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Technical activity from April 1 through June 30, 1968 involving the problem of developing the mathematics curriculum learning units are covered. The principal goal of the entire Project is to demonstrate increased effectiveness of vocational instruction whose content is derived from an analysis of desired behavior following graduation. During these activities three purposes were assumed for the learning of mathematics: mastery of technical skills, abstract reasoning and insight development, and learning skill development. The curriculum also was based on the use of specific behavioral objectives and individualized instructional techniques. Major steps in the curriculum development phase included: (1) listing of content topics, (2) development of semester objectives, (3) sequencing of content units, (4) preparation of unit syllabuses, (5) selection of available mathematics units, and (6) development of unavailable units. Materials for grades 10, 11, and 12 contained 92 units. The appendix includes samples of the materials and sources of the content. Other related documents are ED 024 749-024 754 and ED 024 767, and VT 008 451. (EM)

TENTH QUARTERLY TECHNICAL REPORT

Project No. 5-0009

Contract No. OE-5-85-019

DEVELOPMENT AND EVALUATION OF AN EXPERIMENTAL CURRICULUM FOR THE NEW QUINCY (MASS.) VOCATIONAL-TECHNICAL SCHOOL

The Mathematics Curriculum

31 May 1968

U. S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

Office of Education
Bureau of Research

DEVELOPMENT AND EVALUATION OF AN EXPERIMENTAL CURRICULUM FOR THE NEW QUINCY (MASS.) VOCATIONAL-TECHNICAL SCHOOL.

The Mathematics Curriculum.

ADD:

Project No. 5-0009
Contract No. OE-5-85-019

F. Coit Butler,
G. Bradley Seager
Charles Loch

31 May 1968

The research reported herein was performed pursuant to a contract with the Office of Education, U. S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official Office of Education position or policy.

American Institutes for Research,
Pittsburgh, Pennsylvania.

ERRATA

P. 8, Line 7, Middle Column.
Should read "Laws of Indices."

P. 10, Line 4, under Spiraling.
Should read "review of prerequisite material."

P. 11, Line 5 of "Second Half Year Semester - 10th Grade"
Should read "$15^{\circ} - \Pi/4^{\circ}$ angles."

P. 29, Line 7
Should read "the Quincy vocational - technical student is not atypical of"

P. 31, Line 4
Should read "in the development are"

P. 35 Should be omitted.

P. 131 e., part 2.
Should read $\frac{9.1 \times 8.65}{3\ 1/4}$

P. 135 d., part 2.
Should read $24\ 3/4 \div 68\ 1/2$
d., part 3.
Should read $\sqrt{52\ 9/16}$

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FOREWARD

This report, submitted in compliance with Article 3 of the contract, report on technical activities of Project ABLE during its tenth quarter of operation, 1 April through 30 June 1968. A brief overview of the project is presented first, followed by a report summary. The major portion of the report addresses the problem of developing the mathematics curriculum learning units.

OVERVIEW: Project ABLE

<u>A Joint Research Project of</u>: Public Schools of Quincy, Massachusetts and American Institutes for Research

<u>Title</u>: DEVELOPMENT AND EVALUATION OF AN EXPERIMENTAL CURRICULUM FOR THE NEW QUINCY (MASS.) VOCATIONAL-TECHNICAL SCHOOL

<u>Objectives</u>: The principal goal of the project is to demonstrate increased effectiveness of instruction whose content is explicitly derived from analysis of desired behavior after graduation, and which, in addition, attempts to apply newly developed educational technology to the design, conduct, and evaluation of vocational education. Included in this new technology are methods of defining educational objectives, deriving topical content for courses, preparation of students in prerequisite knowledges and attitudes, individualizing instruction, measuring student achievement, and establishing a system for evaluating program results in terms of outcomes following graduation.

<u>Procedure</u>: The procedure begins with the collection of vocational information for representative jobs in eleven different vocational areas. Analysis will then be made of the performances required for job execution, resulting in descriptions of essential classes of performance which need to be learned. On the basis of this information, a panel of educational and vocational scholars will develop recommended objectives for a vocational curriculum which incorporates the goals of (a) vocational competence; (b) responsible citizenship; and (c) individual self-fulfillment. A curriculum then will be designed in topic form to provide for comprehensiveness, and also for flexibility of coverage, for each of the vocational areas. Guidance programs and prerequisite instruction to prepare junior high students also will be designed. Selection of instructional materials, methods, and aids, and design of materials, when required, will also be undertaken. An important step will be the development of performance measures tied to the objectives of instruction. Methods of instruction will be devised to make possible individualized student progression and selection of alternative programs, and teacher-training materials will be developed to accomplish inservice teacher education of Quincy School Personnel. A plan will be developed for conducting program evaluation not only in terms of end-of-year examinations, but also in terms of continuing follow-up of outcomes after graduation.

<u>Time Schedule</u>: Begin 1 April 1965

Complete 31 March 1970

Present Contract to 31 December 1968

REPORT SUMMARY

During the present reporting period, technical activity concentrated on (1) crystalization of: curriculum unit topics, semester objectives, sequences of learning units, and syllabi for specific vocational areas, in mathematics, (2) analysis of the verbal and mathematical aptitude, ability and achievement characteristics of the mathematics student at Quincy, and (3) the development and testing of learning units in mathematics. The present report presents the history of this activity. It traces the development of the mathematics curriculum and displays the end product from its theoretical conception, to the identification of learning units, the establishment of semester objectives, the sequencing of the learning units, the formation of syllabi in specific vocational areas, the analysis of the learner population and the actual writing and testing of learning units. This report also includes a rationale for the curriculum as a whole and a rationale for the semester objectives.

During the next quarter, the development of curricula in the form of writing and testing learning units in other academic areas will constitute the major portion of technical activity. In addition, evaluation of the senior class guidance program will continue.

RATIONALE FOR PROJECT ABLE MATHEMATICS CURRICULUM

Learning experiences in mathematics in high school serve at least three purposes. Differences in values lead to different hierarchies of these purposes. The order of discussion here is not intended to imply a particular hierarchy.

One purpose of learning experiences in mathematics in high school is mastery of technical skills that will be used later. Increasing mobility among dissimilar vocations makes it difficult to predict the technical skills each student will use after he leaves school. Therefore, the curriculum should provide opportunities to master technical skills of broad application as well as opportunities for specialization in areas of individual interest.

A second purpose of learning experiences in mathematics in high school derives from the position of mathematics in our culture. Mathematics has been called the queen and servant of science. Mathematics can be studied for its practical applications, but it can also be studied independently of applications to the world of the senses. Abstract reasoning can begin early in the study of mathematics. Work with different number bases and with non-Euclidean geometries is justified by the insights into mathematical structure students can thereby develop. These examples show that mathematical structure is a matter of choice and that anyone is free to invent any mathematical structure that appeals to him, provided the structure is internally consistent.

A third purpose of learning experiences in mathematics applies to students at all levels. This purpose transcends subject matter divisions and can be described as learning how to learn. When a student's formal education is complete, he should be able to continue his education informally as the manager of his own learning. This becomes possible when he has mastered general learning skills and can apply these skills in several academic disciplines. It is convenient to call the skills of learning how to learn "process skills" and the skills associated with particular subject matter "content skills." When a student achieves mastery of process skills he becomes capable of managing his learning of content skills.

Fortunately, many learning experiences in mathematics in high school serve all three of these purposes, especially when these experiences are planned with these purposes in mind. Problem solving is an approach well suited to the achievement of all three purposes. Many realistic problems can be included in the curriculum together with problems that develop skill in abstract reasoning. The skills of learning how to learn are largely problem solving skills.

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There are currently two major trends in curriculum revision that deserve mention here. The first of these trends is toward specification of objectives as behaviors expected of the student. The second is toward individualized instruction. These two trends are complementary. Although the broad, long-range objectives of mathematics education have changed little in recent years, new insights are now available to curriculum developers to help them determine intermediate and short-range objectives. Each activity planned for the student can be matched with some knowledge, skill, or attitude toward which the activity is directed. By specifying the knowledge, skills, and attitudes expected of students in terms of their behavior we gain clarity in the statement of objectives, guidance in the selection of procedures for learning, and a base for evaluation of students' progress toward the objectives.

The same objective can be attained at different levels of mastery. Consequently, the criteria of mastery expected of individual students in connection with each objective should be made explicit. Even when several students work toward the same objective at the same time, we seldom expect each student to attain the same level of mastery of the objective simultaneously. What is important is that each student should be working toward a level of mastery that is not only realistic and appropriate for him, but that he and others accept as deserving of respect.

Usually different students work toward different objectives at the same time, even when they work together in groups. In individualized instruction each student, whether working alone or in a group, is encouraged to follow a plan designed specifically for him in recognition of his needs, interests, abilities, prior experiences, and learning style. The teacher who individualizes instruction must have considerable information about each student concerning the variables just named and must spend a large fraction of total teaching time in planning with and for individual students. Evaluation of student progress toward short-range objectives is almost continuous in individualized instruction. Much of the necessary evaluation can be accomplished by the students after they have learned basic skills of self-evaluation. Much of the time students can receive immediate feedback of results by comparing their solutions of problems to solutions imbedded in the learning materials or by applying criteria for acceptable solutions directly to their own solutions.

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To facilitate individualized instruction the curriculum is subdivided into learning units consisting of one or more modules. Each module represents a set of learning experiences designed to help the student achieve a small number of related behavioral objectives at a specific level of mastery. Each module contains a list of readiness skills the student will need to have mastered before undertaking the module, a diagnostic test, a description of objectives in terms understandable to the student, the actual instructional materials to be used by the student, notes to the teacher, references for the module, and a mastery test (Appendix E).

Each module is self-contained in the sense that the readiness skills and the instructional materials contained in the module should be sufficient for the attainment of the objectives of the module. Therefore, a student may work on a module whenever he has the readiness skills to do so and the objectives of the module are judged to be appropriate for him. For convenience in selection, modules are grouped in learning units that are also self-contained. Learning units encompass varying amounts of related subject matter and are seldom made up of more than ten modules. A student can expect to spend from one to three class periods on a single module and from one to thirty class periods on a learning unit.

Distinctions among the various branches of mathematics at the secondary level are going out of fashion. Thus, concepts of algebra and geometry are intermingled rather than assigned to separate textbooks and years.

The rationale for this intermingling is partly to avoid artificial separations of subject matter in mathematics and partly to make available to the student all the mathematical techniques and concepts that he needs and can use to solve problems that defy artificial separations. It follows that the sequence of learning units should be flexible and that relationships among learning units should be clear to the teacher so that he, in turn, can make these relationships clear to the students.

A History and Summary of Development of the Mathematics Curriculum

1. The foundation for the objectives of the math curriculum was laid in a series of letters, memos, and reports dating back to the inception of Project ABLE. The gist of these documents affirmed that the primary source of objectives should be an analysis of behavior in the mathematical realm necessary for vocational competence. (Appendix A - Early Correspondence Concerning the Mathematics Curriculum)

2. Having established the source (which in turn suggested methods for the collection of relevant data), lists of content topics were obtained using checklists (code sheets), questionnaires, and a search of the current research literature. (Appendix B - Examples of Sources of Content Topics)

3. These lists of content topics were consolidated into titles of curriculum units; and the numbers of modules in each unit was identified.

4. Taking into account the broad, overall aims of Project ABLE, semester objectives were developed.

5. The titles of curriculum units were rank sequenced whenever possible and assigned to half-year grade levels where they would conceivably first appear in any individual student's schedule.

6. On receipt of mathematics requirements from the specific vocational areas and from the science curriculum syllabi for mathematics learning units in each vocational area were prepared.

7. On the basis of a two-year collection of scores on the California Achievement Test, the California Test of Mental Maturity, and the Differential Aptitude Test for the noncollege bound Quincy students, it was decided that the curriculum units would be developed at the normal (average) aptitude and ability levels. Because of signigicant differences between achievement tests scores of Quincy students and the national norms, it was also decided that extensive review facilities will be developed within the units. (Appendix C - Achievement, Aptitude and Ability Profiles)

8. All available catalogs of educational materials have been researched for materials applicable to the mathematics curriculum (including complete learning units) at the appropriate ability, aptitude, and achievement levels. (Appendix D - A list of supplemental materials, including basic texts)

9. Learning units for which no appropriate, already developed materials exist have been assigned for development to staff members. (Appendix E - Learning Unit Contents)

10. A schedule for completion of these units has been set up. Top priority has been given to learning units normally taken in the first half of the 10th grade and 11th grade years.

An Outline of Future Development of the Mathematics Curriculum

1. All learning units normally taken in the first half of the 10th grade and 11th grade years should be written by July, 1968. All other learning units should be written by September, 1968. (Appendix F - Sample Learning Units)

2. Local testing and revision in Pittsburgh, using a population of vocational-technical students similar to that of Quincy, will be initiated with each curriculum units as it is written. (Appendix G - Small Scale Testing of Learning Units)

3. Further testing revision in Quincy during this coming summer school session of 1968 and during the 1968-1969 school year will also take place until a "mastery" criterion level of 80-85 percent can be achieved within a reasonable amount of time by every student attempting the unit.

The Mathematics Curriculum Units

The following list of topics are composed of titles of mathematics curriculum learning units. These unit titles came into being as a result of analytical comparisons between careful observations of on the job behavior in the vocational areas, content topics in existing vocational curricula, opinions of vocational teaching staff gathered by the use of coded check lists and questionnaires, and the results of current research in vocational curricula.

Starting with the observations of on the job behavior (obtained during the job analysis stage of Project ABLE) as "must" topics, lists of topics obtained by each of the other methods were compared and each topic weighted according to its frequency of appearance on the different lists and the reliability and validity of its source. A cutoff point in terms of a weight value was established from a frequency distribution of the topics in terms of their weights. The cutoff point established two categories: "necessary to know" and "nice to know". All topics falling in the "necessary to know" category were converted into unit topics. Some "nice to know" topics were also converted into unit topics dependent upon the needs of the community of Quincy.

The unit topics were then rank sequenced, wherever possible, into half-year grade levels according to projected outlines of content for the vocational areas and the science curriculum, and according to principles of learning theory and structure (Project ABLE, Seventh Quarterly Report).

The Mathematics Curriculum

First Half Year--10th Grade — 78 modules, 22 units

Measurement & Scales

Squares and Square Roots (4)
Metric System (3)
Micrometer (1)
Compl., suppl $\angle$s (1)
Protractor (1)
Ratio and Proportion (4)
Percent (1)
Base 10, incl: place, decimals, digits, powers (10)

Number line (1)
Grouping Symbols (1)
Approximation (2)
Estimation (2)
Order of Magnitude (1)
Significant Figures (2)
Laws of Incices (2)

Averages (1)
Whole numbers (5)
Fractions (7)
Decimals and mixed numbers (4)
Mathematical symbols (7)
Number representation (1)

Second Half Year--10th Grade — 53 modules, 13 units

Word Problems (4)
Area (12)
Pythagorean Δs (1)
30-60-90 isos. rt. Δs (1)
Slide rule (3)
Sketching from descriptions (4)
Graphing, coordination (4)
Graphing, pictorial (4)
Graphing, tables, interpretation (4)

Algebraic Terms (1)
Formulae (3)
Linear Equations (6)
45°, $22\ 1/2^{\circ}$, 15°, $\pi/4$ (6)

First Half Year--11th Grade — 99 modules, 16 units

Regular Polygons (3)
Radical Equations (8)
Laws of Indices (8)*
Base 8, 12 (2)
Base Constructions (3)
Inscribed and Circumscribed Polygons (3)
Factoring
Quadratic Equations, formula (12)

Simultaneous equations, 2, 3, variables (10)
Word Problems (4)*
LCM, Fractional Equations (6)
Squares and Square roots (4)*
Area (12)*
Similarity, Equal Areas (6)
Micrometer (1)*
Quadrilaterals, properties (6)

NOTE: The number in parentheses following each unit title, i.e.:(6), is the suggested number of modules in the unit according to conventional treatment of the units subject matter.

Second Half Year--11th Grade | 75 modules | 13 units

Boolean Algebra, Switching (1)
Variation (6)
Radian Measure (4)
Remainder Theorem (6)
Lever Problems (2)
Linear and Angular Velocity (1)
Ratio of Similitude (2)
Complex numbers, graphs (12)
Arc Measure (2)
Mixture problems (2)
Logs to different bases (4)
Volume (12)
Frustrum of Solids (3)

First Half Year--12th Grade | 73 modules | 11 units

Radian Measure (4)*
Factoring (6)*
Quadratic Equations, formula (12)*
Word Problems (4)*
Multiplication and Division of Ploynomials (6)
Logs to different bases (4)*
Area (12)*
Similarity, Equal Areas (6)*
Volume (12)*
Micrometer (1)*
LCM, Fractional Equations (6)*

Second Half Year--12th Grade | 64 modules | 11 units

Solving Polynomials, 3rd degree and up (1)
Trigonometric Functions (20)
Conic Sections: Properties (6)
Conic Sections: Equations & use of (8)
Ratio of Areas, scale (5)
Slide rule (3)*
Bases 3, 4, 5, 6, 7, 9 (2)
Relations and Functions (4)
Extrapolation from graphs (1)
Boyles Law (2)
Frustrum of Solids (3)*

Semester Objectives

Concurrent with the identification and sequencing of learning unit topics, semester objectives were developed. Four guidelines dominated the construction of these objectives.

Behavioral Form

The semester objectives, like the specific content objectives for each topic, were written in behavioral form, according the Mager(1962). Thus, the same benefits, especially ease of evaluation and revision, falling to individual instructional materials whose objectives were specified in behavioral terms also attach themselves to the larger whole, the curriculum.

Exit Points

Special attention was given to the concept of providing each student exiting from the curriculum at the end of any one of the three high school years marketable skills in mathematics in terms of the three competence goals of Project ABLE, vocational, citizenship, and leisure.

Spiraling

Opportunity for review is provided within every semester. Much of this review is dependent on student or teacher initiative and takes the form of repeating previously mastered units or modules. Within intermediate and advanced units, however, review op prerequisite material is usually provided.

Vocational Priority

The physical number and size of curriculum units and modules is reflective of the increasing demands of time, competance and effort by the vocational areas as the student progresses through the curriculum from semester to semester.

Other, more specific considerations of the semester objectives are provided in the Rationale for Semester Objectives.

Semester Objectives

First Half Year Semester - 10th Grade

1. The student will be able to solve correctly in writing 80-85 percent of the posttest arithmetic problems found in the curriculum materials in all necessary following areas: base 10, number line, whole numbers, fractions, decimals, and mixed numbers, squares and square roots, ratio and proportion, percent, averages, approximation, estimation, order of magnitude, significant figures, and laws of indices.

2. The student will be able to solve correctly in writing 80-85 percent of posttest mathematical abstract reasoning problems found in the curriculum materials in all necessary following areas: mathematical symbols, number representation, and grouping symbols.

3. The student will be able to solve correctly in writing 80-85 percent of introductory measurement posttest problems found in the curriculum materials in all necessary following areas: the metric system, measurement and scales, the micrometer, the protractor, and complementary and supplementary angles.

Second Half Year Semester - 10th Grade

1. The student will be able to solve correctly in writing 80-85 percent of the posttest common geometry and trigonometry problems found in the curriculum materials in all necessary following areas: pythagorean triangles, 30°-60°-90° isosceles righ triangles, and 45°- 22 1/2°- 15°- /4° angles.

2. The student will be able to solve correctly in writing 80-85 percent of the posttest elementary algebra problems found in the curriculum materials in all necessary following areas: algebraic terms, formulae, and linear equations.

3. The student will be able to solve correctly in writing 80-85 percent of the posttest computational aid problems found in the curriculum materials in all necessary following areas: the slide rule.

4. The student will be able to solve correctly in writing 80-85 percent of the posttest practical mathematics applications problems found in the curriculum materials in all necessary following areas: word problems and area.

5. The student will be able to solve correctly in writing 80-85 percent of the posttest mathematical information presentation problems found in the curriculum materials in all necessary following areas: sketching from descriptions, graphing-coordination, graphing-pictorial, and graphing-tables, interpretation.

First Half Year Semester - 11 Grade

1. The student will be able to solve correctly in writing 80-85 percent of the posttest intermediate geometry problems found in the curriculum materials in all necessary following areas: basic constructions, regular polygons, inscribed and circumscribed polygons, properties of quadrilaterals, and similarity and equal areas.

2. The student will be able to solve correctly in writing 80-85 percent of the posttest intermediate algebraic equations problems found in the curriculum materials in all necessary following areas: factoring, LCM and fractional equations, radical equations, quadratic equations and formula, and simultaneous equations with 2 and 3 variables.

3. The student will be able to solve correctly in writing 80-85 percent of the posttest number theory problems found in the curriculum materials in all necessary following areas: base 8 and 12.

4. The student will be able to solve correctly in writing 80-85 percent of the posttest arithmetic problems found in the curriculum materials in all necessary following areas: squares and square roots, and laws of indices.

5. The student will be able to solve correctly in writing 80-85 percent of the posttest practical mathematics applications problems found in the curriculum materials in all necessary following areas: word problems and area.

6. The student will be able to solve correctly in writing 80-85 percent of the posttest introductory measurement problems found in the curriculum materials in all necessary following areas: micrometer.

Second Half Year Semester - 11th Grade

1. The student will be able to solve correctly in writing 80-85 percent of the posttest practical mathematics applications problems found in the curriculum materials in all necessary following areas: volume, mixture problems, lever problems, and linear and angular velocity.

2. The student will be able to solve correctly in writing 80-85 percent of the posttest advanced algebra problems found in the curriculum materials in all necessary following areas: complex numbers and graphs, variation, and boolean algebra and switching.

3. The student will be able to solve correctly in writing 80-85 percent of the posttest advanced geometry problems found in the curriculum materials in all necessary following areas: ratio of similitude, radian measure, arc measure, and frustrum of solids.

4. The student will be able to solve correctly in writing 80-85 percent of the posttest computational aids problems found in the curriculum materials in all necessary following areas: logs to different bases.

5. The student will be able to solve correctly in writing 80-85 percent of the posttest arithmetic problems found in the curriculum materials in all necessary following areas: remainder theorm.

First Half Year Semester - 12th Grade

1. The student will be able to solve correctly in writing 80-85 percent of the posttest intermediate algebra problems found in the curriculum materials in all necessary following areas: LCM and fractional equations, factoring, quadratic equations and formula, and multiplication and division of polynomials.

2. The student will be able to solve correctly in writing 80-85 percent of the posttest practical mathematics applictions problems found in the curriculum in all necessary following areas: area, volumne, and word problems.

3. The student will be able to solve correctly in writing 80-85 percent of the posttest intermediate and advanced geometry problems found in the curriculum materials in all necessary following areas: similarity and equal areas, and radian measure.

4. The student will be able to solve correctly in writing 80-85 percent of the posttest computational aids problems found in the curriculum materials in all the necessary following areas: logs to different bases.

5. The student will be able to solve correctly in writing 80-85 percent of the posttest measurement problems found in the curriculum materials in all necessary following areas: micrometer.

Second Half Year Semester - 12th Grade

1. The student will be able to solve correctly in writing 80-85 percent of the posttest advanced geometry problems found in the curriculum materials in all necessary following areas: ratio of areas and scale, frustrum of solids, properties of conic sections, and equations and use of conic sections.

2. The student will be able to solve correctly in writing 80-85 percent of the postest advanced algebra problems found in the curriculum materials in all necessary following areas: relations and functions, and solving polynomials, 3rd degree and up.

3. The student will be able to solve correctly in writing 80-85 percent of postest trigonometry problems found in the curriculum materials in all necessary following areas: trigonometric functions.

4. The student will be able to solve correctly in writing 80-85 percent of the posttest mathematical informations presentation problems found in the curriculum materials in all necessary following areas: extrapolation from graphs, and ratio of areas and scale.

5. The student will be able to solve correctly in writing 80-85 percent of the posttest practical mathematics applications problems found in the curriculum materials in all necessary following areas: Boyle's Law.

6. The student will be able to solve correctly in writing 80-85 percent of the posttest number theories problems found in the curriculum materials in all necessary following areas: bases 3, 4, 5, 6, 7, and 9.

7. The student will be able to solve correctly in writing 80-85 percent of the posttest computational aids problems found in the curriculum materials in all necessary following areas: the slide rule.

Rationale for the Semester Objectives

Until semester syllabi are available in all other related disciplines (i.e., vocational areas, and science) it is not possible to prepare a time sequence of curriculum units and course objectives more precise than in half-year semesters for typical students in all vocational areas taking the mathematics curriculum. The mathematics curriculum is sequenced for each vocational area rather than as a whole. Curriculum units appear in the half-year semester where they will be first extensively used in the other related disciplines. Some units, therefore, appear in more than one half-year semester and in more than one set of half-year semester objectives. Thus, any one student will not take all units and will not meet all semester objectives in any one half-year semester. The number of units he will not take and objectives he will not meet will generally increase in each vocational area as he advances through the curriculum. The objectives reflect this engineered manifestation of different mathematical needs for each vocational area by the use of the phrase "necessary following area."

Other terms employed in the objectives that need explanation are the words "elementary," "intermediate," and "advanced" in reference to content groupings. In the context of this curriculum, "elementary" refers to content that is without prerequisites in its own area. "Intermediate" and "advanced" differ in the number of prerequisite units.

The objectives contain a mastery criterion on achievement oriented posttests. These and diagnostic pretests are included in every curriculum unit and are the results of a systems development approach to curriculum design (Butler, 1967). Other problems and concerns mentioned in the history of memos and reports generated in the early planning stage of the curriculum have also been met by this approach. Individual differences have been provided for in the variable rates of learning that students will exhibit in completing the curriculum units, rather than in a track system. Remedial work needed by students will be provided by supplementary materials. And calculation skills are inherent in every unit--objectives are behaviorally oriented in terms of <u>problems solved</u> to a mastery criterion.

First half-year semester - 10th grade

Students will take more units grouped into this semester and the next semester than at any other time period in the curriculum. This semester's objectives form a common base for following semesters' curriculum units. The objectives in the arithmetic area is concerned with computational skills and in instilling in students a feeling of being at ease with numbers and their manipulations. The objective in mathematical reasoning complements this end. Both objectives provide for successive approximations of skills that will be used in elementary algebra next semester. The objective on introductory measurement, likewise, assures skills necessary as a base for common geometry and trigonometry next semester. Introductory measurement also provides specialized skills vital at this time to use in the science and vocational curricula.

Second half-year semester - 10th grade

Two types of skills are presented in this semester. A base for more advanced mathematical work is still being built by the objectives in common geometry and trigonometry, and elementary algebra. Also, this semester vocational and scientific practicum is emphasized with the objectives in computational aids, practical mathematics applications and mathematical information presentation. A specific example of student achievement at the end of this semester would be his ability to perform math skills in his 11th grade chemistry class.

First half-year semester - 11th grade

In this semester intermediate geometry becomes less number oriented and more construction and property oriented. This gives the student a base for construction in more advanced geometry and enables the vocational student to develop design and pattern techniques. Emphasis in intermediate algebra objectives is on equation solving, which has direct vocational and scientific application, in addition to serving as a preparation for advanced algebra. The student may also be introduced to number theory in the examination of bases 8 and 12. These have a direct application in the octal and duodecimal arithmetic of computers. Such an objective also gives an added insight into the normal number base of 10 and into mathematical creativity. Provision is also made in this semester for review and late starts in areas of arithmetic, practical mathematics applications and introductory measurement.

Second half-year semester - 11th grade

Practical mathematics applications are expanded and intensified to provide skills necessary in specific vocational areas. Advanced algebra includes units that feature mathematical understanding in general, such as complex numbers and graphs, and variations; and boolean algebra and switching which points directly to computer applications. The student is presented with the additional computional tool of logarithms and the arithmetic tool of the remainder theorem. Advanced geometry is heavily shop-oriented in terms of pattern-making and materials-shaping vocational areas. s

First half-year semester - 12th grade

This is a semester for major review and a final late start presenting objectives in intermediate algebra, practical mathematics applications, intermediate and advanced geometry, computational aids, arithmetic, and measurement.

Second half-year semester - 12th grade

Advanced geometry objectives still concentrate on applications to materials-shaping. Trigonometry is reintroduced with an extreme emphasis on the practical trigonometric functions and slide rule calculations involving these functions. Advanced algebra, on the other hand, has its emphasis on ascertaining generalizations between variables. Solving complex equations is also explored. These units will be of prime importance to students wishing to further their education beyond 12th grade. Final touches in presenting mathematical information are required of the students. The application of mathematics to everyday problems in the sciences is illustrated by an analysis of Boyle's Law. For those students who are interested in exploring number theory, bases 3, 4, 5, 6, 7, and 9 are examined.

The objectives of the curriculum units as they now stand were engineered for peak efficiency by systems development concepts. A limited form of task analysis was employed (Rahmlow, 1968 and Schill and Arnold, 1965) and emphasis is on maximum learning to a criterion level. The factors of usefulness and convenience to the student and maximum integration with all curricula in the system were also weighted heavily. If a student does not meet the objectives we can only assume that it is the fault of the curriculum materials. It is for this reason, that in spite of all the careful planning, the list of curriculum units and objectives should not be considered inflexible and 100 percent final.

Mathematics Requirements for Specific Vocational Areas

Three vocational areas have prepared detailed lists of mathematics content requirements for their Project ABLE materials. Electronics, Power Mechanics-Automobile, and General Woodworking are also developing materials stressing the application of these mathematics skills to specific vocational area situations as supplements to the mathematics curriculum.

The mathematics requirements (not necessarily sequenced) for these vocational areas are as follows:

Math Requirements - Electronics

1. Arithmetic computations
2. Fractions and decimals
3. Square roots
4. Algebraic manipulation of such concepts as Ohmls Law, Kirchoff's Law (transposing numbers in formulae)
5. Solving for unknown sides/angles of triangles (using trig functions)
6. Translating color coding into numerical values (including percent tolerances)
7. Side rule use (senior)
8. Logs (senior)
9. Reading voltmeter and ohmmeter scales
10. Determining wire size (using gage)
11. Computing RMS (route mean square) graphic representation of trig functions
12. Differentiation and intergration (basic)--what is meant by a derivative-integral
13. Conversion of fractions to decimals and vice-versa
14. Drawing graphs
15. Binary and octal systems (advanced)
16. Multiplier prefixes--milli, micro, pico, giga
17. Use of exponents
18. Computing vectors

Math Requirements - Power Mechanics/Auto Mechanics

Unit	Contents
Battery Service	1. Reading voltmeters, hydrometer values (adjust to temp. scale)
Spark Plug	2. Using standared automotives measuring tools, (thickness gauges etc.), decimal system & metric system
Measuring Tools	3. Same as above (English & metric system)
Lubricating	4. Pressures, liquid measurements, (weights of lubricants)
Pump Island	5. Liquid measurements; price per unit
Fasteners	6. Sizes of threads; fractional dimensions; relationship of sizes (English & metric)
Tire Service	7. Reading valves on wheel balance
Cooling System	8. Readings on pressure tester
Other	9. Addition of bills, checking service records
Machinery Practices	10. Use of measuring tools (gages, micrometers, dial indicators, etc.) computation out of round
Alignment Factors	11. Angles, radii (turning), two or three angles, (solid geometry)
Measuring Alignment Factors	12. Same as above
Inspection of Brake	13. Measure thicknesses
Service of Hydraulics	14. Fluid measurements, reading presures in pounds
Engine Compression	15. Reading comp. gauge and relating to specs.
Ignitions Systems	16. Reading volts, amps, meter, thickness gauges
Tune-up Test Equipment	17. Reading gauges, percentage of unburned hydro-carbons, dwell angles, volume, pressures
Tune-up Test Report	18. Clearences, tuning, idle speed, etc.

Math Requirements - General Woods

1. Common fractions (reduction, addition, subtraction, multiplication, and division)
2. Decimal fractions (conversion of decimals to common fractions and common fractions to decimals, table of decimal equivalents, conversion of dimensions, addition, subtraction, multiplication, and division)
3. Percentage (Definitions, applications to business problems and shop problems)
4. Ratio and Proportion (Definitions, direct and inverse ratios, proportion, and averages)
5. Mensuration (Rectangles, square root, triangles, regular plane figures, use of constants, trapezoids, composite plane figures, scale, circle, ellipse, cube root, solid figures, prism, cylinder, pyramid, cone, frustums, sphere, ring section, composite solid figures, volumes and weights, and formulas)
6. Measuring instruments (Micrometer and vernier micrometer, vernier caliper, protractor and vernier protractor)
7. Practical Algebra (Use of letters, substitution, addition, subtraction, multiplication, division, and simple equations)
8. Geometrical Constructions
9. The essentials of Trig. (function of angles, use of tables, right triangles, oblique triangles, and area of triangles)
10. Strength of Materials (Tension, compression, shear, factor of safety, unit stresses, pressure in pipes, and riveted joints)
11. Board measure (flooring, shingles, clapboards, and stairs)
12. Speed Ratios of Pulleys and Gears (Gear trains, idlers, compound gearing, worm and gear, pulley train)
13. Tapers (Taper computation, standard tapers, taper angles, taper turning, offsetting the tail stock, compound rest, and taper attachment)
14. Screw Threads (Pitch, lead, proportions of standard threads, sharp V-thread, American Standard square, Acme, metric, Witworth, pipe, and lathe gearing for cutting screw threads)
15. Cutting speed and Feed (Surface speed, rim speed, lathe, grinder, drill press, milling machine, planer, and shaper)

16. Gears (Spin gears, diametral pitch, tooth properties, use of formulas, table of formulas, rack and pinion, bevel gears, spiral gears, and worm gears)
17. Milling machine work (Indexing, direct, simple, differential, angular, multiple or block, and milling spirals)
18. Use of Tables (Powers, roots, circumferences, areas)
    Natural sines and cosines
    Natural tangents and cotangents
    Natural secants and cosecants
    Weights of materials
    American Standard Coarse-Thread Series
    American Standard Fine-Thread Series
    Metric Conversion
    Decimal and Metric Equivalents

Mathematics Requirements for Tenth Grade Science

In addition to the three vocational areas, the first year science curriculum had identified mathematics content requirements. These are presented in the following memo.

20 October 1967

TO: Helen Sager
COPY: J. Klingensmith
FROM: Audrey Champagne
RE: MATH CURRICULUM

Math topics that would be useful in the 10th grade General Science Program include:

Approximation
Estimation
Order of Magnitude
Significant Figures
Multiplication & Division of Exponents
Slide Rule
Graphs
dependent)-variable
Independent)

Vectors may be a good supplementary program, but it will probably be taught in General Science as such.

AC/pms

Syllabi of Mathematics Learning Units

From the mathematics requirements for Project ABLE materials in vocational areas and science, mathematics learning units were uniquely sequenced to meet the requirements. Math units teaching the necessary behaviors were then arranged into a suggested syllabus for each vocational area.

Three of these sample syllabi in mathematics are:

SYLLABUS OF MATHEMATICS LEARNING UNITS FOR ELECTRONICS

10

Order of Magnitude
Number Line
Base 10
Metric System
Whole Numbers
Fractions
Decimals and Mixed Numbers
Ratio and Proportion
Squares and Square Roots
Laws of Indices
Significant Figures
Estimation and Approximation
The Slide Rule

11

Grouping Symbols
Algebraic Terms
Number Representation
Mathematical Symbols
Linear Equations
Formulae
Radical Equations
Protractor
Complementary and Supplementary $\angle$s
Pythagorean $\triangle$s
30-60-90 Isosceles Right $\triangle$s

12

Graphing (3)
Complex Numbers, Graphs
Extrapolation from Graphs

Trigonometric Functions
Logs to Different Bases
Advanced Slide Rule

Relations and Functions

Boolean Algebra

Bases 8, 12
Bases 2, 3, 4, 5, 6, 7, 9

Post 12

Root Mean Square (Graphic Trigonometry)
Vectors
Differentiation and Integration

SYLLABUS OF MATHEMATICS LEARNING UNITS FOR POWER MECHANICS--AUTOMOBILE

10

- Order of Magnitude
- Number Line
- Base 10
- Metric System
- Whole Numbers
- Fractions
- Decimals and Mixed Numbers
- Measurement and Scales
- Micrometer
- Ratio and Proportion
- Averages
- Significant Figures
- Estimation and Approximation
- Squares and Square Roots
- Laws of Indices
- Slide Rule

11

- Protractor
- Complementary and Supplementary $\angle$s
- 45°, $22\ 1/2^\circ$, 15°, $\pi/4^\circ$ $\angle$s
- Radian Measure
- Arc Measure

- Graphing (3)

- Percentage

12

- Grouping Symbols
- Algebraic Terms
- Number Representation
- Mathematical Symbols
- Linear Equations
- Formulae

- Mixture Problems
- Lever Problems
- Linear and Angular Velocity
- Volume

SYLLABUS OF MATHEMATICS LEARNING UNITS FOR GENERAL WOODWORKING

10

- Order of Magnitude
- Number Line
- Base 10
- Metric System
- Whole Numbers
- Fractions
- Decimals and Mixed Numbers
- Measurement and Scales
- Micrometer
- Ratio and Proportion
- Averages
- Significant Figures
- Estimation and Approximation
- Squares and Square Roots
- Laws of Indices
- Slide Rule

11

- Grouping Symbols
- Algebraic Terms
- Number Representatior
- Mathematical Symbols
- Linear Equations
- Formulae
- Protractor
- Complementary and Supplementary $\angle$s
- Pythagorean $\triangle$s
- 30-60-90 Isosceles Right $\triangle$s

11

- Regular Polygons
- Sketching from Descriptions
- Base Constructions
- Inscribed and Circumscribed Polygons
- Properties of Quadrilaterals

12

- Area
- Similarity, Equal Areas
- Ratio of Similitude
- Ratio of Areas, Scale
- 45°, $22\ 1/2^\circ$, 15°, $\pi/4$ $\angle$s
- Radian Measure
- Arc Measure
- Volume
- Frustrum of Solids
- Trig Functions
- Advanced Slide Rule
- Conic Sections (2)

Writing, Testing and Revising the Learning Units

Level of Writing

The Quincy vocational-technical high school student does not differ in Aptitude, Ability or Achievement from the typical vocational-technical high school student elsewhere in the nation. In terms of Aptitude and Ability, the Quincy vocational-technical student does not differ from the national norm for the typical academic high school student. However, due to common differences in attitude and perceived goals, from academic high school students, the Quincy vocational-technical student* is not typical of all vocational-technical students who score below the national norm in achievement for academic high school students (S. M. S. G., 1964). This information concerning the Quincy vocational-technical student was obtained from a sampling of scores on the California Achievement Test, the Differential Aptitude Test, and the California Test of Mental Maturity. The results of this sampling are reported in graph form in Appendix C.

Conclusions that can be derived from this data are (1) there is no evidence supporting the argument for developing the learning units at any reading level other than the grade level for which it is aimed; and (2) extensive review in arithmetic fundamentals and reasoning should be available for those students requiring it.

Off-the-Shelf Materials

After the learning units had been identified and the level of writing had been determined a search of bibliographies of educational materials was made for the purpose of selecting sources of content for the units and modules. Three major bibliographies researched were: Hendershots' Bibliography of Programmed Learning, Entelek and Northeastern University's Programmed Instruction Guide, and Automated Education Center's Automated Education Handbook. Criteria for consideration of mathematics materials as

*Who is actively pursuing the Vocational-Technical curriculum. Not all students choosing the Vocational-Technical curriculum a year prior to entrance actually enter into it.

content sources were: (1) materials included content on the topic indicated in the title of a learning unit; (2) materials were written at a reading level applicable to the Quincy students; (3) materials were developed to meet behavioral objectives at criterion levels; (4) materials were revised from results of field testing; and (5) materials were field tested on a population possessing similar characteristics to Quincy students. Materials meeting these criteria were then submitted to several mathematicians on the staff of the Department of Curriculum Development, School of Education, University of Pittsburgh for technical review of specific mathematics content. Of the many materials evaluated by this two-step method, a partial list of the basic texts and supplemental texts used as sources of content is reported in Appendix D.

Construction of Learning Units

The format and structure of Project ABLE learning units was determined in the Sixth Quarterly Report (Appendix E). Off-the-shelf materials did not, in any sense, have exactly the same structure as finished Project ABLE learning units. What they did have that was needed, was mathematics content. Learning units were constructed in the appropriate format, around, and using the content of these materials. Instructional sequences from these materials were sometimes incorporated into learning units in varying degrees. If no content source was identified for a learning unit, mathematicians in curriculum development at the University of Pittsburgh were contracted to write the unit. Examples of all of these units have been included in Appendix F.

Time Schedule for the Writing of Learning Units

A tentative schedule for completion of drafts of the learning units has been set up. All learning units normally taken in the first half of the 10th grade and 11th grade years should be available in draft form by July, 1968. All other learning units should be written in this form by September, 1968.

Testing and Revision of Learning Units

The time schedule for the writing of learning units is structured to take advantage of predictable opportunities to field test the units.

The first testing of each unit follows its completion in draft form. The unit is tested locally on a small number of non-college bound and/or vocational-technical students at the appropriate grade level in one of the Pittsburgh Public Schools System High Schools. Revisions in the units that are made from this testing are not based on failures to achieve criterion levels of the objectives. Revisions at this point in the development, are usually concerned with directions, motivation, physical layout of frames, answer blanks, illustrations and phrasing of the text, etc., so that the unit is comprehensible to the student. The few students who attempt the draft unit are usually talked through it by the writer. The student is asked to verbalize his reactions and understandings of the unit. This behavior is noted and the unit is changed until the student can work through the unit smoothly, as the writer had intended. (See Appendix G - "Small Scale Testing of Learning Units.)

The tested draft learning units are reproduced and will be tested against the criterion levels contained in their objectives during the summer school session of July 1 to August 9, 1968 at Quincy Vocational-Technical High School. All learning units not tested during summer school, and/or not achieving the criterion levels for the students, will be tested and revised* during the 1968-1969 school year at Quincy. The curriculum itself will also be revised, if necessary, during this period in terms of sequencing units and adding or subtracting units.

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*Revisions will be made on the basis of an analysis of error patterns.

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APPENDIX A

Early correspondance concerning the development of the mathematics curriculum.

28 May 1965

TO: E. J. Morrison

FROM: R. M. Gagné

SUBJECT: Some thoughts on mathematics in the vocational curriculum

1. Here are three copies of correspondence I have had with three outstanding scholars of mathematics education, in the hope of doing a little advance planning, and to get some idea of what problems we shall be encountering when we get to the point of selecting instructional materials in Project ABLE.

2. I think the most important indication of these replies is that we shall have to make a separate determination of the necessary mathematics knowledge and skill, and of approaches to instruction in these topics. We shall not be able to assume that a set of materials labeled "modern math" will automatically solve our problem for us. Some new pedagogical techniques embodied in these materials will doubtless be of value. But other parts of it will not be.

3. This prospect is interesting from several points of view. Particularly, though, I think it has implications for subjects other than math, as well. In such things as English, science, and social studies we may also be faced with similar choices.

4. You may want to plan some discussion of this matter with people in the Quincy system at some appropriate time. In the meantime, put it in your file. I'm sure we'll have occasion to refer again to this correspondence.

AMERICAN INSTITUTE FOR RESEARCH

28 May 1965

TO: E. J. Morrison

FROM: R. M. Gagné

SUBJECT: Some thoughts on mathematics in the vocational curriculum

1. Here are three copies of correspondence I have had with three outstanding scholars of mathematics education, in the hope of doing a little advance planning, and to get some idea of what problems we shall be encountering when we get to the point of selecting instructional materials in Project ABLE.

2. I think the most important indication of these replies is that we shall have to make a separate determination of the necessary mathematics knowledge and skill, and of approaches to instruction in these topics. We shall not be able to assume that a set of materials labeled "modern math" will automatically solve our problem for us. Some new pedagogical techniques embodied in these materials will doubtless be of value. But other parts of it will not be.

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6 May 1965

Dr. John R. Mayor
American Association for the
Advancement of Science
1515 Massachusetts Avenue, N. W.
Washington, D. C. 20005

Dear John:

I should greatly appreciate having your very general reaction to a problem in mathematics instruction, which itself cannot be described very specifically as yet. I hope you will not mind taking the time to write me briefly about it.

Referring to the report of the SMSG Converence on Mathematics Education for Below Average Achievers, April, 1964, one of the recommendations pertaining to mathematics for students of low ability (p. 216) is: "r. Courses should be similar to the courses for high-ability pupils." This is amplified somewhat under B1 and 2. On the surface, the recommendation seems somewhat unrealistic. I am wondering what your reaction will be to why I think so.

We have started to work with the schools of Quincy, Massachusetts, to develop a new curriculum in vocational-technical education at the high school level. This will be quite a process, and we shall not be asking specific questions about mathematics for perhaps a year and a half. But some preliminary planning about where we are headed needs to be done. Our approach, as you might imagine, is to begin by studying the job, and the future trends in jobs. We hope to be able to analyze this descriptive data in such a way as to reveal what kinds of mathematics understandings are required for the performance of these jobs (quite a variety--auto mechanic, electronic technician, bookkeeper, etc., etc.) I expect we shall consider also other aspects of the individual's life, such as following baseball scores and standing (one of the dullest games I know of) as sources of educational requirements. At any rate, I expect us to be able by this means to define what mathematics the individual needs to know.

At this distance, I would expect the results of such an analysis to reveal that the vocational student needs to know such things as "translating decimals into fractions," "solving problems involving fractions," "stating proportions and percentages in symbolic form," and others, but I should not expect it to reveal a need for such things as "the properties of number systems." Yet the later kind of knowledge, which may perhaps be described in general terms as "the logical derivation of systems of quantification " is what dominates modern mathematics. (I could be wrong about that conclusion, of course.)

It can be argued (and I have heard it argued) that in order to do such things as "solving problems involving fractions," the student <u>really</u> has to know in a formal sense the properties of number systems, and particularly those of rational numbers in relation to whole numbers, etc. I certainly find this difficult to believe--just as difficult as the idea that a student must learn the rules of grammer (or perhaps the rules of phonmic change) in order to learn how to write.

It does seem to me there is a valid difference between a set of principles which help to insure retention and transfer and a set which may not. But this distinction is not necessarily the same one as is implied by the previous paragraph. The fact that certain principles pertaining to the logical derivation of mathematical ideas have now been formulated doesn't necessarily mean that these particular principles are best for pedagogic purposes. The number line, for example, is considered to be advantageous in the teaching of operations with negative numbers, primarily because it promotes retention and also generalization to later topics in analytic geometry. These are reasons for including the number line in courses for low-ability students, just as is the case for high-ability students. It would therefore be my expectation that some of the newer content of what is called "modern math" would be most useful for such students. I'm sure that other examples could be found, e.g., the place value of numerals. What I have doubts about is that one can with any clear rational simply state that the content of mathematics should be the same for low and high ability students. It seems likely that some of the content of modern math cannot be shown to be related to the practical mathematics of the job but only to certain advanced operations in college mathematics. Still another possibility is that certain kinds of content (for example rapid addition and subtraction) may be absent from most modern math, but desirable for vocational students.

It would be most helpful to my own thinking to know what is the general trend of your thoughts on these matters. What is it that I am overlooking, or failing to grasp?

Very best regards,

Robert M. Gagné
Director of Research

rmg:gf

American Association
for the Advancement of Science

1515 MASSACHUSETTS AVENUE, NW, WASHINGTON, D. C., 20005

Phone: 387-7171 (Area Code 202) Cable Address: Advancesci, Washington, D. C.

MAY 13 1965

May 11, 1965

Dr. Robert Gagne
Director of Research
American Institute for Research
410 Amberson [illegible]
Pittsburgh [illegible] nsylvania

Dear Bob:

Your [illegible] of May 6 arrived while I was out of the office for a few days; hence, the delay in replying. It was good to learn of the proposed work in schools of Quincy, Massachusetts, which you are undertaking. I believe that a new curriculum in vocational-technical education is perhaps the most needed curriculum development at this time and I am sure George Brain would also agree with this point of view. Your analysis of the needs in mathematics for various jobs should make an important contribution to the literature.

When we started our work with the University of Maryland Mathematics Project it had been our intention to write a revised edition for students in the lower half of the seventh and eight grade population. However, after tryout for two years the teachers who had been teaching these materials to classes in this group advised that the materials did not need to be rewritten, but rather that the teachers be permitted to cover the seventh grade course in two years. Without pursuing this matter much further we reached the decision that this perhaps was as good as we could do because of our limited staff. For a number of years several of the schools in the Washington area did use the seventh grade book as the text for two grades (7 and 8). I think schools in this area are using the books now. It is my understanding that Montgomery County uses a commercial textbook written by Keedy, Jameson and Johnson, which was based on the University of Maryland materials, but they use this textbook for the upper half of students. I am not sure what they are using for the lower half.

Inspite of our experience at Maryland, and without additional experience on which to base an opinion, I find myself in considerable agreement with what you suggest, both as to topics and the teatment of them. In investigations in mathematics education in the past 30 years, the greatest concern has been about the teaching of fractions. Many have held the hope that to teach properly the properties of the number system and then to introduce rational numbers as a system, would provide the best solution for all intermediate grade students. But I am not at all convinced that the development of number systems including rational numbers has or will solve this problem. In fact, this practice has probably made fractions more difficult. Henry would disagree here, I feel sure, and yet Henry has changed his point of view considerably in the three years working with AAAS. He, as you probably know, first wanted a very abstract mathematical development. Now he seems very much interested in development in terms of applications.

Your comments on the number line seem quite appropriate. Have you seen the materials that SMSG produced for students of lower ability in seventh and eight grade? These might be worth looking at, although they are not directed particularly to the uses of the mathematical competencies to which you refer. You are quite right that in the new mathematics programs there is practically no reference to speed in computation and no encouragement of it.

Right now I do not think that you have overlooked much of anything. I guess my principal concern would be that in the new program one try to avoid, as much as possible, teaching computational skills like teaching the operation of machines. Yet in avoiding this it does not seem necessary to bring in too many of the developments of modern mathematics. There surely is a compromise somewhere between.

Sincerely yours,

John R. Mayor
Director of Education

JRM/nc

SCHOOL MATHEMATICS STUDY GROUP
SCHOOL OF EDUCATION
STANFORD UNIVERSITY
STANFORD, CALIFORNIA

E. G. BEGLE
Director

May 19, 1965

Dr. Robert M. Gagne
Director of Research
American Institute for Research
410 Amberson Avenue
Pittsburgh 32, Pa.

Dear Bob:

I think I can answer your letter with very few words. The statement "Courses for students of low ability should be similar to the courses for high ability students" is the result of a passionate belief that low ability students should not end up as second-class citizens. However, there is essentially no experimental evidence either for or against the statement. In fact, no matter whether we consider the low ability students or the high ability students we have a surfeit of passionate beliefs and an extreme paucity of factual information. For the former group of students we are beginning to collect some information from the SMSG Longitudinal Study. For the low ability students I am afraid it will be a long time before we have any solid information.

Everything you say in your next to the last paragraph of your letter of May 6 is extremely plausible. Nevertheless, we simply don't know what the story is in fact and it is high time that something is done about it.

Best regards,

E. G. Begle

MAY 26 1965

BOEING SCIENTIFIC RESEARCH LABORATORIES
P.O. BOX 3981
SEATTLE, WASHINGTON 98124

May 25, 1965

IN REPLY REFER TO

1-8240-4016

Dr. Robert M. Gagné
Director of Research
American Institute for Research
410 Amberson Avenue
Pittsburgh 32, Pennsylvania

Dear Bob:

I am sorry to be so much *en retard* in answering your query about vocational mathematics. My reactions, admittedly not well thought out, are about as follows:

I think the remark that courses for low ability students should be similar to the courses for high ability pupils was a little bit overdrawn in order to reassure some of the representatives at the conference that no discrimination against minority groups would be permitted.

I think you and I rather agree that an informal approach which does involve real understanding and uses those properties which are essential for manipulation is the kind of thing to work for in a curriculum of the vocational-technical type. My own feeling would be that there is a lot in many modern mathematics treatments which does put undue stress on structure and abstract principles--principles which have nver been clearly shown to aid practical performance of arithmetic operations. I quite agree that there is no convincing proof that the principles pertaining to logical derivation of mathematical ideas necessarily are those best for pedagogic purposes.

I guess what I am saying, Bob, is that I don't know the answer, that I think no one else really does, and I suspect that you are on the right track on what you said.

Best personal regards.

Cordially,

B. H. Colvin
Head, Mathematics Research Laboratory

HEADQUARTERS OFFICES — THE BOEING COMPANY

REPORT OF MEETING OF THE COMMITTEE ON MATHEMATICS

7 May 1966

Quincy, Massachusetts

Attending: Norman C. Harris, Chairman: Robert M. Gagné, Edmund King, Margaret King, Phil Ryan, Gilbert Syme

Group leader Harris called upon Mr. Gagné to qcquaint the group woth the present status of Project ABLE, and we are indebted to Mr. Gagné for his clear concise presentation.

The discussion opened with though given to the desirability of setting up a mathematics curriculum which would enable interchange of students between the college-bound and the non-college-bound. Mr. Pruitt entered our room during this discussion and, when asked his views on the subject, stated that, in his opinion, our group should concern itself only with a mathematics curriculum suitable for the vocational-technical school.

Mr. Harris listed on the board certain guidelines or hypotheses to assist in giving our discussion some direction as follows:

1. A course should have a good balance of practicality and intellectuality. In short, our interest lies in course content and also how the course is taught.

2. A course should have a carefully planned sequential structure.

3. A beginning course should include some of the features of arithmetic, algebra, geometry, and right-angle trigonometry. "Classical content" must be ignored and content selected which will be applicable to needs of technical students.

4. Consideration should be given to re-orientation and in-service training of teachers.

5. Consideration should be given as to the availability of suitable texts.

6. Consider the idea of a "core" nathematics program for all mathematics students in the vocational-technical school.

7. The "core" program itself.

One of the major ideas represented by the proposed mathematics program (for grades 10-12) is that it would be designed to provide certain kinds of "terminal competence" at the end of each year. This means a reorganization of the topics of mathematics, although it was thought that the total coverage of topics throughout the three years might be not very different from that of the standard high school program.

Somehow, our discussion began with a consideration of mathematics at grade 9 level (which some of us assumed to be within the scope of our deliberations) and Mr. Gagné pointed out that the vocational-technical school would start at grade 10 level. Concentrating on this level. Concentrating on this level, the question arose--one course or track for all students, or more than one course? Also, the question of a general entrance testing program to evaluate the abilities of entering students. It was brought to the attention of the group that our sources of supply of students would be from the several 9th grades throughout the city.

The group recommends a basic grade 10 course for all mathematics students and points out that a remedial course may be necessary for some before they enter the basic core course.

With theis basic course in mind, the group discussed subject matter to be included in such a course. (We are indebted to someone on Project ABLE for the excellent code sheets which aided our diliberations.)

The following is the recommendation of the group with respect to a basic grade 10 mathematics course. We wish it understood that this program is not inflexible.

1. Refresher material (arithmetic). Rigorous work in decimals and fractions. Understanding and competency in computations. Applications to typical life problems. Use of multi-media, auto-tutors, programmed materials, etc., to supplement regular calss work.

   An achievement test for all students at the end of 10-12 weeks. Those who qualify will continue into the next section. Those who do not qualify will continue with section (1) material.

2. Measurement - geometric shapes- systems of measurement - units involved - instruments of measurement - kitchen measurements - liquid-dry - length, areas, volumes. Introduction to dimensional analysis (How do different units of measure and areas of various geometric figures compare?). Introduction to the idea of macro and micro--powers of ten notation.

   A time period of 6-8 weeks in section (2) with an achievement test to determine those qualified to continue to section (3).

3. Elementary Geometry - constructions; properties of figures; elementary nature of proof; applications to industrial and technical problems.

   A period of approximately 6 weeks is recommended for section (3).

4. Number Systems - Numbers to bases other than 10; Negative numbers; abacus; irrational numbers (elementary concepts only); complex number (elementary).

5. Algebra - Symbolism and the use of literal numbers; the four fundamental operations; some factoring; equalities; solution of simple linears; use of modern terminology; ratio and proportion; problem solving.

6. Right-angle trigonometry - Introductory material and solution of practical problems.

The mathematics group adjourned at 1:05 p.m.

From A Report on Project ABLE, Dated: 26 May 1966

On the academic side, a Social Science curriculum outline covering a three -year sequence of material has come from several small working group meetings. Using a theme of student centered problem solving and decision making process, specific content items (topic objectives) are now in preparation and some selection of instructional materials is underway. Some portions of this curriculum area are programmed for tryout early in the 1966-67 school year. In Mathematics, some structuring of the kind and degree of content has been accomplished on the basis of estimates made by various faculty members. There appears to be a three-way clustering of jobs in terms of the estimates of mathematics required in the vocational areas and a three-way three-year sequence has been tentatively outlined. The Science and English areas received initial attention during the Project ABLE Advisory Panel working sessions on 7 May attended by some 25 faculty members and Quincy Public Schools staff. These working groups provided some direction and specific purpose.

From the <u>Sixth Quarterly Report</u>, Dated: 30 September 1966

<u>Mathematics and science courses</u>. Objectives for these two sets of courses derive primarily from analysis of the goal of vocational competence. Identification of objectives for mathematics and science began with an analysis of objectives in the vocational areas for prerequisite mathematical and scientific capabilities. A detailed checklist of skills and knowledges was prepared for each of the two subject areas (See Appendix A). These checklists were used by the vocational specialist and the "academic" specialist to identify capabilities as essential, desirable, or unnecessary for the jobs in his area. Data collected from all vocational areas were consolidated. It then was possible to identify a "core" content required for nearly all job sequences and also to define several subsets of additional capabilities, each of which was appropriate only for some job sequences. The topics selected by this procedure provide the minimum content for courses in mathematics and science.

While vocational requirements supply a major portion of the demand for topics in mathematics and science, a few topics appear in response to the goal of responsible citizenship. Thus, for example, no vocational area requires any capability with respect to the concepts of nuclear physics. Yet, nuclear energy is a matter of important concern to today's citizen, so some elementary units in this material hopefully will be included. Several topics in biology are included on the same basis. Of course, many topics included first for vocational reasons also serve the goal of responsible citizenship and the goal of self-fulfillment as well.

Many of the topics being selected for mathematics and science are of a size appropriate as learning units for students with the expected entry capabilities. Some topics are more complex and must be analyzed into their components which are appropriate as learning units. In any case, the learning units are being identified by the familiar process of analyzing selected terminal performance capabilities for their prerequisite capabilities.

Excerpts from a memo to E. J. Morrison from Norman C. Harris on the Conference with Mr. Philip Ryan re Mathematics program on November 18, 1966 follow:

Suggested Action for Winter-Spring

1. Compile data on the actual math level of all stuedents in the general math courses in the junior high school. Find out what the "raw material" is going to be like as it moves toward the 10th grade. The tests used should identify strengths and weaknesses in the various areas of mathematics--fundamental operations, mastery of concepts and definitions, computational skills, simple algebra, elementary geometry, measurement, problems solving (word problems).

2. In cooperation with the trade and technical (and business) instructors of the skill areas, identify the actual mathematics skills and concepts necessary during the 10th grade vocational work. This must be done carefully, not hastily off some checklist. Use Flanagan's "Critical Incident Technique" to force a considered response. Perhaps two categories--"necessary to know" and "nice to know"--might emerge.

3. Purchase, borrow, or requisition massive collection of new textbooks, workbooks, programmed texts, filmstrips, tapes, slides, single concept films, etc., related to the topic of elementary technical mathematics. Purchase (or rent) one each of several pieces of hardware (Autotutor, tape recorder, film viewer, other teaching machines) to use in evaluating and experimenting with the new materials.

4. Select carefully (!) one or two other people and give them 1/4 or 1/2 time on the project to assist in planning and experimentation. Perhaps new leadership will be required. I leave this to your judgment. But a great deal more manpower is essential, at once!

5. Divide the 10th grade core course into four phases, to correspond with the four terms on which the high school now operates. Set the course up in four independent units 10A, 10B, 10C, 10D. Exceptionally able students might start at 10C level. Slow students might have to take 10A twice before going on to 10B. Each phase must be satisfactorily completed (mastery) before going on to the succeeding phase. Achievement tests, yet to be developed, will be given at the end of each phase to determine whether or not a student moves to the next phase.

Great flexibility in scheduling will be essential. For example, it might be necessary in September to start one or more sections of 10C (for able students who could skip 10A and 10B), several sections of 10B, many sections of 10A, and perhaps a few sections of a very low-level course, say 10X, for those not capable of starting out in 10A. Transfer from one phase to another (up or down)should be possible whenever the performance data justify it. (Weekly quizzes could determine this.)

I would submit that a "mathematics learning laboratory" might be essential. This facility could be set up in a standard classroom, fitted wiht carrels for individual study. Each carrel should have one or more pieces of "new media" hardware. The "library" of the learning lab would be stocked with complete sets of slides, strip films, tapes, single concept films, etc., which students could check out with little if any red tape, for use in the learning lab. Teachers would distribute each week the titles and call numbers of the various media available for learning the "lessons" of that week. The learning lab would be open all day at every hour, and its use should be made convenient for students not difficult. My recommendation is that it be in or near the vocational spaces, or near the math rooms, and not in the library. A technician or classified employee could be in charge, or, if the system desired, beginning teachers or practice teachers could be rotated into the lab to assist students. Several copies of each item have to be available. If the item desired is always "out" when students ask for it, they soon get out of the habit of using the learning laboratory.

It should be emphasized that the learning lab and, indeed, all "new media," are adjuncts to learning. They supplement teaching--tehy do not replace it. They allow "slow" students to catch up, and able students to move ahead faster than the class, if they desire.

6. It is far too presumptuous of me to say what context should be included in each of the four phases. This task should be the major effort of a planning group during the next several months. As a very rough suggestion, however, the following might be used as a guide to get their planning started:

10A. Review of fundamental processes of arithmetic. Strong development of decimals and fractions. Place value. Powers of ten. Computations--getting the right answer to practical problems of the trades and business. Measurement and measuring systems. Problems involving formulas. Emphasis on word problems in the use of formulas.

10B. Transition from simple formulas to linear algebraic equations. Literal numbers. Four fundamental processes in algebra. Signed numbers. Word problems involving linear equations. Ratio and proportion. Direct and indirect variation. Word problems with many applications from the shop, labs, and business.

10C. Introduction to geometry and trigonometry: The general triangle; the right triangle; 30°, 60°, and 45°; Pythagorean theorem--analytical and geometric; Geometry of the circle and other common plane figures; Elementary solid figures (formulas); Applied problems on all the above; The trigonometry of the right triangle; Sine, cosine, tangent; Use of the tables; Problems involving right triangle trigonometry.

10D. Introduction to graphics; Making and interpreting graphs; Graphs of linear equations; Directed line segments--elementary vectors; Introduction to exponents, powers, and roots; Elementary logarithms--use of tables; Introduction to slide rule--much practical problem solving.

The above is, of course, just a hasty, off-the-cuff stab at what might be in a basic 10th grade core course. It needs much refinement. It is a skeleton whose bones will need filling out by long hours of planning--accepting this topic and discarding that. There is never time enough for everything--and many topics will have to be treated superficially or not at all in 10A, B, C, D. This course lays the groundwork, however, for treatment in depth during the 11th and 12th years. The one major goal of the 10th grade course should be to insure that all students can engage in rapid, accurate calculations of an arithmetic nature, and get the right answer!

Scheduling will be a problem. So will staffing. Possibly most of the teachers for this program will have to be newly recruited. Articulation with the vocational teachers and with the teachers of the science core course should be continuous.

A big job lies ahead. A start has really not yet been made. These notes are in confidence for your use as you see fit. Best wishes.

Possible Schedule/Harris Suggestion

Time blocks in the day	Plan for students going to work right out of high school			Alternate for students going on to post-high school	
	10th	11th	12th	11th	12th
	Same for all				
1	Explore skill	Skill	Skill	Skill	Skill
2	Explore skill	Skill	Skill	Skill	Skill
3	Math	Skill	Skill	Math	Math
4	English	English	Skill	Science	Science
5	S. Studs.	Elective	Humanities	English	Humanities
6	Study or Elective	Science	Elective	Elective	Elective
7	Phys.Ed.-Elective	Phys.Ed.-Elective	Phys.Ed. Elective	Phys.Ed. Elective	Phys.Ed. Elective

Note: Elective may be study period, drama, art, music, typing, other math, science, other social science such as economics, history, psychology.

APPENDIX B

Examples of Sources of Content Topics

INSTRUCTIONS FOR USE OF THE CODE SHEETS
FOR MATHEMATICAL SKILLS; A-#1 through E-#52.

For each job you have listed for which you will teach in your field----please go through the listings of A-1 up to and including E-52 and if that coded mathematical skill is needed by the student--to complete any task of that particular job---encircle the code as follows:

	A-10
3	A-11
2	A-12

In the column to the left of the encircled codes only, please indicate by the insertion of a 1, 2, 3, if the degree of emphasis required in the teaching of this skill is a:

1. small amount - go over in passing; good but not necessary.
2. average amount - good knowledge; highly desirable
3. great amount - excellent knowledge following much practice; essential

When a code such as A-10 does not apply do nothing.

SELECTED JOB TITLE________________

A-1; Addition as such.

A-2; Subtraction as such.

A-3; Multiplication as such, also large number products.

A-4; Division as such, also long division.

A-5; Involution or raising of a number to a power.

A-6; Evolution or extraction of a square root from a number.

A-7; Square root or finding by method the square root of any number.

A-8; Cube root or finding by method the cube root of any number.

A-9; Axioms i.e. equals added to equals their sums are equal or the shortest distance between two points is a straight line.

A-10; Decimals, definition and describe

A-11; Fractions, definition, place and power i.e. 231 = 200 + 30 + 1 where 200 = hundreds, 30 = tens and 1 = units.

A-13; Base Binary System - 2 digits 0 and 1; 2 =10; 3=11; 4=100

A-14; Base 3; 3=10; 4=11; 5=12; 6=20; 9=100

A-15; Base 4; 4=10; 8=20; 16=100

A-16; Base 5; 5=10; 10=20; 25=100

A-17; Base 6; 6=10; 12=20; 36=100

A-18; Base 7; 7=10; 14=20; 49=100

A-19; Base 8; 8=10; 16=20; 64=100

A-20; Base 9; 9=10; 18=20; 81=100

A-21; Base 10; See A-12 our present number system.

A-22; Base 11; 10=T; 11=10; 12=11; 21=1T; 22=20; 32=2T; 120=TT

A-23; Base 12; Duo-decimal System 10=T; 11=E; 12=10; 22=1T; 23=1E; 24=20; 100=84; 120=T0; 132=E0; 144=200; 143=EE

A-24; Slide Rule - Use of and its accuracy.

A-25; Reciprocals - Denotes different kinds of mutual relation; the reciprocal of a number is 1 divided by that number. This process generally is easy when powers of 10 are used.

A-26; Factor or factors - what are they? - Use of.

A-27; Fractions - in addition

A-28; " " subtraction

A-29; " " multiplication

A-30; " " division

A-31; Decimals - in addition

A-32; " " subtraction

A-33; " " in multiplication

A-34; Decimals - in division

A-35; Construction - bisection of an angle

A-36; Construction - perpendicular to a line point

A-37; Construction - paralled to a line

A-38; Construction - perpendicular to a line from a point not on that line - shortest distance.

A-39; Construction - perpendicular bisector of a line sgment.

A-40; Construction - An angle equal to a given angle

A-41; Consturction - draw a triangle - given three sides.

A-42; Construction - find center of an arc

A-43; Construction - find radius of an arc

A-44; Construction - divide a straight line into any number of equal parts

A-45; Construction - draw a regular polygon - multiples of three

A-46; Construction - draw a regular polygon - multiples of four

A-47; Construction - draw a regular polygon - multiples of five

SELECTED JOB TITLE______________________

B-1; Metric System - know and be able to transfer into inches and feet, etc. from meters and grams etc.

B-2; Micrometer - be able to read and understand the reading.

B-3; Units - what are feet, inches, degrees, square feet, etc. Understand what dimension and know how to convert or transfor linear, area and volumetric units.

B-4; Tables - reading and understanding of any standard table.

B-5; Graph - co-ordinate in what quadrant. Understanding of a 2 dimensional graph.

B-6; Graph - pictoral - read and understanding.

B-7; Interpolation - the ability to read between table limits and to give an approximate reading in the figures of that table.

B-8; Extrapolation - the ability to read beyound the given variable limits of the table and give a good approximation.

B-9; Signed Numbers - to perform all functions with these numbers i.e. (-4)+(+8) = +4, etc.

B-10; Number Line - show that B-9 is correct by use of this.

B-11; Pythagorean Theorem - $a^2 + b^2 = c^2$ or $x^2 ty^2 = r^2$

B-12; 3-4-5, Right triangle - recognize in any form - find 3rd side given any 2

B-13; 5-12-13, " " "

B-14; 7-24-25, " " "

B-15; 8-15-17, " " "

B-16; 9-40-41, " " "

B-17; 30° and 60° angles - construct and use in a right triangle

B-18; 45° and 90° angles - " "

B-19; 22 1/2 and 15° angles - " "

B-20; Pi = π = 3.1416

B-21; $\pi/4$ = .7854

B-22; Complementary angles - sum of 90° for any 2 angles.

B-23; Supplementary angles - sum of 180° for any 2 angles.

B-24; Fraction - 3 signs of and use of; $\left(- \frac{-N}{-D}\right)$

B-25; Protractor - measuring of an angle

B-26; Base e - logarithms.

B-27; Base 10 - logarithms (by table)

B-28; Any Base - logarithms. 2 to the 5th power = 32 or log of 32 to the base 2=5

B-29; Degree - What is a degree of an angle? What is an angle - arc degree and what it is?

B-30; Radian - What it is and degree relation.

SELECTED JOB TITLE_______________________________

C-1; Sketching - a rough design or plan - a brief outline of facts in figures.

C-2; Graph (co-ordinate) - Use in showing relations between lines or curves, etc.

C-3; Graph (pictoral) or (circle) - Use in showing numerical relations; parts of the whole by per cent or fraction.

C-4; Ratio - comparison of 2 measures of the same unit.

C-5; Proportion - expression of equality between any two or more ratios.

C-6; Inversion of a Proportion - invert both ratios of a proportion.

C-7; Mean Alternation - alternating the 2nd and 3rd or mean terms of proportion. 2:4::5:10 - 4 and 5 are mean terms

C-8; Extreme Alternation - alternating the 1 stand 4th or extreme terms of proportion 2 and 10 in C-7.

C-9; Addition of Proportion - the adding of the denominator to the numerator and placing sum over denominator or under the numerator 2:3::4:6 goes to 2:3+2::4:6+4 or 2:5::4:10

C-10; Subtraction of Proportion - as in C-9 except we subtract, not add.

C-11; Product of Means = Product of Extremes - use of advant ages.

C-12; Variation - one of the different linear arrangements that can be made of any number of objects taken from a set. An expression of equality of 3 times 8 divided by 4=4 times 9 divided by 6 where 3 becomes 4, 8 becomes 9 and to [illegible] complete the variation when 4 becomes 6.

C-13; Joint Variation - X varies jointly as y and z or x divided by y times z.

C-14; Direct Variation - X varies directly as y or x divided by y

C-15; Inverse Variation - X varies inversly as y or x times y.

C-16; Per cent - divided by 100 or some part of.

C-17; P = B x R - Given any 2 of these, solve for other in Percentage = Base times rate.

C-18; D = R x T - Given any 2 of these, solve for other in the Distance = Rate times time.

C-19; I = E x R - Electrical the Amperes = Volts times Resistance

C-20; Similitude - Ratio of one triangle of same shape to another.

C-21; Congruency - exactly same size and shape.

C-22; Equality - same area or size but not the same shape.

C-23; Line - ratio of side opposite angle to the hypotenuse.

C-24; Cosecant - ratio of hypotenuse to the side opposite the angle.

C-25; Cosine - ratio of hypostenuse to the side opposite the angle.

C-26; Secant - ratio of hypotenuse to the adjacent side of the angle

C-27; Tangent - ratio of the opposite side of the angle to the adjacent side.

C-28; Cotangent - ratio of the adjacent side of the angle to the opposite side.

C-29; Cofunctions - of complementary angles are equal. Line and Cosine

C-30; Eight Fundamental Relations - such as; Sine squared plus Cosine squared = one.

SELECTED JOB TITLE________________

D-1; Definition - Triangle
D-2; " Circle
D-3; " Ellipse
D-4; " Quadrolateral
D-5; " Rectangle
D-6; " Square
D-7; " Parallelogram
D-8; " Trapezoid
D-9; " Trapezium
D-10; " Polygon
D-11; " Regular Polygon
D-12; " Rhombus
D-13; " Parabola
D-14; " Hyperbola
D-15; Parts of - Circle
D-16; " Ellipse
D-17; " Parabola
D-18; " Hyperbola
D-19; " Trapezoids
D-20; " Quadrilaterials
D-21; " Parallelograms
D-22; " Regular Polygon
D-23; Area - Circle
D-24; " Right Triangle
D-25; " Scalene Triangle (Oblique Triangle)
D-26; " Rectangle
D-27; " Square
D-28; " Parallelogram
D-29; " Trapezoid
D-30; " Regular Polygon
D-31; " Composite Figure - by Parts
D-32; " Scaled Figure - Ratio of Areas
D-33; " Ellipse ab π = A
D-34; Lateral Area - Pyramid
D-35; " Cone
D-36; " Cylinder
D-37; " Frustum of a Cone
D-38; " Frustum of a Defined Triangular Pyramid
D-39; " Frustum of a Defined Square Pyramid

D-40; Surface Area - Cone
D-41; " Cylinder
D-42; " Sphere
D-43; " Cube
D-44; " Rectangular Solid
D-45; " Prism
D-46; " Frustum of a Cone
D-47; " Frustum of a Triangular Pyramid
D-48; " Frustum of a Square Pyramid
D-49; Volume - Sphere
D-50; " Cube
D-51; " Rectangular Solid
D-52; " Prism
D-53; " Cylinder
D-54; " Frustum of a Cone
D-55; " Frustum of a Pyramid
D-56; " Code
D-57; " Pyramid
D-58; " of a Composite Solid Figures

SELECTED JOB TITLE ______________________

E-1; Algebraid terms - Basic names and definitions.
E-2; Formulas - Know some - Know how to use the rest.
E-3; Inverse Functions - if addition exists - subtract that amount in equals subracted from equals, etc.
E-4; Parentheses - How to use.
E-5; Factoring - Common Factor Type
E-6; Factoring - Difference of Perfect Squares
E-7; Factoring - Trinomial Perfect Square
E-8; Factoring - Trinomial Trial and Error.
E-9; Factoring - Sum and Difference of Cubes
E-10; Factoring - Grouping Type
E-11; Factoring - Completion of the Square Type
E-12; Lowest common multiple - solving of fractional equations.
E-13; Fraction Equations - How to solve
E-14; Prime Numbers.
E-15; Polynomial Multiplication of Algebraic Expressions.
E-16; Polynomial Division of Algebraic Expressions.
E-17; Solving Linear Equations
E-18; Solving Quadratic Equations
E-19; Quadratic Formula - Develope and use.
E-20; Solving Equations of Power greater than 2nd.
E-21; The Remainder Theorem - Solving of E-19
E-22; Functional Notation - used in solving E-19
E-23; Functional Notation - as such; $y = f(x)$
E-24; Relation - Each value of x has 2 or more values of y
E-25; Function - Each value of x has 1 and only 1 valvue in y.
E-26; Conic Sections - Relate to algebraic expressions
E-27; Ellipse - equations of and their use.
E-28; Circle - " "
E-29; Parabola - " "
E-30; Hyperbola - " "
E-31; Parabola - Maximum and minumum points
E-32; Simultaneous Equations - of two unknowns
E-33; Simultaneous Equations - of three unknowns
E-34; Radical Equations - Solving
E-35; Extraneous Roots - Definition and Use in E-33
E-36; Principal Value - Root of an Equation use.
E-37; Index Laws or Laws of Exponents - like bases andd exponents, etc.
E-38; The use of "i" or "j" in equations. Definition.

E-39; Imaginary Numbers - Square root of a negative unit and the product of two is equal to minus one.

E-40; Complex numbers - a plus b times i - Use of.

E-41; Boolean Algebra - Use in solving of electrical switch problems and their reduction to the simplest form.

E-42; Mixture Problems - setting up word problem equations.

E-43; Distance = Rate times time - setting up word problem equations.

E-44; Digit Type - setting up word problem equation.

E-45; Numbers Type - setting up word problems eqation - a son is twice as old as his father etc.

E-46; Coin Type - setting up word problem equation.

E-47; Work relation types - setting up word problem equation

E-48; Interest or I = PRT - setting up word problem equation

E-49; Lever Type - setting up word problem equation

E-50; Variation as Boyle's :aw pf Gases - setting up word problem equation.

E-51; Linear Distance Relation to angular distance through radians - setting up word problem equations.

E-52; Linear Velocity relation to angular velocity through radians - setting up word problem equation.

TEXTBOOK SYNOPSIS

Cooke, N.M., Basic Math for Electronics, McGraw-Hill Book Co. Inc., N.Y. Toronto, London, 1960

Electronics

Basic

Math - advantage in mental training. A math problem demands logical thinking to proceed to the solution. Analyzed - to determine best method of solution. Relate Math work to problems (practical) which interest the student.

1. Number System (digit etc.)
   a) Add, -, X, ÷
      Long (X & ÷)
2. Averages
3. Fractions (common) +, -, ÷, X
4. Mixed Nos.
5. Improper fractions
6. Prime nos. or factors
7. L.C.M. & L.C.D.
8. Decimals
9. Ratio
10. Percentage
11. Square Root
12. Terms of Algebra - Definition of words used
13. Signed Nos.
14. Parentheses
15. Graph (number line)
16. Laws of Exponents
17. Multiplication (Algebraic)

INSTRUCTOR OPINION

Course of Study - Mr. Dodd (Sheet metal)

<u>1st yr. Related Math</u>

I. 1. Area of the Rectangle, Square

2. Perimeter of the Rectangle, Square

II. 1. Lateral Area of Square, Rectangle

III. 1. Volume of any st. sided tank

2. Volume in gallons, qts, pts, and gills. Gal = 221 cu. in.

IV. 1. Conversions to cubic feet etc:

2. Weight of contents

1st quarter

V. 1. Circumference - Circle

2. Area of a circle

3. π what is it? $A = \pi r^2$

4. What is .7854 $A = .7854\ D^2$ $\left(\frac{3.1416}{4} = .7854\right)$

VI. 1. Convex Area

2. Total Area

VII. Vol = Area of Bottom X H.

2nd quarter

<u>Review Math - Sheet, Metal</u>

1. Add. and Subt. of fractions
2. Mult. and Div. of fractions
3. L.C.D.
4. Improper fraction and mixed nos.
5. Conversion to decimals - Deci -- fraction
6. Add. and Subt. of decimals
7. Mult. and Div. of decimals
8. Sq. root - How to prove
9. Use of formulas
10. Lever prob.
11. Transposing (equations)
12. Sq. of nos. ending in 5

SECTION 13 MATHEMATICS REQUIRED FOR RADIO, ELECTRONICS, AND TELEVISION TECHNICIANS

The topics listed in this section constitute minimum requirements for students studying in this field. How the mathematics is organized and given will depend on the nature of the school undertaking this program. The time sequence of the various subjects is set forth in Sec. 15.

The student should understand and be able to perform problems involving:

ARITHMETIC

Addition, subtraction, multiplication, and division of decimals and fractions (LCD)

ALGEBRA

1. Addition, subtraction, multiplication and division of
   a. Literal numbers (algebraic)
   b. Positive and negative numbers
   c. Polynomials
2. Powers of 10--for expressing numbers and for operation
3. Substitution in formulas
4. Solution of simple equations with one unknown, including equations involving fractions
5. Solution of simultaneous linear equations
6. Reciprocals
7. Graphs and nomographs
8. Powers and roots
9. Fractional exponents
10. Factoring of polynomials
11. Logarithms--multiplication and division of numbers by logs and application to decibels
12. Graphical additions

GEOMETRY

Right triangles.

TRIGONOMETRY

1. Functions of angle (sin, cos, tan)
2. Use of tables of trigonometric functions
3. Graph of sin and cos functions
4. Graphical addition and subtraction of sin and cos functions

MISCELLANEOUS

1. Vector representation
2. Vector components and addition and subtraction of them
3. Graphical solutions of vector problems
4. Scientific notation

RELATED MATHEMATICS

Sheet Metal Course

Quincy Trade School

Grade 10

1. Reading rule
2. Common and Decimal fractions
3. Square root
4. Mensuration -
5. Areas of plane figures (5 basic shapes) Algebraic expressions
   (a) Square (b) Rectangle (c) Triangle (d) Circle (e) Ellipse
6. Transposing expressions - Algebraic form for above
7. Circumference and Perimeters for 5 basic shapes
8. Transposing expressions for above
9. Volumes straight sided figures of solids whose bases contain 5 basic shapes. (a) cube (b) oblong (c) triangular solid (d) cylinder (e) elliptical solid
10. Transposing expressions algebraically for above solids
11. Reducing volumes to gallons, quarts, pints and gills
12. Surface areas for solids having 5 basic shapes
13. Computing cost of metal
14. Reducing surface areas to sq. feet - sq. feet to lbs.
15. Computing percentage of waste
16. Proof that 3.1416 = mean circumference of circle
17. Proof that .7854 = area of one inch diameter circle
18. Simple algebraic solutions - addition, subtractions, division and multiplications - such as proof that $(a+b)^2 = a^2 + 2ab + b^2$

Grade 11

1. Elbows (round pipe) computing rise of meter line
2. Elbows (round pipe) use of backset and trigonometry for computing meter line
3. Computing weight of metal contained in round pipe and square pipe elbows.
4. Estimating weight of metal in duct work.

5. Taking off quantities from a blue print and estimating cost (use of factor table)
6. Volumes of irregular shaped solids
7. Areas of irregular shaped plane figures
8. (a) use of presmoid formula - (b) use of Simpson's rule
9. Areas of irregular shaped solids
10. Estimating cost of metal for irregular shapes in plane and solid figures
11. Algebraic formulas - transposing any formula in connection with mensuration
12. Relationship between a cylinder, sphere and cone
13. Percentage and Ratio
14. Gravity Warm Air - Gravity "Code"
15. Use of gravity code - sizing pipe and furnace
16. Mechanical "Code" -forced warm air

    (a) computing b+v loss (b) use of formulas (c) use of chart

    (d) equal friction chart - variables c+m - static pressure - pipe drain - velocity

RELATED MATHEMATICS

Machine Course

Quincy Trade School

Grade 10

Fractions - addition, subtraction, multiplication, division. Application of same in practical related problems with drawings.

Decimals - addition, subtraction, multiplication, division, changing fractions to decimals, decimals to fractions. Decimal equivalent chart study, rounding off decimals, explanation of decimals on drawings in combination with blue print study. Application of decimals in practical problems.

The more advanced student will be given additional assignments.

Grade 11

Measurement - units of linear measurement
units of area measurement
units of volume measurement
units of circular length & area measurement
degree of accuracy
applications of all units of measurement
reading micrometer to ten thousandths

Introduction to symbolism - formula, practical application of formula to machine shop problems as:
screw threads, circles, cutting speeds,
pulleys, spur gears

Grade 12

Equations - introduction, definition, solving equations by division, multiplication, addition, subtraction.

Assignments - dimensioning problems, fraction & decimal, precentage problems

Exponents - problems in use of exponents

Ratio & proportion - application of proportion to the typical machine shop problems

Relationships- application of rate of change to cutting speeds, to scale drawings

Individual measurement - geometric designs, shop trig for computing cutting angles for compound turning, also taper attachment

Accelerated students will cover additional assignments in Machinery Handbook and others.

OCCUPATIONAL MATHEMATICS INSTRUCTIONAL SYSTEMS STRUCTURE

- Harold F. Rahmlow, Washington State University, June 1968

Data from our clusters studies and from related studies have been synthesized. Those studies cover mathematical capabilities associated with performance of tasks constituting major portions of below-professional and technicians level of work in the following occupational areas:

10 building trades
  - bricklaying
  - carpentry
  - cement finishing
  - electrical work
  - iron work
  - painting
  - plastering
  - plumbing
  - sheet metal and heating

retailing

office work

food service work

child care work

farm work

Analysis indicates that mathematical capabilities most commonly associated with those categories of work include ability to:

- compute and apply fractions
- utilize decimals
- convert fractions and decimals
- compute and utilize percentages
- compute and apply ratios and proportions

On the basis of the above analyses, in cooperation with vocational educators and mathematicians, we have developed the following structure for development of twenty sequential instructional systems designed to help pupils acquire basic vocational mathematics capabilities.

APPENDIX C

Achievement, Aptitude and Ability Profiles

Two classes of students served as the sample presented in these profiles. Both classes are now within the vocational technical curriculum. Complete data in terms of Achievement, Aptitude and Ability is available for one class only. This is the class that received the C.A.T. in 8th grade and the D.A.T. and C.T.M.M. in 9th grade. Its members were not students within the vocational curriculum at the time of testing 1966. The other class was within the vocational technical curriculum at the time of testing during the tenth grade year. It was also tested in 1966.

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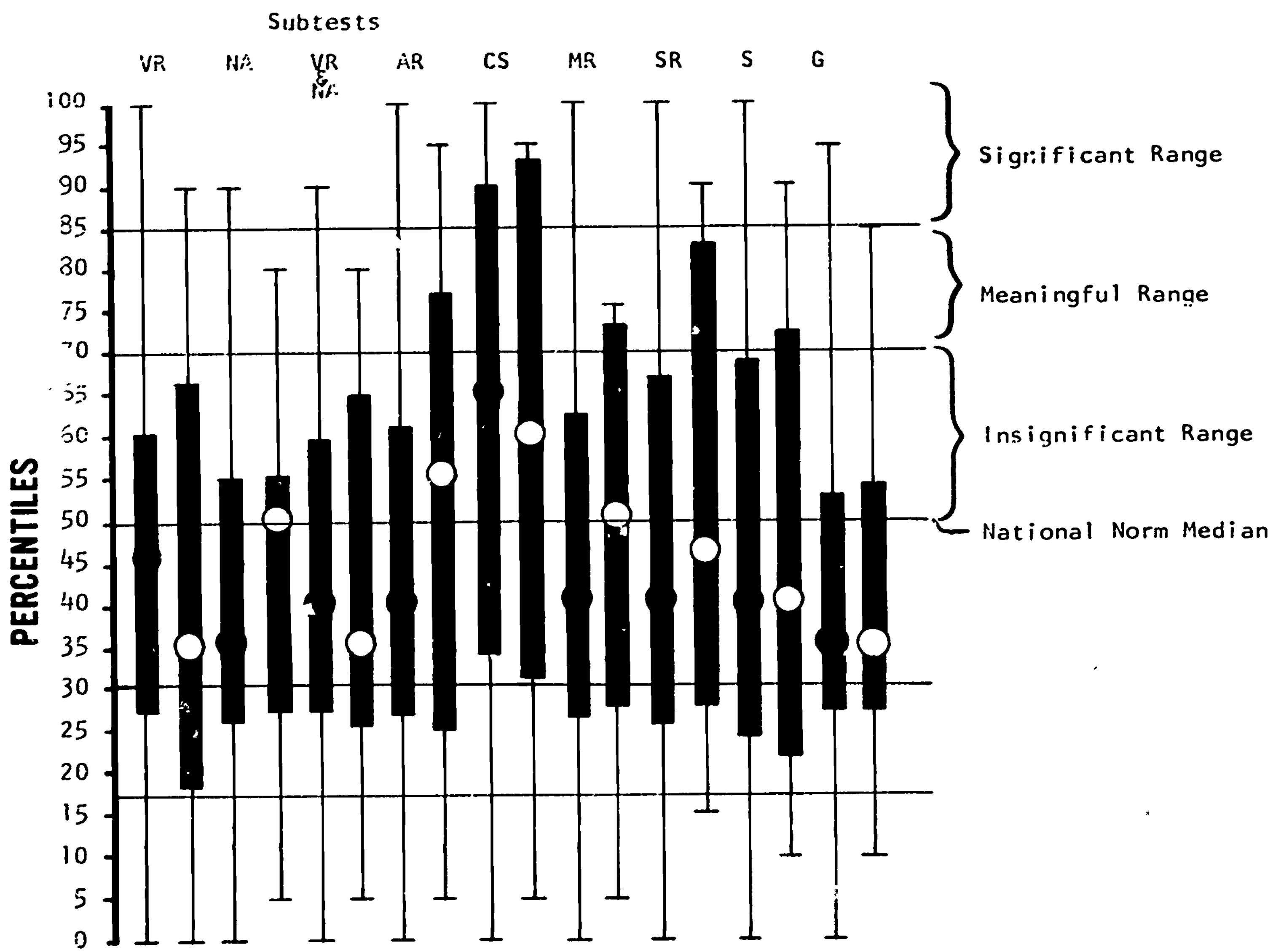
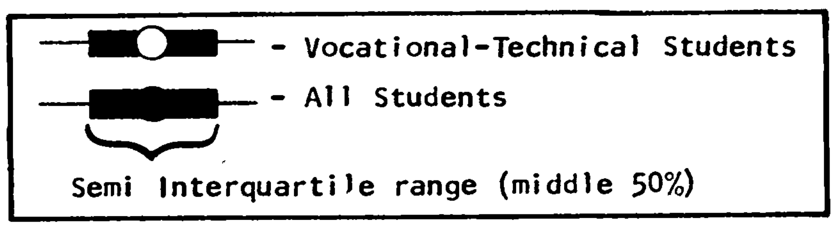


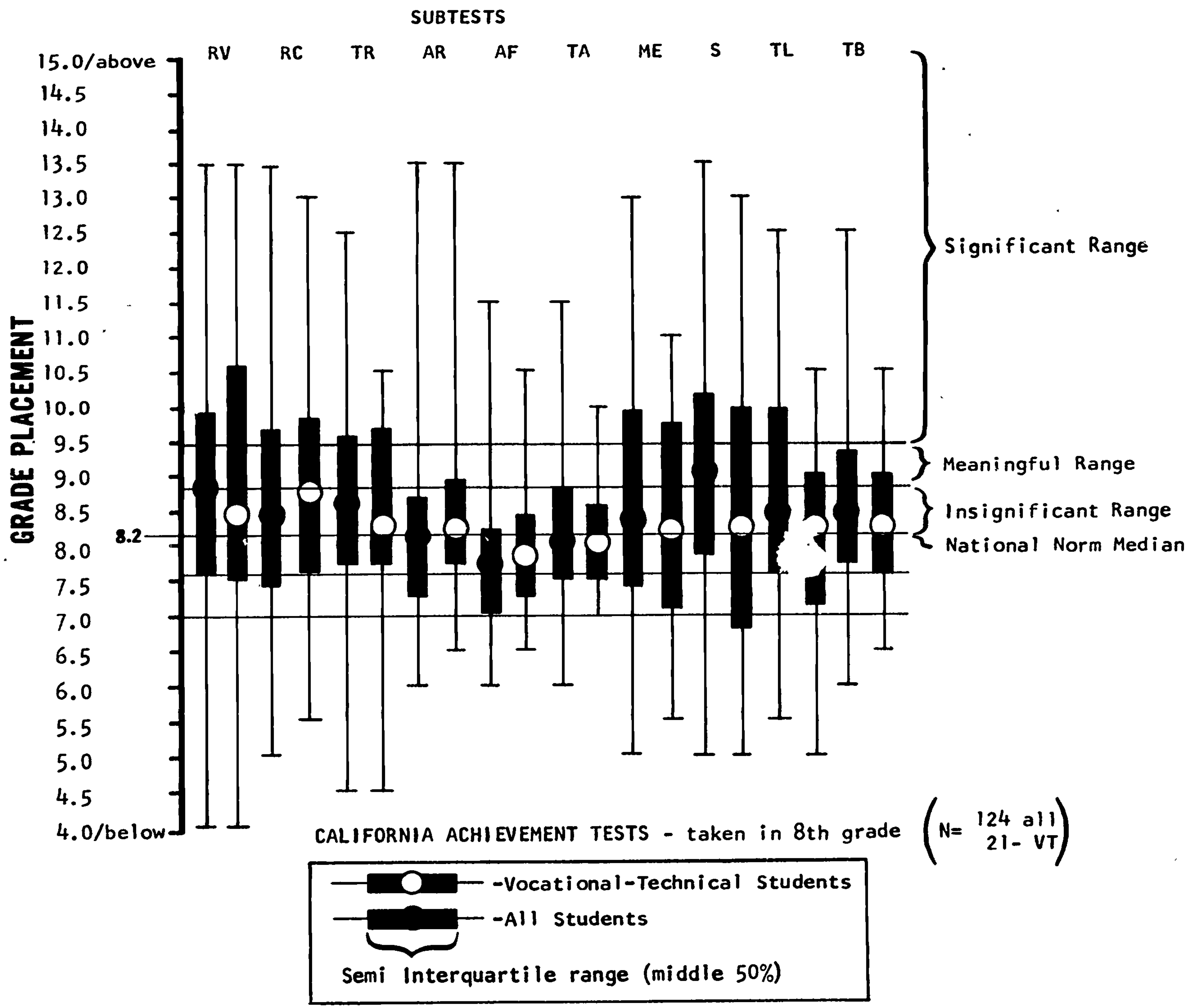
SUBTESTS:

VR - Verbal Reasoning
NA - Numerical Ability
VR & NA - A Measure of Scholastic Aptitude
AR - Abstract Reasoning
CS - Clerical Speed
MR - Mechanical Reasoning
SR - Space Relations

S - Spelling
G - Grammar

DIFFERENTIAL APTITUDE TEST - taken in 9th grade (N = 135-All, 19-VT)
expressed in class medians and ranges





SUBTEST

RV-Reading (Vocabulary)

RC-Reading (Composition)

TR-Total Reading

AR-Arithmetic (Reasoning)

AF-Arithmetic (Fundamentals)

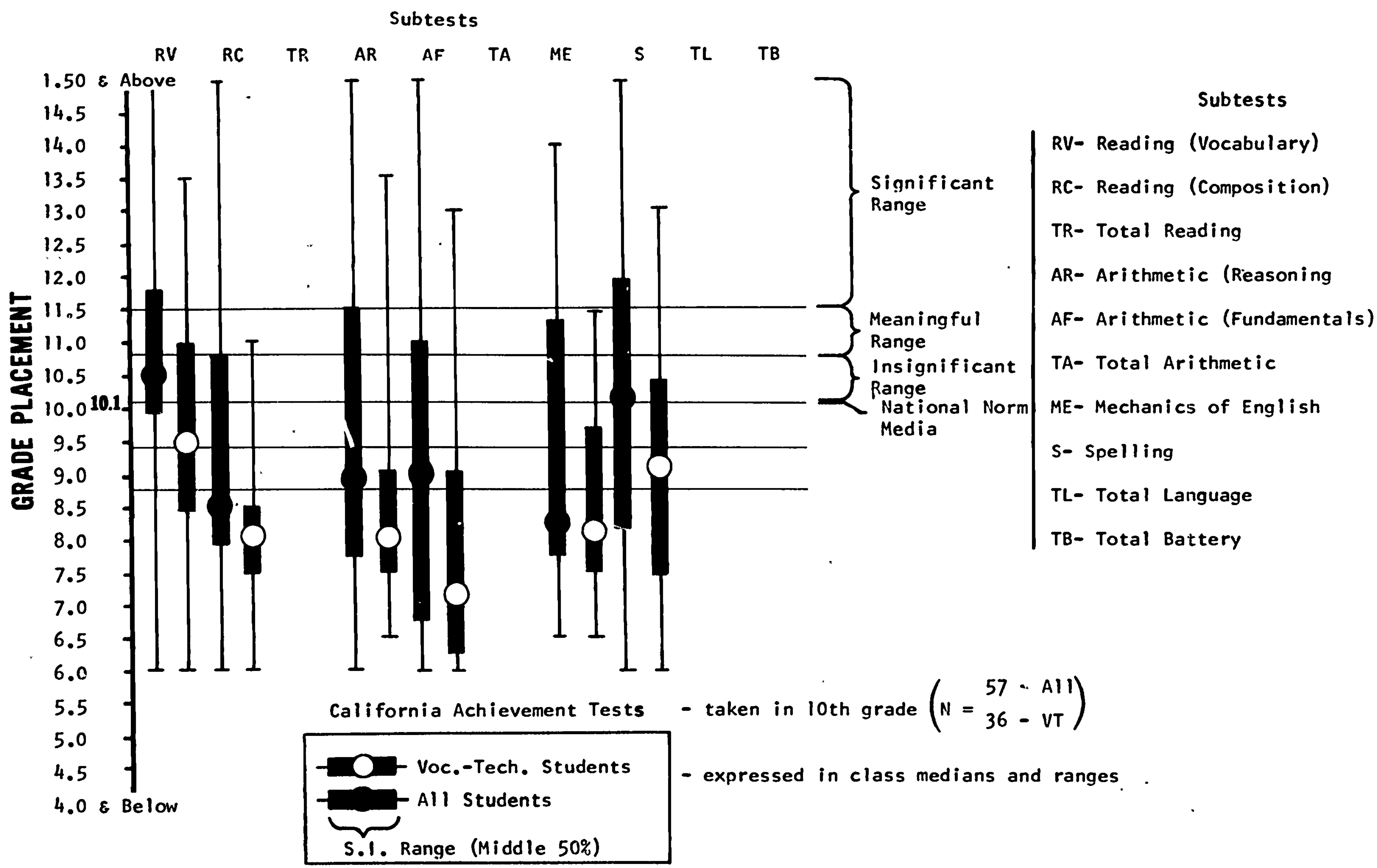
TA-Total Arithmetic

ME-Mechanics of English

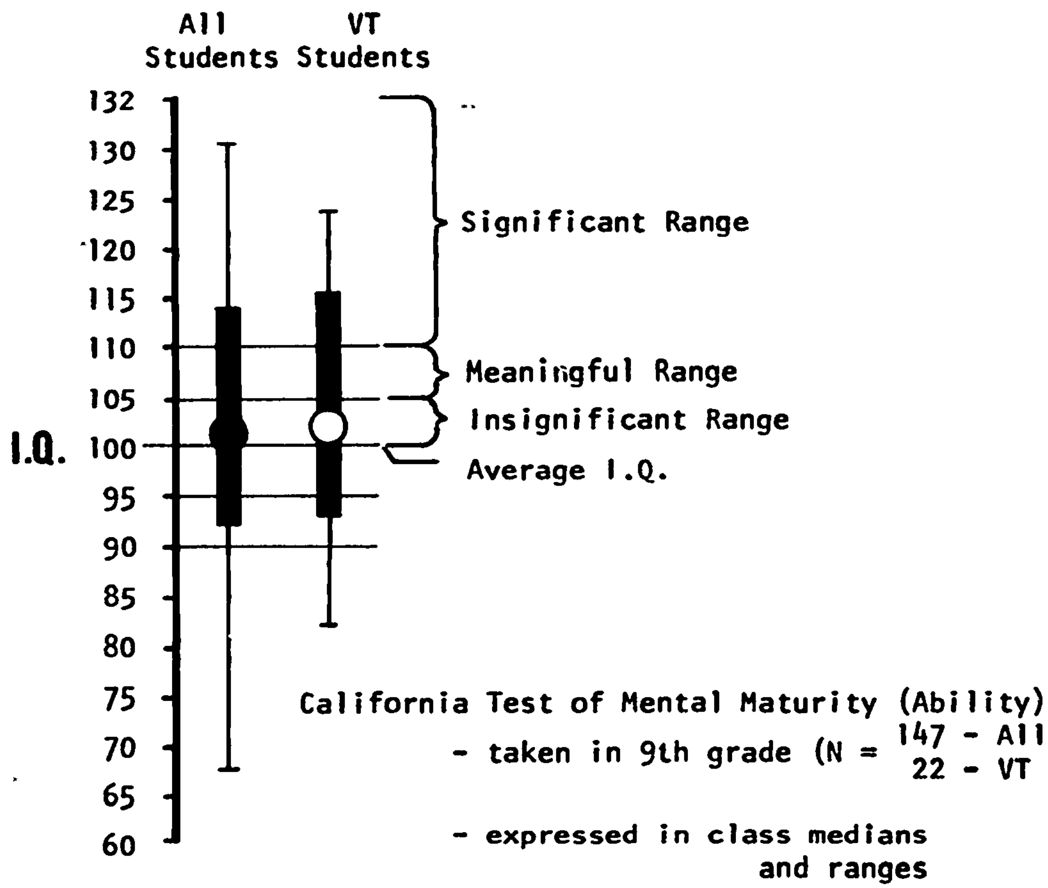
S-Spelling

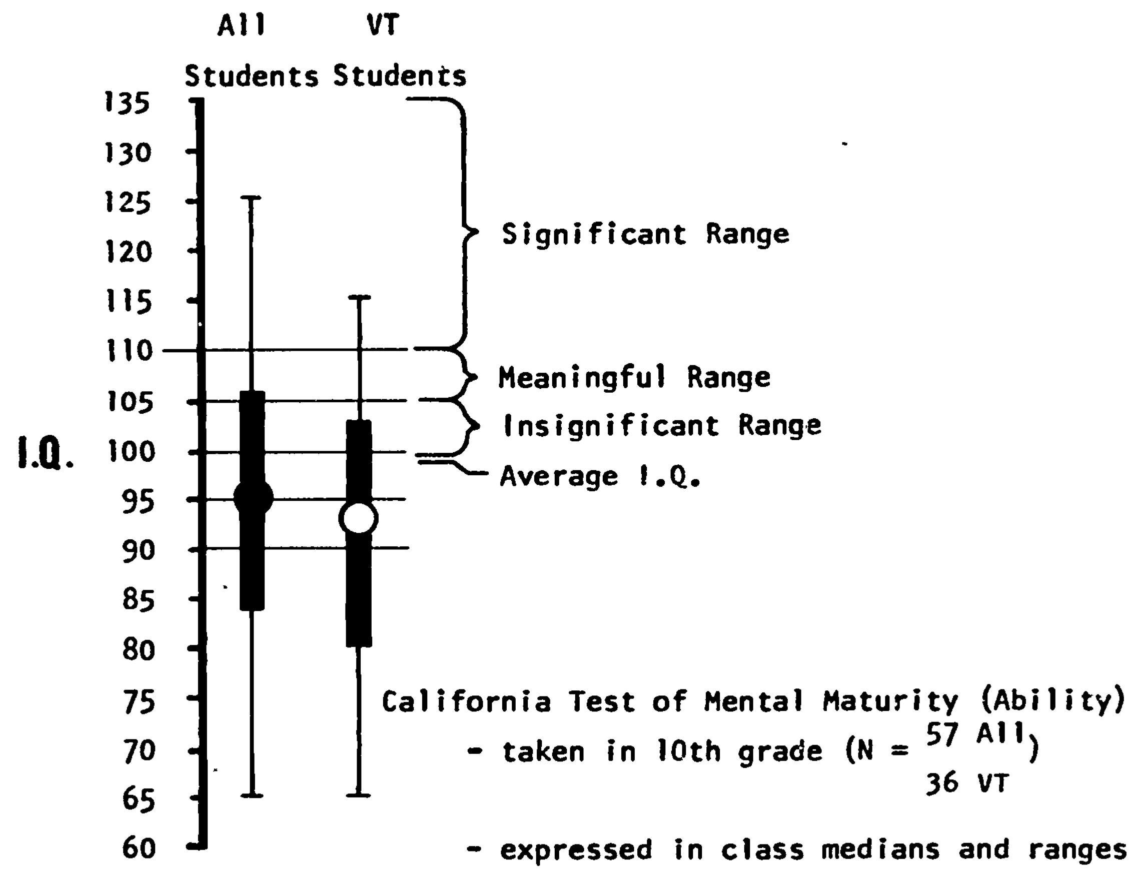
TL-Total Language

TB-Total Battery



Subtests
RV RC TR AR AF TA ME S TL TB
GRADE PLACEMENT
1.50 & Above
14.5
14.0
13.5
13.0
12.5
12.0
11.5
11.0
10.5
10.0
10.1
9.5
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0 & Below
Significant Range
Meaningful Range
Insignificant Range
National Norm Media
Subtests
RV- Reading (Vocabulary)
RC- Reading (Composition)
TR- Total Reading
AR- Arithmetic (Reasoning
AF- Arithmetic (Fundamentals)
TA- Total Arithmetic
ME- Mechanics of English
S- Spelling
TL- Total Language
TB- Total Battery
California Achievement Tests - taken in 10th grade (N = 57 - All, 36 - VT)
- expressed in class medians and ranges
Voc.-Tech. Students
All Students
S.I. Range (Middle 50%)


All Students
VT Students
132
130
125
120
115
110
105
100
95
90
85
80
75
70
65
60
I.Q.
Significant Range
Meaningful Range
Insignificant Range
Average I.Q.
California Test of Mental Maturity (Ability)
- taken in 9th grade (N = 147 - All
22 - VT
- expressed in class medians and ranges


All Students
VT Students
135
130
125
120
115
110
105
100
95
90
85
80
75
70
65
60
I.Q.
Significant Range
Meaningful Range
Insignificant Range
Average I.Q.
California Test of Mental Maturity (Ability)
- taken in 10th grade (N = 57 All)
36 VT
- expressed in class medians and ranges

APPENDIX D

A List of Supplemental Materials Including Basic Texts

A. Basic Texts

1. Klinney, Ruble, and Brown, General Mathematics, A Problem Solving Approach - Book II, Holt, Rinehart, and Winston, Inc., New York, 1968.

Units utilizing content of this text include:

1. Protractor
2. Ratio and Proportion
3. Percent
4. Base 10
5. Measurement and Scales
6. Number line
7. Whole numbers
8. Fractions
9. Decimals and Mixed Numbers
10. Word Problems
11. Area
12. Pythagorean $\triangle$s
13. Graphing, coordination
14. Graphing, pictorial
15. Graphing, tables, interpretation
16. Linear Equations
17. Regular Polygons
18. Radical Equations
19. Base Constructions
20. Base 8, 12
21. Factoring
22. Quadratic Equations, formula
23. Simultaneous Equations, 2,3 variables
24. Quadrilaterals, properties
25. Ratio of Similitude
26. Mixture problems
27. Volume
28. Trigonometric Functions
29. Ratio of Areas, scale
30. Mathematical Symbols

2. Slade, Margolis, and Boyce, Mathematics for Technical and Vocational Schools, John Wiley and Sons, Inc., New York, 1968

Units utilizing content of this text include:

1. Protractor
2. Ratio and Proportion
3. Percent
4. Base 10
5. Measurement and scales
6. Averages
7. Fractions
8. Decimals and Mixed Numbers
9. Number representation
10. Word Problems
11. Area
12. Graphing, coordination
13. Graphing, pictorial
14. Graphing, tables; interpretation
15. Formulae
16. Quadrilaterals, properties
17. Frustrum of Solids
18. Trigonometric Functions
19. Ratio of Areas, scale
20. Relations and Functions

B. Supplemental Texts

1. Drooyan, Wooton, Programmed Beginning Algebra, John Wiley and Sons, Inc., New York, 1963.

Units utilizing content of this text include:

1. Whole numbers
2. Factoring
3. Algebraic Terms
4. Graphing, coordination
5. Linear equations
6. Word problems
7. Grouping symbols

8. Number line
9. Fractions
10. Ratio and Proportion
11. Radical equations
12. Quadratic equations

2. Instructional Systems Development Team, <u>Basic Mathematics</u>, Air Training Command, Lowry Air Force Base, Colorado, 1966.

Units utilizing content of this text include:

1. Decimals and mixed numbers
2. Averages
3. Significant figures
4. Laws of indices
5. Percent
6. Base 10

3. Sackheim, G., Programmed Texts for Junior and Senior High School, <u>How to Use a Slide Rule</u>, and <u>Logarithms</u>, Harper and Row, Evanston, 1966.

Units utilizing content of these texts include:

1. Logarithms to different based
2. The Slide Rule

4. Doane, S., <u>Reading a Micrometer</u>, RDC Program Press, East Lansing, 1966.

Unit utilizing content of this text:

1. The Micrometer

APPENDIX E

Learning Unit Contents

LEARNING UNIT CONTENTS

From Project ABLE Sixth Quarterly Report, 30 September, 1966.

The learning unit material rises from previously specified topic objectives which contained a statement of the capability required of a person in a specific job. This capability represents an achievement or goal which the student is to attain. THE LEARNING UNIT THEN BECOMES THE MAP HE USES TO REACH THE GOAL. It should, therefore, provide all that information which the student needs to "get from here to there." In broad outline, a learning unit should cover the following major items:

1. Objective
2. Overview
3. Learning Experiences
4. Summary
5. References
6. Learning Aids
7. Student Evaluation

In expanded form, these items might be defined as follows:

1. Objective - A brief statement of the goal; this is what the student should know or be able to do after he has had certain experiences. It may be simply a short paraphrase of the topic objective or a shred-out from the topic objective.
2. Overview - This is a statement of why. It provides a focus for the trip the student is about to take and places the learning experiences which follow into a meaningful setting, context or perspective with respect to the goal established by the objective
3. Learning Experiences - These statements are the how by which the student might best go from "here (where he is now) to there (the new capability he is trying to acquire)." They should reflect the specific exposures to facts, procedures, manipulations, explorations, or actions through which the student proceeds. They represent the methods and procedures for gaining the objective. On the part of the student, they may be observations, experiments, discussions, performances, readings, etc. Home assignments can be considered a part of this item.

4. Summary - This is a brief backward look at <u>where</u> the student has been. It should indicate the relationship between what he has just experienced to the need for that experience in terms of later objectives.

5. References - These might well be divided into Basic and Supplementary. The more able students will need a greater range and complexity of material, but all students should have some defined minimum of supporting aids which will provide the route to competency. This division of references and aids is only a device to assure that certain <u>essential</u> tools are available.

6. Learning Aids - Such things as film strips, cutaways, simulators, raw materials, equipment, etc. that must accompany a given learning unit should be listed. If necessary materials are not now available, they should be developed as part of the learning unit. For example, a set of work problems may have to be written.

7. Student Evaluation - This item will include the ways and means by which both the student and director of learning will know whether the capability (goal) has been reached. It will consist of test items, job sample performances, and observations of student actions structured to provide objective, quantitative and/or qualitative assessments. It is the only portion of the learning unit package which the student does <u>not</u> see in advance of proceeding through the unit and its experiences.

APPENDIX F

Sample Learning Units

Unit: Mathematical Relationship Symbols

Module 2: Mathematical Relationship Symbols 27

These Mathematical Relationship Symbols	Have These English Meanings
$=$	is equal to
$>$	is greater than
$<$	is less than
$\geq$	is greater than or equal to
$\leq$	is less than or equal to
$\neq$	is not equal to
$\ngtr$	is not greater than
$\nless$	is not less than
$\ngeq$	is not greater than or equal to
$\nleq$	is not less than or equal to

How many common Mathematical Relationship Symbols are there?

Write your answer here. ______________

Module 3: Positive Negative Symbol Relation and Like Symbol Relation 53

Section II--Like Symbol Relation

Answers: Pair 1 $<$, $\ngeq$; Pair 2 $\geq$, $\nless$; Pair 3 $\leq$, $\ngtr$

In each pair of symbols that mean the same thing, there is a simple symbol and a complex symbol. The simple symbol has less words in the English word phrase that defines it.

Write the simpler symbol of this pair: $>$ (is greater than), $\nleq$ (is not less than or equal to)

__________ .

Mathematical Relationship Symbols

Readiness Skills:

1. Mastery of the use of the Arithmetic Operational symbols: +, -, and x, ÷.
2. Mastery of the material in Kinney, Ruble, and Brown General Mathematics: A Problem Solving Approach, Book II up to page 43.

28

Module 2: Mathematical Relationship Symbols

Answer: 10

Each of the ten common mathematical relationship symbols is made up of one or more of the following basic component symbols:

= an "is equal to" sign.
〉 an empty arrow head (pointing in either direction).
/ a slash.

How many basic relationship symbols are there? Write your answer here ________.
Write the basic relationship symbols here ________, ________, ________.

54

Module 3: Positive Negative Symbol Relation and Like Symbol Relation

Section II--Like Symbol Relation
Answer: 〉 (is greater than)

"The son of your mother's sister" is your cousin. Both "cousin" and "the son of your mother's sister" mean the same thing. But what do you call this relative? Write it here ____________.

Mathematical Relationship Symbols 3

Overview: In order to be able to solve mathematical problems, we must be able to read the language of mathematics as easily as we read the English language. Mathematical language, like English, is made up of symbols. Mathematical language is made up of numerals, operation symbols, and relationship symbols.

Module 2: Mathematical Relationship Symbols 29

Answers: 3, ___=___, ___>___, ___/___

The symbol for "is not equal to" is made from two basic symbols: ______ and ________.

Module 3: Positive Negative Symbol Relation and Like Symbol Relation 55

Section II--Like Symbol Relation

Answer: cousin

You call this relative "cousin" because you don't have to think about what every word in a long phrase like "The son of your mother's sister" means. To make mathematics easier to understand, you should use a simple symbol whenever you can. Draw a circle around the simpler symbol in each of these pairs: Pair 1 $<$, $\not\geq$; Pair 2 $\geq$, $\not<$; Pair 3 $\leq$, $\not>$.

Mathematical Relationship Symbols 4

Overview (cont'd):

Numerals in mathematics are like the nouns of English. They are the names we give to numbers and numbers as ideas. Operational symbols (+, -, x, and ÷) change the name of a number. The mathematical phrase " 1 ± 2" is very much like the English phrase "Aspirin <u>combined with</u> antacid." In other words, operational symbols are very much like verbs in English.

Module 2: Mathematical Relationship Symbols 30

Answers: <u>1</u> and <u>=</u>

The slash in the symbol $\neq$ means the same thing as which word in its English meaning? Write the word here ____________.

Module 3: Positive Negative Symbol Relation and Like Symbol Relation 56

Section II--Like Symbol Relation.

Answers: Pair 1 $<$, $\not\geq$; Pair 2 $\geq$, $\not<$; Pair 3 $\leq$, $\not>$.

Overview (cont'd):

There are many different names for each number. The mathematical sentence "1 + 2 = 3" says that "1 + 2" and "3" are names for the same number. This sentence is very much like the English phrase "Aspirin combined with Antacid is the same as Bufferin." A relationship symbol in mathematics is almost the same thing as an example of a passive verb in English. It shows how numbers are related to one another.

Answer: Not

This is an "is greater than" symbol: $>$.
This is an "is less than" symbol: $<$.

They are both made up of which basic symbol? (page 27)
Write the name of that basic symbol here ____ ____ ____ ____.

Your sister has spots on her face! This statement is true if your sister really does have spots on her face. This statement is false if your sister does not have spots on her face. To find out whether the statement is true or not you have to think of your sister with spots on her face and compare this in your mind to what your sister really looks like. Then you decide if they are the same.

Overview (cont'd):

When nouns and verbs are used together in either English or mathematics, sentences can be formed. These sentences can be either true or false, open (conditional), or closed (unconditional). In this unit, we will learn to read mathematics as we read English, so that we can tell whether mathematical sentences are true or false, open or closed (conditional or unconditional). Using this knowledge, we will then learn to use a specific noun, or numeral to solve problems that contain mathematical symbols of relationship.

Module 2: Mathematical Relationship Symbol 32

Answer: <u>an</u> <u>empty</u> <u>arrow</u> <u>head</u>

The difference between the symbols $>$ (is greater than) and $<$ (is less than) is that the empty arrow head _____ _____ _____ _____.

Module 4: True-False Statement Relation 58

$$2 + 2 = 5$$

To find out whether this statement is true, you have to add 2 and 2 and compare the sum, 4, in your mind to the number 5. Then you make a decision whether or not 4 equals 5. Four does not equal 5, so the mathematical statement is false.

$$2 + 2 - 3$$

This statement is true, false. Circle the correct choice.

Mathematical Relationship Symbols 7

Objectives:

1. Given a common mathematical symbol of relationship, the student will write the English equivalent defining it, in all cases.

Module 2: Mathematical Relationship Symbol 33

Answer: points in different directions.

(or something similar in your own words)

to the left $>$ points: or to the right.

(Draw a line under the correct choice above.)

Module 4: True-False Statement Relation 59

Answer: false

$$2 + 2 = 3$$

If I change the = symbol to a $\neq$ or a $>$ or a $\nless$, the mathematical statement now becomes true.

Change the symbol in 2 + 2 = 5 to the simplest symbol that makes the statement true. Write that symbol here __________.

Objectives:

2. Given an English word phrase defining a common mathematical relationship, the student will be able to write or choose from a list, the equivalent mathematical symbol in all cases.

Module 2: Mathematical Relationship Symbol 34

Answer: to the right.

to the left $<$ points or to the right?

(Draw a line under the correct choice above.)

Module 4: True-False Statement Relation 60

Answer: $<$ (Remember: the simplest symbol has the least number of words in its English word phrase equivalent.)

Is $6 < 3$ a true or false statement? Circle the correct underlined word.

Write the simplest symbol that makes the statement $6 < 3$ true here ________.

Objectives:

3. Given a common mathematical symbol of relationship or its English equivalent, the student will identify in writing or by use of a check list whether it contains a negation with 80 percent accuracy.

Module 2: Mathematical Relationship Symbols 35

Answer: <u>to the left</u>

To help you remember the name and meaning of the symbol $<$ or $>$, determine which way the empty arrow head is pointing. If it is pointing to the left, remember that the first letter of the word "left" is the same as the first letter of: less or greater?

(Draw a line under the correct choice above.)

Moduel 4: True-False Statement Relation 61

Answer: <u>false</u>, $>$.

On a separate sheet of paper, do the fifteen examples numbered 11-26 on page 43 of your text. Follow the directions on page 43 but after every statement that you mark "false," write the simplest symbol that would make the statement "true."

Objectives:

4. Given a list of mathematical symbols of relationship or their English equivalents, the student will identify those that have the same meaning with 80 percent accuracy.

Module 2: Mathematical Relationship Symbols 35

Answer: Less

The name of the $<$ symbol is:

a. "is greater than"

b. "is less than"

(Draw a line under the correct choice above.)

Module 5: Closed-Open Statement Relation, Unconditional-Conditional Statement Relation 62

Your sister is a ______________.

This statement is open and conditional. It is called "open" because it has a word left out that is indicated by the blank. It is called "conditional" because it can be either true or false depending on what word you put in the blank. Make it a closed and unconditionally true statement by circling the correct word that you would put in the blank: a. Girl, b. Dog.

Objectives:

5. Given a simple mathematical relationship, the student will be able to state in writing or by use of a check list whether it is true or false with 80 percent accuracy.

Module 2: Mathematical Relationship Symbols 37

Answer: b. "is less than"

Since > points to the right, and r is also the second letter of less or greater (underline the correct choice), the name of the symbol is " _____ _____ _____" (fill in the blanks).

Module 5: Closed-Open Statement
Relation, Unconditional-Conditional Statement Relation 63

Answer: Your sister is a girl.

You have made the statement closed by filling in a missing word. You have made the statement unconditionally true by filling in the correct word. In mathematics, statements can be either open or closed and unconditional or conditional also. Instead of a blank (__________) a letter of the alphabet after tells you that the statement is open and conditional. Draw a circle around the mathematical statement that is open and conditional: $6 > 3$; $4 < A$

Objectives:

6. Given a simple mathematical relationship, the student will be able to state in writing or by use of a check list whether it is unconditional--a closed mathematical sentence--or conditional--an open mathematical sentence with 80 percent accuracy.

Module 2: Mathematical Relationship Symbols 38

Answers: greater, is greater than

When you use the $<$ and $>$ symbols, to avoid becoming confused remember that the empty arrow head should always point to the smaller number.

If you want to write 6 is greater than 3 using a symbol, the empty arrow head should point to the 6 or 3?

(Draw a line under the correct choice.)

Module 5: Closed-Open Statement Relation, Unconditional-Conditional Statement Relation 64

Answer: $4 < A$

This mathematical statement is "open" because a letter of the alphabet, called a "variable," can be replaced by any of several numbers. It is also conditionally true or false because some numbers you can replace it with make the statement true and others make it false. Draw a circle around any of the following statements that are open and conditional. Draw an "x" through any of the following statements that are closed and unconditional: a. $4A < 6$; b. $2 = 1 + 1$; c. $2 + 3 = B$; d. $9 \not> 10$; e. $c = x$.

Objectives:

7. Given a simple conditional mathematical relationship, the student will find the solution set, stating the correct answer in writing with 80 percent accuracy.

39

Module 2: Mathematical Relationship Symbols

Answer: 3

If you want to write "1 is less than 2" using a symbol, the symbol should point toward the 1 or 2?

(Draw a line under the correct choice.)

Module 5: Closed-Open Statement Relation, Unconditional-Conditional Statement Relation 65

Answers: a. $4a < 6$; b. $2 = \cancel{X} + 1$; c. $2 + 3 = B$; d. $9 \not> \cancel{1}0$; e. $C = X$

Put a check mark in the column that tells whether each of the following mathematical statements is open and conditional or closed and unconditional:

	Conditional Open	Unconditional Closed		Conditional Open	Unconditional Closed
1. $X=5+1$			6. $(99 \div 9A) \neq 22$		
2. $43 \neq (29 \times 180)$			7. $(B+3) \not< 10$		
3. $14 > (2N \div 3)$			8. $(c-8)=(15+4)$		
4. $7y=7$			9. $(2+2) \not> (1+1)$		
5. $(5+6) \not\geq 6$			10. $4+4 \neq 4$		

Unit Contents

14

Module 2: Mathematical Relationship Symbols

40

Answer: 1

Write what the following symbols mean in the blanks next to them:

$4 < 8$ 4 ____ ____ ____ 8

$9 > 2$ 9 ____ ____ ____ 2

Module 5: Closed-Open Statement Relation, Unconditional-Conditional Statement Relation

66

	Conditional Open	Unconditional Closed
1. $X=5+1$	✓	
2. $43 \neq (29 \times 180)$		✓
3. $14 > (2N \div 3)$	✓	
4. $7y=7$	✓	
5. $(5+6) \nleq 6$		✓
6. $(99 \div 9A) \neq 22$	✓	
7. $(B+3) \nless 10$	✓	
8. $(c-8)=(15+4)$	✓	
9. $(2+2) \ngtr (1+1)$		✓
10. $4+4 \neq 4$		✓

Module 1: Diagnostic Pretest 15

Section 1:

1. For examples 1-5 in the list below, write the mathematical relationship symbol in the blank next to the English phrase that means the same thing.
2. For examples 6-10 in the list below, write the English word phrase in the blank next to the relationship symbol that means the same thing.

Examples:	1. is not equal to ____	6. $\not<$ ________________
	2. is less than ____	7. $\geq$ ________________
	3. is not greater than____	8. $=$ ________________
	4. is less than or equal to ____	9. $\not\leq$ ________________
	5. is not greater than or equal to ____	10. $>$ ________________

Module 2: Mathematical Relationship Symbols 41

Answers: 4 is less than 8

9 is greater than 2

What basic symbol must be added to the "is less than" and "is greater than" symbols to change them into "is not less than" and "is not greater than" symbols? Write that symbol here ________.

Write the "is not less than" symbol here ________; and write the "is not greater than" symbol here ________.

Module 6: Solving Conditional (Open) Statements 67

problem	solution
$4\,\underline{X} + 2 = 10$	$4\,(\underline{2}) + 2 = 10$

In an open mathematical statement, if you replace the letter of the alphabet (the variable), with a specific number that makes the statement true, you have solved the statement.

Your sister is a girl.

This statement has been solved (made true) by filling in the blank with the word "girl." It could have been made true by putting the word(s) "female," or "human being." Many different words could have been used to make the statement true.

Module 1--Diagnostic Pretest 16

Section 1--answers:

1. is not equal to $\neq$
2. is less than $<$
3. is not greater than $\not>$
4. is less than or equal to $\leq$
5. is not greater than or equal to $\not\geq$
6. $\not<$ is not less than
7. $\geq$ is greater than or equal to
8. $=$ is equal to
9. $\not\leq$ is not less than or equal to
10. $>$ is greater than

If any of your answers are incorrect, turn to page 27.
If all your answers are correct, turn to page 17.

Module 2: Mathematical Relationship Symbols 42

Answers: 1, $\not<$, $\not>$

(NOTE: When we add a slash (1) to a symbol, we do not change the direction, right or left, that it was originally pointing.)

The $\not<$ symbol means the opposite of the $<$ symbol. Write, using symbols the statement 5 is not less than 3: ___ ___ ___

Module 6: Solving Conditional (Open) Statements 68

The set of words that make the statement true is the solution set of the open or conditional statement. We write the solution set to the open or conditional statement in this form: (all words truthfully describing your sister).

$$6 > A$$

All the whole numbers that make this statement true, are called the solution set of this statement. We write the solution set to this statement, where U = (whole numbers): (5, 4, 3,...); or (all whole numbers less than 6). Write the solution set to the statements: $4 < B$; $2 + Y \geq 3$; $6 \neq C$; $A = 10$.

Module 1: Diagnostic Pretest 17

Section II:

Place a check mark in the column that correctly tells how each of the following relationship symbols and word phrases is stated:

	Negatively Stated	Positively Stated
1. $\neq$		
2. $<$		
3. $\ngtr$		
4. $\leq$		
5. $\ngeq$		

	Negatively Stated	Positively Stated
6. is not less than		
7. is greater than or equal to		
8. is equal to		
9. is not less than or equal to		
10. is greater than		

Module 2: Mathematical Relationship Symbols 43

Answers: $5 \nless 3$

Write "6 is not greater than 6" in symbols here ___ ___ ___.

Module 6: Solving Conditional (Open) Statements 69

Answers: (5,6,7,...) or (all numbers greater than 4); (1,2,3,...) or (all numbers greater than 0); (...3,4,5,7,8,9...) or (all numbers except 6); (10)

If the mathematical relationship symbol in the statement is an = symbol, the solution set will contain only one number. Write the solution sets to the following statements in the blanks next to the statements:

1. X = 1 + 2 __________
2. a + 3 = 9 __________
3. 12 - X = 7 __________
4. (48 3) = 6 + 10 __________

Module 1: Diagnostic Pretest 18

Section II--answers:

	Negatively Stated	Positively Stated
1. $\neq$	✓	
2. $<$		✓
3. $\ngtr$	✓	
4. $\leq$		✓
5. $\ngeq$	✓	

	Negatively Stated	Positively Stated
6. is not less than	✓	
7. is greater than or equal to		✓
8. is equal to		✓
9. is not less than or equal to	✓	
10. is greater than		✓

If fewer than eight answers are correct turn to page 49.
If eight or more answers are correct turn to page 19.

Module 2: Mathematical Relationship Symbols 44

Answers: $\underline{6}\ \underline{\ngtr}\ \underline{6}$

The symbols $\geq$ and $\leq$ are combinations of $>$ and $<$ with $=$. Instead of writing "is greater than or equal to" we write ______. Instead of writing "is less than or equal to" we write _______.

Module 6: Solving Conditional (Open) Statements 70

Answers: 1. (3) 3. (5)
2. (6) 4. (6)

Turn to page 44 of your text and solve, by finding the solution set for, as many of the examples numbered 1-53, as you need to solve to be sure you know how to do this type of problem correctly.

Section III:

From the following mathematical relationship symbols and phrases, write four lists of two phrases or symbols each that mean the same thing.

1. $=$	6. is not greater than or equal to	list # 1. ________, ________
2. $\nless$	7. is less than or equal to	list # 2. ________, ________
3. $\geq$	8. is not greater than	list # 3. ________, ________
4. $\nleq$	9. is not equal to	list # 4. ________, ________
5. $>$	10. is less than	

Answers: $\geq$, $\leq$.

Write the name of the $\geq$ symbol here __ _____ _____ __ _____ __.

Write the name of the $\leq$ symbol here __ _____ _____ __ _____ __.

Section 1:

1. For examples 1-5 in the list below, write the English phrase in the blank next to the relationship symbol that means the same thing.

2. For examples 6-10 in the list below, write the mathematical relationship symbol in the blank next to the English word phrase that means the same thing.

Examples:

1. $\neq$ ________	6. is not less than ________
2. $<$ ________	7. is greater than or equal to ________
3. $\ngtr$ ________	8. is equal to ________
4. $\leq$ ________	9. is not less than or equal to ________
5. $\ngeq$ ________	10. is greater than ________

Section 2: Using the examples above:

1. Write the example numbers of the positively stated word phrases and symbols here: ___, ___, ___, ___, ___.

2. Write the example numbers of the negatively stated word phrases and symbols here: ___, ___, ___, ___, ___.

Section III--answers:

List # 1. $\nless$, $\geq$.

List # 2. $\nleq$, $>$.

List # 3. is not greater than or equal to, is less than.

List # 4. is less than or equal to, is not greater than.

If fewer than four lists are correct, turn to page 51.

If all four lists are correct, turn to page 21.

Module 2: Mathematical Relationship Symbols 46

Answers: is greater than or equal to.

is less than or equal to.

Write the following statements in symbols in the blanks next to the word phrases:

4 is greater than or equal to 4 ____ ____ ____

3 is greater than or equal to 0 ____ ____ ____

2 is less than or equal to 3 ____ ____ ____

2 is less than or equal to 2 ____ ____ ____

Module 7: Criterion Post Test 72

Section 3: Using the examples on page 71, write four lists of two phrases or symbols each that mean the same thing.

List # 1. _____, _____

List # 2. _____, _____

List # 3 _______, _________

List # 4 _______, _________

Section 4: After each of the following mathematical relationship statements check the column marked "true" if the statement is true or check the column marked "False" if the statement is false.

	True	False
1. $6+1 > 0$		
2. $4 \ngeq 8$		
3. $(5+3) < 9$		
4. $2+2=5$		
5. $10+11=23$		

	True	False
6. $12-6 \geq 7$		
7. $4=(6-3)$		
8. $29+16=43$		
9. $(87-6+22) \ngeq 99$		
10. $(4x4)=16$		

Module 1: Diagnostic Pretest 21

Section IV:

After each of the following mathematical relationship statements write a "T" if the statement is true or "F" if the statement is false.

1. $4 = 2$ ____
2. $7 < 9$ ____
3. $8 > 8$ ____
4. $(2 + 16) \geq 18$ ____
5. $12 \leq (10 + 10)$ ____
6. $(1 + 1 + 1) = (3 - 2)$ ____
7. $(4 \times 8) \neq 50$ ____
8. $(6 \div 9) < 1$ ____
9. $(59 + 61) \nless (109 + 2)$ ____-
10. $(4 - 3 + 0) = (1 \times 1)$ ____

Module 2: Mathematical Relationship Symbols 47

Answers: $4 \geq 4$
$3 \geq 0$
$2 \leq 3$
$2 \leq 2$

Write the symbol for "is not greater than or equal to" here _____

Write the symbol for " is not less than or equal to" here _____

Module 7: Criterion Post Test 73

Section 5: In the blank after each of the following mathematical relationship statements, write a "c" if the statement is closed and unconditional or an " ϕ " if the statement is open and conditional:

1. $49C-7=91$ ____
2. $6 \ngeq (1+X)$ ____
3. $5Z \neq 20$ ____
4. $144 > 12A$ ____
5. $16-16=0$ ____
6. $48 \geq (96 \div 6)$ ____
7. $13=(7+6)$ ____
8. $2N \nless (408+408-816)$ ____
9. $(53-3)=(25x2)$ ____
10. $((2+3) \div 4)=1.2$ ____

Section 6--Where U= (whole numbers)

Solve the open and conditional statements identified in Section 5 and write their solution sets here: ______________.

Module 1: Diagnostic Pretest 22

Section IV--answers:

1.	$4 = 2$ F	6.	$(1 + 1 + 1) = (3 - 2)$ F
2.	$7 < 9$ T	7.	$(4 \times 8) \neq 50$ T
3.	$8 > 8$ F	8.	$(6 \div 9) < 1$ T
4.	$(2 + 16) \geq 18$ T	9.	$(59 + 61) \nless (109 + 2)$ T
5.	$12 \nleq (10 + 10)$ F	10.	$(4 - 3 + 0) = (1 \times 1)$ T

If fewer than eight of your answers are correct, turn to page 57.
If eight or more of your answers are correct turn to page 23.

Module 2: Mathematical Relationship Symbols 48

Answers: $\ngtr$, $\nleq$.

On a separate sheet of paper, rewrite, using symbols, the sentences 1-10 on page 43 of your text.

Module 7: Criterion Post Test 74

Answers:

Section 1--1.	is not equal to	6.	$\nless$
2.	is less than	7.	$\geq$
3.	is not greater than	8.	$=$
4.	is less than or equal to	9.	$\nleq$
5.	is not greater than or equal to	10.	$>$

Section 2--positively stated phrases and symbols: 2, 4, 7, 8, 10.
negatively stated phrases and symbols: 1, 3, 5, 6, 9.

Module 1: Diagnostic Pretest

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Section V:

After each of the following mathematical relationship statements, put a check mark in the appropriate column below if the relationship is conditional (the sentence is open) or if the relationship is unconditional (the sentence if closed.)

	Conditional Open	Unconditional Closed
1. $9 > (2+3)$		
2. $(4a-2)=6$		
3. $446 \geq 445$		
4. $0 < 6$		
5. $1N \nless 499$		

	Conditional Open	Unconditional Closed
6. $(6 \div 1)=x$		
7. $(133+29) \leqq 122$		
8. $(73-4) \neq (33 \times 3)$		
9. $(22+22z) \geq 44$		
10. $(y+63 \neq (7 \times 9)$		

Module 3: Positive-Negative Symbol Relation and Like Symbol Relation

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Section I: Positive-Negative Relation

In English, a negatively stated phrase contains a word called a "negative." Negatives are such words as no, not, none, nothing, etc. Mathematical relationships are sometimes stated negatively also. Which of the following mathematical relationship symbols are negatively stated? Check the appropriate column for each symbol.

	Positively Stated	Negatively Stated
1. $=$		
2. $>$		
3. $<$		
4. $\geq$		
5. $\leqq$		

	Positively Stated	Negatively Stated
6. $\neq$		
7. $\ngtr$		
8. $\nless$		
9. $\ngeq$		
10. $\nleqq$		

Module 7: Criterion Post Test

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Answers:

Section3--List # 1 $\leq$, $\ngeq$ — List # 3 is not less than, is greater than or equal to

List # 2 $\ngtr$, $\leqq$ — List # 4 is not less than or equal to, is greater than

(in any order)

Section 4:

The following statement numbers should have been checked as true: 1, 2, 3, 9, 10.

The following statement numbers should have been checked as false: 4, 5, 6, 7, 8.

Module 1: Diagnostic Pretest 24

Section V--answers:

		Open	Closed
1.	$9 > (2 + 3)$		✓
2.	$(4a-2)=6$	✓	
3.	$446 \geq 445$		✓
4.	$0 < 6$		✓
5.	$1N \nless 499$	✓	
6.	$(6 \div 1)=x$	✓	
7.	$(133+29) \nleq 122$		✓
8.	$(73-4) \neq (33 \times 3)$		✓
9.	$(22+22z) \geq 44$	✓	
10.	$(y+63) \neq (7 \times 9)$	✓	

If fewer than eight of your answers are correct, turn to page 62.

If eight or more of your answers are correct, turn to page 25.

Module 3: Positive-Negative Symbol Relation and Like Symbol Relation 50

Section 1: Positive Negative Relation

Answers: symbols 1-5 should be checked in the positive column
symbols 6-10 should be checked in the negative column.

Write the one negative word that appears in the English equivalent of all symbols that are negatively stated. Write that word here ________.

Module 7: Criterion Post Test 76

Answers:

Section 5:

1.	ϕ	6.	C
2.	ϕ	7.	C
3.	ϕ	8.	ϕ
4.	ϕ	9.	C
5.	C	10.	C

Section 6:
1. (2)
2. (6, 7, 8, ...) or (All numbers larger than 5)
3. (...1, 2, 3, 5, 6, 7...) or (All numbers except 4)
4. (11, 10, 9, ..) or (All numbers less than 12)
8. (0, 1, 2,...) or (All nonnegative numbers)

Section VI:

Write the solution set next to the open or conditional mathematical relationship statements listed below: where U = (whole numbers)

1. $4a-2=6$ _______
2. $1N \nless 499$ ______
3. $(f \div 1)=X$ _____
4. $(22+22 \ngeqq) \ngeq 44$ ___
5. $(y+63) \neq (7\times9)$ ___

Section II--Like Symbol Relation

Answer: <u>not</u>

Some negatively stated symbols mean the same thing as some positively stated symbols In fact, the only symbols that do not mean the same thing as other symbols are = and $\neq$. The <u>positively</u> stated symbol of the pair that means the same thing tells you what the number on the left <u>is</u>. Thus, $6>3$ tells you that 6 <u>is</u> greater than 3. The <u>negatively</u> stated symbol of the pair tells you what the number on the left <u>is not</u>. Thus, $6\nleq3$ tells you that 6 <u>is not</u> less than or equal to 3.

Summary:

In this unit, we saw how mathematics was a language of symbols just like English. We learned what the mathematical relationship symbols were and how they are used to state relationships positively or negatively, to make statements true or false, and in closed-unconditional or open-conditional statements. All of this led us to solving simple open and conditional statements. In the next several units we will use this knowledge about mathematical relationship symbols to learn different methods of solving more complex open and conditional statements.

Section VI--answers:

1. $4a-2=6$ (2)
2. $1N \not< 499$ (499, 500, 501, ...) or (all whole numbers greater than 498)
3. $(6 \div 1)=X$ (6)
4. $(22+22Z) \not> 44$ (-1, -2, -3, ...) or (all whole numbers less than 1)
5. $(y+63) \neq (7\times 9)$ (all whole numbers except 0)

If fewer than four of your answers are correct, turn to page 62.
If at least four of your answers are correct, turn to the next unit of study.

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Each of the following symbols means the same thing as another one of the symbols.

Positively Stated		Negatively Stated	
1.	$<$	4.	$\not>$
2.	$\geq$	5.	$\not<$
3.	$\leq$	6.	$\not\geq$

Pair the symbols that mean the same thing and write these pairs down here:

Pair 1 ______, ______; Pair 2 ______, ______; Pair 3 ______, ______.

Mathematical Relationship Symbols 78

References--Kinney, Ruble, and Brown, General Mathematics: A Problem Solving Approach, Book II, Holt, Rinehart, and Winston, New York, 1963.

MODULE 1- ARBITRARY MEASUREMENT SYSTEMS

OVERVIEW

> The history of man is, in part, a history of measurement. Measurement started by comparing things. By comparison, man was able to determine that one herd of animals was larger than another. When numbers were developed, this comparison was made by counting. Later, man developed better units of measure.
>
> The first measurements made by using units of measure were very simple. A man used his foot, hand, thumb, or step to measure length. In this way, he always had his measuring scale handy. He used stones or kernals of grain to measure weight. He used the sun or moon to tell the time or the season.
>
> Now, these units of measurements were not always alike. One man's foot was larger than another's; some stones were heavier than others; and some days were longer than others. So, when man began to build homes, travel in ships, trade products, and divide the land, these natural ways of measuring weren't good enough. Now man needed standard scales for measures.*

But, just for a moment, go back in time and pretend you have never heard of the inch, pound, or foot. You have got to measure an object. What do you do?

In this module you are going to invent your own system of measurement that will be every bit as good as the English system. You will then use your own system to measure objects. Its units of measure will be arbitrarily chosen by you. That means you will use objects chosen at random or by your own free will, as a base. Your system, then, will probably be different from any other person's system in the class.

The purpose of this module is to show you how systems of measurement are invented; and that one system, whose units are chosen at random by one person, is just as good as any other arbitrary system; and how you can accurately measure objects without the use of ordinary measuring devices.

---

*Glenn, W.H., Johnson, D.A., Invitation to Mathematics, Garden City: Doubleday and Company, 1962, p, 67.

MODULE 1- ARBITRARY MEASUREMENT SYSTEMS

OBJECTIVES

After you have finished this unit you will have:

1. Developed and recorded your own system of measurement and been able to tell how you did it, by listing the steps you went through.
2. Measured objects using your system of measurement.
3. Made up a table comparing the units of your system of measurement with feet and inches.
4. Measured the same objects in inches and feet.
5. Converted measurements of the objects from your own units into inches and feet.
6. Recognized and stated that your arbitrary measurement system is just as good as the English system.

MEASUREMENT AND SCALES

Module 1- Arbitrary Measurement Systems

LEARNING EXPERIENCES:

1. Choose one of the following parts of the body or objects that you have:

   a. <u>nose</u>
   b. <u>front tooth</u>
   c. <u>thumb</u> (joint)
   d. <u>finger</u> (joint)
   e. <u>earlobe</u>
   f. <u>fingernail</u>
   g. <u>toe</u> (joint)
   h. <u>toenail</u>
   i. your <u>pencil</u> (width)
   j. a <u>paper clip</u>
   k. a used <u>pencil-eraser</u>
   l. a small piece of <u>paper</u>
   m. a small <u>coin</u> in your pocket
   n. a <u>key</u>
   o. a <u>shoe sole</u> (thickness)
   p. a <u>beltloop</u> (width)

Write the underlined word that is your choice in this blank:______________

2. If you chose an object:
   -cut a piece of string so that it is the same length as the object.

   If you chose a joint or part of the body:
   -cut a piece of string so that it is the same length as the body part or the circumference* of the joint.

3. -Rub the piece of string with wax so that it will lie straight.
   -The piece of string you have just cut and waxed is the same size as one unit in the system of measurement you are inventing. It is called by the same name as the object or joint that you measure. It is called______________________________
   - Pick an even number from two to twenty. Write the number here:______.

* If you do not know what this word means, go look it up in the dictionary.

-Cut off another piece of string that is ______ times as long as the unit. With a pen or pencil mark each length of the smaller unit off on the larger piece of string.

4. Fill in the first item of the table:

Item	Name of Unit	Number of these units in next smaller or larger unit
(Original Unit) 1.		
2.		
3.		

-To find out the name of the next larger unit, change the first letter of the name of the unit in item 1 of the table to a "D". If the name of the unit begins with an "E" or an "A", add a "D" in front of the "E" or "A".

-Write the name that begins with "D" of this new unit in item 2 of the table.

5. If the size of the new unit whose name begins with "D" is larger than one foot, cut off a piece of string that is exactly 1/2 its size. In the table of step 4, place a "1/2" in the last column of item 2.

If the size of the new unit whose name begins with "D" is smaller than one foot, cut off a piece of string that is exactly twice its size. In the table of step 4, place a "2" in the last column of item 2.

-To find the name of the next size unit, replace the "D" of the name of the unit with an "M". Write the name of the new unit that begins with the letter "M", in item 3.

6. Fill in the table below with the names of the units of the table in step 4.

Choose any other 5 objects in your classroom that are much larger than the units you invented.

Write their names in the table below:

Object	Length in ________s	Length in D________s	Length in M________s
1.			
2.			
3.			
4.			
5.			

Measure the 5 objects with the three different lengths of string, filling in the table.

7. How many M__________s are there in the original unit? Write your answers here____________.

-How many original units are there in one M__________?

Write your answer here__________.

8. Measure each of the objects in the table of step 6 with a 12-inch scale. Fill in the new table below:

Object	Length in feet:	Length in inches:
1.		
2.		
3.		
4.		
5.		

9. Fill in the names of your units in this table:

Units	Length in inches	Length in feet
1.		
2D		
3M		

Measure the three different sizes of string with a 12" scale. Fill in the rest of the table.

10. Using the tables in steps 6 and 9, fill in this table:

Object	Length in inches	Length in feet
1.		
2.		
3.		
4.		
5.		

11. How many boxes did you fill in? ____________

12. Compare the table in step 10 with the table in step 8. Of the boxes that you filled in, (step 10), how many are within 10% of the values in the table of step 8?____________

13. Find other people in your class who have finished this module. Compare your system of measurement with the ones they invented.

14. Are the systems that you and your classmates invented as good as the English system that uses the inch and foot for measuring objects in the classroom? _____Yes _____No

15. In step 2, if you didn't have any string could you use something else instead? _____Yes _____No

16. List three other things you could use in step 2 instead of string.
    1.
    2.
    3.

17. Tell, in your own words, what this module was all about:

Criterion Post Test

Questions:

1. Suppose you had to measure a wall's height so you could build another one at a later time that was just as high. You do not have any ordinary measuring devices (that is: rulers, yard sticks, tape measures, etc.). Based on what you did in this module, list the steps you would go through to measure it.

Criterion Post-Test

Answers:

1. The student should include the following ideas in his list: (In his own words)

   a. Determine some unit of measurement.

   b. Lay off these units on a piece of string or some similar device for comparison with the wall's height.

   c. Compare the piece of string to the wall's height and count the number of units the wall is high.

Note: an alternate to steps b. and c. could be:

   b. Cut off a piece of string the same size as the height of the wall.

   c. Count the number of units it takes to equal the length of the piece of string.

2. At least 80% of the boxes in all tables have been filled in.

3. A "yes" answer in step 14.

MODULE 1- ARBITRARY MEASUREMENT SYSTEMS

SUMMARY

You have seen how you can invent a system of measurement when you have no ordinary measuring devices. In the next module you will see that the inch, pound, foot and other units of measurement were invented in much the same way.

Learning Aids

A ball of string
A piece of wax (candle)
Scissors
12 inch scale

Inscribed and Circumscribed Polygons

Readiness Skills

1. Satisfactory completion of the unit on "Basic Constructions."
2. Construction of the perpendicular bisector of a line segment.
3. Identification of a chord, radius, and central angle of a circle.
4. Construction of the midpoint of a line segment.
5. Use of a protractor in construction of an angle of given degree measure.

Objectives

1. Students will construct regular pentagons, hexagons, and octagons with an accuracy of one-sixteenth inch.
2. Students will construct regular polygons inscribed in circles and circumscribed about circles.
3. Students will increase their skills in construction with compass, straightedge, ruler, and protractor.
4. Students will create designs based on regular polygons.
5. Students will discover the relationship between the number of sides in a regular polygon and the degree measure of the central angle of its inscribed circle.

Additional Resource for this Unit

Kinney, Ruble, and Brown, General Mathematics: A Problem Solving Approach, Book II (Holt, Rinehart, and Winston, 1968) pp. 13-16, 230-31.

Notes to the Teacher

Diagnostic Test

1. This skill is developed in the unit on basic constructions.
2. The students should not require a protractor for this construction. The measure of an angle is a number. While the student has not been introduced to radian measure there is no absolute need for the term "degree measure." Nevertheless, by learning this term at this stage the student will be prepared for the introduction of radian measure later.
3. This skill is developed in the unit on basic constructions.
4. This is more than an exercise in accurate construction. The student should be able to state why the degree measure of angle QTR is 60.
5. This skill is developed in the unit on basic constructions. The conclusion in this item is useful in finding the center of a regular polygon and in inscribing a regular polygon in a circle.
6. This item prepares for item 7.
7. Items 6 and 7 together complete the construction of the circumscribed circle.
8. Items 6 and 8 together complete the construction of the inscribed circle.

Students should practice each item on the diagnostic test until they are successful. Students should be encouraged to discuss the purpose of items 6, 7, and 8. They should recognize that the choice of radius for item 1 is arbitrary and affects the size but not the shape of the remaining figures constructed. They might be led to consider what would have happened if any choice other than 60 had been made for the degree measure of angle TOR.

Module 2

The regular pentagon is chosen because of its relative simplicity and representativeness of regular polygons with a greater number of sides.

Confusion can be expected between inscribed and circumscribed polygons and circles. Students should be encouraged to practice using these terms correctly.

Students should repeat the construction of a rhombus if they have difficulty with this cons-uction. They may be asked whether they know other ways to construct a rhombus. If they do, encourage them to demonstrate these ways.

A protractor should be used in the construction of an angle of degree measure 72, although a compass and straightedge are sufficient if the construction is understood by the student.

The student should keep some record of his attempt to discover the relationship between the radius of the circle and the length of one side of an inscribed polygon.

Module 3

Because no answers are given for the first three tasks, the student's work on these tasks should be submitted to the teacher for evaluation. If his work is unsatisfactory at this point he should be encouraged to identify whatever difficulties he has encountered, and he should make some plan of his own for overcoming these difficulties. The teacher may suggest additional practice of previous tasks or may refer him to the section of the text that provides practice with similar tasks.

The student should conclude that the product of the number of sides in a regular polygon and the degree measure of a central angle of the inscribed circle is 360, whatever the number of sides in the regular polygon, and should use this conclusion to complete the table.

The teacher should evaluate the student's performance on tasks 11, 12, and 13 before the student attempts the mastery test. In the mastery test accuracy of construction is less important than the use of correct ideas. If the student shows mastery of ideas he should be given credit for mastery of this unit. Proficiency in accurate construction will increase with practice in later units.

Inscribed and Circumscribed Polygons (R. Nicely, Jr.)

Module 1 Diagnostic Test

1. Use a compass and ruler to draw a circle of radius 1.75 inches.

2. At the center of the circle construct two radii, $\overline{OR}$ and $\overline{OT}$, so that angle TOR has a degree measure of 60.

3. Construct segment RT and measure its length. This length should be 1.75 inches within an accuracy of 0.05 inches.

4. With a protractor find the degree measure of angle OTR. This measure should be 60 within an accuracy of 0.5.

5. Construct the perpendicular bisector of $\overline{RT}$. If your construction is accurate this perpendicular bisector will contain O.

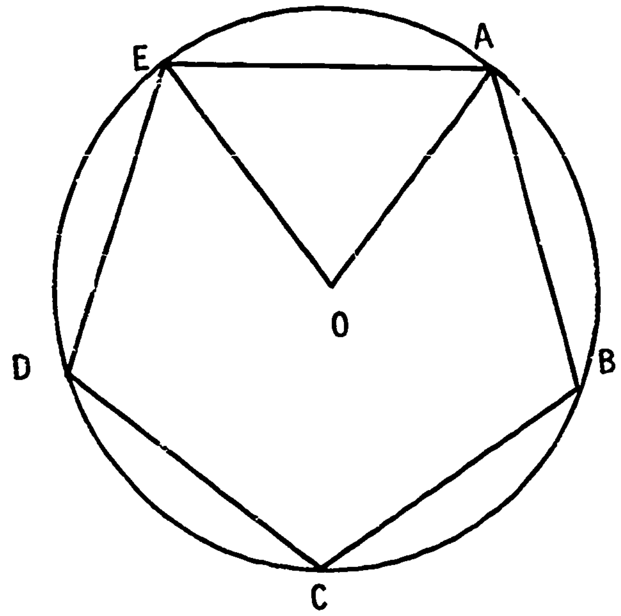
6. Construct the perpendicular bisectors of $\overline{OR}$ and $\overline{OT}$. If your construction is accurate these two perpendicular bisectors will intersect at a point on the perpendicular bisector of $\overline{RT}$. Label this point M.

7. With center M and radius MO construct a circle. If your construction is accurate R and T will be on the circumference of the circle you have just constructed.

8. Construct a circle with center M and radius MQ, where Q is the midpoint of $\overline{RT}$. If your construction is accurate this circle will contain the midpoints of $\overline{OR}$ and $\overline{OT}$.

Inscribed and Circumscribed Polygons

Module 2



ABCDE is a regular pentagon inscribed in the circle with center O. It has the same shape as the Pentagon in Washington, D. C. Any five sided figure is a pentagon. If all five sides are congruent, the pentagon is a regular________ (1).

Any geometric figure that has three or more sides is called a polygon. If all the sides of the polygon are congruent, the polygon is a regular__________ (2).

A regular polygon of three sides is called an equilateral triangle. A regular polygon of ______ sides is called a square. (3) Ahexagon is a ________ploygon of six sides. (4) An octagon is a regular______of eight sides. (5)

In the diagnostic test for this unit you constructed an equilateral triangle that was inscribed in a circle. You also inscribed a circle in that equilateral triangle. Every regular polygon has an inscribed circle and any regular polygon can be inscribed in a circle.

When a geometric figure is inscribed in a circle we can also say that the circle circumscribes the geometric figure. Every regular polygon has a circumscribed circle.

Answers: (1) polygon or pentagon (2) polygon (3) four (4) regular
(5) polygon

Construct $\overline{AB} \perp \overline{BC}$ and AB=BC. With center A and radius AB construct a circle. With center C and radius AB construct a circle. These two circles intersect at B and at another point. Label the other point D. Then ABCD should be a square.

Construct the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$ and label their point of intersection Q.

With center Q and radius QA construct a circle. This is the ________ circle of square ABCD (1).

With center Q and radius from Q to the midpoint of $\overline{AB}$ construct a circle. This is the ________________ circle of square ABCD (2).

* * * * * * * * * *

Every regular polygon has an inscribed circle and a circumscribed circle. It is also true that in any circle a regular polygon can be inscribed and about any circle a regular polygon can be circumscribed.

It is fairly easy to construct an equilateral triangle or a square. How would you construct a regular pentagon? One way is to construct first the inscribed or circumscribed circle. You can start with any circle. The radius of the circle you construct will determine the lengths of the sides of the regular pentagon. If you construct an angle of degree measure 72 at the center of a circle, the chord determined by this angle will be one side of the regular pentagon inscribed in the circle. Now that you know how to get started, try to construct a regular pentagon. Then construct the inscribed circle of that regular pentagon. If this circle is large enough you could even construct another regular pentagon inscribed in this circle.

Construct several inscribed regular pentagons in different circles and, by measurement, try to discover the relationship between the radius of the circle and the length of one side of the regular pentagon. (3)

Answers: (1) circumscribed (2) inscribed (3) The radius is approximately three-fourths the length of a side of the regular pentagon.

Inscribed and Circumscribed Polygons

Module 3

What you have learned about regular pentagons applies almost without change to other regular polygons. Every regular polygon has an inscribed circle and a circumscribed circle. This means that a regular polygon with any given number of sides can be inscribed in a circle or circumscribed about a circle. This knowledge is useful in design.

Use what you have learned about an inscribed pentagon to design a five-pointed star. (1)

A hexagon is a polygon with six sides. To construct a regular hexagon we first construct a circle and a central angle of degree measure 60. Then we proceed in the same way as in constructing a regular pentagon. Try this construction. (2)

Construct a design based on a regular hexagon. (3)

A square is a regular polygon with four sides. The central angle used to construct an inscribed square would have degree measure 90. Complete the table that is started for you below.

Number of sides in a regular polygon (a)		Degree measure of a central angle (b)	Product of (a) and (b)
3		120	______ (4)
4	(square)	90	______ (5)
5	(pentagon)	__	______ (6)
6	(hexagon)	__	______ (7)
8	(octagon)	__	______ (8)
9	(nonagon)	__	______ (9)

Answers: (1), (2), (3) not given (4) 360 (5) 360 (6) 72, 360
(7) 60, 360 (8) 45, 360 (9) 40, 360

The central angle associated with a regular octagon has degree measure ______________ (10). Use this information to construct a regular octagon. (11) Inscribe a circle in the regular octagon you have just constructed. (12) Inscribe a rhombus in this circle. (13) Which is closer to the circumference of the circle, the perimenter of the octagon or the perimeter of the rhombus? ______________ ______________ (14)

Precious stones are sometimes cut in the shape of a regular polygon on some of their faces. These cuts are made with great accuracy and require unusual skill to avoid damaging the stone and to bring out the stone's full brilliance. Design a stone that has at least one equilateral triangle and at least one square among its faces. (15)

Answers: (10) 45 (11), (12), (13) not given (14) The perimeter of the octagon is closer (15) not given

* * * * * * * *

Mastery Test

1. Construct a regular pentagon inscribed in a circle.
2. Inscribe a circle in the pentagon you have just constructed.
3. Construct a regular hexagon with sides 2 inches long. How much difference in length is there between the radius of its inscribed circle and the radius of its circumscribed circle?
4. Construct a seven-pointed star.

THE SLIDE RULE

Prerequisites - Included in the Program How to Use a Slide Rule by George Sackheim; Published by Harper & Row, Evanston, Illinois, 1964.

Overview: There is a story about a foreman on a building construction job who kept on reprimanding one of the laborers for not getting enough done. Finally the man lost his temper.

"Look, you so-and-so," he bellowed, "I'm working as hard as I can."

"I don't want you to work hard," answered the foreman. "I want you to work fast."

Any sensible person when he has a job to do will take time off at the beginning to find how he can get the most results for the least effort. This is not laziness but intelligence and efficiency. A really lazy person often spends more time and trouble trying to avoid the job than would have been needed to do it. Anyone who is likely to be doing much arithmetic will find that the methods explained in this unit can take most of the drudgery out of calculations. The price one has to pay for this increased efficiency is the effort involved in learning and then practicing until these mathematical tools become so familiar that their use is automatic.*

Objectives: After you have completed this program at least 80 percent of the time you will correctly be able to:

1. Set and read the slide rule
2. Multiply numbers using the slide rule
3. Multiply and divide quantities of ten with positive and negative exponents

---

* Hartkopf, R., Maths for Those that Hate It, Adelaide, Rigby Limited, 1965, p. 55.

4. Divide numbers using the slide rule

5. Square numbers using the slide rule

6. Find square roots using the slide rule

7. Cube numbers using the slide rule

8. Find cube roots using the slide rule

9. Solve proportion problems containing one unknown quantity.

DIAGNOSTIC PRETEST

This test is one of two tests that will tell you how much you were able to learn from this unit. It is taken before you start the unit. Another test is taken after you have completed the unit. If you receive a higher score on the second test than you did on this first test, it means you were able to learn from this unit. How much you learned is reflected by how much higher your score on the second test is when you compare it to your score on this test.

DIAGNOSTIC PRETEST - QUESTIONS

1. Set the following numbers on the D scale, one at a time and show them to your instructor:

   a. 1628

   b. 3175

   c. 5525

   d. 650000

   e. 0.01885

   Follow the directions in the answer section.

2. Solve the following problems using the slide rule and write the answers in the blank:

   a. $1.06 \times 0.18 =$ ________________

   b. $1.75 \times 3.625 =$ ________________

   c. $1.4 \div 3.5 =$ ________________

   d. $0.621 \div 82.6 =$ ________________

   e. $\dfrac{9.1 \times 8.65}{3\text{-}1/4} =$ ________________

   Check your answers in the answer section.

3. Square the following numbers using the slide rule:

   a. $(16)^2 =$ ________________

   b. $(0.19)^2 =$ ________________

   Find the square root of the following numbers using the slide rule:

   c. $\sqrt{625} =$ ________________

   d. $\sqrt{0.0365} =$ ________________

   e. $\dfrac{\sqrt{(18)^2}}{14} =$ ________________

   Check your answers in the answer section.

4. Cube the following numbers using the slide rule:

a. $(90)^3 =$ ______________________

b. $(0.007)^3 =$ ______________________

Find the cube root of the following numbers using the slide rule:

a. $\sqrt[3]{216} =$ ______________________

b. $\sqrt[3]{242.62} =$ ______________________

c. $\sqrt[3]{\dfrac{18 \times (14)^3}{6.23}} =$ ______________________

Check your answers in the answer section.

5. Solve the following proportion problems for X using the slide rule:

a. $\dfrac{4.5}{2.10} = \dfrac{x}{1.50}$, X = ______________________

b. $\dfrac{31}{x} = \dfrac{3}{4}$, X = ______________________

Check your answers in the answer section.

DIAGNOSTIC PRETEST - ANSWERS

1. If 4 or more of your answers are correct, do section 2 of the Pretest.

   If less than 4 of your answers are correct, read the sections "How to Learn with this Program" and "Look at your Slide Rule"; then, start with frame 1 and work the program.

2. a. 0.191 d. .0075

   b. 6.35 e. 24.3

   c. .4

   If 4 or more of your answers are correct, do section 3 of the Pretest.

   If less than 4 of your answers are correct, read the sections "How to learn with this Program" and "Look at your Slide Rule"; then, start with frame 78 and work the program.

3. a. 256 d. .605

   b. .000358 e. 15.2

   c. 25

   If 4 or more of your answers are correct, do section 4 of the Pretest.

   If less than 4 of your answers are correct, read the sections "How to Learn with this Program" and "Look at your Slide Rule"; then, start with frame 225 and work the program.

4. a. 730,000 d. 6.24

   b. .000000344 e. 19.8

   c. 6

   If 4 or more of your answers are correct, do section 5 of the Pretest.

   If less than 4 of your answers are correct, read the sections "How to Learn with this Program" and "Look at your Slide Rule"; then, start with frame 293 and work the program.

5. a. 3.21

   b. 41.4

If both of your answers are correct, get your next unit from your instructor.

If either of your answers are incorrect, read the sections "How to Learn with this Program" and "Look at your Slide Rule"; then, start with frame 338 and work the program.

CRITERION POST TEST - QUESTIONS

1. Set the following numbers on the D scale, one at a time, and show them to your instructor:

   a. 1495
   b. 3725
   c. 6577
   d. 220000
   e. 0.01191

2. Solve the following problems using the slide rule and write the answers in the blank:

   a. $340 \times 65 =$ ____________________
   b. $1.875 \times 27.3 =$ ____________________
   c. $55 \div 13 =$ ____________________
   d. $24\text{-}3/4 \div 68\text{-}1/2 =$ ____________________
   e. $\dfrac{3.1416 \times 0.75 \times 0.75}{.022} =$ ____________________

3. Square the following numbers using the slide rule:

   a. $(101)^2 =$ ____________________
   b. $(1.85)^2 =$ ____________________

   Find the square root of the following numbers using the slide rule:

   c. $\sqrt{1000} =$ ____________________
   d. $\sqrt{52\text{-}9/16} =$ ____________________
   e. $\sqrt{\dfrac{2}{(3)^2}} =$ ____________________

4. Cube the following numbers using the slide rule:

   a. $(1.07)^3 =$ ____________________
   b. $(4.75)^3 =$ ____________________

Find the cube root of the following numbers using the slide rule:

c. $\sqrt[3]{2116}$ = ____________________

d. $\sqrt[3]{0.01}$ = ____________________

e. $\sqrt[3]{\frac{22 \times (7)^3}{7.8}}$ = ____________________

5. Solve the following proportion problems using the slide rule:

a. $\frac{9}{57} = \frac{4}{x}$, X = ____________________

b. $\frac{x}{18} = \frac{5}{12}$, X = ____________________

CRITERION POST TEST - ANSWERS

2.	a. 22100	d. .361
	b. 51.1	e. 80.3
	c. 4.23	
3.	a. 10200	d. 7.25
	b. 3.4	e. .47
	c. 31.6	
4.	a. 1.23	d. .215
	b. 106	e. 9.76
	c. 12.83	
5.	a. 42	
	b. 7.5	

SUMMARY: Now that you have learned how to use the slide rule, it can save you a lot of work. In other units you will take, you will be asked to multiply, divide, square, and cube numbers and find their square roots and cube roots. This is a lot of work. If you use your slide rule to make these computations, you will save time and effort. A slide rule sitting on the shelf can't help you. A slide rule helps make mathematics easy and enjoyable, but you must (fill in the blank in your own words)

_____ _____ !

Answer: Use it !

References: Sackhein, George, How to Use a Slide Rule, Evanston, Illinois: Harper and Row, 1964.

Learning Aids: A standard slide rule.

LOGARITHMS

Readiness Skills

Knowledge of:

1. Laws of exponents.
2. Scientific notation.
3. Ratio and proportion.
4. Significant figures.
5. Laws of approximate numbers.

Notes to the Teacher

This unit attempts to extend the concept of exponents which the students know and understand to the concept of logarithms. The approach is to illustrate that logarithms are merely special exponents and that, therefore, the laws of exponents automatically apply to logarithms.

Although knowledge of Scientific Notation is listed in the Readiness Skills, it is not necessary that the student be familiar with scientific notation; it is necessary that he be able to write a number as the product of two factors: a number between 1 and 10 and an integral power of 10. He may be able to do the latter without knowing the terms "scientific notation" or exponential notation."

It is hoped that the individual student will develop his own method of finding the characteristic of a logarithm quickly, since it will be more meaningful to him if he discovers it for himself.

Students are encouraged to work in pairs or in small groups on this unit, so they may use one another for resources.

The students have been instructed to report to you after they have comp' ted each set of exercises. If the student is having difficulty, help him diagnose his problems and have him review the unit and/or refer him to the resource materials for this unit which will aid him.

The answers for the exercises follow:

Answers to Exercises--Logarithms

1. 2.5105
2. 4.5105
3. 7.5105 -10
4. 1.8149
5. 3.6294
6. 66000
7. 2020000
8. 14.0
9. 267
10. 510
11. 5.00
12. 129
13. .00890
14. 420
15. 6.63
16. 28200
17. .198
18. 111.2
19. .003190
20. 2.746

Pretest

1. $a^{\frac{1}{2}} =$ ____________
2. $a^2 \cdot a^3$ ____________

3. $(a^2)^3$ = ________

4. a^o = ________

5. (a^3) = ________

6. $\sqrt{a^3}$ = ________

7. a^{-1} = ________

8. $a^3 \div a^2$ = ________

9. $a^2 \div a^3$ = ________

10. 10^2 = ________

11. $1000 = 10^{\square}$

12. $324 = 3.24 \cdot$ ______

13. $52.4 = 5.24 \cdot 10^{\square}$

14. $5.36 = 5.36 \cdot 10^{\square}$

15. $.536 = 5.36 \cdot 10^{\square}$

16. Write 1652 as the product of a number between 1 and 10 and an integer power of 10. How many significant figures are there in:

17. 4006 ________

18. .006 ________

19. 4600 ________

Round off the following numbers to 3 significant figures:

20. 4.783________

21. 7.865 ________

22. 68346 ________

Perform the indicated operations and round off properly, assuming the numbers involved to be approximate:

23. $.43 \cdot 4.05$ ______

24. $16.642 \cdot 16.1$ ______

25. $13.6 \div 21$

26. if $\frac{.16}{.42} = \frac{S}{14}$, S = ______

OBJECTIVES

The student will:

1. See that a table of exponents may be used to perform multiplication, division, to raise a number to a power and to take a root of a number.

2. Recognize that logarithms are exponents, so that the laws of exponents apply to logarithms.

3. Change the exponential form $b^x = N$ to the logarithmic form $\log_b N = X$, and vice versa.

4. Recognize that tables of logarithms may be used just as the table of exponents was used to multiply, divide, raise to powers, and take roots.

5. Use the table of common (or base 10) logarithms to find the logarithm of any real number to four decimal places.

6. Use the common logarithms to find the product and/or quotient of two numbers, to raise a number to a power, and to find a root of a number, correct to 3 significant figures.

Additional Resources for this Unit

1. Table of Four Place Common Logarithms.

2. Slade, Margolis, Boyce, Mathematics for Technical and Vocational Schools, Fifth Edition, (John Wiley & Sons, Inc., 1968) pp. 188-205.

OVERVIEW

One of the greatest time-saving discoveries in mathematics is the method of logarithms, which was invented by John Napier, in the year 1614. The word logarithms is a special name for exponents. These special exponents are useful in solving problems such as $\frac{(453.44)^4 \times \sqrt[3]{36.22}}{0.0012 \times 786.13}$ with relative ease.

In previous units, you have learned to use the laws of exponents. Since logarithms are exponents, we will see that computation with logarithms can be done quickly and with little difficulty using the laws of exponents.

After you complete each set of exercises, report to your instructor so he can check your progress.

Consider the following table of exponents which lists some powers of 3, and the corresponding numbers.

Power of 3	Number	Power of 3	Number	Power of 3	Number
3^{-4}	$\frac{1}{81}$	3^2	9	3^8	6561
3^{-3}	$\frac{1}{27}$	3^3	27	3^9	19683
3^{-2}	$\frac{1}{9}$	3^4	81	3^{10}	59049
3^{-1}	$\frac{1}{3}$	3^5	243		
3^0	1	3^6	729		
3^1	3	3^7	2187		

To multiply $27 \cdot 81$, we note from the table that $27 = 3^3$ and $81 = 3^4$. Thus, $27 \cdot 81 = 3^3 \cdot 3^4 = 3^7$, which by the table is 2187. So $27 \cdot 81 = 2187$.

Find, by use of the table, $729 \div 2187$. $729 \div 2187 = 3^{\square} \div 3 = 3 = \underline{\square}$ (1)

Find by use of the table, $27^2 \cdot \sqrt[3]{729}$

$27 = 3^{\square}$ $\quad 27^2 = 3^{\square}$

$729 = 3^{\square}$ $\quad \sqrt[3]{729} = \sqrt[3]{3^{\square}} = (3^6)^{\square} = 3^{\square}$

Thus, $27^2 \cdot \sqrt[3]{729} = 3^{\square} =$ ________(2)

Answers: (1) $3^6 \div 3^7 = 3^{-1} = \frac{1}{3}$

(2) $27^2 \sqrt{729} = (3^3)^2 \cdot (3^6)^{\frac{1}{3}} = 3^6 \cdot 3^2 = 3^8 = 6561$

Consider the following table of powers of 10:

Power of 10	Number	Power of 10	Number
10^{-4}	.0001	10^1	10
10^{-3}	.001	10^2	100
10^{-2}	.01	10^3	1000
10^{-1}	.1	10^4	10000
10^0	1	10^5	100000

To multiply .0001 x 100, we could write .0 $= 10^{-4}$ and $100 = 10^2$, so $.0001 \times 100 = 10^{-4} \times 10^2 = 10^{-2} = .01$

$$\frac{\sqrt{10000} \cdot 100^2}{.001 \cdot .10^4} = \frac{(10^4)^{\frac{1}{2}} \cdot (10^2)^2}{10^{-3} \cdot 10^4}$$

$$= \frac{10^2 \cdot 10^4}{10^1}$$

$$= \frac{10^6}{10^1}$$

$$= 10^5$$

$$= 1000{,}000$$

The logarithm of a number N to the base b is the power (or exponent) to which th base must be raised to obtain N. For example, since $3^4 = 81$, the logarithm of 81 to the base 3 is __________ (3). Also $5^2 = 25$ can be written "the logarithm of 25 to the base 5 is ________(4). In symbols, we write this as $\log_5 25 = 2$.

What is the meaning of $\log_{10} 100 = 2$? $10^{\square} =$ _________ (5).

We can use logarithms to any base, but since our number system is based on 10, we will begin our discussion with logarithms to the base 10, called common logarithms. It is customary to omit the symbol for the base when referring to common (or base 10) logarithms. So long 1000 = 3 means $\square^3 = 1000$ (6).

Write the following in logarithmic form:

$10^{-2} = 0.01$	log __________ = ________	(7)
$10^{-1} = 0.1$	____________________	(8)
10^{0} = ____	____________________	(9)
10^{1} = ____	____________________	(10)
10^{2} = ____	____________________	(11)
10^{3} = ____	____________________	(12)

Answers: (3) 4 (4) 2 (5) $10^2 = 100$ (6) $10^3 = 1000$ (7) log 0.01 = -2
(8) log 0.1 = -1 (9) $10^0 = 1$ log 1 = 0 (10) $10^1 = 10$
log 10 = 1 (11) $10^2 = 100$ log 100 = 2 (12) $10^3 = 1000$
log 1000 = 3

Write the following in exponential form:

log 1000 = 3	10^3 = <u>1000</u>	
log 10000 = 4	____________________	(13)
log .001 = ____	____________________	(14)

We have seen how to find the common logarithm of a number if it is an integral power of 10. If the number is not a power of 10, we must (in most cases) refer to a table of common logarithms. (See Table 1).

In the Table of Four Place Common Logarithms, we see the first number in the left hand column is 10. The last number in the left-hand column is __________(15). This table gives us the common logarithms of numbers between 1 and 10. Since log 1 = 0 and log 10 = ________(16), the logarithms in the table are between 0 and 1. It is assumed, therefore, that every entry is preceded by a decimal point. The values in the table are called mantissas.

Answers: (13) $10^4 = 10000$ (14) log .001 = -3 $10^{-3} = .001$ (15) 54
(16) 1

Thus to find log 5.42, we look down the left hand column until we reach 54. We then move to the right until we reach the column headed by the Figure 2. The entry is 7340. Thus, log 5.42 = .7340 = 0.7340

log 3.23 = ________ (17)

log 1.04 = ________ (18)

Now consider log 54.2. We know log 54.2 = 0.7340, and log 54.2 = log (5.42 • 10). But since logarithms are exponents, log (5.42 • 10) = log 5.42 + log 10^1 = 0.7340 + 1 = 1.7340. Note that the mantissa is the same for both log 5.42 and log 54.2. Only the whole number part, or <u>characteristic</u> is different.

log 542 = log (5.42 • 100) = log 5.42 + log ________ (19)

= ________(20 + ________ (21) = ________ (22)

log 32.3 = log ________ (23) + log ________ (24)

= ________ + log ________(25) = ________ (26)

log 104 = ________ (27)

Answers: (17) .5092 (18) .0170 (19) log 100 = log 10^2 (20) 0.7340

(21) log 100 = x is written in exponential form as $10^x = 100 = 10^2$

x = 2. Thus <u>log 100 = 2</u> (22) 0.7340 + 2.7340

Thus, to find the common logarithm of a number N, we first write the number as the product of two factors, one of them a number between 1 and 10, and the other an integral power of 10. <u>$N = a \cdot 10^c$</u>

The <u>mantissa</u> which is found in the table is the logarithm of a (the number between 1 and 10) and the <u>characteristic</u> is c (the power to which 10 has been raised).

Thus, 3580 = $3.58 \cdot 10^3$, so

log 3580 = 3.58 + log 10^3

= ______ + 3

= ______ (28)

log .00723 = log (______ • $10^{\square}$)(29)

= ________ + (-3) (30)

Answers (28) .5539 + 3. (29) $7.23 \cdot 10^{-3}$ (30) log 7.23 + (-3) = <u>0.8591</u> + (-3)

But adding 0.8591 to -3 gives -2.1409, which means we must deal with a negative number. Another alternative is to write -3 as 7 - 10. Thus, 0.8591 + (-3) = 7.8591 - 10, and the logarithm can be used as a positive number. This will make it unnecessary to add positive and negative logarithms in the computational work that follows

Add the log .0369 to log 369:

log .0369 = ________ (31a)

log 369 = ________ (31b)

log (.0369) + log 369 = ________ (31c)

Answers: (31a) log .369 = log 3.69 + log 10^{-2}

= 0.5670 + (-2)

= <u>8.5670 -10</u>

(31b) log .369 = log 3.69 + log 10^{2}

= <u>2.5670</u>

(31c) 11.1340 - 10 = 1.1340

<u>Exercises</u>

Find:

1. log 324
2. log 32400
3. log .00324
4. log 65.3
5. log 4260

Can you find a shorter method to find the characteristic?

If you can't find a method, but would like to see faster methods of finding the characteristic, check your resources for this unit.

To multiply 166 · 20.6;

Find log 166 = 2.2201

Find log 20.6 = 1.3139

Add the logs ÷ log (166 · 20.6) = 3.5340

We now have the log of our product. We know that this product has mantissa .5340, which can be found in the table under the column headed by 2 and in the row headed by 34. Thus, our product is 3.42×10^3. We multiply by 10^3 since the characteristic is 3.

Find the product: .0263 · 3.19

log .0263 = ________ <u>-10</u>(32)

log 3.19 = ________ (33)

log (.0263 · 3.19 = ________ (34)

Thus, .0263 · 3.19 = ________ · 10 (35)

= ________ (36)

Answers: (32) 8.4200 -10 (33) <u>0.5038</u> (34) 8.9238 - 10

(35) $8.39 \cdot 10^{-2}$ (36) .0839

To find the logarithm of the product of two numbers, find the sum of the logarithms of the two numbers.

Exercises

Find the product, using logarithms:

6. $124 \cdot 532$
7. $.432 \cdot 4670$
8. $.0021 \cdot 6670$
9. $.421 \cdot 201 \cdot 3.16$

To divide 83.9 by 25.3:

(a) find log 83.9 = _________ (37)

(b) find log 25.3 = _________ (38)

Find the difference (a) - (b)

$\log (83.9 \div 25.3) =$ _________ (39)

The mantissa closest to .5156 is _________ (40)

Thus, since the characteristic is 0, $83.9 \div 25.3 =$ _________ (41) to three significant figures.

To divide 16.1 by 432,

(a) find log 16.1 = 1.2068

(b) find log 432 = 2.6355

Clearly we can't subtract without getting a negative logarithm. So we rewrite (a) as:

(a) log 16.1 = 10.2068 -10

(b) log 432 = 2.6355

Now subtracting (b) from (a) we have:

$\log (16.1 \div 432) = 7.613 \quad -10$

The mantissa closest to .6713 is _________ (41a)

So $16.1 \div 432 = 4.69 \cdot 10^{-3}$

= _________ (41b)

Answers: (37) 1.9187 (38) 1.4031 (39) 0.5156 (40) .5159 (41) 3.28
(41a) .6712 (41b) .00469

To find the logarithm of the quotient of two numbers, subract the logarithm of the divisor from the logarithm of the dividend.

Exercises

Use logarithms to find:

10. $763 \div 125$

11. $750 \div 15.0$

12. $17600 \div 136$

13. $16.5 \div 1860$

To find the nth root of a number N, we recall $\sqrt[n]{N} = N^{\frac{1}{n}}$

But $\log N^{\frac{1}{n}} = \frac{1}{n} \log N$.

Therefore, to find 81.7: $\sqrt[3]{81.7}$:

Find $\log 81.7 = 1.9122$, Multiply by $\frac{1}{3}$

$\log (81.7)^{\frac{1}{3}} = \frac{1}{3} \log (81.7) =$ __________ (42)

Thus $\sqrt[3]{81.7} = (81.7)^{\frac{1}{3}} = 4.34$, to 3 significant figures.

Find $\sqrt[4]{0.241}$

$\log 0.241 =$ __________ (43)

$\frac{1}{4} \log 0.241 =$ __________ (44)

Thus $\frac{1}{4} \log 0.241 = 9.8455 - 10$,

Answers: (42) 0.6374 (43) 9.3820 -10 (44) $\frac{1}{4}(9.3820 - 10) = \frac{1}{4}(39.3820 - 40)$

$= \underline{9.8455 - 10}$

Note our answer is written with characteristic $9-10 = -1$

So $(0.241)^{\frac{1}{4}} =$ __________ (45)

to 3 significant figures.

Find 417^2 :

log 417 = __________

2 log 417 = __________

$417^2 =$ __________ (46)

Find N, where $N = \dfrac{2.31^3 \cdot 31.6}{\sqrt{3.61 \cdot .0666}}$

Numerator

log 2.31^3 = 3 log 2.31 = ________

log 31.6 = ________

log numerator = ________

Denominator

log 3.61 = ________

log .0666 = ________

log 3.61 · .0666 = ________

log $\sqrt{3.61 \cdot .0666}$ = ________

log denominator = ________

log numerator = ________

log denominator = ________

log N = ________

N = ________ (47)

Answers: (45) .701 (46) 2 log 412 = 2 · 2.6149

= 5.2298

$412^2 = 1.70 \times 10^5$

= 170,000

(47) log 2.31^3 = 3 · 0.3636 = 1.0908

log 31.6 = 1.4997

log numerator = 2.5905

log 3.61 = 0.5575

log .666 = 8.8235 -10

log 3.61 · .666 = 9.3810 -10

log denom. $= \frac{1}{2} \cdot 9.3810 - 10$

$= \frac{1}{2} \cdot 19.3810 - 20$

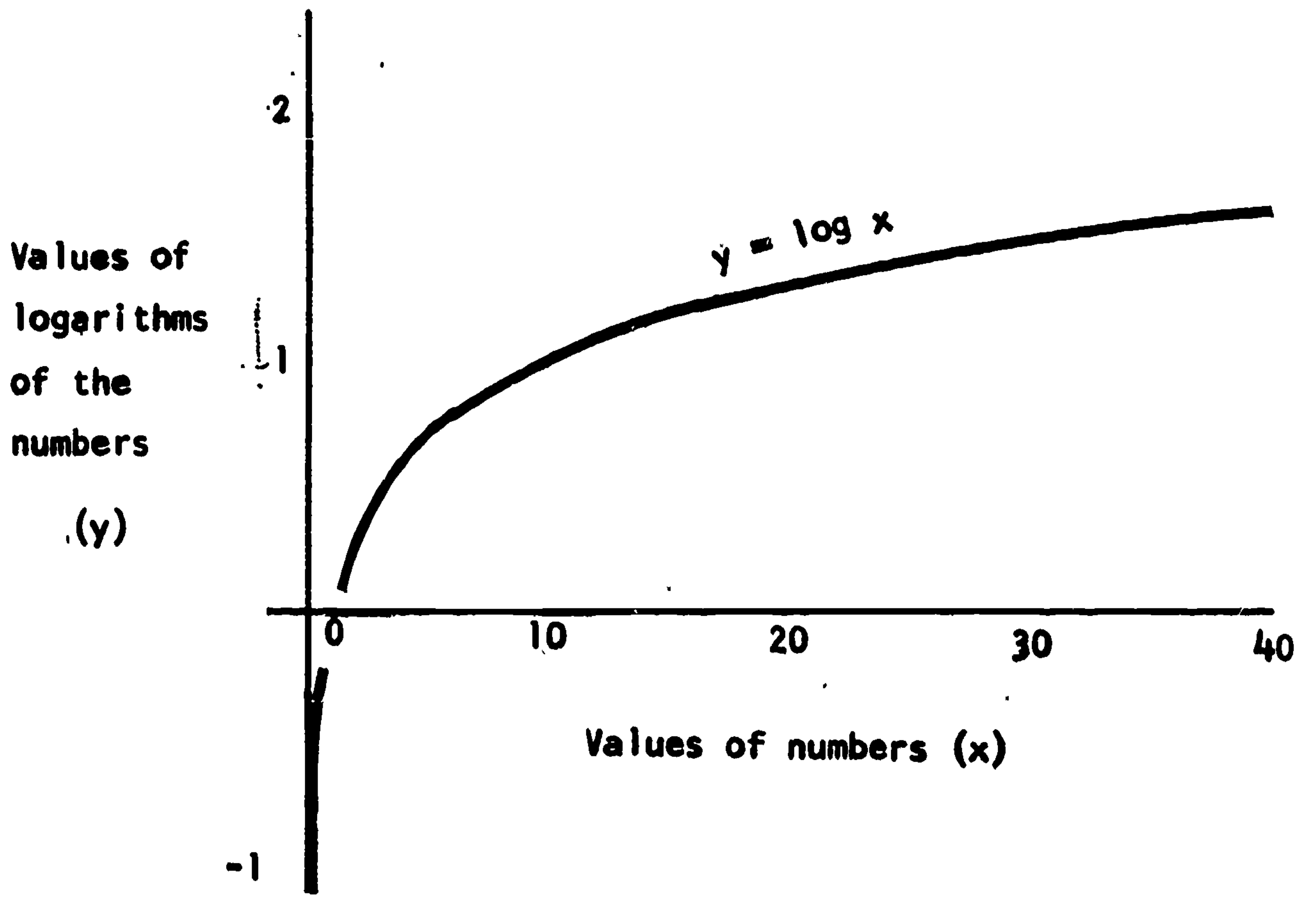
= 9.905 -10

log numerator	2.5905	= 12.5905 -10
log denominator	9.6905 -10	= 9.6905 -10
log N =		2.9000

$N = 7.94 \cdot 10^2$

N = 794

Consider $y = \log_{10} x$. We may find the values of x and y in the Four-Place Table of Common Logarithms and plot a graph of the function $y = \log_{10} x$.



This graph shows we can find the logarithm of <u>any</u> number between, say, 1 and 30 even though the table of logarithms only gives the logarithm of <u>some</u> of the numbers between 1 and 30. That is, we see from the graph that 21.85 has a common logarithm, but we cannot find log 21.85 exactly from the table. However, we can find log 21.80 and log 21.90 from the table. We next assume, quite reasonably, that a straight line drawn from (21,80, log 21.80) to (21.90, log 21.90) will closely approximate the graph of log x between these two points. That is, if 21.85 is halfway between 21.80 and 21.90, then log 21.85 must be halfway between log 21.80 and log 21.90. Thus we use a method called <u>linear interpolation</u> to find log 21.85:

Exercises

Use logarithms to find:

14. 16.1^3

15. $\sqrt[3]{291}$

16. $\sqrt{165 \cdot 13^3}$

17. $\dfrac{\sqrt{.165 \cdot 128}}{16.2^2}$

log 21.90 = 1.3404

log 21.85 .05 .10 = S? .0019

log 21.80 = 1.3385

21.85 is $\frac{.05}{.10} = \frac{5}{10}$ of the way from 21.80 to 21.90, so log 21.85 is $\frac{5}{10}$ of the way from 1.3385 to 1.3404.

The <u>tabular difference</u> from 1.3385 to 1,3404 is .0019.

Thus log 21.85 $= 1.3385 + \frac{5}{10} \cdot .0019$

$= 1.3395$

Note the value $S = \frac{5}{10} \cdot .0019$ or $\frac{5}{10} = \frac{S}{.0019}$

Find log 21.83

log 21.90 = __________

log 21.83 .03 .10 = S .0019

log 21.80 = __________

$$\frac{.03}{.10} = \frac{S}{.0019}$$

S = ____________(48)

∴ log 21.83 = 1.3385 + S

= ____________(49)

Find log 321.8

log 322.0 = 2.________

log 321.8 .8 1.0

log 321.0 =

$$\frac{.8}{1.0} = \frac{s}{\boxed{}}$$

s = ____________ (50)

log 321.8 = ____________ (51)

Answers: (48) s = .00057 = .0006 (49) 1.3385 + .0006 = 1.3391

(50) log 322.0 = 2.5211

log 321.8 .8 1.0 = s .0013

log 321.0 = 2.5198

$$\frac{.8}{1.0} = \frac{s}{\boxed{.0013}}$$

.8 · .0013 = s, so s = .0010

(51) log 321.8 = 2.5198 + s

= 2.5208

Next consider the problem 321.8 · 21.83.

We have found log 321.8 = 2.5208

log 21.83 = 1.3391

So log (321.6 · 21.83) = 3.8599

The product should be accurate to four significant digits, because each of the factors is accurate to four significant digits. We use interpolation to find the product. We find .8599 is between the entries .8597 and .8603 in the table of logarithms.

Thus we have:

$$\begin{array}{lcl}
\log 7250 & & = 3.8603 \\
\log \text{PRODUCT} & 10 & = 3.8599 \\
 & r & \quad\quad .0002 \quad\quad .0006 \\
\log 7240 & & = 3.8597
\end{array}$$

$$\text{So} \quad \frac{r}{10} = \frac{.0002}{.0006}$$

$$r = 10 \cdot \frac{.0002}{.0006} = 3$$

$\therefore$ PRODUCT is 7240 + 3 = 7243

Thus 321.8 · 21.83 = 7243, to four significant digits.

Find the Quotient: $\dfrac{\sqrt{98.66}}{7.880}$

First note that both the dividend and the divisor are accurate to four significant digits, so the quotient should be accurate to four significant digits.

$$\text{Log } N = \log\left(\frac{\sqrt{98.66}}{7.880}\right) = \frac{1}{2}\log 98.66 - \log 7.880$$

$$\begin{array}{lcl}
\log 98.70 & & = 1.9943 \\
\log 98.66 & .10 & \quad ? \quad\quad .0004 \\
 & d & \quad\quad s \\
\log 98.60 & & = 1.9939
\end{array}$$

log 7.880 = __________(52)

$$\frac{.06}{.10} = \frac{s}{.0004}$$

log 98.66 = ________________(53)

$\frac{1}{2}$ log 98.66 = ________________

log 7.880 = 0.8965

log N = __________________(54)

So

$$\left.\begin{array}{l}\log 1.270 \\ \left.\begin{array}{l}\log N \\ \\ \log 1.260\end{array}\right] r\end{array}\right] .010 \qquad \left.\begin{array}{l} = 0.1038 \\ \left.\begin{array}{l} = 0.1014 \\ \\ = 0.1004\end{array}\right] .0010\end{array}\right] .0034$$

So $\dfrac{r}{.010} = \dfrac{.0010}{.0034}$, which implies $r = .003$

Thus $N = 1.260 + r$

$= \underline{\qquad\qquad\qquad}$ (55)

Answers: (52) 0.8961 (53) 1.9941 (54) $\frac{1}{2}\log 98.66 = 0.9970$ (55) N=1.263

$$\begin{array}{rl} \log 7.880 & = 0.8956 \\ \hline \log N & = 0.1014 \end{array}$$

Exercises

Perform the computations using logarithms.

1. $\sqrt{12370}$

2. $\sqrt{\dfrac{0.07530}{86.02}}$

3. $\sqrt[5]{156.2}$

Summary

You have learned to work with logarithms using the laws of exponents, and to handle fairly difficult computations such as:

$$\sqrt[5]{256.1}\ , \quad \underline{\sqrt{.165} \cdot 128}\ , \quad \text{and} \quad \sqrt[3]{291}.$$

You may have found that you need more practice before you feel you have mastered this unit. If this is the case, be sure to work more problems from the resources for this unit. It is interesting to note that logarithms have uses millions of times each day, directly or indirectly. The sliderule, which you may study in another unit, is a device which may be used to solve problems involving many different types of computations. It enables us to multiply and divide by mechanically adding and subtracting logarithms and aids us in performing other operations with logarithms very rapidly. Computers,which work electronically with lightening speed, also use logarithmic procedures in some of their computations.

APPENDIX G

Small Scale Testing of Learning Units

Small Scale Testing of Sample Learning Units

A. Units tested, location, and date:

1. Conic Sections - Beaver, Pennsylvania Campus of Penn State University; January, 1968
2. Inscribed and Circumscribed Polygons, Module 1, and Mathematical Relationship Symbols - Gladstone High School, Pittsburgh, Pa.; March, 1968.

B. Student (subject) sample characteristics:

1. Conic Sections - entering freshmen members of a class in remedial mathematics; average I.Q.
2. Inscribed and Circumscribed Polygons - 2 tenth grade girls, 1 ninth grade boy; average I.Q.; presently taking plane geometry.
3. Mathematical Relationship Symbols - 2 eighth grade boys, 1 eighth grade girl; average I.Q.; presently taking general mathematics.

C. Results:

1. Conic Sections

a. 13 students found the unit interesting and informative.
4 students found the unit better than class.
5 students said there was no difference between regular class and the unit.
8 students said it could have been better.
0 students said it was terrible.

b. 0 students said their test scores were much better on this unit.
2 students said their test scores were better than average on this unit.
19 students said their grades were the same as they usually get in math.
11 students said their grades were worse than their average.
1 student said his grade was terrible.

c. 15 said they would have preferred regular classes in place of the method used.

0 said they would have preferred the unit without an instructor.

1 said he would have preferred working in groups but not meeting in class.

15 preferred the method used rather than any alternative.

d. 6 students did more than 80 unassigned problems.

2 students did 60 - 80 unassigned problems.

11 students did 50 - 59 unassigned problems.

3 students did 20 - 39 unassigned problems.

11 students did less than 20 unassigned problems.

1 student did 0 problems.

e. 6 said they would have gotten done faster and done as well working alone rather than in groups.

0 said they would have gotten done faster.

7 said working in groups or not made no difference.

11 said it would have taken longer if they hadn't worked in groups.

12 said they would not have done as well working alone.

f. 1 student said the circle needed more information in the unit.

2 students said the ellipse needed more information in the unit.

7 said the hyperbola needed more information in the unit.

17 said the parabola needed more information in the unit.

10 said none of the conics needed more information in the unit.

g. 17 said the circle was presented best.

13 said the ellipse was presented best.

4 said the ellipse was presented best.

0 said the parabola was presented best.

2 said none was presented better than the others.

h. 0 said the circle caused them the most trouble.
4 said the ellipse caused them the most trouble.
9 said the hyperbola caused them the most trouble.
24 said the parabola caused them the most trouble.
4 said none of the conics caused them trouble.

i. 0 said the circle is the hardest for them.
2 said the ellipse is the hardest for them.
12 said the hyperbola is the hardest for them.
19 said the parabola is the hardest for them.
3 said none of the conics was hard.

j. 0 said the circle was the most confusing for them.
1 said the ellipse was the most confusing for him.
10 said the hyperbola was the most confusing for them.
24 said the parabola was the most confusing for them.
3 said none of the conics was confusing.

k. 3 said finding the coordinates of the center was the hardest.
2 said finding the coordinates of the vertices was hardest.
8 said finding the coordinates of the foci was hardest.
14 said changing to the general form was the hardest.
10 said changing to the standard form was the hardest.
1 said none of the above was hard.

l. 2 said they had the most difficulty with the major axis of the ellipse.
16 said they had the most difficulty with the transverse axis of the hyperbola.
14 said they had the most difficulty with the axis of the parabola.
1 said he had no difficulty with axes.

m. 30 said the objectives were reasonable and they were tested on those objectives.
4 said they were not tested on the objectives of the unit.

n. 22 said they would like to study other topics in math using this method.
12 said they would not like to study other topics using this method.

Conclusions:

1. The parabola module needs to be revised.
2. More examples seem necessary, especially in changing from general to standard forms and vice versa.
3. Stüdents were more actively involved in this unit than in regular.
4. The question of whether the groups should be selected or left to free choice is unresolved. (A sociogram may be the answer here.)
5. The number of homework problems done was independent of the scores on the test.
6. The fact that many test scores were not better than the average of their math scores on other topics may be due to the relative difficulty of conic sections compared to other topics.

2. <u>Inscribed and Circumscribed Polygons</u> - module 1, the diagnostic pretest, is most effective in uncovering weakness in readiness skills rather than in content that the unit teaches; on second day of unit a student asked where to start -- directions should be included that tell students to start where they left off the day before; rest of directions are sufficient; answers in plain sight are a temptation for students to cheat -- they should be on another page (perhaps an eraseable answer sheet/); everywhere the word "rhombus" appears in the unit it should be changed to "square"; all comments by students are favorable -- they especially liked the provisions for individual differences, i.e., last paragraph on page 2.

3. <u>Mathematical Relationship Symbols</u> - the directions for the diagnostic pretest are inadequate and resulted in confusion-- the students looked ahead to the answers and filled them in the blanks of the test, and once the student completed a section of the test he either went onto the next section illegally or turned to and read only the first page of the material that he was told to turn to. Then he turned back to the test questions without con-

tinuing on in the material. This might be alleviated by having the teacher score the pretests. Once beyond the first page of the material, however, no difficulties were experienced. The answer blanks are not always large enough to contain a handwritten answer. The students' comments were mixed toward the material stemming from the inadequate directions (they are also inexperienced in taking programs).

1. Page 5 - Will pupils of this level in ABLE <u>know</u> what is a passive verb? Seems this whole sentence is superfluous.
2. Page 19 - What is to be paired? Seems to be that 1 - 5 should be paired with 6 - 10. The directions are not complete. They are confusing. Thus doubt is added to pupils' thoughts.
3. a) Page 25 - answers on page 26: Subtitle #4 gives answer of [-1, -2, -3, ...] or [all whole numbers less than 1]. All whole numbers less than one is 0 (zero) only as defined by Kinney, Ruble and Brown in their General Mathematics - Book I (pg 26); Book II (pg 55) - a problem solving approach.
   b) Page 25 - answers on page 26: Subtitle #5. Is it not better and certainly more explicit to give [1, 2, 3, ...] or (all Natural Numbers) not (all whole numbers except 0). The reason for this would be to be consistent with work completed, on having pupils use the most simple answer, as best, in Mathematical Relationship Symbols. This also shows pupil as one understanding that whole numbers = zero + Natural Numbers.
4. Page 52 - In pairing symbols <u>what</u> should be used on answer sheet. The numbers 1 - 6 or the actual symbols. If symbols are required why number them 1 - 6. Again are # 1 - 6 superfluous.
5. Page 58 - Should be $2 + 2 = 3$, not $2 + 2 - 3$
6. Page 62 - Suggest perhaps another word for choice b), besides, "dog." The nomenclature of the youth today includes this word description of a girl - one who is homely, poor figure, etc.
7. Page 68 - $6 > A$ $A = [5, 4, 3, \ldots]$ should be the set $A = [5, 4, 3, 2, 1, 0]$. This is <u>not</u> an infinite set. It is

finite as defined by Kinney, Ruble and Brown in the text. (All whole numbers less than 6) is correct verbally.

8. Page 68 - answers on page 69: 4 3, answer (all <u>whole</u> numbers greater than 0). Again, (Natural Numbers), for 2 = y 3 or (all <u>whole</u> numbers greater than 0). For 6 ≠ c, (0, 1, 2, 3, 4, 5, 7, 8, 9,...) not (... 3, 4, 5, 7, 8, 9...) because of definition of [....] implies infinite set before 6 and after 6 which is not true of <u>whole</u> numbers. All <u>whole</u> numbers except 6, also.

9. Page 69 - Answers on page 70: (48 3) = 6 + 10 solution set 67. Could it be possible $\frac{(48 \div 3) X}{6} = 6 + 10$?

10. Page 70 - Reference made to page 44, #1 - 53. What text? not text referred to previously - that being Book II - Kinney, Ruble and Brown.

11. Page 73 - Section 6 U = (whole numbers). Therefore 3) Answers page 76 is (0, 1, 2, 3, 5, 6, 7...) not (... 1, 2, 3, 5, 6, 7...) or (All <u>whole</u> numbers except 4); 4) (11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0) not (11, 10, 9,...) implies set and (All <u>whole</u> numbers less than 12); 8) (0, 1, 2,...) is fine but why not (all whole numbers) not' [All nonnegative numbers) which introduces a new concept for the whole number if it were (nonnegative integer.)