DOCUMENT RESUME

ED 021 478 56 EM 006 291

By-Rosen, Ellen F., Stolurow, Lawrence M.

IN PURSUIT OF A SIMPLEX IN UICSM HIGH SCHOOL ALGEBRA. COMPARATIVE STUDIES OF PRINCIPLES FOR PROGRAMMING MATHEMATICS IN AUTOMATED INSTRUCTION. TECHNICAL REPORT NUMBER 5.

Illinois Univ., Urbana.

Spons Agency-Office of Education (DHEW), Washington, D.C.

Report No-NDEA-7A-806

Pub Date Jul 64

Contract-OEC-711151-01

Note-9p.

EDRS Price MF-$0.25 HC-$0.44

Descriptors-ALGEBRA, AUTOINSTRUCTIONAL PROGRAMS, *CONCEPT FORMATION, *LEARNING PROCESSES, *MATHEMATICAL CONCEPTS, SECONDARY SCHOOL MATHEMATICS, STATISTICAL ANALYSIS

To test whether the process of learning mathematics involves the effect of a hierarchical concept (as opposed to a "snowball" effect, in which one set of concepts becomes the basis for a new set), a matrix of correlations of progress test scores from a self-instructional high school algebra course was analyzed. The form of the matrix did not demonstrate a hierarchical structure, but this, of course, does not demonstrate the validity of the snowball effect. (LH)

UNIVERSITY OF ILLINOIS
Urbana, Illinois

In Pursuit of a Simplex in UICSM High School Algebra

Ellen F. Rosen and Lawrence M. Stolurow

COMPARATIVE STUDIES OF PRINCIPLES FOR PROGRAMMING MATHEMATICS IN AUTOMATED INSTRUCTION

Technical Report No. 5

July, 1964

Co-Investigators:

Lawrence M. Stolurow
Professor, Department of Psychology
Training Research Laboratory

Max Beberman
Professor, College of Education
University of Illinois Committee
on School Mathematics (UICSM)

Project Sponsor:

Educational Media Branch
U. S. Office of Education
Title VII

Project No. 711151.01

U. S. Office of Education
Title VII

COMPARATIVE STUDIES OF PRINCIPLES FOR PROGRAMMING MATHEMATICS IN AUTOMATED INSTRUCTION

Technical Report No. 5

In Pursuit of a Simplex in UICSM High School Algebra

Ellen F. Rosen and Lawrence M. Stolurow

July, 1964

In Pursuit of a Simplex in UICSM High School Algebra

Ellen F. Rosen and Lawrence M. Stolurow

Popular belief maintains that the process of learning mathematics involves a "snowball" effect. That is, the learner begins with a set of concepts which in turn become the basis for a new set of concepts.

Recently Gagne and Paradise (1961) analyzed a section of the Maryland Mathematics Project materials on linear equations and reported that it had a hierarchical structure. While their data on immediate retention supported the analysis of a tree pattern, their data on delayed retention did not.

The question raised in the present study is whether the beginning UICSM algebra course, which was written in self-instructional form (see Beberman and Stolurow, 1964), would reveal a structure of inclusion in the pattern of intercorrelations among the test scores. If this pattern were found, it would suggest the existence of an associative structure which could be analyzed to determine its specific properties.

This type of thinking leads to a postulated structure for a series of mathematical progress tests throughout a course in beginning algebra as given below:

$$X_i = \sum_{g=1}^{i} f_g + S_i$$

for all $i \in I$ where I is the set of tests, X_i is the score on the ith test, f_i the portion of the score X_i has in common with all the following tests, and s_i the error specific to test X_i. Thus

$$X_1 \subset X_2 \subset \ldots \subset X_n$$

when the s_i's are zero. This is called the principle of inclusion (Jones, 1961) and leads to the simplical form of molar analysis.

An intercorrelation matrix, calculated from data assumed to have such an underlying structure, should have certain properties when corrected for unreliability (Jones, 1959):

(1) the largest correlations should be in the superdiagonal and the correlations should decrease as the cells get further away from it (superdiagonal form);

(2) the partial correlations --

$$r_{ik.j} = 0$$

for all $i < j < k$

or

$$r_{ik.j} = \frac{r_{ik} - r_{ij} \cdot r_{jk}}{\sqrt{1 - r_{ij}^2}\ \sqrt{1 - r_{jk}^2}} = 0$$

or $r_{ik} = r_{ij} \cdot r_{jk}$

(3) If the structure involves an error term, the error (unreliability) can be corrected for by using the correction for attenuation --

$$r'_{ik} = \frac{r_{ik}}{\sqrt{r_{ii}\ r_{kk}}}$$

Then the r'_{ij} will have simplical form, and the r_{ij} will have quasi-simplical form.

Assume that an m by m matrix of intercorrelations has perfect simplical form; then $r_{ik} = r_{i,\ i+1} \cdot r_{i+1,\ k}$

$$= r_{i,\ i+1} \cdot r_{i+1,\ i+2} \cdot r_{i+2,\ k}$$

where $r_{i,\ i+1}$ are called gradations for all i.

Continuing in this same manner,

$$r_{ik} = r_{i,\ i+1} \cdot r_{i+1,\ i+2} \ldots r_{k-1,\ k},$$

Thus if the simplical form is obtained, given the correlations in the super-diagonal, the other correlations can be generated.

A further relation of interest falls out from the fact that the last variable in a perfect simplex, in this case, is the most complex. Assign it (X_m) a value of 1 for its complexity. The complexity of a variable (X_i) is defined as its correlation with X_m. From the defining properties of the simplex

$$r_{im} = r_{ij}\ r_{jm}$$

or

$$r_{ij} = r_{im}/r_{jm}\ ,\ i < j < m.$$

Correcting the relationship of the gradations and the complexities for error by using the correction for attenuation, one finds the relations

$$r_{ij} = \rho_{i,\ i+1} \cdots \rho_{j-1,\ j} \sqrt{r_{ii}\ r_{ji}}$$

$$r_{ij} = \frac{\rho_{im}}{\rho_{jm}} \sqrt{r_{ii}\ r_{jj}}$$

where $\rho_{i,\,i+1}$ for all $i < m$ are the gradations of the quasi-simplical form, and ρ_{im} for all i are the complexities of the quasi-simplical form, and r_{ii}, r_{jj} are the reliabilities of X_i and X_j, respectively (Jones, 1959, p. 15).

HYPOTHESIS

Jones (1961) suggests the manipulation of variables to find locations which lead to simplical form, but the purpose here is different from his usage. Jones is looking for a hypothesis, here a hypothesis is being tested. The hypothesis being tested is whether the variables in the matrix can be described by the notion of inclusion. If they can be so described, the matrix will be simplical in form; if they can't, the matrix will not be a simplex.

RESULTS

Applying these defining properties to the matrix of correlations presented in Table 1, the quasi-simplex was generated by using ρ_{im} and ρ_{ii} as listed in the table. The data are from the 1962 administration of the UICSM programed texts which were used in the "pure mode" (see Beberman and Stolurow, 1964).

The observed correlations corrected for attenuation are presented in Table 2. As is apparent from the inversions of the ordering of the correlations, the criterion of superdiagonal form is not met. There is no need to pursue the simplex any further for the purposes of this study. The structure of inclusion is not adequate to describe the function of the beginning UICSM algebra course.

Table 1

Observed Correlations*

Test	Part Test Number 101	103	104	107	114	115	116	Complexity**
101	.39	.45	.48	.46	.50	.46	.44	.44
103		.72	.60	.36	.47	.51	.41	.41
104			.65	.50	.55	.44	.53	.53
107				.70	.51	.37	.49	.49
114					.62	.50	.61	.61
115						.70	.47	.47
116							.79	.79

*Reliabilities given in main diagonal.

**Correlation of the variable with Part Test 116 (see text, p. 3).

Table 2

Observed Correlations Corrected for Attenuation (Unreliability)

Test	Part Test Number						
	101	103	104	107	114	115	116
101		.86	.96	.88	1.00	.88	.80
103			.88	.51	.70	.72	.55
104				.74	.86	.65	.74
107					.77	.53	.66
114						.76	.87
115							.64
116							

References

Beberman, Max and Stolurow, L. M. Semi-Annual Report

Gagne, R. M. and Paradise, N. E. Abilities and learning sets in knowledge acquisition. Psychol. Monogr., 1961, 75 (14), Whole No. 518.

Jones, Marshall B. Simplex Theory. Pensacola, Fla.: U. S. Naval Aviation Medical Center, U. S. Naval School of Aviation Medicine. Monograph Series, No. 3, 1959.

Jones, Marshall B. Molar Correlational Analysis. Pensacola, Fla.: U. S. Naval Aviation Medical Center, U. S. Naval School of Aviation Medicine, Monograph Series No. 4, 1961.