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OCCUPATIONAL MATHEMATICS; PROPORTIONS. REPORT NO. 16-T.FINAL REPORT.

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This programed mathematics textbook is for student use in vocational education courses. It was developed as part of a programed series covering 21 mathematical competencies which were identified by university researchers through task analysis of several occupational clusters. The development of a sequential content structure was also based on these mathematics competencies. After completion of this program the student should be able to distinguish between correct and incorrect proportions, solve a proportion for one unknown quantity when given values for the other three, and solve specific problems involving proportions. The material is to be used by individual students under teacher supervision. Twenty-six other programed texts and an introductory volume are available as VT 006 882-VT 006 909, and VT 006 975. (EM)

FINAL REPORT
Project No. OE7-0031
Contract No. OEG-4-7-070031-1626
Report No. 16-T

Occupational Mathematics

PROPORTIONS

June 1968

U.S. DEPARTMENT OF
HEALTH, EDUCATION AND WELFARE

Office of Education
Bureau of Research

Occupational Mathematics

PROPORTIONS

Project No. 0E7-0031
Contract No. OEG-4-7-070031-1626
Report No. 16-T

by
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June 1968

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Washington State University, Department of Education, Pullman, Washington
State Coordinating Council for Occupational Education, Olympia, Washington

OBJECTIVES

1. The student should be able to distinguish between correct and incorrect proportions.

2. The student should be able to solve a proportion for one unknown quantity (given values for the other three).

3. The student should be able to solve specific problems involving proportions.

Greetings! You are about to begin improving your knowledge of basic mathematics. There are many important uses for the mathematics you are learning.

This booklet is not like your ordinary books. It is designed to help you learn as an individual. On the following pages you will find some information about mathematics. After the information is presented, you will be asked a question. Your answers to these questions will determine how you proceed through this booklet. When you have selected your answer to the question, turn to the page you are told to.

Do not write in this booklet. You may wish to have a pencil and some paper handy so you can write when you want to.

Remember this is not an ordinary book.

1. Study the material on the page.
2. Read the question on the page (you may want to restudy the material on the page).
3. Select the answer you believe is correct.
4. Turn to the page indicated by your answer.

Are you ready to begin?

(a)	Yes	Turn to page 1
(b)	No	Turn to page C
(c)	HELP	Go see your teacher

Your answer was (b) No.

Well, this booklet is a little different.

Go back and read page B again. After you have read it, you will probably be ready to begin.

In this unit we are going to study an important section of mathematics which is used mainly for problem solving. This section is called Proportions.

The definition of a proportion is that it is a statement of equality between two ratios. In more simple language we would say that it is two ratios that are equal to each other. To have a proportion, then, we must have an equal sign and two ratios. Example:
1/3 = 2/6 is a proportion.

Have you forgotten what a ratio is? Well, just in case, let's give you the definition. A RATIO IS ANY NUMBER OF THE FORM a/b WHERE b $\neq$ 0.

(Continue on next page)

There are two ways in which proportions are written. Let's demonstrate by way of an example. We will use the two equivalent fractions 2/5 and 4/10. The most common way of writing a proportion is simply setting the two equivalent fractions equal to each other. We then have 2/5 = 4/10. The other way is to write each ratio with a colon (:) and the equal sign with a double colon (::). Then we have 2:5::4:10. This is read as 2 is to 5 as 4 is to 10.

Remember now, 2/5 = 4/10 and 2:5::4:10 are two different ways of writing the SAME relationship.

Turn to page 2 now and continue.

Here is your first question.

Is the fraction 5/6 also a ratio?

(a) Yes Turn to page 5

(b) No Turn to page 4

Incorrect!

While we do have two ratios and an equal sign, you should notice that the two ratios CANNOT be set equal to each other. In other words, 2/3 = 5/6 is not a true equality since 2/3 = 4/6, NOT 5/6.

Remember, to have a proportion, the two ratios must be equivalent to each other.

Return to page 5 and try again.

Your answer is incorrect!

The fraction 5/6 is in the form a/b where $b \neq 0$, therefore, it is also a ratio by definition. In fact all integers and fractions are also ratios.

Now you should make a choice.

(a) I would like a little more work on ratios.
Turn to page 6

(b) I'm ready to continue the unit now.
Turn to page 1

Okay, 5/6 is a ratio. In fact any fraction or integer is a ratio.

Now, let's continue with this question.

Is 2/3 = 5/6 a proportion?

(a) Yes Turn to page 3

(b) No Turn to page 9

Ratios are a very important part of this unit. You really should become quite familiar with them.

Therefore, go to Unit 4 and study the concept of ratio.

Return to page 1 of this Unit when you have finished Unit 4.

Very good! 4/2 = 2 is a proportion.

Is 2:3::9:4 a proportion?

(a) Yes Turn to page 8

(b) No Turn to page 11

Incorrect!

While the numbers are written in the form of a proportion, you should see that 2/3 and 9/4 are not equivalent. Therefore, it can't be a proportion.

Return to page 7 and try again.

Correct, 2/3 = 5/6 is not a proportion.

Is 4/2 = 2 a proportion?

(a) Yes Turn to page 7

(b) No Turn to page 10

Wrong answer!

Since I'm sure you know that 4/2 does equal 2, I feel that you thought that 2 was not a ratio. However, 2 can be written as the fraction 2/1 and therefore it fits the definition of a ratio. In fact all rational numbers (whole numbers and fractions) are ratios since they all are of the form a/b, where $b \neq 0$.

Return to page 9 and try again.

Very good! Your answer is correct.

To continue, we need to know some of the terminology that pertains to proportions. First of all, there are 4 terms to a proportion. Let's look at the terms by way of an example. Consider the proportion 1/2 = 3/6 (1:2::3:6); the 1 is the first term, the 2 is the second term, the 3 is the third term, and the 6 is the fourth term.

To go one step further, the second and third terms are called the means, and the first and fourth terms are called the extremes. Using the example above, the extremes are 1 and 6, while the means are 2 and 3. Now, let's use what we have just learned.

What are the means of the proportion 2/3 = 4/6 ?

(a) 2 and 3 Turn to page 13

(b) 4 and 6 Turn to page 20

(c) 2 and 6 Turn to page 17

(d) 3 and 4 Turn to page 15

You seem to be having trouble identifying the terms of a proportion.

Go back to page 11 and read the material there carefully. Then continue from there.

Incorrect!

The means of a proportion are the second and third terms of the proportion. Remember the example: 1:2::3:6 (1/2 = 3/6), well the 2 and the 3 are the means. In other words, the means, 1: 2::3 :6, refer to the two middle terms.

Try this problem.

What are the extremes of the proportion 2:4::3:6 ?

(a) 2 and 6 Turn to page 15

(b) 3 and 6 Turn to page 16

(c) 4 and 3 Turn to page 12

Incorrect! 5 is the second term of the proportion 3/5 = 6/10.

It should be real easy to see the second term of the proportion if you wrote 3/5 = 6/10 as 3:5::6:10.

Just count down the line. (3) : (5) :: (6) : (10)

1st 2nd 3rd 4th

term

Answer this question.

What is the third term of the proportion a/b = c/d ?

(a) b — Turn to page 12

(b) d — Turn to page 19

(c) Neither of them — Turn to page 18

Your answer is correct!

Let's continue with this problem.

What is the second term of the proportion 3/5 = 6/10 ?

(a)	6	Turn to page 21
(b)	5	Turn to page 18
(c)	Neither of them	Turn to page 14

Wait a minute! The 3 and the 6 form one of the ratios, thus they are not the means or the extremes.

Return to page 17 and try again.

Incorrect!

The means of a proportion are the second and third terms of the proportion. Remember the example: 1:2::3:6 (1/2 = 3/6), well the 2 and the 3 are the means. In other words, the means, 1:2::3:6, refer to the two middle terms.

Try this problem.

What are the extremes of the proportion 2:4::3:6 ?

(a) 2 and 6 Turn to page 15

(b) 3 and 6 Turn to page 16

(c) 4 and 3 Turn to page 12

Your last answer was correct! Let's now look at the more useful side of proportions, namely problem solutions.

The main idea involved in solving a proportion is: "THE PRODUCT OF THE MEANS IS EQUAL TO THE PRODUCT OF THE EXTREMES."

Let's look at some examples:

(a) If $1/2 = 3/6$, then $1 \cdot 6 = 2 \cdot 3$ and $6 = 6$

(b) If $3/5 = 6/10$, then $3 \cdot 10 = 5 \cdot 6$ and $30 = 30$

(c) If $\frac{2\ 1/2}{3\ 1/3} = 3/4$, then $2\ 1/2 \cdot 4 = 3\ 1/3 \cdot 3$ and $10 = 10$

You should see that the product of the means is always equal to the product of the extremes if we have a proportion.

(Continue on next page)

Now, how do we solve a problem with this idea? Very simple. Let's look at an example problem, namely x/3 = 4/6. The question simply is: What number does x stand for if x times 6 must be equal to 4 times 3 ? Well, 4 times 3 = 12. What times 6 will equal 12 ? 2, of course.

You try one.

What number for x will make x/5 = 4/10 a proportion?

(a) 4 Turn to page 27

(b) 2 Turn to page 25

(c) $12\frac{1}{2}$ Turn to page 23

Wait a minute!

d was the last or the fourth term of the proportion.

you were asked to find the third term.

Go back to page 21 and make another selection.

Incorrect!

The means of a proportion are the second and third terms of the proportion. Remember the example: 1:2::3:6 (1/2 = 3/6), well the 2 and the 3 are the means. In other words, the means, 1:2::3:6, refer to the two middle terms.

Try this problem.

What are the extremes of the proportion 2:4::3:6 ?

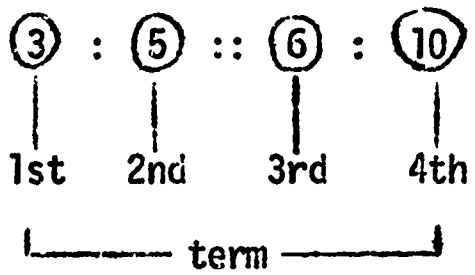
(a) 2 and 6 Turn to page 15

(b) 3 and 6 Turn to page 16

(c) 4 and 3 Turn to page 12

Incorrect! 5 is the second term of the proportion 3/5 = 6/10.

It should be real easy to see the second term of the proportion if you wrote 3/5 = 6/10 as 3:5::6:10. Just count down the line.



Answer this question.

What is the third term of the proportion a/b = c/d ?

(a) b — Turn to page 12

(b) d — Turn to page 19

(c) Neither of them — Turn to page 18

Your answer is incorrect! Here is how to reason out the solution.

The problem was $1/x = 3/18$. Now, from the Rule that the "product of the means equals the product of the extremes," we see that x times 3 must equal 1 x 18 which is 18. So, the question simply becomes, what number times 3 = 18 ?

The answer to this question is _____?

(a) 9 Turn to page 28

(b) 6 Turn to page 32

Your answer is incorrect! Let's look at how to work it.

Given the proportion $x/5 = 4/10$, we wish to find a number for x which will make x times 10 = 4 times 5.

Since 4 times 5 = 20, what number times 10 will give you 20 ?

(a) 4 Turn to page 28

(b) 2 Turn to page 29

Your answer is incorrect! Here is how to reason out the solution.

The problem was 1/x = 3/18. Now, from the Rule that the "product of the means equals the product of the extremes," we see that x times 3 must equal 1 x 18 which is 18. So, the question simply becomes, what number times 3 = 18 ?

The answer to this question is _____?

(a) 9 Turn to page 28

(b) 6 Turn to page 32

Very good! Correct answer.

Work this problem.

What is the value of x in the proportion 1/x = 3/18 ?

(a) 9 Turn to page 22

(b) 18 Turn to page 24

(c) 6 Turn to page 26

Very good! Your answer is correct.

You should have observed that in solving any proportion by the rule that the product of the means equals the product of the extremes we have <u>always</u> ended up with an equation of the form a = bc where a and b are numbers.

You should know that the equation a = bc solved for c is c = a/b. (Solutions of the equation a = bc are covered in Unit 14.)

Therefore, <u>all</u> solutions of a proportion involve just two steps.

Step 1: Apply the rule that the product of the means equals the product of the extremes.

Step 2: Solve the equation of the form a = bc for c.

(Continued on next page)

Look at this example.

Example: $x/3 = 5/6$

Step 1: $6x = 15$ (Product of means = product of extremes)

Step 2: $x = 15/6$ (If $a = bc$, then $c = a/b$)

Then reduce your answer if necessary. In this case we get $x = 5/2$ or $2\frac{1}{2}$ for our final answer.

Okay, now you solve $8/12 = x/1.5$ for x.

(a) 1 Turn to page 39

(b) 8 Turn to page 35

(c) $2\frac{1}{4}$ Turn to page 41

Your answer is incorrect! Let's look at how to work it.

Given the proportion x/5 = 4/10, we wish to find a number for x which will make x times 10 = 4 times 5.

Since 4 times 5 = 20, what number times 10 will give you 20 ?

(a) 4 Turn to page 28

(b) 2 Turn to page 29

Incorrect! Come now, it's not that hard.

Try to reason the problem out instead of dividing or multiplying by whatever number comes to your head.

What number times 4 is <u>equal to</u> 8 ?

(a) 4 Turn to page 30

(b) 2 Turn to page 29

Correct!

Work this problem now.

If 7/2 = x/6, what is the value of x ?

(a) 42 Turn to page 31

(b) 84 Turn to page 28

(c) 21 Turn to page 25

You seem to be having trouble solving equivalent relationships.

Go study the concepts presented in Unit 14, and then return to page 1 of this Unit.

Whoops! 42/6 = 7 not 7/2!

Return to page 29 and try again.

Correct!

Try this problem now.

If 3/1 = 24/x, then x = _____?

(a) 1 Turn to page 38

(b) 8 Turn to page 26

(c) 72 Turn to page 28

Correct!

Let's continue.

The value of x in the proportion 4/x = 2/.25 is _____.

(a) 50 Turn to page 48

(b) 1/2 Turn to page 39

(c) 32 Turn to page 37

(d) 2 Turn to page 45

Your answer is wrong! Here is the step by step method for solving it.

We were given the proportion: $4/x = \frac{3\ 1/3}{2\ 1/2}$

Step 1: $3\ 1/3(x) = 4(2\ 1/2)$ [Product of the extremes = product of the means]

Step 2: $3\ 1/3(x) = 10$ [Multiplied 2 1/2 x 4.]

Step 3: $x = \frac{10}{3\ 1/3}$ [Divided both sides of the equal sign by 3 1/3. This is the solution of a = bc for c.]

Step 4: $x = 3$ [Divided 10 by 3 1/3.]

Okay, now you work this problem.

What is the value of x in the proportion $x/4 = \frac{4\ 1/2}{3}$?

(a) 6 Turn to page 36

(b) $\frac{27}{8}$ Turn to page 44

(c) 12 Turn to page 47

Your answer is incorrect! Let's look at the solution to the problem.

8/12 = x/1.5 tells us that 8(1.5) must equal 12(x). Well, 8 times 1.5 = 12. So, 12 times some number (x) must equal 12. Therefore, x = 1.

Let's examine another way to work this problem. Since we know that the Product of the Means = the Product of the Extremes, we can write 12(x) = 8(1.5) immediately from the problem. Now, the problem is in the form of a = bc.

(Continued on next page)

Let's look at this solution step by step from our problem of 8/12 = x/1.5

Step 1: 12(x) = 8(1.5) [Product of the means = product of the extremes.]

Step 2: 12(x) = 12 [Multiply 8 x 1.5]

Step 3: x = 12/12 [Divide both sides of the equal sign by 12.]

Step 4: x = 1 [Reduce]

You try a problem now.

What is the value of a in a/3 = 15/5 ?

(a) 1 Turn to page 37

(b) 3 Turn to page 42

(c) 9 Turn to page 33

Your answer is correct!

Here is your next problem.

What is the value of c in the proportion $\frac{2\ 1/2}{5} = \frac{c}{20}$?

(a) 250 Turn to page 47

(b) 5/8 Turn to page 46

(c) 10 Turn to page 42

No! Wrong answer.

Remember: THE PRODUCT OF THE MEANS MUST EQUAL THE PRODUCT OF THE EXTREMES.

Check your answer to be sure you are right.

Let's try this problem.

If $x/12 = 3/4$ then by the rule above we know that $4(x) = 12(3)$. Now, what does $x = ?$

(a) 16 Turn to page 49

(b) 36 Turn to page 45

(c) 9 Turn to page 33

What?? 1 x 3 = 24 ?! Come on now, you are not trying very hard.

Return to page 32 and work the problem again.

Correct! Keep up the good work.

Try this problem.

If $4/x = \frac{3\ 1/3}{2\ 1/2}$, then what is the value of x ?

(a) 25/12 Turn to page 34

(b) 16/3 Turn to page 40

(c) 3 Turn to page 43

Your answer is wrong! Here is the step by step method for solving it.

We were given the proportion: $4/x = \frac{3\ 1/3}{2\ 1/2}$

Step 1: $3\ 1/3(x) = 4(2\ 1/2)$ [Product of the extremes = Product of the means.]

Step 2: $3\ 1/3(x) = 10$ [Multiplied 2 1/2 x 4]

Step 3: $x = \frac{10}{3\ 1/3}$ [Divided both sides of the equal sign by 3 1/3. This is the solution of a = bc for c.]

Step 4: $x = 3$ [Divided 10 by 3 1/3]

Okay, now you work this problem.

What is the value of x in the proportion $x/4 = \frac{4\ 1/2}{3}$?

(a) 6 Turn to page 36

(b) 27/8 Turn to page 44

(c) 12 Turn to page 47

Your answer is incorrect! Let's look at the solution to the problem.

8/12 = x/1.5 tells us that 8(1.5) must equal 12(x). Well, 8 times 1.5 = 12. So, 12 times some number (x) must equal 12. Therefore, x = 1.

Let's examine another way to work this problem. Since we know that the Product of the Means = the Product of the Extremes, we can write 12(x) = 8(1.5) immediately from the problem. Now, the problem is in the form of a = bc.

(Continued on next page)

Let's look at this solution step by step from our problem of 8/12 = x/1.5

Step 1: 12(x) = 8(1.5) [Product of the means = Product of the extremes.]

Step 2: 12(x) = 12 [Multiply 8 x 1.5]

Step 3: x = 12/12 [Divide both sides of the equal sign by 12.]

Step 4: x = 1 [Reduce]

You try a problem now.

What is the value of a in a/3 = 15/5 ?

(a) 1 Turn to page 37

(b) 3 Turn to page 42

(c) 9 Turn to page 33

No: Wrong answer.

Remember: THE PRODUCT OF THE MEANS MUST EQUAL THE PRODUCT OF THE EXTREMES.

Check your answer to be sure you are right.

Let's try this problem.

If $x/12 = 3/4$ then by the rule above we know that $4(x) = 12(3)$. Now, what does x = ?

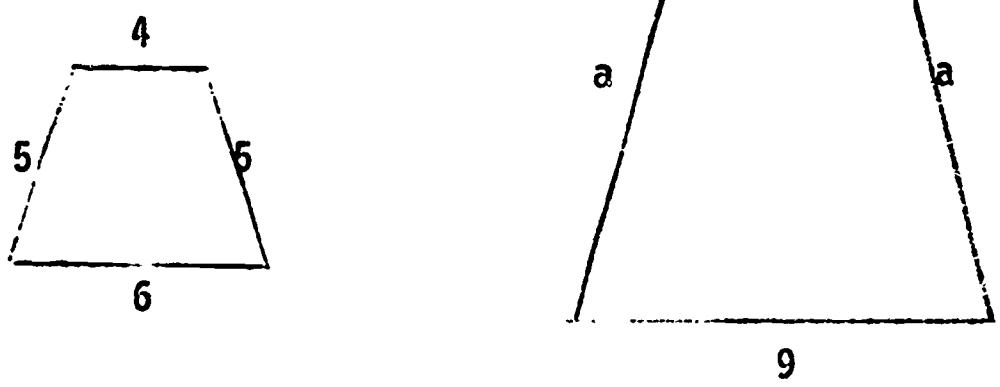
(a) 16 Turn to page 49

(b) 36 Turn to page 45

(c) 9 Turn to page 33

Correct! You're doing fine. Let's continue now by working problems that use proportions. These problems come from the fields of Chemistry, Electronics, Geometry, etc. Here is an interesting one for you.

If any two figures are similar, then their corresponding sides are proportional. Look at these two similar figures.



What is the value of c in the larger figure if the smaller figure has sides as listed in the figure?

(a) $7\frac{1}{2}$ Turn to page 57

(b) 4 Turn to page 53

(c) 6 Turn to page 50

No! Did the product of the means equal the product of the extremes? You can always check your answer that way.

Try this problem.

The value of P in $\frac{1/2}{P} = \frac{1/4}{7}$ is _____.

(a) 7/8 Turn to page 37

(b) 7 Turn to page 49

(c) 14 Turn to page 36

You seem to be having problems solving equivalent relationships.

You will find that Unit 14 (Solutions of A = BC) will help you learn the concepts needed to solve these problems.

Finish Unit 14 before returning to page 18 of this Unit.

Wait a minute! (5/8) (5) doesn't equal (2 $\frac{1}{2}$) (20).

Remember, check your answer using the rule of proportions. That is, the product of the means equals the product of the extremes.

Return to page 36 and try again.

No: Did the product of the means equal the product of the extremes? You can always check your answer that way.

Try this problem.

The value of P in $\frac{1/2}{P} = \frac{1/4}{7}$ is _____.

(a) 7/8 Turn to page 37

(b) 7 Turn to page 49

(c) 14 Turn to page 36

Whoops! You forgot that there was a decimal point in your problem.

Return to page 33 and try again.

You seem to be having problems solving equivalent relationships.

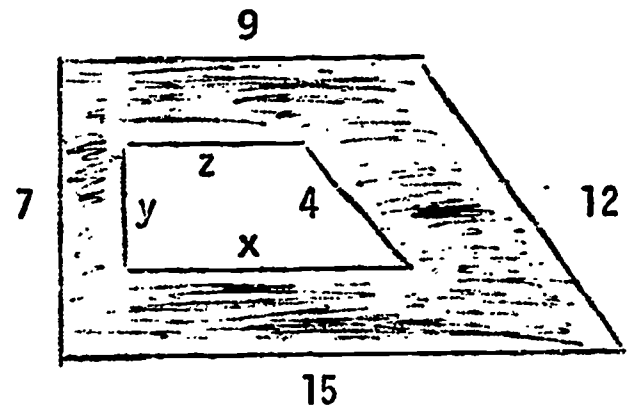
You will find that Unit 14 (Solutions of A = BC) will help you learn the concepts needed to solve these problems.

Finish Unit 14 before returning to page 18 of this Unit.

You're correct. Good work.

Try this problem.

Look at the figure to the right.



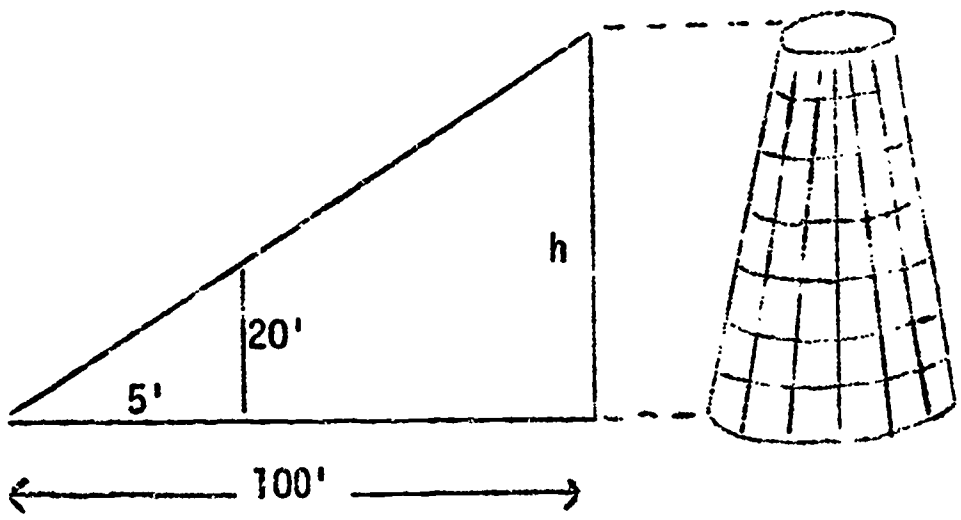
If the figure on the inside is similar to the figure on the outside, then what must be the value of x ?

(a) 12 Turn to page 54

(b) 5 Turn to page 56

(c) 3 Turn to page 62

Very good! That's correct. Now work this one.



A surveyor sights over the top of a 20 foot stick to the top of an old brick chimney. If the surveyor is 100 feet from the chimney and 5 feet from the stick, how high is the chimney? (Hint: we have two similar triangles, so their sides are proportional)

(a) 1 foot — Turn to page 60

(b) 25 feet — Turn to page 55

(c) 400 feet — Turn to page 50

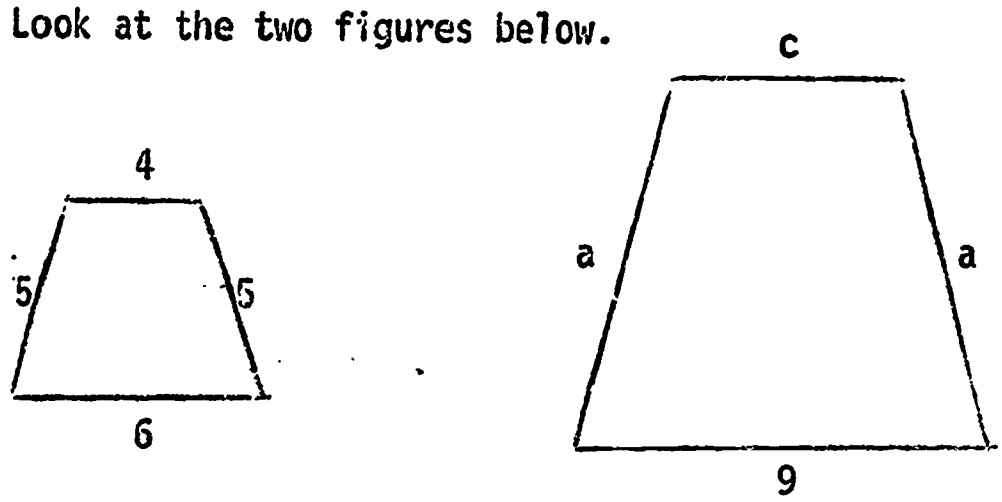
Incorrect! You set up the wrong proportion.

Remember, only CORRESPONDING sides are PROPORTIONAL.

Return to page 58 and try again.

No, you didn't quite get it.

Look at the two figures below.



By the corresponding sides being proportional we mean that 5/a = 4/c = 5/a = 6/9.

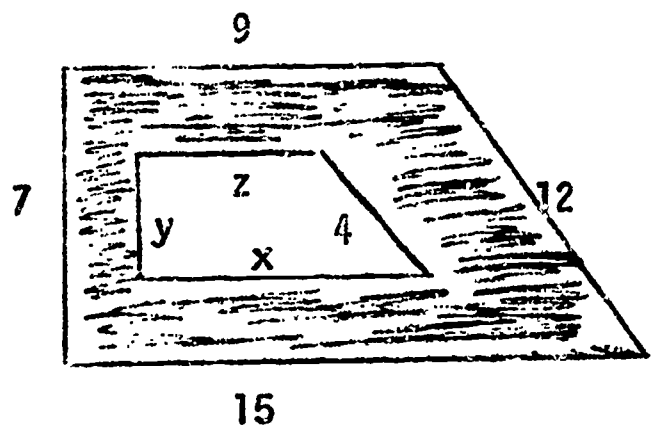
Now since we wish to solve for c, we take the ratio that has c in it. Also, since we only know how to solve for one unknown, we take the ratio with no unknowns in it. Thus, we have the proportion 4:c::6:9.

Now, what is the value of c when 4/c = 6/9 ?

(a) 6 Turn to page 51

(b) 8/3 Turn to page 61

Incorrect! Here is how the problem is set up.



Since two of the corresponding sides are in a ratio of 4/12 then x and 15 must be in the ratio of 4/12. Therefore, 4/12 = x/15. (Notice that the two numerators belong to the same figure and the two denominators belong to the same figure.)

Now, answer this question.

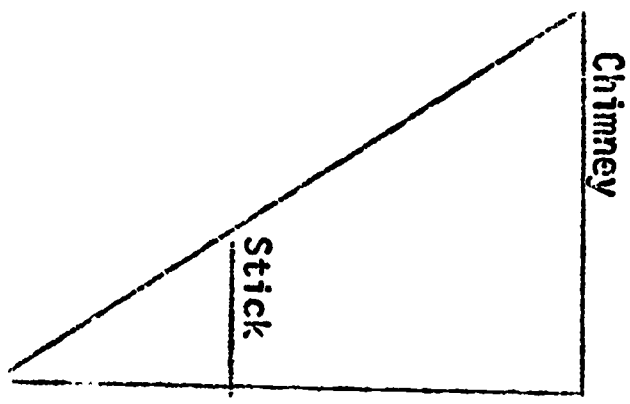
What is the value of x in the proportion 4/12 = x/15 ?

(a) 5 Turn to page 58

(c) 4 Turn to page 61

Incorrect!

Since the triangles are similar, we know that the corresponding sides are proportional. The corresponding sides of the two triangles are (1) the heighth of the chimney, (2) the heighth of the stick, (3) the distance to the chimney, and (4) the distance to the stick. Now, we must set up the proportion.



Which of the following proportions is correct?

(a) $\frac{\text{heighth of stick}}{\text{heighth of chimney}} = \frac{\text{distance to chimney}}{\text{distance to stick}}$

Turn to page 63

(b) $\frac{\text{Heighth of stick}}{\text{distance to stick}} = \frac{\text{heighth of chimney}}{\text{distance to chimney}}$

Turn to page 51

Okay. That is the correct answer. Let's try a problem from Chemistry.

If we have a balanced chemical equation, the amount of chemicals used or made is <u>proportional</u> to the weights of the chemicals. Let's consider this balanced chemical equation:

$$Zn + S \longrightarrow ZnS$$

Since Zinc (Zn) weighs 65 and Sulfur (S) weighs 32, then Zinc Sulfide (ZnS) weighs 97.

Now, how much Zinc Sulfide (ZnS) would be made from 130 pounds of Zinc (Zn)?

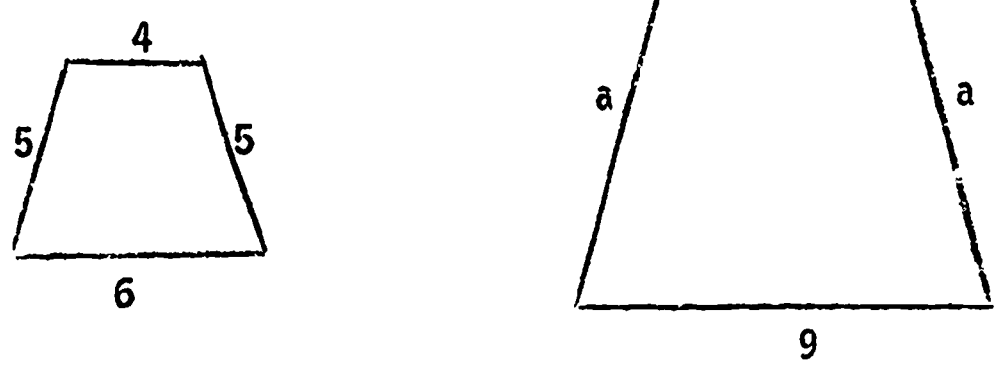
(a) 130 Turn to page 59

(b) 97 Turn to page 67

(c) 194 Turn to page 70

No! You didn't quite get it.

Look at the two figures below.



By the corresponding sides being proportional we mean that 5/a = 4/c = 5/a = 6/9.

Now, since we wish to solve for c, we take the ratio that has c in it. Also, since we only know how to solve for one unknown, we take the ratio with no unknowns in it. Thus, we have the proportion: 4:c::6:9.

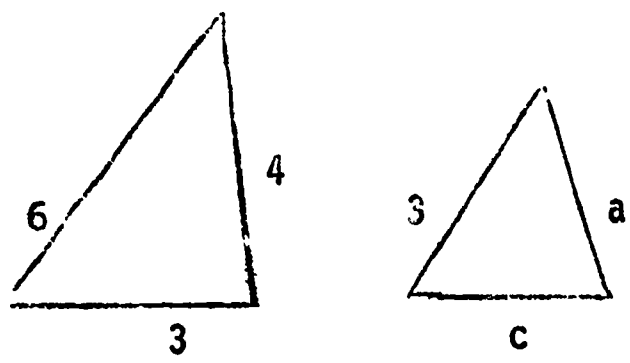
Now, what is the value of c when 4/c = 6/9 ?

(a) 6 Turn to page 51

(b) 8/3 Turn to page 61

Correct!

Now work this one.



If the two triangles above are similar, then the value of a is _____?

(a) 4 Turn to page 52

(b) $1\frac{1}{2}$ Turn to page 64

(c) 2 Turn to page 56

Incorrect!

The balanced equation with its weights looks like this:

Now, everything is proportional, so what must be the value of x ?

(a) 97 Turn to page 71

(b) 194 Turn to page 73

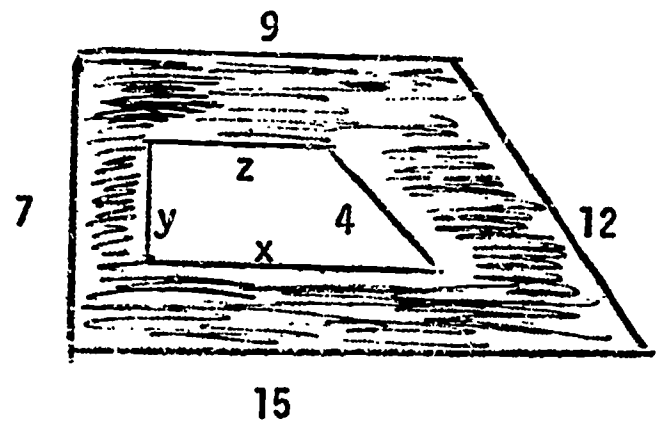
What? A 1 foot chimney. Be serious now. Our answer should be reasonable.

Return to page 51 and make a different choice.

You seem to be having trouble solving a basic proportion.

Return to page 18 and continue from there.

Incorrect: Here is how the problem is set up.



Since two of the corresponding sides are in a ratio of 4/12 then x and 15 must be in the ratio of 4/12. Therefore, 4/12 = x/15. (Notice that the two numerators belong to the same figure and the two denominators belong to the same figure.)

Now answer this question.

What is the value of x in the proportion 4/12 = x/15 ?

(a) 5 Turn to page 58

(c) 4 Turn to page 61

Incorrect!

"Heighth of stick" does not correspond to the "distance to chimney." Remember only corresponding sides of similar figures are proportional.

Return to page 55 and make another selection.

Incorrect! You set up the wrong proportion.

Remember, only CORRESPONDING sides are PROPORTIONAL.

Return to page 58 and try again.

Your answer is incorrect! Let's take a closer look at the problem.

When you substituted into the equation $\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$ you came up with the equation $\frac{1}{R_t} = \frac{1}{2} + \frac{1}{3}$. Now, $\frac{1}{Rt} = \frac{1}{2} + \frac{1}{3}$ is <u>not</u> a proportion. Why? Because we don't have TWO ratios set equal to each other. The 1/2 + 1/3 is <u>NOT</u> a ratio. However, if we add the 1/2 and 1/3 together to get 5/6, then we will have a ratio and $\frac{1}{R_t} = \frac{5}{6}$ will be a proportion.

Remember: Add together the resistances to form a proportion. Then solve the proportion.

Now, let's solve $\frac{1}{R_t} = \frac{1}{2} + \frac{1}{3}$ for R_t. Your answer will be:

(a) $1\frac{1}{5}$ Turn to page 69

(b) 6 Turn to page 71

Incorrect!

The amount of oxygen to its weight is proportional to the amount of water to its weight.

Return to page 73 and set up the proportion and work the problem again.

Incorrect!

The balanced equation with its weights looks like this:

$$\underset{65}{Zn} + S \text{ - - - - - - - - } \rightarrow \underset{97}{ZnS}$$

(130 lbs. above Zn; x above ZnS)

Now, everything is proportional, so what must be the value of x ?

(a) 97 Turn to page 71

(b) 194 Turn to page 73

Incorrect!

The amount of oxygen to its weight is proportional to the amount of water to its weight.

Return to page 73 and set up the proportion and work the problem again.

Correct!

Now work this one.

What will be the value of R_t in $\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ if we have three resistances with $R_1 = 6$, $R_2 = 2$, and $R_3 = 3$?

(a) 1 Turn to page 76

(b) 11 Turn to page 75

(c) $\frac{1}{11}$ Turn to page 72

Very good! You are correct again. Let's extend our problem solving to the field of electronics now.

In electricity the total resistance of a parallel circuit is given by the relationship $\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

where R_t = total resistance, R_1 = resistance of resistor 1, R_2 = resistance of resistor 2, etc.

Now, what is the value of R_t if there are only two resistors and R_1 = 2 ohms and R_2 = 3 ohms.

(a) 1.2 Turn to page 76

(b) 5/6 Turn to page 74

(c) 5 Turn to page 65

You seem to be having trouble solving a basic proportion.

Return to page 18 and continue from there.

No! You combine (add) the resistances to form a proportion. Then you solve the proportion for R_t.

Now, when you add the resistances from $\frac{1}{R_t} = \frac{1}{6} + \frac{1}{2} + \frac{1}{3}$

you get _____?

(a) $\frac{1}{R_t} = \frac{1}{11}$ Turn to page 79

(b) $\frac{1}{R_t} = 1$ Turn to page 77

Correct!

Now work this one.

How many pounds of oxygen are needed to produce 9 pounds of water if the balanced chemical equation looks like this:

$$2H_2 + O_2 \dashrightarrow 2H_2O$$

where Oxygen (O_2) weighs 32 and water ($2H_2O$) weighs 36?

(a) 128 Turn to page 66

(b) 9 Turn to page 68

(c) 8 Turn to page 70

Your answer is incorrect! Let's take a closer look at the problem.

When you substituted into the equation $\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$ you came up with the equation $\frac{1}{R_t} = \frac{1}{2} + \frac{1}{3}$. Now, $\frac{1}{R_t} = \frac{1}{2} + \frac{1}{3}$ is NOT a proportion. Why? Because we don't have TWO ratios set equal to each other. The 1/2 + 1/3 is NOT a ratio. However, if we add the 1/2 and 1/3 together to get 5/6, then we will have a ratio and $\frac{1}{R_t} = \frac{5}{6}$ will be a proportion.

Remember: Add together the resistances to form a proportion. Then solve the proportion.

Now, let's solve $\frac{1}{R_t} = \frac{1}{2} + \frac{1}{3}$ for R_t. Your answer will be:

(a) $1\frac{1}{5}$ Turn to page 69

(b) 6 Turn to page 71

No! You combine (add) the resistances to form a proportion. Then you solve the proportion for R_t.

Now, when you add the resistances from $\frac{1}{R_t} = \frac{1}{6} + \frac{1}{2} + \frac{1}{3}$

you get _____?

(a) $\frac{1}{R_t} = \frac{1}{11}$ Turn to page 79

(b) $\frac{1}{R_t} = 1$ Turn to page 77

Correct again! Now, we are going to solve percentage problems using proportions.

This type of problem comes from one of the fundamental relationships of percentage (Unit 15), namely that Percent = $\frac{\text{Amount}}{\text{Base}}$. All we have to do is write percent as a common fraction with a denominator of 100 and we have a proportion problem.

Let's see how you do with this problem.

14 is _____% of 42.

(a) 300 Turn to page 89

(b) 5.88 Turn to page 78

(c) $33\frac{1}{3}$ Turn to page 85

That's correct!

Now, go to page 69 and complete the problem.

Incorrect. Let's look at the solution to this problem.

Since Percent = $\frac{\text{Amount}}{\text{Base}}$, the exact number, call it N, that we are looking for will be $\frac{N}{100} = \frac{\text{Amount}}{\text{Base}}$. Now, in the problem, 14 is $\underline{N}$ % of 42, the amount is 14, the base is 42 and we are looking for N.

Thus, $\frac{N}{100} = \frac{14}{42}$

$42N = 1400$

and, $N = \frac{1400}{42} = 33\frac{1}{3}$

Try this problem now.

15 = _____% of 60.

(a) 25 Turn to page 83

(b) 40 Turn to page 80

(c) 9 Turn to page 86

You seem to be having trouble adding fractions.

Go to Unit 5A, page 29, Booklet #I and review the concepts presented there. Then return to page 18 of this Unit.

Your answer is incorrect! Come now, it's not that hard.

Since $\frac{N}{100} = \frac{\text{Amount}}{\text{Base}}$, we can solve this proportion by using the rule that the product of the means equals the product of the extremes. This gives the equation N(Base) = 100(Amount). Then given two of the unknowns, we can solve for the third by the methods we used in solving the equation a = bc.

Try this problem now.

4% of 16 is _____.

(a) 6.4 Turn to page 83

(b) 4 Turn to page 90

Very good! You're doing fine.

Continue with this problem.

What is the value of x in the proportion 3x/8 = 18/24 ?

(a) 2 Turn to page 92

(b) 6 Turn to page 94

(c) 32/9 Turn to page 97

Incorrect! You must have set the proportion up incorrectly.

The problem read: "If 40% of 25 students received a "C" grade, how many students received "C's"?

The proportion you should have used is _____.

(a) $\frac{40}{100} = \frac{25}{B}$ Turn to page 80

(b) $\frac{N}{100} = \frac{25}{40}$ Turn to page 93

(c) $\frac{40}{100} = \frac{A}{25}$ Turn to page 91

Correct!

Now work this problem.

15 = 30% of _____.

(a) 45 Turn to page 87

(b) 50 Turn to page 85

(c) 20 Turn to page 80

Whoops! That is the amount of the discount. What did he actually pay for the suit? Because what you actually pay is the price of the suit minus the discount.

Return to page 91 and make another choice.

Your answer is correct!

Let's continue with this problem.

If 40% of 25 students in a mathematics class received a "C" grade, how many students received a "C" grade?

(a) 6 Turn to page 88

(b) 10 Turn to page 81

(c) 16 Turn to page 82

Incorrect!

Percentage = $\frac{\text{Amount}}{\text{Base}}$, NOT Amount x Base.

Go back to page 78 and work the problem again.

Incorrect!

$\underline{\text{Percentage}} = \frac{\text{Amount}}{\text{Base}}$, <u>NOT</u> <u>times</u> Amount or Base.

Return to page 83 and work the problem again.

Incorrect! You must have set the proportion up incorrectly. The problem read: "If 40% of 25 students received a "C" grade, how many students received "C's"?

The proportion you should have used is _____.

(a) $\frac{40}{100} = \frac{25}{B}$ Turn to page 80

(b) $\frac{N}{100} = \frac{25}{40}$ Turn to page 93

(c) $\frac{40}{100} = \frac{A}{25}$ Turn to page 91

Incorrect! Let's look at the solution to this problem.

Since Percent = $\frac{\text{Amount}}{\text{Base}}$, the exact number, call it N, that we are looking for will be $\frac{N}{100} = \frac{\text{Amount}}{\text{Base}}$. Now, in the problem, 14 is __N__ % of 42, the amount is 14, the base is 42 and we are looking for N.

Thus, $\frac{N}{100} = \frac{14}{42}$

$42N = 1400$

and, $N = \frac{1400}{42} = 33\frac{1}{3}$

Try this problem now.

15 = _____ % of 60.

(a) 25 Turn to page 83

(b) 40 Turn to page 80

(c) 9 Turn to page 86

You seem to be having trouble solving percentage problems.

Go to Unit 15 and study percentage and solutions of percentage problems. Then return to page 76 of this Unit.

Go to Unit 15.

Correct!

Work this problem now.

Joe Doe bought a suit at a discount sale where the discount was 20%. If the original price tag was $85.00, how much did he pay for the suit?

(a) $17.00 Turn to page 84

(b) $68.00 Turn to page 81

(c) $42.50 Turn to page 80

Very Good!

Continue with this problem.

If 7/3 = 4a/6, what is the value of a?

(a) $3\frac{1}{2}$ Turn to page 100

(b) $1\frac{5}{9}$ Turn to page 96

(c) 42 Turn to page 105

Wait a minute!

You were given that the percentage was 40. Therefore, you are not solving for N, because N is the percentage and is 40.

Go back to page 82 and try again.

You seem to be having a little trouble. Let's look at solutions of proportions in detail.

First of all, to solve <u>any</u> proportion, we multiply the two means and the two extremes and set them equal to each other. This will always give us an equation of the form a = bc where a and b are numbers.

Then we simply solve a = bc for c by the methods used in Unit 14.

Let's look at solving these proportions in a step by step manner.

Step 1: $a/b = c/x$ (This represents our original proportion where x is unknown and a, b, and c are numbers)

Step 2: $(bc) = ax$ (The product of the means = product of the extremes. You should now notice that the equation is in the form of a = bc)

Step 3: $x = \frac{(bc)}{a}$ (Solving the equation a = bc for c, we will <u>always</u> obtain c = a/b)

(Continue on next page)

Step 4: Reduce your answer if necessary.

Let's look at a couple of examples now.

Example 1:

Step 1: $x/3 = 5/6$ (Original Equation)

Step 2: $6x = 15$ (Product of means = product of extremes)

Step 3: $x = 15/6$ (Solving equation of the form $a = bc$ for c)

Step 4: $x = 5/2$ or $2\frac{1}{2}$ (Reduced Answer)

Example 2:

Step 1: $4/x = 2/5$

Step 2: $2x = 20$

Step 3: $x = 20/2 = 10$

Now you solve $15/w = 5/2$ for w.

(a) 1/6 Turn to page 106

(b) 30 Turn to page 98

(c) 6 Turn to page 101

Oops! Divided incorrectly.

Return to page 101 and work the problem again.

Your answer is incorrect! Here is the step by step solution of the problem: $7/3 = 4a/6$.

Step 1: $12a = 42$ (Product of means = product of extremes)

Step 2: $a = 42/12$ (Divided both sides of equal sign by 12)

Step 3: $a = 7/2$ or $3\frac{1}{2}$ (Reduced)

Okay, let's see if you can solve $5/3 = 8/R$ for R.

(a) 24 Turn to page 98

(b) $4\frac{4}{5}$ Turn to page 99

(c) 5/24 Turn to page 103

You seem to be having a little trouble. Let's look at solutions of proportions in detail.

First of all, to solve <u>any</u> proportion, we multiply the two means and the two extremes and set them equal to each other. This will always give us an equation of the form a = bc where a and b are numbers.

Then we simply solve a = bc for c by the methods used in Unit 14.

Let's look at solving these proportions in a step by step manner.

Step 1: $a/b = c/x$ (This represents our original proportion where x is unknown and a, b, and c are numbers)

Step 2: $(bc) = ax$ (The product of the means = product of the extremes. You should now notice that the equation is in the form of a = bc)

Step 3: $x = \frac{(bc)}{a}$ (Solving the equation a = bc for c, we will <u>always</u> obtain c = a/b)

(Continue on next page)

Step 4: Reduce your answer if necessary.

Let's look at a couple of examples now.

Example 1:

Step 1: $x/3 = 5/6$ (Original Equation)

Step 2: $6x = 15$ (Product of means = product of extremes)

Step 3: $x = 15/6$ (Solving equation of the form a = bc for c)

Step 4: $x = 5/2$ or $2\frac{1}{2}$ (Reduced Answer)

Example 2:

Step 1: $4/x = 2/5$

Step 2: $2x = 20$

Step 3: $x = 20/2 = 10$

Now you solve $15/w = 5/2$ for w.

(a) 1/6 Turn to page 106

(b) 30 Turn to page 98

(c) 6 Turn to page 101

Incorrect!

You're having trouble solving the equations which are in the form a = bc for c. It's really not that hard. Look at this example:

Example: $3x = 12$

To solve for x, we divide both sides of the equation by 3. Thus, $x = 12/3$ or 4.

Remember! Solving the equation a = bc for c will always give us c equal to a divided by b.

Try this problem.

The value of x in $x/2 = 1/2$ is _____.

(a) 2 Turn to page 104

(b) 1 Turn to page 101

$4\frac{4}{5}$ is the correct answer!

Here is another problem for you.

Find the value of c in the proportion 1/6c = 2/16.

(a) 16 Turn to page 98

(b) 3/4 Turn to page 102

(c) $1\frac{1}{3}$ Turn to page 100

Excellent! You have completed this Unit.

Let's review what you have learned.

1. You have learned to recognize proportions and to determine whether they are correct.

2. You have learned to solve any proportion containing only one unknown.

Now you are ready for a test on this Unit.

Tell your teacher that you have finished.

Your answer is correct!

Continue with this one.

What is the value of Q in the proportion 4/7 = 8/20?

(a) 56 Turn to page 98

(b) 7 Turn to page 92

(c) $\frac{1}{7}$ Turn to page 95

Whoops!

Whey you solve the equation a = bc for c, you obtain c = a/b. Remember you must <u>divide</u> both sides of the equation by the same number.

Return to page 99 and work the problem again.

Wrong answer!

Solutions of the equation a = bc for the variable c will always be found by DIVIDING a by b.

Now, return to page 96 and apply this generality to the problem.

Turn to page 96.

You seem to be having problems solving basic proportions.

Turn to page 94 and study the material there carefully.

Then continue the Unit from there.

Turn to page 94.

Your answer is incorrect!

Turn to page 94 and study the material on that page.

Then continue from there.

Turn to page 94.

Incorrect!

After you multiply the means and extremes, you should have an equation in the form of a = bc where a and b are numbers.

Now, c is equal to a divided by b. Remember?

Step 1: a = bc (Original equation)

Step 2: a/b = bc/b (Divided both sides of the equal sign by b)

Step 3: a/b = c (Reduce)

Now, return to page 94 and work the problem again.

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CAI MATHEMATICS

TEST QUESTIONS

UNIT 19 - PROPORTIONS

Directions: The correct answers will always be expressed in lowest terms.

1. 1/2 = 4/8 is a proportion

   a) No

   b) Yes

. In the proportion x/3 = 4/9, x equals

   a) 4/27

   b) 12

   c) 1 1/3

3. The value of p in $\frac{1/4}{p} = \frac{1/8}{5}$ is

   a) 10

   b) 5/8

   c) 5/32

4. Given two similar triangles with sides of 2,3, and 4 and x,12, and y respectively. The value of x is

   a) 16

   b) 8

   c) 3

5. [illegible] :: 9:2 is true when K equals

   a) 13 1/2

   b) 2/3

   c) 6

6. If 3/2 = 15/a, the value of a is

   a) 22 1/2

   b) 2

   c) 10

7. If $1/R_t = 1/R_1 + 1/R_2$ and $R_1 = 3$ and $R_2 = 6$ then R_t =

   a) 2

   b) 9

   c) 1/2

8. What is the value of q in the proportion 4/5 = 8/2q

   a) 5

   b) 4

   c) 3 1/5

9. 1:3::7:R is a correct proportion when R equals

   a) 2 1/3

   b) 3/7

   c) 21

10. 3:5::5:6 is a correct proportion

   a) Yes

   b) No

11. If $1/R_t = 7/6$, then R_t equals

   a) 67

   b) 7/6

   c) 6/7

12. If 5/3 = 10/R, then R equals

    a) 6

    b) 50/3

    c) 1 1/2

13. If 15% of 80 students received an "A" grade, how many students received "A's"?

    a) 3

    b) 12

    c) 5

14. Solve 1/3d = 2/27 for d

    a) 2/81

    b) 2/9

    c) 4 1/2

15. Is 4/x = 12/3x a prcportion?

    a) No

    b) Yes

16. If Percentage = Amount ÷ Base, then

    a) Amount = Percentage ÷ Base

    b) Amount = Percentage times Base

    c) Base = Percentage times Amount

17. Given that n/100 = 2 1/5, then n equals

    a) 250

    b) 220

    c) 500/11

18. If a man received a 10% discount on a $90 suit, how much did he pay for the suit?

    a) $9

    b) $67.78

    c) $81

19. Which of the following is <u>not</u> a correct proportion?

a) 4/2 = 2

b) 1/2 = 2/4

c) 2:6::3:1

20. 12 = _____% of 60

a) 30

b) 20

c) 50

21. What is the value of a in 2a/3 = 25/6 ?

a) 6 1/4

b) 4/25

c) 2 7/9

22. 1:x::3:18 is true for x equal to

a) 6

b) 18

c) 1/6

23. If two rectangles are proportional and the sides of one are 3 and 6 and the sides of the other are 2 and x then x equals

a) 9

b) 4

c) 1

24. Which of the following proportions is correct?

a) 1:3::3:1

b) 6:6::1:2

c) 2:5::6:15

25. In a balanced equation the amounts of chemicals used are proportional to the weight of the elements. So if it takes 46 pounds of Sodium to make 100 pounds of salt and the atomic weight of Sodium is 23, then the atomic weight of salt is

    a) 50

    b) 200

    c) 125

ANSWER SHEET

UNIT 19 - PROPORTIONS

1. b
2. c
3. a
4. b
5. b
6. c
7. a
8. a
9. c
10. b
11. c
12. a
13. b
14. c
15. b
16. b
17. b
18. c
19. c
20. b
21. a
22. a
23. b
24. c
25. a

To the instructor: The above problems are related to the objectives as follows:

OBJECTIVE	QUESTIONS
1	1,10,15,19,24
2	2,3,5,6,8,9,12,14,17,21,22
3	4,7,11,13,16,18,20,23,25