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This study is concerned with the problem of measuring rate of learning in individualized instructional situations, and with the relationship between rate and a number of variables. Pupil aptitude, achievement, and classroom performance were measures correlated with learning rates in six different Individually Prescribed Instruction (IPI) mathematics units. The predictor variables (verbal and non-verbal I.Q., mental age, mathematics and reading achievement scores, number of skills learned in previous terms, attention score, and reaction to the learning situation and learning materials) analyzed singly, or as a composite predictor, did not correlate significantly with any or all of the criterion variables. No single predictor consistently predicted any rate measure. Multiple correlations showed that lack of rate measure comprehensiveness did not cause poor rate predictability. Multiple correlations and regression analysis showed the relative importance of various predictors as a function of specific units studied. Canonical correlations indicated some type of substantial relationship between the rate domain and the predictor variables domain. (WR)

AN INVESTIGATION OF SELECTED PROCEDURES FOR MEASURING AND PREDICTING RATE OF LEARNING IN CLASSROOMS OPERATING UNDER A PROGRAM OF INDIVIDUALIZED INSTRUCTION

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FOREWORD

The research and development reported herein was performed pursuant to a contract with the United States Office of Education, Department of Health, Education and Welfare, under provisions of the Cooperative Research Program.

The writer wishes to express her deepest gratitude to her major advisors, Dr. C. M. Lindvall and Dr. Margaret B. MacFarland, without whose advice, inspiration, and constant guidance during the years of the writer's graduate studies, this work would not have been possible. The writer also would like to take this opportunity to extend her appreciation and gratitude to the members of the doctoral committee, Dr. William W. Cooley, Dr. Michael Gladis, and Dr. John L. Yeager.

The writer extends appreciation and gratitude to:

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CHAPTER I

INTRODUCTION AND REVIEW OF RELATED RESEARCH

The problem of how to improve the effectiveness of instruction in our schools has always been a major topic of concern among teachers, parents and other persons in the field of education. It is generally agreed, both by educational theorists and practitioners, that there are a host of factors affecting the pupil's learning performance. That is, the pupil's level of achievement and the speed with which he learns depend upon the unique individual characteristics that the pupil brings with him to the learning situation, the nature of the task to be learned, and the conditions under which the particular learning takes place.

This recognition of the multiplicity of factors that influence the pupil's learning performance, and the fact that there is a noticeable difference in pupils' levels of achievement, as well as differences in the amount of time each pupil needs to achieve the learning objectives, have focused the emphasis on the importance of making provisions for individual differences among pupils in our schools. The work on programmed instruction and other plans for individualized instruction represent some of the attempts made to meet this need for individualization. The essential goal of programs of this type is to permit each pupil to progress at his individual rate. In other words, they permit each pupil to progress through a learning sequence at a pace determined by his own work habits and by his ability to master the designated instructional objectives.

Because of this changing trend in the instructional procedures and the emphasis on the individualization of instruction, methods of assessing pupil's learning performance also are in need of modification. If school programs are to permit pupils to progress at individual rates, it is important to identify reliable procedures for measuring rate of learning and to investigate factors related to it.

The multiplicity of factors that influence the pupil's learning performance has been investigated in recent studies on "learning." According to Piaget's[1] developmental theory and the findings from his intensive research studies on the cognitive development of children, the environmental background as well as various personal factors are related to the child's performance on learning tasks. In other words, both the maturation process and the learning process are basic to the child's cognitive development. Maturation opens up possibilities for cognitive development but is not sufficient in itself to the actualization of these potentials.

Piaget's well-known formulation of the mental process is based on two adaptation processes of the learner; that of "assimilation" and "accommodation." Assimilation describes the capability of the child in handling new situations and new problems on the basis of what he has already learned; it is the process of the inner organization of information. Accommodation, on the other hand, describes the process of change through which the child becomes able to manage situations that are at first too difficult for him. This is the process through which the child

[1]Jean Piaget, The Origins of Intelligence in Children (New York: International University Press, 1963).

modifies his existing patterns of learning behavior to conform to the outer reality in the learning situation.[1]

The child's learning performance, according to Piaget's cognitive theory then, is based upon the interplay between the child's continuous mental development and the environmental factor that he encounters. In other words, learning is a function of the interaction between those factors within the learner and those external factors in the learning environment.

Many researchers have explored the relationship between human abilities and success in different types of learning situations. Allison[2] attempted to assess the relationship between achievement and human abilities and to explore the interrelationships among learning parameters and measures of human abilities in thirteen different learning situations. He concluded that learning is not a unitary trait or ability but involves several factors and is dependent upon the learning task, the content of the material, and the psychological process of learning.

Duncanson[3] administered a battery of ability tests and learning tasks to 102 sixth-grade pupils. The learning tasks included concept formation, paired associations, and rote memory tasks with verbal, numerical, and figural material. He factor analyzed the results of the ability tests to examine the contribution of each component to each person's performance for that particular learning task. Duncanson was interested in determining

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[1] Alfred Baldwin, Theories of Child Development (New York: John Wiley and Sons, Inc., 1967), pp. 177-178.

[2] R. B. Allison, "Learning Parameters and Human Abilities" (unpublished doctoral dissertation, Princeton University, June, 1966).

[3] J. P. Duncanson, "Learning and Measured Abilities," Journal of Educational Psychology, 57:220-239, 1966.

(1) the number of factors involved in learning a given task and (2) the importance or weight of each of the factors for a given individual. His research results also support the theory that not one but a multitude of factors are related to learning performance.

Other investigators have been concerned with the relative contribution of various measures in the prediction of success in classroom learning. The study of Guilford, Hoepener and Peterson[1] on predicting achievement in ninth-grade mathematics suggests the multiplicity of factors that influence the pupil's learning in school. They administered a battery of twenty-five factor tests and three standardized tests (the Diagnostic Aptitude Test, the California Test of Mental Maturity, and the Iowa Test of Basic Skills) to 600 subjects in the study. Using factor analysis, they concluded from the results that batteries of factor scores were better predictors of achievement in mathematics than the scores from the standardized tests. A composite of thirteen factor scores increased precision in prediction when added to each of the three standardized test combinations, such as a combination of the DAT-numerical and the Iowa scores, the DAT-numerical and the CTMM scores, and so forth.

Smith and his associates[2] studied the relationship between academic success and some non-intellectual variables using 116 fifth and sixth grade students as their subjects. They found that these non-intellectual

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[1]J. P. Guilford, Ralph Hoepener, and Hugh Peterson, "Predicting Achievement in Ninth-Grade Mathematics from Measures of Intellectual Aptitude Factors," Educational and Psychological Measurement, 25:659-682, 1965.

[2]John T. Smith, Maxine D. Ruter, Frank M. Lackner and Donna S. Kwall, "Academic, Sociometric and Personality Variables in the Prediction of Elementary School Achievement." Proceedings of the 75th Annual Convention of American Psychological Association. 2:339-340, 1967.

factors do contribute to the accuracy of predicting academic success. This finding was in agreement with Spies[1] who concluded from his study that non-intellectual measures can be of value in combination with intellectual measures in predicting academic success.

Whitman[2] studied "free recall learning" in the classroom situation and found, that level of achievement in classroom learning is a function of task, method of presentation and practice variables. Most of the studies cited thus far, indicating that human learning is a function of a variety of types of variables, have used some measure of level of achievement as the criterion measure. However, other investigators have been concerned with the rate at which learning takes place and have studied factors related to rate.

One of the classical studies was carried out by Lyon[3] when he presented the relationship between amount to be learned and time to learn (rate of learning) by showing how length of time to learn increases as items are added. The results of Lyon's experiment on the time required to learn lists with different numbers of nonsense syllables show that with small numbers of syllables the addition of a few more syllables makes a large difference in time per syllable, but with a large number the addition of the same number of items makes little difference in time to learn. Lyon also studied the amount of time that it took to memorize poetry; the

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[1]C. J. Spies, "Some Non-intellectual Predictors of Classroom Success," Technical Report No. 10, Office of Naval Research, Contract No. N R816 (14) Naval Air Technical Training, 1966.

[2]James R. Whitman, "Classroom Learning as a Function of Task, Method of Presentation and Practice Variable." Proceedings of the 75th Annual Convention of the American Psychological Association. 2:315-316, 1967.

[3]D. O. Lyon, Memory and the Learning Process (Baltimore, Marlyand: Warwick and New York, 1917).

results were similar to those he found with nonsense syllables. Although the material is much longer, each syllable of the poetry took less time to memorize than each individual nonsense syllable. Therefore, he concluded that the additional time per comparable item should be less for meaningful material than for some non-redundant material like random lists of nonsense syllables. This suggests that learning rate is a function of the type of material to be learned.

Woodrow[1] studied rate of learning through controlled experiments in laboratory situations and concluded that there exists no general factor acting as the single determining condition of the rate of learning, especially in learning situations where variety of learning behavior is required.

Underwood[2] listed the following series of factors as determiners of rate of learning: (1) Massed versus distributed practice; (2) Type of material, which includes intra-list similarity and meaningfulness; and (3) Affectivity, which includes such variables as knowledge of performance, whole versus part learning, active recitation, sense modality, and the amount of material to be learned. However, among those factors, Underwood[3] pointed out in his later work that according to available evidence, meaningfulness, intratask similarity, intertask similarity, active recitation versus passive study are the most important factors for verbal learning.

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[1]Herbert Woodrow, "Interrelations of Measures of Learning," The Journal of Psychology, 10:49-72, 1940.

[2]B. J. Underwood, Experimental Psychology: An Introduction (New York: Appleton-Century-Crofts, 1949), pp. 398-419.

[3]B.J. Underwood, "Verbal Learning in the Education Processes," ed. John P. Dececco, Educational Technology: Readings in Programmed Instruction (New York: Holt Rinehart and Winston, 1964), pp. 56-67.

The controlled experimental studies conducted by Woodrow and Underwood, cited above, both suggest that the rate of learning is a function of many factors intrinsic to the individual, to the learning task, and to the situation. Since these studies were conducted in restricted laboratory situations, the implications obtained from them cannot be regarded as conclusive evidence in explaining the factors that are related to classroom learning. It would seem to be essential that these same types of experiments be carried out in school situations so that results from the laboratory can be verified and information be gathered concerning the factors that are related to rate of classroom learning.

The variable "rate of learning" however, cannot be appropriately studied in the typical school. Under conventional instructional procedures pupils are required to learn particular lessons in a given interval of time and everyone is expected to proceed at essentially the same pace. Under this procedure, possible individual differences in rate of progress are not permitted to operate. In other words, a pupil's success or failure in school learning is judged in terms of the amount or the degree he has achieved when learning time is held constant.

The relatively recent emphasis on programs with provisions for individual differences has resulted in a number of procedures which permit pupils to proceed through a given set of learning tasks at individual rates. This emphasis has resulted in some attention being focused on the measurement of rate of learning. Under these individualized instructional plans, the level of mastery does not provide a valid indication of the individual pupil's learning ability, since each pupil is required to go through a set of learning experience until he demonstrates mastery of the task to be learned, and the required level of achievement for the particular learning experience is the same for all students. Therefore, under these programs the pupil's learning performance is measured through his rate of progress. The rate of learning, then, becomes a measure of the pupil's achievement.

Suppes[1] studied differences in students' rate of learning by setting up a very rigidly controlled experiment in the classroom learning situation. Using forty first-grade pupils in his study, he found that in seven weeks of learning to solve mathematics problems, the fast child covered 3,400 problems whereas the slow child covered only 2,200 problems. In another study of thirty-eight kindergarten children, he again found different rates. The fastest child performed the learning tasks in 196 trials, while the slowest child needed 2,506 trials to learn the same tasks.

Kalin[2] conducted a research study with an experimental program in mathematics, using two groups of students in the fifth and the sixth grades. The results of the study supported Suppes'[3] conclusion that there is a difference between two groups (the experimental and the control groups) in terms of "time needed" to complete the learning task, with the experimental group requiring twenty percent less time to complete the work. Nicholas[4] found a significant difference on the post-test results among

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[1]Patrick Suppes, "Modern Learning Theory and the Elementary School Curriculum," American Educational Research Journal, 1:79-94, 1964.

[2]Robert Kalin, "Development and Evaluation of a Programmed Text in an Advanced Mathematical Topic of Intellectually Superior Fifth and Sixth Grade Pupils" (unpublished doctoral dissertation, Florida State University, 1962).

[3]Suppes, op. cit.

[4]Donald L. Nicholas, "The Effect of Pacing Rate on the Efficiency of Learning Four Programmed Instructional Material" (unpublished doctoral dissertation, Indiana State University, 1967).

four treatment groups in learning programmed materials at different assigned pacing rate. He concluded from his study that, the rate of learning is related to the learner's achievement.

Several researchers have been interested in finding answers to the question, "What are the important factors that are associated with the pupil's rate of learning?" Jensen[1] investigated the learning ability of three groups of junior high school students grouped according to their mental abilities (Stanford Binet I.Q. scores were used). He developed an index of learning to indicate each student's rate of progress, and found highly significant differences in the rate of progress among the groups, indicating that intelligence is related to pupil's rate of learning.

Glaser, Reynolds, and Fullick[2] studied the effects of programmed instruction under a variety of conditions with students in different grades. They found, in agreement with Jensen, that intelligence appears to be related to the rate at which each student works through the program.

In another study, Gropper and Kress[3] report results on the relationship between pacing mode and performance. The experimental results indicate that (1) there are variations in the rate at which the individuals achieve the particular learning task, (2) when low ability students do work at a pace appropriate to their ability level, they are more likely to reach high achievement levels, and (3) fast workers appear to be capable

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[1]Arthur Jensen, "Learning Ability in Retarded, Average, and Gifted Children," ed. John P. DeCecco, Educational Technology: Readings in Programmed Instruction (New York: Holt, Rinehart and Winston, 1964), p. 375.

[2]Robert Glaser, James H. Reynolds, and Margaret G. Fullick, Programmed Instruction in the Intact Classroom, Project No. 1342, Cooperative Research, U. S. Office of Education, December, 1963, p. 25.

[3]George L. Gropper and Gerald C. Kress, "Individualizing Instruction Through Pacing Procedures," Audiovisual Communication Review, 8:165-182, Spring 1965.

of tolerating fast fixed tempos; slow fixed tempos would be even more inefficient and ineffective for them. These findings suggest that when pupils are permitted to work at their own pace, and when learning programs are tailored to the needs of each learner, the learning performance may become more efficient and effective.

Carroll[1] has developed a conceptual model of factors affecting success in school learning and the way they interact. Carroll included five major factors in his learning model. The factors are listed under two headings: (1) Determinants of time needed for learning and (2) Determinants of time spent in learning. By combining these elements in his model, Carroll has developed a formula to express the degree of learning, in quasi-mathematical terms, that Degree of Learning =

$$F\left(\frac{\text{Time spent--Opportunity, Perseverance, Aptitude}}{\text{Time needed--Ability to Understand Instruction, Quality of Instruction}}\right)$$

Sjogren[2] used the data obtained from the results of a learning experiment conducted by Sjogren and Knox[3] to test Carroll's model of school learning. The original purpose of the experiment conducted by Sjogren and Knox was to test whether the imposition of different speeded conditions in a learning task interacted with age in affecting performance on the learning task. Three different sets of programmed materials were used in the experiment and two hundred and eight adults were selected as subjects. Through analyzing the data, Sjogren found that in all cases,

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[1]John B. Carroll, "A Model of School Learning," Teachers College Record, 64:723-732, 1963.

[2]Douglas D. Sjogren, "Achievement as a Function of Study Time," American Educational Research Journal, 4:337-343, 1967.

[3]Douglas D. Sjogren and Alan B. Knox, "The Influence of Speed and Prior Knowledge and Experience on Adult Learning," Cooperative Research Project No. 2233, U. S. Office of Education, University of Nebraska, 1965.

the linear relationship between the ratio (time spent and time needed) and the degree of learning (as indicated by achievement test scores) was statistically significant. He found that approximately 15 to 25 percent of the variance in the achievement test score was explained by this ratio. Therefore, Sjogren concluded that the results of the study do support Carroll's model in that a measure of degree of learning (an assessment of student's achievement) was significantly related to the ratio of time taken to time needed for the study of the programmed materials.

Yeager and Lindvall[1] have reported on three possible measures of the rate of learning under a program of individually prescribed instruction. They concluded that the rates of learning are not consistent for individual students over various units of classroom lesson material but are specific to the learning task. This conclusion is in agreement with Carroll's learning model, which suggests the complexity in studying the problem of "learning" and the factors that are related to pupil's learning performance.

A rather intensive study of rate of learning in a school situation was conducted by Yeager.[2] He studied the pupil's rate of learning under a program of individualized instruction in a public school system. Yeager investigated three measures of learning rate in terms of the consistency of each measure and the relationship of each to student intelligence and

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[1]John L. Yeager and C. M. Lindvall, "An Exploratory Investigation of Selected Measures of Rate of Learning," Journal of Experimental Education, 36(2):78-81, Winter 1967.

[2]John L. Yeager, "Measures of Learning Rates for Elementary School Students in Mathematics and Reading under a Program of Individually Prescribed Instruction" (unpublished doctoral dissertation, University of Pittsburgh, 1966).

level of reading achievement. The investigator concluded that (1) there is a lack of consistency in the three rate measures and that (2) they do not correlate highly with the pupil's intelligence and the level of his reading achievement. These results may be attributable to the unreliability of the rate measures used. Yeager suggests that a composite measure of pupil's rate of learning may perhaps provide a more effective way of studying the factors that are associated with this variable and that a more meaningful and useful result may be obtained from this combination. This conclusion also suggests the applicability of the proposal of Hotelling:[1] "When it is desirable to predict the non-measurable variable by means of a second set of observable quantities, . . . no single regression equation can provide a fully adequate solution." He pointed out further that any combination of criteria may be used as the dependent variate in a regression equation and that generally not one, but several regression equations must be used to give a proper picture.

From the results of the various research studies on "learning" in general and on "rate of learning" in particular, one can draw the following conclusions:

1. There are substantial differences in level of achievement and in learning rates among pupils in school learning situations, as well as under experimental laboratory conditions. (Suppes, Bolvin, Kalin, Nicholas, and Yeager)

2. These individual differences are a function of the characteristics of the learner and many other variables that are closely related to the learning task and the learning environment. (Piaget, Spies, Whitman, Smith, Allison, Guilford, Carroll, and Sjogren)

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[1]H. Hotelling, "The Most Predictable Criterion," Journal of Educational Psychology, 26:139-142, 1935.

3. Although some studies indicate that rate of learning is related to typical aptitude measures (Jensen, Glaser and others, and Gropper and associated), investigations carried out in less controlled classroom situations indicate that rate is not consistent over various units of materials nor does it have any simple relationship to measures of aptitude. (Yeager)

These later results suggest several possible reasons why rate has not been found to be consistent, and why factors related to it, or that can predict it, have not been clearly identified.

1. The unreliable criterion measures (rate measures) used by the researchers resulted in inconsistent correlations.

2. The "incompleteness" of the criterion measure resulted in an invalid criterion measure. A single rate measure may cover only one dimension of the pupil's rate of learning and therefore, provide only a partial criterion measure. Since other dimensions of the rate of learning may have been left out, no comprehensive results can be obtained from these research studies.

3. The difficulties in identifying and obtaining measures of those personal characteristics of the individual learner as well as those situation factors that are relevant to the pupil's learning performance resulted in attempts to oversimplify the analysis of the determiners of rate. Many variables that are experimentally controllable in the learning laboratories must be accounted for by statistical procedures in studying rate in the classroom learning situation.

The lack of clarification of the nature and the operation of the pupil's learning rate is apparent in the research studies cited, hence, suggest the need for further research in this area. The present study endeavored to overcome some of the weaknesses of earlier studies through careful attention to the following eps: (1) Identification of a rather comprehensive and more meaningful rate measure, or measures, in evaluating pupil's learning performance; (2) Specification of the independent predictors that are associated with the rate measures; (3) Determination of the relative contributions made by each of these predictors to the prediction of the criterion measure--the rate measures; and (4) Identification of the interrelationships among the predictors--factors that are associated with learning rate.

CHAPTER II

PROBLEM AND HYPOTHESES

This study is concerned with the problem of identifying a useful and comprehensive measure of "rate of learning" in individualized classroom learning situations. To do this, it will investigate certain single measure of rate, each of which measures some aspect of the variable, as well as a composite measure, which should provide a more complete measure of a pupil's rate. The predictability of the rate measures, singly and combined, will then be examined.

This research will also examine a number of variables that are hypothesized as being ralated to rate. These variables may be useful as predictors of rate or, in some cases, they may have to be partialed out if a meaningful rate measure is to be obtained.

Statement of the Problem

The research question to be answered through this proposed research is: What is the most useful measure of "rate of learning" in Individually Prescribed Instruction classrooms in terms of the comprehensiveness of the rate measure and its predictability from selected student characteristics?

Specific Problems

1. What is the correlation between certain single measures of "rate of learning" for students in E-Level units of the IPI mathematics curriculum and such predictors as verbal and non-verbal I.Q., mathematics

achievement score, reading achievement score, total number of skills mastered during the previous school year, student's attention score, mental age, and pupil's reaction to the learning activities and materials?

2. What is the correlation between a composite rate measure (made up of the best combination of single rate measures) and such predictors as verbal and non-verbal I.Q., mathematics achievement score, reading achievement score, total number of skills mastered during the previous school year, student's attention score, student's reaction to learning activities and materials, and the mental age of the student?

3. What is the relative contribution of each of these selected predictors to the prediction of each rate measure?

4. What is the correlation between the best linear combination of the predictors and a linear composite of the rate measures?

5. How are the variables identified in Carroll's learning model involved in the measurement and the prediction of "rate of learning?"

Hypotheses

The foregoing questions, based on results from previous studies reviewed in Chapter I and, most specifically, on the work of Yeager[1] in investigating rate of learning in IPI, will be studied by testing the following hypotheses.

Hypothesis I: There will be no significant relationship between any one single predictor and a single rate measure.

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[1] John L. Yeager, "Measures of Learning Rates for Elementary School Students in Mathematics and Reading Under a Program of Individually Prescribed Instruction" (unpublished doctoral dissertation, University of Pittsburgh, 1966).

Hypothesis II: The predictability of each of the rate measures will increase when all the predictors are included in the regression equation.

Hypothesis III: There will be no significant relationship between a composite of the rate measure and one single predictor.

Hypothesis IV: The power of prediction of the rate measure will increase when a composite of the single rate measures is used in the regression equation together with a composite of the selected predictors.

Hypothesis V: Combining four learning rates as a criterion variable results in a more valid measure than using a single rate measure.

Hypothesis VI: The predictor variables selected in our research study for predicting pupil's rate of learning are equivalent to factors identified in the elements of the conceptual learning model that Carroll has developed to express degree of learning.

Definition of Terms

1. Individually Prescribed Instruction: Individually Prescribed Instruction (IPI) is a method of instruction that permits the assignment of new learning experiences based on the student's entering behavior and provides a structure that enables a student to progress at a rate commensurate with his ability. The basic design of this procedure is to provide an effective and workable plan for individual differences among students.[1]

[1]C. M. Lindvall and Robert Glaser, "The Role of Evaluation in Individually Prescribed Instruction" (paper presented at the 15th Annual Conference of Directors of State Testing Programs at Princeton, N. J., October 31, 1965).

2. IPI Mathematics Curriculum Sequence: The IPI mathematics curriculum is organized in terms of topic areas and levels of complexity. As can be seen from the chart below, the levels currently extend from level A through I and cover such topics as numeration, place value, addition, subtraction, and so on. A given topic at one level, such as D-Numeration, is known as a unit. Each unit, in turn, is made up of several related skills, varying in number from unit to unit.[1] For this study, pupil performance in six units at level E served as the focus for investigation.

IPI MATHEMATICS CURRICULUM

Topics	Levels								
	A	B	C	D	E	F	G	H	I
Numeration									
Place Value									
Addition									
Subtraction									
Multiplication									
Division									
Combination of Processes									
Fractions									
Money									
Time									
Systems of Measurement									
Geometry									
Special Topics									

[1] C. M. Lindvall and Richard Cox, "A Rationale and Plan for the Evaluation of the Individually Prescribed Instruction Project" (paper presented at the American Educational Research Association Convention, New York, February 1967).

3. Criterion Variable: Four rate measures are used as the criterion set of variables for this study. The rate measures are:

$Rate_1 = \frac{\text{100-pretest scores}}{\text{Days worked on the unit}}$: $Rate_1$ indicates the points the pupil has earned per day. The larger the quotient the faster the pupil learns.

$Rate_2 = \frac{\text{Number of pages worked}}{\text{Days worked on the unit}}$: $Rate_2$ indicates the number of pages the pupil has worked per day. The larger the quotient the faster the pupil learns.

$Rate_3 = \frac{\text{Number of skills learned}}{\text{Days spent on the unit}}$: $Rate_3$ indicates the number of skills the pupil has mastered per day. The larger the quotient, the faster the pupil learns.

$Rate_4$ = Total number of skills the pupil has worked between September 1967 and January 1968.

4. Predictor Variable: Selected measures of student characteristics are used as variables in the predictor set.

Verbal and non-verbal I.Q.: As measured by the Lorge-Thorndike Intelligence Test.

Mental Age: As measured by the Lorge-Thorndike Intelligence Test.

Mathematics and Reading Achievement: As measured by the Stanford Achievement Test.

Total number of skills worked during the previous school year.

Attention: Scores obtained through direct observations in the classroom.

Pupil's reaction to the learning activities: Obtained through a questionnaire prepared for this purpose. ($Question_1$)

Pupil's reaction to the learning materials: Obtained through a questionnaire prepared for this purpose. ($Question_2$)

CHAPTER III

RESEARCH DESIGN

The Setting for the Study

To investigate problems in the measurement of rate of learning and identify factors that are associated with differences in rate, it is necessary to work in situations where variations in rate actually exist. For this reason, schools in which Individually Prescribed Instruction is used offer a useful field laboratory for this type of research. Oakleaf and McAnnulty Elementary Schools in the Baldwin-Whitehall School District, both of which employ the IPI procedure, were used in this study.

The IPI procedure is a program designed to achieve a certain type of individualized instruction in Grades K through six in the subject areas of reading, arithmetic, and science. Its basic elements included (1) detailed sequences of behavioral objectives which define the abilities that each pupil is to acquire, (2) study materials, that are largely self-instructional in nature, to teach each objective, (3) a testing program for placing each pupil at the proper point in the curriculum sequence and for monitoring his progress, and (4) classroom management procedures that permit each pupil to proceed at a rate best suited to his needs. In mathematics, the curriculum is organized in terms of topics (numeration, place value, addition, subtraction, multiplication, division, combination of processes, fractions, money, time, system of measurement, geometry and special topics) and levels (A,B,C,D,E,F,G,H, and I). This means that a pupil will typically work through the series of topics at one level before moving on to the next. The work in a given topic at a given level, such as Level E-Numeration or Level C-Addition, is identified as a unit and is defined on the basis of a certain number of objectives and the study materials developed to enable the pupil to master these objectives.

The IPI procedure also makes rather detailed provisions for diagnosis of pupil skills and abilities, and for continous monitoring of pupil progress. A series of tests were constructed specifically for this purpose.[1]

1. Placement tests: These tests are given to the students at the beginning of the school year. The placement tests provide information essential to the proper placement of each pupil at the appropriate level of the curriculum sequence.

2. Pretests: Pretests provide exact information concerning the pupil's command of any material in the level, and they serve as a basis for developing his prescription describing what materials he needs to study.

3. Curriculum-embedded tests: A student takes these tests when he completes his study of each objective. These tests are basic instruments for use in determining when a pupil is ready to move on to a new objective. They are also essential for preparing prescriptions.

4. Unit post-tests: These tests are given when a pupil has completed work in a unit. This provides an overall survey of his command of the unit and is the basis for deciding whether he needs more work in it or is ready to go to the next unit.

For the most part, the curriculum materials are self-study materials, that is, materials with which a pupil can work by himself with a

[1]C. M. Lindvall and Robert Glaser, "The Role of Evaluation in Individually Prescribed Instruction" (paper presented at the 15th Annual Conference of Directors of State Testing Programs at Princeton, N. J., October 31, 1965).

minimum of assistance from the teacher. However, the plan also involves some small and large group instruction as well as ind: idual instruction by a teacher. (For detailed description of IPI see Yeager's dissertation.[1])

Research Population

Because of the amount of time needed to obtain all of the observation data needed for the present study, it was necessary to limit it to a certain number of units in the curriculum rather than to attempt to study all units and all pupils. The research population of this study consisted of the students from Oakleaf and McAnnulty Schools, who worked on the following units of the Level E-Mathematics Curriculum between September 1967 and January 1968.

TABLE 1

NUMBER OF STUDENTS IN GRADES TWO THROUGH SIX IN OAKLEAF AND McANNULTY SCHOOLS WORKING IN SIX SELECTED UNITS IN THE MATHEMATICS CURRICULUM AND CONSTITUTING THE SAMPLE FOR THIS STUDY

Grade	Units in Level E-Mathematics Curriculum					
	Numeration	Place Value	Addition	Subtraction	Multiplication	Comb. of Processes
2	1	1	1			
3	4	5	2	3		
4	34	26	6	20	19	7
5	67	41	16	44	56	18
6	76	36	17	36	36	37
Total	182	109	42	103	111	62

[1]John L. Yeager, "Measures of Learning Rates for Elementary School Students in Mathematics and Reading under a Program of Individually Prescribed Instruction" (unpublished doctoral dissertation, University of Pittsburgh, 1966).

Measures of Rate of Learning

Due to the complexity of what takes place in school learning situations, there are many problems in measuring rate of learning in such settings. These problems generally can be classified into two major categories: (1) problems that are associated with the measure used and (2) problems associated with the great number of variables that probably affect rate of learning.

A major problem that has been encountered in measuring rate of learning in the Individually Prescribed Instruction program arises from the definition of "rate." A general definition of rate may be expressed as the amount or degree of anything in relation to units of time, i. e., $\text{rate} = \frac{\text{amount or degree}}{\text{time}}$. The main problem here becomes that of determining which measures should be used to determine accurately the amount or degree of learning (the numerator in the above equation).

Since the instructional program of Individually Prescribed Instruction is based on the detailed specification of the sequences of instructional objectives of each unit through which each pupil is to proceed at his individual pace, and since the pupil's progress through these sequences is monitored through the use of special tests specifically designed to measure the stated objectives, one obvious way of expressing the amount of learning which has taken place is the pupil's score on these achievement measures.

However, among the available achievement measures, there is the problem of differentiating the actual amount of learning that has taken place from the amount of work that the pupil has done during the specific time period. This is to say, the amount of work required within a unit may vary from one pupil to another even when both pupils have to master exactly the same objective. For example, the amount of work they have to do may be different due to differences in the number of work pages that are assigned to them. Student A may need to work through eight work pages to learn the objective while student B may only need to work through two work pages to achieve mastery of the same objective. To resolve the difficulties and the limitations inherent in measuring the amount of learning and work done by each pupil, the investigator proposed using a composite of several rate measures to increase the comprehensiveness and reliability of this criterion measure.

One of the rate measures Yeager[1] used for his study was "the difference of the criterion and the pretest score divided by the number of days to complete the unit -- $\frac{\text{100 - pretest score}}{\text{days worked on unit}}$." Here we have a measure of the pupil's rate of learning in terms of how fast he worked to master the amount of material he did not know initially. However, under the Individually Prescribed Instruction program, each pupil goes through the unit in a different sequence, this sequence being determined individually according to the pupil's specific learning needs and the level of achievement he had when entering the unit. Therefore, pupils working on the same unit and having the same percentage of content left to master do not necessarily work on the same work pages, nor even the same number of work pages. It is for this reason that using this rate measure alone may not give an accurate picture of how fast a pupil really learns. With the rate measure proposed by Yeager, a pupil may be classified as slower than others who had the same amount to learn in the unit merely because more pages were assigned to him. Therefore, in addition to the rate measure proposed by Yeager (1966),

[1]John L. Yeager, "Measures of Learning Rates for Elementary School Students in Mathematics and Reading under a Program of Individually Prescribed Instruction" (unpublished doctoral dissertation, University of Pittsburgh, 1966).

$\text{Rate}_1 = \frac{100 - \text{pretest score}}{\text{days worked on unit}}$, a measure was included to take into consideration the number of work pages the pupil has worked per day. Hence, a second measure of rate was formulated to supplement Rate_1;

$$\text{Rate}_2 = \frac{\text{number of pages worked}}{\text{days worked on the unit}}.$$

Under the Individually Prescribed Instruction program, the pupil can master a unit without having to work through each skill within it. Pretest results may show that one student has five skills to master, another three skills, and still another only two skills. Furthermore, pupils may differ in the number of skills they have to study in a unit even though they have the same total pretest score. In order to meet this problem, which is not covered by the first two rate measures, an additional rate measure which will help to contribute to a more complete picture of the pupil's rate of learning was employed; namely,

$$\text{Rate}_3 = \frac{\text{number of skills learned}}{\text{days spent on the unit}}.$$

However, some units in the curriculum may be more difficult for one pupil than the others will be; thus, if we only have the above three measures to account for a particular pupil's rate of learning, our result still may not be accurate since the pupil may have worked faster or slower in this particular unit than on the other units. Therefore, an account of the number of skills in all the units in which he has worked will provide still another dimension of his rate of learning. This fourth rate measure, Rate_4, is the number of skills learned during the research period (September 1967 through January 1968). All of the above four proposed measures were used in this study as criterion variables.

Factors Associated with Learning Rate

To understand learning rate it is important to determine those factors that covary with it. For this study they are referred to as predictor variables. The following are the predictor variables used in this study together with the reasons for their being selected as factors hypothesized to be associated with rate of learning.

1. Verbal I.Q. and 2. Non-Verbal I.Q.: As measured by the Lorge-Thorndike Intelligence Test. It appears logical that the pupil's intellectual ability should be considered as one of the factors associated with how fast he learns. Many research studies on this topic of the relationship between I.Q. and the pupil's learning in school agree on this point. Carroll[1] puts this variable under the element of "Ability to Understand Instruction" in his model. A measure of general intelligence, the I.Q. score, or some index of achievement may be useful for predicting performance on a great range of intellectual or cognitive tasks. As pointed out by Ferguson,[2] Spearman strived to show that a general intellective factor operated in the performance of many mental tasks.

Carver and DuBois[3] investigated the relationship between learning and intelligence by giving 269 U. S. Navy enlisted men a battery of seven learning tests and the General Classification Test. Their results indicated that there is a significant relationship between intelligence and the gain scores on the learning tasks.

Yeager[4] found in his study that the student's intelligence quotient is significantly correlated with the number of mathematics units completed

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[1]John B. Carroll, "A Model of School Learning," Teachers College Record, 64:723-732, 1963.

[2]George A Ferguson, "On Learning and Human Ability," Canadian Journal of Psychology, 8:95-112, 1954.

[3]Ronald P. Carver and Philip H. DuBois, "The Relationship Between Learning and Intelligence," Journal of Educational Measurement, 41:133-136, Fall 1967.

[4]Yeager, op. cit.

in one year. Glaser, Reynolds, and Fullick[1] also suggest that there is a relationship between the intelligence of the student and the pace with which he works through the learning program.

Coleman and Cureton[2] suggested from the results of their study that a group I.Q. test and selected subtests from a school achievement battery measures are substantially identical functions. However, the investigators pointed out that in measuring general intelligence with a view to making inferences about differences in native capacity, there is a good reason to separate the functions tested into two dimensions: the verbal and non-verbal (arithmetical) factors.

Frost[3] cites in his article the findings of the Gudersen and Feldts study, which are in agreement with Coleman and Cureton's conclusion. They found in their study of fourth-grade groups with different verbal and non-verbal I.Q. scores that the verbal groups ranked first in each area of achievement, while the non-verbal groups ranked last. Cronbach[4] also suggests that non-verbal I.Q. has some special function in school learning. It gives indications of pupils who have good reasoning ability but who are below standard in reading and verbal development.

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[1]Robert Glaser, James H. Reynolds, and Margaret G. Fullick, Programmed Instruction in the Intact Classroom, Project No. 1342, Cooperative Research, U.S. Office of Education, Decemver 1963, p. 25.

[2]W. Coleman and E. E. Cureton, "Intelligence and Achievement: The Jungle Fallacy," Educational and Psychological Measurement, 14:347-351, 1954.

[3]B. P. Frost, "Some Conditions of Scholastic Achievement," Canadian Education and Research Digest, 5:267-284, December 1965; 6:5-17, March 1966.

[4]L. J. Cronbach, Essentials of Psychological Testing, (New York: Harper Brothers, 1960), pp. 169-171.

These latter sources all suggest the advisability of looking at both verbal and non-verbal aptitude. Thus, verbal and non-verbal I.Q. scores should be used as two separate predictors rather than using general I.Q. as a single predictor.

3. Mental Age: As measured by the Lorge-Thorndike Intelligence Test. Mental Age, one indicator of the extent of a child's mental development, is established through intelligence testing on the basis of the raw scores typically achieved by children of a particular chronological age. In this way, it provides information about the level of ability at which a pupil is operating.

Cronbach[1] explains that the "M.A. is an estimate of present performance and of promise in the immediate future." He points out that when making educational decisions for a group of pupils of varied age, the mental age rather than the I.Q. gives the most relevant information. Furthermore, he points out that in research studies, if it is desired to correlate some other learning variables with mental ability, the mental age should be used rather than the I.Q. This is because the correlation of I.Q. with another variable is lower than that of M.A. with the same variable in a group of mixed age. According to Cronbach's analysis, I.Q., then, is not a measure of the subject's present learning ability but rather a mathematical statement of the rate at which that learning ability will change in time.

However, House and Zeaman[2] state that theories of intelligence are often vague or inconsistent about whether I.Q., M.A., or both are related

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[1]Cronbach, op. cit.

[2]Betty J. House and David Zeaman, "Visual Discrimination Learning and Intelligence in Defectives of Low Mental Age," American Journal of Mental Deficiency, 65:51-58, 1960-1961.

to learning. These investigators concluded that M.A. is a measure of how much has been learned, while I.Q. is a measure of present learning ability --a crucial variable in this case. The investigators also pointed out that M.A. and I.Q. are independent variables that correlate with learning rate provided that C.A. can be ruled out as a relevant variable. Hence, both I.Q. and M.A. are correlated with the learning performance of the pupil. In view of the widespread disagreement concerning the relative importance of mental age and I.Q. as variables related to pupil's learning performance, both indices (M.A. and I.Q.) will be used as predictors in the present study.

4. Mathematics Achievement Score: As measured by the Stanford Achievement Test. Many studies have shown that an important predictor of future achievement is some measure of past achievement in the same, or a related, subject.

Townsend[1] has shown a high relationship between previous and later achievement. This result is supported by Piaget's[2] theory of mental development in children. According to Piaget's formulation of the process of 'assimilation' in the sequence of mental development, the learner acts on an environmental object in relation to his previous experience with some similar object and imposes some of his own conceptions on it. As a result, new activities are incorporated into the child's repertoire in response to the demands of the environment. Therefore, in addition to the I.Q. score, the mathematics achievement score gives a general indication as to how

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[1]Agatha Townsend, "Growth of Independent School Pupils in Achievement of the Standard Achievement Test," Educational Research Bulletin, 56:61-71, 1951.

[2]J. Piaget, The Origins of Intelligence in Children, (New York: International University Press, 1952).

well this pupil learns in the area of mathematics, since previous knowledge of the subject is an important factor in determining success in learning the task at hand. It also seems logical to consider the fact that how well a pupil has learned a subject will have some relation to how fast and how well he will master the subsequent learning tasks in the same subject.

5. Reading Achievement Scores: As measured by the Stanford Achievement Test. After reviewing the results of research studies that have been conducted in school learning, it seems reasonable to assume that at different levels of the educational process, different factors affect the learning performance. This is due to the different learning behavior that is required by the different learning tasks as well as the different levels of the learning tasks. It is a well-recognized fact that during the years of the pupil's formal education in the schools, the learning objectives that are set up for him change as he proceeds through the sequence of learning behavior. The skills and the ability required of him also differs from task to task and level to level. For example, reading is obviously of paramount importance in elementary school subjects such as spelling, arithmetic and science. Reading achievement score is used here as a predictor variable on the assumption that it is a determiner of the pupil's ability to understand instruction. Bloom[1] indicated in his analysis of Carroll's model for school learning that this variable should be directly related to the amount of time required by the pupil to learn a particular task.

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[1]B. S. Bloom, "Mastery Learning for All" (invited address presented at the American Educational Research Association Annual Meeting, February 7-10, 1968).

6. Total number of skills: This measure includes the total number of skills mastered by the pupil during the previous school year (1966-1967). It is included as one of the predictors for the same reason that mathematics achievement score is included in this study. Past rate of learning in a given subject should be a predictor of how fast the pupil will learn in the future.

7. Attention: This variable measures how much attention a pupil gives to the learning of the unit. The measure is obtained through direct observation in the classroom.

Carroll[1] explains in his learning model that a learner who, in view of his aptitude, the quality of the instruction, and his ability to understand instruction, needs a certain amount of time to learn a task, may or may not be willing to perservere for that amount of time in trying to learn. Therefore, he postulates perserverance as a determiner of rate of learning.

Frost[2] cited the findings of Kern, who, from a study of under-achievers, concluded that the pupil's difficulties with study habits were the main obstacle to attainment of educational goals. Hence, it is not enough that each pupil is provided with the opportunity to learn at his own ability level and with an adequate amount of time to learn the task; we must see to it that he utilizes the opportunity appropriately. Therefore, a measure of the amount of time he actually spends on learning the task is crucial to the accurate prediction of the outcome of his learning behavior. In other words, the amount of time he actually spends on learning is

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[1]Carroll, op. cit.

[2]Frost, op. cit.

important as a determiner of his rate of learning and as an indication of his learning behavior.

Three ten-minute-period observations were used to account for the attention score for each pupil working in each of the selected E-level units in the Mathematics Curriculum.

8. The pupil's reaction to the difficulty of the learning activities: A questionnaire was constructed for this purpose. The question raised was, "Was this work hard or difficult for you?" This question ($Question_1$) was presented to the pupil by an aide after each CET. The pupil's answer was then recorded by the aide on a five-point scale, ranging from "very easy" to "very difficult."

The student's perception of the difficulty of the task to be learned is important. In terms of Carroll's[1] learning model, this variable may be a measure of the quality of instruction. The simplest and the most direct way to obtain a meaningful measure of the perceived difficulty of the learning task is to ask the pupil about it.

As Carroll has pointed out, it is the school's responsibility to organize and present the task to be learned to the pupil in such a way that he will learn it as rapidly and as efficiently as he is able. Quality of instruction, according to Carroll, in addition to teacher performance, also includes the nature and the type of the instructional material used to help the learner to learn the task. Therefore, when the quality of instruction is at the optimal level, the learner should be able to learn the task without much difficulty. As Bloom[2] points out, the pupil's aptitude for learning a particular task may be modified by appropriate learning

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[1]Carroll, op. cit.

[2]Bloom, op. cit.

experience. He hypothesizes that more efficient learning conditions reduce the amount of time required for the pupil to learn a particular task.

Although this variable (pupil's perception of the difficulty of his work) has many possible implications and is almost impossible to specify, it is hypothesized that it is in some way associated with the pupil's rate of learning. Since the main interest of this research study is in determining the extent of the relationship between this readily available measure and rate of learning, rather than the exact nature of the variable, the inclusion of this variable as one of the predictors would seem to be appropriate.

9. The pupil's reaction to learning materials: This measure was obtained from Question_2 of the questionnaire. The question was, "How well did you like the things you did in this skill?" This question is scored by the aide on a five-point scale. Here the possible pupil response ranges from "like very much" to "dislike very much."

Question_2 was included as one of the predictors to obtain some information about the pupil's reaction to the instructional procedures and materials he has to use to master the learning task. Here the investigator is concerned with the student's liking for his learning task and, hence, the extent of his motivation for pursuing it. The emotional expression of "like" or "dislike" for the things an individual has to do in the learning task should be related to the level of his performance. High motivation should lead to better learning performance and to a faster rate of learning.

Carroll's learning model[1] includes the motivation factor under the element of "perserverance." He explains that the maximum amount of time for which an individual will apply himself to the learning of a task

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[1]Carroll, op. cit.

depends partly on the level of his motivation. $Question_2$ provides us also with some information about the effectiveness of material in interesting the pupil, and in motivating him to learn the task at hand. It is hypothesized that this expressed degree of interest will be related to the pupil's rate of learning.

Procedures of Data Gathering

All pupils worked on the selected E-Level units of the Mathematics Curriculum and were observed during the months from September 1967 through January 1968.

The following information was obtained and recorded for each pupil included in the study:

Pretest score of each unit

Verbal and non-verbal I.Q. scores

Mental Age

Mathematics and reading achievement scores

Total number of skills mastered during the previous school year (1966-1967)

Days spent on each unit

Pages worked on each unit

Skills worked on each unit

Total number of skills mastered during the research period

Attention score

Scores from the questionnaire.

CHAPTER IV

PRESENTATION AND THE ANALYSIS OF DATA

The basic purpose of this study has been to investigate characteristics of measures of rate of learning in individualized instruction situations. In particular it has studied the relationship of selected pupil aptitude and achievement measures as well as certain classroom performance measures to rate of learning. In studying the relationship of these variables to rate, four different rate measures have been employed both singly and in combination. To examine these relationships as they are found with different types of specific mathematic content, student performance in six different units of IPI mathematics was investigated.

Previous research by Yeager[1] has shown the inadequacy of certain measures of rate of learning in IPI situations, and also suggested some new measures that might be employed. The current study then, can be viewed as an extension of the work initiated by Yeager. As the first step in the assessment of the functioning of the measures involved, which were detailed in Chapter III, the Pearson product-moment correlation coefficient was computed between each individual predictor and each individual rate measure. This analysis was used to test research hypothesis I.

Since one of the implications of Yeager's study was that his finding of a consistent lack of correlation between predictors and rate measures was due to a lack of comprehensiveness in any single measure,

[1]John L. Yeager, "Measures of Learning Rates for Elementary School Students in Mathematics and Reading under a Program of Individually Prescribed Instruction" (unpublished doctoral dissertation, University of Pittsburgh, 1966).

the present study investigated the relationship between each predictor and a composite rate measure. This was done by determining the multiple correlation between each predictor and a linear function of four rate measures. This analysis was used to test research hypothesis III.

To further examine the relationship between the predictors and the measures of learning rate, multiple correlations and multiple regression equations were determined for the prediction of each rate measure from the best linear combination of the nine predictors. This permits an examination of the strength of this relationship and of the relative contribution of each predictor. This analysis was used in testing research hypothesis II.

A logical next step in the analysis seemed to be to ask the question "What is the relationship between the best linear combination of the predictors and a linear composite of the rate measures?" This question can be investigated through the use of canonical correlation[1] analysis. This type of analysis provides us with canonical correlation coefficients, which show us the degree of relationship between such linear composites (the canonical variates), and also provides indices of the relative contribution of each of the original variables to the composite of which it is a part. This technique was used to test research hypothesis IV.

All of the analyses described in the foregoing discussion are presented in the following sections of this chapter. The statistical analysis was carried out on the IBM 7090 computer at the Computation and

[1] H. Hotelling, "The Most Predictable Criterion," Journal of Educational Psychology, 26:139-192, 1935, and William W. Cooley and P. R. Lohnes, Multivariate Procedures for the Behavioral Science (New York: John Wiley, 1962), pp. 37-44.

Data Processing Center of the University of Pittsburgh.[1] The computer programs used in this study were:

1. Correlation with Item Deletion BMD03D.[2]
2. Multiple Correlation Analysis by Cooley and Lohnes.[3]
3. Canonical Correlation Analysis by Cooley and Lohnes.[4]

The Analysis of Data

1. The Relationship Between Individual Predictors and Individual Rate Measures

The relationship between each of the criterion measures and each predictor was analyzed for each of the selected E-level units of the IPI mathematics curriculum.

Tables 2 through 7 summarize the results for each of the E-level units.[5] Only significant correlation coefficients are listed.

In looking at the significant Pearson product-moment correlation coefficients, it is seen that none of the predictors correlate consistantly with any single rate measure for all six units of the IPI mathematics curriculum included for this study. Although all of the predictors are correlated with one rate measure or another in one or several of the units,

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[1]The University of Pittsburgh Computing and Data Processing Center is partially supported by the National Science Foundation Grant G-11309.

[2]W. J. Dixon (ed.), Biomedical Computer Programs (Health Sciences Computing Facility, Department of Preventive Medicine and Public Health, School of Medicine, Los Angeles: University of California, 1965), pp. 60-66. (See Appendix A for range of N's involved by units.)

[3]William W. Cooley and P. R. Lohnes, Multivariate Procedures for the Behavioral Science, (New York: John Wiley, 1962), pp. 31-59.

[4]Ibid.

[5]See Appendix A for Means and Standard Deviations for all variables.

these predictors are not found to be related generally to all rate measures in all situations. In most cases, the correlations between the rate measures and the predictors for each unit are not significantly different from zero. Among 216 correlation coefficients 16 were significant at the .05 level and 12 at the .01 level.

The results clearly support hypothesis I, that no single rate measure nor any single predictor can explain the complex nature of rate of school learning. For example, in Table 2, none of the predictors correlated significantly with $rate_1$, only one variable (the total number of skills completed during the previous school year) is correlated with $rate_2$ at the .01 level of significance. There is no significant relationship between $rate_3$ and any of the predictors. $Rate_4$ is correlated with $question_2$ at the .05 level of significance, and it is correlated with the attention measure and $question_1$ at the .01 level of significance. Out of the 36 correlations for the numeration unit, only two were significant at the .01 level and two at the .05 level.

TABLE 2

PRODUCT-MOMENT CORRELATION COEFFICIENT BETWEEN THE RATE MEASURES AND THE PREDICTORS FOR THE E-NUMERATION UNIT OF THE IPI MATHEMATICS CURRICULUM (N = 182)

	Verb. I.Q. (X_1)	Non-Verb. I.Q. (X_2)	M.A. (X_3)	Math. Achiev. (X_4)	Read. Achiev. (X_5)	Skills 1967 (X_6)	Atten. (X_7)	$Ques._1$ (X_8)	$Ques._2$ (X_9)
$Rate_1$									
$Rate_2$						.193**			
$Rate_3$									
$Rate_4$							.165*	.183**	.161*

(Only significant correlations are listed)

**.01 level of significance, critical R = .181, d.f. = 200
*.05 level of significance, critical R = .138, d.f. = 200

For the place value unit of the E-level mathematics curriculum, the results indicate that mental age and mathematics achievement score correlated with $rate_1$ at the .01 level of significance (See Table 3) while reading achievement score and the total number of skills worked during the previous school year correlated with $rate_1$ at the .05 level. The total number of skills worked also correlated at the .01 level with $rate_2$ for this unit. None of the predictors correlate with $rate_3$ or $rate_4$ for this particular unit. Out of 36 correlations for the place value unit three correlations are significant at the .01 level and two are significant at the .05 level.

TABLE 3

PRODUCT-MOMENT CORRELATION COEFFICIENT BETWEEN EACH RATE MEASURE AND EACH PREDICTOR FOR THE E-PLACE VALUE UNIT OF THE IPI MATHEMATICS CURRICULUM (N = 109)

	Verb. I.Q. (X_1)	Non-Verb. I.Q. (X_2)	M.A. (X_3)	Math. Achiev. (X_4)	Read. Achiev. (X_5)	Skills 1967 (X_6)	Atten. (X_7)	$Ques._1$ (X_8)	$Ques._2$ (X_9)
$Rate_1$			.311**	.345**	.234*	.245*			
$Rate_2$						.335**			
$Rate_3$									
$Rate_4$									

(Only significant correlations are listed)

**.01 level of significance, critical R = .254, d.f. = 100
*.05 level of significance, critical R = .195, d.f. = 100

For the E-addition unit of the IPI mathematics curriculum, two predictors correlated with $rate_1$ at the .01 level and one is correlated with $rate_1$ at the .05 level, while total number of skills worked during the previous school year is correlated with $rate_2$ at the .05 level. (See Table 4)

For the E-subtraction unit, 10 out of 36 correlations were significant (See Table 5) while only two correlations were significant for the E-multiplication unit (Table 6). For the E-combination of process unit, three out of the 36 correlations were significant at the .05 level. (See Table 7)

TABLE 4

PRODUCT-MOMENT CORRELATION COEFFICIENT BETWEEN THE RATE MEASURES AND THE PREDICTORS FOR E-ADDITION UNIT OF THE IPI MATHEMATICS CURRICULUM (N = 42)

	Verb. I.Q. (X_1)	Non-Verb. I.Q. (X_2)	M.A. (X_3)	Math. Achiev. (X_4)	Read. Achiev. (X_5)	Skills 1967 (X_6)	Atten. (X_7)	$Ques._1$ (X_8)	$Ques._2$ (X_9)
$Rate_1$					.386*	.403**	.412**		
$Rate_2$						.314*			
$Rate_3$									
$Rate_4$									

(Only significant correlations are listed)

**.01 level of significance, critical R = .393, d.f. = 40
*.05 level of significance, critical R = .304, d.f. = 40

TABLE 5

PRODUCT-MOMENT CORRELATION COEFFICIENT BETWEEN THE RATE MEASURES AND THE PREDICTORS FOR E-SUBTRACTION UNIT OF THE IPI MATHEMATICS CURRICULUM (N = 103)

	Verb. I.Q. (X_1)	Non-Verb. I.Q. (X_2)	M.A. (X_3)	Math. Achiev. (X_4)	Read. Achiev. (X_5)	Skills 1967 (X_6)	Atten. (X_7)	$Ques._1$ (X_8)	$Ques._2$ (X_9)
$Rate_1$				.223*	.219*	.255**			
$Rate_2$		.242*				.341**			
$Rate_3$			.217*		.317**	.247**		.233*	
$Rate_4$							.260**		

(Only significant correlations are listed)

**.01 level of significance, critical R = .254, d.f. = 100
*.05 level of significance, critical R = .195, d.f. = 100

TABLE 6

PRODUCT-MOMENT CORRELATION COEFFICIENT BETWEEN THE RATE MEASURES AND THE PREDICTORS FOR THE MULTIPLICATION UNIT OF THE IPI MATHEMATICS CURRICULUM (N = 111)

	Verb. I.Q. (X_1)	Non-Verb. I.Q. (X_2)	M.A. (X_3)	Math. Achiev. (X_4)	Read. Achiev. (X_5)	Skills 1967 (X_6)	Atten. (X_7)	Ques.$_1$ (X_8)	Ques.$_2$ (X_9)
Rate$_1$	.215*								
Rate$_2$									
Rate$_3$						.293**			
Rate$_4$									

(Only significant correlations are listed)

**.01 level of significance, critical R = .254, d.f. = 100
*.05 level of significance, critical R = .195, d.f. = 100

TABLE 7

PRODUCT-MOMENT CORRELATION COEFFICIENT BETWEEN THE RATE MEASURES AND THE PREDICTORS FOR THE E-COMBINATION OF PROCESS UNIT OF THE IPI MATHEMATICS CURRICULUM (N = 69)

	Verb. I.Q. (X_1)	Non-Verb. I.Q. (X_2)	M.A. (X_3)	Math. Achiev. (X_4)	Read. Achiev. (X_5)	Skills 1967 (X_6)	Atten. (X_7)	Ques.$_1$ (X_8)	Ques.$_2$ (X_9)
Rate$_1$				.284*					
Rate$_2$					.262*				
Rate$_3$									
Rate$_4$						.288*			

(Only significant correlations are listed)

**.01 level of significance, critical R = .302, d.f. = 70
*.05 level of significance, critical R = .233, d.f. = 70

The results of the simple correlation analysis of the data for this study is in agreement with Yeager's study[1] on the rate of learning--

[1]Yeager, *op. cit.*

the study from which some of the hypotheses for the present investigation was formulated. Yeager studied the consistency of three measures of student learning rate and the relationship between each rate measure and each of the student characteristics--student's I.Q. and reading achievement score. Based on the results of his study, Yeager suggests that perhaps the reason that no consistent relationship was found between each of the student characteristics studied and each rate measure is the lack of comprehensiveness in the measures. Therefore, based on the results of Yeager's investigation and Carroll's[1] conceptual model of school learning, hypotheses II, III and IV of this study were formulated.

2. Relationship of Individual Predictors and a Composite of the Four Rate Measures.

To investigate the possibility that individual prediction might be more highly related to rate if a more comprehensive rate index were used, the multiple correlation of each predictor with a composite of the four rate measures was determined. The results of this multiple correlation analysis as presented in Table 8 show that, as would be expected, the correlation between each predictor variable and the rate measures increases when all four rate measures are combined. The table shows, for example that in the E-Numeration unit the multiple R for $question_1$ and the four rate measures combined is .247 which is larger than .183, the only significant correlation between $question_1$ and a single rate measure as reported in Table 2.

Among the multiple R's tested for each predictor and the four rate measures, the majority of them were not significantly different from zero. For example, for the numeration unit, only $question_1$ is significantly

[1]John B. Carroll, "A Model of School Learning," Teachers College Record, 64:723-732, 1963.

related to the rate measures. This general lack of an increase in number of significant correlations with single predictors when a composite rate measure is used suggests that the lack of relationship between predictors and rate is not due primarily to the lack of comprehensiveness in the rate measure.

TABLE 8

MULTIPLE CORRELATION COEFFICIENTS BETWEEN EACH MEASURE OF STUDENT CHARACTERISTICS AND THE RATE MEASURES FOR THE SELECTED E-LEVEL UNITS OF THE IPI MATHEMATICS CURRICULUM

Student Characteristics	Numeration (N=181)	Place Value (N=42)	Addition (N=109)	Subtraction (N=102)	Multiplication (N=111)	Comb. of Processes (N=61)
Verb I.Q. (X_1)	.227	.149	.271	.227	.274	.352
Non-Verb I.Q. (X_2)	.197	.236	.232	.263	.139	.263
M.A. (X_3)	.187	.396	.301	.235	.214	.251
Math Achiev. (X_4)	.179	.290	.187	.267	.310	.304
Read Achiev. (X_5)	.151	.290	.398	.348*	.145	.347
Skills 1967 (X_6)	.220	.352	.479	.429**	.348*	.387
Attention (X_7)	.187	.165	.452	.283	.206	.262
Question_1 (X_8)	.247*	.221	.104	.347*	.135	.177
Question_2 (X_9)	.222	.221	.293	.213	.076	.150

**.01 level of significance
*.05 level of significance

3. The Relationship of Each Rate Measure and the Best Composite of the Nine Predictors

After analyzing the relationship between rate measures and each of the predictors, the correlation between each rate measure and all of the predictors combined was investigated. These multiple R's are shown in Table 9. A comparison of the correlation coefficients obtained from the simple correlation analysis and the multiple correlation analysis shows, as would be expected, that all the multiple R's are larger than the Pearson product-moment r's, and a much larger percent of the multiple R's are found to be significant. This result supports hypothesis II that the predictability of rate measures increases when all the predictors are included in the regression analysis. For example, the Pearson product-moment correlation coefficient between $rate_4$ and the predictors (verbal I.Q., non-verbal I.Q., M.A., mathematics achievement, reading achievement, skills, attention, $question_1$, and $question_2$) for the numeration unit are: .054, .069, .124, .099, .068, .023, .165*, .183**, and .161* respectively; the multiple R for $rate_4$ and all the predictors combined for the same unit is .377**. The Pearson r between $rate_2$ and the predictors are: -.048, -.112, -.088, -.066, -.119, .193**, -.025, -.131, and -.125 respectively, while the multiple R for $rate_2$ and all the predictors combined for the same unit is .343**.

Nevertheless, among the multiple R's tested for each rate measure and all the predictors for the selected E-level units of the mathematics curriculum, many are not significantly different from zero. (See Table 9) For example, for $rate_1$ and $rate_2$ only 50 percent of the units studied showed significant multiple R's; for $rate_3$, 33 percent of the units had significant multiple R's and for $rate_4$ only 17 percent of the units studied showed significant multiple R's. Therefore, among the 24 multiple R's listed in Table 9 only nine are significant--37.5 percent.

TABLE 9

MULTIPLE CORRELATION COEFFICIENT BETWEEN EACH RATE MEASURE AND THE PREDICTORS (STUDENT CHARACTERISTICS) FOR THE SELECTED E-LEVEL UNITS OF THE IPI MATHEMATICS CURRICULUM

Rate Measures	Unit					
	Numeration	Place Value	Addition	Subtraction	Multiplication	Comb. of Processes
$Rate_1$	.203	.433**	.642*	.527**	.344	.327
$Rate_2$	.343**	.450**	.598	.502**	.339	.392
$Rate_3$	.293	.302	.495	.517**	.416*	.438
$Rate_4$	.337**	.257	.247	.379	.099	.433

**.01 level of significance
*.05 level of significance

In those cases where the multiple correlation coefficients are significant, it is of interest to examine the relative contribution of the various predictors. One way of interpreting the individual contribution of the predictor variables is by examining the beta weights, the partial regression coefficients. The contribution of most predictors in this study can be expected to be quite small, since the multiple R's are relatively small. This is seen in the data of Table 10.

Because of the small size of the betas and because of their known inconsistency from sample to sample when the sample size is small, it is not surprising to note the lack of any consistent pattern in the betas from one regression equation to another. This problem can be investigated further by examining the structure R's for these same relationships. These structure R's indicate the correlations between the original predictors and the derived linear composite of the predictors. The structure R's are more stable in that they do not fluctuate as much from sample to sample as the betas do. For this reason, it is more meaningful to use the

TABLE 10

BETA COEFFICIENTS FOR MULTIPLE REGRESSION EQUATIONS FOR PREDICTING INDICATED RATE MEASURE IN MATHEMATICS UNITS FOR CASES WHERE THE MULTIPLE CORRELATION IS SIGNIFICANT

Rate Measure	Unit	Multi. R	Verb I.Q. (X_1)	Non-verb I.Q. (X_2)	M.A. (X_3)	Math Achiev. (X_4)	Read Achiev. (X_5)	Skills 1967 (X_6)	Atten. (X_7)	Ques.[1] (X_8)	Ques.[2] (X_9)
1	Place Value	.433**	-.093	-.024	-.172	.247	.022	.189	-.059	-.049	-.033
1	Addition	.642*	.325	-.494	.131	-.122	-.203	.353	.292	.081	-.217
1	Subtraction	.524**	.219	-.467	.219	.113	.086	.171	.159	.194	-.056
2	Numeration	.343**	.213	-.133	-.077	-.027	-.194	.280	-.049	-.113	-.088
2	Place Value	.450**	.016	-.132	-.040	.193	.044	.371	-.032	-.209	-.004
2	Subtraction	.502**	-.025	-.380	.275	.008	.033	.243	.205	.048	.083
3	Subtraction	.517**	.187	-.370	.247	.107	.263	.138	.035	.290	-.084
3	Multiplication	.416*	.064	.037	.072	.210	-.046	.241	.156	-.099	.026
4	Numeration	.337**	-.024	-.139	.265	.073	-.029	-.041	.213	.166	.133

**.01 level of significance
*.05 level of significance

structure R's to examine the relative contribution of the various predictors for interpretative purposes. The structure R's for the significant multiple correlations are listed in Table 11.

In general, the structure R's show the same inconsistency from unit to unit and from one rate measure to another as do the betas. However, it is significant to note that in all except one case the structure R's associated with number of skills mastered in 1967 are of substantial size and positive. This does suggest that further investigations of predictors of rate of learning should give attention to past rate as a possible useful measure.

The results of the simple correlation analysis and the multiple correlation analysis indicate that no one predictor variable is a major determiner of rate of pupil learning, that rate is determined by a complex of factors, and that the relative importance of each factor varies greatly from one learning task to another. In order to obtain a more complete assessment of pupil's learning in school under the IPI program, more than one measure of rate of learning should be used. This general hypothesis will be examined further in the canonical correlation analysis.

4. The Relationship Between a Composite of the Four Rate Measures and the Composite of the Nine Predictors

The foregoing analyses suggested the desirability of examining the relationship between a linear combination of the predictors and a linear combination of the rate measures. Canonical correlation analysis determines the correlation between linear composites of two sets of variables, where these composites are derived in such a way as to yield the maximum relationship.

TABLE 11

STRUCTURE R'S OF EACH VARIABLE FOR THE INDICATED RATE MEASURE IN MATHEMATIC UNITS FOR CASES WHERE THE MULTIPLE CORRELATION IS SIGNIFICANT

		Structure R								
Rate Mea-sure	Unit	Verb I.Q. (X_1)	Non-verb I.Q. (X_2)	M.A. (X_3)	Math Achiev. (X_4)	Read Achiev. (X_5)	Skills 1967 (X_6)	Atten. (X_7)	Ques. 1 (X_8)	Ques. 2 (X_9)
1	Place Value	.084	.150	.718	.796	.539	.567	-.272	.141	-.112
1	Addition	.230	-.077	.304	.208	.602	.628	.643	.075	.110
1	Subtraction	.146	-.265	.292	.426	.418	.487	.384	.384	-.024
2	Numeration	-.139	-.328	-.257	-.193	-.349	.562	-.073	-.383	-.363
2	Place Value	-.105	-.261	.221	.388	.257	.743	-.131	-.286	-.334
2	Subtraction	-.304	-.483	.125	.185	.177	.679	.366	.088	-.302
3	Subtraction	.246	-.091	.420	.230	.613	.477	-.014	-.450	-.013
3	Multiplication	.594	.081	.369	.038	-.157	.484	-.166	.132	.181
4	Numeration	.160	.204	.369	.293	.203	.068	.489	.543	.479

Two sets of variables can contain very similar information even though pairs of variables selected one from each set may not be particularly highly correlated. This is because of the fact that what one variable is measuring in one set may be measured by a combination of variables in the other set or vice versa. Therefore, the underlying traits may be reflected in several variables, yet only when these several variables are examined in combination are the relationships between two sets of variables made evident.[1]

The number of canonical correlations is equal to the rank of the intercorrelation matrix among the two sets of variables. For example, the rank of the matrix for the present investigation is four, the number of rate measures. In canonical analysis it is algebraically impossible to have more non-zero canonical correlations than the number of variables in the smaller of the two sets.[2] In general the complete factor structure of a set of variables contains as many factors as there are variables. Hence it is obvious that if the larger set is composed of nine variables as in the case of this study, and the smaller set is composed of four variables, only four factors can be extracted from the smaller set. As a result, canonical R's are available between four of the factors of the larger set and the four factors of the smaller set. The remaining factors in the predictor set have no counterpart in the criterion set and do not enter into the canonical solution.

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[1]For a discussion of this point see William W. Cooley, "Further Relationships with the TALENT Battery," Personnel and Guidance Journal, 44:295-303, November 1965.

[2]Cooley and Lohnes, op. cit.

According to Hotelling,[1] a canonical solution can be explained in terms of factor analysis. That is, that the two sets of variables could be factor analyzed independently first and then weights developed to rotate the two factor structures to maximum correlation. In the case of canonical analysis, the factors extracted (the linear combination of the predictor set and the linear combination of the criterion set) are called canonical variates, and the correlation between the first canonical variate (factor) of the criterion set and first canonical variate of the predictor set is the first canonical correlation. Therefore, for the present study, by using the canonical correlation analysis technique, two new scores were developed through the combination of the four rate measures and the combination of the measures of student characteristics. The derived combinations (factors), one from each set of variables, are the canonical variates. The correlation between the first two canonical variates (one from each set) is the first canonical correlation. This canonical correlation indicates the similarities between the two sets of variables.

The first canonical variate for the rate measure was obtained through weighting the pupil's original rate scores by the corresponding coefficients, and the first canonical variate of the predictor variables was obtained from weighting the pupil's original student characteristics scores by the corresponding coefficients. Each of these two derived first canonical variates are uncorrelated with the other derived canonical variates (other combination of the variables) in it's own set but each has maximum correlation with its paired variate from the other set.[2]

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[1]Hotelling, op. cit.

[2]William W. Cooley and Judy D. Miller, "The Project TALENT Test as a National Standard," Personnel and Guidance Journal, 44:1038-1044, June 1965.

Tables 12 through 17 indicate the latent roots, the corresponding canonical correlations and their associated tests of significance. The canonical correlations were tested for significance by a Chi square approximation.[1]

For the numeration unit, the first canonical correlation of .438 was significant at the .01 level while the remaining three canonical correlations were not significant. (See Table 12) This means that there is one dimension of the student characteristic set which is significantly related to a corresponding dimension of the set of rate measures.

TABLE 12

CANONICAL CORRELATIONS AND THEIR ASSOCIATED TEST OF SIGNIFICANCE FOR THE NUMERATION UNIT

N	Root Removed	Canonical R	λ R-Squared	Chi Square	d.f.	Lambda Λ	Test of Significance
182	0	.438	.192	67.68	36	.6793	< .01
	1	.309	.091	30.35	24	.8408	> .05
	2	.240	.058	13.35	14	.9250	> .05
	3	.136	.018	3.26	6	.9816	> .05

The first canonical R's for three other units of the IPI mathematics curriculum are also significant. For the place value unit, the first canonical R is .486, which is significant at the .05 level. (Table 13) The first canonical R for the subtraction unit, .656, is significant at the .01 level. (Table 15) For the multiplication unit, the first canonical R is .474, which is significant at the .05 level. (Table 16)

[1]William W. Cooley, "Canonical Correlation" (paper presented at the APA Symposium on Application of Multivariate Analysis, September 7, 1965).

The first canonical R's of the addition and the combination of processes unit are not significant. This may be due to the small sample size used in the analysis of these two units, since the canonical R's are quite large, .656 and .549 respectively.

In general, the results of the canonical analysis for each of the six units selected for this study indicate that there is a significant relationship between the domain of rate measures and the domain of the predictors.

TABLE 13

CANONICAL CORRELATIONS AND THEIR ASSOCIATED TEST OF SIGNIFICANCE FOR THE PLACE VALUE UNIT

N	Root Removed	Canonical R	λ R-Squared	Chi Square	d.f.	Lambda Λ	Test of Significance
109	0	.486	.236	53.28	36	.5931	< .05
	1	.408	.166	25.86	24	.7760	> .05
	2	.218	.047	7.34	14	.8306	> .05
	3	.152	.023	2.38	6	.9769	> .05

TABLE 14

CANONICAL CORRELATIONS AND THEIR ASSOCIATED TEST OF SIGNIFICANCE FOR THE ADDITION UNIT

N	Root Removed	Canonical R	λ R-Squared	Chi Square	d.f.	Lambda Λ	Test of Significance
42	0	.689	.476	43.23	26	.291	> .05
	1	.580	.337	20.64	24	.555	> .05
	2	.359	.129	6.27	14	.836	> .05
	3	.199	.040	1.42	6	.960	> .05

TABLE 15

CANONICAL CORRELATIONS AND THEIR ASSOCIATED TEST OF SIGNIFICANCE FOR THE SUBTRACTION UNIT

N	Root Removed	Canonical R	λ R-Squared	Chi Square	d.f.	Lambda Λ	Test of Significance
103	0	.656	.431	84.10	36	.4164	< .01
	1	.424	.179	30.03	24	.7314	> .05
	2	.289	.083	11.56	14	.8912	> .05
	3	.107	.028	2.71	6	.9722	> .05

TABLE 16

CANONICAL CORRELATIONS AND THEIR ASSOCIATED TEST OF SIGNIFICANCE FOR THE MULTIPLICATION UNIT

N	Root Removed	Canonical R	λ R-Squared	Chi Square	d.f.	Lambda Λ	Test of Significance
111	0	.474	.224	50.27	36	.6167	< .06
	1	.373	.139	23.83	24	.7952	> .05
	2	.206	.042	8.29	14	.4034	> .05
	3	.189	.036	3.78	6	.9643	> .05

TABLE 17

CANONICAL CORRELATIONS AND THEIR ASSOCIATED TEST OF SIGNIFICANCE FOR THE COMBINATION OF PROCESSES UNIT

N	Root Removed	Canonical R	λ R-Squared	Chi Square	d.f.	Lambda Λ	Test of Significance
69	0	.529	.280	37.56	36	.5051	> .05
	1	.389	.158	19.49	24	.7017	> .05
	2	.366	.134	10.02	14	.8335	> .05
	3	.194	.038	2.11	6	.9624	> .05

The nature of these pairs of significant canonical variates can be inferred by examining the vectors (the partial regression coefficients) associated with them. The beta weights listed in Table 18 determine the canonical variates which are associated with the largest R for each unit of the mathematics curriculum selected for this study. For example, the first canonical variates for the numeration unit can be expressed as equations as follows:

First canonical variate for the criterion set =

$$.124z_{rate_1} - .752z_{rate_2} - .072z_{rate_3} + .710z_{rate_4}$$

First canonical variate for the predictor set =

$$- .466z_{x_1} + .025z_{x_2} + .025z_{x_2} + .596z_{x_3} +$$

$$.198z_{x_4} + .251z_{x_5} - .528z_{x_6} + .433z_{x_7} + .475z_{x_8}$$

$$+ .364z_{x_9}$$

Where the z's are the variable scores expressed as standard scores.

The size and the sign of these weights must be taken into account in examining the nature of the canonical variates. For example, a positive sign for a weight means that a high standard score for its variable would make a positive contribution to the size of the canonical variate and a low score would make a negative contribution. The converse is also true. Therefore, the size of these weights is an index of the relative importance of each variable whereas the signs indicate the nature of the contribution made by the variables. For the numeration unit for example, mental age, reading score, attention score, $question_1$ and $question_2$ made positive contributions to the pupil's rate of learning while pupil's verbal I.Q. and the total number of skills worked during the previous school year contribute

TABLE 18

THE CANONICAL VECTORS (BETA WEIGHTS) ASSOCIATED WITH THE FIRST CANONICAL VARIATES OF EACH OF THE SELECTED E-LEVEL UNITS OF THE IPI MATHEMATICS CURRICULUM

	Numeration	Place Value	Addition	Subtraction	Multiplication	Comb. of Processes
Criterion Set						
$Rate_1$	.124	.619	-.780	.577	-.062	-.041
$Rate_2$	-.752	.476	-.451	.461	-.471	.111
$Rate_3$	-.072	.191	-.062	.339	-.730	.658
$Rate_4$	.710	.071	.177	.162	-.233	.651
Canonical R	.4383**	.4855*	.6897	.6562**	.4738*	.5292
Predictor Set						
Verb I.Q.	-.446	-.128	-.741	.234	-.233	-.727
Non-Verb I.Q.	.025	-.160	1.081	-.876	-.033	.169
M.A.	.596	.286	-.497	.528	-.049	.228
Math Achiev	.198	.519	.133	.053	-.619	-.155
Read Achiev	.251	.067	.136	.266	.258	.518
Skills 1967	-.528	.622	-.590	.343	-.624	.696
Attention	.433	-.057	-.424	.258	-.272	-.088
$Question_1$	.475	-.258	.153	.399	.247	.404
$Question_2$	.364	-.003	-.073	-.137	-.164	.409

**.01 level of significance
*.05 level of significance

negatively to the pupil's learning rate. (Table 18) Mental age seemed to be the largest contributor to the first canonical R for the numeration unit.

The interpretation of the betas has the same problems associated with it as are found in the case of multiple regression analysis, in that they are typically quite unstable from sample to sample particularly where relatively small samples are used. However, the interpretation of a given canonical correlation is greatly aided when the structure R's (factor structure loadings) are used to examine the relative contributions made by the individual variables. The structure R indicates the strength of relationship (correlation) between the original variable and the derived canonical variate (factor), hence, it does not change as radically from sample to sample as the betas do. Therefore, it seemed to be more appropriate and meaningful to use the structure R's to examine the strength of association between the predictor variables and rate of learning. Table 19 lists the structure R's for the first canonical variates of each unit selected for this study.

Table 20 presents the proportions of the variance of the predictor set extracted by the first canonical variate (factor) for each of the selected units of IPI math curriculum; and Table 21 presents the proportions of the variance of the criterion set extracted by the first canonical variate for each of the units.

The first column of Tables 20 and 21 lists the first canonical correlation of each unit, the second column lists the R^2 (λ) of the first canonical R which represents the proportion of the variance of the composites shared by the first canonical vectors of the two sets of variables. Column three of the tables presents the proportion of the variance of the total set extracted by the first canonical variate. The fourth column lists the amount of redundant variance attributed to the first canonical factor and the last column expresses the values in the fourth column as proportions of the total redundancy. The redundancy is obtained by finding the product of R-squared (λ) and the variance extracted and, hence, tells us what proportion of the variance extracted from the total predictor

TABLE 19

CORRELATION BETWEEN ORIGINAL VARIABLES AND DERIVED FIRST CANONICAL VARIATES FOR EACH OF THE SELECTED E-LEVEL UNITS (STRUCTURE R'S) OF THE IPI MATHEMATICS CURRICULUM

Unit	Canon-ical R	Criterion Set (Rate Measure)				Predictor Set (Student Characteristics)								
		$Rate_1$	$Rate_2$	$Rate_3$	$Rate_4$	Verb I.Q. (X_1)	Non-verb I.Q. (X_2)	M.A. (X_3)	Math Achiev. (X_4)	Read Achiev. (X_5)	Skills 1967 (X_6)	Atten. (X_7)	$Ques._1$ (X_8)	$Ques._2$ (X_9)
Numer-ation	.438**	.087	-.678	-.053	.669	.152	.320	.380	.310	-.313	-.260	.309	.531	.469
Place Value	.485*	.849	.318	.486	-.104	.024	-.053	.594	.682	.473	.684	-.180	-.041	-.256
Addi-tion	.689	-.866	-.689	-.449	-.080	-.168	.191	-.279	-.233	-.420	-.658	-.572	-.121	-.256
Subtrac-tion	.656**	.760	.672	.690	.361	.014	.315	.299	.342	[illegible]	.554	.301	.372	-.072
Multi-plica-tion	.473*	-.466	-.588	-.851	-.316	-.220	-.166	-.265	-.607	-.097	-.724	-.355	-.120	-.010
Comb.of Processes	.529	.128	.086	-.767	.754	-.225	.147	.282	.087	.161	.720	.166	.321	.276

**.01 level of significance
*.05 level of significance

TABLE 20

THE COMPONENTS OF REDUNDANCY MEASURES FOR THE FIRST CANONICAL VECTOR OF THE PREDICTOR SET (STUDENT CHARACTERISTICS) FOR EACH OF THE SIX MATHEMATICS UNITS INDICATED IN THIS STUDY

Unit	Canonical R	R-Squared λ	Variance Extracted v_p	Redundancy $\lambda \cdot v_p$	Proportion of Total Redundancy
Numeration	.438**	.192	.125	.024	.585
Place Value	.486*	.236	.174	.042	.532
Addition	.689	.476	.135	.064	.609
Subtraction	.656**	.431	.116	.050	.561
Multiplication	.747*	.224	.132	.030	.612
Comb. of Processes	.529	.280	.101	.028	.341

**.01 level of significance
*.05 level of significance

TABLE 21

THE COMPONENTS OF REDUNDANCY MEASURES FOR THE FIRST CANONICAL VECTOR OF THE CRITERION SET (RATE MEASURES) FOR THE SIX MATHEMATICS UNITS INDICATED IN THIS STUDY

Unit	Canonical R	R-Squared λ	Variance Extracted v_c	Redundancy $\lambda \cdot v_c$	Proportion of Total Redundancy
Numeration	.438	.192	.230	.004	.494
Place Value	.486	.236	.409	.096	.701
Addition	.689	.476	.358	.170	.632
Subtraction	.656	.431	.416	.179	.765
Multiplication	.474	.224	.347	.078	.619
Comb. of Processes	.529	.280	.295	.083	.519

set by the first canonical vector of that set is associated with the first canonical vector in the criterion set. It will be noted that these proportions, the redundancy, are very small for all six units. However, the proportions in the last column, showing what proportion the redundancy for the first canonical vector is of the total redundancy for all canonical vectors of the predictor set, indicate that in five of the six units the first canonical variate accounts for over fifty percent of the total associated variance.

As an example, in interpreting Table 20 the first canonical R for the numeration unit is .438, which is significant at the .01 level, and λ = .192 which means that 19 percent of the variance in the first canonical variate of the predictor set is predictable from the first canonical variate of the criterion set. The proportion of variance of the predictor set extracted by the first canonical variate is .125. This value is obtained from the column sum of the squared loadings (squared structure R) of variables within the predictor set on the first canonical variate divided by the number of variables. Therefore, it indicates the variance extracted by the first canonical factor for the numeration unit. The redundancy of the first canonical variate of the predictor set for the numeration unit is .024. This value is obtained by multiplying the λ and the proportion of variance extracted by the first canonical factor (columns 2 and 3 of Table 20). Therefore, it [illegible]ates the amount of redundant variance attributed to the first canonic[illegible] variate of the predictor set. The last column of Table 20 indicates that, [illegible] the numeration unit, 59 percent of the total variance of the predictor s[illegible] predicted by the variance in the criterion set is associated with the [illegible] [illegible]nonical variate. This value is obtained by dividing the redunda[illegible] [illegible] the first canonical factor (column 4 of Table 20) by the sum of t[illegible] [illegible] variance of all the possible canonical factors--four in this case. The data presented in Tables 20 and 21 seem to indicate that the proportion of the redundancy associated with the first canonical factor is quite consistent throughout the six sample groups (six units). In the predictor set, (Table 20) the proportion of the total redundancy associated with the first canonical variate is very similar except for the combination of processes unit. Although the components of the six units for the criterion set are not as consistent as the predictor set (Table 21), the differences are not great. Therefore, in general, it can be concluded that there is a relationship between the two domains of measures being studied and that the percentages of associated variance related to the first canonical R are quite similar for the six sample groups included in this study.

The predictor variables selected for the research study are compared with the variables listed under the elements of Carroll's[1] conceptual model for the learning process. The result of this logical comparison is reported in Table 22.

Opportunity is listed as one of the conditions of school learning. Carroll defines this element as the amount of time the pupil is allowed to have for learning. Since time allowed for each student in learning the tasks under the IPI curriculum is whatever amount of time he needs to reach the particular instructional goal, the opportunity for learning is the same for every student in the sense that each student would have his own maximum opportunity. Therefore, this variable would not contribute to the prediction of pupil's learning performance, and it was not included as a variable in this study. Perserverance is defined by Carroll as the amount of time the pupil is willing to engage actively in learning

[1]Carroll, op. cit.

TABLE 22

COMPARISON BETWEEN THE ELEMENTS IN CARROLL'S MODEL FOR LEARNING AND THE PREDICTOR VARIABLES SELECTED FOR THIS STUDY

Elements of Carroll's Model	Predictor Variables of this Stu
1. Opportunity: The amount of time allowed for learning a task.	1. Not included as a predictor variable in this study.
2. Perseverance: The amount of time the learner is willing to devote to learning.	2. Attention scores.
3. Aptitude: The amount of time required by the learner to attain mastery of a task.	3. Reading and mathematics achie ment scores, the total numbe of skills mastered during th previous school year.
4. Ability to understand instruction: Some combination of general intelligence and verbal ability.	4. Verbal I.Q., Non-verbal I.Q. and mental age.
5. Quality of Instruction: The effectiveness of procedures employed to communicate knowledge to the learner.	5. Scores obtained from the questionnaire.

activities during the period that he is working to achieve the instruc- tional objective. For this study, the percent of time that the pupil actually spent in learning the task was determined by a sample of thre separate 10-minute observations of each child while he was working in each of the selected units of mathematics. The data of the present st seemed to contradict the general hypothesis of a positive relationship between this variable and the criterion measures proposed. The result of the multiple regression analysis and the canonical regression analy as shown in Tables 11 and 19 indicate that there is an inconsistency f unit to unit in the degree of association between this predictor and t rate measures. This would seem to indicate that the pupil attention variable may interact with the nature of the content and other aspects of the learning situation associated with a given unit in such a way as to involve a relatively complex relationship to rate.

The pupil's ability to understand instruction is defined by Carroll as involving some combination of general intelligence and verbal ability. For the present study, verbal and non-verbal I.Q. scores as well as the mental age of the pupil were used as predictors of pupil's learning rate and may be interpreted as representing indices of ability to understand instruction. These variables also did not make any consistent contribution to the rate measures. This may be explained by the different type of skills required by each unit of the mathematics curriculum. The complexity of the skills involved, the amount of abstract thinking required by the type of task to be learned as well as the materials and methods used in presenting the learning task to the pupil could have been a factor in causing the inconsistent results. However, the inconsistencies in the structure R's as shown in Tables 11 and 19 for the same unit when different rate measures are being predicted makes it inadvisable to generalize as to what the specific nature of these differences may be.

Quality of instruction is another element included in Carroll's model. This includes such variables as the effectiveness of the procedures employed to communicate knowledge by teachers, textbooks, workbooks, laboratory exercises, programmed instructional materials and so forth. For the present study, a questionnaire was constructed for this purpose. The answers provide some indication of pupils' perceptions of the quality of the instructional material and the nature of the learning task. Again, an inconsistent result was obtained. This may be explained by the different needs of the pupils. It may also be related to the relatively incomplete nature of the data provided by these questions in providing any type of fully valid measure of quality of instruction.

The element of aptitude is defined by Carroll as the amount of time the pupil requires to learn the specific task. In this study, measures of time required enter into both the predictor and criterion measures. That is, the number of units completed last year (one year being the "time required" to master that number of units) is one of the predictors while the four rate measures, which all indicate "time required," are elements of the criterion. Time required, as measured by number of units completed last year, was taken as an indication of aptitude for learning, together with such more conventional measures of aptitude as intelligence and past achievement in reading and mathematics. Again, these measures showed no consistent pattern in their relationship to rate.

As has been discussed above, the predictor measures showed no consistent correlation with rate of learning over various learning tasks --units of IPI mathematics. This lack of consistency seems to support Carroll's conclusion that rate of learning is specific to the task to be learned.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

This study has been concerned with the problem of measuring rate of learning in individualized instructional situations, and has investigated the relationship between rate and a number of other variables. Some indication of the complex nature of measures of learning rate in school classrooms was suggested by Yeager,[1] and this study has endeavored to increase understanding of the concept of rate of learning by exploring the relationships suggested by his work. The study may also be viewed as an attempt to investigate the variables influencing learning suggested by Carroll[2] and Bloom.[3]

As measures of rate of learning, this study has employed four different measures: (1) $\text{Rate}_1 = \frac{\text{100 - pretest score}}{\text{days spent on the unit}}$, (2) $\text{Rate}_2 = \frac{\text{number of pages worked}}{\text{days spent on the unit}}$, (3) $\text{Rate}_3 = \frac{\text{number of skills learned}}{\text{days spent on the unit}}$, and (4) Rate_4 = total number of skills the pupil has worked between September 1967 and January 1968. These rate measures have been investigated singly and in combination.

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[1]John L. Yeager, "Measures of Learning Rates for Elementary School Students in Mathematics and Reading Under a Program of Individually Prescribed Instruction" (unpublished doctoral dissertation, University of Pittsburgh, 1966).

[2]John B. Carroll, "A Model of School Learning," Teachers College Record, 64:723-732, 1963.

[3]B. S. Bloom, "Mastery Learning for all" (invited address presented at the American Educational Research Association Annual Meeting, February 7-10, 1968).

As variables that should have an influence on rate, or that should be related to rate, it has investigated certain measures of the pupil's scholastic aptitude, academic achievement, and certain classroom performance characteristics that past work had shown to be related to learning. Nine measures were used in this study. They are: (1) verbal I.Q., (2) non-verbal I.Q., (3) mental age, (4) mathematics achievement score, (5) reading achievement score, (6) total number of skills worked during the previous school year, (7) attention score, (8) $question_1$, student's reaction to the difficulty level of the work, and (9) $question_2$, student's reaction to the type of learning material used.

Summary

The results of this study can be summarized in terms of four major types of analysis.

1. As a first step in investigating the relationship between rate of learning and the selected predictors, Pearson product-moment correlations were computed between each individual predictor and each individual rate measure for six E-level units of the IPI mathematics curriculum. Few of these correlation coefficients were statistically significant and even these were quite small in absolute value, the largest correlation coefficient being .412. Furthermore, the size of these coefficients was not at all consistent from unit to unit. This indicated that any single predictor would be of little use in predicting any one of the rate measures.

2. On the assumption that one reason for the lack of correlation between the rate measures and the predictors was the limitation in lack of comprehensiveness in any single rate measure, the next logical step in investigating this relationship was to use a combination of the four rate

measures in a multiple correlation analysis to determine the relationship between the rate measures and each of the predictors. The multiple correlation coefficient between each predictor and linear function of the four rate measures was determined for each of the selected E-level mathematics units. Although such multiple R's were, rather naturally, larger than the correlation coefficients between a single rate measure and any given predictor, few of them were significant. This indicated that lack of comprehensiveness in the rate measures was not a major cause of the poor predictability of rate.

3. Since the foregoing analysis showed that any single predictor did not have a substantial relationship with rate of learning, a study was made of the effectiveness of using a composite predictor. This was carried out by determining the multiple correlation and the multiple regression equation for each rate measure and the nine predictors for each E-level mathematics unit selected for this study. The multiple correlation and regression analysis showed that in several instances prediction could be improved substantially through this procedure.

However, the partial regression coefficients produced by the multiple regression analysis showed an apparent inconsistent pattern in the relative contribution of the predictors. The contributions made by the predictors to rate of learning differ from unit to unit, suggesting that the relative importance of the various predictors is a function of the specific unit or topic being studied.

4. Since the results of the foregoing multiple correlation and multiple regression analyses indicated that the relationship between the two sets of variables being studied was inconsistent over different rate measures, this suggested the desirability of studying this relationship

with a composite of the predictors and a composite of rate measures. This was investigated through a canonical correlation analysis. This analysis yielded more consistent results in terms of the relationship between the two domains as indicated by the first canonical R. These first canonical correlation coefficients for the six E-level mathematics units were, .438, .486, .689, .656, .473, and .529. These results suggest that there is some type of substantial relationship between the rate domain and the domain of variables represented by the nine predictors.

Looking at the canonical weights and the structure R's, the data suggest that the relative contribution made by the predictors to the canonical correlation is not consistent from unit to unit, even though the relationship between the composite of the rate measures and the composite of the predictors seemed to be substantial for all units. This finding tends to substantiate the conclusion suggested by the results of the multiple correlation analysis that the relative importance of the predictor variables is a function of the specific unit or topic being studied.

Conclusions and Recommendations for Further Study

The analysis of the data of this study have yielded no clear cut answers concerning the most satisfactory measures of rate of learning in Individually Prescribed Instruction classrooms or concerning the nature of the relationship between rate and other student characteristics. As a result, its major contribution may be as a warning against making the too easy assumption that such simple measures as number of units or number of skills completed per unit of time are a valid measure of the learning rate of a pupil. Evidence presented here also indicates that currently there is no valid way (such as by using measures of past rate, of past achievement, of academic aptitude, of attention in the classroom, etc.) of predicting how quickly a student will work through a unit of instruction in IPI. Much more work needs to be done before such predictions will be possible or before it will be feasible to group students in terms of how fast they can be expected to move through a unit. The most promising prediction variable revealed by the data of this study seemed to be previous rate as measured by number of skills mastered last year. Although the predictive value of this variable in the present study was not large, its general contribution was such as to indicate that it may be the most promising variable in further investigations of rate.

In the writer's judgment it is unlikely that the kind of prediction of rate attempted here can be made more effective by adding additional measures of student characteristics. The predictors used in this study appear to be quite comprehensive in covering those characteristics which past research and current theory suggest should be the determiners of rate. The exception to this would be background data gained from a developmental history of the child. These kinds of data have not been investigated directly in this study.

It is recognized that the child's family and social environment as well as his childhood experiences will influence his physical and emotional development. Therefore, research studies which examine the family and social factors in the learner's developmental history may reveal some information about the characteristics of the learner that are not accounted for by the predictors included in this study. Consequently, some additional information which could be helpful in the prediction of a pupil's rate of learning might be gained by investigating these earlier developmental stages of the child. For instance, a study of the dependency relationship between the pupil and his parents might contribute to our understanding of the relationship between rate and the emotional and social development of the child, since his early interaction with his parents contributes to the self-confidence and self-concept he now has. Since IPI is an instructional program that requires a great deal of independent work in the part of the pupil, his dependency relationship with his parents might have some influence on how fast he learns under the IPI program. Of course, the study of the predictive value of such data would also have to include the investigation of the extent to which they reveal themselves in measures already used in this study such as intelligence, achievement, and attention.

The writer also doubts the fruitfulness of further extended work in the attempt to develop a more refined measure of rate of learning. It would seem that the four measures used in this study either singly or in some combination, should provide a valid and reliable measure of rate. If these measures are inconsistent over different units in terms of the relative rate of a student, which may be one inference to be made from the data of this study, it would seem that reasons should be sought other than a lack of reliability in the measures themselves.

With respect to this problem, it might be worthwhile to explore the influence of any variablity in level of mastery achieved while working at a given rate. That is, although the IPI procedure requires that a pupil achieve "mastery" of a given objective before moving on to another, this criterion of mastery involves any score between 85 and 100 percent. Although this provides for only minor variability in level of achievement, it may be enlightening to take this measure into account by using a criterion measure which associates the pupil's rate with his achievement.

Because of the involved nature of the relationships among rate measures and between these rate measures and various predictors, it should also be interesting to study the relationships of these variables to some relatively pure measure of rate of pupil learning. For example, one might obtain a measure of time required for mastery of simple verbal learning tasks for a sample of IPI students. A study of the relationship of this variable, both to IPI rate measures and to the predictors used here could be enlightening with respect to our understanding of the nature of both sets of variables.

The fluctuation of the relative importance of the predictors from unit to unit suggests that Carroll[1] may be correct in stating that learning rate is not a general characteristic of a student but is specific to the learning task. Therefore, before any consistent relationship between the rate measures and the predictors can be established, a careful study of the characteristic nature of the task to be learned should be carried out. A logical extension of this study would be to further examine the nature of the learning objectives and the types of skills to be learned within each of the units of the IPI curriculum, so that relationship between pupil's rate of learning and the type of tasks to be learned can be specified. In this connection it might be useful to examine the relationship among several successive units on the same topic (such as B Addition, C Addition, D Addition, and E Addition) in terms of any consistencies in values of given predictors. This suggestion is based on the assumption that such units, dealing with the same basic topic, would be similar in types of skills involved.

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[1]Carroll, op. cit.

It also recognized that within the specific research situation of this study, the extent to which various student variables are related to rate of learning may be influenced by the effectiveness of the instructional system. It is possible that one reason for a lack of higher correlation in the present study is the fact that the IPI system is not operating in a manner which permits each student to progress at a rate in keeping with his real aptitude. This is a problem of being able to provide for those conditions under which maximal learning can take place.

With respect to this point of view, Carroll has suggested in his model that the effectiveness of instruction is a determiner of rate. In other words, the pupil's learning rate will vary depending upon the nature and the quality of instruction provided for him. Further refinement of the system leading to optimal conditions for learning for each pupil under the IPI program might provide a more controlled setting for investigating the nature of the relationship between the selected predictor variables and pupil's rate of learning in school situations.

In this respect, the results of this study also may support the logic behind this additional factor in Carroll's conceptual learning model for the learning process. Further study of his model in actual classroom learning situations may have to await the development of instructional programs that are known to be effective and efficient or at least the development of a procedure for assessing the effectiveness of such programs.

APPENDIX

APPENDIX: MEANS AND STANDARD DEVIATIONS FOR THE PREDICTOR AND CRITERION VARIABLES

Variable	Unit											
	Numeration		Place Value		Addition		Subtraction		Multiplication		Combination of Processes	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
Verbal I.Q.	109	13.6	110	13.6	109	13.3	110	12.3	118	16.3	110	13.1
Non-Verb I.Q.	114	13.8	113	14.4	114	15.7	114	13.4	116	12.2	116	12.2
M.A.	11.6	1.4	11.3	1.3	11.3	1.3	11.6	1.2	11.9	1.0	13.16	8.1
Math Achiev.	5.1	1.1	4.8	.85	4.9	.79	5.1	.88	5.3	.85	5.6	1.1
Read. Achiev.	5.4	1.1	5.2	1.0	5.2	.94	5.4	.94	5.7	.95	5.9	1.3
No. of Skills	36	10.3	35	9.3	34	9.7	36	9.7	34	9.2	38	9.3
Attention	86	.14	37	.14	86	.14	86	.14	86	.16	88	.12
$Question_1$	3.7	.81	3.9	.77	3.9	.67	3.6	.89	3.67	.82	3.6	.79
$Question_2$	3.6	.89	3.7	.82	3.7	.90	3.4	.99	3.58	.94	3.5	.91
$Rate_1$ $\frac{pretest}{days}$	7.01	9.8	17.32	15.4	11.77	8.1	10.58	8.8	6.60	9.1	10.25	7.2
$Rate_2$ $\frac{pages}{days}$	3.96	2.0	4.66	2.6	3.29	2.2	3.49	2.0	5.68	13.1	6.29	10.8
$Rate_3$ $\frac{skills}{days}$	.99	4.85	.906	1.0	.769	.99	.508	.374	.495	.45	.69	.85
$Rate_4$ skills	23	6.9	23	6.4	22	6.2	22	7.5	22	6.7	24	7.5

Actual range of N's as used in missing data program

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