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GENERAL REPORT OF MATHEMATICS CONFERENCE AND TWO SPECIFIC REPORTS. (TITLE SUPPLIED).

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THE FIRST PAPER, "REPORT OF MATHEMATICS CONFERENCE," IS A SUMMARY OF DISCUSSIONS BY 29 PARTICIPANTS IN A CONFERENCE ON CURRENT PROBLEMS IN MATHEMATICS EDUCATION RESEARCH. REPORTED ARE (1) RECENT PROGRESS, PROBLEMS, AND PLANS OF CURRICULUM DEVELOPMENT GROUPS, (2) GENERAL FORMULATION OF CURRICULUM AND METHODS, (3) TEACHER TRAINING, (4) COMMUNICATION, AND (5) RESEARCH MANPOWER. THE SECOND PAPER, "ELEMENTARY MODERN MATHEMATICS FROM THE ADVANCED STANDPOINT," GIVES A RECOMMENDED SYLLABUS FOR THE FIRST FIVE SEMESTERS OF A SIX-SEMESTER COLLEGE MATHEMATICS COURSE TO BE TAKEN BY POTENTIAL HIGH SCHOOL TEACHERS. FINALLY, THE THIRD PAPER, "SCHOOL MATHEMATICS STUDY GROUP (SMSG) AND THE GIFTED CHILD," REPORTS THE REASON FOR INCLUDING EXPLICIT RECOMMENDATIONS FOR EDUCATING THE GIFTED CHILD IN ANY LONG-RANGE CURRICULUM REFORM. (RP)

REPORT OF MATHEMATICS CONFERENCE

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CONFERENCE ON CURRENT PROBLEMS IN MATHEMATICS EDUCATION RESEARCH

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INTRODUCTION

For about a decade there has been important activity in the development of new methods of teaching mathematics in the elementary and high schools. The number of groups doing research in the field has grown steadily. While some degree of awareness of each other's activities was important to the groups from the beginning, this need has grown faster than the number of groups. Two groups in particular, the SMSG and the CCSM, have pressing requirements for a thorough knowledge of up-to-date developments in mathematics education. The School Mathematics Study Group (SMSG) is engaged in the development of new mathematics material for all grade levels, to be widely used throughout the Nation's schools. The Cambridge Conference on School Mathematics (CCSM) has been concerned with the delineation of long range goals for mathematics education, based on the innate abilities of the student and on his needs in our society.

With N.S.F. support these two groups arranged a conference in March 1965. Invitations were accepted by representatives of most of the research groups and those concerned with teacher training in mathematics education. The intention of the meeting was to learn of each group's past experiences, present research program and immediate plans. It was hoped that a clear picture of difficulties being faced at this stage would emerge. It was intended that regular means of communication would be initiated for the future. Communication with the public and those in education was discussed. In addition, time was scheduled to consider the problem of teacher training, the training of teachers of teachers, and the recruitment of mathematicians to the research groups. A resume of the three days of reports and discussions is presented here. No attempt is made to keep to the chronological order of the discussion. Unfortunately, many interesting details have been omitted to allow the spectrum of discussion to emerge better.

RECENT PROGRESS, PROBLEMS AND PLANS OF CURRICULUM DEVELOPMENT GROUPS

The bulk of the reports were concerned with the description of specific new curricular material as tried on a small number of students. But many groups have had experience with the use of materials for large numbers of children. Their reports were also much concerned with evaluation of past results and their present ideas on a change of direction. Professor Edward G. Begle and Dr. J. Fred Weaver stressed that SMSG had completed its first round of writing directed at supplying the college bound with a more interesting and deeper approach to subject matter already in the curriculum. They have more recently been going beyond that framework in, for instance, the geometric material for K-3 and in probability for 7-12. Special attention is being given to trying modified material on low ability classes. They are now evaluating response to determine whether the assumption that SMSG type curriculum is most applicable to the college bound or to above average student is justified. The more specifically mathematical questions, such as the effectiveness of set theory, or the use of number line in early grades are also to be examined in the light of studies of past achievement. The problems of teaching the culturally deprived are being considered.

Professor Max Beberman reported on the development by UCISM of units designed to develop the basic arithmetic skills among the general student population. The use of "stretcher" and "shrinker" machines introduces an operational concept that appears to overcome some of the difficulties involved in the thorough learning of multiplication and division of the reals. Professor Beberman's concern with thorough acquirement of basic needs was expressed by his plea for careful testing and evaluation of new programs before their use on a large scale.

Mrs. Lore Rasmussen stressed the need for designing curricula for young children which met their need for concrete representation and for an open-ended discovery approach geared to their individual interests and abilities. She pointed out that often advantage is not taken of the wealth of early concepts and intuition brought to the classroom by youngsters, as for instance their concept of real number connected with size.

The combining of a mathematics and a science program was a need that emerged from the work of Minnimath and led to Minnimast. Dr. Paul C.

Rosenbloom reported on the writing and testing of an integrated program now completed to fourth grade in mathematics and to second grade in science. He also described several units of an advanced nature for use in the high school. The topics included number theory, and approach to harmonic motion through finite difference equations and computer programming.

The Madison Project work for elementary schools was summarized by Professor Robert B. Davis. Their curriculum is being modified in the direction of deduction and formal proof. While stressing formality less than the familiar Suppes' program, Professor Davis believes that the child benefits from the more clear-cut approach to establishing true from false. Furthermore, the child is impressed with the power of tools that permit many deductions from a few axioms or "short lists".

In the absence of Professor Patrick Suppes, Dr. Shirley Hill described some of his work with computer controlled programmed learning. The advantages were that the student controlled the pace and times of utilization. Also the teachers were left free to chat individually with students who come to question. It is inferred that this may tend to humanize rather than dehumanize education.

Professor Bryan Thwaites, a visitor from the U.K., pointed out that their new high school curriculum utilized "motion geometry" in place of Euclid. This meant that the burden of teaching the deductive approach to proofs now fell onto the algebraic studies.

The many representatives of CCSM took turns in describing their experiments since the Cambridge Conference of June-August 1963. Some of this took place in the 1964 summer session at Morse School in Cambridge, while other work was done during the school year. The projects were unconnected to each other but were designed to test the hypotheses or work out some details of the "Goals for School Mathematics" which came out of the 1963 conference. Mr. Henry Pollak and Professor Ellis Kolchin were impressed by the materials developed, and the children's activities at Miss Mason's school in Princeton, New Jersey. A set of integrated units on the number line, the base systems and geometry have been written up for kindergarten classes. Detailed instructions for the teachers' handling of the youngsters are included, so that a teacher with no experience in the mathematics was able to teach it well.

With the active participation of several teachers at Estabrook School in Lexington, Mass., Professor Earle E. Lomon had tried inequalities and number line work in first and second grades, motion geometry and group algebra in third and fourth grades, and sequences, algebra, graphing and slopes, and experimental probability in fifth and sixth grades. The 1964 summer session at Morse School was used to try inequalities on pre-first graders, and slopes on pre-seventh graders. He believed that this material showed promise in giving motivation and practice in the basic skills. The logical and deductive abilities of the children had been pushed too hard but much could be done in this direction.

In the summer session at the Morse School, Professor Andrew M. Gleason had developed symmetry, multiplication of large numbers on graph paper and chip trading (base 3) in grades two and three. In grade two the manipulative skill of the children was found to be limiting. Miss Walter had developed further her intuitive symmetry units with mirror cards and the construction of figures from triangles and squares. She found that these evoked interest and an able response at grade levels from K to 6. Professor Kolchin's work with pre-sixth graders used a graphical approach to max-min problems to foster measurement and arithmetic as well as geometric ideas. At the same grade level a sound development of vector geometry was evolved by Professor Shlomo Z. Sternberg. With Professor Griffiths, Professor Hilton had found modular arithmetic on a useful approach to algebraic ideas with pre-seventh graders.

With seventh grade students Professor Bernard Friedman had developed an alternative to the conventional approaches to geometry. The basis was reflection symmetry. To give substance to the more obvious results, the geometry of the sphere was built up simultaneously with that of the plane.

GENERAL FORMULATION OF CURRICULA AND METHODS

There was a discussion of the criteria which could be applied to whether the schools were doing a good job in mathematics or not, and to the design of new curricula.

Professor Gleason's dictum that the student should find out how ideas influence things and events seemed to provide a starting point. This implied that abstract aspects of mathematics most nearly relevant to answering questions about the external world were most relevant. This would emphasize the real number system, rather than abstract set theory. Professor Begle pointed out that material, such as sets, was often needed pedagogically as a precursor to some mathematics of more direct use.

Logical reasoning, inventiveness and the courage to think ahead, attributes advocated by Dr. Jerrold R. Zacharias, could be fostered in the young by intuitive and concrete representations. The work of Miss Walter and of Mrs. Rasmussen exemplified this in many ways. Professor Hilton urged the importance of honest teaching at every stage so that the student need never rebel at unlearning anything.

There was considerable discussion of the weight that our present computer age should have in determining the curriculum. It was agreed that teaching programming introduced step-by-step logic in a valuable way. It was also proposed that this approach could be used in teaching the culturally deprived, as it presented strong motivation in glamour and for job training.

Many groups voiced their present concern with the below average achiever and with the culturally deprived.

Another area in which stress is being put is the overlap in the curricula of mathematics and science. Various groups were trying different approaches to this.

TEACHER TRAINING

It was noted that further use of materials in elementary school was severely limited by the lack of teachers with sufficient mathematical background. As long as there was not specialization of elementary school staff the training problem was enormous, involving all elementary teachers. There was some discussion of whether approaches such as "team teaching" could utilize a degree of specialization without presenting the young child with a too heterogeneous format.

On the other hand many high school teachers were already suitably trained and one could contemplate the task of teaching all the mathematics specialists.

Professor Gail Young made an estimate that $30,000,000 over a three year period was required for the task -- as much as for a mile of the Massachusetts Turnpike! A study of a group of teachers' colleges had shown that the bulk of those graduated went straight into teaching with a very inadequate mathematics background.

Dr. Rosenbloom contended that the presently organized teacher training programs had slots, methods courses and teacher training, which could be redirected and improved to satisfy many of our training needs. The methods courses could i) develop the correspondence of the school level subject matter to the college level matter, and ii) treat learning problems and procedures in the relevant context of the mathematics and science material. The methods' texts should provide a sort of laboratory manual for the teacher, with which to initiate experiments in modern mathematics teaching during student teaching experience.

Dr. Zacharias spoke of the need to attract bright young people into teaching. (The prestige question came up several times). New materials for teacher's colleges are badly needed. E.S.I. has made an arrangement with ten Massachusetts State Colleges for the development of material and the trading of people.

CCSM announced that it is organizing a meeting on the goals for the training of teachers of mathematics for the summer of 1966.

COMMUNICATION

Communication among research groups, between the groups and those responsible for adoption in schools, and between the groups and the public were all discussed. Not many generally approved proposals were made.

With respect to intergroup communication, it was felt that a meeting of the present type was of value and should be held about once a year. It was agreed that some form of journal communication was required in addition. The specific preferences for this type of communication ranged from newsletters, to sections of existing journals, to an information sheet gathered by a "wandering reporter", to a journal dedicated to research of a professional standard in mathematics curricula. The requirements of speed in learning of even tentative results, the need for prestige publications to enhance

the career of the researcher, and the necessity of avoiding the implication of any national consensus on the suitability of relatively new material, conflicted inextricably. For the meantime, it was decided to explore further the use of portions of existing journals.

Some of the above journal forms would also be suitable for informing educational systems of the available material. In many cases repositories of such material, including films, exist, but their presence is not generally recognized and their own distribution system not adapted to all needs.

The need for several competent reviews of new texts was stressed. The usefulness of extending this to reviews of curricula was debated. The encouragement of reviews was recommended to NCTM as an urgent matter for the math curriculum reform movement.

Dr. Zacharias pointed out that the opening of ten regional educational laboratories throughout the U.S., under the Office of Education, would be a new means of disseminating, as well as developing, curricular and teacher training material. He also advocated greater contact with publishers and with science writers. More T.V. programs on the subject of education are in the making.

RESEARCH MANPOWER

The ever-increasing demands on able mathematicians interested in school education were ruefully noted. New projects throughout the world, as well as the increasing pace of reform in the U.S., contribute to a shortage. Young people find the area attractive but are fully occupied with academic duties and mathematics research. Much discussion was devoted to the problem of giving due credit for curricular research, for academic advancement. While it seemed that a few individual departments may give such recognition, in general, they do not. To further his professional career, a junior faculty member must devote considerable time to research in his field. It was suggested, as stated above, that a suitably professional journal may alleviate this problem. Professor Edwin E. Moise was asked to write department chairmen of universities, asking them to encourage research mathematicians to do work in education and to give prestige to the endeavor.

IN CONCLUSION

The conference finished with many debates left incomplete and many viewpoints left unsaid. It had clearly opened up many more questions than it had answered, even tentatively. It was felt by the majority that such a meeting had been necessary as a new point of departure, in spite of the frustrating lack of time to delve further into problems. Perhaps the ready agreement of people hard pressed for time to meet again in a year testifies best to the value of the conference.

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Elementary Modern Mathematics from the Advanced Standpoint

In order that any syllabus devised for the high school should be teachable, there must be those able and available to teach it. Thus, we recommend that the syllabus should form the first 5 semesters of a 6-semester college math course to be taken by potential high school teachers. Of course, its content should be familiar to all who have been through a college math course. The 6th semester would be taken up with more advanced material and material omitted from the course as proposed on the grounds of shortage of time (e.g., multiple integrals, Stokes' theorem, elementary group theory, linear programming).

We have aimed at achieving the right level for high school mathematics, the material should be suitable, and it is of supreme importance that the attitude inculcated should be right. We place less emphasis on actual content than on level and point of view. Nor do we insist in all cases on the precise order of presentation laid down in our draft although there are certain principles (algebra before analysis as preparation for the calculus, for example) which we would not sacrifice. But, we believe this curriculum may be regarded as a l.u.b. in future discussions. We can make no valid prediction about the time required in the high school for the topics discussed in the syllabus; but we are quite confident that they can be taught and absorbed in 5 semesters in college. (We would enunciate one principle to guide the choice of omissions from the syllabus; we should not sacrifice practice to conserve theory, since it is no use understanding what you would be talking about but having nothing to say).

Our advocacy of the suitability in college teaching of the proposed curriculum should not be interpreted as a retreat from our claim that the level, point of view and nature of the material are, in fact, appropriate to a high school syllabus. But the realities of the educational situation require that in the first instance we must train the teachers to teach it -- and that we have in any case to wait many years before students will reach the 7th grade equipped, as we would wish them to be equipped, for launching on the course. However, we would immodestly claim that in fact the course is superior in many ways to that currently given in many colleges. We point out that the part of the algebra course, which would be completed in the first semester, by presenting the formal differentiation of polynomials,

behaviour at infinity, orders of magnitude, etc., would provide a good preparation for elementary physics.

The course would almost certainly require the writing of special textbooks. However, rather than wait on the production of such textbooks--and in the meanwhile introduce no reform into college courses--it would clearly be preferable to introduce certain reforms immediately and utilize such high-quality texts as do in fact exist. In this connection, we might instance the texts by Moise (geometry), Mostow, etc., (algebra) and Courant (calculus). We believe it would be difficult, if not impossible, to devise textbooks suitable for the transmission of this material to both college and high school students. It might be added that the provision of special textbooks may incline colleges to adopt the syllabus, at least experimentally; and it avoids misunderstanding.

We offer some comments on the impressions we have gained from our deliberations. We have been aware of the danger of putting ourselves-- or, rather, our contemporaries--in the position of the students for whom we have been trying to cater. There is a tendency in judging the difficulty of certain mathematical concepts and pieces of mathematical reasoning, to suppose that the student approaches these things equipped as our contemporaries were equipped at the same educational grade. Nowhere, perhaps, is this potential fallacy more glaring than in estimating the relative difficulties of algebra, geometry and calculus. There is a feeling among some that the algrebraic content of the course is too stiff, that for example, the notion of a polynomial is too sophisticated to be described to a 7th grader. The fact is that this notion is one of the few which can be described honestly and precisely at that level, and it is certainly mathematically useful at the stage at which one is interested in factorization; we happen also to believe it will be quite sufficiently 'real' to the 7th grader. We acknowledge that the notion of plane Euclidean geometry is also useful to the student at that stage and has enormous intuitive content--but converting it into a precise analytical concept, replete with coordination procedures, from an intuitive synthetic-geometrical concept, presents formidable difficulties. Not least of these is the quite general difficulty of passing from the intuitive to the precise; for, in the nature of things, no mathematics demonstration can validate the passage. Yet there may be some who maintain that it would be better to suppress polynomials, rather because there is no vague notion, familiar to the students at this stage, to be replaced by a

precise one; it seems to be held (elsewhere than in this subgroup) that advantage should be taken of the familiarity of the function concept in order to confuse the issue. In fact the greatest difficulty we have had in clarifying our own mathematical thinking about the topics appropriate to a high school course have centered around the problem of tying up intuitive with precise concepts. Apart from geometry, the examples which naturally come to mind are logarithms, the trigonometric functions, and concepts of length and area associated with curves. Certainly, we do not deny that these topics inevitably appear first in imprecise form, it would be sterilizing to await the mathematical sophistication needed to render them quite precise. But the very fact of their arrival in the course in immature form poses problems of teaching and mathematical understanding from which algebra is very largely free. Thus, we are unrepentant in giving the prominence we have to algebra, though we readily admit, as stated earlier, that it may be preferable to change the order of presentation in certain instances and to interleave the algebra with some geometry and elementary probability theory in the 7th and 8th grades.

A second - and related - point concerns the amount of unlearning that should figure in a well-designed course. The process of unlearning is frequently painful to both student and teacher; in the absence of really excellent rapport between student and teacher it can destroy the student's confidence. Thus, it is seen to be particularly important to present the student with an intuitive and imprecise approximation only when he is not yet ready for the real thing.

Third, we feel it is essential to show the greatest of pedagogical skill and insight in helping the student to decide the extent to which he should require proof of validity before feeling entitled to use a technique. It is very important not to inhibit the student's enthusiasm and facility for solving problems by preoccupying him excessively with scruples about rigour. The essential point here seems to be to develop intellectual honesty, so that the student knows what is being assumed and what has been proved, where the concept is quite precise and where it is imperfect. This is, of course, an end in itself; but it also has the immensely desirable effect of enabling the student to steer a middle course between the extremes of glib indifference to mathematical principles and paralyzing obsession with mathematical rigour. Discovery and proof are both vital in mathematics.

A fourth point concerns the relation of application to mathematics. It has seemed throughout the preparation of this syllabus that the need to develop proficiency in certain mathematical techniques by reason of their applicability has led inevitably to the discussion of important mathematical concepts (e.g., function, polynomial, vector space, limit); and, conversely, that the introduction of good mathematical topics, judged by the criteria of coherence and power, has led easily and smoothly to significant applications. Thus, we find no justification, in our own thinking, for the view that, at the high school level, at any rate, the needs of the potential professional mathematician are different from those of the potential professional user of mathematics and even more different from those of the intelligent citizen of the 21st century. Of course speeds and and styles of presentation will differ and so too will the facility of absorption, even between individuals belonging to the same broad category. But there seems to be a compelling and inescapable quality about good mathematics --and this must, in the last analysis, constitute the justification of the curriculum.

SMSG and the "gifted" child

It is generally recognized that, with the possible exception of music, intellectual precociousness and sophistication as well as the onset of intellectual maturity appear at a younger age for the mathematically talented than for any other intellectual discipline. Valid contributions to the mathematical literature are not infrequently made but a few years beyond the typical age for completion of the curriculum being here discussed. (This same point is made to the students in SMSG Jr. High Math, 6, pg. 57). Because of this, any long range curriculum reform, as a matter of principle, must take cognizance of these facts by including within its program explicit recommendations for educating children so gifted.

The standard arguments for enrichment being preferable to advancement certainly apply with regard to these children when in a public school setting. Enrichment not only broadens the child in depth and in scope, but also eases the pragmatics of teaching the more advanced levels.

Since it appears to be the consensus of this study group that providing text and problem material is probably the most important contribution to the standard curriculum that the "professionals" can make, it may well be that the best procedure vis-a-vis the gifted child is to make available to all grades enrichment material similar to the monographs that SMSG has sponsored for the high school level. Such material should be generally available in the classroom to those children who would like to read it. However, this material should be for pure pleasure; it should not be for advanced standing or better grades in the class. It should simply exist and its existence should be made known.

I don't feel that any of the above is either new or contrary to the views of most. However, I do feel that from the start more thought should be given to this group of students then has so far been given.

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