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Two factor analysis algorithms, previously described by P. Horst, have been programed for use on the General Electric Time-Sharing Computer System. The first of these, Principal Components Analysis (PCA), uses the Basic Structure Successive Factor Method With Residual Matrices algorithm to obtain the principal component vectors of a correlation matrix. The program will accept up to fifty variables in the correlation matrix and will successively compute up to thirty component vectors; it terminates upon finding a latent root less than one. Varimax Rotation (VARR) uses the Successive Factor Varimax Solution algorithm to produce a normalized rotated factor matrix, given no more than 10 principal component vectors of up to 50 variables. FORTRAN listings and sample runs of both programs are appended. (RM)

NATIONAL CENTER FOR EDUCATIONAL STATISTICS
Division of Operations Analysis

TWO COMPUTER PROGRAMS FOR FACTOR ANALYSIS

by

Carl E. Wisler

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Introduction

This note describes two computer programs which are available for factor analysis using the General Electric Time-Sharing Computer System. One program obtains the principal components of a correlation matrix; the other makes an orthogonal transformation of the principal components using a varimax algorithm. The primary purpose of this paper is to facilitate use of the programs; for theoretical interpretations and mathematical detail the reader should refer to the books listed in the bibliography section. With regard to computational aspects, some familiarity with the Time-Sharing Computer System is assumed.

The factor analysis package consists of two completely separate programs named PCA (Principal Components Analysis) and VARR (Varimax Rotation). In most applications the analysis would proceed according to the following sequence: 1. Run PCA, 2. Visually inspect the output of PCA and choose the principal components to be rotated, 3. Run VARR. Either program, however, may be used alone if desired.

The basic input to PCA is the correlation matrix which summarizes the original data. The terminal output includes the principal component vectors, listed in order of the variance accounted for, and the latent roots of the correlation matrix. Some additional output is optionally available. PCA also writes the principal component vectors (referred to collectively as the principal component matrix) in a permanent file named PC (Principal Components) where they may be used later by the program VARR.

The basic input to VARR is a set of principal component vectors. The terminal output includes the rotated factor matrix, the percent of variance which is accounted for by each vector and some additional optional output.

Principal Components Analysis

There are a number of possible procedures for obtaining the principal components of a correlation matrix. The particular algorithm followed by PCA is one which Horst[1/] calls the Basic Structure Successive Factor Method With Residual Matrices. The method is a step-wise procedure which solves for one principal component factor at a time. The effect of a given factor is removed from the correlation matrix leaving a residual matrix from which the next factor is then obtained. PCA continues to extract factors from the residual matrices until a latent root less than one is obtained; the factor associated with that root then becomes the last principal component.

The procedure is iterative with respect to the solution for each principal component. That is, the solution begins with an initial approximation vector and proceeds through successive iterations with each cycle yielding a better approximation to the correct factor. Because

1/ P. Horst, Factor Analysis of Data Matrices, (New York: Holt, Rinehart and Winston, 1965, pp. 156-167).

the procedure is iterative, information must be supplied to the program to control termination. The detailed requirements are discussed in the section on input and output.

An exact basic structure solution yields factors which are mutually orthogonal to one another. The iterative method of PCA gives factors which are approximately orthogonal if enough iterations are carried out.

The first principal component obtained by PCA tends to account for the maximum variance which can be accounted for by a single factor. If enough iterations are carried out, each successive factor accounts for the maximum variance not previously accounted for.

Other properties of the solution and an outline of the procedure are given by Horst[2/]. The purely computational parts of the program are almost identical to those provided by Horst[3/]. The dimension and input/output statements have, of course, been changed to conform to Time-Sharing FORTRAN. A listing of the program may be found in Appendix A.

Varimax Rotation

Varimax is the name given to one analytical procedure for making an orthogonal transformation of principal component factors. As a consequence of the varimax rotation the number of large and small loadings

---

2/ Ibid., pp. 157-167.
3/ Ibid., Appendix, p.607.

tend to be maximized and the number of intermediate loadings tend to be minimized. In the foregoing sense the final factor matrix is regarded as a simple structure.

There are several possible variations of the varimax computational procedure. The one used in VARR is referred to by Horst[4/] as the Successive Factor Varimax Solution. The new factor vectors are obtained in a step-wise fashion. The first factor tends to be that one for which the variance of the squared loadings is maximized. Subsequent factors are mutually orthogonal and, subject to this constraint, tend to come out in the order of the variance of their squared loadings.

The procedure followed in VARR is the so-called normal varimax in that each row of the principal components matrix is divided by the corresponding communality. The final varimax rotated matrix is thus normalized. It should be noted, however, that the Successive Factor Varimax Solution does not give the same answer as the varimax method originally suggested by Kaiser[5/].

As in PCA, each vector is obtained by an iterative procedure and information must be supplied to the program to control truncation. Details will be found in the input/output section.

4/ Ibid., pp. 423-428
5/ H.F.Kaiser, "Computer Program for Varimax Rotation in Factor Analysis," Educational and Psychological Measurement, Vol. 19, No. 3, Autum 1959, pp. 413-420.

Restrictions

The principal components analysis will accept up to fifty variables in the correlation matrix and will compute up to thirty principal components. Extraction of principal components is automatically stopped, however, after appearance of the first latent root less than one.

The varimax analysis will accept up to ten principal components with up to fifty variables.

Input and Output

PCA Program

Part of the input to PCA is via a data file named RMAT (R Matrix) and part is entered on-line during execution of the program.

Prior to running PCA, the upper triangle of a correlation matrix is entered in RMAT in Time-Sharing FORTRAN standard input format. The contents of a sample RMAT file are shown in Appendix B. Only the sequence of values is important; the arrangement of data with respect to line numbers is optional.

After the correlation matrix has been entered in RMAT, PCA is ready to RUN. During execution of PCA a request for five more items of information will appear on the teletypewriter in the following form:

```
N,INMAT,IRES,P,LI=
?
```

The user must then enter five numerical values from the keyboard. The symbols have the following meanings.

N — The order of the correlation matrix entered in RMAT.

INMAT — A programming variable which controls the output. If INMAT=1, the correlation matrix is printed as part of the output; if INMAT=0, printing of the correlation matrix is suppressed.

IRES — A programming variable which controls the output. If IRES=1 (and INMAT=1), each residual matrix is printed out; if IRES=0, the residual matrices are not printed. If IRES has a value of one then so must INMAT.

P — A stabilization limit on the number of iterations performed to extract any given principal component. After each iterative cycle a "measure of improvement"[6/] is computed and compared to the value of P.

6/ For a details regarding the "measure of improvement" see the listing of PCA in Appendix A or Horst, op.cit.

When the "measure" is less than the value of P, the iterative procedure is truncated. The user may enter a positive value if he chooses or by entering a zero the program automatically sets P equal to .00001, a value which will probably be satisfactory for most purposes.

LI An absolute limit on the number of iterations performed to extract any given principal component. The user may enter a positive integer if he chooses or by entering a zero the program will automatically set LI equal to 30.

After entering the required data the printout at the teletypewriter might then appear as follows (user supplied information is underlined):

```
N,INMAT,IRES,P,LI=
? 9,1,1,0,0
```

The output of PCA is as follows:

CORRELATION MATRIX.(optional) The lower triangle of the input correlation matrix.

PRINCIPAL COMPONENT i. The i^{th} principal component written as a row vector. The loading on the first variable appears first and so on.

RESIDUAL MATRIX i. (Optional) The residual matrix after the i^{th} principal component has been extracted. Written in lower triangular form.

NUMBER OF ITERATIONS. The number of iterations required to extract each principal component. Written in the order of extraction.

LATENT ROOTS. The latent root associated with each principal component. Written in order of extraction.

CUMULATIVE PERCENT OF VARIANCE ACCOUNTED FOR BY COMPONENTS. The cumulative percent of total data variance which is accounted by the first i components.

See Appendix C for a sample output.

PC Data File

The principal components extracted by PCA are stored in a file called PC for later use as input to VARR. The components are stored in Time-Sharing FORTRAN standard format in the order in which they were extracted, i.e., the first component extracted is first in the file and so on. Contents of a sample PC file may be found in Appendix C.

Prior to running PCA, a dummy PC file must be set up large enough to contain the principal components which are output from PCA. The Time-Sharing Library contains items of six different sizes which may be used to set up the dummy file.[7]

A set of principal components may also be entered directly into PC rather than as output from PCA.

VARR Program

Part of the input (viz., the principal components) to VARR is from the file PC and part is entered on-line during execution of the program.

A set of principal components must be available in PC prior to running VARR. Procedures pertaining to PC have already been discussed.

During execution of VARR a request for six more items of information will appear on the teletypewriter in the following form:

```
M,N,INMAT,ITRANS,P,NL=
?
```

7 General Electric Company, Time-Sharing FORTRAN Reference Manual, (General Electric Company, October 1966, pp. 65-66).

The user must then enter six numerical values from the keyboard. The symbols have the following meanings:

M	The number of principal components to be rotated. The first M principal components in PC will be rotated.
N	The number of variables represented in each principal component.
INMAT	A programming variable which controls the output. If INMAT=1, the M principal components are printed out; if INMAT=0, printing is suppressed.
ITRANS	A programming variable which controls the output. If ITRANS=1, the final transformation matrix is printed out; if ITRANS=0, printing is suppressed.
P	A stabilization limit on the number of iterations performed to obtain any given rotated vector. After each iterative cycle a "measure of improvement" 8/ is computed and compared to the value of P. When the "measure" is less than the value of P, the iterative procedure

8/ For details regarding the "measure of improvement" see the listing of VARR in Appendix D or Horst, op.cit.

is truncated. The user may enter a positive value if he chooses or by entering a zero the program automatically sets P equal to .0001, a value which will probably be satisfactory for most purposes.

NL An absolute limit on the number of iterations performed to rotate any given principal component. The user may enter a positive integer if he chooses or by entering a zero the program will automatically set NL equal to 40.

After entering the required data the printout at the teletypewriter might then appear as follows (user supplied information is underlined):

```
M,N,INMAT,ITRANS,P,NL=
? 4,9,1,1,0,0
```

The output of VARR is as follows:

PRINCIPAL COMPONENT MATRIX. (optional). The M principal components selected for rotation. Each component is in column vector form.

NUMBER OF ITERATIONS. The number of iterations required to obtain each rotated factor. Written in order of their transformation.

PERCENT VARIANCE ACCOUNTED FOR BY EACH FACTOR. The percent variance accounted for by each factor with respect to only the variance accounted for by the M principal components. That is, the sum of these percentages add to 100% neglecting round-off error.

FINAL TRANSFORMATION MATRIX. (Optional) The matrix which transforms the principal components matrix to the varimax rotated factor matrix. Written as a series of column vectors.

VARIMAX ROTATED FACTOR MATRIX. The new factors written as column vectors in the order of their transformation.

Bibliography

1. Anderson, T.W. (1958). Introduction to Multivariate Statistical Analysis. New York: John Wiley and Sons.

2. Cooley, W.W. and Lohnes, P.R. (1962). Multivariate Procedures for the Behavioral Sciences. John Wiley and Sons.

3. Harman, H.H. (1960). Modern Factor Analysis. Chicago: The University of Chicago Press.

4. Horst, P. (1965). Factor Analysis of Data Matrices. New York: Holt, Rinehart and Winston.

5. Morrison, D.F. (1967). Multivariate Statistical Methods. New York: McGraw-Hill.

APPENDIX A

Listing of PCA Program

```
100 DIMENSION R(1275),A(50),U(50),KV(30),EV(30),PER(30)
110 $FILE RMAT,PC
120 PRINT"N,INMAT,IRES,P,LI="
130 INPUT,N,INMAT,IRES,P,LI
140 IF(P) 5,5,6
150 5 P=.00001
160 6 IF(LI) 7,7,8
170 7 LI=30
180 8 M=(N*(N+1))/2
190 READ(1) (R(I),I=1,M)
200 E=0.
210 DO 502 L=1,30
220 IF(INMAT) 10,10
230 IF(L-1) 80,80,9
240 80 PRINT↑"CORRELATION MATRIX"
250 GO TO 10
260 9 IF(IRES) 10,10
270 PRINT↑"RESIDUAL MATRIX",
280 PRINT,L-1
290 10 LK=L
300 DO 102 I=1,N
310 DO 101 J=1,I
320 IJ=I+((J-1)*(N*2-J))/2
330 101 U(J)=R(IJ)
340 IF(INMAT) 102,102
350 IF(L-1) 1020,1020
360 IF(IRES) 102,102
370 1020 PRINT,↑(U(J),J=1,I)
380 102 CONTINUE
390 DO 18 I=1,N
400 18 A(I)=1.
410 DO 41 K=1,LI
420 DO 28 I=1,N
430 I1=I+1
440 U(I)=0.
450 DO 24 J=1,I
460 IJ=I+((J-1)*(N*2-J))/2
470 24 U(I)=U(I)+R(IJ)*A(J)
480 DO 28 J=I1,N
490 IJ=((I-1)*(N*2-I))/2+J
500 28 U(I)= U(I)+R(IJ)*A(J)
510 S=0.
520 DO 31 I=1,N
530 31 S=S+U(I)*A(I)
540 S=1./SQRTF(S)
550 DO 34 I=1,N
560 34 A(I)=U(I)*S
570 S=0.
580 DO 37 I=1,N
590 37 S=S+A(I)**2
```

```
600 E1=E
610 E=SQRTF(S)
620 IF((ABSF(E1/E-1.))-P) 42,41,41
630 41 CONTINUE
640 KV(L)=K-1
650 GO TO 43
660 42 KV(L)=K
670 43 EV(L)=S
680 PRINT↑"PRINCIPAL COMPONENT",
690 PRINT,L
700 PRINT,↑(A(I),I=1,N)
710 WRITE (2) (A(I),I=1,N)
720 IF (E-1.) 52,47,47
730 47 DO 50 I=1,N
740 DO 50 J=1,I
750 IJ=I+((J-1)*(N*2-J))/2
760 50 R(IJ)=R(IJ)-A(I)*A(J)
770 IF(1.-S)502,52,52
780 502 CONTINUE
790 52 PRINT↑"NUMBER OF ITERATIONS"
800 PRINT,↑(KV(I),I=1,LK)
810 PRINT↑"LATENT ROOTS"
820 PRINT,↑(EV(I),I=1,LK)
830 PER(1)=100.0*EV(1)/N
840 DO 60 I=2,LK
850 60 PER(I)=PER(I-1)+100.0*EV(I)/N
860 PRINT↑"CUMULATIVE PERCENT VARIANCE ACCOUNTED FOR BY COMPONENTS
870 PRINT,↑(PER(I),I=1,LK)
880 STOP
890 END
```

APPENDIX B

Sample PCA Problem

The sample problem is one given by Horst[9/]. The correlation matrix is:

1.000	.829	.768	.108	.033	.108	.298	.309	.351
	1.000	.775	.115	.061	.125	.323	.347	.369
		1.000	.272	.205	.238	.296	.271	.385
			1.000	.636	.626	.249	.183	.369
				1.000	.709	.138	.091	.254
					1.000	.190	.103	.291
						1.000	.654	.527
							1.000	.541
								1.000

A listing of the input as it appears in the data file RMAT is as follows:

```
110 1.,.829,.768,.108,.033,.108,.298,.309,.351,1.,.775
120 .115,.061,.125,.323,.347,.369,1.000,.272,.205,.238,
130 .296,.271,.385,1.000,.636,.626,.249,.183,.369,1.000,.709
140 .138,.091,.254,1.000,.190,.103,.291,1.000,.654,.527,1.000
150 .541,1.000
```

9/ Horst, Ibid., p.122.

The values for N, INMAT, IRES, P and LI are 9,1,1,0,0, respectively and are supplied on-line.

APPENDIX C

Sample PCA Output

Printed below is the teletypewriter output from a principal components analysis of the problem given in Appendix B. Information supplied on-line by the user is underlined.

```
PCA        14:03    W1 THU 10/05/67

N,INMAT,IRES,P,LI=
? 9,1,1,0,0

CORRELATION MATRIX

      1.00

      .829          1.00

      .768          .775          1.00

      .108          .115          .272          1.00

      .033          .061          .205          .636          1.00

      .108          .125          .238          .626          .709
      1.00

      .298          .323          .296          .249          .138
      .19           1.00

      .309          .347          .271          .183          .091
      .103          .654          1.00

      .351          .369          .385          .369          .254
      .291          .527          .541          1.00
```

PRINCIPAL COMPONENT 1

.7152	.7385	.772	.5574	.4654
.5196	.6403	.6145	.7151	

RESIDUAL MATRIX 1

.4885				
.3008	.4546			
.2159	.2049	.4041		
-.2906	-.2966	-.1583	.6893	
-.2993	-.2827	-.1542	.3766	.7834
-.2636	-.2588	-.1631	.3364	.4672
.73				
-.1599	-.1499	-.1983	-.1079	-.16
-.1427	.59			
-.1305	-.1068	-.2034	-.1595	-.1949
-.2163	.2605	.6224		
-.1604	-.1591	-.1671	-.0296	-.0788
-.0806	.0691	.1016	.4886	

PRINCIPAL COMPONENT 2

-.4965	-.4818	-.30	.647	.7418
.6912	-.0784	-.1645	.0335	

RESIDUAL MATRIX 2

.242				
.0616	.2225			
.067	.0604	.3141		
.0306	.0151	.0358	.2708	
.0685	.0747	.0683	-.1033	.2332
.0795	.0743	.0442	-.1108	-.0455
.2522				
-.1989	-.1877	-.2218	-.0572	-.1018
-.0385	.5838			
-.2121	-.1861	-.2527	-.053	-.0729
-.1026	.2476	.5954		
-.1438	-.143	-.157	-.0513	-.1037
-.1036	.0717	.1071	.4875	

PRINCIPAL COMPONENT 3

-.3478	-.319	-.4048	-.071	-.1844
-.1918	.5883	.6222	.3677	

RESIDUAL MATRIX 3

.1211				
-.0493	.1207			
-.0738	-.0688	.1502		
.0059	-.0076	.007	.2657	
.0043	.0159	-.0064	-.1164	.1992
.0128	.0131	-.0335	-.1244	-.0809
.2155				
.0057	2.61904E-05	.0163	-.0154	.0067
.0243	.2378			
.0042	.0124	-8.18963E-04	-.0089	.0419
.0168	-.1184	.2082		
-.0159	-.0257	-.0082	-.0252	-.0359
-.0333	-.1445	-.1216	.3523	

PRINCIPAL COMPONENT 4

.0305	.057	-.0094	-.1074	.1175
.1312	.2845	.1661	-.5648	

NUMBER OF ITERATIONS

7	8	8	16

LATENT ROOTS

3.7491	2.0495	1.3308	.4744

CUMULATIVE PERCENT VARIANCE ACCOUNTED FOR BY COMPONENTS

41.6562	64.4286	79.2154	84.4866

AT LINE NO. 860: STOP RAN 172/6 SEC.

A listing of the data file PC as written by the program PCA appears below.

1000	.7152	.7385	.772	.5574	.4654
1010	.5196	.6403	.6145	.7151	
1020	-.4965	-.4818	-.30	.647	.7418
1030	.6912	-.0784	-.1645	.0335	
1040	-.3478	-.319	-.4048	-.071	-.1844
1050	-.1918	.5883	.6222	.3677	
1060	.0305	.057	-.0094	-.1074	.1175
1070	.1312	.2845	1661	-.5648	

APPENDIX D

Listing of VARR Program

Several comment lines in the listing below have only numbers on them. These numbers may be used to relate the subsequent FORTRAN statements to the numbered equations in Horst's text[10/]. For example, line 460 has the number 1; this means that the next few statements pertain to equation 18.3.1 of the text.

```
100 DIMENSION A(50,10),B(50,10),H(10,10),C(50),V(50),KV(10)
110 +BS(10),W(10),U(10),PER(10)
120 SFILE PC
130 PRINT"M,N,INMAT,ITRANS,P,NL="
140 INPUT, M,N,INMAT,ITRANS,P,NL
150 IF(P) 10,10,11
160 10 P=.0001
170 11 IF(NL) 12,12,13
180 12 NL=40
190 13 READ(1) ((A(I,J),I=1,N),J=1,M)
200 IF(INMAT) 25,25
210 PRINT↑"PRINCIPAL COMPONENT MATRIX"
220 JA=1
230 IF(M-5) 20,20
240 JB=5
250 GO TO 21
260 20 JB=M
270 21 DO 22 I=1,N
280 22 PRINT,(A(I,J),J=JA,JB)
290 IF(JB-M) 23,25,25
300 23 PRINT,↑
310 JA=JB+1
320 JB=JB+5
330 IF(JB-M) 21
340 JB=M
350 GO TO 21
360'ROW NORMALIZATION
370 25 DO 115 I=1,N
```

10/ P. Horst, Ibid, pp. 424-425.

```
380 C(I)=0.
390 DO 115 J=1,M
400 115 C(I)=C(I)+A(I,J)**2
410 DO 117 I=1,N
420 117 C(I)=1./SQRTF(C(I))
430 DO 120 I=1,N
440 DO 120 J=1,M
450 120 A(I,J)=C(I)*A(I,J)
460'1
470 DO 127 J=1,M
480 W(J)=0.
490 DO 127 I=1,N
500 127 W(J)=W(J)+A(I,J)
510 FN=N
520 AL=0.
530 DO 131 J=1,M
540 131 AL=AL+W(J)**2
550 AL=1./SQRTF(AL)
560 DO 134 J=1,M
570 134 W(J)=W(J)*AL
580'2
590 DO 138 I=1,N
600 V(I)=0.
610 DO 138 J=1,M
620 138 V(I)=V(I)+A(I,J)*W(J)
630'3
640 DO 178 L=1,M
650 LA=1
660 DO 144 I=2,N
670 IF(V(LA)-V(I)) 144,144,143
680 143 LA=I
690 144 CONTINUE
700'4
710 DO 146 J=1,M
720 146 H(J,L)=A(LA,J)
730 E=0.
740 DO 173 K=1,NL
750 KV(L)=K
760'5
770 DO 153 I=1,N
780 B(I,L)=0.
790 DO 153 J=1,M
800 153 B(I,L)=B(I,L)+A(I,J)*H(J,L)
810'6
820 BS(L)=0.
830 DO 156 I=1,N
840 156 BS(L)=BS(L)+B(I,L)**2
850 BS(L)=BS(L)/FN
860 E1=E
870 E=AL
880 IF(P-ABSF(E1/E-1.)) 158,158,174
890 158 DO 159 I=1,N
900 159 B(I,L)=B(I,L)**3-B(I,L)*BS(L)
```

```
910'7
920 DO 163 J=1,M
930 U(J)=0.
940 DO 163 I=1,N
950 163 U(J)=U(J)+B(I,L)*A(I,J)
960'8
970 AL=0.
980 DO 166 J=1,M
990 166 AL=AL+U(J)**2
1000 AL=SQRTF(AL)
1010'9
1020 DO 169 J=1,M
1030 169 H(J,L)=U(J)/AL
1040 173 CONTINUE
1050'16
1060 PRINT
1070 174 DO 176 I=1,N
1080 DO 176 J=1,M
1090 176 A(I,J)=A(I,J)-B(I,L)*H(J,L)
1100'17
1110 DO 178 I=1,N
1120 178 V(I)=V(I)+B(I,L)
1130 PRINT :"NUMBER OF ITERATIONS"
1140 PRINT, (KV(L),L=1,M)
1150 DO 179 L=1,M
1160 179 PER(L)=100.0*BS(L)
1170 PRINT:"PERCENT VARIANCE ACCOUNTED FOR BY EACH FACTOR"
1180 PRINT,(PER(L),L=1,M)
1190 IF(ITRANS) 190,190
1200 PRINT :"FINAL TRANSFORMATION MATRIX"
1210 JA=1
1220 IF(M-5) 184,184
1230 JB=5
1240 GO TO 185
1250 184 JB=M
1260 185 DO 186 I=1,M
1270 186 PRINT,(H(I,J),J=JA,JB)
1280 IF(JB-M) 187,190 ,190
1290 187 PRINT,:
1300 JA=JB+1
1310 JB=JB+5
1320 IF(JB-M) 185
1330 JB=M
1340 GO TO 185
1350 190 CALL SINCHA(A,B,W,V,N,M,NL,P)
1360 STOP
1370 END
```

```
1380 SUBROUTINE SINCHA (A,B,W,V,N,M,NL,P)
1390 DIMENSION A(50,10),B(50,10),W(10),V(10)
1400 DO 6 I=1,M
1410 DO 4 J=I,M
1420 A(I,J)=0.
1430 DO 2 K=1,N
1440 2 A(I,J)=A(I,J)+B(K,I)*B(K,J)
1450 4 A(J,I)=A(I,J)
1460 6 V(I)=1.
1470 E=0.
1480 DO 14 L=1,NL
1490 DO 8 I=1,M
1500 W(I)=0.
1510 DO 8 J=1,M
1520 8 W(I)=W(I)+A(I,J)*V(J)
1530 AL=0.
1540 DO 10 I=1,M
1550 10 AL=AL+W(I)*V(I)
1560 AL=SQRTF(AL)
1570 E1=E
1580 E=AL
1590 IF(P-ABSF(E1/E-1.)) 12,12,15
1600 12 DO 13 I=1,M
1610 13 V(I)=W(I)/AL
1620 14 CONTINUE
1630 15 DO 16 I=1,N
1640 W(I)=0.
1650 DO 16 J=1,M
1660 16 W(I)=W(I)+B(I,J)*V(J)
1670 DO 18 I=1,N
1680 DO 18 J=1,M
1690 S=SIGNF(1.,W(I))*SIGNF(1.,V(J))
1700 18 B(I,J)=B(I,J)*S
1710 PRINT :"VARIMAX ROTATED FACTOR MATRIX"
1720 JA=1
1730 IF(M-5) 20,20
1740 JB=5
1750 GO TO 21
1760 20 JB=M
1770 21 DO 22 I=1,N
1780 22 PRINT,(B(I,J),J=JA,JB)
1790 IF(JB-M) 23,25,25
1800 23 PRINT,:
1810 JA=JB+1
1820 JB=JB+5
1830 IF(JB-M) 21
1840 JB=M
1850 GO TO 21
1860 25 RETURN
1870 END
```

APPENDIX E

Sample VARR Problem

The problem for VARR is to rotate the following principal components:

Principal Component 1	Principal Component 2	Principal Component 3	Principal Component 4
.7152	-.4965	-.3478	.0305
.7385	-.4818	-.3190	.0570
.7720	-.3000	-.4048	-.0094
.5574	.6470	-.0710	-.1074
.4654	.7418	-.1844	.1175
.5196	.6912	-.1918	.1312
.6403	-.0784	.5883	.2845
.6145	-.1645	.6222	.1661
.7151	.0335	.3677	-.5648

A listing of this input as it appears in the data file PC is given below.

1000	.7152	.7385	.772	.5574	.46
1010	.5196	.6403	.6145	.7151	
1020	-.4965	-.4818	-.30	.647	.74
1030	.6912	-.0784	-.1645	.0335	
1040	-.3478	-.319	-.4048	-.071	-.18
1050	-.1918	.5883	.6222	.3677	
1060	.0305	.057	-.0094	-.1074	.11
1070	.1312	.2845	.1661	-.5648	

The values for M, N, INMAT, ITRANS, P, NL are 4,9,1,1,0,0, respectively, and are supplied on-line.

APPENDIX F

Sample VARR Output

Printed below is the output from a varimax rotation of the principal components given in Appendix E. Information supplied on-line by the user is underlined.

```
VARR        14:12    W1 THU 10/05/67

M,N,INMAT,ITRANS,P,NL=
? 4,9,1,1,0,0

PRINCIPAL COMPONENT MATRIX
            .7152        -.4965        -.3478        .0305
            .7385        -.4818        -.319         .057
            .772         -.30          -.4048        -.0094
            .5574         .647         -.071         -.1074
            .4654         .7418        -.1844         .1175
            .5196         .6912        -.1918         .1312
            .6403        -.0784         .5883         .2845
            .6145        -.1645         .6222         .1661
            .7151         .0335         .3677        -.5648

NUMBER OF ITERATIONS
            5             5             5             4

PERCENT VARIANCE ACCOUNTED FOR BY EACH FACTOR
            34.1572       34.9125       23.831        7.0993

FINAL TRANSFORMATION MATRIX
            .5054         .7039         .4673        -.1754
            .8359        -.5393        -.0999        -.0221
           -.2011        -.4612         .8462        -.1757
            .0741         .0315         .2358         .9685

VARIMAX ROTATED FACTOR MATRIX
            .0199         .9942         .1031         .0254
            .0414         .9885         .1455         .0081
            .2388         .9671         .0498         .0724
            .9597         .0842         .1279         .2357
            .9983         .018          .0167        -.0534
            .9924         .0955         .0474        -.0607
            .1752         .2512         .9496        -.0671
            .0665         .2647         .9603         .0581
            .2783         .303          .5175         .7503

AT LINE NO. 1360: STOP        ,RAN 184/6 SEC.
```