R E P O R T R E S U M E S

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A DESCRIPTION OF VARIABLES AND THEIR IMPLEMENTATION IN STUDIES OF PRINCIPLES FOR THE PROGRAMING OF HIGH SCHOOL ALGEBRA. COMPARATIVE STUDIES OF PRINCIPLES FOR PROGRAMING MATHEMATICS IN AUTOMATED INSTRUCTION, TECHNICAL REPORT NO. 8.

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DESCRIPTORS- *TIME FACTORS (LEARNING), *PROGRAMED TEXTS, *LEARNING THEORIES, MATHEMATICS INSTRUCTION, CONVENTIONAL INSTRUCTION, PROGRAMED INSTRUCTION, LEARNING ACTIVITIES, COMPARATIVE ANALYSIS, UICSM UNIT I SERIES

MANIPULATION OF FRAMES WITHIN PROGRAMED MATHEMATICS TEXTS IN ORDER TO STUDY FOUR VARIABLES YIELDED, IN A PREVIOUSLY REPORTED PAPER, CORRELATIONAL DATA (FOR RELATIVELY SMALL TREATMENT GROUPS) THAT ARE PRESENTED HERE. FIRST, THE ORDER OF PRESENTATION OF PROGRAMED MATERIAL (BEFORE, AFTER, AND WITHOUT CONVENTIONAL INSTRUCTION) HAD NO EFFECT ON ACHIEVEMENT. (THE CONDITION OF "CONVENTIONAL INSTRUCTION ONLY" WAS ABSENT.) SECOND, AN INTERMEDIATE PRACTICE TASK WAS FELT TO CONTRIBUTE MORE THAN EITHER A STIMULUS OR A RESPONSE COMPONENT TO LEARNING EVEN THOUGH STATISTICAL ANALYSIS SHOWED THAT ALL THREE TREATMENT GROUPS WERE EQUAL. THIRD, CONTRARY TO HYPOTHESIS, LEARNING FROM A RANDOM FRAME SEQUENCE WAS SIGNIFICANTLY SUPERIOR TO LEARNING FROM A LOGICAL SEQUENCE, BUT LEARNING TIME WAS ABOUT THE SAME FOR BOTH GROUPS. FINALLY, BY OMITTING SELECTED FRAMES IN AN ATTEMPT TO VARY SIZE OF STEP, IT WAS FOUND THAT LEARNING WAS EQUAL FOR LARGE AND SMALL STEP SEQUENCES. ALTHOUGH THE LARGE STEP SEQUENCE HAD A TIME-SAVING EFFECT, TIME PER PAGE WAS ABOUT EQUAL FOR BOTH GROUPS. (LH)

UNIVERSITY OF ILLINOIS
Urbana, Illinois

A Description of Variables and Their Implementation in Studies of Principles for the Programing of High School Algebra

Ellen F. Rosen and Lawrence M. Stolurow

COMPARATIVE STUDIES OF PRINCIPLES FOR PROGRAMMING MATHEMATICS IN AUTOMATED INSTRUCTION

Technical Report No. 8

July, 1964

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In order to study the variables important in programing, certain manipulations were performed on selected programed textbooks from the UICSM Unit I series. These manipulations resulted in new forms of the textbooks which were then used in studies aimed at the formulation of general guidelines for writing such textbooks. The purpose of this report is to further clarify what these variables are by operationally describing the manipulations performed and to present representative examples and some data relating to their effectiveness.

The data presented are from a study performed in the Champaign school system in the fall of 1963 (See L. M. Stolurow and M. Beberman, Comparative Studies of Principles for Programming Mathematics in Automated Instruction. Urbana, Ill: Univer. of Illinois, USOE Contr. No. 711151.01, Quarterly Reports 9 and 19, June - December, 1963.). They consist of item difficulties of each of the sample frames and the point biserial correlation coefficient of success on the item with score on the end test. This report is intended as a supplement to the studies which were performed and as a guide both for people interested in this type of research and for those already engaged in such research who wish to compare the outcomes of their studies with those of this series.

The general format of this report is (a) to present a description of the variable, then (b) to present sample frames with related descriptive statistics, and where possible (c) to indicate some effects of the variable on learning.

The topics covered are as follows:

1. Conventional instruction followed by programed instruction or the reverse order.
2. The implementation of discovery.
3. The lack of association.
4. The size of the step between frames.

Variable: The Sequential Relationships -- Either Conventional followed by Programed Instruction or the Reverse.

The study of this variable requires a good teacher, a good programed text, both of which teach the same topic. The positioning of teacher-led lectures and program-guided study in time is manipulated. In our series (See Stolurow and Beberman, 1963, Quarterly Report 9 and 10, Chapters, 8 and 9.) the textbook selected was Part 104 of the UICSM programed mathematics materials which presents the multiplication of real numbers. The teacher either covered the same material before the booklet was used (program-following condition), after the student had studied the topic via the booklet (program-lead condition), or the teacher did not present the

material at all and the topic was presented only via the programed version (pure condition).

No sample frames are presented for this variable since no manipulation of the programed material was necessary. The law of repetition applied to this study states that the program lead and follow conditions should produce better learning since the student has more chance for practice and gets more reinforcement. However, in the Champaign studies with Part 104, no difference was found ($F < 1$).

THE IMPLEMENTATION OF DISCOVERY

This study involved the use of UICSM programed mathematics materials, Parts 106 and 107. Part 107 was used intact. Part 106 is an introduction to Part 107; thus, variations in that which is taught in Part 106 should affect subsequent learning of the task presented in Part 107. Part 106 and 107 teach the conventions of grouping and simplification of punctuated and unpunctuated expressions.

The variable of interest here is the effect of either response, stimulus, or mediator practice (Part 106R, 106S, or 106M, respectively) upon the learning of a task (Part 107). The response component of the task in this case, was simplification or finding the value of the expression; the stimulus component was familiarity with the symbols involved, the operation and the punctuation used in expressions; and the mediator component was the rules

and practice of the punctuation of sentences. The following sample frames from these three variable booklets were chosen to show how the groups were given practice with the component assigned to their condition.

Tables 1 through 5 give descriptive statistics for the subsequent frames.

The condition which was felt to be best for later learning of the task was the mediator condition. However, all groups were equal ($F < 1$).

Table 1[a]

Descriptive Statistics for Items on

Pages 1-2 of Part 106S

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	1.000	undefined[b]
Program Leading (Programed followed by conventional instruction)	9	.777	-.262
Program Following (Conventional followed by programed instruction)	8	.750	.900*

[a]These data relate to the program frames indicated and reproduced on the following two pages.

[b]Var. (item) = 0.000

*Significant at the .01 level with appropriate degrees of freedom (number of students minus two)

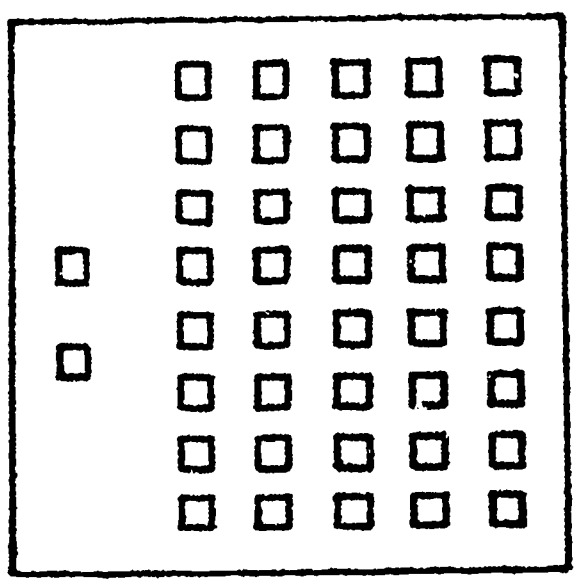
Bill was asked to find out how many seats there were in Room 207. He reported to the class that there were 2+8x5 seats in Room 207. He wrote the following on the board:

$$2+8\times5$$

and said that the class could check his computation.

Steve objected. He said that when he worked it out, he got (2+8)x5.

Here is the floor plan for Room 207, the room Bill went to.

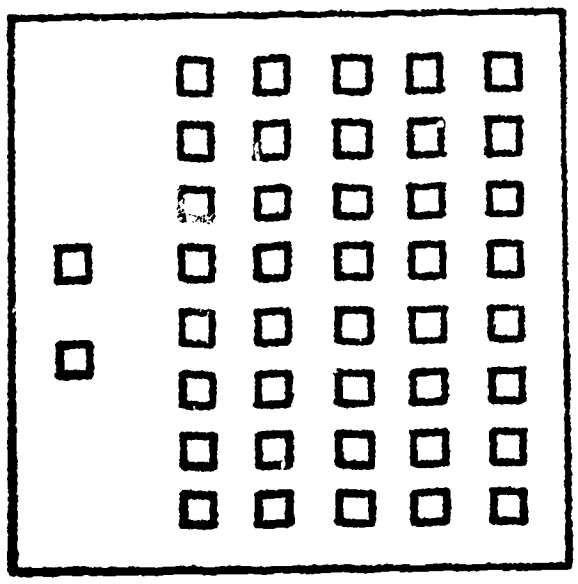


Who was right, Bill or Steve?

Circle the answer on your work sheet.

Turn to PAGE 2.

Check your answer.



Bill was right because there are 2+8x5 seats in Room 207.

Record your result.

* * *

Turn to PAGE 5.

Table 2[a]

Descriptive Statistics for Items on

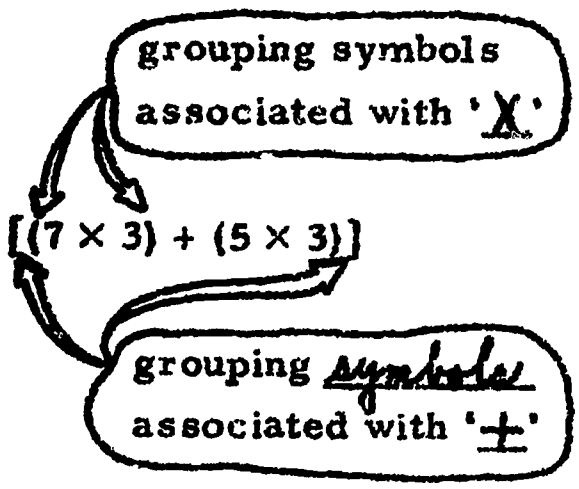
Pages 23-24 Part 106S

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.687	-.061
Program Leading (Programed followed by conventional instruction)	9	.333	.432
Program Following (Conventional followed by programed instruction)	8	.625	.587

[a]These data relate to the program frames indicated and reproduced on the following two pages.

Check your answers.

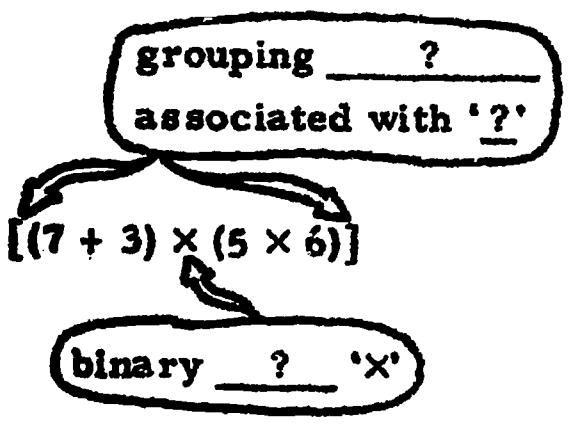


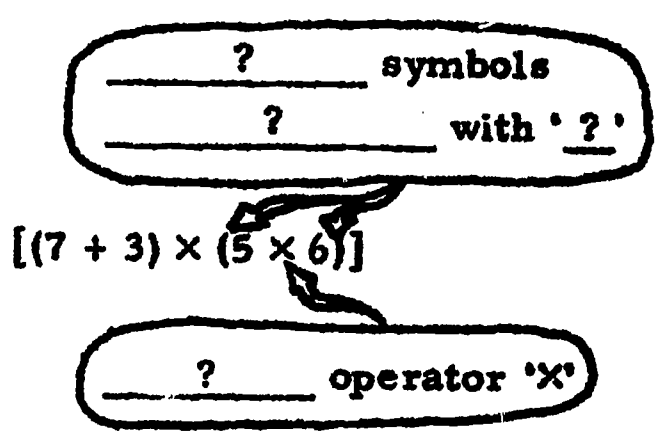
The last operation to be performed in order to simplify this expression is additions.

Record your results.

* * *

Consider the expression '[(7 + 3) × (5 × 6)]'.

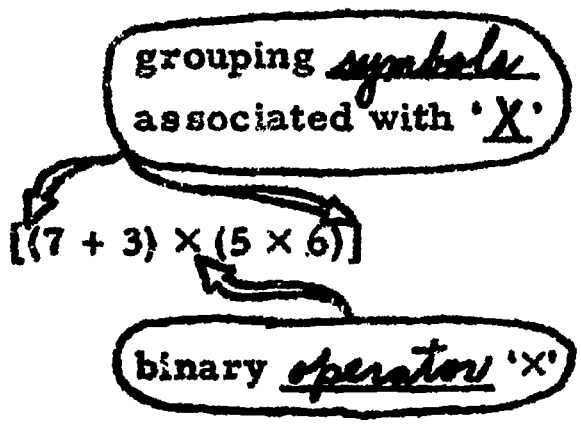


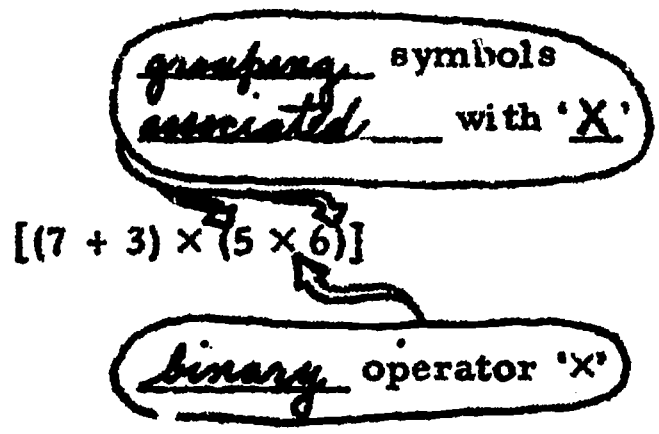


The last operation to be performed in order to simplify this expression is ? . Fill the blanks on your work sheet.

Turn to PAGE 24.

Check your answers.





The last operation to be performed in order to simplify this expression is multiplication.

Record your results.

This is the end of Part 106. Put your work sheet under the front cover of this booklet, and return it to your teacher.

Table 3[a]

Descriptive Statistics for Items on Pages 13-14 of Part 106M

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	15	.933	-.233
Program Leading (Program followed by conventional instruction)	9	.888	.445
Program Following (Conventional followed by programed instruction)	7	.857	-.527

[a]These data relate to the program frames indicated and reproduced on the following two pages.

Check your answers.

	Principal operator
(1) $[(7 + 8) - 3] - 2$	'$-$'
(2) $(^{-}2 \times 5) + (5 \times 6)$	'$+$'
(3) $^{-}2 \times (20 + ^{-}20)$	'$\times$'
(4) $(44 \times 2) \div (8 \times 11)$	'$\div$'

Record your results.

* * *

Punctuate; that is, insert grouping symbols. [Remember that we agreed to omit the outermost grouping symbols.] The arrow indicates the principal operator.

Sample. $5 + 5 \times 3$
$\uparrow$

Answer. $(5 + 5) \times 3$
$\uparrow$

(1) $3 + 4 \times 2$
$\uparrow$

(2) $^{-}1 \times ^{-}6 + 4$
$\uparrow$

(3) $8 + 2 \div 5$
$\uparrow$

Fill the blanks on your work sheet.

Turn to PAGE 14.

Check your answers.

(1) $(3 + 4) \times 2$

(2) $^{-}1 \times (^{-}6 + 4)$

(3) $(8 + 2) + 5$

Record your results.

* * *

Punctuate each of the following; the arrow indicates the principal operator.

(1) $9 \div 3 + 3$

(2) $6 \times 5 \div 5$

(3) $^{-}2 + 2 \times ^{-}12$

Fill the blanks on your work sheet.

Turn to PAGE 15.

Table 4[a]

Descriptive Statistics for Items on

Pages 1-2 of Part 106R

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	12	.916	-.487
Program Leading (Program followed by conventional instruction)	9	1.00	undefined[b]
Program Following (Conventional followed by programed instruction)	5	.800	-.211

[a] These data relate to the program frames indicated and reproduced on the following pages.

[b] var. (item) = 0.000

Bill was asked to find out how many seats there were in Room 207. He reported to the class that there were 42 seats in Room 207. He wrote the following on the board:

$$[2 + (8 \times 5)]$$

and said that the class could check his computation.

Steve objected. He said that when he worked it out, he got 50. He justified his answer in the following manner:

"First, I found the sum of __?__ and __?__.

Then I multiplied that sum by __?__."

Fill the blanks on your work sheet to complete the sentences.

Turn to PAGE 2.

Check your answers.

Bill claimed that there were 42 seats in Room 207.

$[2 + (8 \times 5)]$ is 42.

Steve said that $(2 + 8) \times 5$ is 50.

Steve first found the sum of 2 and 8, and then he multiplied this sum by 5.

Record your results.

Table 5[a]

Descriptive Statistics for Items on

Pages 20-21 of Part 106R

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	12	.666	.106
Program Leading (Program followed by conventional instruction)	9	.555	-.072
Program Following (Conventional followed by programed instruction)	5	.400	.488

[a]These data relate to the program frames indicated and reproduced on the following pages.

Check your answers.

		Result
(1)	$[(3 + 41) \times 6]$	$= 44 \times 6$
		$= 264$
(2)	$[(16 \times 5) + (7 \times 5)]$	$= [80 + 35]$
		$= 115$
(3)	$\{[(8 + 12) \times 3] + 5\}$	$= \{[20 \times 3] + 5\}$
		$= \{60 + 5\}$
		$= 65$

Record your results.

* * *

Here are some more expressions for you. Simplify them.

		Result
(1)	$[(5 + {}^{-}1) \times (8 + {}^{-}2)]$	?
(2)	$\{[(5 + {}^{-}1) \times 8] + {}^{-}2\}$	?
(3)	$[(9 \times 3) + (5 + 13)]$	?
(4)	$[(3 + 5) \times (4 + 7)]$	?
(5)	$[(16 + 7) \times 5]$	?

Fill in the blanks on your work sheet.

Turn to PAGE 21

Check your answers.

		Result
(1)	$[(5 + {}^{-}1) \times (8 + {}^{-}2)] = [4 \times 6]$	$= 24$
(2)	$\{[(5 + {}^{-}1) \times 8] + {}^{-}2\} = \{[4 \times 8] + {}^{-}2\}$	
	$= \{32 + {}^{-}2\}$	$= 30$
(3)	$[(9 \times 3) + (5 + 13)] = [27 + 18]$	$= 45$
(4)	$[(3 + 5) \times (4 + 7)] = [8 \times 11]$	$= 88$
(5)	$[(16 + 7) \times 5] = [23 \times 5]$	$= 115$

Record your results.

* * *

THE WEB OF ASSOCIATION

In this study Part 110. 5 was used. It covers the notion of logical consequence, the difference between truth and validity, the recognition and discrimination of the principles of multiplication by one and zero and the principles of adding zero for numbers of arithmetic and real numbers.

A web of association is the associative network or set of "bonds" relating particular stimuli to responses. Conventional texts have always been written with some logical development but while they are organized they do not necessarily have a "built in" capability of producing a "web of associations" which the student necessarily learns and uses. Thus, any logical ordering of frames in a program attempts to build up a web for the student, but different orderings presumably produce different webs. Conversely, any nonlogical or near chance presentation of frames in a program makes no attempt to build up such a web; thus, it is left for the student to either develop the interrelationships himself or to take things frame by frame and not attempt such associations.

Thus, the modified booklet (Part 110. 5) is an essentially random sequence of frames (pages) of the sequence developed by the UICSM staff (Part 110. 5). The first ten frames from both booklets and some descriptive statistics about those frames are presented here as a sample.

Tables 6 through 25 give descriptive statistics for the subsequent frames.

The logical version (Part 110. 5) should produce better learning. If it does not produce better learning, then the students who have the random sequence should take more time to finish the booklet since they must do their

own integration. The data gathered indicated a significant sequence effect ($F = 4.59$) at .05 level (random group superior) with the random group taking about the same amount of time per page to finish the booklet.

Table 6[a]

Descriptive Statistics for Items on

Page 1 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	18	.888	.108
Program Leading (Program followed by conventional instruction)	7	.714	-.016
Program Following (Conventional followed by programed instruction)	12	1.00	undefined[b]

[a]These data relate to the program frames indicated and reproduced on the following page.

[b]var. (item) = 0.00

One day Mr. Wilson, the history teacher, was telling his class about some famous American authors. Vera wrote these statements in her notebook to help her remember what Mr. Wilson said:

> The author of *Huckleberry Finn* is Samuel Clemens.
> The author of *Tom Sawyer* is Mark Twain.
> Mark Twain is the same person as Samuel Clemens.
> Samuel Clemens is an American author.

Later, she read *Tom Sawyer* and wrote a book report about it. Here are some of the statements in her report.

(1) Samuel Clemens is the author of *Tom Sawyer*.

(2) Mark Twain is an American author.

(3) The author of *Tom Sawyer* is also the author of *Huckleberry Finn*.

(4) *Tom Sawyer* was written before *Huckleberry Finn*.

Look at each of these statements from Vera's book report and decide whether she could have known it without any more information than her notes from Mr. Wilson's history class which we showed you. That is, decide whether or not the statements (1), (2), (3), and (4) are <u>logical consequences</u> of the four statements from Vera's history-class notes. [Write 'yes' or 'no' on your work sheet.]

Turn to PAGE 2.

Table 7[a]

Descriptive Statistics for Items on

Page 2 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Piserial Correlation of item score with test score
Pure (Program only)	18	.888	.148
Program Leading (Program followed by conventional instruction)	7	.857	.372
Program Following (Conventional followed by programed instruction)	12	.833	.705**

[a]These data relate to the program frames indicated and reproduced on the following page.

[b]var. (item) = 0.00

**Significant at the .05 level with appropriate degrees of freedom (number of students minus two).

Check your answers and record your results.

The statements in Vera's history class notes are:

> The author of *Huckleberry Finn* is Samuel Clemens.
> The author of *Tom Sawyer* is Mark Twain.
> Mark Twain is the same person as Samuel Clemens.
> Samuel Clemens is an American author.

(1) Yes A logical consequence of the above statements is:

> Samuel Clemens is the author of *Tom Sawyer*.

because the statements in Vera's notes tell us that

> the author of *Tom Sawyer* is Mark Twain,
> and he is the same person as Samuel Clemens.

(2) Yes Another logical consequence of these statements is:

> Mark Twain is an American author.

because

> Mark Twain is the same person as Samuel Clemens,
> and this person is an American author.

(3) Yes This statement from Vera's book report:

> The author of *Tom Sawyer* is also the author of *Huckleberry Finn*.

is another logical consequence of her history class notes because they tell us that

> the author of *Tom Sawyer* is Mark Twain, who is the same person as Samuel Clemens, who is the same person as the author of *Huckleberry Finn*.

(4) No The statement:

> *Tom Sawyer* was written before *Huckleberry Finn*.

is not a logical consequence of just what is given in Vera's history class notes. That is, you would need to know more in order to conclude it.

* * *

There is exactly one statement in Vera's notes at the top of this page such that, if it had been deleted, none of the statements in her book report would have been a logical consequence of the remaining statements. Write that statement on your work sheet. [Reread the answer discussions if you need help.]

Turn to PAGE 3.

Table 8[a]

Descriptive Statistics for Items on

Page 3 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	18	.833	.034
Program Leading (Program followed by conventional instruction)	7	1.00	undefined[b]
Program Following (Conventional followed by programed instruction)	12	.916	.694**

[a]These data relate to the program frames indicated and reproduced on the following page.

[b]var. (item) = 0.00.

**Significant at the .05 level with appropriate degrees of freedom (number of students minus two).

Check your answer.

If the following statement from Vera's history class notes:

Mark Twain is the same person as Samuel Clemens

were deleted, none of the statements in her book report would be a logical consequence of the remaining statements from her notes.

Record your result.

* * *

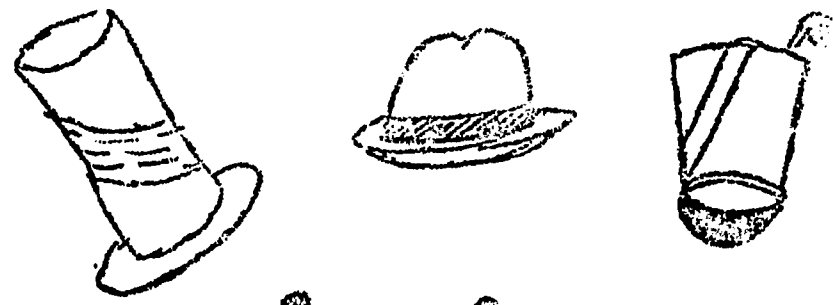
Read these statements about Zabranchburg High School:

Richard is the Student Council president.

The Student Council president is the chairman of the Assemblies Committee.

The chairman of Assemblies Committee is the band major at each pep rally.

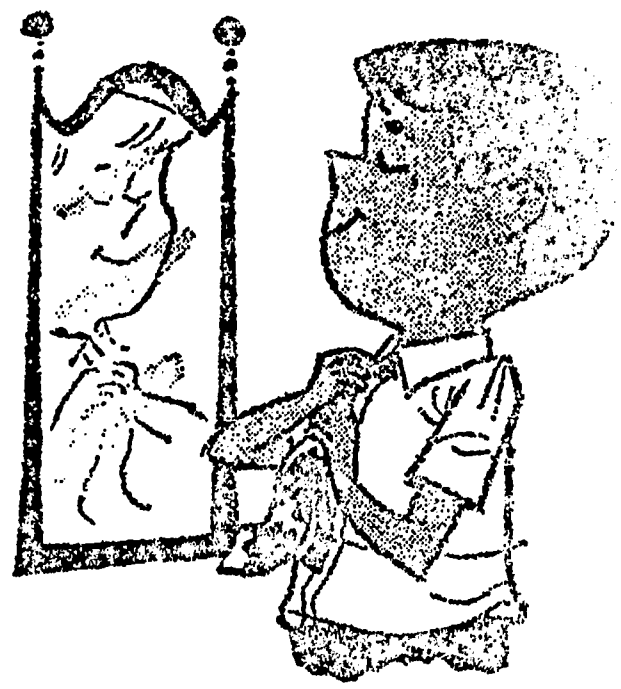
Tell which of the following statements are logical consequences of these. [Yes/No]



(1) Richard is the band major at each pep rally.

(2) The band major at each pep rally is Richard.

(3) Richard was elected by a majority of the students.



Turn to PAGE 4.

Table 9[a]

Descriptive Statistics for Items on

Page 4 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test scores
Pure (Program only)	18	.833	.478**
Program Leading (Program followed by conventional instruction)	7	.571	.572
Program Following (Conventional followed by programed instruction)	12	.666	-.107

[a]These data relate to the program frames indicated and reproduced on the following page.

**Significant at the .05 level with appropriate degrees of freedom (Number of students minus two).

Check your answers.

The statements about Zabranchburg High School tell us that the following titles name the same person:

Richard

Student Council President

the chairman of the Assemblies Committee

the band major at each pep rally

Therefore,

(1) [Yes] **the statement:**

Richard is the band major at each pep rally.

is a logical consequence of the given statements. And furthermore,

(2) [Yes] **the statement:**

The band major at each pep rally is Richard.

is also a logical consequence. However,

(3) [No] **the statement:**

Richard was elected by the majority of the students.

is not a logical consequence, regardless of how we might feel about Richard and the election.

Record your results.

* * *

One more statement may be added to those already given about Zabranchburg High School, so that the statements in (3), above:

Richard was elected by the majority of the students.

will be a logical consequence of the statements about Zabranchburg.

Here is the start of such a statement. Finish it on your work sheet.

The Student Council president ? .

Turn to PAGE 5.

Table 10[a]

Descriptive Statistics for Items on

Page 5 of Part 110. 5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test scores
Pure (Program only)	18	.444	.316
Program Leading (Program followed by conventional instruction)	17	.571	.139
Program Following (Conventional followed by programed instruction)	12	.500	undefined[b]

[a]These data relate to the program frames indicated and reproduced on the following page.

[b]var. (item) = 0. 00.

Check your answer.

The statements:

The Student Council president *was elected by the majority of the students.*

and:

Richard is the Student Council president.

give the desired logical consequence:

Richard was elected by the majority of the students.

Record your result.

* * *

Notice these statements about numbers of arithmetic:

$$87 \times 4899 \times 79 = 79 \times 87 \times 4899$$

$$79 \times 87 \times 4899 = 6783 \times 4899$$

$$6783 \times 4899 = 33,229,917$$

Tell which of the following are logical consequences of the statements above: [Yes/No]

(1) $87 \times 4899 \times 79 = 79 \times 4899 \times 87$

(2) $79 \times 4899 \times 87 = 33,229,917$

(3) $87 \times 4899 \times 79 = 6783 \times 4899$

Turn to PAGE 6.

Table 11[a]

Descriptive Statistics for Items on

Page 6 of Part 110. 5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test scores
Pure (Program only)	18	.833	-. 205
Program Leading (Program followed by conventional instruction)	7	.714	-.135
Program Following (Conventional followed by programed instruction)	12	.833	.135

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers and record your results.

Here are the statements about numbers of **arithmetic:**

$$87 \times 4899 \times 79 = 79 \times 87 \times 4899$$

$$79 \times 87 \times 4899 = 6783 \times 4899$$

$$6783 \times 4899 = 33,229,917$$

(1) [No] The statement:

$87 \times 4899 \times 79 = 79 \times 4899 \times 87$

is not a logical consequence of just these statements shown above.

(2) [No] The statement:

$79 \times 4899 \times 87 = 33,229,917$

is not a logical consequence of these statements, either.

(3) [Yes] The statement:

(☆) $87 \times 4899 \times 79 = 6783 \times 4899$

is a logical consequence of the first two given statements. That is:

87 × 7899 × 79 is the same as 79 × 87 × 4899,

which is the same as 6783 × 4899.

* * *

Let us reexamine the three statements at the top of this page. Mark each one to show whether you think it is 'True', 'False', or '?' [if you are not sure whether it is a true or a false statement]

(1) $87 \times 4899 \times 79 = 79 \times 87 \times 4899$

(2) $79 \times 87 \times 4899 = 6783 \times 4899$

(3) $6783 \times 4899 = 33,229,917$

Next, fill the blanks in the following sentence on your work sheet:

The statement from Exercise 3, above:

(☆) $87 \times 4899 \times 79 = 6783 \times 4899$

___?___ a logical consequence of the statements (1), (2), and (3).

[is/is not]

Turn to PAGE 7.

Table 12[a]

Descriptive Statistics for Items on

Page 7 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	18	.777	.470**
Program Leading (Program followed by conventional instruction)	7	1.00	undefined[b]
Program Following (Conventional followed by programed instruction)	12	.916	-.073

[a]These data relate to the program frames indicated and reproduced on the following page.

[b]var. (item) = 0.00.

**Significant at the .05 level with appropriate degrees of freedom (Number of students minus two).

Check your answers.

We asked you to mark what you THOUGHT about each of the statements:

(1) $87 \times 4899 \times 79 = 79 \times 87 \times 4899$

(2) $79 \times 87 \times 4899 = 6783 \times 4899$

(3) $6783 \times 4899 = 33,229,917$

You are correct if you answered what you THOUGHT. But, **regardless of** whether you thought statements (1), (2), and (3) were true or false, **the** statement:

(☆) $87 \times 4899 \times 79 = 6783 \times 4899$

is a logical consequence of the statements (1) and (2), above.

Record your results.

* * *

You may be surprised at what we discovered about the statements above.

(1) $87 \times 4899 \times 79 = 79 \times 87 \times 4899$ [True]

(2) $79 \times 87 \times 4899 = 6783 \times 4899$ [False]

(3) $6783 \times 4899 = 33,229,917$ [True]

Here is why statement (2) is false.

$$\begin{array}{r} 79 \\ \times 87 \\ \hline 553 \\ 632 \\ \hline 6873 \end{array}$$

[not '6783' as shown in (2): $79 \times 87 \times 4899 = 6783 \times 4899$]

Does this discovery now mean that it is FALSE to say that the statement:

(☆) $87 \times 4899 \times 79 = 6783 \times 4899$

is a logical consequence of statements (1) and (2), above? [Yes/No]

Turn to PAGE 8.

Table 13[a]

Descriptive Statistics for Items on

Page 8 of Part 110. 5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	18	.944	-.092
Program Leading (Program followed by conventional instruction)	7	.857	.372
Program Following (Conventional followed by programed instruction)	12	.750	.070

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answer.

No You can decide whether or not a given statement is a logical consequence of other statements just by observing the forms of the statements. The truth or falsity of the statements has nothing to do with it [except that a false statement will never be a logical consequence of true ones].

Record your result.

* * *

Here is a statement:

(☆) 9853 + 1582 = 4831 + 6524

and here is another statement:

(⚓) 4831 + 6524 = 9853 + 1582

Now, answer the following questions on your work sheet.

(1) Is (⚓) a logical consequence of (☆)?

(2) Is (☆) a logical consequence of (⚓)?

Turn to PAGE 9.

Table 14[a]

Descriptive Statistics for Items on

Page 9 of 110. 5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	18	.333	.216
Program Leading (Program followed by conventional instruction)	7	.428	-.139
Program Following (Conventional followed by programed instruction)	12	.500	.242

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

Yes { (Φ) is a logical consequence of (☆).
(☆) is a logical consequence of (Φ).

(☆) says that 9853 + 1582 is the same thing as 4831 + 6524, and (Φ) says that 4831 + 6524 is 9853 + 1582.

If you are given (☆), you may logically conclude (Φ); and if you are given (Φ), you may logically conclude (☆).

Record your results.

* * *

Consider the following statement:

(☆) (1846 + 93) + 75 = 1846 + (93 + 75)

As you know, (☆) is an instance of the associative principle for addition because it fits the pattern-sentence. Since (☆) is an instance, (☆) is a logical consequence of the apa.

Now look at:

(Φ) 1846 + (93 + 75) = (1846 + 93) + 75

(1) Is (Φ) an instance of the apa?

(2) Is (Φ) a logical consequence of (☆)?

(3) Is (☆) a logical consequence of the apa?

(4) Is (Φ) a logical consequence of the apa?

Turn to PAGE 10.

Table 15[a]

Descriptive Statistics for Items on Pages 10 and 11 of Part 110. 5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	18	.944	-.092
Program Leading (Program followed by conventional instruction)	7	.857	.372
Program Following (Conventional followed by programed instruction)	12	.916	-.182

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

(☆) $(1846 + 93) + 75 = 1846 + (93 + 75)$

(✠) $1846 + (93 + 75) = (1846 + 93) + 75$

(1) No ; (✠) is not an instance of the apa. But,

(2) Yes ; (✠) is a logical consequence of (☆), and

(3) Yes ; (☆) is a logical consequence of the apa. (An instance of a principle is a logical consequence of that principle.) So,

(4) Yes ; (✠) is a logical consequence of the apa because (✠) is a logical consequence of (☆) as stated in Exercise (2), and (☆) is a logical consequence of the apa as stated in Exercise (3).

Record your results.

* * *

Here is a pattern-sentence for the distributive principle for multiplication over addition:

___ × ..... + ~~~ × ..... = (___ + ~~~) × .....

Now, consider the following statements:

(☆) $(5 \times 8) + (7 \times 8) = (5 + 7) \times 8$

(✠) $(5 + 7) \times 8 = (5 \times 8) + (7 \times 8)$

Only one of the statements (☆) and (✠) is an instance of the dpma. Which one is it?

Turn to PAGE 11.

Check your answer.

Just by looking at the two statements:

(☆) $(5 \times 8) + (7 \times 8) = (5 + 7) \times 8$

(⚓) $(5 + 7) \times 8 = (5 \times 8) + (7 \times 8)$

you can easily see that statement (☆) fits the pattern-sentence for the dpma, but (⚓) does not. Here is the pattern-sentence for the dpma:

___×..... + ~~~×..... = (___+~~~)×.....

So, (☆) is an instance of the dpma and (⚓) is not.

Record your result.

* * *

Turn to PAGE 12.

Table 16[a]

Descriptive Statistics for Items on

Page 1 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.937	-.139
Program Leading (Program followed by conventional instruction)	16	.625	.416
Program Following (Conventional followed by programed instruction)	12	.750	.493

[a]These data relate to the program frames indicated and reproduced on the following page.

One day Mr. Wilson, the history teacher, was telling his class about some famous American authors. Vera wrote these statements in her notebook to help her remember what Mr. Wilson said:

> The author of *Huckleberry Finn* is Samuel Clemens.
> The author of *Tom Sawyer* is Mark Twain.
> Mark Twain is the same person as Samuel Clemens.
> Samuel Clemens is an American author.

Later, she read *Tom Sawyer* and wrote a book report about it. Here are some of the statements in her report.

(1) Samuel Clemens is the author of *Tom Sawyer*.

(2) Mark Twain is ar American author.

(3) The author of *Tom Sawyer* is also the author of *Huckleberry Finn*.

(4) *Tom Sawyer* was written before *Huckleberry Finn*.

Look at each of these statements from Vera's book report and decide whether she could have known it without any more information than her notes from Mr. Wilson's history class which we showed you. That is, decide whether or not the statements (1), (2), (3), and (4) are logical consequences of the four statements from Vera's history-class notes. [Write 'yes' or 'no' on your work sheet.]

Turn to PAGE 2.

Table 17[a]

Descriptive Statistics for Items on

Page 2 of Part 110. 5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.812	-.301
Program Leading (Program followed by conventional instruction)	16	.625	-.208
Program Following (Conventional followed by programed instruction)	8	.875	-.035

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers and record your results.

The statements in Vera's history class notes are:

> The author of *Huckleberry Finn* is Samuel Clemens.
> The author of *Tom Sawyer* is Mark Twain.
> Mark Twain is the same person as Samuel Clemens.
> Samuel Clemens is an American author.

(1) Yes A logical consequence of the above statements is:

> Samuel Clemens is the author of *Tom Sawyer*.

because the statements in Vera's notes tell us that

> the author of *Tom Sawyer* is Mark Twain,
> and he is the same person as Samuel Clemens.

(2) Yes Another logical consequence of these statements is:

> Mark Twain is an American author.

because

> Mark Twain is the same person as Samuel Clemens,
> and this person is an American author.

(3) Yes This statement from Vera's book report:

> The author of *Tom Sawyer* is also the author of *Huckleberry Finn*.

is another logical consequence of her history class notes because they tell us that

> the author of *Tom Sawyer* is Mark Twain, who is the same person as Samuel Clemens, who is the same person as the author of *Huckleberry Finn*.

(4) No The statement:

> *Tom Sawyer* was written before *Huckleberry Finn*.

is not a logical consequence of just what is given in Vera's history class notes. That is, you would need to know more in order to conclude it.

* * *

There is exactly one statement in Vera's notes at the top of this page such that, if it had been deleted, none of the statements in her book report would have been a logical consequence of the remaining statements. Write that statement on your work sheet. [Reread the answer discussions if you need help.]

Turn to PAGE 3.

Table 18[a]

Descriptive Statistics for Items on

Page 3 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.937	.198
Program Leading (Program followed by conventional instruction)	16	.937	.166
Program Following (Conventional followed by programed instruction)	8	1.000	undefined[b]

[a]These data relate to the program frames indicated and reproduced on the following page.

[b]var. (item) = 0.00.

Check your answer.

If the following statement from Vera's history class notes:

Mark Twain is the same person as Samuel Clemens

were deleted, none of the statements in her book report would be a logical consequence of the remaining statements from her notes.

Record your result.

* * *

Exactly one of the sentences below is not an instance of the principle for multiplying by zero. Which one is it? [Circle your answer on the work sheet.]

(1) $0 + 0 = 0$

(2) $0 \times 0 = 0$

(3) $3\frac{1}{2} \times 0 = 0$

(4) $0.25 \times 0 = 0$

Turn to PAGE 4.

Table 19[a]

Descriptive Statistics for Items on

Page 4 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.937	-.139
Program Leading (Program followed by conventional instruction)	16	.937	.166
Program Following (Conventional followed by programed instruction)	8	1.00	undefined[b]

[a]These data relate to the program frames indicated and reproduced on the following page.

[b]var. (item) = 0.00.

Check your answer.

(1) '0 + 0' is not an instance of the principle for multiplying by zero.

Record your result.

* * *

Let's check to see whether this statement is true or false:

$$\underbrace{17 \times 0}_{(1)} + \underbrace{83 \times 0}_{(2)} = \underbrace{2 \times 0}_{(4)}$$

(1) $17 \times 0 = ?$

(2) $83 \times 0 = ?$

(3) $(17 + 83) \times 0 =$

(4) $2 \times 0 = ?$

If you answered these questions correctly you can see that the statement is true. Now, answer the following questions.

(5) $\frac{1}{2} \times 0 = ?$

(6) $\pi \times 0 = ?$

(7) $1 \times 0 = ?$

(8) $56980261593475 58\frac{17}{352} \times 0 = ?$

Turn to PAGE 5.

Table 20[a]

Descriptive Statistics for Items on

Page 5 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.812	.327
Program Leading (Program followed by conventional instruction)	16	.562	.284
Program Following (Conventional followed by programed instruction)	8	.500	.094

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

(1) $17 \times 0 =$ 0

(2) $83 \times 0 =$ 0

(3) $(17 + 83) \times 0 =$ 0

(4) $2 \times 0 =$ 0

(5) $\frac{1}{2} \times 0 =$ 0

(6) $\pi \times 0 =$ 0

(7) $1 \times 0 =$ 0

(8) $569802615934755\frac{17}{352} \times 0 =$ 0

Record your results.

* * *

Now, do these problems. Write just the answers on your work sheet.

(1) $^{+}8 + {}^{-}5 = ?$

(2) $^{-}7 + {}^{-}4 = ?$

(3) $^{-}5 + {}^{+}8 = ?$

(4) $^{-}4 + {}^{-}7 = ?$

(5) $^{+}7 + {}^{-}18 = ?$

(6) $({}^{-}74 + {}^{+}7) + {}^{-}18 = ?$

(7) $^{-}86 + {}^{+}99 = ?$

(8) $({}^{+}100 + {}^{-}86) + {}^{+}99 = ?$

Turn to PAGE 6.

Table 21[a]

Descriptive Statistics for Items on

Page 6 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.875	-.055
Program Leading (Program followed by conventional instruction)	16	.562	.081
Program Following (Conventional followed by programed instruction)	8	.875	-.035

[a]These data relate to the program frames indicated and reproduced on the following page.

[Part 110.5]

Check your answers.

(1) $^{+}8 + {}^{-}5 = {}^{+}3$

(2) $^{-}7 + {}^{-}4 = {}^{-}11$

(3) $^{-}5 + {}^{+}8 = {}^{+}3$

(4) $^{-}4 + {}^{-}7 = {}^{-}11$

(5) $^{+}7 + {}^{-}18 = {}^{-}11$

(6) $(^{-}74 + {}^{+}7) + {}^{-}18 = {}^{-}85$

(7) $^{-}86 + {}^{+}99 = {}^{+}13$

(8) $(^{+}100 + {}^{-}86) + {}^{+}99 = {}^{+}113$

Record your result.

* * *

Read these statements about Zabranchburg High School:

Richard is the Student Council president.

The Student Council president is the chairman of the Assemblies Committee.

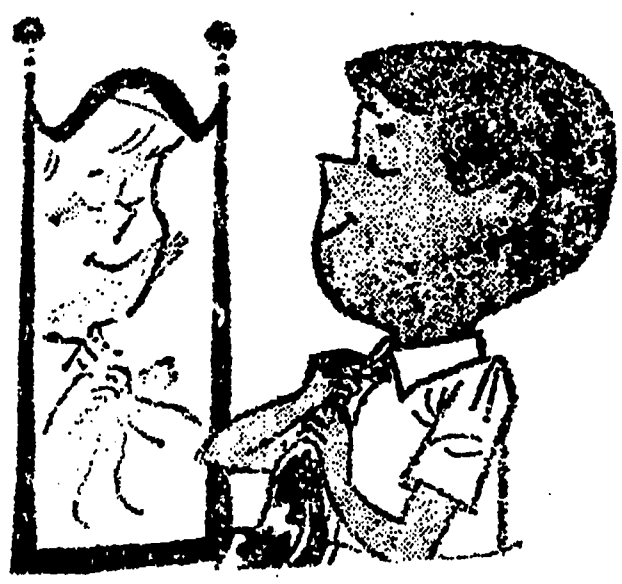
The chairman of Assemblies Committee is the band major at each pep rally.

Tell which of the following statements are logical consequences of these. [Yes/No]

(1) Richard is the band major at each pep rally.

(2) The band major at each pep rally is Richard.

(3) Richard was elected by a majority of the students.



Turn to PAGE 7.

Table 22[a]

Descriptive Statistics for Items on

Page 7 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.375	-.291
Program Leading (Program followed by conventional instruction)	16	.437	.081
Program Following (Conventional followed by programed instruction)	8	.375	-.318

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

The statements about Zabranchburg High School tell us that the following titles name the same person:

Richard

Student Council President

the chairman of the Assemblies Committee

the band major at each pep rally

Therefore,

(1) [Yes] the statement:

Richard is the band major at each pep rally.

is a logical consequence of the given statements. And furthermore,

(2) [Yes] the statement:

The band major at each pep rally is Richard.

is also a logical consequence. However,

(3) [No] the statement:

Richard was elected by the majority of the students.

is not a logical consequence, regardless of how we might feel about Richard and the election.

Record your results.

* * *

Simplify:

$$\pi \times 34 + \pi$$

Turn to PAGE 8

Table 23[a]

Descriptive Statistics for Items on

Page 8 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	1.000	undefined[b]
Program Leading (Program followed by conventional instruction)	16	.812	.103
Program Following (Conventional followed by programed instruction)	8	1.000	undefined[b]

[a]These data relate to the program frames indicated and reproduced on the following page.

[b]var. (item) = 0.00.

Check your answer.

$\pi \times 34 + \pi = 77 \times 35$

Record your result.

* * *

The answer we gave to the problem above can be justified by the use of another principle which is quite simple. In fact, you are probably already aware of it. Use it to complete the sentences below. If your answers are correct, the completed sentences should be instances of this new principle.

(1) $8 \times 1 =$ _____

(2) $3 \times 1 =$ _____

(3) $48 \times 1 =$ _____

(4) $0 \times 1 =$ _____

(5) $1 \times 1 =$ _____

(6) $1776\frac{7}{8} \times 1 =$ _____

Turn to PAGE 9.

Table 24[a]

Descriptive Statistics for Items on

Page 9 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.312	-.200
Program Leading (Program followed by conventional instruction)	16	.375	-.166
Program Following (Conventional followed by programed instruction)	8	0.000	undefined[b]

[a]These data relate to the program frames indicated and reproduced on the following page.

[b]var. (item) = 0.00.

Check your answers.

(1) $8 \times 1 = 8$

(2) $3 \times 1 = 3$

(3) $48 \times 1 = 48$

(4) $0 \times 1 = 0$

(5) $1 \times 1 = 1$

(6) $1776\frac{7}{8} \times 1 = 1776\frac{7}{8}$

Record your results.

* * *

Complete the following proof of the statement:

$$(25 \times 50) \times (4 \times 2) = (25 \times 4) \times (50 \times 2)$$

(1)	$(25 \times 50) \times (4 \times 2) =$	$[($? $) \times 4] \times 2$		[?]
(2)	?	$= [25 \times ($? $)] \times 2$		[?]
(3)	?	$= [25 \times ($? $)] \times 2$		[cpm]
(4)	?	=	?	[?]
(5)	?	=	?	[?]

Therefore,

_____ ? _____ .

Turn to PAGE 10.

Table 25[a]

Descriptive Statistics for Items on Pages 10-11 of Part 110.5

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.437	.047
Program Leading (Program followed by conventional instruction)	16	.625	.083
Program Following (Conventional followed by programed instruction)	8	.750	.712**

[a]These data relate to the program frames indicated and reproduced on the following pages.

**Significant at the .05 level with appropriate degrees of freedom (number of students minus two).

Check your answers.

Here is a proof of the statement:

$$(25 \times 50) \times (4 \times 2) = (25 \times 4) \times (50 \times 2)$$

(1) $(25 \times 50) \times (4 \times 2) = [(25 \times 50) \times 4] \times 2$ [apm]

(2) $[(25 \times 50) \times 4] \times 2 = [25 \times (50 \times 4)] \times 2$ [apm]

(3) $[25 \times (50 \times 4)] \times 2 = [25 \times (4 \times 50)] \times 2$ [cpm]

(4) $[25 \times (4 \times 50)] \times 2 = [(25 \times 4) \times 50] \times 2$ [apm]

(5) $[(25 \times 4) \times 50] \times 2 = (25 \times 4) \times (50 \times 2)$ [apm]

Therefore,

$$(25 \times 50) \times (4 \times 2) = (25 \times 4) \times (50 \times 2)$$

Record your results.

* * *

Use the associative principle for addition of real numbers to simplify the following:

(1) $(^-75 + ^+9) + ^+91$

(2) $(^+50 + ^-33) + ^-67$

(3) $(^+31 + ^-34) + ^+134$

(4) $(^-287 + ^-25) + ^+24$

(5) $^-28 + (^+28 + ^-75)$

(6) $^-25 + (^-50 + ^+75)$

(7) $^+397 + (^+189 + ^-397)$

(8) $(^-237 + ^+626) + (^-626 + ^+237)$

Turn to PAGE 11

Check your answers.

(1) $(^-75 + {}^+9) + {}^+91 = \underbrace{^-75 + \underbrace{(^+9 + {}^+91)}_{^+100}}_{^+25}$

(2) $(^+50 + {}^-33) + {}^-67 = \underbrace{^+50 + \underbrace{(^-33 + {}^-67)}_{^-100}}_{^-50}$

(3) $(^+31 + {}^-34) + {}^+134 = \underbrace{^+31 + \underbrace{(^-34 + {}^+134)}_{^+100}}_{^+131}$

(4) $(^-287 + {}^-25) + {}^+24 = \underbrace{^-287 + \underbrace{(^-25 + {}^+24)}_{^-1}}_{^-288}$

(5) $^-28 + (^+28 + {}^-75) = \underbrace{\underbrace{(^-28 + {}^+28)}_{0} + {}^-75}_{^-75}$

(6) $^-25 + (^-50 + {}^+75) = \underbrace{\underbrace{(^-25 + {}^-50)}_{^-75} + {}^+75}_{0}$

(7) $^+397 + (^+189 + {}^-397) = {}^+397 + (^-397 + {}^+189)$

$= \underbrace{\underbrace{(^+397 + {}^-397)}_{0} + {}^+189}_{^+189}$

(8) $(^-237 + {}^+626) + (^-626 + {}^+237) = [(^-237 + {}^+626) + {}^-626] + {}^+237$

$= [^-237 + (^+626 + {}^-626)] + {}^+237$

$= [(^+626 + {}^-626) + {}^-237] + {}^+237$

$= \underbrace{\underbrace{(^+626 + {}^-626)}_{0} + \underbrace{(^-237 + {}^+237)}_{0}}_{0}$

Record your results.

* * *

Turn to PAGE 12.

THE SIZE OF THE STEP BETWEEN FRAMES

In theory, the size of the step between frames is an easy variable to manipulate, by simply omitting intervening frames in an already existant program. The difficult part is to determine whether, in fact, the size of the step has been changed. The large step size version (Part 112L) and the small step size version (Part 112) in this series were produced by the method of simple omission of non essential frames. To find out whether step size actually had been changed, item difficulties were compared. To determine the effect of step size upon learning, the end test scores of students who had different booklets were compared.

The portions selected for presentation here are corresponding in that each covers a segment. They are the same except for the fact that the 112 segment has a few more frames. The item difficulties here have special significance. The interested reader is referred to L. M. Stolurow and M. Beberman, Comparative Studies of Principles for Programming Mathematics in Automated Instruction. Urbana, Ill: Univer. of Illinois, USOE Contr. No. 711151.01, Quarterly Reports 9 and 19, June - December, 1963, for more details and to the forthcoming final report.

Tables 26 through 38 give descriptive statistics for the subsequent frames.

The small step sequence (112) should produce learning which is at least as good as that produced by the large step size sequence (112L). However, if learning is equal, there should be a time saving in the use of the 112L book since it is shorter. Learning was found to be equal ($F = 1.68$, $df = 1, 72$) and the large step version produced a time saving, however, time per page was about equal for both groups.

Table 26[a]

Descriptive Statistics for Items on

Page 32 of Part 112

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	20	.800	.123
Program Leading (Program followed by conventional instruction)	11	.909	-.197
Program Following (Conventional followed by programed instruction)	8	.875	.644

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

(1) The principle for *subtraction* tells us how to convert a problem in subtracting a real number into a problem in *adding the opposite* of that real number.

(2) *No*; it is not possible to solve each problem in subtracting a number of arithmetic.

Record your results.

* * *

Here is a problem in subtracting a number of arithmetic.

$$3 - 8 = ?$$

This problem can't be solved. Let's see why. Subtracting the number 8 of arithmetic is the inverse of adding the number 8 of arithmetic. So, to solve this problem, amounts to finding the number of arithmetic to which we can add 8 and get 3. Is there such a number of arithmetic? [Answer 'yes' or 'no' on your work sheet.]

Turn to PAGE 33.

Table 27[a]

Descriptive Statistics for Items on

Page 33 of Part 112

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	20	.750	.291
Program Leading (Program followed by conventional instruction)	11	.818	.413
Program Following (Conventional followed by programed instruction)	8	.625	.193

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answer.

No ; it is not possible to find a number of arithmetic to which we can add 8 and get 3.

Record your result.

* * *

Here are some subtraction problems with numbers of arithmetic. Which ones can be solved? [On your work sheet, circle the 'yes' for those which can, and circle the 'no' for those which can't.]

(1) 9 - 2 = ? (2) 8 - 11 = ? (3) 11 - 8 = ? (4) 9 - 9 = ?

(5) 6 - 5 = ? (6) 6 - 5.9 = ? (7) 6 - 5.99 = ? (8) 6 - 6.1 = ?

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Table 28[a]

Descriptive Statistics for Items on

Page 34 of Part 112

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	20	.800	.005
Program Leading (Program followed by conventional instruction)	11	.818	-.445
Program Following (Conventional followed by programed instruction)	8	.875	-.592

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

Subtraction problems for numbers of arithmetic.

(1) 9 - 2 = ?; *yes* (2) 8 - 11 = ?; *no*

(3) 11 - 8 = ?; *yes* (4) 9 - 9 = ?; *yes*

(5) 6 - 5 = ?; *yes* (6) 6 - 5.9 = ?; *yes*

(7) 6 - 5.99 = ?; *yes* (8) 6 - 6.1 = ?; *no*

Record your results.

* * *

There is no such thing as an impossible subtraction problem for real numbers. The fact that we can find the opposite of any real number and the fact that we can always add this opposite assure us that we can do any subtraction problem.

Do the following subtraction problems. Write the answers on your work sheet.

Sample. $^{+}4 - {}^{+}9 = {}^{+}4 + {}^{-}9 =$ *-5* ← Answer.

(1) $^{+}9 - {}^{+}2 =$? (2) $^{+}8 - {}^{+}11 =$? (3) $^{+}11 - {}^{+}8 =$? (4) $^{+}9 - {}^{+}9 =$?

(5) $^{+}6 - {}^{+}5 =$? (6) $^{+}6 - {}^{+}5.9 =$? (7) $^{+}6 - {}^{+}5.99 =$? (8) $^{+}6 - {}^{+}6.1 =$?

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Table 29[a]

Descriptive Statistics for Items on

Page 35 of Part 112

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	20	.750	.372
Program Leading (Program followed by conventional instruction)	11	.909	-.061
Program Following (Conventional followed by programed instruction)	8	1.000	undefined[b]

[a]These data relate to the program frames indicated and reproduced on the following page.

[b]var. (item) = 0.00.

Check your answers.

(1) $^{+}9 - {}^{+}2$

$= {}^{+}9 + {}^{-}2$

$= {}^{+}7$

(2) $^{+}8 - {}^{+}11$

$= {}^{+}8 + {}^{-}11$

$= {}^{-}3$

(3) $^{+}11 - {}^{+}8 = {}^{+}3$

(4) $^{+}9 - {}^{+}9 = 0$

(5) $^{+}6 - {}^{+}5 = {}^{+}1$

(6) $^{+}6 - {}^{+}5.9 = {}^{+}0.1$

(7) $^{+}6 - {}^{+}5.99 = {}^{+}0.01$

(8) $^{+}6 - {}^{+}6.1 = {}^{-}0.1$

Record your results.

* * *

Complete each of the following. [Do NOT use the '$\mathcal{O}(...)$' notation.]

Sample. $^{+}6 - {}^{-}9 = {}^{+}6 + \underline{\ ?\ }$

Solution. $^{+}6 - {}^{-}9 = {}^{+}6 + {}^{+}9$

(1) $^{+}9 - {}^{-}3 = {}^{+}9 + \underline{\ ?\ }$

(2) $^{-}5 - {}^{-}3 = {}^{-}5 + \underline{\ ?\ }$

(3) $0 - {}^{+}3 = 0 + \underline{\ ?\ }$

(4) $^{-}6 - {}^{+}8 = \underline{\ ?\ } + {}^{-}8$

(5) $^{-}7 - 0 = {}^{-}7 + \underline{\ ?\ }$

(6) $^{-}14 - {}^{-}3 = \underline{\ ?\ } + {}^{+}3$

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Table 30[a]

Descriptive Statistics for Items on

Page 36 of Part 112

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	20	.750	.047
Program Leading (Program followed by conventional instruction)	11	.727	.143
Program Following (Conventional followed by programed instruction)	8	.500	.340

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

(1) $^{+}9 - {}^{-}3 = {}^{+}9 + \underline{{}^{+}3}$

(2) $^{-}5 - {}^{-}3 = {}^{-}5 + \underline{{}^{+}3}$

(3) $0 - {}^{+}3 = 0 + \underline{{}^{-}3}$

(4) $^{-}6 - {}^{+}8 = \underline{{}^{-}6} + {}^{-}8$

(5) $^{-}7 - 0 = {}^{-}7 + \underline{0}$

(6) $^{-}14 - {}^{-}3 = \underline{{}^{-}14} + {}^{+}3$

Record your results.

* * *

Now, solve these subtraction problems.

(1) $^{+}9 - {}^{-}3 =$?

(2) $^{-}5 - {}^{-}3 =$?

(3) $0 - {}^{+}3 =$?

(4) $^{-}6 - {}^{+}8 =$?

(5) $^{-}7 - 0 =$?

(6) $^{-}14 - {}^{-}3 =$?

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Table 31[a]

Descriptive Statistics for Items on

Page 37 of Part 112

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	20	.750	.426
Program Leading (Program followed by conventional instruction)	11	.818	.110
Program Following (Conventional followed by programed instruction)	8	.625	.475

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

(1) $^{+}9 - {}^{-}3 = {}^{+}9 + {}^{+}3 =$ $^{+}12$

(2) $^{-}5 - {}^{-}3 = {}^{-}5 + {}^{+}3 =$ $^{-}2$

(3) $0 - {}^{+}3 = 0 + {}^{-}3 =$ $^{-}3$

(4) $^{-}6 - {}^{+}8 = {}^{-}6 + {}^{-}8 =$ $^{-}14$

(5) $^{-}7 - 0 = {}^{-}7 + 0 =$ $^{-}7$

(6) $^{-}14 - {}^{-}3 = {}^{-}14 + {}^{+}3 =$ $^{-}11$

Record your results.

* * *

Answer the following on your work sheet.

(1) $^{+}11 - {}^{+}2 =$?

(2) $^{+}18 - {}^{-}4 =$?

(3) $0 - {}^{+}7 =$?

(4) $^{+}3 - {}^{+}8 =$?

(5) $^{-}65 - 0 =$?

(6) $^{-}56 - {}^{-}4 =$?

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Table 32[a]

Descriptive Statistics for Items on

Page 38 of Part 112

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	20	.600	.321
Program Leading (Program followed by conventional instruction)	11	.727	.187
Program Following (Conventional followed by programed instruction)	8	.625	.193

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

(1) $^{+}11 - {}^{+}2 = {}^{+}11 + {}^{-}2 =$ $^{+}9$

(2) $^{+}18 - {}^{-}4 = {}^{+}18 + {}^{+}4 =$ $^{+}22$

(3) $0 - {}^{+}7 = 0 + {}^{-}7 =$ $^{-}7$

(4) $^{+}3 - {}^{+}8 = {}^{+}3 + {}^{-}8 =$ $^{-}5$

(5) $^{-}65 - 0 = {}^{-}65 + 0 =$ $^{-}65$

(6) $^{-}56 - {}^{-}4 = {}^{-}56 + {}^{+}4 =$ $^{-}52$

Record your results.

* * *

Fill in the blanks on your work sheet.

(1) $^{-}23 - {}^{-}30 =$?

(2) $^{+}7 - {}^{-}20 =$?

(3) $^{-}19 - {}^{+}69 =$?

(4) $^{+}19 - {}^{-}69 =$?

(5) ? $- {}^{-}11 = {}^{-}6$

(6) $^{-}5 -$? $= {}^{-}6$

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Table 33[a]

Descriptive Statistics for Items on

Page 39 of Part 112

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	20	.600	.321
Program Leading (Program followed by conventional instruction)	11	.636	-.188
Program Following (Conventional followed by programed instruction)	8	.750	.354

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

(1) $^{-}23 - {}^{-}30 = {}^{-}23 + {}^{+}30 =$ $^{+}7$ (2) $^{+}7 - {}^{-}20 = {}^{+}7 + {}^{+}20 =$ $^{+}27$

(3) $^{-}19 - {}^{+}69 = {}^{-}19 + {}^{-}69 =$ $^{-}88$ (4) $^{+}19 - {}^{-}69 = {}^{+}19 + {}^{+}69 =$ $^{+}88$

(5) $^{-}17 - {}^{-}11 = {}^{-}6$ [Check: $^{-}17 - {}^{-}11 = {}^{-}17 + {}^{+}11 = {}^{-}6$]

(6) $^{-}5 - {}^{+}1 = {}^{-}6$ [Check: $^{-}5 - {}^{+}1 = {}^{-}5 + {}^{-}1 = {}^{-}6$]

Record your results.

* * *

Write the answers to these questions on your work sheet.

(1) $^{+}8 - {}^{-}3 =$? (2) $^{-}4 - {}^{+}7 =$?

(3) ? $- {}^{-}3 = {}^{+}5$ (4) ? $- {}^{+}7 = {}^{-}2$

(5) $^{-}2 -$? $= {}^{-}7$ (6) $^{+}8 -$? $= {}^{-}8$

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Table 34[a]

Descriptive Statistics for Items on

Page 22 of Part 112L

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.750	.371
Program Leading (Program followed by conventional instruction)	13	.923	-.088
Program Following (Conventional followed by programed instruction)	10	1.000	undefined[b]

[a]These data relate to the program frames indicated and reproduced on the following page.

[b]var. (item) = 0.00.

Check your answers.

(1) The principle for subtraction tells us how to convert a problem in subtracting a real number into a problem in adding the opposite of that real number.

(2) No; it is not possible to solve each problem in subtracting a number of arithmetic.

Record your results.

* * *

Here is a problem in subtracting a number of arithmetic.

$$3 - 8 = ?$$

This problem can't be solved. Let's see why. Subtracting the number 8 of arithmetic is the inverse of adding the number 8 of arithmetic. So, to solve this problem, amounts to finding the number of arithmetic to which we can add 8 and get 3. Is there such a number of arithmetic? [Answer 'yes' or 'no' on your work sheet.]

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Table 35[a]

Descriptive Statistics for Items on

Page 23 of Part 112L

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.437	.216
Program Leading (Program followed by conventional instruction)	13	.615	.706**
Program Following (Conventional followed by programed instruction)	10	.600	-.174

[a]These data relate to the program frames indicated and reproduced on the following page.

*Significant at the .01 level with appropriate degrees of freedom (number of students minus two).

**Significant at the .05 level with appropriate degrees of freedom (number of students minus two).

Check your answer.

No ; it is not possible to find a number of arithmetic to which we can add 8 and get 3.

Record your result.

* * *

There is no such thing as an impossible subtraction problem for real numbers. The fact that we can find the opposite of any real number and the fact that we can always add this opposite assure us that we can do any subtraction problem.

Do the following subtraction problems. Write the answers on your work sheet.

Sample. $^{+}4 - {}^{+}9 = {}^{+}4 + {}^{-}9 =$ 5 ← **Answer.**

(1) $^{+}9 - {}^{+}2 =$? **(2)** $^{+}8 - {}^{+}11 =$? **(3)** $^{+}11 - {}^{+}8 =$? **(4)** $^{+}9 - {}^{+}9 =$?

(5) $^{+}6 - {}^{+}5 =$? **(6)** $^{+}6 - {}^{+}5.9 =$? **(7)** $^{+}6 - {}^{+}5.99 =$? **(8)** $^{+}6 - {}^{+}6.1 =$?

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Table 36[a]

Descriptive Statistics for Items on

Page 24 of Part 112L

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.750	-.059
Program Leading (Program followed by conventional instruction)	13	.615	-.062
Program Following (Conventional followed by programed instruction)	10	.500	.494

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

(1) $^{+}9 - {}^{+}2$

$= {}^{+}9 + {}^{-}2$

$= {}^{+}7$

(2) $^{+}8 - {}^{+}11$

$= {}^{+}8 + {}^{-}11$

$= {}^{-}3$

(3) $^{+}11 - {}^{+}8 = {}^{+}3$

(4) $^{+}9 - {}^{+}9 = 0$

(5) $^{+}6 - {}^{+}5 = {}^{+}1$

(6) $^{+}6 - {}^{+}5.9 = {}^{+}0.1$

(7) $^{+}6 - {}^{+}5.99 = {}^{+}0.01$

(8) $^{+}6 - {}^{+}6.1 = {}^{-}0.1$

Record your results.

* * *

Now, solve these subtraction problems.

(1) $^{+}9 - {}^{-}3 =$?

(2) $^{-}5 - {}^{-}3 =$?

(3) $0 - {}^{+}3 =$?

(4) $^{-}6 - {}^{+}8 =$?

(5) $^{-}7 - 0 =$?

(6) $^{-}14 - {}^{-}3 =$?

Turn to PAGE 25.

Table 37[a]

Descriptive Statistics for Items on

Page 25 of Part 112L

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.687	.109
Program Leading (Program followed by conventional instruction)	13	.769	-.325
Program Following (Conventional followed by programed instruction)	10	.800	.033

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

(1) $^{+}9 - {}^{-}3 = {}^{+}9 + {}^{+}3 = {}^{+}12$

(2) $^{-}5 - {}^{-}3 = {}^{-}5 + {}^{+}3 = {}^{-}2$

(3) $0 - {}^{+}3 = 0 + {}^{-}3 = {}^{-}3$

(4) $^{-}6 - {}^{+}8 = {}^{-}6 + {}^{-}8 = {}^{-}14$

(5) $^{-}7 - 0 = {}^{-}7 + 0 = {}^{-}7$

(6) $^{-}14 - {}^{-}3 = {}^{-}14 + {}^{+}3 = {}^{-}11$

Record your results.

* * *

Fill in the blanks on your work sheet.

(1) $^{-}23 - {}^{-}30 =$?

(2) $^{+}7 - {}^{-}20 =$?

(3) $^{-}19 - {}^{+}69 =$?

(4) $^{+}19 - {}^{-}69 =$?

(5) ? $- {}^{-}11 = {}^{-}6$

(6) $^{-}5 -$? $= {}^{-}6$

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Table 38[a]

Descriptive Statistics for Items on

Page 26 of Part 112L

Proportion Correct and Correlation with End Test

Mode	Number of Students	Proportion Correct	Point Biserial Correlation of item score with test score
Pure (Program only)	16	.625	-.080
Program Leading (Program followed by conventional instruction)	13	.692	.223
Program Following (Conventional followed by programed instruction)	10	.700	.068

[a]These data relate to the program frames indicated and reproduced on the following page.

Check your answers.

(1) $^{-}23 - {}^{-}30 = {}^{-}23 + {}^{+}30 =$ $^{+}7$ (2) $^{+}7 - {}^{-}20 = {}^{+}7 + {}^{+}20 =$ $^{+}27$

(3) $^{-}19 - {}^{+}69 = {}^{-}19 + {}^{-}69 =$ $^{-}88$ (4) $^{+}19 - {}^{-}69 = {}^{+}19 + {}^{+}69 =$ $^{+}88$

(5) $^{-}17$ $- {}^{-}11 = {}^{-}6$ [Check: $^{-}17 - {}^{-}11 = {}^{-}17 + {}^{+}11 = {}^{-}6$]

(6) $^{-}5 -$ $^{+}1$ $= {}^{-}6$ [Check: $^{-}5 - {}^{+}1 = {}^{-}5 + {}^{-}1 = {}^{-}6$]

Record your results.

* * *

Fill in the blanks on your work sheet.

(1) $^{-}2 - {}^{-}19 =$?

(2) ? $- {}^{-}7 = {}^{-}10$

(3) $^{-}14 -$? $= {}^{-}20$

(4) $^{-}6 - {}^{+}16 =$?

(5) ? $- {}^{+}8 = {}^{+}4$

(6) $^{-}7 -$? $= {}^{+}10$

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