are more dots put an X beside the dots. If there are more squares put an X beside the squares. If you think that there is just as many squares as dots, put an X beside both the squares and dots. (CHECK TO MAKE SURE THEY DO SO.)

# $TEST_2$

1. Classifying objects

   Materials: Logical Blocks

   Procedure: a. Group the blocks by color. Ask the child to state the rule which was used.

   *b. Ask the child to group the blocks and to state the rule he used (other than by color).

2. Comparing and equalizing objects

   Materials: Two bars of Unifix cubes (3 and 5)

   Procedure: Present the bars to the child and ask him which is longer. Ask him to make the bars equal in length.

3. Comparing representations of objects

   Materials: Plastic chain and two red discs mounted on strips of paper

   Procedure: After you have placed the discs on the table in front of the child give him the suggestion to use the chain. Ask him to determine which red disc is closer to him.

4. Comparing and equalizing sets

   Materials: Beans

   Procedure: a. Present the child with two sets of beans (4 and 6). Ask the child which set has more objects.

   *b. Ask the child to make the sets equal in number.

5. Comparing representations of sets

   Materials: Beans, stars of paper

   Procedure: Place a set of stars on the table. Explain to the child that these stars will disappear and another set will come out. He must be able to tell you which set of stars was larger. Suggest that he use the beans as an aid.

*1b. and 4b. were omitted from the test in Experiment 3.

APPENDIX B

PRESENTATION 1

Introducing Herman

Materials needed: HERMAN (Stuffed Toy Duck)

Vocabulary and Labels to be Introduced: Describe property. Names of some properties: shape, size, color, material, etc.

Introduction: Herman is visiting us today. We want you to get to know him so that if you meet him again you will recognize him.... When you go home you are going to have to describe Herman to your mother, so that she will recognize him if she sees him. How can we describe him?... Well, there are certain things which make Herman, and help us to recognize and describe him. We will call these things properties. Some of the properties of Herman which we will talk about are: colors, shapes, material, feel, size, and so on.

Activity: Have children talk about Herman, and describe him. Mention property of color, property of feel, etc. and talk about these. Then do the same thing with one or more children. "How would we describe Mary?" "What properties can we talk about, etc?" Make informal comparison between children, and between children and Herman, comparison always being made on a single property.

Notes by Teacher:

## PRESENTATION 2

### Examining Objects and Noting Their Properties

Materials needed: One ball, one box, one sheet of construction paper

Vocabulary and Labels to be Introduced: Objects - things

Introduction: Yesterday we talked about Herman. We found that we could describe him by telling about his different properties, like color, shapes and size. How many described Herman to your mother? "How did you do it?" Today we will talk about other objects, other things and we will see that they can be described by telling about their properties.

Activities: Start with the ball. Pass it around among the children. Have them describe it. Note (if they don't) that its shape is round, it is smooth, fairly soft, green, made of plastic, etc. Then do the same with the box. It is square on all sides, pink, has pink lines, small squares, paper, it opens at both ends, etc. Note properties on which an informal comparison between the objects can be made. Finally, pass around a piece of construction paper and go through the same process.

Notes by Teacher:

## PRESENTATION 3

### Further Practice on Properties

Materials needed: Flannel board, one set of construction paper cut-outs: strips, long and short, wide and narrow, green and yellow, with dot and without dot.

Vocabulary and Labels to be Introduced: How things are alike/different

Introduction: We have been talking about describing objects by telling their properties. This is a very useful wayof telling about things. It helps especially if things are very much alike and we need to tell them apart. In a case like that, we can tell how things are like each other (e.g., orange and ball are round), if they have the same properties, or how they are different from each other (e.g., orange and apple have different taste, etc.).

Activity: Put the cut-outs one by one on the flannel board and have a child describe each in terms of its properties. After the first one is described, each successive one can be partly described by reference to another, e.g., describing one as fat, the other thin, long-short, etc.

Notes by Teacher:

## PRESENTATION 4

### Classifying

Materials needed: Set of construction paper cut-outs, one envelope of cut-outs per student. Envelope contains large and small, red and blue circles, squares and triangles. Also cut-outs from Presentation 3

Vocabulary and Labels to be Introduced: Groups, classes, classify, all - ones

Introduction: (Set of cut-outs randomly placed on the flannel board) Is there any way that we can put these in some sort of groups or classes? Perhaps, we could put all short ones in this side and all long ones on this. This is called classifying. We can classify things by putting objects which have the same property together in groups. Is there another way in which we could classify this group of papers? All greens here, all yellows there! etc.

Activity: Have children suggest ways in which the group of cut-outs can be rearranged and classified. Have individual students arrange the cut-outs according to a rule you state, e.g., "Put all those with dots here, all those without over here."

Notes by Teacher:

## PRESENTATION 5

### Classification Rule

Materials needed: Construction paper cut-outs and envelope of strips for each student, as in Presentation 4

Vocabulary and Labels to be Introduced: Rule, classification

Introduction: We have been classifying these objects....putting them into classes. Every time we put things into classes we decided beforehand how we were going to put them together. We said, "All the yellow ones on this side and all the green ones on this side." This is called a rule. We know what rules are for. They tell us what to do. School rules tell us what to do at school. Classification rules tell us what to do when we are classifying. For instance, if our rule is "all yellows together, and all greens together," then we know that we put all the yellow strips together in a bunch or group, and all the greens in another, and so on. The rule will tell us that every object in the group will have the same property, at least one. Now when we see a class we can look at each object to see what property all of the objects have. Then we can tell the classification rule.

Activity: Have the children open their envelopes and classify the strips on the basis of a rule they can verbalize. It may be necessary to start by specifying a rule which they can then implement. After children have classified the strips on the basis of one rule, then have them rearrange them on the basis of another rule.

Next the students can be teamed up in pairs and they can play the game "Guess the rule" wherein one student groups the objects and the other tries to determine the rule. After the children have attained some proficiency at this, try grouping the children themselves and have them determine the rule by which they were classified.

Teacher's Notes:

## PRESENTATION 6

Comparing Objects on Property Length

Materials Needed:

Vocabulary Introduced: Length, height, long, tall, tallness

Introduction: Last week we talked about objects and properties of objects and we found out that we can use what we learned about properties to make classes of things, using our classification rules. This week we are going to use what we know about objects and their properties to do something different. We are going to compare things. We will be talking first of the property length. Does everyone know what I mean by how long something is?: how far it is from one spot (point, place, etc.) to another. We sometimes use other words as well as "length" such as "height," which means how long something is, or how far it is from the bottom of something to the top; or how tall it is, for instance how long we are from the bottoms of our feet to the tops of our heads. Now we can compare Bill with Mary on the property of tallness, and we can say which is taller or longer. We can compare the length of our fingers, our arms, our shoes, etc. and our pencils.

Activity: Have the children make these comparisons.

## PRESENTATION 7

### Representing Length

Materials Needed: Links, twine

Vocabulary and Labels to be Introduced: Represent

Introduction: In talking about comparing lengths of two objects we found that we could place them side by side and look at them to find out which is longer. This is easy but sometimes we cannot do this. For instance: you can compare Jim's crayon with Billy's to see which is longer etc. but if we want to see whether the crayon Jim has at school is longer than the one he has at home we must find another way of comparing them. The trick is to take something which we can move from one place to the other and use that. We will call this representing the object. Let us see.

Activity: Compare pencils or crayons. Place one pencil or crayon on one side of the room and one on the other (Jim's at school and at home). Now compare. Children can use their hands, piece of paper, etc. Have them compare other objects in the room which cannot be moved. Introduce links. Ask children to compare waists, etc.

Teacher's Notes:

## PRESENTATION 8

### Manyness: A Property of Sets

Materials Needed: Discs, wads, foam balls

Vocabulary and Labels to be Introduced: Group, set, manyness, how many, number

Introduction: We talked about objects and properties of objects. We saw that we could describe Herman and many other things, by telling about their properties. Now we will look at groups of objects and see if we can do the same thing.

Activities: Group the red discs (or wads) in threes and fours and have children talk about the properties of the groups. They can note the various properties. Try to elicit the additional property of "manyness" from them by, if necessary, using a single disc as a contrast. Do enough of these activities so that the children note manyness of the set as a property.

Teacher Remarks:

## PRESENTATION 9

Comparing Sets on the Property Manyness

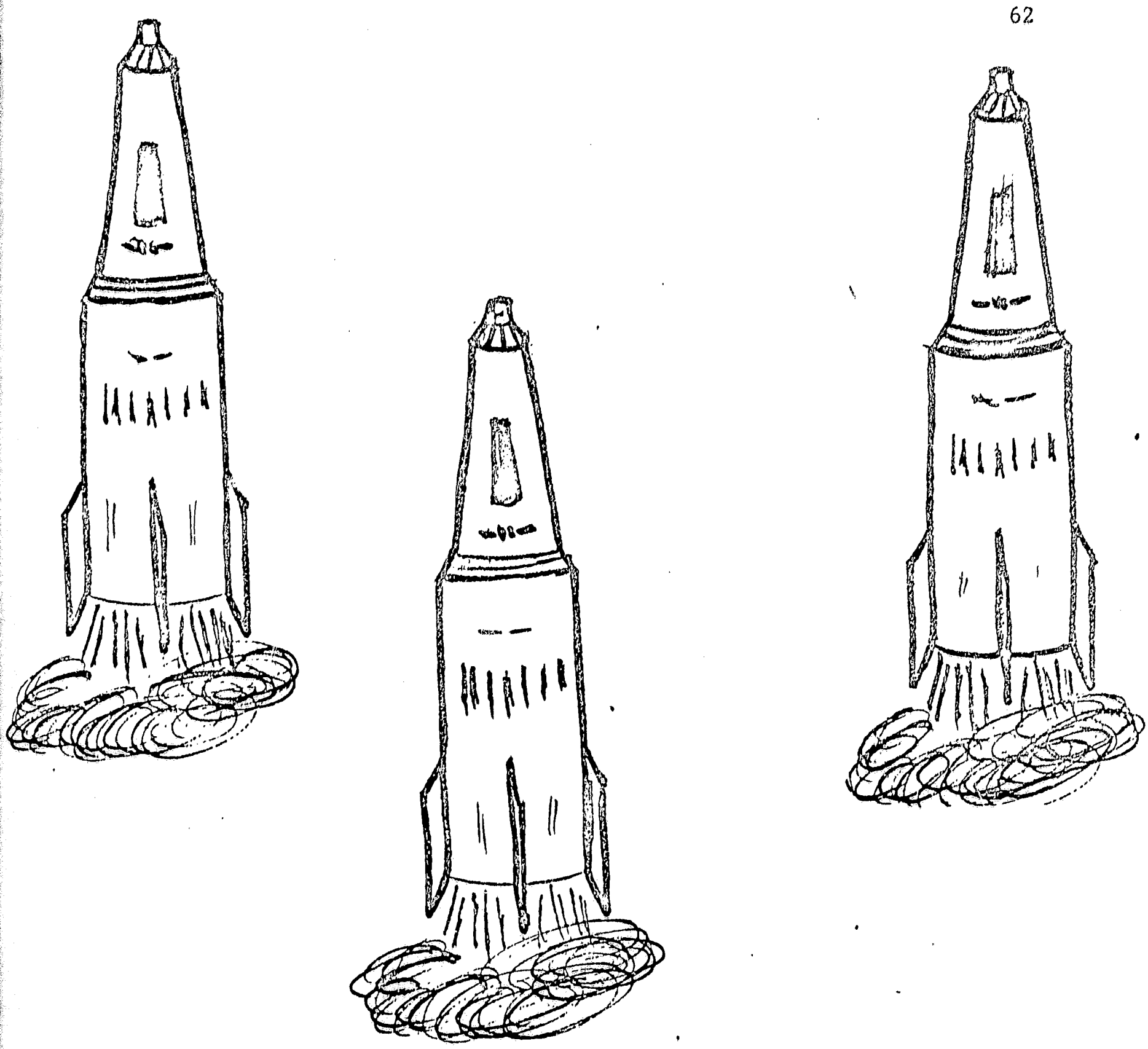
Materials: Cut-outs, worksheets, felt board figures

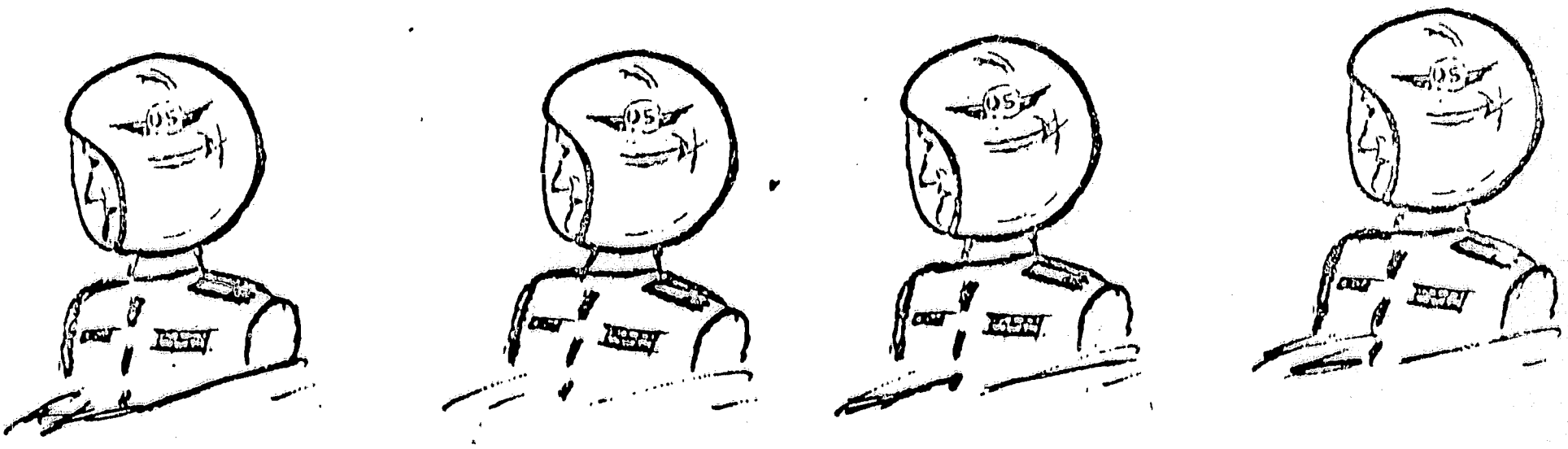
Vocabulary and Labels to be Introduced: Matching, as many as, more than, less than

Introduction: We have just found out that manyness or number is a property of a set or group of objects. You remember that when we compared objects last week we compared them on the property length. Now we will compare groups on the property manyness to see if the two have the same number of objects or if one has more objects than the other. One way of comparing groups on the property "manyness" is to count the number of objects in each set. But if we cannot count or cannot count very well there is an easier way. We call it matching. This is how we do it. (Use felt board, set two groups of objects on the felt board and match them by moving them in pairs. First equal then unequal sets.) When one group has one or more left and there is nothing in the other to match it with then we say that group has more.

Activities: Have the children group the cut-outs by various properties and match them on the basis of manyness. Have the children match the astronauts and the rockets.

Teacher Remarks:

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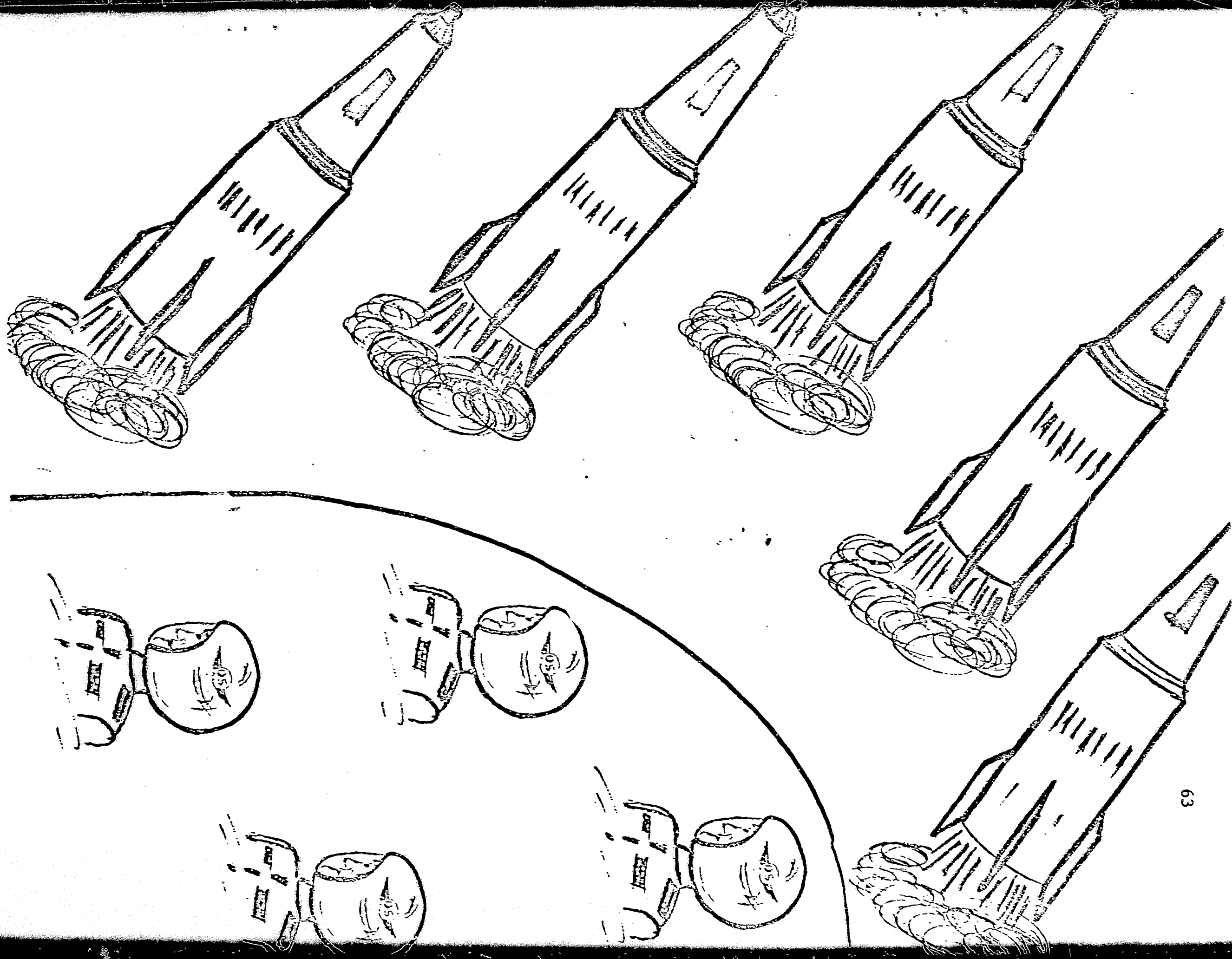

## PRESENTATION 10

Representing Sets and Comparing Representations

Materials Needed: Felt board, felt cut-outs

Vocabulary and Labels to be Introduced:

Introduction: We have talked about objects and comparing objects on the property length by putting them beside each other. Then we found that we could represent objects and compare them in that way. Now we have talked about comparing (groups) by matching. So next we will talk about representing sets. Can anyone think of how we might represent groups of things? For instance, how could we tell if you have as many chairs in your house at home as Mrs. Cook has here. [Try to elicit the notion of making a tally mark, notch, or one-to-one correspondence with stones or marbles, etc.] Let us match some sets by first representing them and then matching the representations.

Activities: Place two sets of cut-outs on the felt board and have the children first represent one, and then match the representation with the other set. Have them represent sets which will disappear, and then match the representation with the set when it reappears.

## PRESENTATION 11

### Comparing Sets on Two or More Properties

Materials Needed: Two sets of cubes, one set having larger cubes than the other. Two sets of discs, one set having larger discs than the other. Felt board and felt discs.

Vocabulary and Labels to be Introduced:

Introduction: When we talked about comparing groups we only talked about comparing them on the property manyness. Some people compare them on other properties and then may get confused. For instance, we could have a group like this [put felt discs on the board close together in a line] and another like this [put another set of discs (same number) on the board underneath the first, but more separated]. Now we could compare these groups on the property manyness. And what do we find?... What other property could we compare them on [elicit or point out length]? So we see that the groups can be the same on the property manyness, but one is bigger than the other if we compare on the property length.

Activity: Have children put discs (and blocks) in rows, touching each other. Have them name properties and make comparisons, especially on length and manyness.