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DIVISIBILITY TESTS.

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THIS BOOKLET, ONE OF A SERIES, HAS BEEN DEVELOPED FOR THE PROJECT, A PROGRAM FOR MATHEMATICALLY UNDERDEVELOPED PUPILS. A PROJECT TEAM, INCLUDING INSERVICE TEACHERS, IS BEING USED TO WRITE AND DEVELOP THE MATERIALS FOR THIS PROGRAM. THE MATERIALS DEVELOPED IN THIS BOOKLET INCLUDE SUCH CONCEPTS AS (1) DIVISIBILITY TESTS, (2) CHECKING THE FUNDAMENTAL OPERATIONS BY CASTING OUT NINES AND ELEVENS, AND (3) APPLICATION OF DIVISIBILITY. ACCOMPANYING THESE BOOKLETS WILL BE A "TEACHING STRATEGY BOOKLET" WHICH WILL INCLUDE A DESCRIPTION OF TEACHER TECHNIQUES, METHODS, SUGGESTED SEQUENCES, ACADEMIC GAMES, AND SUGGESTED VISUAL MATERIALS. (RP)

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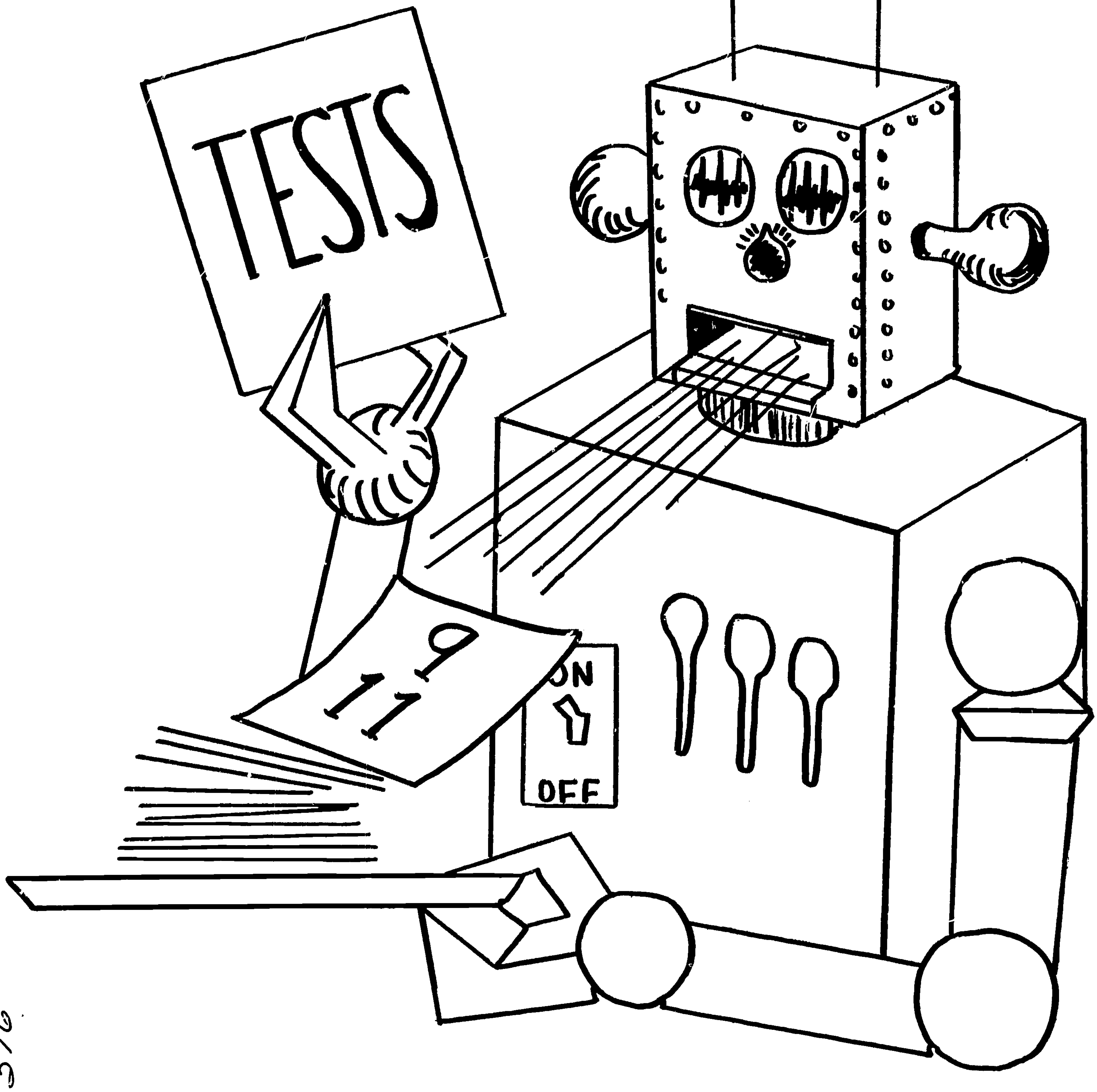
DIVISIBILITY



SE 004 516

ESEA Title III

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DIVISIBILITY TESTS

Table of Contents

DIVISIBILITY TESTS

There are divisibility tests for some numbers. These tests can be used to eliminate long trial divisions when you are looking for a number that will divide evenly (zero remainder) into another number. Some numbers greater than one can be divided evenly only by themselves and by one. This type of number can be found more easily using divisibility tests.

<u>TEST FOR 2</u> - If the last digit of a number is divisible by 2, then the number is divisible by 2.

Example: Is 2 a divisor of 234? Yes - since 2 is a divisor of 4.

Example: Is 2 a divisor of 157? No - since 2 is not a divisor of 7.

Look above and read the <u>Test For 2</u> again. Then draw a circle (◯) around each number that is divisible by 2.

12	75	326	1007	499
21	891	488	2016	10,000
43	654	515	8790	21,152

<u>TEST FOR 3</u> - If the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

Example: Is 3 a divisor of 267? Sum of the digits of 267 is 2 + 6 + 7 = 15. 1 + 5 = 6. Yes - since 3 is a divisor of 15 (also 6).

Example: Is 3 a divisor of 157? Sum of the digits of 157 is 1 + 5 + 7 = 13. Also 1 + 3 = 4. No - since 3 is not a divisor of 13 (or of 4).

Note: You can continue adding the digits until you arrive at a single digit number - as in the case of getting 13 and then 4. Any of the sums can be used for the test.

Draw a rectangle ($\boxed{}$) around the numbers that are divisible by 3.

999	5100	1515	1935	2444
399	4350	1908	6969	9493
464	89	666	24	3624

TEST FOR 4 - If the last two digits form a number that is divisible by 4, then the entire number is divisible by 4.

Example: Is 4 a divisor of 624? Yes - since 4 is a divisor of 24.

Example: Is 4 a divisor of 157? No - since 4 is not a divisor of 57.

Draw two circles (◎) around the numbers that are divisible by 4.

3623	1228	7172	1137	6648
1111	0008	2222	585	1964
8	144	1221	732	5824

<u>TEST</u> <u>FOR</u> <u>5</u> - If the last digit of a number is 0 or 5, then the number is divisible by 5.

Example: Is 5 a divisor of 225? Yes - since 225 ends in a 5.

Example: Is 5 a divisor of 157? No - since 157 does not end in 0 or 5.

Make two checks (✓✓) next to the numbers that are divisible by 5.

254	4807	362636	13	954
500	950	9055	225	1894
6890	4005	121212	705	2141

<u>TEST</u> <u>FOR</u> <u>6</u> - If a number is divisible by 2 and 3, then it is divisible by 6.

Example: Is 2232 divisible by 6? Divisible by 2? Yes - since 2 is a divisor of 2. Divisible by 3? Sum of the digits of 2232 is 2 + 2 + 3 + 2 = 9. Yes - the number is divisible by 6 since it is divisible by 2 and 3.

Draw a circle and a rectangle ($\boxed{\bigcirc}$) around the numbers that are divisible by 6.

2436	39	21	1022	585
36	5	72	2159	8640
4928	42612	36240	7172	642

<u>TEST</u> <u>FOR</u> <u>7</u> - Note: Tests exist for divisibility by 7, but it is usually easier and faster to actually divide.

<u>TEST</u> <u>FOR</u> <u>8</u> - If the last 3 digits form a number that is divisible by 8, then the number is divisible by 8.

Example: Is 8 a divisor of 3120? Yes - since 8 is a divisor of 120.

Example: Is 8 a divisor of 157? 157/8 = 19 r. 5. No - since 8 will leave a remainder of 5 and not 0.

Note: The <u>Test</u> <u>For</u> <u>8</u> is not useful for numbers with less than 4 digits. Actually you have to divide by 8 if the number is less than 4 digits.

Draw three circles () around the numbers that are divisible by 8.

1660	3168	6864	2456	80840
99888	6169	3072	7824	6424
3320	4645	5840	98040	36016

<u>TEST</u> <u>FOR</u> <u>9</u> - If the sum of the digits of a number is divisible by 9, then the number is divisible by 9.

Example: Is 9 a divisor of 693? Sum of the digits of 693 is 6 + 9 + 3 = 18. Also 8 + 1 = 9. Yes - since 9 is a divisor of 18 (also 9 is a divisor of 9).

Example: Is 9 a divisor of 157? Sum of the digits of 157 is: 1 + 5 + 7 = 13. Also 1 + 3 = 4. No - since 9 is not a divisor of 13 (or a divisor of 4). Actually we should already know before this test that 9 will not divide 157 evenly. Why? (What test have we made that would give us this information?)

Draw two rectangles ($\boxed{\boxed{}}$) around the numbers that are divisible by 9.

639	108	109	843	9991
900	36	105	810	6126
333	29	3456	15363	261

<u>TEST FOR 10</u> - If the last digit of a number is 0, then the number is divisible by 10.

Example: Is 10 a divisor of 650? Yes - since 650 ends in 0.

Example: Is 10 a divisor of 157? No - since 157 does not end in 0.

Check (✓) the numbers that are divisible by 10.

760	112	900	378
87460	784670	109876	328
225	1003	4550	16595

TEST FOR 11 - The divisibility test for 11 is easy to do but not easy to explain. It will be given in steps.

Step I - Starting with the ones position, place an x over every other digit in the number.

Step II - Add up all the digits with an x above them; then add up all the digits without an x above them.

Step III - Find the difference between the two sums you found in Step II. If this difference is divisible by 11, then the number is divisible by 11 (remember that 11 is a divisor of 0).

Example: Is 11 a divisor of 6391? Step I: $\overset{}{6}\overset{x}{3}\overset{}{9}\overset{x}{1}$ Step II: 3 + 1 = 4 and 6 + 9 = 15 Step III: 15 - 4 = 11. Yes - since 11 is a divisor of 11.

Example: Is 11 a divisor of 157? Step I: $\overset{x}{1}\overset{}{5}\overset{x}{7}$ Step II: $\overset{x}{7}$ + $\overset{x}{1}$ = 8 Step III: 8 - 5 = 3. No - since 11 is not a divisor of 3.

Draw an X beside the numbers that are divisible by 11.

2077	121	1320	57463	1234
14161	4455	9999	8930	45556
3249	3643	87641	7664	19836

Now we have checked up to 11 and have not yet found a divisor of 157. What is the largest number we will check in our search for a divisor? If we check 13 and it is not a divisor, could we say without going on that 157 can be divided evenly only by itself and by one? The answer is yes. Can you figure out why? What is the product of 13 x 13? ______________

ACTIVITIES

1. Use the given symbol to show each number in the corresponding set (use divisibility test):

   A. Divisible by 2 (○) - { 25, 77, 86, 149}

   B. Divisible by 3 (▭) - { 144, 151, 231, 1239 }

   C. Divisible by 4 (◎) - { 394, 7076, 160, 258 }

   D. Divisible by 5 (✓✓) - { 5785, 6070, 8326, 579 }

   E. Divisible by 6 ($\boxed{\bigcirc}$) - { 146, 294, 1266, 387 }

   F. Divisible by 8 (◎) - { 5264, 7158, 3994, 60112 }

   G. Divisible by 9 ($\boxed{\boxed{}}$) - { 63864, 147, 153, 693 }

   H. Divisible by 11 (X) - { 121671, 121, 243, 6765 }

2. If a 3 digit number is repeated to form a 6 digit number, the 6 digit number is divisible by 11. Test this idea and see if you can find out why.

   Example: 123 repeat: $1\overset{x}{2}3\ \overset{x}{1}2\overset{x}{3}$; $\overset{x}{2} + \overset{x}{1} + \overset{x}{3} = 6$ and $1 + 3 + 2 = 6$;
   $6 - 6 = 0$.
   Therefore 11 is a divisor of 123123.

3. Is 4566 divisible by 6? Show why, using the <u>Test For 6</u>.

4. Is 95,832 divisible by 4? Show why, using the <u>Test For 4</u>.

5. Is 142,142 divisible by 11? Show why, using the <u>Test For 11</u>.

6. Show whether the numbers in the left column are divisible by the numbers across the top. Put an X in the table to show divisibility.

	2	3	4	5	6	8	9	10	11
24									
729									
644									
201									
1560									
81									
162									
255									
1111									
3248									

APPLICATION OF DIVISION

CASTING OUT NINES

Look back at the divisibility test for nine. "Casting out nines" is an application of the nine divisibility test. The idea of "casting out nines" provides some sort of check for addition, subtraction, multiplication, and division. This check is not "fool-proof", as an answer can check and still be wrong. However, if an answer does not check, it is definitely wrong.

In "casting out nines" we make use of our nine divisibility test. For example, if the 9's were "cast out" of 23, we would have a remainder by simply adding the digits: $2 + 3 = 5$.

Addition

Remainders

Example 1. $623 \longrightarrow 6 + 2 + 3 = 11 \longrightarrow 1 + 1 = 2 \longrightarrow 2$

$+\ 964 \longrightarrow 9 + 6 + 4 = 19 \longrightarrow 9 + 1 = 10 \longrightarrow 1 + 0 = 1 \rightarrow +1$

$1587 \longrightarrow 1 + 5 + 8 + 7 = 21 \rightarrow 2 + 1 = 3 \longleftarrow \boxed{\text{check}} \longrightarrow 3$

If 623 is divided by 9, the remainder is 2.

If 964 is divided by 9, the remainder is 1.

Then the sum of 623 and 964 must have a remainder of 3 when all 9's are divided out or "cast out."

Remainders

Example 2. $361 \longrightarrow 3 + 6 + 1 = 10 \longrightarrow 1 + 0 = 1 \longrightarrow 1$

$822 \longrightarrow 8 + 2 + 2 = 12 \longrightarrow 1 + 2 = 3 \longrightarrow 3$

$+\ 541 \longrightarrow 5 + 4 + 1 = 10 \longrightarrow 1 + 0 = 1 \longrightarrow +\ 1$

$1724 \longrightarrow 1 + 7 + 2 + 4 = 14 \longrightarrow 1 + 4 = 5 \longleftarrow \boxed{\text{check}} \longrightarrow 5$

Work the following problems and check by casting out nines.

$$\begin{array}{r} 569 \\ +\ 965 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 8953 \\ +\ 6487 \\ \hline \end{array}$$

$$\begin{array}{r} 4672 \\ 8341 \\ +\ 2133 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 62 \\ 478 \\ +\ 949 \\ \hline \end{array}$$

Subtraction

					Remainders
Example 1.	624 --->	6 + 2 + 4 = 12 --->	1 + 2 = 3	--------------->	3
	− 163 --->	1 + 6 + 3 = 10 --->	1 + 0 = 1	--------------->	−1
	461 --->	4 + 6 + 1 = 11 --->	1 + 1 = 2	← check →	2

If the remainder of the minuend is less than the remainder of the subtrahend, then add 9 to the remainder of the minuend before subtracting.

Remainders

Example 2. 622 ⟶ 6 + 2 + 2 = 10 ⟶ 1 + 0 = 1 ⟶ 1 + 9 = 10 ⟶ 10

- 161 ⟶ 1 + 6 + 1 = 8 ⟶ - 8

461 ⟶ 4 + 6 + 1 = 11 ⟶ 1 + 1 = 2 ⟵ $\boxed{\text{check}}$ ⟶ 2

Work the following problems and check by casting out nines.

$$\begin{array}{r} 264 \\ -\ 178 \\ \hline \end{array} \qquad \begin{array}{r} 3782 \\ -\ 2891 \\ \hline \end{array}$$

$$\begin{array}{r} 965 \\ -\ 569 \\ \hline \end{array}$$

$$\begin{array}{r} 4821 \\ -\ 2362 \\ \hline \end{array} \qquad \begin{array}{r} 632 \\ -\ 489 \\ \hline \end{array}$$

Multiplication

Remainders

Example 1.

$$62 \rightarrow 6 + 2 = 8 \longrightarrow 8$$

$$\times 68 \rightarrow 6 + 8 = 14 \rightarrow 1 + 4 = 5 \longrightarrow \times 5$$

$$4216 \rightarrow 4 + 2 + 1 + 6 \rightarrow 13 \rightarrow 1 + 3 = 4 \quad \text{check} \quad 40 = 4 + 0 = 4$$

Example 2.

$$96 \rightarrow 9 + 6 = 15 \rightarrow 1 + 5 = 6 \longrightarrow 6$$

$$\times 94 \rightarrow 9 + 4 = 13 \rightarrow 1 + 3 = 4 \longrightarrow \times 4$$

$$9024 \rightarrow 9 + 0 + 2 + 4 = 15 \rightarrow 1 + 5 = 6 \quad \text{check} \quad 24 = 2 + 4 = 6$$

Work the following problems and check by casting out nines.

$$\begin{array}{r} 79 \\ \underline{\times\ 72} \end{array} \qquad \begin{array}{r} 5431 \\ \underline{\times\ 60} \end{array}$$

$$\begin{array}{r} 842 \\ \underline{\times\ 267} \end{array}$$

$$\begin{array}{r} 680 \\ \underline{\times\ 56} \end{array} \qquad \begin{array}{r} 4973 \\ \underline{\times\ 2046} \end{array}$$

Division

After completing a division problem, set it up in multiplication form for a regular check. Then use "casting out nines" in this form.

Example 1.

```
          297
    23 | 6831
         46
         223
         207
          161
          161
```

```
                                                     Remainders
297   -->  2 + 9 + 7 = 18  -->  1 + 8 = 9  -------->   9
x 23  -->  2 + 3 = 5  -------------------------------> x 5
----                                                  ---
6831 -> 6 + 8 + 3 + 1 = 18 --> 1 + 8 = 9    check    45 = 4 + 5 = 9
```

In the example above, the remainder was 0. If the remainder is <u>not</u> 0, it must be accounted for in this way.

Example 2.

```
          3260
    15 | 48912
         45
          39
          30
           91
           90
           12
```

```
                                                          Remainders
3260  -->  3 + 2 + 6 + 0 = 11  -->  1 + 1 = 2  -->          2
x 15  -->  1 + 5 = 6  -->  1 + 5 = 6  -->                  x 6
----                                                       ---
48900                                                       12

+ 12  -->  1 + 2 = 3  ----------------------------------->  + 3
----                                                       ---
48912 -> 4 + 8 + 9 + 1 + 2 = 24 --> 2 + 4 = 6   check   15 = 1 + 5 = 6
```

Work the following problems and check by "casting out nines."

$23\overline{)6754}$

$52\overline{)8996}$

$29\overline{)178}$

$42\overline{)871}$

CASTING OUT ELEVENS

The idea of "casting out elevens" is very similar to "casting out nines." The major difference is the way we get the remainders. After we get our remainder, the process is exactly the same.

Use the divisibility test for 11 to get the remainder. As you recall, place an x over every other digit, beginning at the ones position. Sum up the digits with an x above them and sum up the digits without an x above them. Now we must subtract the sum of the digits <u>without</u> x's above them <u>from</u> the sum of the digits <u>with</u> x's above them. This difference is the remainder when the number is divided by 11.

Example 1:

$$
\begin{array}{r@{\,}r}
 & 79 \\
11 & \overline{)876} \\
 & \underline{77} \\
 & 106 \\
 & \underline{99} \\
 & 7
\end{array}
= \text{remainder}
$$

$$\text{Or}\quad \overset{x\ \ x}{876} \ \text{—}\ (8 + 6) - 7 = 14 - 7 = 7 \ \text{remainder}$$

Example 2:

$$
\begin{array}{r@{\,}r}
 & 96 \\
11 & \overline{)1062} \\
 & \underline{99} \\
 & 72 \\
 & \underline{66} \\
 & 6
\end{array}
= \text{remainder}
$$

$$\text{Or}\quad \overset{x\ \ x}{1062} \ \text{—}\ (0 + 2) - (1 + 6) = \overset{13}{\cancel{2}} - 7 = 6 \ \text{remainder}$$

add an eleven when you can't subtract

Addition

Remainders

Example 1. $\overset{x\,x}{642} \longrightarrow (6 + 2) - 4 \longrightarrow 8 - 4 = 4$

$\overset{x\,x}{+\ 890} \longrightarrow (8 + 0) - 9 \longrightarrow 19 - 9 = +\ 10$

$14 \longrightarrow 4 - 1 = 3$

$(5 + 2) - (1 + 3) \longrightarrow 7 - 4 = 3 \leftarrow$ check

Shorten the process by adding mentally.

Remainders

Example 2. $\overset{x\,x}{8603} \longrightarrow 9 - 8 = 1$

$\overset{x\,x}{+\ 3915} \longrightarrow 14 - 4 = +\ 10$

$12{,}518$

$11 \longrightarrow 1 - 1 = 0$

$14 - 3 = 11$

and $1 - 1 = 0 \leftarrow$ check

Work the following problems and check by casting out elevens.

721	4514
+ 631	+ 3121

327	567
+ 683	+ 892

Subtraction

Remainders

Example 1.

```
 x x                    x   x
 621  ─────────────→   (6 + 1) - 2 = 7 - 2 = 5
 x x                    x   x
- 137 ─────────────→   (1 + 7) - 3 = 8 - 3 = 5
 484                                         0

(4 + 4) - 8 = 8 - 8 = 0 ←── check ──→ 0
```

Example 2.

```
  x x
 9543  ──────→  8 - 13 ──→ 19 - 13 = 6 ──→  6
  x x
- 5329 ──────→  12 - 7 = 5 ─────────────→ - 5
 4214                                       1

6 - 5 = 1 ←──────── check ──→ 1
```

Work the following subtraction problems, and check by casting out elevens.

```
  934            723
- 625          - 327


  834            962
- 567          - 479
```

Multiplication

Example 1.

				Remainders
x 54	→	4 - 6 →	15 - 5 = 10 →	10
x x 56 3024	→	6 - 5 = 1 →		x 1 10

4 - 5 → 15 - 5 = 10 ← check

Example 2.

				Remainders
x x 252	→	4 - 5 →	15 - 5 = 10 →	10
x x x 314 1008 252 756 79,128	→	7 - 1 = 6 →		x 6 60 — 0 - 6 (add 11 to 0) 11 - 6 = 5

16 - 11 = 5 ← check

Work the following problems and check by casting out elevens.

956
x 74

327
x 723

432
x 265

382
x 68

Division

Use the same form as was used for "casting out nines." (The same problems will be illustrated as were for nines.)

Example 1.

```
       297
23 |6831
    46
    223
    207
     161
     161
```

```
                                           Remainder
x x
297  ————→  9 - 9 = 0  ————→  0
  x          x
x 23 ————→  3 - 2 = 1  ————→  x 1
6831                              0
6831 ————→  9 - 9 = 0 ← check
```

As you have learned about <u>nines</u>, the remainder must be accounted for if it is <u>not</u> <u>zero</u>.

Example 2.

```
       3260
15 |48912
    45
     39
     30
      91
      90
      12
```

```
                                                        Remainder
 x x
 3260   ————→  2 - 9  ————→  13 - 9 = 4  ————→  4
    x
 x  15  ————→  5 - 1 = 4  ————————————→  x 4
 48,900                                    16
                                          +12
                                           28  ————→  8 - 2 = 6
+    12
 x  x x
 48,912 ————→  15 - 9 = 6  ←———— check
```

Work the following problems and check by "casting out elevens."

$27\overline{)72549}$

$35\overline{)97615}$

$65\overline{)478530}$

$93\overline{)6278}$

ACTIVITIES

1. A "Magic Square" is a square array of numbers in which the sum of each row, column, and diagonal is the same. Two of the three arrays below are "magic squares." Check your addition by "casting out nines" or "casting out elevens."

123	53	103
73	93	113
83	133	63

59	39	89
109	79	99
69	119	49

81	11	61
31	51	71
41	91	21

2. Subtraction and addition must be done to make each of these arrays a "magic square." Check your subtraction by "casting out nines" or "elevens."

14		12
9	11	
10		8

	18	20
17	22	15

3. Select any three digit number in which the digit in the hundreds' position is greater than the digit in the ones' position (example: 531, as 5 is greater than 1). Use this number as a minuend. Obtain the subtrahend by interchanging the digits in ones' position and hundreds' position. Subtract and check your answer by "casting out nines." Check each of these; then do four more examples, and compare answers.

Examples:

521	681	891
- 125	- 186	- 198

Is nine always in the tens' position of your answer?

In your answers, does the digit in the ones' position and the digit in the hundreds' position always add up to 9?

Your Four Examples:

4. Make up three division problems which have a **remainder** **<u>other than zero</u>**. **Check your answer by "casting out nines."**

5. If a number is divided by 10, the remainder will be the digit in the <u>ones' position</u>. Use a check for <u>addition</u> and <u>multiplication</u> by "casting out tens."

(remainder when divided by 10)

Example:

$$
\begin{array}{lrcr}
 & 48 & \longrightarrow & 8 \\
 & \underline{+\ 53} & \longrightarrow & \underline{+\ 3} \\
\text{Sums} & 101 & & 11
\end{array}
$$

Remainders (1) $\xleftarrow{}$ $\boxed{\text{check}}$ $\xrightarrow{}$ (1) (remainder after each sum is divided by 10)

Example:

$$\begin{array}{lrcr} & 98 & \longrightarrow & 8 \\ & \underline{x\ 92} & \longrightarrow & \underline{x\ 2} \\ \text{Product} & 9016 & \longrightarrow & 16 \\ \text{Remainder} & 6 & \boxed{\text{check}} & 6 \end{array}$$

(Remainder after each product is divided by 10.)

Now check these problems by "casting out tens."

A.
$$\begin{array}{r} 369 \\ \underline{+\ 124} \\ 493 \end{array}$$

B.
$$\begin{array}{r} 682 \\ \underline{+\ 961} \\ 1743 \end{array}$$

C.
$$\begin{array}{r} 86 \\ \underline{x\ 84} \\ 7224 \end{array}$$

D.
$$\begin{array}{r} 132 \\ \underline{x\ 22} \\ 2904 \end{array}$$

6. Make up an addition problem and check it by "casting out eights." Since you could use many different numbers to "cast out," why do you think that "casting out nines" is the most popular? Is it because it is easy to find the remainder when a number is divided by 9?

Example:

$$\begin{array}{rcrcr} 537 & \longrightarrow & 67\ \text{r. } 1 & \longrightarrow & 1 \\ \underline{+\ 928} & \longrightarrow & 116\ \text{r. } 0 & \longrightarrow & \underline{+\ 0} \\ 1465 & \longrightarrow & 58\ \text{r. } 1 & \longrightarrow 1 \quad \boxed{\text{check}} & 1 \end{array}$$

7. Select any number having <u>two or more digits</u>. Sum the digits and subtract this sum from the number you started with. Is your result divisible by 9? Try several examples other than the ones shown below.

Number	Sum of the Digits	Difference	9 "is a divisor of"
38	$3 + 8 = 11$	$38 - 11 = 27$	27
328	$3 + 2 + 8 = 13$	$328 - 13 = 315$	315
6218	$6 + 2 + 1 + 8 = 17$	$6218 - 17 = 6201$	6201
____	____________	____________	____
____	____________	____________	____
____	____________	____________	____
____	____________	____________	____
____	____________	____________	____
____	____________	____________	____

APPLICATION OF DIVISIBILITY

Can you name many objects that are produced by machines? How about cars, shoes, television sets, books, and planes? Machines are used to produce toys. Now let's test your imagination and also see if you can apply some of the ideas presented about divisibility.

Suppose you worked for a company that produces toys. They have three machines that produce toy robots. A machine stamps a number on each robot it produces. You are to examine some robots that are produced by these machines and make sure they are working. If a robot does not work as it should, you are to stop the machine that produced it. We shall label the machines I, II, and III.

Below is the order the machines follow to stamp their numbers. As you can see, each machine stamps a number, skips two numbers, and stamps the next number. This process continues on.

Machine	Robot Numbers
I	1, 4, 7, 10, 13, 16, . . .
II	2, 5, 8, 11, 14, 17, . . .
III	3, 6, 9, 12, 15, 18, . . .

Are you ready to examine some robots? The first robot you examine that will not work is Number 47. Which machine produced this robot? Can you figure it out without counting? It was Machine II. Here are some more numbers that Machine II stamped: 74, 56, 65, 128, 281, 92. Since there are three machines, divide the robot number by 3. The remainder for each number (stamped by Machine II) is 2. Use the space given on the next page to try doing this yourself with the numbers given above.

Here are the numbers again that Machine II used: 74, 56, 65, 128, 281, 92. Divide each number by 3 (there are 3 different machines). What is your remainder each time?

Divide 3 into some robot numbers that Machine I produced. The remainder for each number is 1. Turn back one page to look again at the order Machine I uses to stamp its numbers. Try dividing 3 into these numbers: 1, 4, 7, 10, 13, 16.

When you divide 3 into a number produced by Machine III, the remainder will be 0. Try dividing 3 into the numbers produced by Machine III on the preceding page (3, 6, 9, 12, 15, 18).

In the diagram on the next page, the remainder is shown beside each number stamped on the robots that Machines I, II, and III produced.

Machine I		Machine II		Machine III	
Stamped Number	Remainder	Stamped Number	Remainder	Stamped Number	Remainder
1	1	2	2	3	0
4	1	5	2	6	0
7	1	8	2	9	0
10	1	11	2	12	0
13	1	14	2	15	0
16	1	17	2	18	0
.	.	.	.	.	.
.	.	.	.	.	.

To find which machine made the stamp, you could also use your divisibility test for 3, since there are 3 machines. The Test For 3 shows us that if the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

Sum the digits of a number on any robot you choose, and continue summing until you get a robot number that is listed above. (You would only have to make your list up to 9. Do you know why?) Try this using a robot stamped 5298.

Here is an example of a robot problem. Study this problem and then turn to the next page, where you will find two problems for you to figure out by yourself.

Example:

Which machine produced robot 9875? $9 + 8 + 7 + 5 = 29$ and $2 + 9 = 11$ and $1 + 1 = 2$. Also, $9875/3 = 3291$ r. 2

Answer: Machine II, since 29, 11, and 2 are all under Machine II and 9875 divided by 3 leaves a remainder of 2.

The answers to these two problems, and an explanation of how to find them, are given below. See if you can figure them out without looking ahead. You already have enough information to outsmart the robots if you think carefully!

Problem 1: Which machine produced robot 394?

Problem 2: Which machine produced robot 6823?

Problem 1: What machine produced robot 394? $3 + 9 + 4 = 16$ and $1 + 6 = 7$.
Answer: Machine I, since 16 and 7 are robot numbers of Machine I.

Problem 2: What machine produced robot 6823? $6 + 8 + 2 + 3 = 19$ and $9 + 1 = 10$ and $1 + 0 = 1$.
Answer: Machine I since 19, 10, and 1 are all under Machine I. Also, if you divide 394 or 6823 by 3, you get a remainder of 1, and these are robot number remainders of Machine I.

Problem 3: The machines have been working many hours. These are the numbers of the robots produced just before several mechanical failures occurred. Which machine has to be checked in each case.

Robot	Machine
528964	_____
748236	_____
965278	_____
8777777	_____

Could you do problems involving many more machines, like 9 or 23 perhaps? Yes; all you need to do is divide the robot number by the number of machines, and your remainder will be the machine number that produced that particular robot.

Production needed to be increased. Eleven machines have been built. Can you keep your job? Which machine stamped 567?

Machine	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
Numbers Stamped	1	2	3	4	5	6	7	8	9	10	11
	12	13	14	15	16	17	18	19	20	21	22
	.	.	.	.	.	.	.	.	.	.	.

Which machine stamped $\overset{x\,x}{56}7$?   12 - 6 = 6.   Machine VI

Which machine stamped these numbers:

53

7765

6897

98654

454369