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USES OF SYMMETRY IN DESIGN EDUCATION. FINAL REPORT.

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A THEORY OF STRUCTURE IS ESSENTIAL TO AN OBJECTIVE ORGANIZATION OF BASIC PEDAGOGIES IN DESIGN. THE PURPOSE OF THIS STUDY WAS TO ASSESS THE STRUCTURAL THEORY OF MATHEMATICIAN K.L. WOLF AND TO TRANSLATE THIS THEORY INTO A VISUAL PRODUCT THAT COULD BE USED BY BEGINNING DESIGN STUDENTS. WOLF DESCRIBES 6 ISOMORPHIC COVERAGE OPERATIONS AND 7 HOMOEOMORPHIC COVERAGE OPERATIONS. TRANSLATION (T), ROTATION (R), AND MIRROR-REFLECTIONS (M) ARE THE THREE PRIME ISOMORPHIC OPERATIONS, AND COUPLED, THE THREE COMBINE INTO THREE MORE ISOMORPHIC OPERATIONS, (T + M), (T + R), AND (M + R). DILATION (D) IS THE BASIC HOMOEOMETRIC OPERATION WHICH IN TURN IS COMBINED WITH THE SIX ISOMORPHS TO COMPLETE THE LIST OF 13 COVERAGE OPERATIONS. IT WAS POSSIBLE TO JUSTIFY ALL 13 OPERATIONS ON VISUAL TERMS AND TO FIND EXTANT NATURAL OR MAN-MADE EXAMPLES OF 12 OF THESE. THE 13TH IS SO COMPLEX THAT IT CAN BE REPRESENTED IN DRAWING BUT MAY NOT EXIST IN ANY KNOWN OBJECT. FURTHER, A THEORY OF DOMAINS WHICH ARE RULED BY ELEMENTS WAS DEVELOPED. IN ANY ISOMORPHIC OR HOMOEOMORPHIC STRUCTURE, EACH DOMAIN IS OF THE SAME OR SIMILAR SHAPE, AND A TOTALITY OF THEM COMPLETELY FILLS SPACE, PLANAR OR THREE-DIMENSIONAL. A PRIMER FOR FIRST-YEAR DESIGN STUDENTS WHICH TRANSLATES THE RATHER ABSTRACT MATHEMATICAL CONCEPT INTO VIVID VISUAL IMAGES HAS BEEN DEVELOPED FROM THIS MATERIAL. (HC)

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Uses of Symmetry in Design Education

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Uses of Symmetry in Design Education

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William S. Huff

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3. CONTENTS

4. <u>ACKNOWLEDGEMENTS</u>

1. Dr. K. L. Wolf, author of SYMMETRIE and consultant

2. Hochschule fur Gestaltung's Tomas Maldonado, who introduced Dr. Wolf's material into Ulm's curriculum

3. C.I.T. Hunt Botanical Library; the Henry Clay Frick Library of Fine Arts, University of Pittsburgh; Yale's Sterling and Biencke Libraries; Fogg Museum, Harvard University; the Rare Books and Special Collections, Firestone Library, Princeton University; and a few of the staff of Carnegie Tech's central library

4. Jay-Bee Photographic Studios for quality and patience.

5. INTRODUCTION

A theory of structure is essential to an objective organization of basic pedagogics in design. Speiser, Weyl, Wolf, Coxeter, Toth are amongst the foremost mathematicians identified with the study of symmetry, an area employing group theory; but only Wolf suggests the expansion of symmetry into a full-blown structure theory.

Ulm's Hochschule fur Gestaltung brought Wolf's rich exposition of symmetry into its design curriculum in 1955; and in 1960, Carnegie Tech's Department of Architecture incorporated the same into its design program.

One day, after several weeks of classroom work in symmetry, a rather intent, introspective, contemplative (Zen-oriented) student arose to declare: I don't believe in SYMMETRY. In fact, he actually meant: firstly, nothing is perfect, and secondly, no one thing is exactly like another. Yet this student pointed out the necessity for a broader effort that can reveal symmetry as central to many of man's conceptualizations, perceptions, observations, to his aesthetic responses and to his production of devices and designs.

Symmetry, in fact is at the very base of our objectivity: Scientific method depends on it, for it is the basis of predictions and the foundation of the faith that same or similar conditions will produce same or similar results.

Furthermore, symmetry represents the realization of the improbable--of which man himself is a phenomenal product.

The unthinking, inarticulate, yet sensitive warning of the student is a persuasive reminder that the seeking of perfection is specifically Pythagorean, yet the recognition of symmetries is generally human.

This study was made possible through a Federal Government Grant from the Office of Education (HEW). It has allowed the investigation and compilation of certain material already uncovered and searching for and/or the researching of new material. The intent has been to present a mathematical concept in visual terms--a concept that has had a long history along intuitive lines, yet a firm foundation only as recent as the mid 19th century with the genius of Galois. The intent has also been to make a broad encompassing statement concerning the nature of man's concern with symmetry along with the hope of avoiding the fatuous claim of definitiveness. History, science, philosophy have provided us with many cues; but the work of Dr. K. L. Wolf has provided us with an objective setting for this subject.

At a glance, the symmetry study appeared to be a rather neat package. On closer inspection we have found many discrepancies: discrepancies amongst the mathematicians and mathematicians' apparent contradictions of themselves. There is even a sloppiness about 2-d and 3-d conditions; and though this is not the case with Dr. Wolf, at least one of his concepts has still not been cleared.

Mathematics deals with structures, most of which can be represented in the physical world of our own awareness; yet the very ability of the mathematician to think abstractly often hinders him to see, to visualize, to translate his concepts into visual images.

Our task as we have set it, is to attempt to bridge the chasm. In producing this work of a minimal verbiage, coupled with a multitude of far ranging visual images, we precariously verge on an encyclopedic endeavor that possesses a certain uniqueness, a certain honesty, a certain expression of concern, and a great deal of naivete. We do intend, by this effort, to commend to others the consideration of symmetry as a slice of our mortal being--whether that be fantasy or fact, sought or rejected, perceived or conceived.

6. METHOD

This project has two aspects; research and design:

On the research end, our task, which in the beginning seemed simple, proved to be difficult. Our job was to assess the structural theory of mathematician Wolf and to translate his abstract presentation into a visual product that could be used by beginning design students. We concentrated on his 13 isometric and homoeometric "coverage operations." On the design end, our task was to find vivid visual material which would convey our intent.

Our first difficulty occurred when, on the one hand, we found that Wolf's exposition was murky in certain areas (both on the visual and verbal levels of his book SYMMETRIE) and on the other hand that we began to suspect that other 'authorities' were equally murky on these same items. We tried corresponding with Dr. Wolf, but that was not very satisfactory; and then Ulm's Hochschule fur Gestaltung arranged a meeting between us in early 1966. Wherefore Wolf's book was murky, Wolf himself was not; and he very quickly cleared away the many seeming discrepancies of his book (discrepancies, however, that other mathematicians were unable to resolve). So much for responsibility to the material itself.

In designing a teaching tool, our first thought was to make an animated film which could show the development of structures according to each of the 13 symmetry operations. Two things eventually took us away from film towards a visual primer.

Firstly, the section concerning the pure manipulation of the 13 symmetric operations shrank, while a historio-philosophic section grew; for we began to feel the necessity to demonstrate how man's concepts, observations, and responses have been affected by this abstraction we have made into the construct: symmetry theory. And the visual material we began to gather, coming from 14th to 19th century graphic sources, proved to be too static to justify animated film.

Secondly, the advent of computer-graphics, which sprang upon us only shortly after we received our Government Grant, suggested that it could handle magnificently the section dealing with the 13 operations and in turn made hand animation seem absurd, cumbersome, and unworkable--especially in the three dimensional situations. On the other hand, our budget was not prepared to cover computer animation.

7. RESULTS

We have been able to verify Wolf's 13 operations, translate them into strong visual images, and find natural and/or man-made devices that represented 12 of the 13 phenomenon.

The major point that we established with Dr. Wolf, a point that was missed by his mathematical colleagues, is that 4 of the 13 coverage operations can exist only in 3 dimensional space. The other 9 can be described on the plane; 8 have 3 dimensional analogs.

In the course of our work, we developed (perhaps of not insignificant importance) a theory of domains, the clues of which came from both Wolf and Coxeter. We discovered that an element exists or dominates a region or domain, that in a symmetric (isomorphic or homeomorphic) structure all domains are like or similar, and that the total of them completely fill space.

Our hope also is to have suggested how symmetry touches man in the full range of his life.

We have designed a primer of minimal text and maximal visual material that should be able to communicate this material to first year design students, pre-college students, and interested laymen. The graphic layout is of high aesthetic quality.

8. DISCUSSION

Certain authorities on symmetry recognize only about half of Dr. Wolf's coverage operations. Under isometric operations, #6 mirror rotation would be perhaps the only one in question. It is rare, but it does exist in crystals and can be shown to be the only symmetric quality of many of them.

In greater question amongst the mathematicians are the seven dilatation operations. Most would recognize only a couple, perhaps pure dilatation and the spiral. They argue that pure D (dilatation) is the same as D + T (dilatative translation); and that it is only a matter of the vantage point in respect to the center of dilatation. Abstractly, they may be correct; yet we, of the visual world, certainly interpret the structure of the onion to be rather different from the structure of a tiered wedding cake, while some mathematicians may not. For us, Dr. Wolf is then, meaningful. The same goes for D + M (dilatative mirror) which others would say is but a two fold spiral--but not in all cases! The most complex operation is dilatation + mirror + rotation. It is so complex that, while it can be comprehended, it is doubtful that an extant example can be found; and its rendering, while possible, is even cumbersome.

Wolf's 14th symmetry operation, called Krempung or turning inside-out, is not included, since we either do not understand Dr. Wolf or do not agree with him on it. We would say that it is a little different from Klappung or flopping over, which is an aspect of mere Rotation. Only that Krempung turns 180° on a circular axis and Klapping turns 180° on a straight axis!

Again, in the area of our product, the printed primer, we have collected 90% of the visual material with the missing 10% of visual images (which have been defined but not found) keeping us from printing the work in full, as of this date. The last 10% is, of course, the hardest.

Our original design included three sections in the second part: man and his universe; man and his natural environment; man and his aesthetic response. We have subsequently come to think that there should be a fourth section: man and his devices and designs (extensions of his muscle, senses, and mind). This section may be developed through other funds to join the work done under this grant.

9. CONCLUSION

Since we have established a firm visual foundation for 13 symmetrical operations, as proposed by mathematician Dr. Wolf, we are close to printing a limited edition of the primers. One of six sections is complete and can be printed by itself. Other sections, as their missing visual material is found, will also be printed over the next period of time until the whole is completed. The Government will receive a full set of sketch layouts of each and every section in the form in which it is to be printed.

Notes, bibliography, descriptions of illustrations will come after all else is done.

The text of this book roughly follows the material I have been presenting to first year architectural students at Carnegie Tech for seven years and to first year graphics students at the Ulm School of Design for four years. Over the next year we should like to conduct experiments with this primer to establish its effectiveness as a teaching tool. (Details yet incomplete)

1. To give a section of students the primer as an implementation to my usual lectures on symmetry.
2. To give a section of students the primer as their only source of information on symmetry.
3. To give a section of students my usual lectures as the only source of information on symmetry.
4. To give a section of students the primer as an implementation to another instructor's guidance on the subject of symmetry.

Then, of course, to attempt to discern a pattern in the students' competence to employ symmetry in basic design exercises.

We have had, already, some preliminary testing of the material. Firstly, there has been great interest and enthusiasm from some of our most important colleagues (designers and artists): Louis Kahn, Philadelphia architect; Jerzy Soltan, Polish architect; Tomas Maldonado, Ulm-Argentinian designer; Robert Engman, sculptor; Neil Weliver, painter--as well as our Carnegie Tech Architectural Chairman and architect, Paul Schweikher. These people have given me strong moral support for my work on this project.

We have also introduced this material (in a most unscientific manner) to the student bodies of: the University of New Mexico; Harvard University; Princeton University; Ulm's Hochschule fur Gestaltung; and again, of course, Carnegie Tech. The closer we come to the intellectual centers, the keener the interest is amongst the student body. But with faculty, Ulm excepted, there is perhaps an inversion in the degree of interest, i.e. there was a certain reactionary tendency amongst some of the intellectuals against the introduction of objective material into a design curriculum--especially at Harvard. And since my target is students rather than highly intellectually formalized instructors, I consider I have some measure of success here also. (The high school may very well be the optimal place for dissemination of our material.)

10. SUMMARY

With the purpose of developing a theory of structure, Ulm's Hochschule fur Gestaltung (Ulm School of Design) in 1955 introduced the mathematical concept of SYMMETRY into its design curriculum. The main source of information came from Symmetrie by K. L. Wolf and R. Wolff. No other mathematician, including Weyl, Coxeter, Speiser, and Toth have outlined so complete a theory.

Wolf's book however, presents the designers with certain problems:

1. It is a mathematical work which operates in the very abstract realm of group theory and the author is most interested to analyse the numbers of different symmetrical operations that can be performed on any given geometric body. The designer, on the other hand, is interested in the synthetic rather than the analytic; he builds structures out of parts, rather than breaking down wholes into pieces.
2. While Wolf's book is profusely illustrated, the visual communication of many of his examples do not work. Many who have worked with his book have been led astray and have transmitted his information (even in books) in an erroneous manner.

One of our tasks, then, was to clarify Wolf's material, test it in terms of visual images (found either in nature or in man made objects), and, finally, to present it in a form that is palpable for designers and, in particular, for students of design.

A second task was to suggest its relevance to man in presenting man's historio-philosophic conern with symmetry: how man conceptualized his universe in terms of symmetry; how he observed it in natural morphology; how he responds to it on the aesthetic level; and how he employs it in his devices and designs.

Wolf describes 6 isomorphic coverage operations and 7 homoeomorphic coverage operations. Translation (T), Rotation (R), and Mirror-Reflection (M) are the three prime isomorphic operations; and, coupled, the three combine into three more isomorphic operations: (T + M), (T + R), (M + R). Dilation (D) is the basic homoeometric operation which in turn is combined with the six isomorphs in order to complete the list of the 13 coverage-operations with which we have concerned ourselves.

Some operations caused great difficulties, namely, T + R, M + R, D + T + R, D + M + R.

Both the verbal and visual explanations of Wolf's work are obscure, which contributed to the confusion created by others who worked with his theory. Wolf himself, is clear and sure; and, in a meeting with him in January 1966, the difficult points were hastily resolved. What seemed to have been missed by others is that the four difficult operations exist only in three dimensional space, while the rest can be described on a plane. From this revelation and clues of both Wolf and Coxeter, we developed the idea of domains which are described by the elements, but which themselves must have the characteristics of (1) being the same or similar and of (2) completely filling space. (The development of this concept alone makes the whole study more than worthwhile).

We verified, then, to our satisfaction, the thirteen coverage operations and found extant natural and/or man-made examples of 12 of them. D + M + R is apparently so complex that, while it can be represented graphically, it is not manifest in existing objects (in a pure form); in fact D + M itself is rarified.

Working on the historio-philosophic implication of symmetry threw us into a sort of encyclopedic venture ranging from Classical to Modern Civilization, from atomic to cosmic sciences, and from aesthetics to methodologies.

In the beginning, we thought to fashion oru material into a teaching tool, involving animated film. We soon turned to the idea of a printed primer, since the material we gathered was evidentally rather static in nature, and animation would have to be strained. Also, the section, most obviously calling for animation, was the one displaying the 13 symmetries in operation. Unfortunately, the grant was funded, in accord with our budgetary requirements, only a few months before the apparent advent of computer graphics - which had the effect of making hand animation absurd.

The primer is designed to contain a minimum of text with a maximal richness in visual imagery. The illustrations and text will have lengthy supplemental notations which are not relevant to the intent of the book, but which should satisfy the curiosity of those who possess it. The notes and a very complete bibliography will appear in separate sections.

11. REFERENCES

Many books have been used in this study: see remarks in 12. Bibliograph.

These sources are primary:

Book

Thompson, Sir D'Arcy Wentworth

ON GROWTH AND FORM
Cambridge University Press
1959

Wold, K. Lothar and Wolff, Robert

SYMMETRIE
Bohlau-Verlag
Munster/Koln
1956

Individual

Wolf, K. Lothar

Grenzflacheninstitut
Marienthal (Rheinpfalz)
Germany

12. BIBLIOGRAPHY

The books used in preparing these teaching materials fall into three general categories:

1. The Mathematical exposition of SYMMETRY
2. Texts ranging from Plato, to organic chemistry, to writings of Frank Lloyd Wright.
3. Sources of visual images.

The task of compiling a list of these works is a monumental one in itself. Such a list will be prepared and printed as the last booklet of the series.

13. GLOSSARY

As in the case of the bibliography, a glossary is a large task and will be developed in time.

14. APPENDICES

There are no appendices, unless the sketch layouts and one finished printed section of the primer may be considered as such.